Miners and modals[1]

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Abstract. I generalise Kolodny and MacFarlane's miners puzzle by showing epistemic analogues of their case exist. After motivating various conservative approaches to the original problem, I show how they fail to solve the problem in its epistemic guise. I argue that a probabilistic approach to information-sensitivity gives a general solution to the problem.

Keywords: deontic modals, miners puzzle, epistemic 'should', probability.

1. Introduction

Kolodny and MacFarlane introduced the infamous miners problem to the literature on deontic modals. I show that this semantic puzzle runs deeper than previously thought: there are epistemic analogues of Kolodny and MacFarlane's case and they have a variety of upshots for our understanding of the problem.

After outlining the classic semantics and the problem it faces in section 1, I clarify what questions are at stake in section 2. Miners cases motivate not just a more expressive semantics but also the use of orderings based on measure-theoretic notions like expected utility and probability in our semantics for 'ought' and 'should'. I show in section 3 that epistemic miners cases pose a major stumbling block for responses that try to avoid appealing either to information-sensitivity or measure-theoretic tools. Classic responses like Kratzer's and Cariani, Kaufman, and Kaufman's are geared explicitly towards the deontic case and do not generalise naturally. In section 4, I argue that information-sensitivity should be understood as a probabilistic phenomenon. I give an emendation of the classic semantics that can access probabilistic orderings and is sensitive to conditionalisation.

2. The problem

Take the following case from Kolodny and MacFarlane (2010):

> **Miners.** Ten miners are trapped either in shaft A or in shaft B, but we do not know which. Flood waters threaten to flood the shafts. We have enough sandbags to block one shaft, but not both. If we block one shaft, all the water will go into the other shaft, killing any miners inside it. If we block neither shaft, both shafts will fill halfway with water, and just one miner, the lowest in the shaft, will be killed.

The following sentences seem true here:

(1) I ought to block neither shaft.

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(2) If the miners are in shaft A, I ought to block shaft A.

(3) If the miners are in shaft B, I ought to block shaft B.

Surprisingly, it has been shown that the classic view of ‘ought’ and ‘should’ cannot predict the joint truth of (1) – (3).[2]

2.1. Information and the classical theory

The classic view, so-called in von Fintel (2012), assumes that ‘ought’ and ‘should’ are necessity modals:[3] ⌜ought ϕ⌝ is true just in case for any world w in the modal’s domain ϕ is true at w. More precisely:

$$[\![ought\ \phi]\!]^{c,i} = 1 \text{ iff } \forall w' \in BEST(i) : [\![\phi]\!]^{c,i[w_i \rightarrow w']} = 1^4$$

This aspect of the classic view will not be under dispute here.

The classic view also says how the domain, $BEST(i)$, is determined. Following Kratzer,[5] it is constrained by two ingredients, a modal base, f, and an ordering source, g. The modal base is a function from worlds to sets of propositions.[6] These propositions represent the information we take to be held fixed in the background. The relevant body of information might be what a given agent knows, in which case the modal base is *epistemic*. Or it might simply be what is compatible with some relevant set of facts, in which case the modal base is *circumstantial*. On the classic theory, the role of the modal base is simply to restrict the domain of quantification: only worlds in the intersection of the modal base can feature in the domain of quantification.

The ordering source is used to construct an ordering on worlds. Its job is to represent, for each world, what the relevant priorities are. To do this, we let the ordering source be a function from worlds to sets of propositions, a function that, when given a world, yields us the set of priorities at that world. We generate an ordering from this as follows:

$$w_1 \leq_{w,f,g} w_2 \text{ iff } \{p \in g(w) : w_1 \in p\} \supseteq \{p \in g(w) : w_2 \in p\}$$

In other words, w_1 is at least as good as w_2 relative to $\langle w, f, g \rangle$ just in case w_1 makes true all the propositions in $g(w)$ w_2 does and possibly more.

The domain of quantification of the modal is just the set of top $\leq$-ranked worlds compatible

[2] See, for instance, Charlow (2013), Cariani et al. (2013), Silk (2014).

[3] This semantics has been challenged by many: see, for instance, Lassiter (2011) and Cariani (2013). However, such challenges are orthogonal to the problem discussed here and so we can safely use the above semantics as our working theory.

[4] Here i is a variable over indices and $i[w_i \rightarrow w']$ is the index formed by replacing the world in i with w'.

[5] Kratzer (1977), Kratzer (1991).

[6] When it does not cause confusion, I sometimes use the term ‘modal base’ to pick out what is strictly speaking the intersection of modal base.

with the information in the modal base.[7] In other worlds,

$$BEST(w, f, g) = \{w \in \bigcap f(w) : \neg\exists w' \in \bigcap f(w) : w' <_{w,f,g} w\}$$

For us, the important feature of the classic semantics is that it rules out any interaction between these parameters: the ordering does not vary as we vary the modal base (but keep the other parameters fixed). In other words, on the classic semantics we have

> **No f-shifting**: For any modal bases f_1 and f_2, given a world w and ordering source g, $w_1 \leq_{w,f_1,g} w_2$ iff $w_1 \leq_{w,f_2,g} w_2$.[8]

Given this principle, the only role for the modal base is to direct our attention to a certain portion of the ordering.

This is the crucial feature of the classic semantics: even as we add information to the modal base, the classic semantics will keep the background ordering on possibilities fixed.

2.2. The need for information-sensitivity

Miners challenges **No f-shifting**: Kolodny and MacFarlane (2010) have argued that, on their deontic readings, adding information can change the relevant ordering for ‘ought’ and ‘should’. In particular, they think **Miners** shows that worlds can move up in the ordering as we add more information to the modal base.

To see why the classic semantics struggles here, we will need a theory of conditionals. I adopt throughout Kratzer’s restrictor theory of conditionals.[9,10] On this theory, ‘if’-clauses restrict the domain of the modal in the consequent. More formally, where $f + \phi$ is the modal base such that $f + \phi(w) = f(w) \cup \{\phi\}$, we have:

(4) $[\![\textit{if } \phi \textit{ then MODAL } \psi]\!]^{c,w,f,g} = 1$ iff $[\![\textit{MODAL } \psi]\!]^{c,w,f+\phi,g} = 1$

Conditionals like (2) and (3) then have the following truth-conditions:

$$[\![\textit{if } \phi \textit{ then ought } \psi]\!]^{c,w,f,g} = 1 \text{ iff } \forall w' \in BEST(w, f + \phi, g) : [\![\psi]\!]^{c,w',f+\phi,g} = 1$$

So ⌜if ϕ then ought ψ⌝ will be true just in case all the best worlds which are ϕ-worlds are also ψ-worlds.

[7] Here and throughout I make the limit assumption in stating the classic semantics.

[8] It is straightforward to see that this holds on the classic semantics. While f is an argument for $\leq$, it actually does not appear on the right-hand side of the definition. Hence, on the classic semantics it is a redundant argument. I include it as an argument to emphasise the point that the classic semantics does not allow the order to shift as the modal base changes.

[9] Kratzer (1991), Kratzer (2012).

[10] As Charlow (2013) shows, the problem still arises even if we adopt other theories of the conditional, such as those of Stalnaker (1968) and Lewis (1973).

To see why **Miners** creates a problem, it will help to get some parameters on the table. I do not know the location of the miners, so $\bigcap f(w)$ will contain worlds where they are in shaft A and worlds where they are in shaft B.[11] Given that I have not made up my mind about what to do, the modal base will also contain worlds where I block shaft A, where I block shaft B and where I block neither. Since this is the only relevant information here, we can simplify and represent my knowledge with this set of worlds:

$$\bigcap f(w) = \{\textit{(A, blA), (A, bl B), (A, blN), (B, blA), (B, blB), (B, blN)}\}$$

We'll take $g(w)$ to say that I should save as many miners as I can; or in other words,

$$g(w) = \{\textit{I save 10 miners, I save 9 miners, ... , I save 1 miner}\}$$[12]

Given these parameters, we can see that the best worlds will be ones where I block the correct shaft. So the ranking will be

$$(A, blA), (B, blB) < (A, blN), (B, blN) < (A, blB)(B, blA)$$

This will give us the right predictions for (2) and (3). $BEST(w, f+A, g)$ will be $\{(A, blA)\}$ and $BEST(w, f+B, g)$ will be $\{(B, blB)\}$. But we fail to predict the truth of (1). $BEST(w, f, g)$ will be a superset of $BEST(w, f+A, g)$, namely $\{(A, blA), (B, blB)\}$. In both of these worlds I block one of the shafts. This forces us to predict that (1) is false, the wrong prediction.

We can also consider what happens if we pick an ordering source which predicts the truth of (1).[13] Suppose $BEST(w, f, g)$ is $\{(A, blN), (B, blN)\}$. This predicts that (1) is true: both worlds are ones where I block neither shaft. But now notice that $BEST(w, f, g)$ contains worlds where the miners are in A; so $BEST(w, f+A, g)$ will be $\{(A, blN)\}$. But then, (2) is false: all the best worlds where the miners are in A are worlds where I still block neither shaft. For similar reasons, we will predict (3) is false.

In either case, we have a problem: we cannot both keep the background ordering of worlds fixed and predict the truth of (1), (2) and (3). Kolodny and MacFarlane's diagnosis is that, to make the right predictions, $BEST$ must be defined in such a way that makes it *information-sensitive*:

$BEST$ is information-sensitive iff there exist f_1, f_2 and w such that:

1. $\bigcap f_1(w) \supseteq \bigcap f_2(w)$
2. $BEST(w, f_1, g) \cap \bigcap f_2(w) \neq \varnothing$
3. $\exists w' : w' \in BEST(w, f_2, g) \ \&\ w' \notin BEST(w, f_1, g)$

[11]It is shown in Cariani et al. (2013) how the problem arises for a circumstantial modal base. In fact, as we are about to see, the problem is independent of the particular choice of parameters.
[12]I use italicisation to refer to propositions i.e. '*p*' denotes the propositions that p.
[13]From the results in Lewis (1981) we know there will have to be some such ordering source. But we will also see an example of an ordering source which makes similar predictions in section 3.1.

To see why the miners case seems to involve information-sensitivity, let us show that each condition appears to be met in **Miners**. Condition 1 follows from the set-up of the case and the restrictor semantics: we leave open possibilities where the miners are in A, so $\bigcap f(w) \supseteq \bigcap f + A(w)$. Condition 2 follows also from the set-up of the case: the best worlds, the ones where I block neither, include worlds where the miners are in A and worlds where they are in B. The crucial condition is condition 3. This condition is met just in case $BEST(w, f + A, g)$ contains something that was not originally in $BEST(w, f, g)$. And indeed, if (2) is true, then there must be such a world, (A, blA).[14]

Information-sensitivity is incompatible with **No-*f*-shifting**. It is a consequence of **No-*f*-shifting** that, when there are ϕ-worlds in $BEST(w, f, g)$, then $BEST(w, f + \phi, g)$ is $BEST(w, f, g) \cap \phi$. That is, whenever we add a proposition ϕ to the modal base that is true some of the best worlds, the new best worlds are always the old ones where ϕ is true. **Miners** appears to be a counterexample: the conditionals add a new proposition to the modal base that is true in some of the best worlds; but the new set of best worlds in fact must be *disjoint* from the old one.

Thus it looks like we need some new way of defining $BEST$ which allows the ordering to shift as we add information to the modal base. This is the semantic challenge of the miners case.[15]

3. What is at stake

The literature has gone in different ways from this point, taking various morals from the case. I will try to carve out what seem to me the key questions here. In doing so, I will try to get clear on what reasons there might be to favour the various conservative impulses the literature has displayed.

3.1. A pragmatic solution

The first, most straightforward question is whether we really need to add information-sensitivity to our semantics. When semantic explanations fail, it is natural to turn to pragmatics for an answer. By doing so, we might explain the judgments in **Miners** without altering the classic semantics. We'll call a theory that tries to do without any information-sensitivity a *very conservative* theory.

Adding information-sensitivity has met with strong resistance in some quarters.[16] For some, information-sensitivity is a deeply dubious property. Charlow (2013) for instance asks how

[14]In fact, something somewhat stronger will have to be true: $BEST(w, f, g)$ and $BEST(w, f + A, g)$ will have to be *disjoint*. However, the weaker principle, information-sensitivity, captures the main conceptual contrast with the classic semantics, the idea that possibilities get ranked higher as we get more information.

[15]The semantic challenge is to be distinguished from what we might call the *inferential* challenge. As Kolodny and MacFarlane note, (1), (2) and (3) together give us a counterexample to modus ponens. This feature of the case will not concern us here. Moreover, as has been shown in Khoo (2013), our background theory of conditionals, the restrictor view, does not validate modus ponens anyway.

[16]See, for instance, von Fintel (2012).

it could be possible that certain worlds get *better* as more information is added.[17] But this reads too much into the semantics: even when the modal is deontic, our ordering need not represent how good worlds themselves are. Preference orderings can surely change as we get more information: which possibilities seem best to me can change as I gain more information.

That being said, resistance here *is* well-motivated, even if not by the reasons that have been given. Adding information-sensitivity would result in a theory more expressive than the classic theory. As well as the readings provided by the classic semantics, we now predict new possible interpretations of modals where shifting the modal base shifts the ordering. But we should prefer less expressive theories where possible: if we can postulate fewer possible readings and still capture the data, then that is what we should do. In this case, we should wonder if we can capture the appearance of information-sensitivity using some pragmatic mechanisms.

The main kind of very conservative response denies that (1), (2) and (3) are all evaluated within the same context. In particular, it claims that the ordering source used to evaluate (1), the 'subjective' ought, is different from that used to evaluated (2) and (3), the objective 'ought'. As outlined in von Fintel (2012),[18] such a strategy can successfully predict the judgements. Suppose the ordering source for (1) were

> $g_{(2)}(w)$ = {*If we know where the miners are, our chosen action yields the optimal outcome for the miners*, *If we do not know where the miners are, our chosen action yields a still acceptable outcome for the miners and would not yield a less acceptable outcome if they weren't where they in fact are*}

We then get the result that (1) is true. If we suppose that the ordering source for (2) and (3) is

> $g_{(2)}(w)$ = {*I save 10 miners, I save 9 miners, ... , I save 1 miner*}

we predict true readings for both.

Context-shifting strategies are only as plausible as the claim that context might supply those parameters. But these particular parameters are plausible. There is a genuine difference between the subjective and the objective 'ought': the former tracks what we should do given what we know, and the latter tracks what would be best for us to do given all the facts. What's more, it gives us an understanding of the case which is intuitively satisfying. This approach cannot be accused of dreaming up *ad hoc* parameters to solve the problem.

3.2. A non-probabilistic solution?

There is another aspect of the classic semantics at stake, even if we admit information-sensitivity. If 'ought' is information-sensitive, there is a serious question about where the information-

[17]It is not clear that this is Charlow's final view on the matter. (See, for instance, Charlow (2016).) But this thought does seem to account for some of the suspicion of information-sensitivity in the literature.
[18]Von Fintel attributes it to unpublished notes by Kratzer.

sensitivity comes from. MacFarlane and Kolodny give no clear guidance here — nothing in their system tells us anything about how it is to be generated. But our semantics should be predictive. Given a plausible story about the context, it should tell us *why* information-sensitivity comes into play in cases like miners.

The classic semantics gives us a very clear story about where our orderings come from: they are constructed out of sets of propositions by appeal to entailment. Something like this story might yet hold up, even if the classic semantics must be altered in other ways. This brings us to our second question: can miners cases be explained using only possible worlds machinery? This question is an important one about the structure of our theory of modal vocabulary and its relations to other important concepts. We'll call a theory that answers *no* to this question a *moderately conservative* account.

It is striking that the judgements in the miners case track natural judgements about the expected utilities: blocking neither shaft has the highest expected utility; and conditional on the miners being in A, blocking A has the highest expected utility (and similarly for B). But such measure-theoretic notions carry far more information than measure-theoretic tools: they tell us not just how possibilities are ranked, but carry information about *how much* better certain possibilities are than others. Before allowing these kinds of structures to access our semantics for modals, we should want good reason to think they are needed.

A leading moderately conservative theory is that of Cariani et al. (2013). This semantics allows information-sensitivity but remains close to the spirit of the Kratzer framework in constructing its orderings. Cariani, Kaufman, and Kaufman (CKK from henceforth) add a decision problem to the Kratzer semantics, a set of propositions representing the actions available to an agent in a given scenario. For instance, in the miners case, the decision problem δ would be $\{$*I block A*,*I block B*,*I block neither*$\}$. What ends up being important is not just the decision problem but also the decision problem as restricted by the modal base. Such a restriction is obtained by intersecting each member of the decision problem with the modal base. In our example, the decision problem restricted by f would be $\{$*I block A and the miners are either in A or B, I block B and the miners are either in A or B, I block neither and the miners are either in A or B*$\}$.

Importantly, the relevant orderings on worlds, though information-sensitive, are still generated by means of entailment. An ordering is defined on the members of the restricted decision problem and used to create a corresponding ordering on worlds. A member of the decision problem p is at least as good as another q just in case p entails all the same ordering source propositions as q and maybe more. More precisely:

$$p \precsim_{f,g,w} q \text{ iff } \{r \in g(w) : p \subseteq r\} \supseteq \{s \in g(w) : q \subseteq s\}$$

A world is then taken to be just as good as the restricted decision problem proposition of which it is a member. Where $\Delta_{\delta,f}(w)$ denotes the decision problem proposition (as restricted to f) containing w, we say that $w' \leq_{w,f,g,\delta} w''$ just in case $\Delta_{\delta,f}(w') \precsim_{f,g,w} \Delta_{\delta,f}(w'')$. Our clause for the modal is more or less as before:

$$[\![ought\ \phi]\!]^{c,w,f,g} \text{ iff } \forall w' \in BEST(w,f,g,\delta) : [\![\phi]\!]^{c,w',f,g} = 1$$

where the $BEST$, like before, is:

$$BEST(w,f,g,\delta) = \{w \in \bigcap f(w) : \neg\exists w' \in \bigcap f(w) : w' <_{w,f,g,\delta} w\}$$

4. Epistemic miners cases

I have shown that conservativity at each point is well-motivated. But now that we have built it up, I intend to knock it down. Both kinds of conservativity are insufficiently general. There are epistemic analogues of **Miners** and conservative solutions cannot account for then.

4.1. The case

So far we have seen only deontic 'ought's. But 'ought' can also be read *epistemically*. For example, suppose that Jane has been told the bus left 30 minutes ago and it usually takes 40 minutes to get to her bus stop. Jane might truly say

(5) The bus ought to arrive in 10 minutes.

This sentence says that it is probable, given Jane's evidence, that the bus will arrive in 10 minutes. More generally, ⌜ought ϕ⌝ seems to communicate that ϕ is probable, given the relevant agent's evidence.

Once we have isolated the epistemic 'ought', it becomes natural to ask whether it too is (apparently) information-sensitive. If so, then we should be able to generate cases analogous to **Miners** for the epistemic 'ought'. In fact we can. Take the following case:

> **Exam**. Alex and Billy are the top math students in their class and will take their weekly algebra exam tomorrow.
>
> - Alex does best in 66% of the exams.
> - Given that Billy studies tonight, Billy will probably get the best grade: out of exams he studied for, Billy did best in 66% of them.
> - Given that Billy doesn't study, Billy will certainly not do best. Alex did better in all of the exams that Billy didn't study for.
> - Billy always lets a fair coin toss decide whether he will study. He studies just in case it comes up heads.

Imagine we are asked who will do best and consider the following replies:

(6) Alex should do best.

(7) But, if turns out that Billy studied, then he should do best.

Both seem true here. The first seems true because, given what we know, it is more likely that Alex will do best. The second seems true because, were we to learn that the coin came up heads, we would think it more likely that Billy will do best.

Just as in **Miners**, the classic semantics cannot predict the truth of both (6) and (7). We can see that $BEST(w, f, g)$ should both contain worlds where Billy studies and worlds where he doesn't. After all, it's neither likely that he will nor likely that he won't.[19] So $BEST(w, f, g) \nsubseteq$ *Billy studies* and $BEST(w, f, g) \nsubseteq$ *Billy doesn't study*. To predict (6), we need the set of best worlds to entail the proposition that Alex does best. So we want $BEST(w, f, g) \subseteq$ *Alex does best*. To predict (7), we want the set of best worlds which are worlds where Billy studies to be ones where Billy does best. In other words, we want $BEST(w, f + \textit{Billy studies}, g) \subseteq$ *Billy does best*.

Suppose we have $BEST(w, f, g) \subseteq$ *Alex does best*, $BEST(w, f, g) \nsubseteq$ *Billy studies* and $\nsubseteq$ *Billy doesn't study*. This means that the updated modal base we use to evaluate (7) is consistent with $BEST(w, f, g)$: as we said, $BEST(w, f, g)$ neither entails that Billy studies nor that Billy doesn't study. This means that $BEST(w, f + \textit{Billy studies}, g)$ must be a subset of $BEST(w, f, g)$. But if $BEST(w, f + \textit{Billy studies}, g)$ is a subset of $BEST(w, f, g)$, then $BEST(w, f + \textit{Billy studies}, g)$ also entails that Alex studies. We then fail to predict that (7) is true. So whenever $BEST(w, f, g)$ contains both worlds where Billy studies and ones where he doesn't, if we make (6) true, we are forced to make (7) false.

Information-sensitivity looks to be needed here too. Condition 1 is met because of the restrictor semantics: the modal base used to evaluate (7) is a subset of that used to evaluate (6). Condition 2 is met: $\bigcap f + \textit{Billy studies}(w)$ is consistent with $BEST(w, f, g)$; in other words, the set of best worlds is consistent with the antecedent of (7), Finally, the third condition is satisfied. $BEST(w, f, g)$ and $BEST(w, f + \textit{Billy studies}, g)$ must be disjoint. If (6) is true, then all worlds in $BEST(w, f, g)$ are ones where Alex does best; if (7) is true, all worlds in $BEST(w, f + \textit{Billy studies}, g)$ are ones where Billy studies; and, of course, in no worlds do they both do best.

By running it through the classic semantics, we can see that **Exam** has the same problematic structure as **Miners**. We shall now see that unlike the original case, our epistemic miners case is also problematic for conservative solutions.

4.2. Against context-shifting

As we saw, the most natural very conservative strategy posits a context-change in **Miners**: the ordering source used to evaluate (1) is different to that used for (2) and (3). It will have to say

[19]What's more, neither of the sentences

(i) Billy should study.

(ii) Billy should fail to study.

has a true reading here.

something similar about **Exam**. The ordering source used to evaluate (6) (call it $g_{(6)}$) is not that used to evaluate (7) (call it $g_{(7)}$).

But notice that if $g_{(6)}$ is available in the context, then we predict that it should be available to evaluate the conditional:

(8) (Even) if Billy studied, Alex should get the best results.

If this were the case, (8) should have a true reading. It would be heard to say:

(9) Even if Billy studied, it is still the case that, just given what we know *now*, Alex should get the best results.

But this is not the case: (8) has no true reading here. The context-shifting strategy thus overgenerates here: it predicts that, in addition to (7), we should also have a true reading of a conditional like (8). This is a bad prediction for the context-shifting view. Overgeneration is the hallmark of too much context-sensitivity.

The proponent of this strategy will have to say that, for some reason, the ordering source used to evaluate (6) is not available for (8). This is puzzling, particularly when (6) and (7) are uttered in sequence. They would be claiming the context shifts in such a way that, instead of giving (8) a true and non-trivial reading, it delivers instead a *false* reading of the sentence. None of the familiar mechanisms of context-change, such as accommodation in the sense of Lewis (1979), fit this profile. When context change happens, very often it does so to interpret a speaker charitably. Accordingly, it rarely changes to make utterances *false*. There is no obvious reason why the steadfast reading of (8) should be inaccessible.

Note that things get worse when we look back to **Miners**. Consider the following conditional:

(10) Even if the miners are in shaft A, I ought to block neither shaft.

This conditional is structurally analogous to (8); but unlike (8), this conditional is actually true here. This disparity poses an extra challenge for the very conservative theorist. Whatever way we try to explain the overgeneration here, we do *not* want it to carry over to the original miners case. As we noted, a steadfast reading is genuinely accessible there and so the context-shifting strategy must walk a fine line: its story must be strong enough to secure that there is no true reading of (8), but must not rule out a steadfast reading of (10). It is not clear how this could be done.

This issue does not put the same pressure on a non-conservative view. Unlike the context-shifting view, it is need not say that some available ordering source makes (8) true. It can posit an ordering source in **Miners** to make (10) true. But such a view is under no obligation to say the same thing for (8). This is a considerable advantage: when we posit just one ordering source to explain the truth of (6) and (7) we never open up the question of how we avoid predicting a true reading of (8).

4.3. Against moderate conservativity

Exam poses a separate and severe challenge for the approach in Cariani et al. (2013). Their account relies heavily on deontic features of the scenario in **Miners** to predict the consistency of the original miners sentences. But just this feature makes it hard to see how their theory can be adapted to the epistemic case.[20]

The first problem is how to interpret the decision problem parameter. Decision problems model the choices an agent must make in a given scenario; but in the scenario we outlined, there is no such choice at issue. In such a case, CKK say that the decision parameter should be set to the set of all worlds. This is designed to make the decision problem redundant, as they suppose the decision parameter will not be needed outside of deontic contexts. But naturally the decision problem for **Exam** must be non-trivial.

Probably the best way to generalise the view here is to think of the decision problem more generally as some salient partition of the modal base. In **Exam** we might let the decision problem be

(11) $\{$*Alex does best*,*Billy does best*$\}$.

Even still, when we give the semantics plausible ordering sources, it does not make the right predictions.

Take a probability based ordering source:

(12) $g(w) = \{\phi{:}\phi \text{ is probable in } w.\}$

To simplify things, suppose that the only things probable on our evidence are that Alex does best, that if Billy studies, Billy does best and that if Billy does not study, Alex does best. This gives us the following:

(13) $g(w) = \{$*Alex does best*,*If Billy studies, Billy does best*,*If Billy doesn't study, Alex does best*$\}$

To predict the truth of (6) we want this ordering on decision problem cells:

(14) *Alex does best* < *Billy does best*.

Our current choice of ordering source delivers this. Only *Alex does best* entails any ordering source proposition (namely itself). To predict (7) we want a new ordering on decision problem cells:

[20] The solution in Charlow (2013) seems to face similar issues. For him information-sensitivity is generated by the interaction of two ordering sources, one tracking what is deontically best and another tracking what is actionable. Information-sensitivity is generated by the fact that, against different modal bases, different propositions will be actionable. Again, it's not clear how to extend this idea to epistemic cases, as there is no obvious parallel for the actionable propositions.

(15) *Billy studies and Billy does best* < *Billy studies and Alex does best*.

But we do not get this. Grant that *Billy studies and Billy does best* entails the conditional *If Billy studies, Billy does best*. The proposition *Billy studies and Alex does best* also entails an ordering source proposition, namely *Alex does best*. Neither cell of the decision problem entails all the ordering source propositions of the other and more besides. This means that, rather than giving us (15), the two cells are *incomparable*.

The CKK approach yields information-sensitivity, but not in all of the right places. When we chose a plausible ordering source for **Exam**, one that tracks the probabilities of the case, refining the decision problem is not enough to get the change in ordering we need. Here too conservativity looks unpromising because it fails to generalise.

5. A solution

Cases like **Exam** are important evidence that information-sensitivity is more prevalent than previously thought. It appears not just in the deontic realm, but in the epistemic too. Conservativism fails because it is too narrow in scope. It cannot explain away information-sensitivity, as the very conservative theorist hopes. Nor can it be explained with possible worlds machinery alone, as the moderate conservative hopes.

If instead we account for information-sensitivity by appeal to probability, we do better. I will start by outlining a connection between the set of best worlds and probabilistic notions and show that if this connection were to hold, we would predict our data. Crucially, the role played by conditionalisation is what allows the orderings to shift. Then I will outline a semantics which delivers those principles and so predicts what we want in **Miners** and **Exam**.

5.1. The role of probability

I suggest that, in the relevant miners cases, we want our semantics to predict the following:

Deontic: $\forall w' \in BEST(w,f,g) : [\![\phi]\!]^{c,w',f,g} = 1$ iff $\neg\exists\psi$: $EU(\psi|\bigcap f(w)) > EU(\phi|\bigcap f(w))$.

Epistemic: $\forall w' \in BEST(w,f,g) : [\![\phi]\!]^{c,w',f,g} = 1$ iff for the contextually supplied threshold probability $\theta : P(\phi|\bigcap f(w)) > \theta$.

In each case, conditionalisation generates information-sensitivity. The expected utility of ϕ might be overtaken by that of some other option entailing $\neg\phi$ whenever we conditionalise on some other proposition ψ. When the set of best worlds tracks expected utilities, updating the modal base with ψ will change the relevant best worlds: now they include $\neg\phi$ worlds that were not there before. Similarly for probabilities: conditionalising on some proposition ψ may cause the probability of ϕ to drop below the threshold and push that of $\neg\phi$ above it. This will mean that updating the modal base with ψ will change the ordering on worlds: they will now include $\neg\phi$ worlds that were not there before. So in each case, conditionalisation can lead to worlds getting a higher position in the ordering.

Let's now see this in action. Recall our sentences from **Miners**:

(1) I ought to block neither shaft.

(2) If the miners are in shaft A, I ought to block shaft A.

(3) If the miners are in shaft B, I ought to block shaft B.

We can fill in the details of the case to see how **Deontic** will give us the right results. The miners are just as likely to be in A as they are to be in B. Outcomes where I save more miners have higher utility than those where I save less. So let's imagine that P and U are as follows:

$$P(A) = P(B) = 0.5$$

$$U(A \wedge blA) = U(B \wedge blB) = 1$$

$$U(A \wedge blB) = U(B \wedge blA) = 0$$

$$U(A \wedge (\neg blA \wedge \neg blB)) = U(B \wedge (\neg blB \wedge \neg blA)) = 0.9$$

When we conditionalize P on $\bigcap f(w)$, this will not change the probabilities above. When we do the expected utility calculations,[21] the resulting order on propositions is

block neither < *block A* $\approxeq$ *block B*

Thus *block neither* has the highest expected utility and so, given **Deontic** we predict (1) to be true in this context.

When we conditionalise on $\bigcap(f + A(w))$, the probabilities change. The ordering on propositions shifts accordingly:[22]

[21] We can see that conditionalising P on $\bigcap f(w)$ will make no difference to any of the values of P which we have specified. So the value assigned to blA will be

$$U(A \wedge blA)Pr(A) + U(B \wedge blA)Pr(B) =$$
$$1.(0.5) + 0.(0.5) = 0.5$$

which will be the same as the value assigned to blB; whereas as the value assigned to $(\neg blA \wedge \neg blB)$ will be

$$U(A \wedge (\neg blA \wedge \neg blB))Pr(A) + U(B \wedge (\neg blB \wedge \neg blA))Pr(B) =$$
$$(0.9).(0.5) + (0.9).(0.5) = 0.9.$$

[22] Our new probabilities will be

$$P(A) = 1$$
$$P(B) = 0$$

Recalculating the expected utilities, the value assigned to $(\neg blA \wedge \neg blB)$ will be equal to

$$block\ A < block\ neither < block\ B$$

block A now has the highest expected utility. Hence, given **Deontic**, when the modal base restricted to the worlds where the miners are in A, all the worlds in $BEST(w, f + A, g)$ will be ones where we block shaft A. Given the restrictor view of conditionals, it follows that (2) is true here. By similar reasoning, we also predict the truth of (3).

Let's turn now to **Exam** to see how **Epistemic** predicts the right results there. Our sentences there were:

(6) Alex should do best.

(7) But, if turns out that Billy studied, then he should do best.

Given the set up, the probabilities should be

$$P(Alex\ does\ best) = 0.66$$

$$P(Billy\ does\ best) = 0.33$$

$$P(Alex\ does\ best \mid Billy\ studies) = 0.33$$

$$P(Billy\ does\ best \mid Billy\ studies) = 0.66$$

Suppose now that the threshold probability is 0.5. Conditionalising on $\bigcap f(w)$ here will make no difference to the probabilities assigned to the above propositions. Hence, the proposition that Alex does best will pass the threshold and, by **Epistemic**, the best worlds will be ones where Alex does best. Hence (6) will be true.

When we conditionalise on $\bigcap(f(w)+Billy\ studies)$, the probabilities do change. In fact the probabilities of *Alex does best* and *Billy does best* are now equal to the conditional probabilities given above and the proposition *Billy does best* will now pass the 0.5 threshold. So, relative to our more restricted modal base $f(w)+Billy\ studies$, **Epistemic** tells us that all the best worlds are ones where Billy does best. Given the restrictor analysis of conditionals, we then predict that (7) is true in this context.

$$U(A \wedge (\neg blA \wedge \neg blB))Pr'(A) + U(B \wedge (\neg blB \wedge \neg blA))Pr'(B) =$$
$$(0.9).1 + (0.9).0 = 0.9.$$

but the value assigned to *blA* will be

$$U(A \wedge blA)Pr'(A) + U(B \wedge blA)Pr'(B) =$$
$$1.1 + 0.0 = 1.$$

The value assigned to *block B* will

$$U(A \wedge block\ B)P(A) + U(B \wedge block\ B)P(B) =$$
$$0.1 + 1.0 = 0.$$

5.2. Implementation

We've seen that allowing probabilities into our semantics gives us a good general picture of where information-sensitivity comes from. Now I outline a more general definition of *BEST* that, when combined with a plausible selection of parameters supplied by context, delivers the desired connection.

Earlier we entertained the question of whether all the necessary orderings for modal semantics can be generated using just propositions. If we want probability to play a serious role, this will be difficult to maintain. Probabilistic notions are notoriously difficult to recover from purely qualitative information. As shown in Lassiter (2015), attempts to do so (like that in Kratzer (1981)), tend to have undesirable logical properties: for instance, Kratzer's approach predicts that whenever ϕ is as likely as ψ and as χ, it is as likely as $\psi \vee \chi$; but probabilistic orderings do not in general have this property.[23] Thus, if probability is to be used in our semantics for 'ought', it is hard to see how it could be moderately conservative in the sense that we outlined earlier.

We will make the classic semantics more flexible so that it can access the kinds of orderings we need. We keep the modal base parameter without any changes: it is still a function from worlds to propositions and intuitively represents the information we are holding fixed. However, we change how ordering sources work. Firstly, we want ordering sources to have, among other things, modal bases as arguments: this is essential to any solution that allows information to shift the relevant ordering.[24] Secondly, we want to allow ordering sources to exploit orders on propositions. The final ordering on worlds should track an expected utility ordering in the deontic case and a probability ordering in the epistemic case. We modify our definition of an ordering source accordingly: now an ordering source g is a function from a world and a modal base to an ordering $\precsim_{w,f,g}$ on propositions.

In the deontic case, the ordering will straightforwardly track the relevant expected utility ordering. That is we will have

$\phi \precsim_{w,f,g} \psi$ just in case $EU(\phi|\bigcap f(w)) \geq EU(\psi|\bigcap f(w))$.

In the epistemic case, we want the ordering to reflect whether or not a proposition passes a contextually supplied threshold. That is, we want it to be the case that no proposition is strictly better than ϕ whenever the probability of ϕ passes the given threshold. To secure this, we will define the ordering as follows:

$\phi \precsim_{w,f,g} \psi$ iff, where θ_c is the contextually determined threshold, one of the following conditions holds:

1. $P(\phi|\bigcap f(w)) > \theta_c$; or

[23] One exception to this is the semantics in Holliday and Icard (2013); but as Lassiter (2015) points out, that semantics will have issues validating entailments between probabilistic 'should' and other epistemic auxilliaries.

[24] As such, it is not distinctive of the approach pursued here: other information-sensitive solutions such as those in Cariani et al. (2013), Silk (2014) and Carr (2015) also suggest this move.

2. $P(\phi|\bigcap f(w)) \geq P(\psi|\bigcap f(w))$

The first clause helps deliver the constraint we outlined. For once ϕ passes the relevant threshold no other proposition will be strictly better than it.

We have an ordering on propositions and our aim now is to define $BEST$ from this ordering. We will form $BEST(w, f, g)$ by simply taking the $\precsim_{w,f,g}$-best propositions consistent with the modal base and intersecting them. More formally, letting the set of best propositions be

$$PBEST(w, f, g) = \{p \subseteq \bigcap f(w) : \neg\exists q \subseteq \bigcap f(w) : q \prec_{w,f,g} p\}$$

we then say that

$$BEST(w, f, g) = \bigcap PBEST(w, f, g)$$

That is, the domain for 'ought' is the intersection of the best propositions.[25]

This will predict **Deontic**, given plausible assumptions. In **Miners**, it is plausible that we are deciding based on the expected utilities of the various options. Conditional on only the information in the modal base, blocking neither shaft has the unique best expected utility. In that case, *we block neither shaft* is the unique best proposition and so the set of best worlds is simply the worlds in the modal base where we block neither shaft. But once we add to the modal base the proposition that the miners are in shaft A, *we block shaft A* is the unique best proposition and so all the best worlds are ones where we block shaft A.

We also predict **Epistemic**, given plausible assumptions. Suppose again that the threshold is 0.5. The proposition that Alex gets the best results has 0.66 probability and so will be one of the best propositions. $BEST(w, f, g)$, being the intersection of $PBEST(w, f, g)$ and $\bigcap f(w)$ will contain only worlds where Alex gets the best results. Moreover, when we add to the modal base the proposition that Billy studied, then *Billy gets the best results* will be among the best propositions and so, given our semantics, all the best worlds will be ones where Billy gets the best results.

[25]In fact, this semantics is only really a first pass, as it will deliver the wrong results in cases where the set of best propositions is inconsistent. What we need is a generalisation of the intersecting method for cases like these.

Here is one way to generalise it. We still construct $BEST$ from $PBEST$ but this time the procedure is somewhat more complicated. First, say that a maximal consistent intersection S of $PBEST$ is a set S that has the following properties:

1. S is the intersection of some members $S_1, \ldots, S_n$ of $PBEST$
2. The result of intersecting S with any further members of $PBEST$ results in the empty set.

In other words, we form a maximal consistent intersection of $PBEST$ by intersecting as many propositions in $PBEST$ as we can before getting the empty set.

We then form $BEST$ by forming the union of the maximal consistent intersections of $PBEST$:

$$BEST(w, f, g) = \bigcup\{S : S \text{ is a maximal consistent intersection of elements of elements of } PBEST(w, f, g)\}$$

6. Conclusion

The problem raised by Kolodny and MacFarlane's case runs deeper than previously thought. Miners cases arise not just in the deontic realm, but also in the epistemic realm. This has important ramifications for the ensuing debate. Conservativity, while well-motivated and plausible for the original cases, fails to generalise. This failure, I have argued, should lead us to see miners cases not as a deontic phenomenon, but as a probabilisitic one. The classic semantics, in its original form, cannot accommodate this. But I have shown that, by dropping certain assumptions about how orderings are generated, we get a more flexible theory that can give a properly general solution to the miners problem.

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