The universal measurer[1]

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Abstract. It is well-known that numerically modified container phrases such as *four glasses of water* are ambiguous between individuating and measure interpretations. We show that this ambiguity arises for numerically modified atomic predicates generally, including e.g. *four oranges* and *four grains of rice*, but not for measure phrases like *four ounces of water*. We develop an analysis that accounts for this generalization in terms of a type-shifting principle, the Universal Measurer.

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1. Individuating and Measuring

Many have observed that utterances with numerically modified CONTAINER PHRASES such as *four glasses of water* are ambiguous (Selkirk 1977, Chierchia 1998, Landman 2004, Rothstein 2009, 2010, Scontras 2014). (1a), for example, has an INDIVIDUATING INTERPRETATION (II), which is paraphrased in (1b), and a MEASURE INTERPRETATION (MI), paraphrased in (1c).

(1) a. Mary put four glasses of water in the soup.
b. There's a group of four glasses x such that each of x is filled with water and Mary put x in the soup. (II)
c. There's an amount of water x such that x measures four glasses worth and Mary put x in the soup. (MI)

For the II, suppose that Mary heats water for coffee in an odd way: she places glasses full of water in boiling soup. In this situation, (1a) is true even though no water touches the soup. For the MI, suppose instead that Mary is making soup, and that the recipe calls for four glassfuls of water. Mary takes a certain glass, fills it with water, empties the water into the soup, and then repeats the process three more times. In this situation, (1a) is true even though no glass touches the soup.

The literature on individuating/measure (I/M) ambiguities has tended to focus exclusively on container phrases, perhaps suggesting that I/M ambiguities arise due to the meanings of container nouns. This paper's primary empirical contribution is to show that I/M ambiguities are not limited to container phrases. Rather, other countable predicates such as *grain of rice* and *orange* also give rise to I/M ambiguities. For example, (2a) is ambiguous between the II in (2b) and the MI in (2c).

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(2) a. Mary put four oranges in the punch.
b. There's a group of four oranges x such that Mary put each of x in the punch. (II)
c. There's an amount of orange x such that x measures four oranges worth and Mary put x in the punch. (MI)

For the II, suppose that Mary wants to decorate already made punch. She thinks that floating fruit would look nice, so she places a few apples, some pears, and four oranges in the punch. For the MI, suppose instead that Mary's punch recipe calls for four oranges worth of pulverized orange. Mary has no oranges handy, but she does have some prepackaged orange pulp. She pours a certain amount into the punch, estimating it to be approximately four oranges worth. (2a) is an acceptable, true answer to the question *How many oranges did Mary put in the punch?* in both scenarios.

Container nouns like *glass* belong to a larger category of nouns Scontras (2014) calls QUANTIZING NOUNS, or nouns which help facilitate counting and other forms of measurement. These also include MEASURE NOUNS like *ounce* and ATOMIZER NOUNS like *grain*. All three combine with an *of*-phrase to form what Rothstein (2009) calls a "classifier phrase", e.g. *glass of wine*, *ounce of water*, or *grain of rice*. However, we will argue here that there is an important asymmetry between these different sorts of nouns: while container nouns and atomizer nouns give rise to I/M ambiguities, measure nouns do not. Rather, measure phrases like *four ounces of water* only have MIs.[2] As evidence, consider the difference in acceptability between the quantizing nouns in (3).

(3) [Context: There are four glasses filled with wine, four grains of rice, and a bowl containing four ounces of water on a table. Pointing at them, Mary says:]
Each of those four {glasses of wine / grains of rice / ??ounces of water} is for John.

According to Rothstein (2010), distributive expressions such as *each* presuppose a domain of individuated, and thus countable, objects, or "atoms" in the sense of e.g. Krifka (1989). (3) shows that atomizer phrases and container phrases on the II are ATOMIC PREDICATES: when singular, they denote sets of atomic individuals. Measure phrases, on the other hand, do not.

The theoretical contribution of this paper is to account for the generalization that atomic predicates of various sorts, including *orange*, give rise to I/M ambiguities, but measure phrases do not. We begin with the observation that both quantizing nouns and ordinary count nouns are QUANTIZED PREDICATES in Krifka's sense: they are predicates such that if an individual satisfies the predicate, no proper part of the individual does. For example, an arbitrary part of an ounce of water is not itself an ounce of water, and an arbitrary part of an orange is not itself an orange. For atomic predicates, this is expected because atomicity entails quantization. Since atoms do not have proper parts, they trivially do not have proper parts having certain properties. In contrast, quantization

[2] cf. Champollion 2010. Apparent exceptions to this generalization include measure nouns used as container nouns, e.g. *liter of water* or *ounce of cocaine*. See Scontras 2014 for discussion.

needn't imply atomicity. For instance, suppose x and y are both quantities of water measuring an ounce. Then both satisfy *ounce of water*, yet they needn't be atoms on account of that. After all, they may have overlapping parts; it could be e.g. that y is the result of replacing half of x with a different half ounce of water. Rather, measure phrases are more plausibly what Krifka calls STRICTLY QUANTIZED PREDICATES: they are quantized but not atomic.

Our view is that I/M ambiguities arise for atomic predicates in general because it is always possible to shift an atomic predicate to a strictly quantized predicate. On this view, IIs arise due to the atomicity of the predicates in question. For example, *glass of water* denotes a set of atomic glasses by default, and the II of *four glasses of water* results from *four* functioning as a cardinality modifier, i.e. it counts the number of atomic glasses constituting a certain plurality. In contrast, MIs arise thanks to a type-shifting principle we dub THE UNIVERSAL MEASURER (UM), in homage to Pelletier's (1975) Universal Grinder and Universal Packager. UM applies "universally" to atomic predicates, effectively transforming them into measure expressions. For example, it shifts the meaning of *glass* from a predicate true of atomic glasses to a non-standard measure, or what Partee and Borschev (2012) call an AD HOC MEASURE. As a result, *four glasses of water* comes to denote quantities of water measuring four glasses worth, thus resulting in an MI. Like *ounce of water*, *glass of water* on the MI is strictly quantized since quantities of water satisfying the predicate may share overlapping parts. MIs of e.g. *four oranges* arise thanks to a "reflexivized" version of UM. Applying it to *orange* returns quantities of orange measuring so many oranges worth, again rendering *orange* strictly quantized. As a result, UM accounts for the fact that atomic predicates of various sorts are I/M ambiguous. Because UM does not have an inverse, the analysis predicts that it is not possible to shift from MIs to IIs. And since measure phrases denote (standardized) measures by default, this also correctly predicts that they do not generally have IIs.

The rest of the paper is organized as follows. In §2 we argue that I/M ambiguities arise for various sorts of atomic predicates, not just container phrases. To do so, we generalize Rothstein (2010)'s diagnostics for I/M-ambiguous container phrases and also develop novel diagnostics. Applying the diagnostics reveals that I/M ambiguities are more prevalent than has been previously recognized. §3 presents our analysis of I/M-ambiguities. We show that supplementing Scontras (2014)'s semantics for quantizing nouns with UM makes it possible to account for the pervasiveness of I/M ambiguities. Consequently, the resulting analysis both builds on and improves previous analyses.

2. Diagnosing I/M-Ambiguities

The purpose of this section is to show that I/M ambiguities are not limited to just container phrases. We present diagnostics for IIs and MIs, beginning with three diagnostics from Rothstein (2010). Though all three are applicable to container phrases, not all are applicable to atomic predicates more generally. Therefore, we develop some additional heuristics to supplement Rothstein's. Taken together, these reveal that I/M ambiguities are more prevalent than previously recognized.

2.1. Rothstein's Diagnostics

Rothstein (2010)'s first diagnostic for disambiguating container phrases involves distributive expressions such as *each*. On the II, *four glasses of water* denotes groups of four atomic glasses, each of which is filled with water. On the MI, it instead denotes quantities of water measuring four glassfuls. Because *each* presupposes a domain of atoms, it is thus compatible only with IIs:

(4) [Context: Mary has a strange way of heating up water for coffee. She fills individual glasses with water and then places those glasses in boiling soup.]

(5) [Context: Mary is making soup. Following the recipe, she fills a certain glass four times with water, pouring the contents each time into the soup.]

(6) Mary put each of the four glasses of water in the soup.

The fact that (6) is acceptable in (4) but odd in (5) shows that the former induces an II of *four glasses of water*, the latter an MI. Accordingly, we call contexts like (4) INDIVIDUATING CONTEXTS and those like (5) MEASURE CONTEXTS.

Rothstein's second diagnostic involves the possibility of *-ful* suffixation. According to it, *-ful* can be acceptably suffixed to a container noun in measure contexts but not individuating contexts. And, indeed, an utterance of (7) is acceptable in (5) but not (4).

(7) Mary put four glassfuls of water in the soup.

According to Rothstein, that's because the function of *-ful* is to transform a container noun such as *glass* into a measure noun. It denotes quantities of a substance (e.g. water) measured in terms of an ad hoc glass-unit. Consequently, *-ful* suffixation effectively forces an MI.

Rothstein's third diagnostic involves DEGREE RELATIVES. The diagnostic relies on Carlson's (1977) observation that relative clauses denoting sets of individuals can be headed by either *which* or *that*, unlike relative clauses denoting measured quantities, which are necessarily headed by *that* or a null complementizer. This diagnostic is applied to measure nouns and container nouns in (8).

(8) a. I wanted to inspect the four ounces of water {∅/ ??which / that} Mary put in the soup.
b. I wanted to inspect the four glassfuls of water {∅/ ??which / that} Mary put in the soup.
c. I wanted to inspect the four glasses of water {∅/ which / that} Mary put in the soup.

(8a) shows that the measure phrase *four ounces of water* denotes only measured quantities of water. Similarly, (8b) demonstrates that *four glassfuls of water* denotes only measured quantities of water,

as expected given the meaning Rothstein attributes to *-ful*. In contrast, (8c) shows that *four glasses of water* does have an II, since it is acceptable with both kinds of relative clauses. However, because *four glasses of water* has an MI paraphrased as *four glassfuls of water*, we should expect relative clauses like the one in (8c) to be acceptably headed by *which* only in individuating contexts such as (4). This prediction is also correct.

Two of Rothstein's diagnostics can be used to support our claim that I/M ambiguities extend beyond just container phrases. Note first that *each* is acceptable with the atomizer phrase *four drops of blood* when uttered in (9) – an individuating context – but not (10) – a measure context.

(9) [Context: John and Mary are detectives at a crime scene, where rain has recently washed away four drops of blood that were on the sidewalk. Mary says:]
I saw each of the four drops of blood before the rain started.

(10) [Context: Mary is making soup. The recipe calls for four drops of pig blood. Mary does not have a dropper, so she puts four drops worth of blood in a teaspoon and then pours it into the soup. Later, Fred asks if the soup really contains four drops of blood. John says:]
I saw (#each of) the four drops of blood while Mary was making the soup.

This is to be expected if *four drops of blood* is I/M-ambiguous, and (9) induces an II of the predicate while (10) induces an MI. Secondly, notice that while *that* in (11) is acceptable in both (9) and (10), *which* is only acceptable in (9).

(11) Earlier, John inspected the four drops of blood {which/that} Mary {saw/put in the soup.}

Again, this is to be expected if *four drops of blood* is I/M-ambiguous, and if relative clauses headed by *which* presuppose a domain of individuated objects. Applying Rothstein's diagnostics to the numerically modified atomic predicate *four oranges* yields similar results:

(12) [Context: John and Mary are at a party with punch that was decorated using whole, fresh fruit. Now there are only three oranges in the punch, and there is an argument about whether originally there were four. Mary says:]
Before the party started, I saw each of the four oranges that Bill put in the punch.

(13) [Context: John and Mary are at a party, and there is an argument about how many oranges were used to make the punch. Mary was there when Bill made the punch, and saw him measure out four oranges worth of pulp from a store-bought container. She says:]
Before the party started, I saw (#each of) the four oranges that Bill put in the soup.

(14) John wanted to inspect the four oranges {which/that} Bill put in the punch.

Four oranges is acceptable with *each* in the individuating context (12) but not in measure context (13). Similarly, *which* in (14) is acceptable only in the individuating context.

These results show that like the numerically modified atomizer phrase *four drops of blood*, *four oranges* is I/M-ambiguous. However, because neither *drop* nor *orange* denotes a container, unlike e.g. *glass* or *box*, Rothstein's *-ful* suffixation heuristic is not applicable to these nouns. Nevertheless, there is a way of naturally extending Rothstein's diagnostic to include all three categories. We do this in the next section, where we also introduce some novel diagnostics intended to supplement Rothstein's.

2.2. Some Additional Diagnostics

We begin by generalizing Rothstein's *-ful* suffixation diagnostic. Rothstein (2010) proposes that *-ful* transforms a container noun like *glass* into a measure noun, one denoting measured quantities of a substance. We propose that *-ful* is a special case of *worth* in this respect. More generally, *worth* transforms atomic predicates into measure expressions. For example, *four glassfuls of water* and *four glasses worth of water* are synonymous: both denote quantities of water measuring four ad hoc glass-units. Similarly, *four oranges worth of orange* denotes quantities of orange measuring four ad hoc orange-units, while *four grains worth of rice* denotes quantities of rice measuring four ad hoc grain-units. In all three cases, *worth* expresses a relation between substances and their measures. *-ful* also expresses a relation between substances and measures, only that relation is restricted to containment. Hence, *glassful of water* measures how much of a substance would be contained in a certain glass. On the other hand, *worth* is far more liberal with regard to how measures are determined, as shown in (15).

(15) [Context: John and Mary are planning a dog sledding trip. John is out buying supplies for the trip but can't remember how many dogs they planned to bring. He calls Mary, asking her how much dog food to buy. Mary responds:]
We need four dogs #(worth) of dog food.

In (15), the function of *worth* is to transform *dog* into a measure noun, one measuring how much dog food a certain dog can eat for the duration of John and Mary's trip.

Thus, our first diagnostic is a natural extension of Rothstein's *-ful* diagnostic: I/M-ambiguous phrases are acceptable with *worth* in measure contexts but not individuating contexts. This is illustrated by (18), which is odd in the individuating context (16) but acceptable in the measure context (17).

(16) [Context: Mary has made some punch for the party. She wants to decorate it, and she thinks floating fruit would look nice. She places a few apples, some pears, and four oranges in the punch. John asks 'How many oranges did Mary put in the punch?'. Fred replies:]

(17) [Context: Mary is making punch for the party. The recipe calls for four processed oranges, but Mary is out of oranges. She pours a certain amount of prepackaged orange pulp into the punch, estimating that it is roughly equal to how much orange pulp four typical oranges would produce. John asks 'How many oranges did Mary put in the punch?'. Fred replies:]

(18) Mary put four oranges worth (of orange) in the punch.

This result makes sense if *four oranges* is I/M ambiguous, if *worth* in (18) transforms *orange* into a measure noun, thus resulting in an MI, and if an utterance of (18) implicates that what was put in the punch was something other than individual oranges, like e.g. orange pulp.

Our second diagnostic involves the nouns *number* and *amount*. Scontras (2014) calls *amount* a DEGREE NOUN. It denotes a relation between kinds and degrees, specifically between instances of a kind and measures of those instances. For example, *amount* in (19a) denotes a relation between a certain group of apples and their collective weight, their cardinality, or some other contextually salient measure.

(19) [Context: Pointing at four 1 lb. apples in a bowl.]

a. John ate that amount of apples every day for a year.

b. John ate that number of apples every day for a year.

(19a) is ambiguous: it can mean that every day for a year John ate apples whose collective weight equals four pounds, or else that every day for a year John ate a total of four apples, regardless of their weight. On the other hand, (19b) can only mean the latter. That's plausibly because *number* is a special case of *amount*: it too is a degree noun, but it relates pluralities to their cardinalities.

This difference between *amount* and *number* can be used to demonstrate I/M ambiguities. Notice that (20a) is true in the context given, unlike (20b).

(20) [Context: Mary places four glasses filled with water in her soup. John places eight glasses filled with water in his soup. John's glasses are exactly half the size of Mary's.]

a. There are four glasses of water in Mary's soup, and there's the same amount of water in John's soup. (true)

b. There are four glasses of water in Mary's soup, and there's the same number #(of glasses) of water in John's soup. (false)

Again, *four glasses of water* denotes quantities of water measuring four glasses worth on the MI. In (20), *the same amount* anaphorically refers to this abstract measure. Consequently, (20) is true only if the amount of water Mary put in her soup is equal to the amount of water John put in his, which is indeed the case. On the II, *four glasses of water* denotes pluralities of four atomic glasses, each filled with water. In (20b), *the same number* anaphorically refers to this abstract cardinality. Consequently, (20b) will be true only if the number of glasses Mary placed in her soup is equal to the number of glasses placed in his, which is not the case. Now consider (21).

(21) [Context: Mary places four glasses filled with water into her soup. John places four glasses filled with water into his soup. John's glasses are exactly half the size of Mary's.]
   a. There are four glasses of water in Mary's soup, and there's the same amount of water in John's soup. (false)
   b. There are four glasses of water in Mary's soup, and there's the same number of glasses in John's soup. (true)

These judgments make sense only if *four glasses of water* receives a MI in (21a) and an II in (21b). That's because there are four glassfuls of water in Mary's soup but not in John's, even though there are just as many glasses filled with water in both soups.[3]

Applying this diagnostic to atomizer phrases such as *four grains of rice* reveals that they too are I/M-ambiguous.

(22) [Context: Mary and John are making soup. Mary adds four grains of rice to her soup. John adds eight grains of rice to his. John's grains are exactly half the size of Mary's.]
   a. There are four grains of rice in Mary's soup, and there's the same amount of rice in John's soup. (true)
   b. There are four grains of rice in Mary's soup, and there's the same number of grains in John's soup. (false)

(22a) is true only if *four grains of rice* receives an MI since there are in fact four grains worth of rice in both soups. However, (22b) is false since there are twice as many grains of rice in John's soup. This shows that in (22b), *four grains of rice* gives rise to an II. These conclusions are confirmed by the examples in (23), where the evaluations are reversed. (23a) is false because the volume of rice in John's soup is half of that in Mary's, while (23b) is true because there are just as many rice grains in both soups.

[3]Greg Scontras (p.c.) proposes that (21a) also has a reading that is true in (21), and likewise for (23a) and (25a) below. While we ourselves have trouble getting this interpretation of (21a), the crucial observation related to defining this diagnostic is that it has false interpretation in the context provided, unlike (21b).

(23) [Context: Mary and John are making soup. Mary adds four grains of rice to her soup, and John does the same. John's grains are exactly half the size of Mary's.]

a. There are four grains of rice in Mary's soup, and there's the same amount of rice in John's soup. (false)

b. There are four grains of rice in Mary's soup, and there's the same number of grains in John's soup. (true)

These examples show that numerically modified atomizer phrases, like numerically modified container phrases, are I/M ambiguous. Applying the diagnostic *four oranges* yields similar results:

(24) [Context: Mary and John are making punch. Mary adds four oranges to her punch. John adds eight oranges to his. John's oranges are exactly half the size of Mary's.]

a. There are four oranges in Mary's punch, and there's the same amount orange in John's punch. (true)

b. There are four oranges in Mary's punch, and there's the same number of oranges in John's punch. (false)

(25) [Context: Mary and John are making punch. Mary adds four oranges to her punch. John adds four oranges to his. John's oranges are exactly half the size of Mary's.]

a. There are four oranges in Mary's punch, and there's the same amount of orange in John's punch. (false)

b. There are four oranges in Mary's punch, and there's the same number of oranges in John's punch. (true)

Our final additional heuristic involves the (un)acceptability of modifiers such as *approximately* and *roughly*, or what Lasersohn (1999) calls SLACK REGULATORS. We illustrate the diagnostic in (26) using a container phrase. The individuating context in (4) and measure context in (5) are repeated for convenience.

(4) [Context: Mary has a strange way of heating up water for coffee. She fills individual glasses with water and then places those glasses in boiling soup.]

(5) [Context: Mary is making soup. Following the recipe, she fills a certain glass four times with water, pouring the contents each time into the soup.]

(26) [Context: John, who was watching Mary the whole time, says:] Mary put approximately four glasses of water in the soup.

John's utterance of (26) is acceptable in the measure context but not the individuating context. That's plausibly thanks to an implicature carried by a use of *approximately*: it implicates that the speaker is unsure whether the amount indicated is the amount which actually obtains. On the MI, Mary's utterance of (26) implicates that for all she knows, the amount of water she poured into the soup is not exactly four glassfuls. This sort of uncertainty is normal with measurement. For instance, whether a given bowl contains exactly four ounces of water is something that we can only know to a certain degree of precision. Most everyday purposes do not require a great deal of precision, and it's only when more precision is required that we need to use slack regulators. However, it is hard see how a use of *approximately* could be appropriate in (26) if an II of *four glasses of water* is intended. After all, Mary just placed the four glasses in the soup, and so there would appear to be little room left for uncertainty concerning their exact cardinality. Thus, I/M-ambiguous expressions are generally acceptable with slack regulators on MIs but not IIs, at least when the cardinality is question is relatively small.[4]

In (27), we apply this diagnostic to an example with *four oranges*. The individuating context (16) and measure context (17) are repeated for convenience.

(16) [Context: Mary has made some punch for the party. She wants to decorate it, and she thinks floating fruit would look nice. She places a few apples, some pears, and four oranges in the punch. John asks 'How many oranges did Mary put in the punch?'. Fred replies:]

(17) [Context: Mary is making punch for the party. The recipe calls for four processed oranges, but Mary is out of oranges. She pours a certain amount of prepackaged orange pulp into the punch, estimating that it is roughly equal to how much orange pulp four typical oranges would produce. John asks 'How many oranges did Mary put in the punch?'. Fred replies:]

(27) Mary put approximately four oranges in the punch.

Fred's utterance of (27) is acceptable in the measure context but not the individuating context. This result is predicted if *four oranges* is I/M ambiguous, and if slack regulators are generally acceptable only with MIs in the case of small numbers.

Taken together, the examples in §2 show that I/M ambiguities are not limited to just container nouns. Atomic nouns are in general I/M-ambiguous. In the next section, we argue that this is because a type-shifting principle we call "the Universal Measurer" shifts atomic predicates to measure expressions, thus resulting in MIs.

3. The Universal Measurer

Perhaps because of the tendency to focus on container phrases, some previous analyses of I/M ambiguities have located their source in features peculiar to the meanings of container nouns.

[4]Slack regulators are generally acceptable with IIs involving large cardinalities, where an exact measure is not so easily determined. It is for this reason that we use only small numbers when diagnosing I/M ambiguities.

Consider for instance the analyses of Rothstein (2009, 2010) and Scontras (2014). They claim that *glass* is by default a monadic predicate true of glasses, i.e. (28a), which can then be shifted into a relational noun either via the CONSTRUCT STATE SHIFT (CSS), a type-shifting operation proposed by Rothstein in her discussion of the Hebrew construct state, or else via a similar meaning Scontras attributes to *of*.

(28) a. $[\![\text{glass}]\!] = \lambda x.\, \texttt{glass}(x)$
b. $\lambda P.\lambda Q.\lambda x.\exists y.\, P(x) \wedge Q(y) \wedge R(x,y)$ (CSS)
c. CSS($[\![\text{glass}]\!]$) = $\lambda Q.\lambda x.\exists y.\, \texttt{glass}(x) \wedge Q(y) \wedge R(x,y)$
d. CSS($[\![\text{glass}]\!]$)($[\![\text{of water}]\!]$) = $\lambda x.\exists y.\, \texttt{glass}(x) \wedge \texttt{water}(y) \wedge R(x,y)$

The relation variable R in (28c) is free because its value is supplied by context. In the case of container phrases like *glass of water*, R is naturally interpreted as the relation of containment, or being filled with (c.f. Partee and Borschev 2012) Thus, as demonstrated in (28d), applying CSS to *glass* and combining the result with *of water* returns a predicate true of glasses filled with water. This predicate can then combine with a cardinality modifier such as the one in (29c), where '$\mu_{\#}$' is a cardinality measure measuring the number of atoms constituting a plurality.[5]

(29) a. $[\![\text{four}]\!] = 4$
b. $[\![\text{CARD}]\!] = \lambda n.\lambda P.\lambda x.\, \mu_{\#}(x) = 4 \wedge P(x)$
c. $[\![\text{CARD}]\!]([\![\text{four}]\!]) = \lambda P.\lambda x.\, \mu_{\#}(x) = 4 \wedge P(x)$
d. $[\![\text{four glasses of water}]\!]$ = $\lambda x.\exists y.\, \mu_{\#}(x) = 4 \wedge \texttt{glasses}(x) \wedge \texttt{water}(y) \wedge R(x,y)$

The result is the the meaning of *four glasses of water* in (29d), a predicate true of those pluralities consisting of four atomic glasses, each of which is filled with water, thus leading to an II.

The MI is said to result from *glass* taking on the meaning of *glassful*. This is given in (30b), where 'μ_{glass}' is an ad hoc glass-measure; it measures how much a given quantity of some substance, e.g. water, would fill a certain glass.

(30) a. $[\![\text{-ful}]\!] = \lambda P.\lambda Q.\lambda n.\lambda x.\, Q(x) \wedge \mu_P(x) = n$
b. $[\![\text{glassful}]\!]$ = $\lambda Q.\lambda n.\lambda x.\, Q(x) \wedge \mu_{glass}(x) = n$
c. $[\![\text{glassful of water}]\!]$ = $\lambda n.\lambda x.\, \texttt{water}(x) \wedge \mu_{glass}(x) = n$
d. $[\![\text{four glassfuls of water}]\!]$ = $\lambda x.\, \texttt{water}(x) \wedge \mu_{glass}(x) = 4$

[5] As (29a) suggests, we assume that *four* is a numeral referring to the number four. There are numerous proposals available in the literature for getting from this numeral denotation to the cardinality modifier in (29c). Following Kennedy (2013), we assume for convenience that it results from combining with something like CARD in (29b).

In effect, adding *-ful* to a container noun like *glass* transforms it into a measure noun, one denoting quantities of a substance measuring a certain amount in terms of an ad hoc glass-unit. Consequently, *four glassfuls of water* will denote those quantities of water measuring four glassfuls, i.e. those whose volume is equal to the amount of water which would result from filling a certain glass four times. On the analyses in question, the MI of *four glasses of water* results from combining the default, atomic meaning of *glass* given in (28a) with the meaning of *-ful* given in (30a), perhaps through a process of silent *-ful* suffixation (Scontras 2014:80-81; Rothstein 2010:32).

Though these analyses account for I/M ambiguities in container phrases, in their current form they do not account for examples with atomizer nouns or ordinary count nouns. Since the latter do not denote containers, they have the wrong sort of meaning to combine with *-ful*, either overtly or covertly. Nevertheless, these analyses can be extended to account for the generalization that I/M-ambiguities hold for various atomic predicates. We show how in what follows.

3.1. The semantics of measure nouns

We take as our starting point Scontras' semantics for measure nouns, which itself presupposes the broadly "Neo-Carlsonian" perspective of Chierchia (1998). For our purposes, at least, the important fact about Chierchia's semantics is that there is a systematic correspondence holding between KINDS, or the referents of bare mass and plural nouns, their instances, and corresponding properties. This is related to the well-known fact that bare nouns have both referential and predicative uses, as shown in (31). Chierchia (1998:350-1) relates the meanings involved in these different uses via the two operators in (32), where '$\sqsubseteq$' is a mereological relation.

(31) a. Water is widespread.
b. Mary drank (some) water.

(32) a. For any property P and world w, ${}^{\cap}P = \lambda w'.\ \iota x[P_{w'}(x)]$, if defined
b. For any kind k and world w, ${}^{\cup}k = \lambda x.\ x \sqsubseteq k_w$, if defined

In (31a), *water* refers to the water-kind, but in (31b) it denotes a predicate true of quantities of water. Chierchia relates the two by analyzing kinds as individual concepts, i.e. functions from worlds to the maximal sum of instances of that kind in that world. For example, the water-kind $\mathtt{W}$ is a function that takes a world to the maximal sum of quantities of water in that world. Applying the ${}^{\cup}$-operator to $\mathtt{W}$ returns the set of all quantities of water in a world, thus providing a suitable denotation for predicative uses of *water* like (31b). Conversely, applying the ${}^{\cap}$-operator to the corresponding property nominalizes it, thus returning the original kind.

On Scontras' semantics, measure nouns make it possible to measure instances of a kind. For instance, *ounce* denotes a relation between a kind k and a number n such that instances of k measure n-ounces. This is the meaning given in (33), where 'μ_{oz}' is an ounce-measure.

(33) a. $[\![\text{ounce}]\!] = \lambda k.\lambda n.\lambda x.\ ^{\cup}k(x) \wedge \mu_{oz}(x) = n$
b. $[\![\text{ounce of water}]\!] = \lambda n.\lambda x.\ ^{\cup}\mathtt{W}(x) \wedge \mu_{oz}(x) = n$
c. $[\![\text{four ounces of water}]\!] = \lambda x.\ ^{\cup}\mathtt{W}(x) \wedge \mu_{oz}(x) = 4$

According to (33c), *four ounces of water* denotes those quantities of water measuring four ounces. In the next section, we'll show how adopting UM makes it possible to derive similar sorts of MIs for *four glasses of water*, *four grains of rice*, and *four oranges*.

3.2. The Universal Measurer

We begin with IIs. Following Ladusaw (1982), we assume that *of* denotes a mereological relation, as in (34a). Since this relates two individuals of type $\langle e \rangle$ but kinds are individual concepts of type $\langle s, e \rangle$, it follows that combining *of* with the bare mass noun *water* creates a type-mismatch. However, this is easily remedied by applying Montague (1974)'s extensionalizing $^{\vee}$-operator, which when applied to a kind returns the maximal sum of the corresponding substance in the world of evaluation. For example, applying $^{\vee}$ to the kind $\mathtt{W}$ returns the maximal sum of actual quantities of water. Consequently, this has the appropriate type to combine with *of*, namely $\langle e \rangle$. The result is the denotation for *of water* given in (34b), namely the set of all parts of the maximal quantity of water, or more simply the set of all quantities of water.

(34) a. $[\![\text{of}]\!] = \lambda x, y.\ y \sqsubseteq x$
b. $[\![\text{of water}]\!] = \lambda x.\ y \sqsubseteq\ ^{\vee}\mathtt{W}$

Following Rothstein, we assume that IIs for container phrases such as *four glasses of water* arise from applying CSS to the meaning of *glass* and combining the result with the denotation of *of water* in (34b), thus yielding (35a). Since glasses are atomic individuals, the predicate in (35a) can combine with a cardinality modifier such as (29a) to return a predicate true of pluralities consisting of a certain number of glasses filled with water, ultimately resulting in (35b).

(35) a. $\text{CSS}([\![\text{glass}]\!])([\![\text{of water}]\!]) = \lambda x.\exists y.\ \mathtt{glass}(x) \wedge y \sqsubseteq\ ^{\vee}\mathtt{W} \wedge R(x, y)$
b. $[\![\text{four glasses of water}]\!] = \lambda x.\exists y.\ \mu_{\#}(x) = 4 \wedge \mathtt{glasses}(x) \wedge y \sqsubseteq\ ^{\vee}\mathtt{W} \wedge R(x, y)$

Consequently, *Mary put four glasses of water in the soup* is true if there is a plurality of four glasses, each of which was filled with water and put in the soup by Mary. This is the II.

For MIs, we assume that there is a general type-shifting principle – "the Universal Measurer" – which shifts the meaning of an atomic predicate to that of a measure expression specifically within measure contexts.[6] For instance, UM shifts *glass* into a measure noun which can then compose with the meanings of *of water* and *four* given above.[7]

(36) a. $\lambda P.\lambda k.\lambda n.\lambda x.\ ^{\cup}k(x) \wedge \mu_P(x) = n$ (UM)

b. UM($[\![\text{glass}]\!]$) = $\lambda k.\lambda n.\lambda x.\ ^{\cup}k(x) \wedge \mu_{glass}(x) = n$

c. UM($[\![\text{glass}]\!]$)($[\![\text{of water}]\!]$) = $\lambda n.\lambda x.\ ^{\cup}[^{\cap}\lambda y.\ y \sqsubseteq\ ^{\vee}\mathtt{W}](x) \wedge \mu_{glass}(x) = n$

d. $[\![\text{four glasses of water}]\!]$ = $\lambda x.\ ^{\cup}[^{\cap}\lambda y.\ y \sqsubseteq\ ^{\vee}\mathtt{W}](x) \wedge \mu_{glass}(x) = 4$

According to (36d), *four glasses of water* denotes those quantities of water measuring four ad hoc glass-units, and so *Mary put four glasses of water in the soup* will be true if Mary put an amount of water in the soup equal to four glasses worth. This is the MI.

The crucial difference between this analysis and previous approaches lies in the potential generality of UM. Nothing in the definition of UM, which we hypothesize is lexicalized as *worth*, restricts its application to just those nouns expressing containment relations.[8,9] As a result, UM can apply to various sorts of atomic predicates. However, one significant problem with this suggestion is that, as stated, UM is not sufficient to derive MIs for *four grains of rice* or *four oranges*. Its first argument – P – is a monadic predicate which is transformed into a measure expression. However, unlike container nouns, atomizers such as *grain* are inherently relational (Scontras 2014). Consequently, they have the wrong type to function as ad hoc measures. Furthermore, on the MI of *four oranges*, the substance measured – orange – is necessarily of the same kind as the ad hoc unit of measurement – an orange. Yet nothing in (36a) guarantees that the measure argument – P – and the substance argument – k – are of the same kind, and with good reason. After all, if that were generally the case, then UM would not suffice to derive the MI of e.g. *four glasses of water*.

[6] See Barker (1998) for arguments that this kind of type-shifting is generally available and necessary.

[7] Note that although *of water* supplies the kind argument of (36b), it denotes a predicate, not a kind. This mismatch triggers the application of $^{\cap}$ to the predicate $\lambda y.\ y \sqsubseteq\ ^{\vee}\mathtt{W}$, which results in a kind.

[8] The relationship between UM and *worth* may not be quite so straightforward, however, because the two have different distributions. Specifically, there are times when *worth* is obligatory. For example, *four dogs worth of dog food* is acceptable in (15) but #*four dogs of dog food* is not. One possibility is that UM restricts the way in which the ad hoc measure is determined to "natural" relations (Vikner and Jensen 2002) between the individual determining the unit of measure and the substance measured, allowing for containment in the case of container nouns and material constitution in the case of ordinary count nouns (as in e.g. *four oranges [worth of orange matter]*). On this view, *worth*, UM, and *-ful* create a continuum based on the degree to which the measure relation is lexically restricted. However, in general, more work is needed to understand exactly how UM and *worth* differ with respect to the constraints they place on substances and their measures.

[9] See Schwarzchild 2002 for the related idea that *worth* denotes a "scale function" used in measuring.

Our proposed solution to both problems involves "reflexivizing" UM. To do that, we adapt a simplified version of a common approach to analyzing transitive verbs such as *bathe* in (37a), which, when made intransitive, are necessarily reflexive.

(37) a. John bathed the baby.
b. John bathed.

(37b) cannot mean that John bathed just anyone; it can only mean that he bathed himself. A common explanation posits an operation that transforms a transitive verb into a intransitive, reflexive verb (see Reinhart and Siloni 2005 and references therein). This operation is represented in (38a) as "VREF".

(38) a. $\lambda R.\lambda x.\, R(x, x)$ (VREF)
b. VREF($[\![\text{bathe}]\!]$) $= \lambda x.\, \texttt{bathe}(x, x)$

By setting both arguments of the verb to be identical, VREF effectively guarantees that intransitive uses of e.g. *bathe* are reflexive.

We assume that VREF is a special case of a more general reflexivization operation which takes relational expressions of various types and returns reflexivized versions of those expressions. One particular instantiation of this principle is "REFL" in (39a), where $\mathfrak{Q}$ has the same type as UM.

(39) a. $\lambda\mathfrak{Q}.\lambda P.\lambda n.\lambda x.\mathfrak{Q}(P)(P)(n)(x)$ (REFL)
b. REFL(UM) = $\lambda P.\lambda n.\lambda x.\, {}^{\cup\cap}P(x) \wedge \mu_P(x) = n$ (RUM)

In effect, applying REFL to UM resets the first two arguments of UM to be identical, leaving the last two arguments intact. Note that since UM takes a kind as its second argument, applying REFL to UM leads to a type-mismatch. This is remedied by applying Chierchia's $^{\cap}$-operator to the predicate, thus returning a kind ${}^{\cap}P$. The latter is of the same sort as the ad hoc measure μ_P, as desired. The result is THE REFLEXIVIZED UNIVERSAL MEASURER (RUM) stated in (39b).

We propose that MIs of *four oranges* and *four grains of rice* result from applying RUM to the default denotations of *oranges* and *grains of rice*, as shown in e.g. (40b).

(40) a. Mary put four oranges in the punch.
b. RUM($[\![\text{oranges}]\!]$) = $\lambda n.\lambda x.\, {}^{\cup}[{}^{\cap}\lambda y.\, \texttt{oranges}(y)](x) \wedge \mu_{orange}(x) = n$
c. RUM($[\![\text{oranges}]\!]$)($[\![\text{four}]\!]$) $= \lambda x.\, {}^{\cup}[{}^{\cap}\lambda y.\, \texttt{oranges}(y)](x) \wedge \mu_{orange}(x) = 4$

According to (40c), *four oranges* denotes those quantities of orange measuring four ad hoc orange-units, and so (40a) will be true just in case the amount of orange Mary put in the punch equals four oranges worth.[10] And this, of course, is the desired MI.

MIs for atomizer phrases can be derived similarly. On Scontras' semantics for atomizer nouns, *grain* partitions the rice-kind into countable, atomic grains. Assuming a denotation for *of rice* similar to *of water* from above, combining (41b) with the former creates another type-mismatch resolvable by applying Chierchia's kind-forming operator $^{\cap}$. Combining (41b) with this kind results in (41c), or the set of rice grains, where R names the rice-kind. Finally, applying RUM to this set yields a measure of rice given in terms of an ad hoc grain-of-rice-unit, or (41d).

(41) a. Mary put four grains of rice in the soup.
b. $[\![\text{grain}]\!] = \lambda k.\lambda x.\ x \in \pi_{grain}(k)$
c. $[\![\text{grain of rice}]\!] = \lambda x.\ x \in \pi_{grain}(^{\cap}\lambda y.\ y \sqsubseteq\ ^{\vee}\mathtt{R})$
d. $\text{RUM}([\![\text{grains of rice}]\!]) = \lambda n.\lambda x.\ ^{\cup}[^{\cap}\lambda y.\ \mathtt{grains\text{-}of\text{-}rice}(y)](x) \wedge \mu_{grain-of-rice}(x) = n$

As a result, *four grains of rice* denotes those quantities of rice measuring four grains worth, and so (41a) is true just in case Mary put an amount of rice equal to four grains worth in the soup, or the desired MI.

In sum, positing UM allows us to account for the fact that all atomic predicates, not just container nouns, are I/M-ambiguous. One important prediction of this approach is that UM applies not just to individual lexical items but also to phrasal constituents. The same holds for *worth*. It too can take phrasal arguments, as witnessed by e.g. *Mary put 500 grains of rice but only 400 grains of rice worth of water in the pot, so we'll need more water.*

4. Conclusion

In this paper, we have proposed an account of the novel empirical generalization that atomic predicates generally, including atomizers and container nouns, give rise to I/M ambiguities, unlike measure nouns. We argued that these ambiguities arise due to a universally available type-shifting operation, the Universal Measurer. It shifts the meaning of an atomic predicate to that of a strictly quantized predicate. Crucially, this shift in meaning is unidirectional. Since UM does not have an inverse, and since MIs only arise thanks to an application of UM, it is in general impossible to recover IIs from MIs. Similarly, because measure nouns denote standardized measures of substances by default, they generally fail to give rise to IIs.

[10] Note that in the system of Chierchia (1998), mass nouns qua predicates denote the closure of atoms plus all pluralities formed from them. On the other hand, plural nouns like *oranges* strictly denote pluralities. Consequently, $^{\cup}[^{\cap}\lambda y.\ \mathtt{oranges}(y)]$ effectively massifies the plural predicate, as Chierchia points out.

Though the discussion here has focused only on the nominal domain, we expect our analysis to apply to atomic predicates across all domains, just as certain influential analyses of the mass/count distinction apply across multiple domains (Bach 1986, Krifka 1989, Zwarts 2005). For instance, consider the verbal predicate *flew for four days* in (42a), which is also plausibly ambiguous between the II suggested in (42b) and the MI in (42c).

(42) a. Mary flew for four days.
b. There's a group of four days x such that Mary flew on each of x (II)
c. There's an interval of time x s.t. x measures 96 hours and Mary flew for x. (MI)

For the II, imagine that Mary has a private plane. On some days, she flies to work, and on other days, she drives. The flight is only 30 minutes one way, so each day she flies she gets about an hour of flying time. In this scenario, John can truly utter (42a) to describe Mary's behavior over a given four day period. This is the II of *flew for four days*. For the MI, suppose instead that Mary has had numerous business flights over the past month, each lasting different intervals of time. After calculating the total amount of time Mary has spent flying over the past month, John, who is in charge of reimbursing Mary for her travel costs, truly utters (42a), meaning that Mary flew 96 hours in total over the past month. We leave showing how the analysis of I/M ambiguities sketched here can be extended to account for (42) and similar examples as a task for future research.

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