Severing maximality from *fewer than*: evidence from genericity[1]

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Abstract. This paper presents new evidence suggesting that the downward entailingness of a quantifier like *fewer than four people* is due not simply to the lexical meaning of the quantifier, but also to a separate, and in principle optional, maximization operation that occurs in the scope of the quantifier, a decomposition already posited for independent reasons by Spector (2014).

Keywords: quantification, plurality, modified numerals, distributivity, collectivity, genericity

1. Introduction

This paper concerns the role that maximality plays in the semantics of (sentences containing) the numeral modifier *fewer than*. On the basis of new data involving generic readings of quantificational noun phrases of the form *fewer than n NP*, I argue that the maximality component normally associated with *fewer than* should be viewed as an optional component that is separate from the meaning of *fewer than*. I start by presenting what I call 'maximal' and 'non-maximal' readings of sentences in which *fewer than n NP* contributes existential force, and the puzzle that the existence of such readings gives rise to. In section 2, I present two recent theories of the puzzle and show that, for the core existential data, the two theories are on a par. In section 3, I present new data from the generic domain, which, in section 4, I show can only be captured by one of the two theories. Section 5 discusses some extensions and predictions. Section 6 concludes.

1.1. Maximal readings

One main intuitive difference between *more than* and *fewer than* is that the latter, but not the former, normally conveys an upper bound. For example, (1a), but not (1b), conveys an upper bound of three on the number of students who (may have) attended. Put differently, (1a), but not (1b), intuitively entails (1c).

(1) a. Fewer than four students attended.
  b. More than three students attended.
  c. It is not the case that more than three students attended.

In addition, (1a), but not (1b), does not entail any lower bound: it is compatible with no students having attended. To be sure, (1a) may implicate that some student(s) attended, but, in line with Generalized Quantifier Theory (Barwise and Cooper, 1981), I do not take this to be part of the

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literal meaning of (1a). For example, *Every colloquium was attended by fewer than four students* is judged true even if some colloquia were attended by no students at all. That the lower-bound inference disappears in such environments is unsurprising if it is a kind of implicature (cf. *Every colloquium was attended by some of the students*, which is true even if some colloquia were attended all of the students).

The following is thus an appropriate representation of the meaning of (1a) (cf. Hackl, 2000; Nouwen, 2010; Kennedy, 2015).[2, 3]

(2) a. $\max(\lambda n\,.\,\exists x[\#x = n \wedge \mathbf{students}(x) \wedge \mathbf{attended}(x)]) < 4$
b. 'The maximum number of students who attended, if any, is less than 4.'

Let us call such a reading – one that conveys an upper bound – a MAXIMAL reading, and note that no other reading is available for (1a).

1.2. Non-maximal readings

Surprisingly, when *fewer than four* combines with (certain) non-distributive predicates, we do not get the same kind of maximal reading (Buccola, 2015b; Spector, 2014; Solt, 2007; Ben-Avi and Winter, 2003; Winter, 2001; Van der Does, 1992; Scha, 1981). For example, in (3a), a collective interpretation of *lifted the piano* is forced by *together*, and (3a) does *not* entail (3b): in a context where, say, three semantics students lifted the piano together, and seven phonology students lifted the piano together, (3a) is true, while (3b) is false.

(3) a. Fewer than four students lifted the piano together.
b. It is not the case that more than three students lifted the piano together.

This case is thus markedly different from that of (1a): if three semantics students attended, and seven phonology students attended, then (1a) is false despite the existence of a salient group of fewer than four attending students.

Moreover, (3a), unlike (1a), appears to entail a lower bound, viz. that some student(s) lifted the piano. This entailment explains why a sentence like *Fewer than four babies lifted the piano together* feels false (or, if true, then extremely surprising) in normal contexts. By contrast, *Fewer than four babies were smoking*, though admittedly odd (due to the implicature), nevertheless does not feel false in most normal contexts; for instance, it can be followed up with, *Yes, I agree, because no babies were smoking (thankfully).*

[2]Here, x ranges over sums of individuals (Link, 1983), and # is a function that maps a sum x to the cardinality of x, i.e. to that number n such that x has n atomic parts. There are, of course, other ways to represent this meaning.

[3]I use the terms *sum*, *plurality*, and *group* interchangeably, with no theoretical distinction between them.

In sum, (3a) simply means that a group of fewer than four students lifted the piano, which we may represent as follows.

(4) a. $\exists x[\#x < 4 \wedge \textbf{students}(x) \wedge \textbf{lifted}(x)]$
b. 'A group of fewer than four students lifted the piano.'

In a completely parallel way, in (5a), a cumulative interpretation of *drank more than twenty beers* is forced by the expression *between them*, and (5a) does *not* entail (5b): in a context where, say, three semantics students drank 21 beers between them, and all the students together drank 30 beers between them, (5a) is true, while (5b) is false.

(5) a. Fewer than four students drank more than twenty beers between them.
b. It is not the case that more than three students drank more than twenty beers bw. them.

And once again, (5a) entails a lower bound, viz. that at least some student(s) drank more than twenty beers between them. In sum, (5a) simply means that a group of fewer than four students drank more than twenty beers, which we again may represent as follows.[4]

(6) a. $\exists x[\#x < 4 \wedge \textbf{students}(x) \wedge \textbf{drank\_more\_than\_20\_beers}(x)]$
b. 'A group of fewer than four students drank more than twenty beers.'

Let us call such readings – ones that make a simple existential statement, without conveying any overall upper bound – NON-MAXIMAL (or EXISTENTIAL) readings.

1.3. Inadequate representations

Clearly, the non-maximal readings of (3a) and (5a) cannot be represented on analogy with the maximal reading of (1a), by using a maximality operator, as below, for these representations incorrectly predict that (3a) and (5a) should entail (3b) and (5b), respectively, and they incorrectly predict that (3a) and (5a) should not entail any lower bound.[5]

(7) a. $\max(\lambda n\,.\,\exists x[\#x = n \wedge \textbf{students}(x) \wedge \textbf{lifted}(x)]) < 4$
b. 'No group of more than three students lifted the piano.'

[4] I will say nothing more in this paper about cumulatively interpreted predicates. The point here is just to give another example of a non-maximal reading of a sentence involving *fewer than*. For more on cumulative interpretations of transitive predicates in this regard, see Buccola and Spector (2016).

[5] Whether or not (3a) and (5a) *also* have these maximal readings (i.e. are ambiguous between a maximal reading and a non-maximal one) is an open empirical question; see Buccola and Spector (2016) for discussion. The point here is that they clearly have non-maximal readings, which cannot be represented by a formula involving a (wide-scope) maximality operator.

(8) a. $\max(\lambda n\,.\,\exists x[\#x = n \wedge \mathbf{students}(x) \wedge \mathbf{drank\_more\_than\_20\_beers}(x)]) < 4$
b. 'No group of more than three students drank more than twenty beers.'

Conversely, the maximal reading of (1a) cannot be represented on analogy with the non-maximal readings of (3a) and (5a), viz. as a simple existential statement about groups, since this leads to what is known as Van Benthem's problem (Van Benthem, 1986): saying that a group of fewer than four students attended amounts to saying that at least one student attended.[6, 7]

(9) a. Fewer than four students attended.
b. $\exists x[\#x < 4 \wedge \mathbf{students}(x) \wedge \mathbf{attended}(x)]$
$\equiv \exists x[\mathbf{students}(x) \wedge \mathbf{attended}(x)]$
c. 'At least one student attended.'

1.4. Puzzle and roadmap

As the above discussion illustrates, the presence of maximality in sentences involving *fewer than* is, in some sense, variable, depending on the types of predicates that *fewer than n* combines with. The puzzle, then, is: What explains the variable presence of maximality? In the next section, I review two recent accounts of the puzzle. The first account, which I call 'Lexical Maximality' (LMax), proposes that *fewer than* lexically encodes reference to maximality, and that the variable presence of maximality is due to the variable scope of *fewer than* relative to covert existential quantification (Buccola, 2015b). The second account, which I call 'Separate Maximality' (SMax), proposes that the variable presence of maximality is due to the optional application of a maximization operation that is separate from the meaning of *fewer than* (Spector, 2014). I will show that, for the core data above, LMax and SMax are completely on a par: they generate exactly the same set of readings, up to truth-conditional equivalence. In section 3, I introduce data from the generic domain (generically interpreted sentences that contain *fewer than*), which I argue support SMax over LMax. Section 5 explores some extensions and predictions. Section 6 concludes.

2. Two theories

Both Buccola (2015b) and Spector (2014) propose that both maximal and non-maximal (existential) readings are grammatically generated across the board, and that certain 'weak' readings, in which

[6]Proof: ($\Rightarrow$) Let z be a group of fewer than four students who attended. Then, on the standard assumption that there is no empty group, it follows that one or more students attended. ($\Leftarrow$) Let z be a group of one or more students who attended. Then every atomic part of z is a student who attended, which means there is a group of fewer than four students who attended (namely, any atomic part of z).

[7]This equivalence relies on the assumption that a plural noun phrase like *students* may contain atomic individuals in its extension, but this assumption in unnecessary for the argument at hand: if the extension of *students* contains only plural individuals of cardinality 2 or more, then $\exists x[\#x < 4 \wedge \mathbf{students}(x) \wedge \mathbf{attended}(x)]$ is equivalent to $\exists x[\#x = 2 \wedge \mathbf{students}(x) \wedge \mathbf{attended}(x)]$, i.e. 'At least two students attended', which is just as inadequate.

the modified numeral makes no semantic contribution, are systematically ruled out by pragmatic blocking constraint, a version of which is provided in (10).

(10) **Pragmatic blocking constraint**
If an LF ϕ contains a numeral n, then for any numeral m distinct from n, substituting m for n in ϕ must yield different truth conditions.

The two theories differ mainly in their method of overgeneration, as explained shortly. For both theories, let us assume that a numeral n denotes a degree (number) but can also optionally be interpreted as an intersective adjective (see Landman, 2004), which I write as $n_{\texttt{isCard}}$.

(11) a. $[\![\text{one}]\!] = 1$, $[\![\text{two}]\!] = 2$, $[\![\text{three}]\!] = 3, \ldots$
b. $[\![n_{\texttt{isCard}}]\!] = \lambda x \,.\, \#x = [\![n]\!]$
c. $[\![n_{\texttt{isCard}}\ \text{students}]\!] = \lambda x \,.\, \#x = [\![n]\!] \wedge \textbf{students}(x)$

2.1. Lexical maximality and scope ambiguity

On the Lexical Maximality (LMax) approach of Buccola (2015b), a modified numeral like *fewer than four* denotes a generalized quantifier over degrees, which lexically encodes a maximality operator (cf. Heim, 2000; Hackl, 2000), and numerical indefinites like *fewer than four students* are headed by a silent existential determiner, $\varnothing_{\exists}$ (cf. Link, 1984, 1987; Krifka, 1999).

(12) $[\![\text{fewer than four}]\!] = \lambda P_{dt} \,.\, \max(P) < 4$

(13) $[\![\varnothing_{\exists}]\!] = \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \exists x[P(x) \wedge Q(x)]$

In an expression like (14), *fewer than four* is uninterpretable and must move. The basic insight of the LMax approach is that *fewer than four* may interact scopally with $\varnothing_{\exists}$ to derive the two kinds of readings (maximal and non-maximal) that we are interested in.

(14) $[_{\text{DP}}\ \varnothing_{\exists}\ [_{\text{NP}}\ [_{\text{AP}}\ \text{fewer than four}] \ldots]]\ [_{\text{VP}} \ldots]$

When *fewer than four* scopes above $\varnothing_{\exists}$, i.e. adjoins to S, we get a maximal reading.

(15) a. [fewer than four] [λn [[$\varnothing_{\exists}$ [$n_{\texttt{isCard}}$ NP]] VP]]
b. $\max(\lambda n \,.\, \exists x[\#x = n \wedge [\![\text{NP}]\!](x) \wedge [\![\text{VP}]\!](x)]) < 4$

This is precisely the reading we want for a sentence like (1a) with a distributive predicate.[8]

[8]Again, it is unclear whether maximal readings are also available for sentences with collectively interpreted predicates (see footnote 5). If they are not, then LMax and SMax both face a problem, since the constraint in (10) does not exclude

(16) a. [fewer than four] [λn [[$\varnothing_\exists$ [$n_{\texttt{isCard}}$ students]] attended]]
b. $\max(\lambda n\,.\,\exists x[\#x = n \wedge \textbf{students}(x) \wedge \textbf{attended}(x)]) < 4$
c. 'The total number of students who attended is less than 4.' ✓

When *fewer than four* scopes below $\varnothing_\exists$, i.e. quantifies into AP (shown below) or (equivalently, but not shown) into NP (Heim and Kratzer, 1998), we get a non-maximal, i.e. existential, reading.

(17) a. [$\varnothing_\exists$ [[$_{\text{AP}}$ λx [[fewer than four] [λn [x $n_{\texttt{isCard}}$]]]] NP]] VP
b. $\exists x[\max(\lambda n\,.\,\#x = n) < 4 \wedge [\![\text{NP}]\!](x) \wedge [\![\text{VP}]\!](x)]$
$\equiv \exists x[\#x < 4 \wedge [\![\text{NP}]\!](x) \wedge [\![\text{VP}]\!](x)]$

For a sentence like (1a) with a distributive predicate, this reading is blocked by the constraint in (10) because replacing *four* by any other numeral yields the same weak reading, following the logic of Van Benthem's problem.

(18) a. [$\varnothing_\exists$ [[λx [[fewer than four] [λn [x $n_{\texttt{isCard}}$]]]] students]] attended
b. $\exists x[\#x < 4 \wedge \textbf{students}(x) \wedge \textbf{attended}(x)]$
$\equiv \exists x[\#x < 3 \wedge \textbf{students}(x) \wedge \textbf{attended}(x)]$
$\equiv \exists x[\#x < 5 \wedge \textbf{students}(x) \wedge \textbf{attended}(x)]$
$\equiv \exists x[\textbf{students}(x) \wedge \textbf{attended}(x)]$
c. 'A group of fewer than four students attended.'
$\rightsquigarrow$ 'At least one student attended.' BLOCKED

Crucially, for a sentence like (3a) with a non-distributive predicate, the non-maximal reading is correctly *not* blocked: replacing *four* by *three* yields a stronger meaning, and replacing *four* by *five* yields a weaker meaning.

(19) a. [$\varnothing_\exists$ [[λx [[fewer than four] [λn [x $n_{\texttt{isCard}}$]]]] students]] [lifted the piano together]
b. $\exists x[\#x < 4 \wedge \textbf{students}(x) \wedge \textbf{lifted}(x)]$
$\not\equiv \exists x[\#x < 3 \wedge \textbf{students}(x) \wedge \textbf{lifted}(x)]$
$\not\equiv \exists x[\#x < 5 \wedge \textbf{students}(x) \wedge \textbf{lifted}(x)]$
$\not\equiv \exists x[\textbf{students}(x) \wedge \textbf{lifted}(x)]$
c. 'A group of fewer than four students lifted the piano.' ✓

On the LMax approach, the 'absence' of maximality here is due to the fact in (20), which in turn is due to the fact that every plural individual has exactly one cardinality.

(20) **Fact.** For all individuals x, $\max(\lambda n\,.\,\#x = n) = \#x$.

them.

2.2. Separate and optional maximality

The Separate Maximality (SMax) approach of Spector (2014) posits that a separate and optional maximization operation is responsible for the maximal readings we (sometimes) perceive with *fewer than n* as well as with bare numerals (cf. Kennedy, 2013, 2015). Specifically, maximality is taken to be part of the meanings of numerals and numerical traces, so that a numeral n may not only be interpreted (as before) as a degree or as an intersective adjective ($n_{\texttt{isCard}}$), but also as a generalized quantifier over degrees, notated here by $n_{\texttt{isMax}}$, which denotes the set of all degree predicates whose maximum is equal to n.

(21) $[\![n_{\texttt{isMax}}]\!] = \lambda P_{dt} \,.\, \max(P) = [\![n]\!]$

This approach allows us to derive both maximal (or ‘exactly’, or ‘two-sided’) and non-maximal (or ‘at least’, or ‘one-sided’) readings of bare numerals (in distributive contexts), depending on whether the numeral n is interpreted as $n_{\texttt{isMax}}$ or as $n_{\texttt{isCard}}$, respectively (see Spector 2013 for a survey of the issues). For example, (22) receives two different parses, depending on how *three* is interpreted.

(22) Three students attended.

(23)
a. [$\varnothing_\exists$ [three$_{\texttt{isCard}}$ students]] attended
b. $\exists x[\#x = 3 \wedge \textbf{students}(x) \wedge \textbf{attended}(x)]$
c. ‘At least three students attended.’

(24)
a. three$_{\texttt{isMax}}$ [λn [[$\varnothing_\exists$ [$n_{\texttt{isCard}}$ students]] attended]]
b. $\max(\lambda n \,.\, \exists x[\#x = n \wedge \textbf{students}(x) \wedge \textbf{attended}(x)]) = 3$
c. ‘Exactly three students attended.’

Since maximality is considered a separate component (part of the meaning of numerals and numerical traces), *fewer than four* simply makes an existential statement about degrees.

(25) $[\![\text{fewer than four}]\!] = \lambda P_{dt} \,.\, \exists n[n < 4 \wedge P(n)]$

For the moment, let us continue to assume that, even on the SMax approach, existential force is contributed by the silent determiner $\varnothing_\exists$ (we return to this assumption in section 5.2). Then, for a sentence of the form *Fewer than four NP VP*, there are four LFs to consider, depending on (i) the relative scope of *fewer than four* and $\varnothing_\exists$, and (ii) whether or not maximization applies. When *fewer than four* scopes above $\varnothing_\exists$, and its numerical trace n is *not* interpreted as $n_{\texttt{isMax}}$, we get a non-maximal (existential) reading. Just like for LMax, this reading is appropriate (and not blocked) for a sentence like (3a) with a non-distributive predicate, and for a sentence like (1a) with a distributive predicate, this reading is blocked by the constraint in (10).

(26) a. [fewer than four] [λn [[$\varnothing_\exists$ [$n_{\texttt{isCard}}$ NP]] VP]]

b. $\exists n[n < 4 \wedge \exists x[\#x = n \wedge [\![\mathrm{NP}]\!](x) \wedge [\![\mathrm{VP}]\!](x)]]$
$\equiv \exists x[\#x < 4 \wedge [\![\mathrm{NP}]\!](x) \wedge [\![\mathrm{VP}]\!](x)]$

When *fewer than four* scopes above $\varnothing_\exists$, and its numerical trace *n is* interpreted as $n_{\texttt{isMax}}$ (which itself must also move), then we get a maximal reading, which is appropriate for a sentence like (1a) with a distributive predicate.

(27) a. [fewer than four] [λn [$n_{\texttt{isMax}}$ [λm [[$\varnothing_\exists$ [$m_{\texttt{isCard}}$ NP]] VP]]]]

b. $\exists n[n < 4 \wedge \max(\lambda m\,.\,\exists x[\#x = m \wedge [\![\mathrm{NP}]\!](x) \wedge [\![\mathrm{VP}]\!](x)]) = n]$
$\equiv \max(\lambda m\,.\,\exists x[\#x = m \wedge [\![\mathrm{NP}]\!](x) \wedge [\![\mathrm{VP}]\!](x)]) < 4$

SMax therefore need not rely on scope ambiguity to generate both types of readings (maximal and non-maximal). Moreover, it turns out that allowing *fewer than four* to scope below $\varnothing_\exists$, with or without maximization, does not generate any new readings. Specifically, when *fewer than four* scopes below $\varnothing_\exists$, and maximization does *not* apply, we get the non-maximal (existential) reading.

(28) a. [$\varnothing_\exists$ [[$_{\mathrm{AP}}$ λx [[fewer than four] [λn [x $n_{\texttt{isCard}}$]]]] NP]] VP

b. $\exists x[\exists n[n < 4 \wedge \#x = n] \wedge [\![\mathrm{NP}]\!](x) \wedge [\![\mathrm{VP}]\!](x)]$
$\equiv \exists x[\#x < 4 \wedge [\![\mathrm{NP}]\!](x) \wedge [\![\mathrm{VP}]\!](x)]$

And when *fewer than four* scopes below $\varnothing_\exists$, and maximization *does* apply, it applies vacuously (cf. (20)), and we again get the non-maximal (existential) reading.

(29) a. [$\varnothing_\exists$ [[$_{\mathrm{AP}}$ λx [[fewer than four] [λn [$n_{\texttt{isMax}}$ [λm [x $m_{\texttt{isCard}}$]]]]]] NP]] VP

b. $\exists x[\exists n[n < 4 \wedge \max(\lambda m\,.\,\#x = m) = n] \wedge [\![\mathrm{NP}]\!](x) \wedge [\![\mathrm{VP}]\!](x)]$
$\equiv \exists x[\exists n[n < 4 \wedge \#x = n] \wedge [\![\mathrm{NP}]\!](x) \wedge [\![\mathrm{VP}]\!](x)]$
$\equiv \exists x[\#x < 4 \wedge [\![\mathrm{NP}]\!](x) \wedge [\![\mathrm{VP}]\!](x)]$

Importantly, although SMax generates a greater number of LFs than LMax does, it generates exactly the same set of readings for the sentences discussed so far. The reason is that the existential quantifier (over individuals) of $\varnothing_\exists$ and the existential quantifier (over degrees) of *fewer than four* commute. As a result, (26) and (28) are equivalent. Moreover, given fact (20), it follows that (28) and (29) are equivalent. Thus, (26), (28), and (29) are all equivalent, and are in turn equivalent to LMax's (17). If, however, we can find a case where commutativity between *fewer than four* and the null determiner does not arise, then we might be able to distinguish SMax from LMax.

3. Evidence from genericity

We now move to generic (more specifically, characterizing) sentences, which involve a kind of quasi-universal force, rather than existential force. We will see that the commutativity observed above in the existential domain does not arise for quasi-universally interpreted sentences containing *fewer than n*. As a result, SMax will turn out to generate a reading which LMax cannot, and which, I claim, is indeed the salient reading we want to capture. I start with a very simple description and theory of basic characterizing sentences, followed by an extension to characterizing sentences with bare numerals, and finally to characterizing sentences with numerals modified by *fewer than*.

3.1. Basic characterizing sentences

Sentence (30) is a CHARACTERIZING, or generalizing, or simply generic, sentence (Krifka et al., 1995): it expresses a generalization of some kind (in this case, about cats).

(30) Cats have fur.

In particular, (30) means something like 'Any/every typical cat has fur'. I will represent this reading as follows, where '$\forall_{\text{Gen}}$' should be understood as a kind of restricted universal quantifier, which quantifies over all 'typical' individuals (of some sort or other).[9]

(31) $\forall_{\text{Gen}} x[\mathbf{cats}(x) \rightarrow \mathbf{have\_fur}(x)]$

Note that, since *cats* and *have fur* both have distributive reference, the postulated meaning ('Any (typical) group of cats has the property of having fur') entails that any (typical) individual cat has fur, as desired. We can capture this reading on analogy with existential indefinites simply by positing a silent generic determiner, $\varnothing_{\text{Gen}}$, as shown below.[10]

(32) $[\![\varnothing_{\text{Gen}}]\!] = \lambda P_{et} \,.\, \lambda Q_{et} \,.\, \forall_{\text{Gen}} x[P(x) \rightarrow Q(x)]$

(33) a. [$\varnothing_{\text{Gen}}$ cats] [have fur]
b. $\forall_{\text{Gen}} x[\mathbf{cats}(x) \rightarrow \mathbf{have\_fur}(x)]$

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[9] I leave open the question of how exactly $\forall_{\text{Gen}}$ is interpreted, and in particular what it means for an individual to be 'typical'. See Krifka et al. (1995) for a survey of a number of proposals, any one of which could be employed here. The exact treatment of $\forall_{\text{Gen}}$, in particular how exceptions are allowed for, is an important issue in the semantics of genericity, but is, as far as I can tell, not very important for the analysis of *fewer than* (though see footnotes 13 and 15). As I hope the reader will see, all that seems to matter for the issues at hand is that $\forall_{\text{Gen}}$ is non-existential.

[10] The generic operator should probably really be a sentential operator. The choice of making it a quantificational determiner is simply to make the discussion here as close as possible to that of existential numerical indefinites, where I posited a silent existential determiner, $\varnothing_{\exists}$. As far as I can tell, everything I will say is compatible with a sentential generic operator, provided that quantifiers like *fewer than four* may scope above the operator (see section 4.2).

3.2. Characterizing sentences with bare numerals

Characterizing sentences with bare numerals appear to work exactly as expected: (34), taken from Link (1987), expresses the generalization that any typical group of three men can lift the piano.

(34) Three men can lift the piano. (Link, 1987)

This reading falls out naturally from the adjectival analysis of numerals and the null determiner analysis of genericity developed so far.[11]

(35) a. [$\varnothing_{\text{Gen}}$ [three$_{\texttt{isCard}}$ men]] [can lift the piano]
b. $\forall_{\text{Gen}} x[[\#x = 3 \wedge \mathbf{men}(x)] \rightarrow \mathbf{can\_lift}(x)]$

As an important side remark, the reader may wonder whether the characterizing reading of (34) should instead be represented by an LF where the modal *can* takes wide scope and the propositional argument of the modal has existential force, as shown below.

(36) a. can [[$\varnothing_{\exists}$ [three$_{\texttt{isCard}}$ men]] [lift the piano]]
b. $\Diamond \exists x[\#x = 3 \wedge \mathbf{men}(x) \wedge \mathbf{lift}(x)]$

This interpretation can be paraphrased as 'There is an accessible world *w* such that, in *w*, there is a group of three men who lift the piano'. I have three arguments against such an analysis.

First, and perhaps most importantly, the interpretation derived on this analysis is too weak to represent the quasi-universality of the characterizing reading of (34). On its characterizing reading, (34) entails that if, say, Al, Bill, and Carl are three men (of typical/average strength), then Al, Bill, and Carl can lift the piano. By contrast, (36) simply says that in *some* accessible world, there is *some* group of three men (perhaps three extraordinarily strong men, i.e. not necessarily Al, Bill, and Carl, who are only of average strength) who lift the piano.

Second, observe that weak negative polarity items (NPIs) are licensed inside of generically interpreted numerical nominals, as (37) illustrates. This observation is fully expected on the analysis in (35), because there *any* occurs in a downward-entailing environment, just like the restrictor of *every* (Ladusaw, 1979); however, it is not expected on the analysis in (36), because there *any* occurs in an upward-entailing environment.

(37) Three men with any experience in the moving business can lift the piano.

[11]This analysis is precisely the one that Link (1987) gives, too. However, his main concern in that paper is not the precise analysis of numerals or of genericity, but rather the search for what he calls 'genuine' (as opposed to 'spurious') plural quantification in natural language, an example of which is (34). He does not discuss characterizing sentences containing modified numerals.

Finally, note that (38a) is perfectly acceptable and interpretable, and presumably has a representation like (36) (with *can* replaced by *possible*). Crucially, (38a) is intuitively weaker than (34) (on its generic reading). Moreover, it fails to license weak NPIs. That (34) and (38a) differ intuitively both in meaning and in NPI licensing is strong evidence for different representations, along the lines of (35) and (36), respectively.

(38) a. It is possible for three men to lift the piano.
b. It is possible for three men with {some / #any} experience in the moving business to lift the piano.

In sum, while (36) may represent one reading of (34), it does not represent what I, and Link (1987), take to be the characterizing reading of (34), which henceforth I assume involves wide-scope $\varnothing_{\text{Gen}}$, as in (35).[12]

3.3. Characterizing sentences with modified numerals

Finally, consider a characterizing sentence with *fewer than four*, modeled on (34).

(39) Fewer than four men can lift the piano.

Sentence (39) appears to have available the reading given in (40).

(40) a. $\exists n[n < 4 \wedge \forall_{\text{Gen}} x[[\#x = n \wedge \mathbf{men}(x)] \rightarrow \mathbf{can\_lift}(x)]]$
b. 'There is a number $n < 4$ such that, in general, any group of n men can lift the piano.'

This reading is most natural in a kind of 'speaker ignorance' context: it can be brought out by prepending to (39) something like *I'm not sure exactly how many men it takes, but I'm certain that* . . . . However, it is also natural in a dialog like the following.

(41) A: We'd like to buy the piano, but we are only four people. Will we be able to lift it?
B: Absolutely. In fact, fewer than four people can lift the piano.

Notice once again the lack of any reference to maximality in the representation in (40), despite the presence of *fewer than*. This reading is thus another kind of non-maximal reading, but let us refer more specifically to this reading as an INTERMEDIATE generic reading, for reasons that will soon become clear. As we will see, only SMax can generate this reading, namely by scoping *fewer than four* above $\varnothing_{\text{Gen}}$, and without applying maximization.[13]

[12] For further discussion of these issues, see Buccola (2015a).

[13] We should ask ourselves whether (39) entails any lower bound or not. Recall from section 1 that in distributive existential cases, e.g. (1a) (*Fewer than four students attended*), there is no lower-bound entailment – only an implicature

4. Two theories revisited

We now revisit the LMax and SMax accounts to see what readings they predict for (39).

4.1. Lexical maximality revisited

Just as in the existential domain, the LMax account has two scope possibilities available, depending on the relative scope of *fewer than four* and $\varnothing_{\mathrm{Gen}}$. When *fewer than four* scopes above $\varnothing_{\mathrm{Gen}}$, we get what I will call a MAXIMAL generic reading.

(42) a. [fewer than four] [λn [[$\varnothing_{\mathrm{Gen}}$ [$n_{\mathtt{isCard}}$ NP]] VP]]
b. $\max(\lambda n \,.\, \forall_{\mathrm{Gen}} x[[\#x = n \wedge [\![\mathrm{NP}]\!](x)] \rightarrow [\![\mathrm{VP}]\!]) < 4$

And when *fewer than four* scopes below $\varnothing_{\mathrm{Gen}}$, we get what I will call a STRONG UNIVERSAL generic reading.

(43) a. [$\varnothing_{\mathrm{Gen}}$ [[λx [fewer than four] [λn [x $n_{\mathtt{isCard}}$]]] NP]] VP
b. $\forall_{\mathrm{Gen}} x[[\max(\lambda n \,.\, \#x = n) < 4 \wedge [\![\mathrm{NP}]\!](x)] \rightarrow [\![\mathrm{VP}]\!](x)]$
$\equiv \forall_{\mathrm{Gen}} x[[\#x < 4 \wedge [\![\mathrm{NP}]\!](x)] \rightarrow [\![\mathrm{VP}]\!](x)]$

Turning now to (39), the problem is that neither scope possibility yields the right reading; that is, neither the maximal reading nor the strong universal reading corresponds to the intermediate generic reading in (40). The maximal reading, for example, entails that it is not the case that, say, (any group of) four men can lift the piano, whereas the reading in (40) does not.

(44) a. [fewer than four] [λn [[$\varnothing_{\mathrm{Gen}}$ [$n_{\mathtt{isCard}}$ men]] [can lift the piano]]]
b. $\max(\lambda n \,.\, \forall_{\mathrm{Gen}} x[[\#x = n \wedge \mathbf{men}(x)] \rightarrow \mathbf{can\_lift}(x)]) < 4$
c. 'The maximum number *n* such that, in general, any group of *n* men can lift the piano is less than 4.'

– whereas in non-distributive existential cases, e.g. (3a) (*Fewer than four students lifted the piano together*) and (5a) (*Fewer than four students drank more than twenty beers between them*), there is a lower-bound entailment. It seems to me that (39) *does* entail that there is some *non-zero* number n such that, in general, n people can lift the piano. For example, *Fewer than four babies can lift the piano* feels false (or, if true, then extremely surprising) in most normal contexts, just like *Fewer than four babies lifted the piano* does. Similarly, *Every piano can be lifted by fewer than four men* feels false if some pianos cannot be lifted at all. It is unclear whether the representation I have given in (40) predicts this lower-bound inference. If $\forall_{\mathrm{Gen}}$ is interpreted similarly to the standard quantifier $\forall$, then it would seem not to: choosing $n = 0$ renders the formula true, since the restrictor of $\forall_{\mathrm{Gen}}$ is false for all x (there is no x such that $\#x = 0$). My hope, however, is that a more sophisticated theory of genericity can handle this edge case. See Buccola (2015a) for further discussion, as well as footnote 15.

And the strong universal reading is strictly stronger than (40): it entails that (any group of) three men, two men, and even one man can lift the piano, whereas (40) does not.[14]

(45) a. $[\varnothing_{\text{Gen}}\ [[\lambda x\ [[\text{fewer than four}]\ [\lambda n\ [x\ n_{\texttt{isCard}}]]]]\ \text{men}]]\ [\text{can lift the piano}]$
b. $\forall_{\text{Gen}} x[[\#x < 4 \wedge \textbf{men}(x)] \rightarrow \textbf{can\_lift}(x)]$
c. 'In general, any group of fewer than four men can lift the piano.'

I will address the question of whether maximal generic readings and strong universal generic readings are (ever) available in section 5. The point for now is that LMax is unable to generate the intermediate generic reading in (40), which I claimed to be an available reading of (39).

4.2. Separate maximality revisited

Just as in the existential domain, the SMax account has four possibilities, depending on (i) the relative scope of *fewer than four* and $\varnothing_{\text{Gen}}$, and (ii) whether or not maximization applies. When *fewer than four* scopes above $\varnothing_{\text{Gen}}$, and maximization does *not* apply, then we get the non-maximal, intermediate reading that we want for a sentence like (39).

(46) a. $[\text{fewer than four}]\ [\lambda n\ [[\varnothing_{\text{Gen}}\ [n_{\texttt{isCard}}\ \text{NP}]]\ \text{VP}]]$
b. $\exists n[n < 4 \wedge \forall_{\text{Gen}} x[[\#x = n \wedge [\![\text{NP}]\!]] \rightarrow [\![\text{VP}]\!](x)]]$

(47) a. $[\text{fewer than four}]\ [\lambda n\ [[\varnothing_{\text{Gen}}\ [n_{\texttt{isCard}}\ \text{men}]]\ [\text{can lift the piano}]]]$
b. $\exists n[n < 4 \wedge \forall_{\text{Gen}} x[[\#x = n \wedge \textbf{men}(x)] \rightarrow \textbf{can\_lift}(x)]]$
c. 'There is a number $n < 4$ such that, in general, any group of n men can lift the piano.'

When *fewer than four* scopes above $\varnothing_{\text{Gen}}$, and maximization *does* apply, then we get a maximal reading. This is the same maximal reading that LMax derives when *fewer than four* scopes above $\varnothing_{\text{Gen}}$, i.e. (42).

(48) a. $[\text{fewer than four}]\ [\lambda n\ [n_{\texttt{isMax}}\ [\lambda m\ [[\varnothing_{\text{Gen}}\ [m_{\texttt{isCard}}\ \text{NP}]]\ \text{VP}]]]]$
b. $\exists n[n < 4 \wedge \max(\lambda m\,.\,\forall_{\text{Gen}} x[[\#x = m \wedge [\![\text{NP}]\!](x)] \rightarrow [\![\text{VP}]\!](x)]) = n]$
$\equiv \max(\lambda m\,.\,\forall_{\text{Gen}} x[[\#x = m \wedge [\![\text{NP}]\!](x)] \rightarrow [\![\text{VP}]\!](x)]) < 4$

And when *fewer than four* scopes below $\varnothing_{\text{Gen}}$, then we get a strong universal reading, regardless of whether maximization applies, given once again the fact in (20). This is the same universal reading that LMax derives when *fewer than four* scopes below $\varnothing_{\text{Gen}}$, i.e. (43).

(49) a. $[\varnothing_{\text{Gen}}\ [[\lambda x\ [[\text{fewer than four}]\ [\lambda n\ [x\ n_{\texttt{isCard}}]]]]\ \text{NP}]]\ \text{VP}$

[14] I assume here that plural expressions like *men* contain both atomic and non-atomic individuals in their extension, but this assumption is not crucial: even without it, the derived reading is strictly stronger than (40) (cf. footnote 7).

b. $\quad \forall_{\text{Gen}} x[[\exists n[n < 4 \wedge \#x = n] \wedge [\![\text{NP}]\!](x)] \rightarrow [\![\text{VP}]\!]]$
$\equiv \forall_{\text{Gen}} x[[\#x < 4 \wedge [\![\text{NP}]\!](x)] \rightarrow [\![\text{VP}]\!](x)]$

(50) a. $[\varnothing_{\text{Gen}}$ [[λx [[fewer than four] [λn [$n_{\texttt{isMax}}$ [λm [x $m_{\texttt{isCard}}$]]]]]] NP]] VP
b. $\quad \forall_{\text{Gen}} x[[\exists n[n < 4 \wedge \max(\lambda m\,.\,\#x = m) = n] \wedge [\![\text{NP}]\!](x)] \rightarrow [\![\text{VP}]\!](x)]$
$\equiv \forall_{\text{Gen}} x[[\exists n[n < 4 \wedge \#x = n] \wedge [\![\text{NP}]\!](x)] \rightarrow [\![\text{VP}]\!]]$
$\equiv \forall_{\text{Gen}} x[[\#x < 4 \wedge [\![\text{NP}]\!](x)] \rightarrow [\![\text{VP}]\!](x)]$

Crucially, (46) (the intermediate reading) is not equivalent to (49) (hence, nor to (50)) because the existential degree quantifier of *fewer than four* and the quasi-universal individual quantifier of $\varnothing_{\text{Gen}}$ do not commute. As a result, in the generic domain, SMax generates a reading that LMax does not, which also happens to be the salient reading of (39).

5. Extending the blocking account

5.1. Non-intermediate readings with *can lift the piano*

An immediate question that arises is: Are the two non-intermediate generic readings (i.e. the maximal reading and the strong universal reading) intuitively available for (39)? My answer is that these readings are intuitively *unavailable* to the extent that the inference in (51) is intuitively *valid*. It seems plausible to me that (51) is valid: the more men, the easier it is for them to lift the piano. If so, then, as I will now show, (the LFs corresponding to) the two non-intermediate readings are blocked by the pragmatic blocking constraint in (10), hence are expected to be unavailable.

(51) If n men can lift the piano, then so can $n + 1$ (for any $n \neq 0$).

5.1.1. The maximal generic reading

For the maximal generic reading, given below, the set to which max applies is either empty, or it is non-empty, hence by (51) has no maximum. For the sentence to be true, then, it must be that there is no number n such that n men can lift the piano (otherwise, we get maximality failure). We derive this same reading if *four* is replaced, say, by *three* or by *five*; thus, it is blocked by the constraint in (10).

(52) $\quad \max(\lambda n\,.\,\forall_{\text{Gen}} x[[\#x = n \wedge \textbf{men}(x)] \rightarrow \textbf{can\_lift}(x)]) < 4$
$\equiv \max(\lambda n\,.\,\forall_{\text{Gen}} x[[\#x = n \wedge \textbf{men}(x)] \rightarrow \textbf{can\_lift}(x)]) < 3$
$\equiv \max(\lambda n\,.\,\forall_{\text{Gen}} x[[\#x = n \wedge \textbf{men}(x)] \rightarrow \textbf{can\_lift}(x)]) < 5$
$\equiv [\lambda n\,.\,\forall_{\text{Gen}} x[[\#x = n \wedge \textbf{men}(x)] \rightarrow \textbf{can\_lift}(x)]] = \varnothing$ BLOCKED

5.1.2. The strong universal generic reading

For the strong universal generic reading, given below, the validity of (51) leads to an even stronger reading: any group of men of *any* cardinality can lift the piano. This same reading is derived if *four* is replaced, say, by *three* or by *five*, hence is blocked.

(53)
$$\begin{aligned} &\forall_{\text{Gen}} x[[\#x < 4 \wedge \mathbf{men}(x)] \rightarrow \mathbf{can\_lift}(x)] \\ \equiv\ &\forall_{\text{Gen}} x[[\#x < 3 \wedge \mathbf{men}(x)] \rightarrow \mathbf{can\_lift}(x)] \\ \equiv\ &\forall_{\text{Gen}} x[[\#x < 5 \wedge \mathbf{men}(x)] \rightarrow \mathbf{can\_lift}(x)] \\ \equiv\ &\forall_{\text{Gen}} x[\mathbf{men}(x) \rightarrow \mathbf{can\_lift}(x)] \end{aligned}$$
BLOCKED

5.2. A prediction: *can fit into the elevator*

The view espoused above leads to the following prediction: if the assumption in (51) is invalid, then maximal and strong universal generic readings should be available. Again, it is plausible to me that (51) is valid, but if we move instead to a predicate like *can fit into the elevator*, then the analogous inference is clearly invalid: if n people can fit into the elevator, then it is not necessarily the case that $n + 1$ people can fit into the elevator. In fact, the opposite inference (viz. that $n - 1$ people can fit), given in (54), is valid.

(54) If n people can fit into the elevator, then so can $n - 1$ (for any $n > 1$).

As a result, a blocking account predicts that a sentence like (55) should have both a maximal generic reading and a strong universal reading (and no intermediate generic reading), as I now show.

(55) Fewer four people can fit into the elevator.

5.2.1. The maximal generic reading

Assuming that (54) is intuitively valid, then the maximal reading of (55), given below, does not give rise to any maximality failure, hence is not blocked by the constraint in (10). Indeed, the most salient reading of (55) is the predicted maximal reading, which states that the maximum number of people who can fit into the elevator is less than 4.[15]

[15] Note that, just like maximal reading of a distributive existential sentence like (1a) (*Fewer than four students attended*), the maximal generic reading of (55) is intuitively compatible with no people being able to fit. To the extent that (55) implies that at least one person can fit, I take this to be a pragmatic inference analogous to inference we normally draw from (1a) that at least some student(s) attended. Importantly, however, it is not at all clear how to reconcile this observation with the observation that the intermediate generic reading of (39) (*Fewer than four men can lift the piano*) is *not* compatible with no men being able to lift the piano (see footnote 13). See Buccola (2015a) for further discussion.

(56) $\quad \max(\lambda n\,.\,\forall_{\text{Gen}}x[[\#x = n \wedge \textbf{people}(x)] \rightarrow \textbf{can\_fit}(x)]) < 4$
$\not\equiv \max(\lambda n\,.\,\forall_{\text{Gen}}x[[\#x = n \wedge \textbf{people}(x)] \rightarrow \textbf{can\_fit}(x)]) < 3$
$\not\equiv \max(\lambda n\,.\,\forall_{\text{Gen}}x[[\#x = n \wedge \textbf{people}(x)] \rightarrow \textbf{can\_fit}(x)]) < 5$ ✓

5.2.2. The strong universal generic reading

In addition, the strong universal generic reading is not blocked: replacing *four* with *three* yields a weaker meaning, and replacing *four* with *five* yields a stronger meaning.

(57) $\quad \forall_{\text{Gen}}x[[\#x < 4 \wedge \textbf{people}(x)] \rightarrow \textbf{can\_fit}(x)]$
$\not\equiv \forall_{\text{Gen}}x[[\#x < 3 \wedge \textbf{people}(x)] \rightarrow \textbf{can\_fit}(x)]$
$\not\equiv \forall_{\text{Gen}}x[[\#x < 5 \wedge \textbf{people}(x)] \rightarrow \textbf{can\_fit}(x)]$??/NOT BLOCKED

It is unclear to me whether this reading is indeed available. Note that it is also compatible with (any group of) four, five, . . . people being able to fit. If this reading is unavailable, then the SMax account could probably be modified so that *fewer than four* never scopes below $\varnothing_{\text{Gen}}$ (or $\varnothing_{\exists}$[16]). One way to achieve this would be to replace $\varnothing_{\exists}$ and $\varnothing_{\text{Gen}}$ by the silent counting quantifiers $\langle many_{\exists}\rangle$ and $\langle many_{\text{Gen}}\rangle$ below, inspired by Hackl (2000).

(58) a. $[\![\langle \text{many}_{\exists}\rangle]\!] = \lambda n_d\,.\,\lambda P_{et}\,.\,\lambda Q_{et}\,.\,\exists x[\#x = n \wedge P(x) \wedge Q(x)]$
b. $[\![\langle \text{many}_{\text{Gen}}\rangle]\!] = \lambda n_d\,.\,\lambda P_{et}\,.\,\lambda Q_{et}\,.\,\forall_{\text{Gen}}x[[\#x = n \wedge P(x)] \rightarrow Q(x)]$

The idea here is that, since $\langle many_{\exists}\rangle$ and $\langle many_{\text{Gen}}\rangle$ must first combine with a degree-denoting expression (a numeral or numerical trace), it is impossible for *fewer than four* to ever scope below them, i.e. to quantify into AP (or NP). More precisely, while an LF like (59a) is interpretable, an LF like (59b) is uninterpretable: $\langle many\rangle$ (which here stands ambiguously for $\langle many_{\exists}\rangle$ or $\langle many_{\text{Gen}}\rangle$) requires an argument of type d, but is instead combining with an expression of type et, namely the intersection of the numerical AP and the NP.

(59) a. [fewer than four] [λn [[[n ⟨many⟩] NP] VP]]
b. [⟨many⟩ [[$_{\text{AP}}$ λx [[fewer than four] [λn [x $n_{\texttt{isCard}}$]]]] NP]] VP

5.2.3. The intermediate generic reading

Finally, (55) is predicted not to have a non-maximal, intermediate generic reading: in this case, because of (54), we get an extremely weak reading, which simply states that some number of people

[16]Recall that SMax does not need to rely on scope ambiguity to generate both maximal and non-maximal readings in the existential domain: scoping *fewer than four* below $\varnothing_{\exists}$ yields the same non-maximal, existential reading as scoping *fewer than four* above $\varnothing_{\exists}$ does, without applying maximization, given the commutativity of the existential degree quantifier of *fewer than four* and the existential individual quantifier of $\varnothing_{\exists}$.

can fit into the elevator; the numeral *four* makes no semantic contribution. The logic of this result is exactly the same as that of Van Benthem's problem in the existential domain for distributive predicates, except that here the downward inferences that lead to the result are due to (54) rather than to distributivity.[17] This appears to be a welcome result, as (55) does not seem to have this weak reading.

(60)
$$\exists n[n < 4 \wedge \forall_{\text{Gen}} x[[\#x = n \wedge \textbf{people}(x)] \rightarrow \textbf{can\_fit}(x)]]$$
$$\equiv \exists n[n < 3 \wedge \forall_{\text{Gen}} x[[\#x = n \wedge \textbf{people}(x)] \rightarrow \textbf{can\_fit}(x)]]$$
$$\equiv \exists n[n < 5 \wedge \forall_{\text{Gen}} x[[\#x = n \wedge \textbf{people}(x)] \rightarrow \textbf{can\_fit}(x)]] \qquad \text{BLOCKED}$$

6. Conclusion

I've presented evidence from genericity suggesting that maximality should be separate from the meaning of *fewer than*. That is, reliance on scope ambiguity (LMax) is not enough to generate the range of attested readings of sentences involving *fewer than*. On this view, *fewer than four* is not really a downward-entailing operator. Rather, downward-entailing environments are created under very specific (albeit very common) conditions: namely, when *fewer than four* takes wide scope and its numerical trace n is interpreted as $n_{\texttt{isMax}}$. Moreover, the application of maximality is regulated by a pragmatic constraint that is sensitive to the types of inferences that predicates allow. This explains why the availability of maximal vs. non-maximal readings is only partially related to whether we are in an existential vs. generic context, or whether we have a distributive vs. non-distributive predicate. Finally, we discover that Van Benthem's problem is more pervasive than we once thought, and that the (extremely weak) readings that it gives rise to always seem to be inaccessible.

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[17] Of course, the intuitive validity of (54) probably has something to do with the distributivity of *fit into the elevator*.

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