Ignorance and anti-negativity in the grammar:
***Or*, *some* NP_{SG}, and modified numerals**[*]

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1. Introduction

Given reference to the same domain of individuals, the English disjunction *or* and the English singular indefinite *some* NP_{SG} are truth-conditionally equivalent, (1). However, these items differ in surprising ways with respect to ignorance and polarity sensitivity: While both are able to give rise to a total ignorance effect, (2), only *some* NP_{SG} is also compatible with negative or positive certainty about an element of the domain—what we may call a 'one loser' or 'one winner' scenario—(3-4). And *or* can take scope under negation but *some* NP_{SG} can't, (5), although both are fine in other downward-entailing environments such as the first argument of a conditional/universal, (6-7). Strikingly, comparative-modified numerals (CMNs; e.g., *less than 3*) and superlative-modified numerals (SMNs; e.g., *at most 2*) exhibit the exact same type of patterns, (1'-7'), just that the effects are crossed—with respect to compatibility with certainty CMNs are like *some* NP_{SG} and SMNs like *or*, while with respect to anti-negativity CMNs are like *or* and SMNs like *some* NP_{SG}.

(1) Jo called Alice or Bob / some student$_{\{\text{Alice, Bob}\}}$.[1] (= 1 iff $a \vee b$)

(2) (Who did Jo call?) Alice or Bob / some student. ⇝ (ignorance!)

(3) Jo called # Alice, Bob, or Cindy / ✓some student, but not Alice.

(1') Jo called less than 2 people / at most 1 person. (= 1 iff $0 \vee 1$)

(2') (How many people did Jo call?) Less than 2 / at most 1. (⇝ ignorance!)

(3') Jo called ✓less than 3 / # at most 2 people, but not 1.

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[1]For speakers who feel that *some* NP_{SG} wants a 3-element domain, cf. *A, B, or C*—*some student*$_{A,\ B,\ C}$.

(4) Jo called Alice. So, she called # Alice, Bob, or Cindy / ✓some student.

(4') Jo called 2 people. Therefore, she called ✓less than 3 / # at most 2.

(5) Jo didn't call ✓Alice or Bob / # some student.

(5') Jo didn't call ✓less than 2 people / # at most 1 person.

(6) If Jo called ✓Alice or Bob / ✓some student, she won.

(6') If Jo called ✓less than 2 people / ✓at most 1 person, she won.

(7) Everyone who called ✓Alice or Bob / ✓some student won.

(7') Everyone who called ✓less than 2 people / ✓at most 1 person won.

These paradigms are intriguing on their own, but even more intriguing together: They show that ignorance and polarity sensitivity are connected but vary independently, and are also surprisingly uniform across two very different areas of language — disjunction/indefinites and modified numerals—as summarized in Table 1 below.

		compatibility with certainty	
		no	yes
anti-negativity	no	*or*	CMNs
	yes	SMNs	*some* NP_{SG}

Table 1: Compatibility with certainty about a specific element and anti-negativity

Subsets of the two paradigms, and even some of the similarities between them, have been recognized and analyzed in the literature.[2] However, the complete paradigms, and their remarkable parallelism, have never been recognized or analyzed in full.[3] Thus, an

[2]See, for example, Strawson (1952), Grice (1989), Lauer (2014), Chierchia (2013:251), or Nouwen (2015:250) for observations regarding incompatibility with certainty in *or*, Rips (1994) for the same plus strong experimental support; Nicolae (2017) for ignorance in the French disjunction *ou*; Strawson (1974), Farkas (2002, 2003) for ignorance in *some* NP_{SG}, and Becker (1999) and Alonso-Ovalle and Menéndez-Benito (2003) for hints about compatibility with positive certainty (described however in other terms); Farkas (2003), Szabolcsi (2004), Nicolae (2012), a.o., for anti-negativity in *some*; Westera and Brasoveanu (2014), Alexandropoulou et al. (2015), Cremers et al. (2017) for experimental evidence that both CMNs and SMNs can give rise to ignorance/variation effects and Geurts and Nouwen (2007), Geurts et al. (2010), Cummins and Katsos (2010) for experimental evidence that CMNs are compatible with positive certainty but SMNs are not, or Nouwen et al. (2019) for a recent overview of all of these experimental findings; Geurts and Nouwen (2007), Büring (2008), Nouwen (2010), Geurts et al. (2010), Cummins and Katsos (2010), Coppock and Brochhagen (2013), Westera and Brasoveanu (2014), Nouwen (2015), Kennedy (2015), Spector (2015), Mendia (2015), Schwarz (2016) for theoretical discussions of ignorance in CMNs and SMNs; Nilsen (2007), Geurts and Nouwen (2007), Cohen and Krifka (2014), Spector (2015) for theoretical discussions of anti-negativity in SMNs and Mihoc and Davidson (2017, 2019) for experimental evidence; Büring (2008) and subsequent literature for analogies between SMNs and disjunction and Spector (2014, 2015) for analogies between SMNs and *PPI* disjunction, and Chierchia (2013:251) / Nouwen (2015:250) for the observation that, w.r.t. incompatibility with certainty, *or* / *or* and SMNs are like epistemic indefinites.

[3]For example, the extent of the similarity of SMNs to *or*, and of both to epistemic indefinites; the fact that epistemic indefinites with partial variation may include items that are compatible not just with negative

account that would capture all the patterns for *or*/*some* NP_{SG}, or all the patterns for CMNs/SMNs, or that would fully explain their similarity, is still missing. The goal of this paper is to provide such an account.

I specifically propose an account in terms of alternatives and exhaustification . In §2 I discuss the truth conditions; in §3—the alternatives; and in §4—the implicature calculation mechanism. In §5 and §6 I show how all of these, coupled with two further parameters on subdomain alternative use, help us capture ignorance and polarity sensitivity. In §7 I very briefly discuss scalar implicatures. In §8 I conclude.

2. The truth conditions

Any alternative-based account must first spell out truth conditions. For *or* with more than two disjuncts this typically involves logical forms with multiple $\vee$'s. However, in our example the *or* utterance that was equivalent to the *some* NP_{SG} utterance contained just one occurrence of *or*. I thus adopt the truth conditions in (8) (cf. Mitrović and Sauerland 2016:473 for conjunction). For *some* NP_{SG} I adopt the standard truth conditions in (9).

(8) Jo called a, b, ..., or ...

$$\bigvee_{x \in \{a,b,\ \dots\}} C(j,x) \Leftrightarrow C(j,a) \vee C(j,b) \vee\ \dots \qquad \text{(assertion)}$$

(9) Jo called some student.

$$\exists x \in [\![\text{student}]\!]\,[C(j,x)] \qquad \text{(assertion)}$$

For CMNs/SMNs I adopt the Heim (2000)–Hackl (2000)–Kennedy (2015) view that the modifiers are functions type $\langle d, \langle dt, t\rangle\rangle$ which take in a numeral n and a degree predicate and yield true iff the maximum of the degree predicate is a number in a range defined relative to n. However, I also propose an important revision. On the mentioned view the range is defined using four mathematical primitives, the relations $>$ / $<$ or $\geq$ / $\leq$. These do not faithfully map onto the morphological primitives of these items—*much/little*, the comparative, and the *at*-superlative. As for *or*, I argue that truth conditions that better reflect the morphology are to be preferred. I therefore propose: First, that the *much/little* shared across CMNs/SMNs is a function mapping a number n to its positive/negative extent, defined as the set of degrees below/above n and including n, adapting Seuren (1984), Kennedy (1997, 2001)'s extent-based approach to gradable adjectives to degrees. Second / third, that the comparative / *at*-superlative meaning shared between our CMNs / SMNs is a function that takes in *much/little*, the numeral, and a degree predicate and yields true iff the

certainty, as virtually all the ones discussed as such in the literature, but also items compatible with positive certainty, and even items that are fine with both; the fact that CMNs give rise to ignorance too and are in fact like this latter type of partial variation epistemic indefinites; the fact that the polarity sensitivity effect of SMNs is an important part of their profile and is similar to the polarity sensitivity effect of epistemic indefinites. None of these are widely recognized even though, as mentioned in the previous footnote, evidence and suggestions in this sense can be found scattered throughout the literature.

maximum of the degree predicate they combine with is a number in the complement of the positive/negative extent of *n* / in the positive/negative extent of *n*. For details see Figure 1.

(10) Jo called more/less than n people.

$$\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge C(j,x)]) \in \overbrace{\overline{[\![\text{much/little}]\!]\,(n)}}^{\{n+1,\,...\}/\{...,\,n-1\}}$$

(11) Jo called at most/least n people.

$$\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge C(j,x)]) \in \overbrace{[\![\text{much/little}]\!]\,(n)}^{\{...,\,n\}/\{n,\,...\}}$$

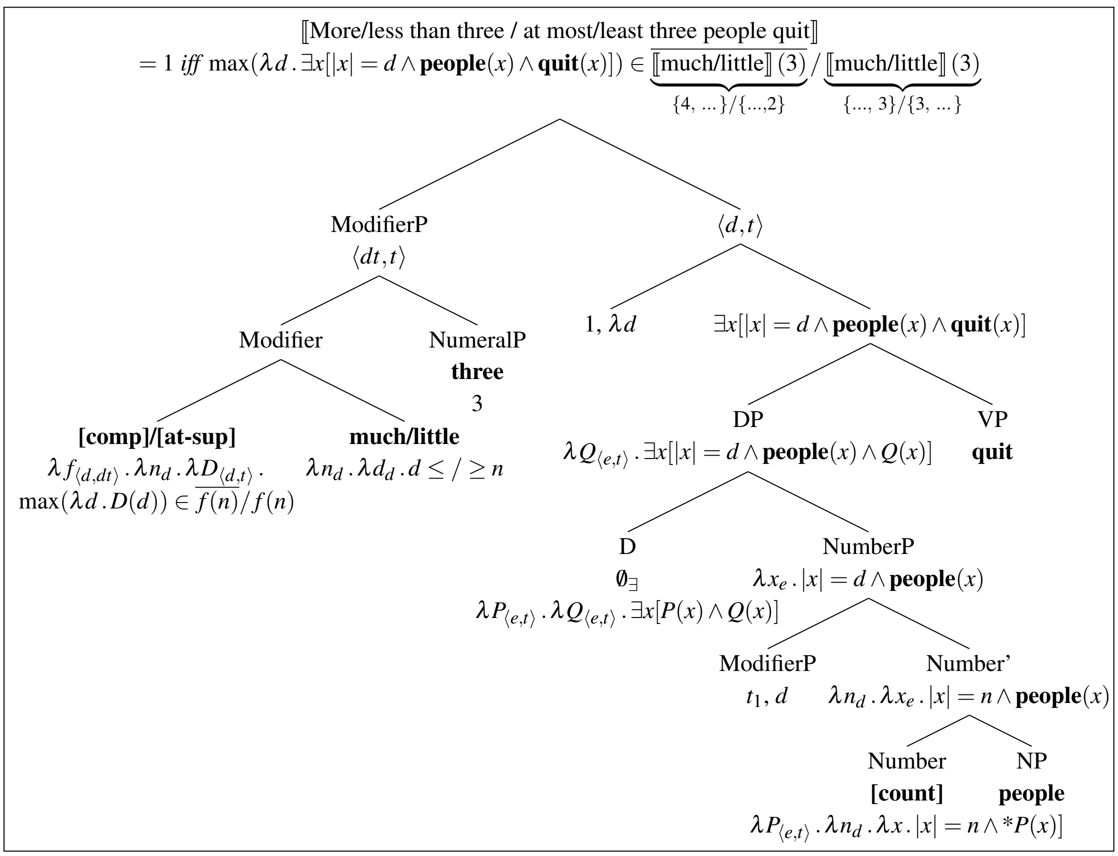

Figure 1: The syntax and semantics of CMNs and SMNs. In NumberP, by replacing ModifierP with NumeralP, one also gets the syntax and semantics of bare numerals (BNs). (I assume that a bare numeral denotes a simple degree; its predicative meaning is derived, for example, via typeshifting, as in Buccola and Spector 2016.) Note: The syntactic assumptions about [count] being the head of a functional projection NumberP intermediary between the DP and the NP and the bare numeral being a phrasal projection NumeralP merged in the specifier of NumberP are as in Zabbal (2005), Scontras (2013), and references therein, though here I extend this assumption to modified numerals and their phrasal projection (what I call 'ModifierP').

Note that the truth conditions in each case make reference to both a scalar element and a domain. *Or*/*some* NP_{SG} and CMN/SMN pairs that refer to the same domain yield equivalent truth conditions. For example:

(12) Jo called Alice or Bob.
Jo called some student$_{\{\text{Alice, Bob}\}}$.
$a \vee b$

(12') Jo called less than 2 people.
Jo called at most 1 person.
$0 \vee 1$

This captures our starting observation about truth-conditional equivalence.

3. The alternatives

Any alternative-based account must also spell out the alternatives. The literature offers many options which sometimes differ quite a bit, especially for CMNs/SMNs. The truth conditions we outlined earlier however invite a very simple and general solution: I propose that, across the board, replacing the domain with its subsets yields subdomain alternatives, DA, and replacing the scalar element with its scalemates yields scalar alternatives, SA.[4]

(13) Jo called a, b, ..., or ...

a. $\bigvee_{x \in \{a,b,\ \dots\}} C(j,x) \Leftrightarrow C(j,a) \vee C(j,b) \vee \ \dots$ (assertion)

b. $\{\bigvee_{x \in D'} C(j,x) \mid D' \subset \{a,b,\ \dots\}\}$ (DA)

c. $\{\bigwedge_{x \in \{a,b,\ \dots\}} C(j,x)\}$ (SA)

(14) Jo called some student.

a. $\exists x \in [\![\text{student}]\!][C(j,x)]$ (assertion)
b. $\{\exists x \in D'[C(j,x)] \mid D' \subset [\![\text{student}]\!]\}$ (DA)
c. $\{\forall x \in D[C(j,x)]\}$ (SA)

(15) Jo called more/less than n people.

a. $\max(\lambda d\,.\,\exists x[|x| = d \wedge C(j,x) \wedge C(j,x)]) \in \overbrace{[\![\text{much/little}]\!]\,(n)}^{\{n+1,\ \dots\}/\{\dots,\ n-1\}}$ (assertion)
b. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge C(j,x)]) \in D' \mid D' \subset \overline{[\![\text{much/little}]\!]\,(n)}\}$ (DA)
c. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge C(j,x)]) \in \overline{[\![\text{much/little}]\!]\,(m)} \mid m \in S\}$ (SA)

(16) Jo called at most/least n people.

a. $\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge C(j,x)]) \in \overbrace{[\![\text{much/little}]\!]\,(n)}^{\{\dots,\ n\}/\{n,\ \dots\}}$ (assertion)
b. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge C(j,x)]) \in D' \mid D' \subset [\![\text{much/little}]\!]\,(n)\}$ (DA)
c. $\{\max(\lambda d\,.\,\exists x[|x| = d \wedge P(x) \wedge C(j,x)]) \in [\![\text{much/little}]\!]\,(m) \mid m \in S\}$ (SA)

[4]Regarding the DA: this is essentially Chierchia (2013)'s DA-generation mechanism, just that here the domain for CMNs/SMNs doesn't correspond to any particular lexical item or even syntactic node but is rather an emergent domain. Regarding the SA: this is essentially a stronger version of Horn (1972)'s SA-generation mechanism, one that derives SA such as *every* but not *many* or *most*. I'm assuming the latter items are SA too, but from a different source. Note: For *or/some* NP_{SG}, SA based on subdomains are also generated $(\forall x \in D'[C(j,x))]$, $D' \subset [\![\text{student}]\!]$). I will assume they are retained only if different from the existing DA/SA. These are crucial for ruling out multiple positive certainty but, for ease of exposition, I will leave them out.

Note that, given reference to the same domain of individuals and, respectively, degrees, not just the truth conditions, but also the alternatives for *or*/*some* NP_{SG} and CMNs/SMNs are, within each pair, equivalent. Schematically, these are as below (assertion in bold, SA horizontally in blue, DA vertically in red; arrows indicate direction of entailment). As later we will need to deal with 2-element and 3-element domains, we spell out both explicitly.

(17) Jo called Alice or Bob.
Jo called some student$_{\{\text{Alice, Bob}\}}$.

$$\begin{array}{lllr} a \quad b & & & \text{(DA)} \\ \downarrow & & & \\ \mathbf{a} \vee \mathbf{b} & \leftarrow & a \wedge b & \text{(SA)} \end{array}$$

(17') Jo called less than 2 people.
Jo called at most 1 person.

$$\begin{array}{lr} \quad\quad 0 \quad 1 & \text{(DA)} \\ \quad\quad \downarrow & \\ 0 \rightarrow \mathbf{0} \vee \mathbf{1} \rightarrow 0 \vee 1 \vee 2 \rightarrow \ldots & \text{(SA)} \end{array}$$

(18) Jo called Alice, Bob, or Cindy.
Jo called some student$_{\{\text{Alice, Bob, Cindy}\}}$.

$$\begin{array}{lr} a \quad b \quad c & \text{(DA)} \\ a \vee b \quad a \vee c \quad b \vee c & \\ \downarrow & \\ \mathbf{a} \vee \mathbf{b} \vee \mathbf{c} \leftarrow a \wedge b \wedge c & \text{(SA)} \end{array}$$

(18') Jo called less than 3 people.
Jo called at most 2 people.

$$\begin{array}{lr} 0 \quad 1 \quad 2 & \text{(DA)} \\ 0 \vee 1 \quad 0 \vee 2 \quad 1 \vee 2 & \\ \downarrow & \\ 0 \rightarrow 0 \vee 1 \rightarrow \mathbf{0} \vee \mathbf{1} \vee \mathbf{2} \rightarrow \cdots \vee 3 \rightarrow \ldots & \text{(SA)} \end{array}$$

4. The implicature calculation mechanism

Finally, any alternatives-and-exhaustification account must spell out an implicature calculation mechanism. The literature again offers many different options. From among these, I will adopt the grammatical view, since it is a view that has already proven to be crucial to disjunction. Between subvariants of the grammatical view such as the contradiction-free and the contradiction-based view, I will adopt the latter, as articulated in Chierchia (2013), since it is a view that unifies epistemic effects with polarity sensitivity, so it is directly relevant to us. On this contradiction-based view, alternatives are factored in via a silent exhaustivity operator O(nly) which asserts the prejacent and negates all the non-entailed alternatives. Some examples of the workings of O are as follows:

(19) $O_{DA}(a \vee b) = (a \vee b) \wedge \neg a \wedge \neg b, = \bot$

(19') $O_{DA}(0 \vee 1) = (0 \vee 1) \wedge \neg 0 \wedge \neg 1, = \bot$

(20) $O_{SA}(a \vee b) = (a \vee b) \wedge \neg(a \wedge b)$

(20') $O_{SA}(0 \vee 1) = (0 \vee 1) \wedge \neg 0, = 1$

5. Ignorance

Recall our starting ignorance patterns: All of *or*/*some* NP_{SG}/CMNs/SMNs can give rise to a speaker ignorance effect, but *some* NP_{SG} and CMNs are also compatible with specific positive or negative certainty. How do we derive these patterns?

First, following the alternative-based approaches to ignorance (Sauerland 2004 a.o.), we will assume that ignorance is obtained from symmetric alternatives—in our case, the DA. Second, following the alternative-based approaches to ignorance *and* polarity sensitivity (e.g., Chierchia 2013, Nicolae 2017), we will assume that for all our items these alternatives are factored in obligatorily. Now, exhaustification via O_{DA} without any intervening operator leads to a crash, as we saw in (19). But, ignorance is usually understood as a silent modal effect. So, let's study what happens when we perform O_{DA} across an overt possibility or necessity modal. As it turns out, O_{DA} across a possibility modal also crashes. Following discussions of O_{DA} across possibility modals (Fox 2007, Chierchia 2013, a.o.), we will assume that O_{DA} actually proceeds relative to the DA interpreted exhaustively—for example, following Chierchia (2013), pre-exhaustified DA, henceforth ExhDA. An ExhDA is a fully grown DA that is prefixed with O, where we will assume that pre-exhaustification is done relative to DA of the same size. Using ExhDA indeed helps—O_{ExhDA} across the possibility modal not longer yields a crash but instead a free choice effect.

(21)
$$O_{ExhDA}\Diamond(a\vee b) = \Diamond(a\vee b)\wedge \underbrace{\neg\underbrace{O\Diamond a}_{\Diamond a\wedge\neg\Diamond b}}_{\Diamond a\to\Diamond b} \wedge \underbrace{\neg\underbrace{O\Diamond b}_{\Diamond b\wedge\neg\Diamond a}}_{\Diamond b\to\Diamond a}$$

(21') [5]
$$O_{ExhDA}\Diamond(0\vee 1) = \Diamond(0\vee 1)\wedge \underbrace{\neg\underbrace{O\Diamond 0}_{\Diamond 0\wedge\neg\Diamond 1}}_{\Diamond 0\to\Diamond 1} \wedge \underbrace{\neg\underbrace{O\Diamond 1}_{\Diamond 1\wedge\neg\Diamond 0}}_{\Diamond 1\to\Diamond 0}$$

Now, analogously, O_{ExhDA} across an overt necessity modal yields the same total variation, free choice effect—and, for the domain of individuals, also total nonvariation.

(22)
$$O_{ExhDA}\Box(a\vee b) = \Box(a\vee b)\wedge \underbrace{\neg\underbrace{O\Box a}_{\Box a\wedge\neg\Box b}}_{\Box a\to\Box b} \wedge \underbrace{\neg\underbrace{O\Box b}_{\Box b\wedge\neg\Box a}}_{\Box b\to\Box a}$$

(22')
$$O_{ExhDA}\Box(0\vee 1) = \Box(0\vee 1)\wedge \underbrace{\neg\underbrace{O\Box 0}_{\Box 0\wedge\neg\Box 1}}_{\Box 0\to\Box 1} \wedge \underbrace{\neg\underbrace{O\Box 1}_{\Box 1\wedge\neg\Box 0}}_{\Box 1\to\Box 0}$$

At this point we will assume with the literature (Kratzer and Shimoyama 2017, Sauerland 2004, Chierchia 2013, Meyer 2013, a.o.) that ignorance occurs because the seemingly episodic sentences are actually embedded under a null matrix-level speaker-oriented epistemic necessity modal, which we will write as $\Box_S$. O_{ExhDA} across $\Box_S$ yields the exact same results as (22), that is, total variation—or, for domains of individuals, also no variation at all—just that this time the total variation effect is epistemic, that is, ignorance.

But if the only ignorance effect we obtained above was total—how do we get specific positive or negative certainty? Following the literature (e.g., Alonso-Ovalle and Menéndez-Benito 2010, Chierchia 2013, Fălăuş 2014), we will assume that, at least for some items, O_{ExhDA} can actually proceed relative to natural subsets of their logical DA set—that is, just the singletons, $O_{ExhSgDA}$, or just the non-singletons, $O_{ExhNonSgDA}$. Now, for a 2-element domain removing any natural subset would destroy the domain, so we will assume this can only happen for domains with 3 or more elements. But the results of exhaustification for a 3-element domain are already a lot harder to evaluate, so to make our jobs easier, we will specifically consider compatibility with the scenarios of interest listed in Table 2.

[5] This is not the end of the story, but we won't be able to discuss this further here.

total ignorance	partial ignorance		no ignorance / total certainty	
‘no winner’	‘one loser’	‘one winner’-1	‘one winner’-2	‘all winners’
e.g.,	e.g.,	e.g.,	e.g.,	e.g.,
w_1: x ~~y~~ ~~z~~	w_1: ~~x~~ y ~~z~~	w_1: x y ~~z~~	w_1: x ~~y~~ ~~z~~	w_1: x y z
w_2: ~~x~~ y ~~z~~	w_2: ~~x~~ ~~y~~ z	w_2: x ~~y~~ z	w_2: x ~~y~~ ~~z~~	w_2: x y z
w_3: ~~x~~ ~~y~~ z	w_3: ~~x~~ y z	w_3: x y z	w_3: x ~~y~~ ~~z~~	w_3: x y z

Table 2: Relevant epistemic state scenarios. Note: *xyz* stand for *abc* or 012.

For a domain of individuals, either example of a ‘one winner’ model makes sense, as does the ‘all winners’ model. However, for a domain of degrees, because positive certainty about one degree entails negative certainty about any other degree, neither the ‘one winner’-1 model nor the ‘all winners’ model is possible. As such, when we abbreviate this grid below to just the second row, combining also the ‘one winner’ cases, any checkmarks in these categories must be understood relative to this; to indicate this, we use an asterisk.

We are now ready to exhaustify, and assess the results, for a 3-element domain.

First, $\mathrm{O}_{\mathrm{ExhSgDA}}$ yields ‘no winner’, ‘one loser’, and ‘all winners’.

(23) $$\begin{aligned}&\mathrm{O}_{\mathrm{ExhSgDA}}(\Box_S(a\vee b\vee c))\\ &=\Box_S(a\vee b\vee c)\wedge\\ &(\Box_S a\rightarrow\Box_S b\vee\Box_S c)\wedge\\ &(\Box_S b\rightarrow\Box_S a\vee\Box_S c)\wedge\\ &(\Box_S c\rightarrow\Box_S a\vee\Box_S b)\end{aligned}$$

(23’) $$\begin{aligned}&\mathrm{O}_{\mathrm{ExhSgDA}}(\Box_S(0\vee 1\vee 2))\\ &=\Box_S(0\vee 1\vee 2)\wedge\\ &(\Box_S 0\rightarrow\Box_S 1\vee\Box_S 2)\wedge\\ &(\Box_S 1\rightarrow\Box_S 0\vee\Box_S 2)\wedge\\ &(\Box_S 2\rightarrow\Box_S 0\vee\Box_S 1)\end{aligned}$$

‘no winner’ ✓	‘one loser’ ✓	‘one winner’ ✗[6]	‘all winners’ ✓*

Second, $\mathrm{O}_{\mathrm{ExhNonSgDA}}$ yields ‘no winner’, ‘one winner’, and ‘all winners’.

(24) $$\begin{aligned}&\mathrm{O}_{\mathrm{ExhNonSgDA}}(\Box_S(a\vee b\vee c))\\ &=\Box_S(a\vee b\vee c)\wedge\\ &(\Box_S(a\vee b)\rightarrow\Box_S(a\vee c)\vee\Box_S(b\vee c))\wedge\\ &(\Box_S(a\vee c)\rightarrow\Box_S(a\vee b)\vee\Box_S(b\vee c))\wedge\\ &(\Box_S(b\vee c)\rightarrow\Box_S(a\vee b)\vee\Box_S(a\vee c))\end{aligned}$$

(24’) $$\begin{aligned}&\mathrm{O}_{\mathrm{ExhNonSgDA}}(\Box_S(0\vee 1\vee 2))\\ &=\Box_S(0\vee 1\vee 2)\wedge\\ &(\Box_S(0\vee 1)\rightarrow\Box_S(0\vee 2)\vee\Box_S(1\vee 2))\wedge\\ &(\Box_S(0\vee 2)\rightarrow\Box_S(0\vee 1)\vee\Box_S(1\vee 2))\wedge\\ &(\Box_S(1\vee 2)\rightarrow\Box_S(0\vee 1)\vee\Box_S(0\vee 2))\end{aligned}$$

‘no winner’ ✓	‘one loser’ ✗[7]	‘one winner’ ✓*	‘all winners’ ✓*

[6] Consider a ‘one winner’ model characterized by $\Box_S a\wedge\neg\Box_S/\Box_S\neg b\wedge\neg\Box_S/\Box_S\neg c$. Such a model would falsify the first implication $\Box_S a\rightarrow\Box_S b\vee\Box_S c$. The same can be verified for $0,1,2$.

[7] Consider a ‘one loser’ model characterized by $\Box_S\neg a\wedge\neg\Box_S b\wedge\neg\Box_S c$. Such a model would mean that the consequent of the third implication is false, which means that the implication itself could be true only if its antecedent $\Box(b\vee c)$ were false as well. But $\Box_S\neg a$ and $\neg\Box_S(b\vee c)$ taken together would contradict the prejacent. The same can be verified for $0,1,2$.

Finally, O_{ExhDA}—the case where we throw all the DA in together—yields the intersection of the previous two cases, that is, just the 'no winner' or the 'all winners' case—precisely the result from ExhDA for a 2-element domain that we started from above.

(25) $O_{ExhDA}(\Box_S(a \vee b \vee c))$ (25') $O_{ExhDA}(\Box_S(0 \vee 1 \vee 2))$

'no winner' ✓	'one loser' ✗	'one winner' ✗	'all winners' ✓*

If all of *or*, *some* NP_{SG}, CMNs, and SMNs have to be interpreted relative to their ExhDA (obligatory ExhDA), but for *some* NP_{SG}/CMNs this requirement can be checked off by using just subsets of the DA-set, while for *or*/SMNs it can only be checked off by using all of the DA-set, this captures all of their ignorance patterns. As for the total certainty result for *or*/*some* NP_{SG}, see §7 for reasons why it might simply be ruled out by their SA-implicatures.

6. Polarity sensitivity

Recall our starting anti-negativity patterns: *or* and CMNs can take scope under negation but *some* NP_{SG} and SMNs cannot; all are however fine in the first argument of a conditional/universal. How do we derive these patterns?

First, note that, across negation, O_{ExhDA} is vacuous—the ExhDA are incompatible with the assertion, so they are already excluded by it, so negating them doesn't strengthen.

(26) $$O_{ExhDA}(\neg(a \vee b)) = \neg(a \vee b) \wedge \neg\underbrace{(\neg a \wedge \neg\neg b)}_{\text{already excluded}} \wedge \neg\underbrace{(\neg b \wedge \neg\neg a)}_{\text{already excluded}}$$

(26') $$O_{ExhDA}(\neg(0 \vee 1)) = \neg(0 \vee 1) \wedge \neg\underbrace{(\neg 0 \wedge \neg\neg 1)}_{\text{already excluded}} \wedge \neg\underbrace{(\neg 1 \wedge \neg\neg 0)}_{\text{already excluded}}$$

O_{ExhDA} vacuous

Now, across *if*/*every* it should be vacuous too. However, note that there is a difference between the scope of negation and the first argument of a conditional / universal—the latter carry an existential presupposition (von Fintel 1999). Suppose now that exhaustification can actually proceed with respect to the presupposition-enriched content of the assertion/alternatives, that is, that it is *strong* exhaustification, O^S, and (non-)vacuity is assessed with respect to this (Gajewski 2011, Chierchia 2013).[8] In that case, the result of O_{ExhDA} is no longer vacuous—instead, it is a free choice effect, as show below for *if*.[9]

[8]Gajewski and Chierchia use this idea for strong NPIs. I follow Spector (2014) and Nicolae (2017) in assuming it is crucial to PPIs also.

[9]For *every* the computation is similar just that, due to the fact that the existential quantification is over individuals rather than worlds, we get strengthening only if O_{ExhDA} happens across $\Box_S$. The insertion of $\Box_S$ can be justified if, as in Chierchia (2013), the null modal is conceptualized as a last resort mechanism rescuing

(27) $O^{S}_{ExhDA} \forall w[(a \vee b)_w \rightarrow W_w]$
$= \forall w[(a \vee b)_w \rightarrow W_w] \wedge \exists w[(a \vee b)_w] \wedge$
$(\ldots \wedge \exists w[a_w]) \rightarrow (\ldots \wedge \exists w[b_w]) \wedge$
$(\ldots \wedge \exists w[b_w]) \rightarrow (\ldots \wedge \exists w[a_w])$

(27') $O^{S}_{ExhDA} \forall w[(0 \vee 1)_w \rightarrow W_w]$
$= \forall w[(0 \vee 1)_w \rightarrow W_w] \wedge \exists w[(0 \vee 1)_w] \wedge$
$(\ldots \wedge \exists w[0_w]) \rightarrow (\ldots \wedge \exists w[1_w]) \wedge$
$(\ldots \wedge \exists w[1_w]) \rightarrow (\ldots \wedge \exists w[0_w])$

O_{ExhDA} not vacuous

If all of *or*/*some* NP_{SG} and CMNs/SMNs have to be interpreted relative to their ExhDA (obligatory ExhDA), but for *or*/CMNs the ExhDA can be used vacuously whereas for *some* NP_{SG}/SMNs they cannot, this captures all of our anti-negativity patterns.

7. Very brief note on scalar implicatures

Our alternative generation mechanism in §3 uniformly generated not just DA but also SA. For both *or*/*some* NP_{SG} and CMNs/SMNs, these are classic Horn-style SA. Are these SA correct, and if so, what is their status?

The SA seem crucial to the meaning of *or* and *some* NP_{SG}. As mentioned in §5, for these items, exhaustification relative to their ExhDA consistently yields, in addition to ignorance, a total certainty meaning. Left unchecked, this effect essentially makes them mean *and* and, respectively, *every*. The SA offer a natural way to keep this effect in check—by adding *not and* and *not every*, they ensure that the overall meaning is just ignorance.

These SA are also crucial to the meaning of bare numerals (BNs). On the classic view on which *three* has an 'at least 3' meaning, it is the strong (below $\Box_S$) SA-implicature 'not at least four' that helps it acquire its standard 'exactly 3' meaning.

However, these SA aren't obviously needed for the meaning of CMNs and SMNs. If anything, through entirely parallel mechanisms as for BNs, they are known to give rise to 'exactly' meanings too—a result that for them is however not desirable (Krifka 1999 and literature since). The literature response to these SA has been to move away from the classic (view of the) SA for CMNs and SMNs. Do we want to take this approach too?

In contrast to the current consensus, I would like to argue that the classic view of the SA of CMNs and SMNs is essentially correct, for the following reasons: First, it makes sense conceptually—just like disjunction, indefinites, or bare numerals, these items entail one bound and implicate another. Second, it makes sense empirically—except for a few wrong 'exactly' meanings, it yields all the right implicatures, including implicatures that the revised views since can't (e.g., either one or both of *Jo called more than 3 people* $\rightsquigarrow$ *not more than 5* or *If Jo called more than 3 people, she won* $\rightsquigarrow$ *not if she calls more than 2*). Third, the wrong 'exactly' meanings can all in fact be ruled out once we (re)examine the interaction between the DA and the SA ($\Box_S O_{SA}(0 \vee 1 \vee 2)$ yields 'exactly 2', but not if we add in the DA—$O_{DA} \Box_S O_{SA}(0 \vee 1 \vee 2)$ yields a crash, presumably avoidable by SA-pruning, yielding ignorance over a truncated scale), or the shape of the SA of a negated

an exhaustification that would otherwise fail. Note: This conceptualization of the null modal also helps avoid the possibly problematic issue of double modalization in cases with an overt modal.

scalar (e.g., $O_{SA}\neg(0 \vee 1 \vee 2)$ yields 'exactly 3', but not if the SA of a scalar under negation include their negation-less variants—in that case what we get is ignorance).

But what is the status of these SA, for both *or*/*some NP$_{SG}$* and CMNs/SMNs? Above we argued that, for all these items, at least some of the DA are always obligatory. In his approach to epistemic indefinites, Chierchia (2013) argues that it is not just the DA but also the SA of these items that are obligatory. Do we want to adopt this here too?

As is well known, an *or*/*some NP$_{SG}$*/BN statement such as *Jo called Alice or Bob / some student / 3 students* can be felicitously followed up with *in fact, both / every student / 4 students*. Thus, the SA-implicatures of these items are cancellable. 'Cancellable' obviously means 'not obligatory'. Are the SA-implicatures then optional?

That is the usual conclusion. However, I would like to argue that it is perhaps too weak. That is, the SA could be cancellable but still a *strong default*. This could come, for example, from a preference for the strongest meaning (see, e.g., Chierchia 2004:§3, for a detailed discussion of this for disjunction and indefinites, including possible mechanisms for default override). Whatever the mechanism(s) that regulate this default use of the SA, we will likely also have to allow for some variation by item (possibly due to features specific to the item, e.g., richness of the scale, lexical competitors, etc.). For example, for BNs, we want to always consider the immediately stronger SA; this will ensure that *3* defaults to *not 4* ⇒ *exactly 3*. However, for CMNs/SMNs, while the immediately stronger SA is not an option due to ignorance, the next one isn't an ideal default either as it would lead to a stronger default scale truncation than we want; instead, we really want to leave the choice of the SA to be negated up to context (Cummins et al. 2012). We won't be able to discuss this any further here. For now my proposal is just this: All of *or*, *some*, BNs, CMNs, and SMNs have classic SA-implicatures, and they are perhaps less optional than previously thought.

8. Conclusion

Or/*some NP$_{SG}$* and CMNs/SMNs exhibit interesting similarities and differences with respect to ignorance and anti-negativity, both between and within pairs. In this paper I propose a fully unified solution on which all the similarities within and between pairs boil down to similar truth conditions, subdomain and scalar alternatives, and basic exhaustification mechanism; all the differences within the pairs—to different requirements on their subdomain alternatives (whether all have to be used, whether they must lead to proper strengthening); and all the differences between the pairs—to the different nature of the domains (individuals vs. degrees). The account builds on many insights from the literature, but it also offers new solutions, especially regarding ignorance with positive specific certainty, the truth conditions of CMNs/SMNs, and a general alternative generation and use mechanism across categories, as well as some new suggestions for scalar implicatures.

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