True Distributivity and the Functional Interpretation of Indefinites

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Abstract Functional interpretations of sentences containing indefinites require expressive power beyond quantification over individuals (Hintikka 1986, Schlenker 2006). Analyses of indefinites as contributing Skolemized choice function variables account for these readings (Chierchia 2001, Winter 2004, Schlenker 2006) , but overgenerate (Chierchia 2001, Schwarz 2001). I show that proposals to solve this problem by restricting the domain of quantification over Skolemized choice functions (Kratzer 1998, Schwarz 2001) are inadequate, as they must either undergenerate or fail to eliminate the overgeneration. I show that these analyses fail to account for the full range of data because they do not distinguish two kinds of functional interpretations of indefinites, which parallel the pair-list and (natural) functional interpretations of questions. I show that unrestricted functional interpretations of indefinites, like pair-list interpretations of questions, arise only when an indefinite is in the scope of a distributive quantifier. I then argue that, contra the Skolemized choice function approach, these readings are not due to a certain interpretive possibility of indefinites, but instead result from the semantic interaction of indefinites and distributive quantification. In particular, functional interpretations arise when a distributive quantifier takes scope over an indefinite because sentences of this form have functional witnesses. This is the semantic content of Skolemization that is lost in on Skolemized choice function approach. Finally, I show how this idea can be implemented compositionally using a variable-free semantics for indefinites (Jäger 2001, Shan 2001, Szabolcsi 2003).

1 Introduction

The scopal behavior of indefinites is characterized by two unusual properties.

1.1 Exceptional wide scope

First, their scope is unbounded (Fodor & Sag 1982). In each of the following, the indefinite occurs within an embedded finite clause (ordinarily a scope island), yet may take scope over the entire sentence, as in the indicated interpretation.

(1) Mary read every book that discussed **a certain problem**.
$(\exists x : \text{problem } x)\,(\forall y : \text{book } y \wedge y \text{ discussed } x)$ Mary read y

(2) If we invite **some philosopher**, Max will be offended.
$(\exists x : \text{philosopher } x)$ if we invite x, Max will be offended

(3) John moved to Stuttgart because **a certain woman** lived there.
$(\exists x : \text{woman } x)$ John moved to Stuttgart because x lived there

Moreover, island-escaping scope is not restricted to sentence-widest scope. An indefinite may take *intermediate* scope outside of its minimal clause, but beneath a superordinate operator (Farkas 1981, Abusch 1994):

(4) No linguist read every book that discussed **some problem**.
$(\neg\exists x : \text{linguist } x)\,(\exists y : \text{problem } y)\,(\forall z : \text{book } z \wedge z \text{ discussed } x)\; x \text{ read } z$

(5) Every man moved to Stuttgart because **a certain woman** lived there.
$(\forall x : \text{man } x)\,(\exists y : \text{woman } y)$ x moved to Stuttgart because y lived there

On the standard assumption that Quantifier Raising is (always) clause-bounded, these facts dictate that a distinct mechanism is responsible for the scope of indefinites. Such a mechanism is given by the Kamp-Heim view of indefinites as free variables (Kamp 1981, Heim 1982), wherein the existential force associated with an indefinite is not contributed by the indefinite itself, but rather by an independent existential closure operator. The location of the existential closure operator then determines the apparent scope of the indefinite. The existential closure operator need not occur in the indefinite's minimal clause, allowing extra-clausal scope. A simple sentence containing an indefinite (6a) receives a Logical Form along the lines of (6b), interpreted as (6c).

(6) a. Mary read a book.
b. $\exists_i$ Mary read [a book]$_i$
c. $\exists x.\, \text{book } x \wedge \text{Mary read } x$

1.2 Choice functions

A technical problem arises regarding the interpretation of the restrictor in cases where the indefinite is embedded (Reinhart 1992). On the classical Kamp-Heim analyses, the restrictor of an indefinite is interpreted as part of the material predicated of the free variable. Then the LF (7a) for (2) is interpreted as (7b).

(7) a. $\exists_i$ [if [we invite [some philosopher]$_i$], Max will be offended]
b. $\exists x.$ if philosopher $x \wedge$ we invite x, Max will be offended
There is an entity x such that if x is a philosopher and we invite x, Max will be offended.

It is clear that this is not the desired interpretation for (2). This problem came to be known as the "Donald Duck problem", after Reinhart's observation that if the conditional is interpreted as material implication, (7b) is verified by any entity that is not a philosopher, for instance Donald Duck.

To deal with the problem, Reinhart (1992, 1997) employs *choice functions*. A choice function is a type $(\mathtt{e} \to \mathtt{t}) \to \mathtt{e}$ function f such that for every non-empty property P, $f(P)$ is an element of P. That is, a choice function "chooses" an entity that satisfies each non-empty property. Reinhart interprets an indefinite *some NP* as a free functional variable applying to the denotation of *NP*, with existential closure over choice functions. Thus for Reinhart an LF like (8a) is interpreted as (8b) (where CF is the set of choice functions).

(8) a. $\exists_i \ldots [\text{some NP}]_i \ldots$
b. $(\exists f \in \text{CF}) \ldots f(\text{NP}) \ldots$

As long as the denotation of *NP* is non-empty,[1] (8b) provides the desired interpretation: (9a) and (9b) are equivalent.

(9) a. $(\exists x : \text{philosopher } x)$ (we invite $x \to$ Max will be offended)
There is a philosopher x such that if we invite x, Max will be offended.
b. $(\exists f \in \text{CF})$ (we invite $f(\text{philosopher}) \to$ Max will be offended)
There is a choice function f such that if we invite the philosopher chosen by f, Max will be offended.

1.3 Functional readings

The challenge posed by the unbounded scope of indefinites is one for the syntax-semantics interface. The semantics of wide-scope indefinites are unproblematic. What is controversial is the mechanism by which they are assigned this scope. Following Kamp and Heim, Reinhart's (1992, 1997) choice function account is one on which indefinites are translated as free variables and existential force is contributed by a separate operator. But as Schwarz (2001) emphasizes, this is only necessary on the assumption that QR (and any QR-like operation) is always clause-bounded. Taking indefinites to be simple existential quantifiers which may take scope outside their minimal clause is sufficient to account for the wide-scope readings of indefinites discussed above.

The second unusual property of indefinites is not so easily dealt with. Sentences with indefinites may have *functional* interpretations which cannot be expressed by means of quantification over individuals. An example is the indicated reading of (10), due to Schlenker

1 See Geurts (2000) and Winter (2004) on complications arising from empty NP denotations.

(2006).

(10) If every student makes progress in **a certain area**, nobody will flunk the exam.
$(\exists f \in [\text{student} \to \text{area}])$ if $(\forall x : \text{student } x)$ x makes progress in $f(x)$, nobody will flunk
There is an assignment of areas to students such that if every student makes progress in his or her assigned area, nobody will flunk the exam.

where $[X \to Y]$ is the set of functions from X to Y, so that $[\text{student} \to \text{area}]$ is the set of functions from students to areas.

The reading in question can be appreciated in the following context. Professor Smith is about to give an exam in his Syntax course. Each of his students is particularly weak in one area that will be covered on the exam. These areas differ from student to student. For instance John is weak in Binding Theory, Mary is weak in Case Theory, and so on. Each student will pass the exam if and only if he or she makes progress in their particular area of weakness. Then (10) can be uttered truly in this situation.

Given that the scope of *every student* is clause-bounded, (10) has three different interpretations given by the three possible scope positions of *a certain area*, interpreted as an existential quantifier over individuals.

(11) a. if > every student > a certain area
if $(\forall x : \text{student } x)\,(\exists y : \text{area } y)$ x makes progress in y, nobody will flunk
b. if > a certain area > every student
if $(\exists y : \text{area } y)\,(\forall x : \text{student } x)$ x makes progress in y, nobody will flunk
c. a certain area > if > every student
$(\exists y : \text{area } y)$ if $(\forall x : \text{student } x)$ x makes progress in y, nobody will flunk

None of these interpretations are true in this situation. (11a) and (11b) place too weak requirements on the students' performance, while (11c) places one too strong. That (10) is nonetheless true in this situation shows that the functional reading is in fact a distinct reading.

Scope paradoxes involving indefinites — that is, interpretations which do not arise from any linear ordering of individual quantifiers — were first discussed by Hintikka (1986). A slightly simpler example can be given along the lines of Hintikka's:

(12) Mary hopes that every guest brings **some dish**.
$(\exists f \in [\text{guest} \to \text{dish}])$ Mary hopes that $(\forall x : \text{guest } x)$ x brings $f(x)$

Imagine that Mary is hosting a dinner party, and that for each of her guests, she is particularly fond of one of the dishes that he or she makes.

1.4 Skolemized choice functions

Thus a theory more expressive than those discussed in §1.1 is needed. Chierchia (2001) develops a generalization of Reinhart's choice function proposal that meets this need. The approach is adopted by, among others, Winter (2004) and Schlenker (2006). Recall that for Reinhart, to avoid the Donald Duck problem, existential quantification essentially over individuals is effected by means of quantification over choice functions. Chierchia (2001) extends this approach by effecting quantification over functions to individuals by means of quantification over functions *to choice functions*. A *Skolemized choice function* of n individual arguments is a function that when applied to n individuals of type $\mathtt{e}$ returns a choice function. Using Skolemized choice functions, the functional reading of (12) can be expressed as:

(13) $(\exists f \in \text{SCF}_1)$ Mary hopes that $(\forall x : \text{guest } x)$ x brings $f(x)(\text{dish})$

where SCF_n is the set of Skolemized choice functions of n individual arguments. Such an interpretation can be derived if (12) is assigned an LF like (14):

(14) $\exists_f$ Mary hopes that [every guest]$_x$ x brings [[f x] dish]

On the simplest view, this is accomplished by allowing an indefinite determiner to freely translate as a Skolemized choice function variable applied to the appropriate number of covert pronouns, along with the possibility of existential closure at any propositional node, or at least at the top.

In addition to generating the scope-paradoxical functional readings, this kind of account also provides for a secondary derivation of some of the intermediate readings discussed in §1.1. The intermediate reading of (5) can now be derived as:

(15) $(\exists f \in \text{SCF}_1)$ $(\forall x : \text{man } x)$ x moved to Stuttgart because $f(x)$ lived there

However, as Chierchia (2001) and Schwarz (2001) discuss, such an account (call this the *free Skolemized choice function (SCF) translation* account) *overgenerates*. Take (16a). In addition to the wide-scope and narrow-scope individual LFs that correspond to the two available interpretations of (16a), the free SCF theory assigns (16a) the LF (16b), interpreted as (16c).

(16) a. No girl read some book.
b. $\exists_f$ [no girl]$_x$ x read [[f x] book]
c. $(\exists f \in \text{SCF}_1)$ $(\neg\exists x : \text{girl } x)$ x read $f(x)(\text{book})$
There is a Skolemized choice function f such that no girl read the book assigned to her by f.

But (16c) is true just in case there is a way of assigning a book to each girl such that she did not read that book, that is, just in case no girl read *every* book. This is clearly not a possible interpretation of (16a).

The problem is general for non-upward-entailing quantifiers binding into the individual argument of a Skolemized choice function. Schwarz (2001) shows that whenever Q is non-upward-entailing, the Skolemized choice function (17) LF generates an illicit reading.

(17) $\exists_f\ Q_x$ [... [[f x] NP] ...]

1.5 Natural functions

Schwarz (2001) identifies the overgeneration as dependent on the monotonicity of the quantifier binding the Skolemized choice function argument, but rejects a solution that makes reference to monotonicity. Instead, Schwarz, following Kratzer (1998), proposes that the illicit readings generated by the SCF theory are ruled out by restricting quantification over Skolemized choice functions to the "natural" ones. The function that verifies (16c) is an aribtrary mapping of girls to books they did not read, and does not count as natural. Since no natural function will verify (16c), the unwanted interpretation is avoided. On the other hand, Schlenker (2006) suggests that the function mapping students to the areas they need to approve in that verifies (10) does count as natural, so that that reading is preserved.

This kind of solution to the overgeneration problem makes a strong predictions with respect to the essentially functional readings discussed in §1.3: the availability of an essentially functional reading for a sentence does not depend on the monotonicity of quantifiers in the sentence. The only constraint on functional readings should be whether the verifying function counts as natural. In the following section I argue that this prediction is incorrect.

2 Two kinds of functional indefinites

2.1 Quantifiers and distributivity

Spector (2004) observes that not even all upward-entailing quantifiers license functional readings. The SCF theory predicts that (18a) should have the functional LF (18b). In the exam situation described above, this LF would be interpreted as true (on the assumption that *few* means *not many*). But (18a) has no reading that need be true in this scenario.

(18) a. If many (of the) students make progress in a certain area, few will flunk.
b. $\exists_f$ if [many students]$_x$ x makes progress in [[f x] area], few will flunk

Crucially, the functional interpretation cannot be ruled out via appeal to natural functions, since the *same* assignment of areas to students that has to count as natural to verify (10) would verify (18b), in the same context. Thus (18b) must be ruled out as an LF for (18a).

Spector retains the SCF translation theory, arguing that (18b) is ruled out by an independent syntactic constraint preventing *many students* from binding a covert variable. Spector's argument is based on the following contrast:

(19) *Every boy saw a (different) stray cat in his neighborhood yesterday.*
  a. #John and Peter adopted the cat.
  b. John and Peter **each** adopted the cat.

Only with overt *each* can cats vary with boys. Spector argues this follows because only overt *each* can bind the covert variable parameterizing the covert domain restriction variable in *the cat*:

(20) a. *John and Peter [$\textsc{dist}_x$ adopted [the [C x]] cat]
  b. John and Peter each$_x$ adopted [the [C x]] cat

In general, Spector argues, only true distributive quantifiers based on *every* and *each* can bind covert variables, and therefore only these quantifiers can license functional readings.

However, it is not the case that binding of covert variables is as restricted as licensing of functional readings. Quantifiers headed by *no* have no problem binding covert variables:

(21) *Every boy saw a (different) stray cat in his neighborhood yesterday.*
  No boy adopted the cat.

But still, *no* does not license functional readings of indefinites:

(22) a. If no student makes progress in a certain area, everybody will flunk.
  b. $\exists_f$ if [no student]$_x$ x makes progress in [[f x] area], everybody will flunk

The free SCF theory predicts that (22a) has a functional LF (22b), which like (18a), would be verified in the same exam scenario by the same function that verifies (10). (22a) is instead most naturally interpreted with *a certain area* taking widest scope, so that there is a single area that every student must make progress in to avoid failing. Neither this nor any other interpretation of (22a) is true in the scenario under discussion.

The lack of a true reading of (22a) in this scenario shows that the grammar must not generate (22b) as an LF for it, as neither the restriction to natural functions or Spector's restriction on covert-variable-binding can otherwise eliminate it.

2.2 Varieties of functional readings

We have concluded that the grammar must not generate (interpretations corresponding to) LFs of the form:

(23) $\exists_f \dots Q_x [\dots [[f\ x]\ \mathrm{NP}] \dots]$

where Q is anything but a distributive quantifier, that is, a quantifier based on *every* or *each*. Accepting the unavailability of such LFs as the explanation of the unavailability of the illicit functional readings predicted by the free SCF theory, the domain restriction to natural functions is no longer needed. In fact, this is a positive result, as a *different* class of functional readings of indefinites call for an account in terms of natural functions.

Winter (1998, 2004) and Schwarz (2001) discuss examples like the following:

(24) No man loves **a certain woman he knows**.

As we have seen, (24) does not have an unrestricted functional reading, which would be equivalent to *No man loves every woman he knows*. But if there is a *salient*, *natural* function f mapping men to women they know such that no man loves the woman f maps him to, (24) can perhaps be uttered truthfully. This can be brought out by continuing the sentence by specifying the (intended) natural function.

(25) No man loves **a certain woman he knows** — namely, his mother-in-law.

So in fact we do need the grammar to generate natural functional LFs of the form:

(26) $\exists f_{\text{natural}} \ldots\ Q_x\ [\ \ldots\ [[f\ x]\ \text{NP}]\ \ldots\ ]$

But these readings are *extremely restricted*. First, they seem to all but require a *namely*-type continuation. The unrestricted functional readings discussed in §1.3 require no such specification. Second, as Schwarz (2001) points out, they are only available with *a certain* indefinites:

(27) #No man loves **some woman he knows** — namely, his mother-in-law.

Unrestricted functional readings licensed by distributive quantifiers are available with *some*.

To conclude, there are two distinct kinds of functional readings of indefinites. Within a Skolemized choice function theory of functional indefinites, they are achieved via the two following kinds of LFs. Unrestricted quantification over Skolemized choice functions is only available when the function's argument is bound by a distributive quantifier (based on *every* or *each*), denoted by δ:

(28) $\exists f\ \ldots\ \delta_x\ [\ \ldots\ [[f\ x]\ \text{NP}]\ \ldots\ ]$

An extremely restricted functional reading, modeled as quantification over "natural" functions, arises from a generally available LF:

(29) $\exists f_{\text{natural}} \ldots\ Q_x\ [\ \ldots\ [[f\ x]\ \text{NP}]\ \ldots\ ]$

An account of how the unrestricted reading arises and why it is only licensed by distributive quantifiers is the topic of the remainder of the paper.

2.3 Against an LF account

One possible move at this point for the proponent of the Skolemized choice function theory would be to maintain that indefinites are freely translated as functional variables, but to impose a condition on LFs so that the grammar filters LFs where a non-distributive quantifier binds the individual argument of a Skolemized choice function variable:

(30) $*\exists f \ldots Q_x [\ldots [[f\ x]\ \text{NP}] \ldots]$
unless $Q = \delta$

As Schwarz (2001) argues, directly adopting an LF constraint of this of this kind would be *ad hoc* and formally unmotivated. But this is a conceptual matter; would it account for the facts? It turns out, not quite. Consider (31) with *each* distributing over the plural indefinite *two students*.

(31) If two students each make progress in a certain area, the exam will be a success.

The Skolemized choice function theory generates two functional interpretations (31), given by the LFs in (32a) and (33a).[2]

(32) a. [two students]$_X$ $\exists_f$ if X each$_x$ x makes progress in [[f x] area], ...
b. $(\exists X : \text{two students } X)\,(\exists f \in \text{SCF}_1)$ if $(\forall x <_{\text{at}} X)$ x makes progress in $f(x)(\text{area})$, the exam will be a success.
There are two students s_1 and s_2 and areas a_1 and a_2 such that if s_1 makes progress in a_1 and s_2 makes progress in s_2, the exam will be a success.

(33) a. $\exists_f$ if [two students]$_X$ X each$_x$ x makes progress in [[f x] area], ...
b. $(\exists f \in \text{SCF}_1)$ if $(\exists X : \text{two students } X)\,(\forall x <_{\text{at}} X)$ x makes progress in $f(x)(\text{area})$, the exam will be a success.
There is an assignment of areas to students such that if any two students each make progress in their assigned area, the exam will be a success.

(31) indeed has a functional interpretation, but only the one in (32b); it cannot be interpreted as (33b). Both these LFs satisfy the condition in (30), so the unavailability of (33b) remains a puzzle for the Skolemized choice function approach. The unavailability of reading (33b) will follow from the analysis I propose in §3.

2.4 Two kinds of functional questions

The two kinds of functional readings of indefinites established above closely parallel the two kinds of functional readings of *questions* with quantifiers (Chierchia 1993, Beghelli

2 For simplicity, the plural indefinite *two students* is treated in the following LFs as an existential quantifier rather than a choice functional indefinite itself. See §5.1.

1997, Szabolcsi 1997): the *pair-list* reading and the *(natural) functional* reading:

(34) a. Which woman does **every** man like?
b. *Pair-list*
John – Mary, Bill – Sue, ...
c. *Natural functional*
His mother.

(35) a. Which woman does **no** man like?
b. *Pair-list*
#John – Mary, Bill – Sue, ...
c. *Natural functions*
His mother-in-law.

The natural functional reading is available equally with any quantifier, while (matrix) pair-list readings are only licensed by *every* and *each*. Thus pair-list readings of questions appear to be the analog our unrestricted functional readings of indefinites. Their distribution should receive a unified account. Compare to the proposal of Schwarz (2001), which unifies *all* functional readings of indefinites with natural functional interpretations of questions.

3 Proposal

3.1 Diagnosis

On the Skolemized choice function theory, functional interpretations of indefinite sentences are due to dedicated functional interpretations of indefinites themselves — to produce a functional reading, an indefinite is interpreted a *functional* variable. The theory builds the potential for functional readings into the semantics of indefinites. It then comes as a surprise that the availability of such readings is sensitive to the semantic properties of the quantifier that binds the Skolemized choice function's argument. The restriction to overt distributors is, on this view, mysterious, and can only be accounted for by an *ad hoc* constraint on LFs.

The facts argue instead that functional interpretations are not, as the Skolemized choice function theory has it, inherent in in the semantics of indefinites. Rather, functional interpretations arise from the *interaction* of indefinites, which make a uniform semantic contribution, and distributive quantification. In the following, I argue that a distributive quantifier scoping over an indefinite gives rise to a functional reading because such sentences have *functional witnesses*. This is the semantic content of Skolemization that is lost on the Skolemized choice function approach. I show how this idea can be implemented compositionally using a variable-free semantics for indefinites.

3.2 Witnesses and possibilities

A sentence containing a widest-scope indefinite, if true, is true *by virtue of* having a *witness*. For instance, consider (36).

(36) Mary read a book.

(36) is true if and only if there is a book x that Mary read. Then, (36) is true *in virtue of* of the truth of the proposition that Mary read x. x is a witness for (36). The set of potential witnesses is the extension of the indefinite's NP restrictor — here the set of books. Each potential witness x gives rise to a different *way the sentence could be true*: by being true in virtue of the fact that Mary read x. The set of propositions whose truth a sentence could be true in virtue of is the sentence's set of *possibilities* in the sense of Inquisitive Semantics (Mascarenhas 2009, Groenendijk & Roelofsen 2009, Ciardelli 2009). A sentence ϕ is true just in case it is true in some way, that is, just in case one of its possibilities is true.

For a sentence ϕ, let $\mathcal{P}(\phi)$ be be the set of possibilities for ϕ.

(37) $\mathcal{P}((36)) = \{\mathbf{read}\, x\, \mathbf{mary} \mid \mathbf{book}\, x\}$

In general, for an indefinite sentence $\phi = [_\phi \ldots$ [a NP] $\ldots$], we have:

(38) $\mathcal{P}(\phi) = \left\{ [\![_\phi \ldots \ulcorner x \urcorner \ldots]\!] \;\middle|\; [\![\mathrm{NP}]\!]\, x \right\}$

where $[\![\ulcorner \alpha \urcorner]\!] = \alpha$ for any α.

Let us now extend these considerations to sentences with a quantifier taking scope over an indefinite. First, consider the case with a distributive quantifier:

(39) Every girl read a book.

Like (36), (39) can be true in a number of different ways: it does not fully specify how the relevant facts have to be in order for it to be true. For example, (39) could be true in virtue of Mary reading *War and Peace*, Sue reading *Anna Karenina*, and so on, or in virtue of Mary reading *Syntactic Structures*, Sue reading *The Minimalist Program*, and so on, In general, for any function f from girls to books, (39) can be true in virtue of every girl x reading fx.

(40) $\mathcal{P}((39)) = \{\forall x \in \mathbf{girl}.\, \mathbf{read}\, (fx)\, x \mid f \in [\mathbf{girl} \to \mathbf{book}]\}$

Notice that whereas the potential witnesses for (36) and for wide-scope indefinite sentences in general are *individuals*, the potential witnesses for (39) are *functions* — each possibility for (39) is determined by a function f from girls to books.

Now compare the case where a negative quantifier takes scope over an indefinite:

(41) No girl read a book.

Unlike (39) (and (36)), (41) (on its surface scope interpretation) specifies fully how the relevant facts have to be in order for it to be true: for every girl y and every book x, it must be that y did not read x. So the set of possibilities for (41) is a singleton, whose only member is the (ordinary) propositional meaning of (41):

(42) $\mathcal{P}((41)) = \{\neg\exists y \in \textbf{girl}.\, \exists x \in \textbf{book}.\, \textbf{read}\; x\; y\} = \{[\![(41)]\!]\}$

(41) can only be true in virtue of, for every book, no girl having read that book.

3.3 Quantifying over possibilities

My proposal is that this contrast between the possibility sets of (39) and (41) explains the distribution of functional readings of indefinites. This follows if wide-scope existential closure is not over individuals (or equivalently, Reinhart's (1992, 1997) choice functions), or freely over Skolemized choice functions, but over *possibilities*. That is, an environment C embedding a clause ϕ can be interpreted as in (43):

(43) *Existential closure over possibilities*
$[\![\exists_i\, [_C \ldots\ \phi_i \ldots\ ]]\!] = \exists p \in \mathcal{P}(\phi).\, [\![_C \ldots\ \lceil p \rceil \ldots\ ]\!]$

This subsumes the ordinary case of existential closure over individuals, as each possibility for an indefinite sentence ϕ corresponds to a choice of an individual witness. This is shown for (2), which receives the LF in (44):

(44) $\exists_i$ if [$_\phi$ we invite some philosopher], Max will be offended.

(45) $\mathcal{P}(\phi) = \{\textbf{invite}\; x\; \textbf{we} \mid \textbf{philosopher}\; x\}$

(46) $[\![(44)]\!] =$
$\exists p \in \mathcal{P}(\phi).\, [\![\text{if } \lceil p \rceil, \text{Max will be offended}]\!]$
$= \exists p \in \{\textbf{invite}\; x\; \textbf{we} \mid \textbf{philosopher}\; x\}.\, [\![\text{if } \lceil p \rceil, \text{Max will be offended}]\!]$
$= \exists x \in \textbf{philosopher}.\, [\![\text{if } \lceil \textbf{invite}\; x\; \textbf{we} \rceil, \text{Max will be offended}]\!]$
$= \exists x \in \textbf{philosopher}.\, \textbf{if}\; (\textbf{invite}\; x\; \textbf{we})\; (\textbf{offended}\; \textbf{max})$

We can now account for our original contrast between (10) and (22a), repeated here as (47) and (48).

(47) If every student makes progress in a certain area, nobody will flunk.

(48) #If no student makes progress in a certain area, everybody will flunk.
(# = no functional reading)

Each possibility antecedent of (47), *every student makes progress in a certain area* is, as (39), determined by a function: in this case, a function from students to areas. This set of potential witnesses yields the possibility set in (50). With this possibility set, the possibility existential closure rule (43) generates the interpretation (51) for the LF (49). This is the functional reading of (47).

(49) $\exists_i$ if [$_\phi$ every student makes progress in a certain area]$_i$, nobody will flunk.

(50) $\mathcal{P}(\phi) = \{\forall x \in \textbf{student}.\, \textbf{progress}\ (f x)\ x \mid f \in [\textbf{student} \to \textbf{area}]\}$

(51) $[\![(49)]\!] =$
$\exists p \in \mathcal{P}(\phi).\, [\![\text{if } \lceil p \rceil, \text{ nobody will flunk}]\!]$
$= \exists p \in \{\forall x \in \textbf{student}.\, \textbf{progress}\ (f x)\ x \mid f \in [\textbf{student} \to \textbf{area}]\}.$
$[\![\text{if } \lceil p \rceil, \text{ nobody will flunk}]\!]$
$= \exists f \in [\textbf{student} \to \textbf{area}].\, [\![\text{if } \lceil \forall x \in \textbf{student}.\, \textbf{progress}\ (f x)\ x \rceil, \text{ nobody will flunk}]\!]$
$= \exists f \in [\textbf{student} \to \textbf{area}].\, \textbf{if}\ (\forall x \in \textbf{student}.\, \textbf{progress}\ (f x)\ x)\ (\textbf{nobody flunk})$

On the other hand, as we discussed for (41), the possibility set of the antecedent of (48), *no student makes progress in a certain area*, is just the singleton set containing its ordinary meaning. So interpreting the parallel LF (52) for (48) yields the same result as interpreting (48) in the ordinary way:[3]

(52) $\exists_i$ if [$_\psi$ no student makes progress in a certain area]$_i$, everybody will flunk.

(53) $\mathcal{P}(\psi) = \{[\![(\psi)]\!]\} = \{\neg\exists y \in \textbf{student}.\, \exists x \in \textbf{area}.\, \textbf{progress}\ y\ x\}$

(54) $\exists p \in \mathcal{P}(\psi).\, [\![\text{if } \lceil p \rceil, \text{ everybody will flunk}]\!]$
$\exists p \in \{\neg\exists y \in \textbf{student}.\, \exists x \in \textbf{area}.\, \textbf{progress}\ y\ x\}.\, [\![\text{if } \lceil p \rceil, \text{ everybody will flunk}]\!]$
$= [\![\text{if } \lceil \neg\exists y \in \textbf{student}.\, \exists x \in \textbf{area}.\, \textbf{progress}\ y\ x \rceil, \text{ everybody will flunk}]\!]$
$= \textbf{if}\ (\neg\exists y \in \textbf{student}.\, \exists x \in \textbf{area}.\, \textbf{progress}\ x\ y)\ (\textbf{everybody flunk})$

Thus this mechanism for generating the functional reading of (47) correctly does not do so for (48). In general, the mechanism will only generate a functional reading for an indefinite in the distributive configuration $\delta >$ Indef because only these configurations give rise to a non-singleton set of possibilities determined by functional witnesses.

In the following, I present a precise, compositional fragment that implements the account developed in this section.

4 Fragment

4.1 Variable-free semantics for indefinites

In the preceding, we mainly talked about propositional possibilities, with individual and functional witnesses playing an essentially expository role. But a shift in perspective

3 In fact, this interpretation is probably not available for (48), as *certain* would have no effect. (48) is more naturally interpreted with *a certain area* taking scope over *no student*.

towards witnesses playing the central role allows us to implement the analysis of §3 in a generalization of the variable-free semantics for indefinites developed by Jäger (2001), Shan (2001), and Szabolcsi (2003).

A sentence that does not present multiple possibilities — that is, whose possibility set is a singleton — will be interpreted as usual as a proposition, of type $\mathtt{t}$. A sentence ϕ that does present multiple possibilities, that can be true in more than one way — for instance, a sentence containing a widest-scope indefinite — will be interpreted as a partial function whose image[4] is the possibility set of ϕ. The domain of the function is the set of potential witnesses for ϕ. The function maps each potential witness x for ϕ into the proposition that x is an actual witness for ϕ, which constitutes a possibility for ϕ. To distinguish such functions, their type is written (notation from Jäger 2001) $\sigma \rightsquigarrow \mathtt{t}$, where σ is the type of witnesses for ϕ.

As an example, (36) *Mary read a book* is interpreted as:

(55) $\lambda x \in \textbf{book}.\, \textbf{read}\, x\, \textbf{mary} : \mathtt{e} \rightsquigarrow \mathtt{t}$

The domain of this function, the set of books, gives the set of potential witnesses for (36). And its image gives the set of possibilities for (36):

(56) $\mathsf{Im}((36)) = \{\textbf{read}\, x\, \textbf{mary} \mid \textbf{book}\, x\} = \mathcal{P}((36))$

The propositional content of a sentence ϕ is given by existentially quantifying over its possibility set: ϕ is true $\leftrightarrow \exists p \in \mathcal{P}(\phi).\, p$. We can retrieve the propositional content of a sentence from its interpretation as a type $\sigma \rightsquigarrow \mathtt{t}$ function by existentially quantifying over the image of the function. The existential closure operator $\downarrow$ can then be defined as follows:

(57) $\downarrow(f) = \exists p \in \mathsf{Im}(f).\, p = \exists x \in \mathsf{Dom}(f).\, fx$

The propositional content of (36) is then given by:

(58) $\downarrow((36)) = \exists p \in \mathsf{Im}(\lambda x \in \textbf{book}.\, \textbf{read}\, x\, \textbf{mary}).p = \exists x \in \textbf{book}.\, \textbf{read}\, x\, \textbf{mary} : \mathtt{t}$

Type $\sigma \rightsquigarrow \mathtt{t}$ meanings like (55) for sentences containing indefinites are derived compositionally by treating indefinites in the same manner as pronouns in the variable-free semantics of Jacobson (1999). That is, an indefinite is interpreted as a partial identity function whose domain is the extension of its restrictor:

(59) $\textbf{a book} = \lambda x \in \textbf{book}.\, x : \mathtt{e} \rightsquigarrow \mathtt{e}$

4 The *image* $\mathsf{Im}(f)$ of a function f is the set of values f takes over its domain: $\mathsf{Im}(f) = \{fx \mid x \in \mathsf{Dom}(f)\}$. This set is also called the *range* of f, but *range* is sometimes also used to refer to a function's *codomain*. Thus the term *image* is preferred to avoid this ambiguity.

Generalizing the type $\sigma \rightsquigarrow \mathtt{t}$ to $\sigma \rightsquigarrow \tau$ for any τ, an function of type $\sigma \rightsquigarrow \tau$ is a meaning of type τ which depends on a witness of type σ. The interpretation of an indefinite determiner (*a (certain)*, *some*) is then the following:

(60) $\mathbf{a} = \lambda P.\, \lambda x \in P.\, x : (\mathtt{e} \to \mathtt{t}) \to (\mathtt{e} \rightsquigarrow \mathtt{e})$

The grammar allows the λ-abstraction introduced by an indefinite to float upward, ultimately taking scope over some propositional expression. This step can be illustrated for the derivation of (36) *Mary read a book* to (55):

(61) $\mathbf{read}\ \boxed{\lambda x \in \mathbf{book}.\, x}\ \mathbf{mary}$
$\longrightarrow \lambda x \in \mathbf{book}.\, \mathbf{read}\ x\ \mathbf{mary} : \mathtt{e} \rightsquigarrow \mathtt{t}$

Shan (2001) and Szabolcsi (2003), following Jacobson (1999), implement this behavior through a family of Geach operators. For maximal generality, we instead follow Jäger (2001) in giving a type-logical presentation.

4.2 Type-logical grammar

We begin with the non-associative Lambek calculus **NL** (for a complete presentation, see Moortgat 1997 or Jäger 2005). Atomic categories are dp (determiner phrase), s (sentence), and n (common noun). Non-atomic categories are built up using the directional slashes $/$ and $\backslash$: if A and B are categories, then so are A/B and $B\backslash A$. Each syntactic category A is associated with a semantic type $[\![A]\!]$ such that an expression of category A denotes an object of type $[\![A]\!]$. This mapping $[\![\cdot]\!]$ is given as follows:

(62) $[\![\mathsf{dp}]\!] = \mathtt{e} \quad [\![\mathsf{s}]\!] = \mathtt{t} \quad [\![\mathsf{n}]\!] = \mathtt{e} \to \mathtt{t}$
$[\![A/B]\!] = [\![B\backslash A]\!] = [\![B]\!] \to [\![A]\!]$

We will also use a syntactically inert (lacking elimination or introduction rules) function connective $\to$ such that $[\![B \to A]\!] = [\![B]\!] \to [\![A]\!]$.

Jäger (2001) extends **NL** a categorial constructor $\rightsquigarrow$ for indefinites. If A and B are categories, then so is $B \rightsquigarrow A$, with $[\![B \rightsquigarrow A]\!] = [\![B]\!] \rightsquigarrow [\![A]\!]$. So an indefinite has category $\mathsf{dp} \rightsquigarrow \mathsf{dp}$, and a sentence containing a widest-scope indefinite has category $\mathsf{dp} \rightsquigarrow \mathsf{s}$. In place of Geach operators, Jäger (2001) gives the $\rightsquigarrow$ rule, which allows an expression of category $B \rightsquigarrow A$ to act locally like an A while passing up its B dependency.[5] As a sequent-style Natural Deduction rule:

$$\frac{\Delta \vdash M : B \rightsquigarrow A \qquad \Gamma[x : A] \vdash N : C}{\Gamma[\Delta] \vdash \lambda y \in \mathsf{Dom}(M).\, N\{x \mapsto My\} : B \rightsquigarrow C} \rightsquigarrow$$

5 The $\rightsquigarrow$ rule given here is a modification of Jäger's (2001) to deal explicitly with the functions being partial.

For existential closure we add a rule[6] $\exists$ to lower a function-denoting $A \rightsquigarrow \mathsf{s}$ to a proposition-denoting s:

$$\dfrac{\Gamma \vdash M : A \rightsquigarrow \mathsf{s}}{\Gamma \vdash \downarrow\!(M) : \mathsf{s}}\,\exists$$

With this we can now derive a simple indefinite sentence like (36).[7]

(63) Mary read a book.

$$\dfrac{\dfrac{\dfrac{\dfrac{\text{Mary}}{\mathbf{mary} : \mathsf{dp}} \quad \dfrac{\dfrac{\text{read}}{\mathbf{read} : (\mathsf{dp}\backslash\mathsf{s})/\mathsf{dp}} \quad \dfrac{\dfrac{\dfrac{\text{a}}{\mathbf{a} : (\mathsf{dp} \rightsquigarrow \mathsf{dp})/\mathsf{n}} \quad \dfrac{\text{book}}{\mathbf{book} : \mathsf{n}}}{\lambda x \in \mathbf{book}.\, x : \mathsf{dp} \rightsquigarrow \mathsf{dp}}\,/E}{x : \mathsf{dp}}\,1}{\mathbf{read}\; x : \mathsf{dp}\backslash\mathsf{s}}\,/E}{\mathbf{read}\; x\; \mathbf{mary} : \mathsf{s}}\,\backslash E}{\lambda x \in \mathbf{book}.\, \mathbf{read}\; x\; \mathbf{mary} : \mathsf{dp} \rightsquigarrow \mathsf{s}}\,\rightsquigarrow, 1}{\exists x \in \mathbf{book}.\, \mathbf{read}\; x\; \mathbf{mary} : \mathsf{s}}\,\exists$$

The application of the $\rightsquigarrow$ rule is not clause-bounded, so we immediately generate wide-scope readings of indefinites:

(64) If we invite some philosopher, Max will be offended.

$$\dfrac{\dfrac{\text{we}}{\mathbf{we} : \mathsf{dp}} \quad \dfrac{\dfrac{\text{invite}}{\mathbf{invite} : (\mathsf{dp}\backslash\mathsf{s})/\mathsf{dp}} \quad \dfrac{\dfrac{\dfrac{\text{some}}{\mathbf{some} : (\mathsf{dp} \rightsquigarrow \mathsf{dp})/\mathsf{n}} \quad \dfrac{\text{philosopher}}{\mathbf{phil} : \mathsf{n}}}{\lambda x \in \mathbf{phil}.\, x : \mathsf{dp} \rightsquigarrow \mathsf{dp}}\,/E}{x : \mathsf{dp}}\,1}{\mathbf{invite}\; x : \mathsf{dp}\backslash\mathsf{s}}\,/E}{\mathbf{invite}\; x\; \mathbf{we} : \mathsf{s}}\,\backslash E$$

$$\vdots$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\text{if}}{\mathbf{if} : (\mathsf{s}/\mathsf{s})/\mathsf{s}} \quad \begin{matrix}\vdots \\ \mathbf{invite}\; x\; \mathbf{we} : \mathsf{s}\end{matrix}}{\mathbf{if}\; (\mathbf{invite}\; x\; \mathbf{we}) : \mathsf{s}/\mathsf{s}}\,/E \quad \dfrac{\begin{matrix}\text{Max will be offended} \\ \vdots\end{matrix}}{\mathbf{offended}\; \mathbf{max} : \mathsf{s}}}{\mathbf{if}\; (\mathbf{invite}\; x\; \mathbf{we})\; (\mathbf{offended}\; \mathbf{max}) : \mathsf{s}}\,/E}{\lambda x \in \mathbf{phil}.\, \mathbf{if}\; (\mathbf{invite}\; x\; \mathbf{we})\; (\mathbf{offended}\; \mathbf{max}) : \mathsf{dp} \rightsquigarrow \mathsf{s}}\,\rightsquigarrow, 1}{\exists x \in \mathbf{phil}.\, \mathbf{if}\; (\mathbf{invite}\; x\; \mathbf{we})\; (\mathbf{offended}\; \mathbf{max}) : \mathsf{s}}\,\exists$$

6 Alternatively, we could introduce $\exists$ as a polymorphically typed lexical item: $\exists : \mathsf{s}/(A \rightsquigarrow \mathsf{s}) = \lambda F.\!\downarrow\!(F)$ for any category A.

7 We give the rules in sequent-style format and set the derivations in tree format. These decisions are purely presentational. See Jäger (2001) for discussion.

Non-maximal application of the $\rightsquigarrow$ and $\exists$ rules gives an indefinite non-maximal scope. For instance, application of these rules to the antecedent $\textbf{invite}\ x\ \textbf{we} : \mathsf{s}$ in (64) will give *some philosopher* local scope:

(65) $\textbf{if}\ (\downarrow(\lambda x \in \textbf{philosopher}.\,\textbf{invite}\ x\ \textbf{we}))\ (\textbf{offended max})$
$= \textbf{if}\ (\exists x \in \textbf{philosopher}.\,\textbf{invite}\ x\ \textbf{we})\ (\textbf{offended max}) : \mathtt{t}$

4.3 Witness-aware *every*

The desired functional reading of (10) is (66):

(66) $\exists f \in [\textbf{student} \to \textbf{area}].\,\textbf{if}\ (\forall x \in \textbf{student}.\,\textbf{progress}\ (f\,x)\ x)\ (\neg\exists x.\,\textbf{flunk}\ x) : \mathtt{t}$

As we saw in §3.3 equivalent to existentially quantifying over the possibility set of the antecedent *every student makes progress in a certain area*. In the variable-free system presented above, we do not quantify directly over a a sentence's possibilities, but achieve the same effect by quantifying over its potential witnesses, as given by its interpretation as a function of type $\sigma \rightsquigarrow \mathtt{t}$. In this case, we should have functional witnesses: $\sigma = \mathtt{e} \to \mathtt{e}$. So we need the antecedent (67) to denote the $(\mathtt{e} \to \mathtt{e}) \rightsquigarrow \mathtt{t}$ function (syntactically $(\mathsf{dp} \to \mathsf{dp}) \rightsquigarrow \mathsf{s}$) in (68).

(67) Every student makes progress in a certain area.

(68) $\lambda f \in [\textbf{student} \to \textbf{area}].\,\forall x \in \textbf{student}.\,\textbf{progress}\ (f\,x)\ x : (\mathtt{e} \to \mathtt{e}) \rightsquigarrow \mathtt{t}$

If we can achieve this, the grammar we have already given will generate the desired functional reading reading for (10).

(69) If every student makes progress in a certain area, nobody will flunk.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\text{if}}{\textbf{if} : (\mathsf{s}/\mathsf{s})/\mathsf{s}} \quad \dfrac{\dfrac{\begin{array}{c}(67)\\ \vdots\end{array}}{(68) : (\mathsf{dp} \to \mathsf{dp}) \rightsquigarrow \mathsf{s}}}{p : \mathsf{s}}\,1}{\textbf{if}\ p : \mathsf{s}/\mathsf{s}}\,/\mathrm{E} \qquad \dfrac{\begin{array}{c}\text{nobody will flunk}\\ \vdots\end{array}}{\neg\exists x.\,\textbf{flunk}\ x : \mathsf{s}}}{\textbf{if}\ p\ (\neg\exists x.\,\textbf{flunk}\ x) : \mathsf{s}}\,/\mathrm{E}}{\lambda f \in [\textbf{student} \to \textbf{area}].\,\textbf{if}\ (\forall x \in \textbf{student}.\,\textbf{progress}\ (f\,x)\ x)\ (\neg\exists x.\,\textbf{flunk}\ x) : (\mathsf{dp} \to \mathsf{dp}) \rightsquigarrow \mathsf{s}}\,\rightsquigarrow,1}{\exists f \in [\textbf{student} \to \textbf{area}].\,\textbf{if}\ (\forall x \in \textbf{student}.\,\textbf{progress}\ (f\,x)\ x)\ (\neg\exists x.\,\textbf{flunk}\ x) : \mathsf{s}}\,\exists$$

The question then is how (67) is interpreted as (68). The answer I propose is that the responsibility lies in the semantics of the distributive quantifier *every student*, an answer whose availability relies crucially on the explicit semantic representation of open indefinites provided by the variable-free semantics.

The standard analysis of a distributive quantifier like *every student* is as an $(\texttt{e} \to \texttt{t}) \to \texttt{t}$ generalized quantifier:

(70) $\textbf{every student} = \lambda k.\, \forall x \in \textbf{student}.\, kx : (\texttt{e} \to \texttt{t}) \to \texttt{t}$

Thus if an indefinite is to take narrow scope with respect to *every student*, it must be existentially closed in its nuclear scope. This fails to capture the fact that if the nuclear scope of a distributive quantifier depends on a witness, then the whole $\delta >$ Indef sentence depends on a functional witness. But we can capture this by generalizing the type of quantifiers based on *every*, from applying only to a nuclear scope of type $\texttt{e} \to \texttt{t}$ and returning a simple proposition of type $\texttt{t}$, to applying to a nuclear scope of type $\texttt{e} \to (\sigma \rightsquigarrow \texttt{t})$ and functionalizing the witnesses, returning an object of type $(\texttt{e} \to \sigma) \to \texttt{t}$: a proposition which depends on a functional witness.

For instance, we want:

(71) $\textbf{every student}\ \overbrace{(\lambda y.\, \lambda x \in \textbf{area}.\, \textbf{progress}\, x\, y)}^{\texttt{e} \to (\texttt{e} \rightsquigarrow \texttt{t})}$
$= \lambda f \in [\textbf{student} \to \textbf{area}].\, \forall y \in \textbf{student}.\, \textbf{progress}\, (fy)\, y : (\texttt{e} \to \texttt{e}) \rightsquigarrow \texttt{t}$

The general case is **every** $P\, K$, where $P : \texttt{e} \to \texttt{t}$ and $K : \texttt{e} \to (\sigma \rightsquigarrow \texttt{t})$. Recall that the set of potential witnesses encoded by a $\sigma \to \texttt{t}$ function f is given by its domain $\mathsf{Dom}(f)$. It turns out in this case, with $K = \lambda y.\, \lambda x \in \textbf{area}.\, \textbf{progress}\, x\, y$, that $\mathsf{Dom}(Ky)$ is the same (**area**) for each student y. But this need not be the case: replace *a certain area* with *a certain area he likes* (with *he* understood as bound by *every student*):

(72) If every student$_i$ makes progress in a certain area he$_i$ likes, nobody will flunk.

A potential witness for (72) is not merely a function from students to areas, but a function that maps each student to an area he likes. We write the set of such functions as (74), using the notation in (73). Crucially x can occur free in N in this notation.

(73) $[M \to_x N] = \{f \mid \mathsf{Dom}(f) = M, \forall x \in \mathsf{Dom}(f).\, fx \in N\}$

(74) $[\textbf{student} \to_x (\lambda z.\, \textbf{area}\, z \wedge \textbf{likes}\, z\, x)]$

The following denotation for *every* then captures how distributive quantifiers functionalize witnesses in their scope.

(75) $\textbf{every} : (\texttt{e} \to \texttt{t}) \to (\texttt{e} \to (\sigma \rightsquigarrow \texttt{t})) \to (\texttt{e} \to \sigma) \rightsquigarrow \texttt{t}$
$= \lambda P.\, \lambda K.\, \lambda f \in [P \to_x \mathsf{Dom}(Kx)].\, \forall x \in P.\, K\, x\, (fx)$

This denotation for *every* gives the result in (71), as desired. We will see in the next section how this semantics can be combined with a standard type-logical account of quantification so that the derivation of the functional reading of (10) in (69) can be completed.

First, what of the case where the nuclear scope of an *every* quantifier does *not* depend on a witness, so that the nuclear scope is simply of the ordinary type $\mathtt{e} \to \mathtt{t}$? Do we need to retain the ordinary, non-witness-aware denotation of *every* to handle these cases? This can be avoided by, following Shan (2001), introducing a *unit type* 1 with a singleton domain $\{*\}$. Then any type ρ is isomorphic to the function type $1 \rightsquigarrow \rho$. Identifying the isomorphic types ρ and $1 \rightsquigarrow \rho$, the ordinary meaning of *every* can be recovered by setting $\sigma = 1$.[8] So we can take (75) to be *the* semantics of *every*; we do not need to post an additional lexical entry (or type-shifting principle) to account for functional readings. It is part of the *meaning* of *every* that it produces sentences which require functional witnesses.

4.4 Quantification in type-logical grammar

We implement quantification in our type-logical grammar with Moortgat's (1997) q type constructor. An expression of category $q(A,B,C)$ is an expression that acts locally (for syntactic selection) as an A, and takes scope over an expression of category B to give an expression of category C. Semantically, $[\![q(A,B,C)]\!] = ([\![A]\!] \to [\![B]\!]) \to [\![C]\!]$. For example, an ordinary generalized quantifier (*no girl*) will have category $q(\mathsf{dp},\mathsf{s},\mathsf{s})$, giving the expected semantic type $(\mathtt{e} \to \mathtt{t}) \to \mathtt{t}$. Scope-taking is achieved by the q elimination rule, analogous to QR:

$$\frac{\Delta \vdash M : q(A,B,C) \qquad \Gamma[x:A] \vdash N : B}{\Delta[\Gamma] \vdash M(\lambda x.N) : C}\, q\mathrm{E}$$

To make use of the semantics for *every* given above, we need do nothing more than assign *every* the appropriate syntactic category: $q(\mathsf{dp}, A \rightsquigarrow \mathsf{s}, (\mathsf{dp} \to A) \rightsquigarrow \mathsf{s})/\mathsf{n}$, for any category A.[9] This category encodes that a quantifier like *every student* does not take scope over a simple sentence (s), but over a sentence that depends on a witness of category A ($A \rightsquigarrow \mathsf{s}$), and instead of producing a simple sentence, it produces a sentence that depends on a functional witness ($(\mathsf{dp} \to A) \rightsquigarrow \mathsf{s}$).

Finally, we can complete the derivation of the functional reading of (10), the latter part of which was given in (69).[10]

(76) If [every student makes progress in a certain area], nobody will flunk.

8 This will result in a sentence like *Every boy left* having an interpretation of type $(\mathtt{e} \to 1) \rightsquigarrow \mathtt{t}$. In the presence of partial functions, $\mathtt{e} \to 1$ is not isomorphic to 1. But for any $P : \mathtt{e} \to \mathtt{t}$, $[P \to \{*\}]$ is guaranteed nonempty, so $\exists f \in [P \to \{*\}].M$ will be equivalent to just $M : \mathtt{t}$ (f will never occur free in M).

9 Here we make use of the syntactically inert connective $\to$ discussed at the outset of §4.2.

10 To conserve space in derivations, I sometimes write $M : A$ across two lines as $\begin{matrix} M \\ A \end{matrix}$.

$$\dfrac{\dfrac{\text{every}}{\textbf{every}: q(\mathsf{dp},\mathsf{dp}\rightsquigarrow\mathsf{s},(\mathsf{dp}\to\mathsf{dp})\rightsquigarrow\mathsf{s})/\mathsf{n}} \quad \dfrac{\text{student}}{\textbf{student}: \mathsf{n}}}{\textbf{every student}: q(\mathsf{dp},\mathsf{dp}\rightsquigarrow\mathsf{s},(\mathsf{dp}\to\mathsf{dp})\rightsquigarrow\mathsf{s})}\,/E$$

$$\dfrac{\textbf{every student}: q(\mathsf{dp},\mathsf{dp}\rightsquigarrow\mathsf{s},(\mathsf{dp}\to\mathsf{dp})\rightsquigarrow\mathsf{s})}{y:\mathsf{dp}}\,2$$

$$\dfrac{\dfrac{\text{a certain}}{\textbf{a}: (\mathsf{dp}\rightsquigarrow\mathsf{dp})/\mathsf{n}} \quad \dfrac{\text{area}}{\textbf{area}: \mathsf{n}}}{\lambda x\in\textbf{area}.\, x : \mathsf{dp}\rightsquigarrow\mathsf{dp}}\,/E \qquad \dfrac{\mathsf{dp}\rightsquigarrow\mathsf{dp}}{x : \mathsf{dp}}\,1$$

$$\dfrac{\dfrac{\text{makes progress in}}{\textbf{progress}: (\mathsf{dp}\backslash\mathsf{s})/\mathsf{dp}} \quad x:\mathsf{dp}}{\textbf{progress}\; x : \mathsf{dp}\backslash\mathsf{s}}\,/E$$

$$\dfrac{\dfrac{\dfrac{y:\mathsf{dp} \quad \textbf{progress}\; x : \mathsf{dp}\backslash\mathsf{s}}{\textbf{progress}\; x\; y : \mathsf{s}}\,\backslash E}{\lambda x\in\textbf{area}.\,\textbf{progress}\; x\; y : \mathsf{dp}\rightsquigarrow\mathsf{s}}\,\rightsquigarrow,1}{\textbf{every student}\;(\lambda y.\,\lambda x\in\textbf{area}.\,\textbf{progress}\; x\; y) : (\mathsf{dp}\to\mathsf{dp})\rightsquigarrow\mathsf{s}}\,qE,2$$

$$=$$

$$\dfrac{\lambda f\in[\textbf{student}\to\textbf{area}].\,\forall y\in\textbf{student}.\,\textbf{progress}\;(f y)\; y : (\mathsf{dp}\to\mathsf{dp})\rightsquigarrow\mathsf{s}}{\exists f\in[\textbf{student}\to\textbf{area}].\,\forall y\in\textbf{student}.\,\textbf{progress}\;(f y)\; y : \mathsf{s}}\,\exists$$

$$\vdots$$

4.5 No functional reading for *no*

As we have seen, *no* (and other negative quantifiers) does not license functional readings. This is immediately captured if, unlike *every*, *no* retains its ordinary generalized quantifier semantics:

(77) $\textbf{no} = \lambda P.\,\lambda k.\,\neg\exists x\in P.\,kx : (\mathtt{e}\to\mathtt{t})\to(\mathtt{e}\to\mathtt{t})\to\mathtt{t}$

This meaning — or rather, the lack of a witness-aware meaning — reflects the fact, discussed in §3.3, that when sentence ϕ has the form of a negative quantifier taking scope over an indefinite, for instance *No girl read a book*, ϕ does not give rise to multiple possibilities — it can only be true in one way. Thus the denotation of such a sentence cannot be a $\sigma\rightsquigarrow\mathtt{t}$ function with a non-singleton image.

Crucially, moving from the standard generalized quantifier denotation for *every* to the witness-aware denotation (75) only leads to new interpretations for a $\delta >$ Indef complex embedded under another operator. In the simple case, existentially quantifying over functional witnesses immediately above a universal quantifier is equivalent to existentially quantifying over individuals immediately beneath it. This is the equivalence behind Skolemization:

(78) $\exists f\in[B\to A].\,\forall y\in B.\,R\,(f y)\,y \quad = \quad \forall y\in B.\,\exists x\in A.\,R\,x\,y$

As we have seen, the fundamental problem for Skolemized choice function theories of indefinites is that this equivalence does *not* hold for other quantifiers, in particular negative ones, including *no* ($\neg\exists$):

(79) $\exists f\in[B\to A].\,\neg\exists y\in B.\,R\,(f y)\,y \quad \neq \quad \neg\exists y\in B.\,\exists x\in A.\,R\,x\,y$

Thus giving *no* a witness-functionalizing denotation $\mathbf{no}^*$ following the template of that given for *every*, would introduce a *new interpretation* for monoclausal sentences of the form *no* $>$ *a*, not obtainable with *no*'s basic generalized quantifier meaning.

(80) $\mathbf{no}^* : (\mathtt{e} \to \mathtt{t}) \to (\mathtt{e} \to (\sigma \rightsquigarrow \mathtt{t})) \to (\mathtt{e} \to \sigma) \rightsquigarrow \mathtt{t}$
$= \lambda P.\, \lambda K.\, \lambda f \in [P \to_x \mathbf{Dom}(Kx)].\, \neg\exists x \in P.\, K\, x\, (fx)$

That is, we would have:

(81) $\downarrow(\mathbf{no}^*\, B\, (\lambda y.\, \lambda x \in A.\, R\, x\, y)) \quad \neq \quad \downarrow(\mathbf{no}^*\, B\, (\lambda y. \downarrow(\lambda x \in A. R\, x\, y)))$

whereas:

(82) $\downarrow(\mathbf{every}\, B\, (\lambda y.\, \lambda x \in A.\, R\, x\, y)) \quad = \quad \downarrow(\mathbf{every}\, B\, (\lambda y. \downarrow(\lambda x \in A. R\, x\, y)))$

Unlike with **every**, existential closure immediately below $\mathbf{no}^*$ does not yield the same result as existential closure immediately above it, and so $\mathbf{no}^*$ yields a spurious reading. Perhaps this is not a strictly impossible lexical item, but it is not one that has the meaning of *no* in English, whose truth-conditional meaning is set by the well-known generalized quantifier semantics. The same is true of *every*, but in this case functional witnesses provide a different way of structuring the *same* generalized quantifier meanings. They only lead to new interpretations when *every* is embedded under another operator, and existential closure over functional witnesses can occur higher. In the unembedded case, the basic generalized quantifier semantics must be preserved.

The meaning of $\phi =$ *No girl read a book* (on its surface scope reading) cannot be a function whose domain is the set of functions from girls to books because such functions are not potential witnesses for ϕ: ϕ is not made true by Mary not reading *War and Peace*, Sue not reading *Anna Karenina*, and so on. This is the fundamental difference between *every* and *no*: a sentence of the form *every* $>$ *a* can be true in many ways, each given by a functional witness. Functional witnesses have no role in the semantics of sentences of the form *no* $>$ *a*; such sentences give rise to a single possibility.

The surface scope (*no girl* $>$ *a book*) reading of (83) can be derived as follows, with existential closure in nuclear scope of *no girl*

(83) No girl read a book.

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\text{no}}{\textbf{no}:q(\mathsf{dp},\mathsf{s},\mathsf{s})}\quad\dfrac{\text{girl}}{\textbf{girl}:\mathsf{n}}}{\textbf{no girl}:q(\mathsf{dp},\mathsf{s},\mathsf{s})}\,/\mathrm{E}}{y:\mathsf{dp}}\,2
\qquad
\dfrac{\dfrac{\text{read}}{\textbf{read}:(\mathsf{dp}\backslash\mathsf{s})/\mathsf{dp}}\quad\dfrac{\dfrac{\dfrac{\text{a}}{\textbf{a}:(\mathsf{dp}\rightsquigarrow\mathsf{dp})/\mathsf{n}}\quad\dfrac{\text{book}}{\textbf{book}:\mathsf{n}}}{\lambda x\in\textbf{book}.\,x:\mathsf{dp}\rightsquigarrow\mathsf{dp}}\,/\mathrm{E}}{x:\mathsf{dp}}\,1}{\textbf{read}\;x:\mathsf{dp}\backslash\mathsf{s}}\,/\mathrm{E}}{\textbf{read}\;x\;y:\mathsf{s}}\,\backslash\mathrm{E}}{\lambda x\in\textbf{book}.\,\textbf{read}\;x\;y:\mathsf{dp}\rightsquigarrow\mathsf{s}}\,\rightsquigarrow,1}{\exists x\in\textbf{book}.\,\textbf{read}\;x\;y:\mathsf{s}}\,\exists}{\textbf{no girl}\;(\lambda y.\,\exists x\in\textbf{book}.\,\textbf{read}\;x\;y):\mathsf{s}}\,q\mathrm{E},2
$$

$$
= \neg\exists y\in\textbf{girl}.\,\exists x\in\textbf{book}.\,\textbf{read}\;x\;y:\mathsf{s}
$$

To recap, on the approach presented here, indefinites always make a uniform semantic contribution (essentially a restricted free variable, implemented compositionally in a variable-free semantics). Functional readings of indefinites in the scope of a distributive quantifier arise because of the witness semantics of distributive quantifiers — Skolemization is built into their lexical meaning. Other quantifiers do not give rise to functional witnesses, and thus do not give rise to functional readings of indefinites.

5 Extending the analysis

5.1 Plurals

As discussed in §2.3, (84) has a functional reading.

(84) If two students each make progress in a certain area, the exam will be a success.

This follows if *DP each VP* has the same meaning as [*every one of DP*] *VP*, which is accomplished by the lexical entry for *each* in (85).

(85) $\textbf{each} : (\mathsf{dp}\backslash((\mathsf{dp}\to A)\rightsquigarrow\mathsf{s}))/(\mathsf{dp}\backslash(A\rightsquigarrow\mathsf{s}))$
$= \lambda P.\lambda X.\,\textbf{every}\;(\textbf{atoms}\,X)\;P$
$= \lambda P.\lambda X.\lambda f\in[(\textbf{atoms}\,X)\to_x \mathsf{Dom}(Px)].\,\forall x\in\textbf{atoms}\,X.\,P\,x\,(fx)$

where $\textbf{atoms}\,X$ is the set of atomic parts of X. We treat *two students* as an indefinite over plural entities:

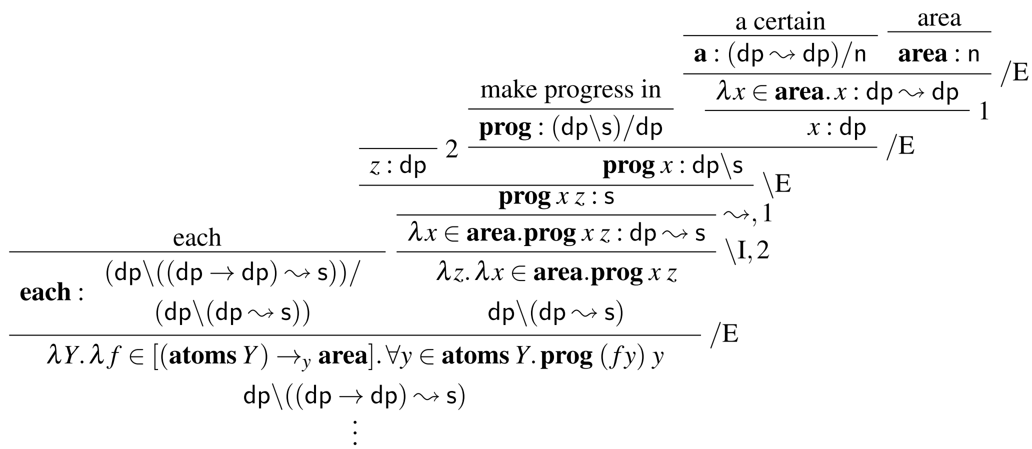
(86) $\textbf{two students} : \mathsf{dp}\rightsquigarrow\mathsf{dp} = \lambda X\in\mathbf{2}\cap\textbf{students}.\,X$

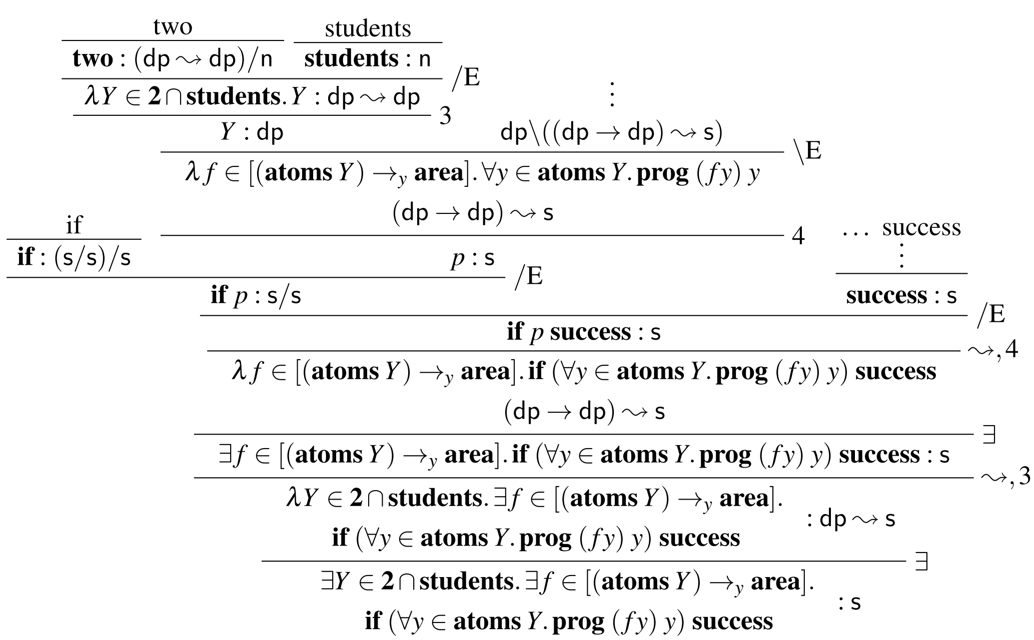
So:

(87) $\textbf{two} : (\mathsf{dp}\rightsquigarrow\mathsf{dp})/\mathsf{n} = \lambda P.\lambda X\in\mathbf{2}\cap P.\,X$

Given these lexical entries, we can derive the functional indefinite interpretation of (84):

(88) If two students each make progress in a certain area, the exam will be a success.





Unlike the Skolemized choice function theory, our account of functional indefinites does not generate the unavailable reading discussed in §2.3. Existential quantification over functions is always quantification over functional *witnesses*, which arise when a distributive quantifier takes scope over an indefinite. Then the domain of the function is the set the distributive quantifier distributes over. In the case of (84), this is always a set of two students. So, without further stipulation, the present account correctly predicts that there is no way to generate a reading of (84) involving quantification over functions with whose domain is all students.

5.2 Plural quantifiers

We have seen that functional readings are unavailable when an indefinite scopes under a negative quantifier because semantically, such sentences do not have functional witnesses. However, this cannot be the explanation for all quantifiers that do not give rise to functional readings. Spector (2004) observes that definite plurals allow functional readings of indefinites only with overt *each* (# indicates no functional reading):

(89) a. #Mary hopes that two guests bring a certain dish.
b. Mary hopes that two guests each bring a certain dish.

Thus overt *each* differs in this respect from the covert distributive operator $dist_{\varnothing}$ implicated by the interpretation of (90a) equivalent to (90b).

(90) a. The students had an espresso.
b. The students each had an espresso.

Thus *each* and $dist_{\varnothing}$ apparently make the same contribution to truth-conditional meaning, but only *each* has a semantics that makes reference to witnesses. $dist_{\varnothing}$ must have the ordinary lexical entry as in (91).

(91) $\mathbf{dist}_{\varnothing} : (\mathsf{dp}\backslash\mathsf{s})/(\mathsf{dp}\backslash\mathsf{s}) = \lambda P.\lambda X.\forall x \in \mathbf{atoms}\ P.Fx$

Moreover, Spector (2004) observes that even upward-entailing plural quantifiers do not give rise to functional readings:

(92) #If $\left\{\begin{array}{c}\text{many}\\ \text{most}\\ \text{all}\end{array}\right\}$ (of the) students make progress in a certain area, the exam will be a success.
(# = no functional reading)

Quantifiers like *many* and *most* could in principle have a witness semantics like plural indefinites (*most of* ≈ *a majority of*), but unlike genuine indefinites, they do not give rise to wide-scope existential readings:

(93) If many of my relatives die, I'll inherit a house.
≠ There is a large group of my relatives such that if they all die, I'll inherit a house.

This behavior follows if these plural quantifiers do not have a witness-aware semantics, but an ordinary generalized quantifier semantics:

(94) **many** : $q(\mathsf{dp}, \mathsf{s}, \mathsf{s})/\mathsf{n} = \lambda P.\, \lambda k.\, \mathbf{large}\ |P \cap k|$

Alternatively, plural quantifiers could be taken to be plural indefinites (or definites in the case of *all*) whose existential scope is specially clause-bounded.

(95) **many** : $(\mathsf{dp} \rightsquigarrow_{\times} \mathsf{dp})/\mathsf{n} = \lambda P.\, \lambda X \in (\lambda Y.\, Y \in P \wedge \mathbf{large}\ |Y|).\, X$

In sentences like (92), distributivity would always come from the covert distributor $\text{dist}_{\varnothing}$, which does not license functional readings.

5.3 *A different*

We have established that only overt distributors *every* and *each* license functional readings of indefinites. In fact, these are the same quantifiers that license sentence-internal readings of *a different* (Beghelli & Stowell 1997, Brasoveanu 2011):

(96) a. Every boy recited a different poem.
b. #Many (of the) boys recited a different poem.

(97) a. The boys each recited a different poem.
b. #The boys recited a different poem.

Only the (a) sentences have a sentence-internal reading (the boys read different poems *from each other*).

Given the semantics I have independently developed for distributives and indefinites to account for functional readings of indefinites, a straightforward semantics for *different* automatically accounts for its distribution. The idea is that *different* contributes a property of sentences with functional witnesses. So (96a) says that the (functional) witness f of *Every boy recited a poem* is one-to-one, that is, that for no two entities x, y in its domain (in this case boys) does $f(x) = f(y)$. This interpretation of (96a) is in (98), with the definition of **one-to-one** in (99)

(98) $\mathbf{one\text{-}to\text{-}one}\ (\iota f \in [\mathbf{boy} \to \mathbf{poem}].\, \forall y \in \mathbf{boy}.\, \mathbf{recited}\ (f y)\ y)$
There do not exist distinct boys x and y such that the poem x recited is the same as the poem y read.

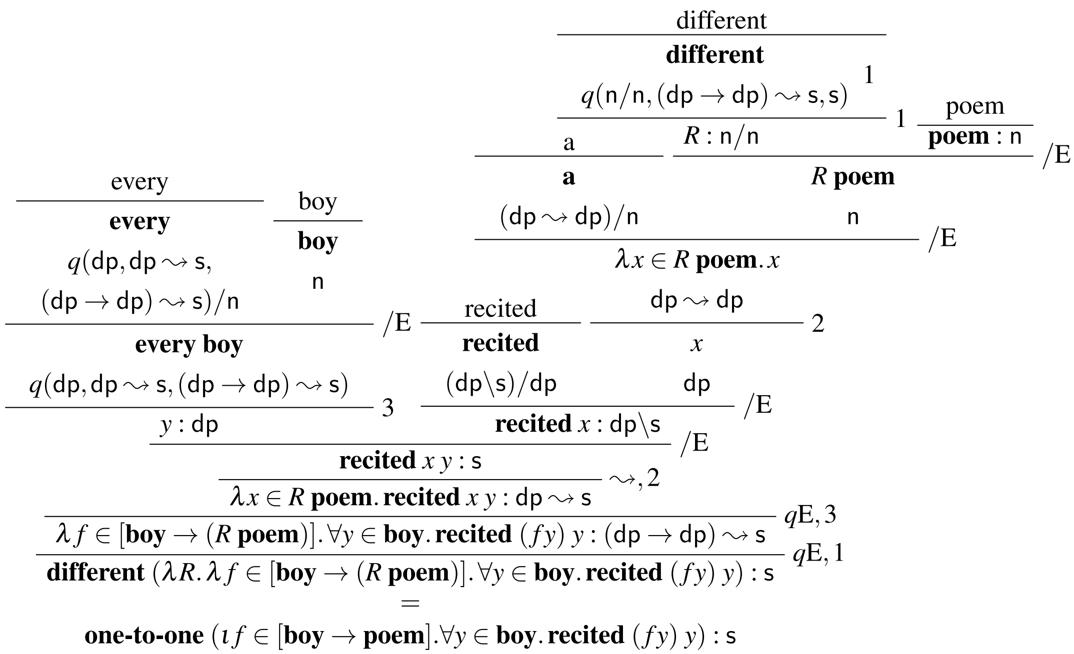
(99) $\mathbf{one\text{-}to\text{-}one} = \lambda f.\, \neg\exists x \in \mathsf{Dom}(f).\, \exists y \in \mathsf{Dom}(f).\, x \neq y \wedge f x = f y$

Compositionally, we can follow Barker (2007) in taking *different* to be a scope-taking adjective (n/n), but that takes scope over a sentence with a functional witness, $(\mathsf{dp} \to A) \rightsquigarrow \mathsf{s}$, to give an s. So *different* has the quantificational category $q(\mathsf{n}/\mathsf{n}, (\mathsf{dp} \to A) \rightsquigarrow \mathsf{s}, \mathsf{s})$, and the semantics in (100) (*different* binds its trace with the identity function).

(100) **different** : $q(\mathsf{n}/\mathsf{n}, (\mathsf{dp} \to A) \rightsquigarrow \mathsf{s}, \mathsf{s})$
$= \lambda F.\, \textbf{one-to-one}\ (\iota f \in \mathsf{Dom}(F(\lambda P.P)).\, F f)$

A complete derivation of (96a) is given in (101).

(101) Every boy recited a different poem.



Given this category assignment, the distribution of singular *different* follows with no further stipulation, since categories of the form $(\mathsf{dp} \to A) \rightsquigarrow \mathsf{s}$ are only created by distributive quantifiers. We take this as evidence that the distinguishing property of (true) distributivity may be tied to functional witnesses.

6 Conclusion

I have argued that existing accounts of functional indefinites in terms of Skolemized choice functions are empirically inadequate. They fail to account for the central fact that unrestricted functional interpretations of indefinites, parallel to pair-list interpretations of questions, only arise when an indefinite scopes under a distributive quantifier. I have argued that functional readings arise in this configuration because such sentences have *functional witnesses*, and existential closure of indefinites is actually existential quantification over potential witnesses. I have shown that this analysis can be straightforwardly implemented using a variable-free semantics for indefinites. On this analysis, indefinites always make a uniform semantic contribution, and it is part of the semantic contribution of a distributive quantifier to turn a dependence on individual witnesses in its nuclear scope into a single dependence on a functional witness. That is, Skolemization resides in the semantics of distributive quantification. In addition to accounting for the distribution of functional interpretations of indefinites, the proposed semantics for indefinites and quantifiers yields a straightforward semantics for singular *a different*, whose sentence-internal reading similarly only arises when in the scope of a distributive quantifier, as a predicate on propositions which require a functional witness. Thus the paper proposes functional witnesses as a core aspect of the semantics of true (*every* and *each*) distributivity, suggesting a new light in which to investigate further phenomena linked to distributivity.

References

Abusch, Dorit. 1994. The scope of indefinites. *Natural Language Semantics* 2(2). 83–135.

Barker, Chris. 2007. Parasitic scope. *Linguistics and Philosophy* 30(4). 407–444.

Beghelli, Filippo. 1997. The syntax of distributivity and pair-list readings. In Anna Szabolcsi (ed.), *Ways of Scope Taking*, 349–408. Dordrecht: Kluwer.

Beghelli, Filippo & Tim Stowell. 1997. Distributivity and negation: The syntax of *each* and *every*. In Anna Szabolcsi (ed.), *Ways of Scope Taking*, 71–108. Dordrecht: Kluwer.

Brasoveanu, Adrian. 2011. Sentence-internal *different* as quantifier-internal anaphora. To appear in *Linguistics and Philosophy*.

Ciardelli, Ivano. 2009. *Inquisitive Semantics and Intermediate Logics*. M.Sc. thesis, ILLC, University of Amsterdam.

Chierchia, Gennaro. 1993. Questions with quantifiers. *Natural Language Semantics* 1(2). 181–234.

Chierchia, Gennaro. 2001. A puzzle about indefinites. In Carlo Cecchetto, Gennaro Chierchia, & Maria Teresa Guasti (eds.), *Semantic Interfaces: Reference, Anaphora and Aspect*, 51–89. CSLI.

Farkas, Donka. 1981. Quantifier scope and syntactic islands. *Proceedings of the Chicago Linguistic Society* 17. 59–66.

Fodor, Janet Dean & Ivan Sag. 1982. Referential and quantificational indefinites. *Linguistics and Philosophy* 5(3). 355–398.

Geurts, Bart. 2000. Indefinites and choice functions. *Linguistic Inquiry* 31(4). 731–738.

Groenendijk, Jeroen & Floris Roelofsen. 2009. Inquisitive semantics and pragmatics. Paper presented at the Stanford Workshop on Language, Communication and Rational Agency.

Heim, Irene. 1982. *The Semantics of Definite and Indefinite Noun Phrases*. Ph.D. dissertation, University of Massachusetts, Amherst.

Hintikka, Jaakko. 1986. The semantics of *a certain*. *Linguistic Inquiry* 17(2). 331–336.

Jacobson, Pauline. 1999. Towards a variable-free semantics. *Linguistics and Philosophy* 22(2). 117–185.

Jäger, Gerhard. 2001. Indefinites and sluicing. A type logical approach. *Proceedings of the Amsterdam Colloquium* 13. 114–119.

Jäger, Gerhard. 2005. *Anaphora and Type Logical Grammar*. Dordrecht: Springer.

Jäger, Gerhard. 2007. Partial variables and specificity. Uli Sauerland & Penka Stateva (eds.), *Presupposition and Implicature in Compositional Semantics*, 121–162. Basingstoke: Palgrave Macmillan.

Kamp, Hans. 1981. A theory of truth and semantic representation. In Jeroen Groenendijk, Theo Janssen & Martin Stokhof (eds.), *Formal Methods in the Study of Language*, 277–322. Mathematical Centre, Amsterdam.

Kratzer, Angelika. 1998. Scope or pseudoscope? Are there wide-scope indefinites? In Susan Rothstein (ed.), *Events and Grammar*, 163–196. Dordrecht: Kluwer.

Krifka, Manfred. 2001. Quantifying into question acts. *Natural Language Semantics* 9(1). 1–40.

Mascarenhas, Salvador. 2009. *Inquisitive Semantics and Logic*. M.Sc. thesis, ILLC, University of Amsterdam.

Moortgat, Michael. 1997. Categorial type logics. In Johan van Benthem & Alice ter Meulen (eds.), *Handbook of Logic and Language*, 93–177. Amsterdam: Elsevier.

Reinhart, Tanya. 1992. Wh-in-situ: An apparent paradox. In *Proceedings of the Amsterdam Colloquium* 8. 483–491.

Reinhart, Tanya. 1997. Quantifier scope: How labor is divided between QR and choice functions. *Linguistics and Philosophy* 20(4). 335–397.

Schlenker, Philippe. 2006. Scopal independence: A note on branching and wide scope readings of indefinites and disjunctions. *Journal of Semantics* 23(3). 281–314.

Schwarz, Bernhard. 2001. Two kinds of long-distance indefinites. Ms., University of Texas, Austin.

Shan, Chung-chieh. 2001. A variable-free dynamic semantics. *Proceedings of the Amsterdam Colloquium* 13. 204–209.

Spector, Benjamin. 2004. Distributivity and specific indefinites. In Sylvia Blaho, Luis Vicente & Mark de Vos (eds.), *Proceedings of ConSOLE XII*.

Szabolcsi, Anna. 1997. Quantifiers in pair-list readings. In Anna Szabolcsi (ed.), *Ways of Scope Taking*, 71–108. Dordrecht: Kluwer.

Szabolcsi, Anna. 2003. Binding on the fly: Cross-sentential anaphora in variable-free semantics. In Geert-Jan M. Kruijff & Richard T. Oehrle (eds.), *Resource Sensitivity in Binding and Anaphora*, 215–229. Dordrecht: Kluwer.

Winter, Yoad. 1998. *Flexible Boolean Semantics*. Ph.D. dissertation, Utrecht University.
Winter, Yoad. 2004. Functional quantification. *Research on Language and Computation* 2(3). 331–363.