Implicatures of modified numerals: quality or quantity?

Liz Coppock Ivano Ciardelli Floris Roelofsen

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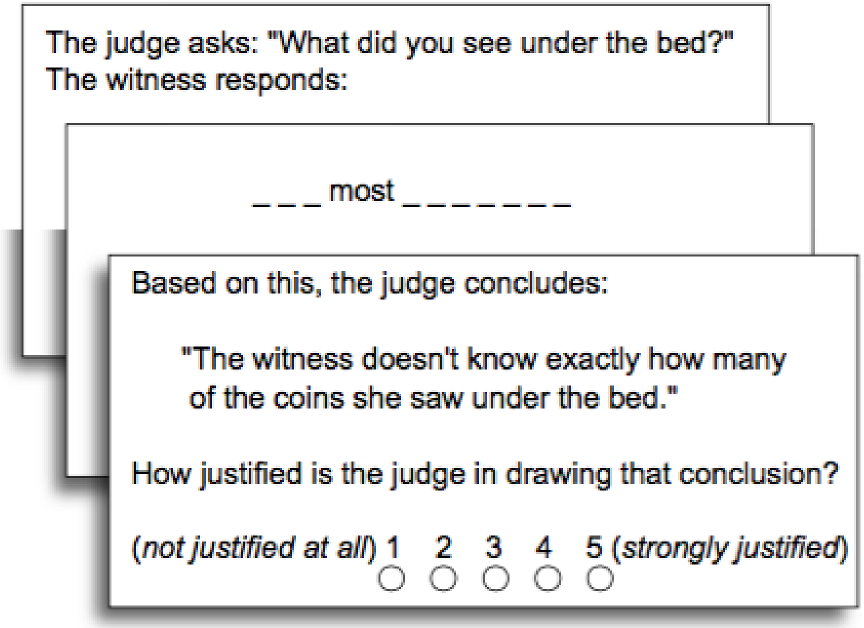
1 Introduction

1.1 Implicatures of modified numerals: the empirical targets

- We will be concerned with three types of modified numerals:
  - **at least** n
  - **more than** n
  - n **or more**
- Many authors have observed that these contrast with each other, as well as with bare numerals, in the implicatures that they give rise to:

(1) a. A hexagon has six sides. $\rightsquigarrow$ exactly six $\not\rightsquigarrow$ ignorance
b. A hexagon has more than five sides. $\not\rightsquigarrow$ exactly six $\not\rightsquigarrow$ ignorance
c. A hexagon has at least six sides. $\not\rightsquigarrow$ exactly six $\rightsquigarrow$ ignorance
d. A hexagon has six or more sides. $\not\rightsquigarrow$ exactly six $\rightsquigarrow$ ignorance

- Note that the ignorance implicature of *at least six* and *six or more* is not just that the speaker does not know exactly how many sides a hexagon has, but also that she considers it **possible** that it has **precisely six** sides.
- Westera & Brasoveanu (2014) argue based on experimental data that this basic empirical picture, which is assumed in most work on the topic, is actually a bit **too simplistic**. See Figure 1.
  - comparative modifiers can signal ignorance (e.g. with 'how many' questions)
  - ignorance can disappear for superlative modifiers (with [certain] polar questions)
- But note:
  - The difference remains in place in some contexts, including 'how many' questions.
  - There is a general trend in favor of ignorance for superlative modifiers across conditions.



POLAR Did you find {at most / less than} ten of the diamonds under the bed?

WHAT What did you find under the bed?

HOWMANY How many of the diamonds did you find under the bed?

APPROX Approximately how many of the diamonds did you find under the bed?

EXACT Exactly how many of the diamonds did you find under the bed?

DISJUNCT Did you find eight, nine, ten, or eleven of the diamonds under the bed?

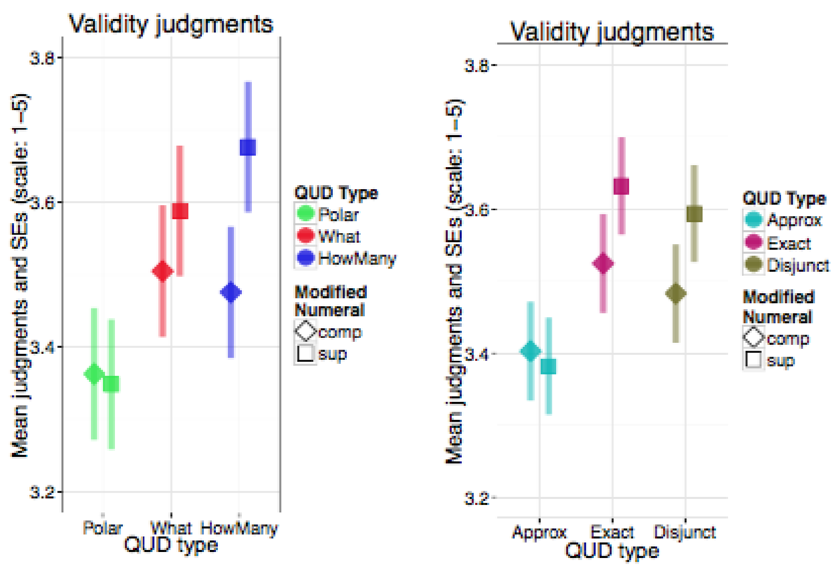


Figure 1: Westera & Brasoveanu's (2014) design and results

- W&B's 'polar question' context involves an echo response.

(2) A: Did you find at most 10 of the diamonds under the bed?
B: I found at most 10 of the diamonds under the bed.

This seems essential for the ignorance implicature not to arise. Compare:

(3) A: Did Johnny eat at least four apples today?
B: Yes, he ate at least four apples. $\not\rightsquigarrow$ ignorance

(4) A: Did Johnny eat more than three apples today?
B: Yes, he ate at least four apples. $\rightsquigarrow$ ignorance

- Moreover, if we compare *more than* and *at least* in non-echo responses to a polar question, the latter implicates ignorance but the former doesn't:

(5) *Context: Johnny's diet prescribes that he eat at most three apples per day.*
A: Did Johnny stick to his diet today?
B: No, he ate more than three apples. $\not\rightsquigarrow$ ignorance
B′: No, he ate at least four apples. $\rightsquigarrow$ ignorance

- Finally, 'out of context', *at least* signals ignorance as well, unlike *more than*:

(6) a. I grew up with more than two parents.
b. ??I grew up with at least three parents.

- So:
  - Yes, *more than* can have an ignorance implicature with a 'how many' question.
  - Yes, the ignorance implicature for *at least* can disappear in the context of certain polar question-answer scenarios (of the echo variety).
  - But the ignorance implicature for *at least* is distinctly more robust than the one for *more than*.

1.2 Quality or quantity?

- At least two approaches have been explored in the literature to explain the ignorance implicatures for superlative modifiers.
  - One approach (e.g., Mayr, 2013b; Kennedy, 2015; Schwarz, 2016) tries to derive ignorance from a particular way of computing **quantity** implicatures.
  - Another approach (Coppock & Brochhagen, 2013) is to derive ignorance as a **quality** implicature.
- Note that in other empirical domains (e.g., free choice effects of disjunction under modals or in the antecedent of a conditional), these two approaches have also both been pursued.

- We will suggest that, in the domain of modified numerals, a **combination** of the two approaches is in fact needed.

2 Previous approaches

2.1 Quantity-based

- We focus on the proposal of Schwarz (2016), but see Mayr (2013a) and Kennedy (2015) for closely related proposals.
- Schwarz is concerned with *at least n* (not with *more than n* or with *n or more*).
- Lexical assumptions:
  - Horn scale: ⟨ *at least*, *only* ⟩
  - Horn scale: ⟨ 1, 2, 3, ... ⟩
  - Alternatives for *Al hired at least two cooks*:

[1	2	3	4	...	)	at least 1
[1]	[2	3	4	...	)	only 1, at least 2
	[2]	[3	4	...	)	only 2, at least 3
		[3]	[4	...	)	only 3, at least 4
			...	...		...

- Background notions and notation:
  - A speaker's information state is a non-empty set of worlds.
  - Implicatures impose constraints on the speaker's information state.
  - A state s **supports** a sentence φ iff $s \subseteq [\![\varphi]\!]$.
  - A state s **does not support** a sentence φ iff $s \not\subseteq [\![\varphi]\!]$.
  - A state s **rejects** a sentence φ iff $s \cap [\![\varphi]\!] = \emptyset$.
  - We use A_φ to denote the set of lexically determined pragmatic alternatives for φ.
- Innocent Exclusion Based Recipe for deriving quantity implicatures:
  - Start with the **quality implicature** that the speaker's state supports φ:

    $0_\varphi = \{s \mid s \text{ supports } [\![\varphi]\!]\}$

  - Now derive **primary quantity implicatures**: The speaker's state does not support any alternative in A_φ that is stronger than φ itself. Let $A_\varphi^\subset$ be the set of those alternatives.

$$A_\varphi^\subset = \{\psi \in A_\varphi \mid [\![\psi]\!] \subset [\![\varphi]\!]\}$$

$$1_\varphi = 0_\varphi \cap \{s \mid s \text{ does not support any } \psi \in A_\varphi^{\subseteq}\}$$

- Now derive **secondary quantity implicatures** by identifying all alternatives $\psi \in A_\varphi^{\subseteq}$ such that:
  1. ψ is **not known** by the speaker according to 1_φ:
     * No $s \in 1_\varphi$ supports ψ
  2. ψ is **innocently excludable** relative to φ (Fox, 2007):
     * There is no subset A' of $A_\varphi^{\subseteq}$ such that:
       · an information state **can** validate the quality implicature and primary quantity implicatures of φ while rejecting every sentence in A':
       $$1_\varphi \cap \{s \mid s \text{ rejects every sentence in } A'\} \text{ is } \textbf{non-empty}$$
       · an information state **cannot** validate the quality implicature and primary quantity implicatures of φ while rejecting every sentence in A' *in addition to* ψ:
       $$1_\varphi \cap \{s \mid s \text{ rejects every sentence in } A' \cup \{\psi\}\} \text{ is } \textbf{empty}$$

  If $\psi \in A_\varphi^{\subseteq}$ is not known by the speaker according to 1_φ and innocently excludable relative to φ, then we say that ψ is **eligible for a secondary quantity implicature**.

$$2_\varphi = 1_\varphi \cap \{s \mid s \text{ rejects any } \psi \in A_\varphi^{\subseteq} \text{ eligible for a secondary quant. impl.}\}$$

- Given the Horn alternatives that Schwarz assumes, no alternative is innocently excludable.
- For instance, in the case of *Al hired at least two cooks*, rejecting 'only two' is not consistent with rejecting 'at least three', given the assumption that 'at least two'.
- So we get primary quantity implicatures, but no secondary ones.
- Hence we predict ignorance, and no 'upper bounding' implicature (exactly n).

Critique

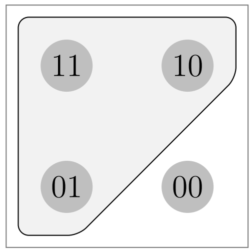
- This approach entails a very tight coupling between ignorance implicatures and upper bounding implicatures.
  - Consequence: Unclear how to distinguish *more than* from *at least*; both lack upper bounding implicatures, but behave differently with respect to ignorance.
- The effects of the QUD documented by Westera & Brasoveanu (2014) are not immediately accounted for.

  (Although Schwarz makes clear that the theory should ultimately be refined so that the alternatives are restricted to those that are relevant, in line with Gricean reasoning).

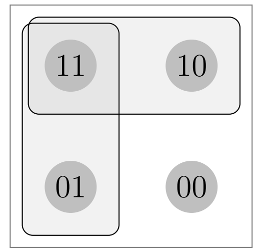
2.2 Quality-based

- Formulated in inquisitive semantics

Traditional disjunction

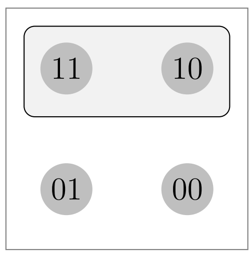


Inquisitive disjunction

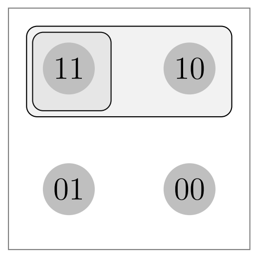


- Coppock & Brochhagen's (2013) analysis: An *at least* clause denotes the set of possibilities that are at least as pragmatically strong as a prejacent possibility.

Ann snores



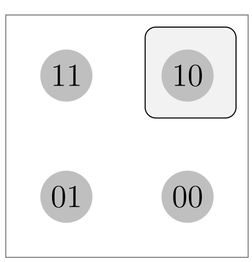
At least Ann snores



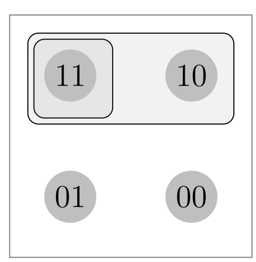
- The analysis of modified numerals depends on how you analyze numerals.
  - Two-sided analysis of numerals:
    - *At least two apples fell*: {[**2,...**), [**3,...**), [**4,...**),...}
    - *At most two apples fell*: {[**0-2**], [**1-2**], [**2**]}
  - One-sided analysis of numerals:
    - *At least two apples fell*: {[**2**], [**3**], [**4**], ...}
    - *At most two apples fell*: {[**0**], [**1**], [**2**]}

- Sincerity Maxim: Don't bring up an issue that you already know how to resolve. If a speaker expresses a proposition with multiple alternatives, then the speaker's information state should not be contained in any of these alternatives.

Fred's information state



At least Ann snores

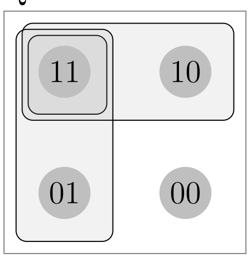


Fred should not assert *At least Ann snores* in this case.

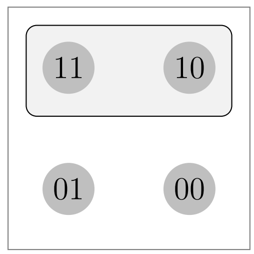
- The effect of the Maxim of Quantity can be computed by "exhaustifying" the proposition; to do this C&B used Balogh's (2009) recipe:
  - To exhaustify a proposition P with respect to a question Q:
    * For every possibility p in P, and every world w in p:
      If w is an element of an answer to Q that is not entailed by p then remove w from p.

  Examples:

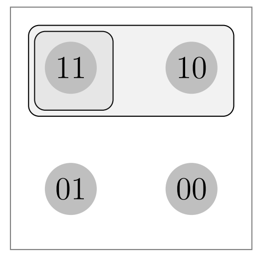
QUD:



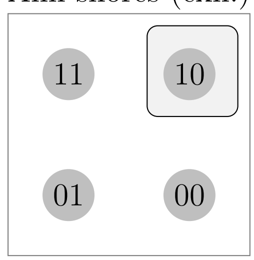
Ann snores



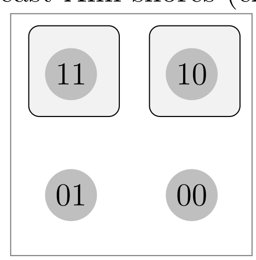
At least Ann snores



Ann snores (exh.)



At least Ann snores (exh.)



Critique

- Coppock & Brochhagen (2013) capture the fact that *at least* generates ignorance implicatures but no upper bounding implicatures, and the fact that *bare numerals* exhibit exactly the opposite pattern.
- They also predict the lack of ignorance implicatures for *more than*.
- The analysis is also QUD-sensitive.
- But they don't predict the **lack of upper bounding implicatures** for *more than*.
- As pointed out by Schwarz (to appear), the **ignorance implicature** that Coppock & Brochhagen (2013) predict **for *at least* n is too weak**. In particular, it does not imply that the speaker should consider n itself a viable option.
- Framework issue:

- C&B formulate their account in 'unrestricted' inquisitive semantics, $\mathsf{Inq_U}$, an extension of the basic inquisitive semantics framework, $\mathsf{Inq_B}$. Technically, the difference between the two is that in $\mathsf{Inq_B}$ propositions are *downward closed*, while in $\mathsf{Inq_U}$ they can be arbitrary sets of states (hence the name 'unrestricted').
- While $\mathsf{Inq_U}$ is richer in expressive power than $\mathsf{Inq_B}$, it is less well-behaved / well-understood from a logical point of view. In particular, it does not come with a suitable notion of entailment. As a consequence, it does not come with the usual algebraic operations on meanings, like *meet* and *join*, either.
- One question, then, is whether an account along the lines of C&B really needs the full expressive power of $\mathsf{Inq_U}$, or whether it could also be formulated in $\mathsf{Inq_B}$.

3 Proposal: a combined approach

3.1 Lexical assumptions

More than

- *More than two apples fell*: $\{$**[3,...)** $\}$
- Horn alternatives for *more than n*: $\{\textit{more than } m \mid m \in \mathbb{N}\} \cup \{\textit{exactly } m \mid m \in \mathbb{N}\}$

At least

- We adopt the proposal from Schwarz's critique of C&B that *at least n* be analyzed more along the lines Buring (2008) suggested, with the same meaning as *n or more*. Example:
  - *At least two apples fell*: $\{$**[2]**, **[3,...)** $\}$
- Horn alternatives for *at least n*: $\{\textit{at least } m \mid m \in \mathbb{N}\}$

3.2 Pragmatic assumptions

Quality

1. **Informative sincerity** (Gricean Quality)[1]

   If a speaker utters a sentence φ, her information state s should be contained in the informative content of φ:[2]

$$s \subseteq \mathsf{info}(\varphi)$$

[1] These maxims are only assumed to be in force in specific types of conversation. What we primarily have in mind here is a conversation in which the participants exchange information in a fully cooperative way.

[2] The informative content of φ, $\mathsf{info}(\varphi)$, is defined as the union of the possibilities in $[\![\varphi]\!]$.

2. **Inquisitive sincerity** (adapted from Groenendijk & Roelofsen, 2009)[3,4]

   - Don't bring up an issue that you already know how to resolve.
   - If a speaker expresses a proposition $[\![\varphi]\!]$ containing multiple semantic alternatives, then her information state s should not be contained in any of these alternatives.
   - More technically, assuming that $[\![\varphi]\!]$ is a downward closed set of information states, and the semantic alternatives introduced by φ are the maximal elements of $[\![\varphi]\!]$:

$$\text{If } \varphi \text{ is inquisitive, then } s \notin [\![\varphi]\!]$$

Combining these together:

$$s \in \mathsf{sincere}(\varphi) \text{ iff } s \subseteq \mathsf{info}(\varphi) \text{ and if } \varphi \text{ is inquisitive, then } s \notin [\![\varphi]\!]$$

3.2.1 Quantity

- We assume that Quantity is about alternative *expressions* that the speaker could have used (as opposed to alternative meanings that the speaker could have expressed, as under Balogh's treatment).
- But **only expressions that are relevant to the QUD are considered**.
- We distinguish **lexical pragmatic alternatives** from **contextual pragmatic alternatives**:
  - The set of **lexical** pragmatic alternatives for φ, A_φ, is determined by Horn scales.
  - The set of **contextual** pragmatic alternatives for φ relative to a question under discussion Q, $A_{\varphi,Q}$, contains only those lexical pragmatic alternatives that are **relevant** to Q.
$$A_{\varphi,Q} = \{\psi \in A_\varphi \mid \psi \text{ is relevant to } Q\}$$
  - We assume that both complete and partial answers are relevant to Q.
  - More precisely: ψ is relevant to Q if and only if the informative content of ψ coincides with the union of a set of semantic alternatives in $[\![Q]\!]$, conceived of as wholly relevant resolving answers.
- Following Schwarz, we adopt an Innocent Exclusion-based recipe for deriving implicatures, but now:

[3] The original formulation of the inquisitive sincerity maxim makes reference to the common ground: "If a speaker utters a sentence φ that is inquisitive w.r.t. the common ground, then φ should be inquisitive w.r.t. the speaker's information state as well." For our current purposes this is not necessary. Thus, for presentational purposes we have simplified the formulation somewhat.

[4] Coppock & Brochhagen proposed a stronger sincerity maxim, which they call the maxim of *interactive sincerity*. On their account this is needed because the predictions that inquisitive sincerity delivers are too weak. On our present account, inquisitive sincerity delivers the right predictions, and interactive sincerity would do so as well.

- The Gricean Quality requirement, that the speaker **believes** φ, is replaced by the requirement that the speaker can **sincerely utter** φ.
- We do not restrict the primary quantity implicatures to those that are consistent with quality.

- So the recipe runs as follows:
  - The first step, as before, is to compute the **quality implicature**:

    $$0_\varphi = \{s \mid s \in \mathsf{sincere}(\varphi)\}$$

  - Next, also as before, we compute **primary quantity implicatures**, based on the assumption that any pragmatic alternative for φ that would have been more informative was apparently not sincerely utterable. Here we restrict the set of pragmatic alternatives to those that are **relevant**, $A_{\varphi,Q}$. Let $A^{\subset}_{\varphi,Q}$ be the set of those alternatives that are stronger than φ itself:

    $$A^{\subset}_{\varphi,Q} = \{\psi \in A_{\varphi,Q} \mid \mathsf{info}(\psi) \subset \mathsf{info}(\varphi)\}$$

    $$1_{\varphi,Q} = \{s \mid \text{for all } \psi \in A^{\subset}_{\varphi,Q} : s \notin \mathsf{sincere}(\psi)\}$$

    'For every contextual pragmatic alternative that is stronger than φ itself, either the speaker doesn't believe it, or the speaker can already resolve the issue that it raises.'
  - Finally, again as before, we compute **secondary quantity implicatures**. The recipe for doing so is the same as on Schwarz's proposal, except that we now take Q into consideration.

    That is, we identify all alternatives ψ in $\boxed{A^{\subset}_{\varphi,Q}}$ such that:

    1. ψ is **not known** by the speaker according to $\boxed{1_{\varphi,Q}}$:
       * No $s \in 1_\varphi$ supports the informative content of ψ
    2. ψ is **innocently excludable** relative to φ $\boxed{\text{and } Q}$:
       * There is no subset A' of $\boxed{A^{\subset}_{\varphi,Q}}$ such that:
         · an information state can validate the quality implicature and primary quantity implicatures of φ while rejecting every sentence in A':

           $0_\varphi \cap 1_{\varphi,Q} \cap \{s \mid s \text{ rejects every sentence in } A'\}$ is **non-empty**

         · an information state *cannot* validate the quality implicature and primary quantity implicatures of φ while rejecting every sentence in A' *in addition to* ψ:

$$0_\varphi \cap 1_{\varphi,Q} \cap \{s \mid s \text{ rejects every sentence in } A' \cup \{\psi\}\} \text{ is } \textbf{empty}$$

If $\psi \in A^{\subset}_{\varphi}$ is not known by the speaker according to $1_{\varphi,Q}$ and innocently excludable relative to φ and Q, then we say that ψ is **eligible for a secondary quantity implicature**.

$$2_{\varphi,Q} = \{s \mid s \text{ rejects any } \psi \in A^{\subset}_{\varphi,Q} \text{ eligible for a secondary quant. implicature}\}$$

- Uttering a sentence φ against the background of a question Q in information state s is **cooperative** only if $s \in 0_\varphi$ (speaker adheres to Quality) and $s \in 1_{\varphi,Q} \cap 2_{\varphi,Q}$ (speaker adheres to Quantity).

4 Examples

4.1 How many? At least three.

(7) Q: How many apples did John eat?
φ: John ate at least three apples.

- $[\![Q]\!] = \{[0], [1], [2], [3], [4], [5], \ldots\}$
- $[\![\varphi]\!] = \{[3], [4, ...)\}$
- Lexical pragmatic alternatives, A_φ:
  - *John ate least four apples* – {[4], [5,...)}
  - *John ate least five apples* – {[5], [6,...)}
  - etc.
- All lexical pragmatic alternatives are relevant to Q, so all of them are contextual pragmatic alternatives:
  $A_{\varphi,Q} = A_\varphi$
- $0_\varphi = \mathsf{sincere}(\varphi) = \{s \mid s \subseteq [3, ...) \text{ and } s \not\subseteq [3] \text{ and } s \not\subseteq [4, ...)\}$

  At this point we have already derived ignorance: the speaker must consider both [3] and $[4, ...)$ possible.
- $1_{\varphi,Q} = \{s \mid \text{for all } \psi \in A^{\subset}_{\varphi,Q} : s \notin \mathsf{sincere}(\psi)\}$

  It can be shown that $0_\varphi \subset 1_{\varphi,Q}$ in this case, so the primary quantity implicatures do not tell us anything more about s than we already know from the quality implicature.
- Now take some $\psi \in A^{\subset}_{\varphi,Q}$. Could ψ be eligible for a secondary quantity implicature?
  - This would require that ψ is not known in any state $s \in 1_{\varphi,Q}$.
  - We have, for every $s \in 1_{\varphi,Q}$, that $s \notin \mathsf{sincere}(\psi)$.

  - But this does not guarantee that ψ is not known in s, because if ψ is inquisitive the reason that $s \notin \textsf{sincere}(\psi)$ may be that s resolves the issue expressed by ψ.
  - For instance, for $\psi =$*at least* 5, we can take the state [5]. This state is not in $\textsf{sincere}(\psi)$ because it resolves the issue expressed by ψ.
  - Yet, the state [5] is contained in $\textsf{info}(\psi)$.
  - Similarly for the other alternatives in $A^{\subseteq}_{\varphi,Q}$.

- Therefore no secondary quantity implicatures arise.

- Hence, we derive:

  - A **quality-based ignorance** implicature.
  - No upper bounding implicature.

4.2 How many? More than two.

(8) Q: How many apples did John eat?
φ: John ate more than two apples.

- $[\![Q]\!] = \{[0], [1], [2], [3], [4], [5], \ldots\}$

- $[\![\varphi]\!] = \{[3, ...)\}$

- Lexical pragmatic alternatives, A_φ:

  - *John ate* ***more than*** *n apples*, for all $n \in \mathbb{N}$,
  - *John ate* ***exactly*** *n apples*, for all $n \in \mathbb{N}$.

- All are relevant, so $A_{\varphi,Q} = A_\varphi$

- $0_\varphi = \textsf{sincere}(\varphi) = \{s \mid s \subseteq [3, ...)\}$

- $1_{\varphi,Q} = \{s \mid \forall\psi \in A^{\subseteq}_{\varphi,Q} : s \notin \textsf{sincere}(\psi)\}$.

- In this case, the pragmatic alternatives are not inquisitive, so the only way the primary implicatures could be satisfied is if the speaker does not believe the informative content of the stronger alternatives.

- So all $\psi \in A^{\subseteq}_{\varphi,Q}$ pass the first check for eligibility for a secondary quantity implicature.

- However, none of them is innocently excludable wrt φ and Q, because excluding one of them forces another one to be true.

- So no secondary quantity implicatures can be drawn.

- This results in:

  - A **quantity-based ignorance** implicature.

- No upper bounding implicature.

- Importantly, the **quantity-based** ignorance implicature of *more than* is predicted to be **less robust** than the **quality-based** ignorance implicature of *at least*.
- This could explain the contrast between the two in W&B's 'how many' condition.
- We now turn to the contrast between the two in non-echoic responses to polar questions.

4.3 [Polar question] More than two.

(9) *Context: John's diet prescribes that he eat at most two apples per day.*
Q: Did John stick to his diet today?
φ: No, he ate more than two apples.

- $[\![Q]\!] = \{[0, ..., 2], [3, ...)\}$
- $[\![\varphi]\!] = \{[3, ...)\}$
- Lexical pragmatic alternatives, A_φ:
  - *John ate **more than** n apples*, for all $n \in \mathbb{N}$,
  - *John ate **exactly** n apples*, for all $n \in \mathbb{N}$.
- But now the only relevant alternative is φ itself, so:
  - $A_{\varphi,Q} = \{\varphi\}$
  - $A^{\subset}_{\varphi,Q} = \emptyset$.
- $0_\varphi = \mathsf{sincere}(\varphi) = \{s : s \subseteq [3, ...)\}$
- Since $A^{\subset}_{\varphi,Q} = \emptyset$, no primary or secondary quantity implicatures arise.
- That is, $1_{\varphi,A}$ and $2_{\varphi,A}$ both contain all possible information states.
- So we derive:
  - **No ignorance** implicature.
  - No upper bounding implicature.

4.4 [Polar question] At least three.

(10) *Context: John's diet prescribes that he eat at most two apples per day.*
Q: Did John stick to his diet today?
φ: No, he ate at least three apples.

- $[\![Q]\!] = \{[0, ..., 2], [3, ...)\}$
- $[\![\varphi]\!] = \{[3], [4, ...)\}$

- Lexical pragmatic alternatives, A_φ:
    - *John ate* ***at least*** *n apples*, for all $n \in \mathbb{N}$
- Again, the only relevant alternative is φ itself, so:
    - $A_{\varphi,Q} = \{\varphi\}$
    - $A^{\subset}_{\varphi,Q} = \emptyset$.
- $0_\varphi = \mathsf{sincere}(\varphi) = \{s \mid s \subseteq [3, ...) \text{ and } s \not\subseteq [3] \text{ and } s \not\subseteq [4, ...)\}$

  Just as in the 'how many' context, we already derive ignorance at this point: the speaker must consider both [3] and $[4, ...)$ possible.
- Since $A^{\subset}_{\varphi,Q} = \emptyset$, no primary or secondary quantity implicatures arise.
- $1_{\varphi,A}$ and $2_{\varphi,A}$ both contain all possible information states.
- So we derive:
    - A **quality-based ignorance** implicature.
    - No upper bounding implicature.

4.5 How many? Three.

(11) Q: How many apples did John eat?
φ: John ate three apples.

- $[\![Q]\!] = \{[0], [1], [2], [3], [4], [5], \ldots\}$
- $[\![\varphi]\!] = \{[3, ...)\}$
- $A^{\subset}_{\varphi,Q}$:
    - *John ate n apples*, for all $n \in \mathbb{N}$.
- $0_\varphi = \mathsf{sincere}(\varphi) = \{s : s \subseteq [3, ...)\}$
- $1_{\varphi,A} = \{s : \forall\psi \in A^{\subset}_{\varphi,Q} : s \notin \mathsf{sincere}(\psi)\}$.
- In this case again, the alternatives are not inquisitive, so the only way the primary implicatures could be satisfied is if the speaker does not believe the informative content of the stronger alternatives. No state that entails one of these stronger alternatives will be included in $1_{\varphi,A}$.
- So all stronger alternatives pass the first check for eligibility for a secondary quantity implicature.
- Moreover, all of them are innocently excludable wrt φ and Q, because each can be excluded without forcing another one to be true.

- So secondary quantity implicatures do arise in this case.
- Hence we derive:
  - No ignorance implicature.
  - An **upper bounding** implicature.

4.6 Additional prediction

- It has been observed that Gricean quality implicatures are not as easily cancellable as quantity implicatures.
- Combined with our proposal, this leads to the prediction that ignorance implicatures triggered by *at least* are more difficult to cancel than the ones triggered by *more than*. Borne out:

(12) a. He has more than 10 cars. In fact, he has 12.
b. He has at least 10 cars. #In fact, he has 12.

5 Conclusion

This proposal allows us to overcome the shortcomings of previous accounts:

- It achieves a three-way contrast between superlative modifiers, comparative modifiers, and numerals without appeal to a two-sided analysis (in contrast to Schwarz)
- It accounts for the QUD-sensitivity observed by Westera and Brasoveanu (in contrast to both Schwarz and C&B).
- It predicts ignorance with respect to the prejacent of *at least* (overcoming Schwarz's critique of C&B)
- It brings C&B's approach in line with recent theorizing on inquisitive semantics using downward-closed possibilities.

Most importantly, we account for the facts that:

- *more than* **can** imply ignorance in *how many* contexts, just like *at least*, as observed by Westera & Brasoveanu.
- But the ignorance implicature of *at least* is **more robust**, accounting for:
  - The fact that it is perceived to be stronger than the ignorance implicature of *more than* in *how many* contexts, as observed by W&B.
  - The fact that it persists in non-echoic responses to polar questions.
  - The fact that it contrasts with *more than* 'out of context'.

- To obtain these results, it is crucial to be able to derive ignorance implicatures both through **quality** (for *at least*) and through **quantity** (for *more than*).

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