Quantity and gradability across categories[*]

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Abstract This paper proposes a unified analysis of scalar modifiers across the adjectival, nominal and verbal domains, with a special focus on the proportional modifier *half* in English. I claim that *half* has a scalar meaning in all the environments in which it appears. Specifically, I show that in partitive and event-modifying uses, *half* targets a QUANTITY-based scale whose scale structure crucially depends on the part structure of a nominal argument, just like many adjectival cases. To formalize the analysis, I extend the degree-based analysis of Kennedy & McNally (2005) for gradable adjectives to partitives and VPs headed by incremental theme verbs, with some discussion of compounding uses of *half* with both nouns and verbs.

Keywords: gradability; scale structure; part structure; event structure

1 Scalar modification across categories

This paper proposes a unified analysis of scalar modifiers across the adjectival, nominal and verbal domains. While there are a variety of previous accounts that deal with scalar modifiers within these individual environments, there is at present no unified analysis across categories. The modifiers I am interested in are proportional scalar modifiers such as *most(ly), part(ly), complete(ly)* and *half*. In particular, in this paper I focus on *half* in English, since it occurs in all three environments with the same phonological shape.[1] As shown in (1)-(3), *half* can occur in a wide range of syntactic environments:

(1)   Gradable adjectives:
        a. The glass is half full.
        b. The cake is half baked.

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[*] I would like to thank the audiences at SALT 20 in Vancouver, SWAMP 2009 in Chicago, and at the University of Göttingen where this work was presented. I am especially grateful for detailed comments I received from Chris Kennedy, Peter Alrenga and Itamar Francez on various incarnations of this work. All remaining errors are of course my own.

1 I otherwise assume that the adjectival and adverbial variants of the other proportional modifiers are morpho-syntactically conditioned variants of individual lexical items.

(2) Partitives:
   a. Jerome ate half (of) the cherries.
   b. Half (of) the books are on the table.

(3) Verb phrases:
   a. The girls half washed the dishes.
   b. John half ate an apple.

I claim that *half* always has a scalar meaning across these different uses. Specifically, I will extend a degree-based analysis that has been proposed for the adjectival case (Kennedy & McNally 2005) to the semantics of *half* in the partitive and verbal cases as well. Furthermore, I will show that QUANTITY-based scale structures in all three environments crucially depend on the part structure of a nominal argument.

The paper proceeds as follows: in section 2 I outline the degree-based analysis of Kennedy & McNally (2005) for scalar modifiers of gradable adjectives. I then extend the degree-based account to partitives in section 3 and verb phrases in section 4. In section 5 I speculate on how such an account may be extended to compounding uses of *half* not otherwise discussed in this paper, while section 6 concludes.

2 Gradable adjectives and *half*

In the degree-based analysis of Kennedy & McNally (2005), *half* and other proportional modifiers are degree terms that modify gradable adjectives. These types of predicates are associated with scales, which are formalized as ordered sets of degrees along some dimension. According to Kennedy & McNally, gradable adjectives may be associated with one of four scale structures that are distinguished based on whether a scale has (or lacks) a maximal or minimal element. Specifically, a scale can be fully open (lacking both minimal and maximal elements), upper closed (having a maximal element but lacking a minimal one), lower closed (having a minimal element but lacking a maximal one), or fully closed (having both maximal and minimal elements). One of the key pieces of evidence for the linguistic significance of scale structure is the distribution of degree modifiers of gradable adjectives. Particularly, *half* can only occur with gradable adjectives that have fully closed scales. Hence the acceptability of (4a) and anomaly of (4b).

(4) a. The glass is half full. / The door is half open.
   b. ?? Taylor is half tall/old.

Intuitively, it is not surprising that *half* should only be acceptable with adjectives that lexicalize fully closed scales. Since *half* should pick out the midpoint of a scale, it requires access to both a maximum and minimum value from which to calculate that midpoint.

Gradable adjectives are analyzed as gradable properties of individuals of type $\langle d, et \rangle$, where the degree argument must be saturated in order to apply the predicate to an individual. The function of *half*, then, is to select the midpoint of the closed scale associated with the adjective and supply the open degree argument with that value, returning a property of individuals. The denotation of *half* is given in (5), where G is a gradable predicate, and $\mathbf{mid}(S_G)$ is the midpoint of the closed scale S associated with G.[2] A derivation of *half full* given in (6).

(5) $[\![\text{half}]\!] = \lambda G_{\langle d,et \rangle} \lambda x_{\langle e \rangle}.G(x)(\mathbf{mid}(S_G))$

(6) $[\![\text{half}]\!]([\![\text{full}]\!]) = \lambda G \lambda x.G(x)(\mathbf{mid}(S_G))[\lambda d \lambda y.\mathbf{full}(y) = d]$
$$= \lambda x.[\lambda d \lambda y.\mathbf{full}(y) = d](x)(\mathbf{mid}(S_{full}))$$
$$= \lambda x.\mathbf{full}(x) = \mathbf{mid}(S_{full})$$

In the absence of a degree modifier, a null degree morpheme *pos* values the degree argument of the gradable predicate based on a contextual standard of comparison. For adjectives with upper-closed scales, including the adjectives that accept modification by *half*, *pos* returns the maximal value on the scale as the contextual standard. This follows from a principle of Interpretive Economy (Kennedy 2007).

Now consider (7), which is ambiguous:

(7)   The meat is half cooked.

The ambiguity stems from the availability of two distinct scales that can be targeted by *half*. On one reading, *half* is targeting the scale that is lexically encoded in the deverbal adjective *cooked* - the cooked-ness scale. On this reading, (7) is true if the degree to which the meat is cooked is half. There is also a second reading, where *half* is targeting a QUANTITY-based scale that is based on the part structure of the nominal argument. On this reading, (7) is true if the proportion of meat that is cooked is half. This type of ambiguity is pervasive among gradable adjectives (Kennedy & McNally 2005, 2010), and the distinction between the two readings becomes especially clear when the adjective is modified by proportional scalar modifiers like *half*.

The quantity-based scale is crucially related to the part structure of the nominal argument. In particular, a bounded nominal argument yields a bounded, fully closed scale. Thus, in (7) the definite, bounded nominal *the meat* corresponds with a fully closed quantity scale that can be targeted by *half*.[3] The point here is that a quantity-based scale is available for modification *half* in the adjectival case. The notion of a quantity-based scale will return during our discussion of *half* in the nominal (partitive) and verbal environments as well.

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2 I use the notation $\mathbf{mid}(S_G)$ as shorthand for a function that calculates the midpoint of a scale by identifying the degree that is equidistant from the maximum and minimum values.

3 A formalization of the mapping between objects and their part structures on one hand and scales and degrees on the other is beyond the scope of this paper.

3 Partitives, quantities and degrees

As we have already seen, *half* also occurs in the partitive environment as in (2), repeated below as (8). Intuitively, in these cases *half* measures the proportion of the QUANTITY of the embedded nominal argument. That is, in (8a) *half* measures a quantity of cherries in the extension of ⟦the cherries⟧, and in (8b) a quantity of books in the extension of ⟦the books⟧.

(8)   a.   Jerome ate half (of) the cherries.
       b.   Half (of) the books are on the table.

It may be instructive to compare the partitive uses of *half* with its use in nominal compounds as in (9).

(9)   a.   half cherries
       b.   half moon

There are two important differences between the interpretations of *half* in (8) and (9). First, when *half* occurs in a partitive, the embedded nominal receives a collective interpretation. That is, in (8a), *half* measures the amount of the collective group of cherries that Jerome ate. Meanwhile in the compounding cases, *half* applies distributively over the atoms of a plurality. Thus (9a) represents a property of individuals that each have the property of being a half cherry - *half* applies within the scope of the plural. Second, in the partitive case, the relevant dimension of measurement is MONOTONIC on the part-whole relation. That is, the quantity measured by *half* crucially depends on the quantity of the embedded nominal. By contrast, in the compounding cases, the dimension is non-monotonic. In (9a), each cherry has the property of being *half*, regardless of the quantity of cherries.

These meaning distinctions mirror the interpretive differences of measure phrases in partitives and nominal compounds as discussed by Schwarzschild (2002, 2006). In particular, Schwarzschild argues that the semantic differences between the partitive and compounding uses of measure phrases is derived from their distinct syntactic representations. Specifically, in the partitive case there is a function that maps entities onto intervals of scale - a scale of cardinality.

The question is how to formally incorporate the connection between the part structure of the embedded nominal and quantity-based scale with which it is associated, specifically with the goal in mind of a unified analysis of proportional modifiers across categories. Since *half* is looking for an open degree argument to value, we need a way to formally introduce a degree argument into the structure, assuming that referential nominals occurring in partitives typically do not lexicalize a degree argument themselves.[4] Taking Schwarzschild's suggestion to its logical end, I propose a function that relates part structure with a quantity/cardinality scale, and that

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4 Proposals have been made, however, that non-referential nominals in English do lexicalize a degree

crucially introduces an open degree argument that can be targeted by proportional modifiers. This function, which I will call μ (for *measure*), relates the QUANTITY of the embedded nominal with a scale, i.e., an ordered set of degrees. Following Schwarzschild (2006), μ_{PRT} is a functional head that is generated between the *of*-PP and the upstairs measure. Its denotation is given in (10).

(10) $[\![\mu_{PRT}]\!] = \lambda P_{\langle e,t\rangle}\lambda d\lambda x.P(x) \wedge \textbf{quantity}(x) = d$

The input of the function μ_{PRT} is the *of*-PP of semantic type $\langle e,t\rangle$, assuming a traditional analysis for partitive *of* as in Ladusaw 1982, where the function of partitive *of* is that of a type shifter that takes an individual and opens up its internal part structure to be made available for quantification or modification. The result of applying μ_{PRT} to a partitive *of*-PP is a gradable predicate of type $\langle d,et\rangle$ with an open degree argument, whose value is determined by the upstairs proportional scalar modifier or measure phrase.

A welcome consequence of this analysis is that we can maintain a uniform semantics for proportional modifiers such as *half* across the adjectival and partitive contexts. In both cases, *half* applies to a gradable predicate with an open degree argument, and values that degree argument with the midpoint of the scale with which the predicate is associated. In the case of partitives, this scale is made available through the application of μ_{PRT} and is necessarily a QUANTITY-based scale relative to the part structure of the nominal within the *of*-PP.

Recall that *half* can only combine with gradable predicates whose scales are fully closed, i.e., ones that have both a minimum and maximum value. This generalization holds in the partitive cases as well. Note that *half* is infelicitous in pseudo-partitives, i.e., ones where the embedded nominal is a bare plural or mass noun.

(11) a. half of the cherries / half of the applesauce
   b. *half of cherries / *half of applesauce

The anomaly of (11b) follows from the fact that the scale that is being targeted by *half* is crucially dependent on the part structure of the embedded nominal. In (11b) the bare nouns are unbounded, corresponding with an unbounded, open scale. By contrast, in (11a) the nominals are linguistically bounded by the presence of the determiner *the*, which is traditionally analyzed as contributing a maximalizing interpretation (e.g. Link 1983; Chierchia 1998). The delimiting nature of determiners means that the nominals in (11a) are bounded, and therefore the scale introduced

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argument (see Cresswell 1976; Krifka 1989). Even if nominals are indeed endowed with a degree argument, this degree argument must be closed off within the DP in order for the nominal to be referential, in which case we still need to introduce a degree argument in partitives as in (8) that can be targeted by *half* or measure phrases.

by μ_{PRT} is also bounded, or fully closed, resulting in the successful application of *half*. The fact that *half* cannot occur in pseudo-partitives is yet another welcome consequence predicted by this analysis. Note that the use of a determiner is not the only means of giving a nominal a bounded interpretation that results in a bounded scale. Other strategies include use of a quantifier such as *all* or a restrictive relative clause.[5]

(12) a. ? Half of Americans own a pet.
      b. Half of all Americans own a pet.
      c. Half of Americans polled own a pet.

Given the semantics of μ_{PRT} in (10) and the denotation of *half* in (5), I give a compositional derivation of *half of the books* in (13).

(13)
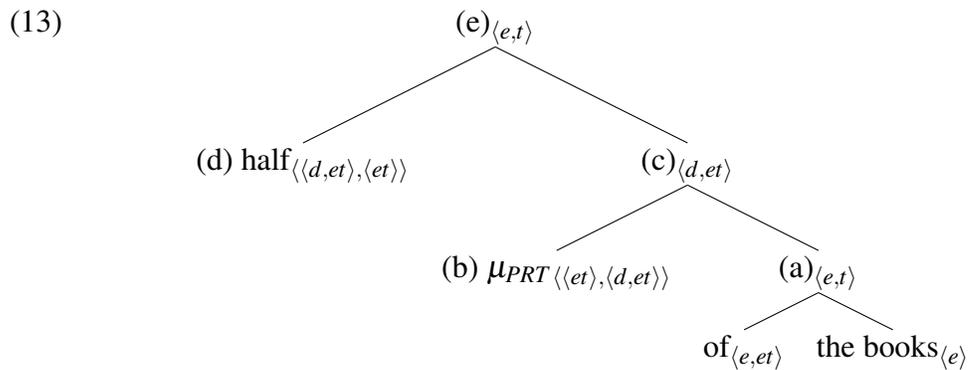
a. $[\![\text{of}]\!]([\![\text{the books}]\!]) = \lambda x.x \leq the.books$
b. $[\![\mu_{PRT}]\!] = \lambda P\lambda d\lambda x.P(x)\wedge\textbf{quantity}(x) = d$
c. $[\![\mu_{PRT}]\!]([\![\text{of the books}]\!]) = \lambda d\lambda x.x \leq the.books\wedge\textbf{quantity}(x) = d$
d. $[\![\text{half}]\!] = \lambda G\lambda x.G(x)(\textbf{mid}(S_G))$
e. $[\![\text{half}]\!]([\![\mu_{PRT}\ \text{of the books}]\!]) =$
    $= \lambda x.[\lambda d\lambda x'.x' \leq the.books\wedge\textbf{quantity}(x') = d](x)(\textbf{mid}(S_{of.the.books}))$
    $= \lambda x.x \leq the.books\wedge\textbf{quantity}(x) = \textbf{mid}(S_{of.the.books})$

The result of the derivation is a property of an individual that is true just in case that individual is a proper part of $[\![\text{the books}]\!]$ and the quantity of that individual[6]

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5 To my ear, (12a) seems somewhat odd (though certainly not as odd as (11b)); however, a quick Google search of the string "half of Americans" yields approximately 13 million results (many of which, to be sure, are followed by a restrictive relative clause, though many are not). I speculate that in this case, real world knowledge tells us that *Americans* is a bounded set, making *half* felicitous despite a lack of overt indication of boundedness. Thanks to Itamar Francez and Hazel Pearson for discussion on these cases.

6 Whether the predicate $\textbf{quantity}(x)$ should technically be a predicate of predicates rather than a predicate of individuals is a detail I set aside for now. Thanks to Matt Husband for discussion on this point.

is equal to the midpoint of the quantity scale of $[\![\mu_{PRT}$ of the books$]\!]$. I assume that existential closure shifts the $\langle e,t \rangle$ property to a type e individual that can be used in argument position.

Recapping this section, a degree-based analysis of partitives allows us to maintain a uniform analysis of proportional scalar modifiers such as *half* across the adjectival and partitive domains.[7] The functional head μ_{PRT} mediates between the part structure of the embedded noun and a quantity-based scale that can be targeted by *half*. Bounded nominals in partitives give rise to fully closed scales over which *half* can operate. As we will see in the next section, these same generalizations will persist with uses of *half* in the verbal domain, where nominal part structure and scale structure will be linked with event structure as well.

4   Degrees, events and *half*

The proportional scalar modifier *half* also occurs as a modifier of verb phrases, as shown in (3), repeated below.

(14)   a.   The girls half washed the dishes.
        b.   John half ate an apple.

Proportional modifiers in these cases have been taken to delimit the extent to which an event is realized (Moltmann 1997; Piñón 2008). That is, in (14a), the extent to which the event of washing the dishes is complete is half. Likewise in (14b), the extent to which the event of eating an apple is also half.

It has been known for some time that in the case of incremental theme verbs (Dowty 1991) such as in (14), it is the incremental theme argument that is in some sense responsible for measuring out the event (see e.g. Tenny 1994). That is, the event of washing the dishes is complete when all the contextually relevant dishes are washed; and the event of eating an apple is complete when all the contextually relevant parts of the apple are eaten. Thus, in measuring out an event described by an incremental theme verb, proportional modifiers such as *half* are in fact measuring out the QUANTITY of the incremental theme argument that is affected during the event.

Further evidence that the object is in fact implicated in the measuring of the event comes from the fact that the use of *half* is only felicitous with fully bounded nominal arguments. Note the contrast in the following examples:

(15)   a.   John half ate the apple.
        b.   ?? John half ate apples / applesauce.

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7 I do not offer a full analysis here of the noun compound uses of *half* such as *half cherries* and *half moon*, though see section 5 for some preliminary discussion of how these cases might be subsumed under this general scalar account.

Here, *half* can only be used to measure the event if the incremental theme argument is bounded. Verb phrases with bare plural or mass nouns as in (15b) do not accept modification by *half* to measure out the event.[8] The facts in (15) mirror the restrictions on *half* in the partitive case. Recall that *half* can only be used in partitives when the downstairs nominal is bounded (*cf.* (11)). The behavior of *half* in (15) is completely expected on the approach to proportional scalar adverbs advocated in this paper. That is, modifiers like *half* operate over a scale of QUANTITY that is related to the part structure of a nominal argument; since *half* requires a fully closed scale over which to operate, it can only be used when the relevant quantity scale is derived from a bounded nominal argument.

Note also that the contrast in acceptability of *half* in (15) also mirrors the telic/atelic distinction with incremental theme verbs. Considering variants of (15) without *half*, we observe that events where the incremental theme is a bounded count noun allow a telic interpretation, whereas events that have bare plurals or mass nouns as the incremental theme can only be interpreted as atelic.

(16)   a.   John ate the apple.   TELIC READING POSSIBLE
       b.   John ate apples / applesauce.   ATELIC READING ONLY

This contrast is also due to the distinction between the boundedness of the incremental theme. Namely, a bounded (or 'quantized') incremental theme allow a telic interpretation, whereas an unbounded (or 'cumulative') incremental theme allows only an atelic interpretation of an event.

This relation between event structure and the part structure of an incremental theme argument has famously been formalized by Krifka (1992) through the OBJECT-EVENT HOMOMORPHISM, which subsumes a mapping to objects and mapping to events as defined below:

(17)   a.   MAPPING TO OBJECTS:
            $\forall R[MAP\text{-}O(R) \leftrightarrow \forall e, e', x[R(e,x) \wedge e' \leq e \rightarrow \exists x'[x' \leq x \wedge R(e', x')]]]$
       b.   MAPPING TO EVENTS:
            $\forall R[MAP\text{-}E(R) \leftrightarrow \forall e, x, x'[R(e,x) \wedge x' \leq x \rightarrow \exists e'[e' \leq e \wedge R(e', x')]]]$

Mapping to objects states that for each sub-event e' of event e with participant x, there is a sub-participant x' that stands in the relation R to e'. That is, for each sub-event e' of eating an apple, there is a part x' of the apple that is consumed in e'. Mapping to events states that for every sub-part x' of participant x in an event e,

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8 There is a reading of *half* which makes (15b) acceptable, namely an evaluative reading that does not measure the event, but rather indicates that the event being described is not considered to be a very good instance of the event type named by the verb. See section 5 for a preliminary sketch of verbal compounding cases of *half* and its evaluative use, as well as Tenny (2000) and Bochnak (2010) for further discussion and analysis.

there is a sub-event e' that stands in the relation R to x'. That is, for each sub-part x' of an apple in the event of eating an apple, there is a sub-event e' of eating x'. These properties, together with summativity, uniqueness of objects and uniqueness of events, constitute the object-event homomorphism. This relation ensures the correspondence between part structure of the incremental theme. In particular, a bounded part structure entails a bounded event, thereby capturing the telicity effects based on the (un)boundedness of the theme argument.

Summarizing this section thus far, we have seen that *half* can modify verb phrases that describe telic events, and has the effect of measuring out the extent to which the event is realized. Since telicity is crucially related to the part structure of the incremental theme argument of the verb, *half* can only modify events whose incremental theme argument is fully bounded. This behavior is completely expected, given the related facts in the adjectival and partitive environments, where *half* operates over a fully closed scale related to the part structure of a nominal argument. The question now is how to relate event structure and scale structure, with the goal of a unified analysis of proportional scalar modification across their adjectival, partitive and event-modifying uses.

This very question has been the object of recent study, notably by Caudal & Nicolas (2005), Kennedy & Levin (2008) and Piñón (2008). In particular, these authors are interested in treating aspectual phenomena within a degree-based framework by relating event structure with scale structure. In an interesting proposal, Caudal & Nicolas (2005) introduce a mapping between events and degrees, which they claim can effectively replace Krifka's (1992) object-event homomorphism. In particular, their system includes a mapping to degrees and a mapping to events:

(18)   a.   MAPPING TO DEGREES:
$$\forall R[\textit{MAP-D(R)} \leftrightarrow$$
$$\forall e, e', d[R(e,d) \wedge INI(e',e) \wedge 0 < d \rightarrow \exists! d'[0 < d' \leq d \wedge R(e',d')]]]$$
   b.   MAPPING TO EVENTS:
$$\forall R[\textit{MAP-E(R)} \leftrightarrow$$
$$\forall e, d, d'[R(e,d) \wedge 0 < d' \leq d \rightarrow \exists! e'[INI(e',e) \wedge R(e',d')]]]$$

Mapping to degrees states that for every event e realized to the degree d and every initial sub-event e' of e, there is a unique degree d' such that e' is realized to the degree d'. Mapping to events states that for every degree d' less than or equal to d - the degree to which the event e is realized - there is an initial sub-event e' such that e' is realized to the degree d'. The predicate $INI(e',e)$ ensures that a sub-event e' is an initial part of an event e.[9]

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9 This is needed to ensure that, for example, reading one page in the middle of a book does not constitute reading a book halfway.

Caudal & Nicolas's (2005) analysis shows that a degree-based account of aspectual phenomena is possible. However, under their system, it remains unclear how the part structure of the nominal is related to the event structure. This is because they claim that telicity is purely a matter of specifying a maximal degree and satisfying mapping to degrees and mapping to events.[10] In addition, under their analysis, as far as I can tell, it is the verbs themselves that are endowed with a degree argument, presumably as part of the lexical entry of the verb, which is not an innocent assumption.[11]

In fact, empirical evidence suggests that incremental theme verbs do not lexicalize a degree argument. As shown by Rappaport-Hovav (2008), incremental theme verbs can appear with resultatives as in (19), which themselves introduce various types of scales. Rappaport-Hovav argues that if incremental theme verbs lexicalized scales themselves, then we would expect them to be unacceptable with resultatives that introduce their own scale, but this turns out not to be the case.

(19)   a.   Keelin steamed the clothes dry/clean/stiff.
        b.   Cinderella scrubbed her knees sore/the dirt off the table/the table clean.

Furthermore, there is evidence against the presence of an open degree argument at the level of the VP. As shown by Gawron (2007), VPs headed by incremental verbs do not accept the full range of degree morphology as one would expect if there was an open degree argument at this level.

(20)   a.   ?? Tim wrote the paper more than Tommy did.
        b.   ?? Tim wrote the paper too much.
        c.   ?? Tim wrote the paper so much that Tommy barely did anything at all.

In fact, proportional modifiers such as *half*, *mostly*, *partially* and *completely* are among the few degree terms that actually seem to be able to modify VPs headed by incremental theme verbs (also intensifier *really*).

Some of the sentences in (20) might receive an interpretation under a reading of comparing events, but what is clear is that the degree morphology here does not target a scale of QUANTITY based on the incremental theme argument. Rather, if one wants to make a comparison based on the relevant quantity scale, the degree morphology

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10 Caudal & Nicolas do propose a predicate QUANTITY(d,x) that relates the quantity of an entity x with degrees on a scale, but then it is not clear how QUANTITY is related to the degree-event homomorphism they propose. It is possible that telicity can be derived from how QUANTITY interacts with the scale, which in turn interacts with event structure, but this idea is not made explicit in their analysis.

11 This is also a feature of Kennedy & Levin (2008) and Piñón's (2008) analyses, though since Kennedy & Levin are concerned only with degree achievement verbs which are derived from gradable adjectives, it seems more plausible that such verbs would lexicalize a degree argument.

appears embedded within the VP, closer to the incremental theme argument itself.

(21) a. Tim wrote more of the paper than Tommy did.
      b. Tim wrote too much of the paper.
      c. Tim wrote so much of the paper that Tommy barely did anything at all.

Therefore degree morphology is possible, just not at the VP level; it occurs within the VP closer to the theme argument. This means that there is indeed evidence for an open degree argument within the VP, but one that is closely related to the theme, and which is closed off below the VP level.

Assuming again, as in the partitive case in section 3, that nominals in these cases are not endowed with a degree argument themselves, we need a way to make available a degree argument within the VP. My proposal then is to introduce a variant of the function μ that relates the part structure of the nominal argument with a quantity scale, and introduces a degree argument within the VP that can be targeted by proportional scalar modifiers (or other degree morphology, as the case may be).[12] The question is how to integrate this function into the VP. I refer our attention back to the examples in (21) for a clue as to how this can be done. Interestingly, in all the degree constructions in (21), we see the obligatory presence of *of*. Taking this fact seriously, I suggest that a partitive element is present in all degree constructions within the VP, including cases of modification by proportional modifiers, where this element is obligatorily not pronounced (*cf. John half ate (*of) an apple*).[13] That a partitive element is involved in the verbal degree constructions we are interested in here should not be surprising. After all, the QUANTITY scale being targeted by *half* and other degree terms is crucially related to the part structure of the embedded nominal argument, and as discussed above in section 3, the partitive element makes available the internal part structure of an individual.

Given these pieces of the analysis, the denotation of the proposed function is given in (22) below:

(22) $[\![\mu_V]\!] = \lambda P \lambda d \lambda e. \exists x [P(x) \wedge \textbf{theme}(e)(x) \wedge \textbf{quantity}(x) = d]$

This variant of μ takes a partitive property argument of type $\langle e,t \rangle$ and returns a gradable property of events, namely one where there is an x that is the theme of the event e and the quantity of x participating in e is equal to the degree d. In some respects, μ is somewhat akin to an agentive v head that introduces an agent outside

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12 The idea of having a functional head introduce the incremental theme argument has also been recently proposed by Stensrud (2009), whose approach incremental theme verbs is an extension of her analysis of resultatives.

13 The correlation between the presence of partitive *of* and degree semantics in both the partitive and VP environments perhaps suggests that the degree semantics is actually encoded in *of* itself. I leave this interesting question to future work.

of the VP (Kratzer 1996) in that it is a functional head that syntactically introduces an event participant.

Under this analysis, a proportional scalar modifier such as *half* combines directly with the [μ_V [$<of>$ DP]] constituent to return an event description, which then combines with a verb via a variant of Event Identification (Kratzer 1996). The derivation of the VP *half eat the apple* would proceed as follows, where I notate the silent partitive element as $<of>$:

(23)
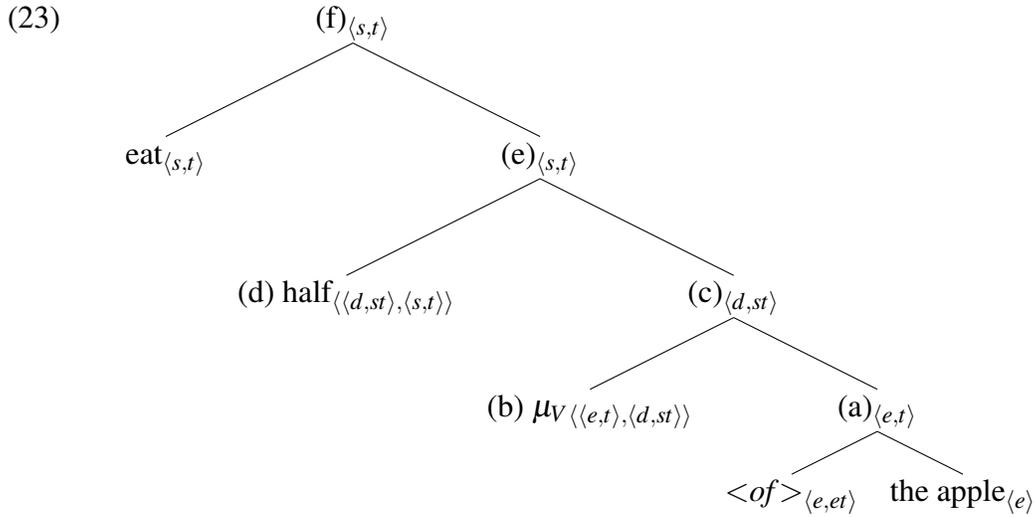
a. $[\![<of>]\!]([\![\text{the apple}]\!]) = \lambda y.y \leq the.apple$
b. $[\![\mu_V]\!] = \lambda P \lambda d \lambda e. \exists x[P(x) \wedge \textbf{theme}(e)(x) \wedge \textbf{quantity}(x) = d]$
c. $[\![\mu_V <of> \text{the apple}]\!] =$
    $\lambda d \lambda e. \exists x[x \leq the.apple \wedge \textbf{theme}(e)(x) \wedge \textbf{quantity}(x) = d]$
d. $[\![\text{half}]\!] = \lambda G \lambda e. G(e)(\textbf{mid}(S_G))$
e. $[\![\text{half } \mu_V <of> \text{the apple}]\!] =$
    $\lambda e. \exists x[x \leq the.apple \wedge \textbf{theme}(e)(x) \wedge \textbf{quantity}(x) = \textbf{mid}(S_{apple})]$
f. $[\![\text{eat half } \mu_V <of> \text{the apple}]\!] =$
    $\lambda e.\textbf{eat}(e) \wedge \exists x[x \leq the.apple \wedge \textbf{theme}(e)(x) \wedge \textbf{quantity}(x) = \textbf{mid}(S_{apple})]$

The result of the derivation in (23) is an event description that is true of an event *e* where there is an *x*, which is a part of an apple, and which is the theme of *e*, and the quantity of which that participates in *e* is equal to the midpoint of the quantity scale of $[\![\mu_V <of> \text{the apple}]\!]$.

Some words are in order to reinforce my claim that *half* should attach below the verb, given that this is not how this structure is pronounced in English. First, the structure in (23) captures structurally the parallels between the event-measuring and partitive uses of *half*. In a sense, the verb phrase *half eat the apple* underlyingly

contains a partitive-like structure that relates the part structure of the embedded nominal with a scale via the function μ. This captures the parallels between the two uses in terms of measuring out a quantity of the nominal argument. Second, under this analysis we avoid the problem of having an open degree argument at the VP level. This is a welcome consequence, since there is a lack of evidence for the presence of a degree argument at the VP level, as shown in (20). In fact, in other degree constructions where it is the quantity of the theme that is at issue, the degree morphology appears within the VP, where we also find an overt partitve *of*, as shown in (21). Third, there is some crosslinguistic evidence that *half* must attach low in the event-measuring cases. For example, the equivalent versions of *half* in Greek (Anastasia Giannakidou, p.c.) and possibly some dialects of European Portuguese (Patricia Amaral, p.c.) are pronounced where they would be generated in a structure like (23). I therefore conclude that the structure in (23) is indeed on the right track and propose a movement operation in English that moves *half* to a preverbal position where it is pronounced. As for the reason why proportional modifiers should be subject to such a movement rule, while other degree terms are not, this is a question I leave to further research.

Under the analysis advocated here, the incremental theme argument is introduced by the function μ_V, which serves to mediate between the part structure of the theme argument and a quantity scale, and introduces an open degree argument. As has already been alluded to, this puts μ_V on par with the *v* head in that both are functional heads that introduce a verb's arguments and assign a thematic role to those arguments. On one hand, *v* introduces the external argument and assigns an agent role, while on the other hand μ_V introduces the internal argument and assigns the theme role. What this means is that the analysis assumes that incremental theme verbs are simple event predicates that do not directly select for an internal argument, and that the syntax and semantics of incremental theme predicates is fully Neo-Davidsonian. While Kratzer (1996) has famously argued against such an approach and has claimed that themes must be directly selected by their verbs, arguments have also been raised that incremental theme verbs do not necessarily select for their themes.

For instance, Rappaport-Hovav (2008) identifies at least two pieces of evidence that incremental theme verbs do not necessarily show a strong attachment to their direct object. First, they can be used intransitively as in (24).

(24)   All last night Cinderella scrubbed/ate/read/drank/wiped and wiped.

Second, they can appear with resultatives that may include objects that are clearly not selected for by the verb itself (*cf.* (19b)). Rappaport-Hovav takes these facts as evidence that incremental theme verbs are simple event predicates that denote activities. Similar arguments are also made in Stensrud (2009). Furthermore, Williams (2009) has argued, based on facts from resultatives in Mandarin, that the

agent and theme roles show certain parallels in their interpretations that go against Kratzer's arguments for separate treatments of these two roles. While adopting a fully Neo-Davidsonian account for introducing theme arguments is far from being uncontroversial, I take the evidence briefly sketched here to mean that the analysis advanced in this paper involving μ_V is at least a plausible one.

The motivation behind the move to introduce a degree argument through a functional head within the VP was to account for the distribution of *half* as an event modifier, as well as the desideratum of a unified analysis of such modifiers across the adjectival, partitive and verbal domains. The analysis involving a new silent functional head may at first seem somewhat costly, but, as I will now show, it does have some welcome benefits if we assume that an incremental theme is always introduced by μ_V.

First, as has already been mentioned, *half* can only occur with a bounded incremental theme argument, and is infelicitous with (unbounded) bare plurals or mass nouns (*cf.* (15)). I claim that this is due to the relation between the part structure of the nominal argument and the scale structure over which modifiers like *half* operate. Namely, a bounded argument yields a bounded scale, and an unbounded argument yields an unbounded scale. This observation also holds for deverbal adjectives derived from incremental theme verbs (see Kennedy & McNally 2005). Since *half* requires a fully closed scale, the contrast in (15) is fully expected.

Second, under the analysis described here, μ_V always introduces an open degree argument that must be saturated. In the absence of a scalar modifier, I propose that this role is fulfilled by a null *pos* morpheme, parallel with the adjectival cases. The resulting interpretations are exactly what are expected given the interpretation of *pos* as outlined in Kennedy 2007. Consider the following examples, repeated from (16):

(25)   John ate the apple.

(26)   John ate apples / applesauce.

In (25), μ_V introduces the incremental theme *the apple* and makes available a degree argument, which is saturated by *pos*. Recall from the discussion in section 2 that, based on a principle of Interpretive Economy, the application of *pos* yields a maximal interpretation when operating over a fully closed scale. Since the theme argument in (25) is bounded, then by hypothesis the corresponding scale should also be bounded, such that the application of *pos* results in a maximal interpretation of the event. This is indeed the case, as the most natural interpretation of (25) is a telic interpretation where the entire apple (i.e., a maximal quantity) is eaten, a welcome prediction of the analysis.

The semantics for (26) are derived in the exact same way. However in this case, the unbounded nominal arguments yield a scale that has no maximal value. When applied to an unbounded scale, *pos* yields a vague interpretation of the sentence that

is based on a contextual standard. That is, the quantity of apple or applesauce eaten in (26) is unspecified. Again, this follows directly from the degree-based analysis presented here and parallels the behavior of degree morphemes in the adjectival domain.

5   Compounding uses of *half*

Throughout the discussion in this paper, I have set aside compounding uses of *half*, where *half* may be used to form either nominal or verbal compounds as in (27).

(27)   a.   John is my half-brother.
         b.   Ann half-sang.

In these cases, *half* directly modifies a nominal or verbal head. Evidence for such an analysis comes from the fact that *half* occurs under the scope of plural marking in the nominal cases (*cf.* (9a)), and the fact that in the verbal cases, the presence of *half* does not correlate with the availability of telic readings (Bochnak 2010).

These uses of *half* differ fundamentally from the other cases discussed in this paper in that the examples in (27) do not target a scale of quantity. That is, in (27a), I am not saying that John is only a half of a person who has the property of being a brother; rather, we only share half of our parents. In (27b) I am not asserting that Ann only completed half an event of singing; rather I am making an evaluative statement about the quality of Ann's singing abilities, and stating that she performed an action that is similar to singing in some respects, but her performance was not a good example of an event of singing.

We may still say that *half* is operating over a scale, but in these cases it seems that it is a scale of *stereotypicality* that is relevant. For instance, while the stereotypical relation of brotherhood means that one shares two parents with a male relative, *half-brother* is used when one shares only one parent with a male relative. Likewise *half-sing* names an event that would not be considered a stereotypical event of singing. There is some evidence that such stereotypicality scales are fully closed scales, in keeping with the distribution of *half* elsewhere. First, in the case of (27a), an expression like *full brother* contrasts with *half-brother* and can be used to mark the maximal endpoint of the scale of stereotypicality of the brotherhood relation. Second, and in my view more convincing, is the possibility of using contrastive focus reduplication to identify the maximal value of a scale of stereotypicality (*cf.* Ghomeshi, Jackendoff, Rosen & Russell 2004). Thus, *brother-brother* identifies the most stereotypical of brotherhood relations, while *sing-sing* identifies the maximal value of a scale of stereotypical singing events. Even though these readings of *half* do not track quantity, the examples in (27) can at least be shown to be scalar in

nature, and related to fully closed scales at that, lending credibility to a unified account of *half* in all its uses.

6 Conclusions

In sum, we have seen that nominal arguments of gradable adjectives, partitives and incremental theme verbs play a crucial role in determining a scale structure that tracks QUANTITY and that can be targeted by proportional modifiers such as *half*. In particular, a bounded nominal argument yields a fully bounded (closed) scale, i.e., one that can be targeted by *half*, while an unbounded nominal argument yields an unbounded scale that disallows modification by *half*. In the partitive and verbal environments, a functional head μ was proposed to relate the nominal argument with the relevant scale and introduce a degree argument into these structures. This move allowed us to maintain a uniform analysis of scalar modifiers across categories and capture the generalizations about their distributions.

While the main goal of the paper was a unified analysis of modifiers like *half* across categories, this study has also brought to light other similarities that point to greater unity between the semantics and syntax of gradable adjectives, partitives and incremental theme predicates. First, I proposed that the semantic derivation of incremental theme predicates involved a covert partitive-like structure, that not only helped draw connections between partitives and VPs, but was also shown to have independent motivation based on amount comparatives and crosslinguistic facts. Second, it was shown that the degree-based analysis for incremental theme verbs makes the right predictions with respect to the behavior of the degree morpheme *pos* that completely parallels its interpretation in the adjectival cases as well.

The pervasiveness of the link between nominal part structure and scale structure suggests that this connection is a core feature of grammar that transcends syntactic category insofar that it occurs in at least the three distinct environments reported on here. Perhaps this is suggestive of the linguistic reality of a nominal-degree mapping, somewhat analogous to Krifka's object-event mapping, but that is more generalized given that it is found across other environments than simply event-denoting predicates. Future study along this vein should focus on a more detailed semantics of the QUANTITY predicate incorporated in the denotation of μ. The beginnings of such a line of research can be found in the degree-based analysis of aspect in Caudal & Nicolas 2005, though I have pointed out certain shortcomings of their account.

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