CONCEPTS   AND   OBJECTS

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0. Introduction

Once upon a time, Pavel Tichý started a dialogue with me. He asked: "Do we define expressions, concepts, or objects?". At that time, a bud of my present theory of concepts was already in my head; I construed definitions as linguistic means for associating concepts with some ("new") simple expressions. Therefore I answered: "Well, for me, definitions concern expressions, so that we define expressions." Tichý laughed. "Imagine", he said, "that you should define the expression 'prime number'. What would you say?" "O.K. Prime numbers are natural numbers divisible exactly by two numbers." "Surely not; if what you define is an expression your definition would be: 'the expression whose first letter is *p*, the second letter *r*, etc." I explained my motivation: an essential feature of (explicit) definitions consists in assigning the *definiendum* with a meaning (i.e., with a concept). Tichý did not assent. He said: "Imagine such a function: its domain is the set of natural numbers, and it returns 1,2 or 3 depending on whether the given argument is divisible by at most two numbers, or by more than two and less than six numbers, or by more than six numbers, respectively. Have I defined this function?" I admitted this. Tichý: "But I have not introduced any new term for denoting this function: there is no *definiendum* here."

This dialogue was characteristic of Tichý's aversion to the dominating trends connected with the s.c. 'linguistic turn'.[1] He even invented a new term: using an etymological paraphrase of *psychologism* (as an old and already defeated enemy of logic) he spoke about *glossologism* (as a rather new modish danger). His philosophy of logic and language was extremely hostile to Quinean behavioristic pseudo-semantics and late Wittgenstein's linguistic games.[2]

I share Tichý's views, resulting in his *transparent intensional logic* (TIL), and I already have made some attempts at applying his conception.[3] The present study belongs to such attempts; its starting intuitions can be formulated as follows:

(I) The (contingent, conventional) connection of an expression with an object ('aboutness') is realized due to the fact that expressions (of a language) encode (Gödel:"fix")[4] abstract entities which may be

*called (their) **meanings** or **concepts** (of the respective objects).[5] Whereas these objects are 'flat' in that they are structureless (at least in the sense that they do not mirror the structure of the respective expression), the meanings (concepts) are structured and their structure is mirrored by the respective expression. Moreover, two distinct languages (on approximately the same level of development) do not differ by encoding distinct meanings (concepts) but by the rules that connect expressions with these meanings. Thus meanings (concepts) are a kind of 'ways' (abstract 'procedures') which contain particular 'intellectual steps'[6] leading (in the best case) to the identification of the respective object. Further, concepts can identify other concepts; the latter play the role of 'quasi-objects'. Finally, objects (and quasi-objects) which are identified by concepts are always identified **a priori:** objects identified by empirical concepts are intensions; they are never the contingent values of these intensions in the actual world. A paradigmatic example: the concept PLANET (encoded, e.g., by the English expression 'planet') identifies a property of individuals; it does not identify the set {Mercury, Venus,...}.*
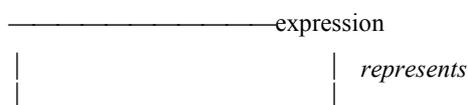
(Maybe that this position could be called 'neo-Fregean position', according to Forbes in [Forbes 1987], p.5-6.)
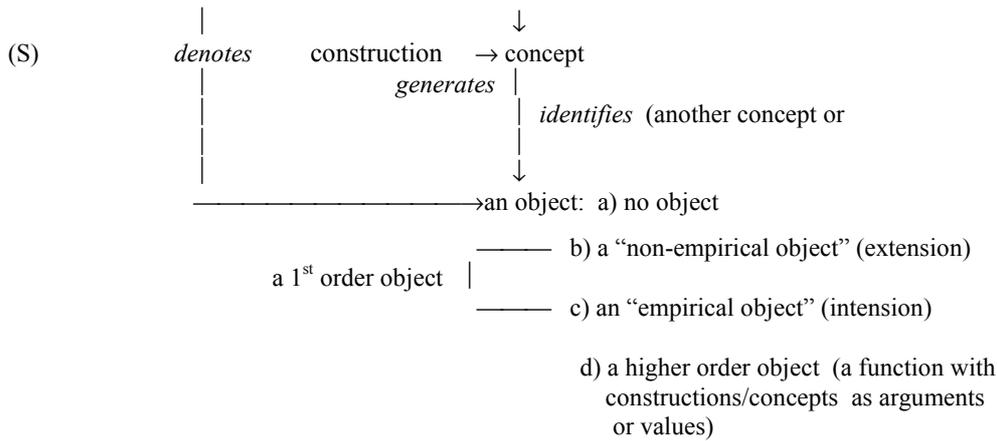
Many points in this 'confession' could be criticized from the very beginning, of course. The main objections would come from Quinean behaviorists[7] and 'late Wittgensteinians[8] ;the *don't ask for meaning, ask for use*-slogan would refuse any assumption that there would be some abstract entity like meaning. Indeed, no nominalist can accept such an assumption. (On the other hand, if an explication of 'concept' ('meaning') were successful, the well-known Quine's argument about circularity[9] connected with the attempts at defining the triple *analyticity-synonymy-meaning* would break down.[10] Now there are two ways of defending the above intuitions. One of them is 'direct': arguments supporting these intuitions could be formulated and allies in the history found. This way is not very effective: where the dissent concerns such basic intuitions as I have introduced in (I) no direct arguments use to be helpful (even if they point out some essential troubles connected with refusing these intuitions). A far more fruitful way consists in applying the following strategy: We start with our intuitions and show that they help us to solve many fundamental semantic problems[11] and that they do not principally block solving other semantic problems.

Indeed, choosing this strategy we do not intend not to use arguments. So on the one hand we do not pretend that we have *proved,* e.g., realism (an unattainable goal, of course), but on the other hand we will try to adduce arguments *supporting*, e.g., realism.

The central theme of the present study is the problem of explicating the notion of **concept**. Yet no survey of all or even of most historically realized theories of concept can be expected: [12] this is an original conception (based on TIL), so that offering a neutral (= non-critical) outline of theories of concept is not our intention.

The arrangement of our study has been inspired by the following scheme (partially derived from (I)):

————————————————expression

| | *represents*

| |

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                    |                    ↓
(S)          denotes     construction   → concept
                    |          generates  |
                    |                     |  identifies  (another concept or
                    |                     |
                    |                     ↓
          ─────────────────────────→an object:  a) no object
                                       ─────  b) a "non-empirical object" (extension)
             a 1st order object   |
                                       ─────  c) an "empirical object" (intension)

                                           d) a higher order object  (a function with
                                              constructions/concepts  as arguments
                                              or values)
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So the contents can be characterized as follows:

Ch.1 justifies the global structure of (S) and concerns *Frege's and Church's semantics* including Tichý's fundamental criticism.

Ch.2 defines what is called *'first order objects'* here. The definitions are based on a simple hierarchy of types and a non-Kripkean possible-world semantics.

Ch.3 is the key chapter. The notion of *construction*, first introduced in TIL, makes it possible to solve many problems which arose due to a basic misunderstanding, viz. to mixing up objects with the way they have been constructed.[13] This notion is the most characteristic one of TIL, and the latter cannot be understood without understanding the notion of construction.[14] The inductive definition of constructions is reproduced here with slight modifications.[15] Constructions not only serve to constructing objects. They can be also construed as a kind of objects (this time no more first order objects). This role of constructions cannot be logically dealt with within the simple hierarchy of types.

Ch.4 reproduces the definition of a version of *ramified hierarchy of types*, as it has been introduced in [Tichý 1988]

Ch.5 defines *concepts*. A brief and very incomplete history of this notion is adduced with a non-standard emphasis on Bolzano's theory of concepts. An important conclusion follows from those "historical" comments: set-theoretical construals of concepts are a blind alley in the theory. A competitive construal of concepts as structural entities[16] can be based on the notion of construction. Then a comparison with some traditional suggestions can be made and some new perspectives demonstrated. Besides, the scheme (S) can be justified and explained more exactly.

Ch.6 introduces a most important notion of *conceptual systems*: this will be needed when definitions will be treated in Ch.8.

Not less important is **Ch.7**, where the connection between abstract concepts and conceptual systems on the one hand and *languages* on the other hand is investigated. This chapter shows that semantics of natural languages can be based on the notion of concept and that the futility of the sceptical objections to the theories of meaning can then be convincingly demonstrated.

The last sections of Ch.7 are a kind of preparation for **Ch.8**, where a possibility of building up a theory of *definitions* based on the present theory of concept is sketched, the *expressive power* of a language is defined and some comments about 'implicit definition', explication and recursive definition are given.

Ch.9 briefly sums up the main points of the study.

To ensure smoothness of the text, references are located in remarks, which are concentrated in a pseudo-chapter following Ch.9.

The main idea of the book has been formulated in some articles[3] and - in Czech - in a book[17]. Unfortunately, I had no opportunity to discuss this idea with Pavel Tichý; he died before his planned return to Czech Republic, and he only knew some global features of my conception. So I am fully responsible for any possible misinterpretation of TIL. On the other hand, I admit that among my views there can be some which Tichý would not approve.

All the same, without him no new theory of concept would be created - at least not by me.

1. Church's scheme

We will probably agree that if the word *concept* were *substituted* for by the word *meaning* or *sense* in the scheme (S), our intuitions would not change (letting aside the up to now enigmatic word *construction*). One of the aims of the present chapter is to show that the possibility of such a substitution has already been explicitly stated and motivated by Alonzo Church[1] and that this Church's departure from Frege's construal of concepts[2] is a progressive step (which has been ignored by most later logicians and philosophers).[3]

To appreciate Church's idea one has to study history from the point where the contemporary **logical analysis of natural language** (**LANL**) (or perhaps: logical semantics of natural language) has begun; I mean Frege.[2] The story of the "Frege's problem" is well-known.[4] If it should be briefly recapitulated, then some new views should accompany this recapitulation. So our first point will specify two important *distinctions* which will dominate our views and affect our approach to Frege's problem and its solution.

Distinction a)   LANL deals with semantic properties and relations of *expressions*. It does not concern *use of expressions*. The latter involves expressions *plus situations* (speakers, circumstances, etc.), i.e., spatial and temporal entities[5]. Expressions themselves are abstract vehicles of 'pragmatic meanings' and as such they possess 'meanings'. Understanding meanings is an *a priori* relation[6] given (at any time point) by a linguistic convention. Knowing meanings we can derive pragmatic meanings dependently on a situation but not *vice versa*.[7] Thus only possible worlds and time points are those *indices*[8] among the Montaguian ones which are relevant for LANL.

Distinction b)   LANL cannot find or even concern the extensions of empirical 'roles', properties, relations etc.: any claim made by LANL is an *a priori* claim. Hence to justify a claim made by LANL we never need experience. LANL is not an empirical discipline.

Whereas the *distinction a)*, explaining the difference between semantics and pragmatics, will be actual only in Ch.7, where Quinean anti-Fregean behaviorism will be criticized, the importance of the *distinction b)* is immediate.

Let us briefly recapitulate Frege's problem. If we compare sentence A of the form $a = a$ with a true sentence B of the form $a = b$, then there is seemingly no semantic difference (as we would formulate it now) between A and B: for if B is true, then a has to denote the same object as b, and this means that B claims the self-identity of the object denoted by a (by b), which is absurd.

Unfortunately, Frege supposed that saying that something is denoted (*bezeichnet*) by an expression a (= something is *Bedeutung* of a) is sufficiently clear. In his classical [Frege 1892] he claims - without any discussion - that the *Bedeutung* of 'morning star' (as well as of 'evening star') is the planet Venus. A thorough criticism of this claim can be found in [Tichý 1988, 92]. Here we can simply state that Frege's construal of *Bedeutung* violates the *distinction b)*: whereas the expression 'morning star' can be understood *a priori* (see [6]; only knowledge of the language is necessary), the extension of the role[9] to be played by what is called (in English) 'morning star' is not *a priori* determined by the meaning of the expression; we can understand the latter without knowing that this role is played just by Venus - this knowledge is a result of an empirical (astronomical) discovery. Intuitively, this is notorious, therefore, Frege's *Bedeutung* is frequently translated as *reference*[10]. *Bedeutung* in the 'normal' use (as 'meaning') is a semantic relation to be investigated by LANL. Frege's denoting relation connecting an expression with its *Bedeutung* is an empirical, contingent relation fully irrelevant for LANL. Frege rightly drew fatal consequences from this careless construal of *Bedeutung*: sentences denote truth-values[11], predicates denote 'concepts' (*Begriffe*), i.e., characteristic functions of classes.[12] Especially in the case of sentences the counterintuitive character of Frege's *Bedeutung* is more than obvious. If the meaning of a sentence should be what makes it possible to understand that sentence, then of course the truth-value is the last thing that could serve this purpose.

The troubles with *Bedeutung* showed Frege that his problem of explaining the distinct semantic character of $a = a$ and $a = b$ would be unsolvable if *Bedeutung* were the only "semantic" entity at our disposal. Frege's way out is well-known: he introduced a new entity called 'sense' (*Sinn*). Unfortunately, no definition of *Sinn* has been formulated by Frege; we only know that sense is *the mode of presentation* ('Art des Gegebenseins'), viz. of *Bedeutung*.[13] The sentence $a = b$ is informative (unlike $a = a$) because the *sense* of a differs from the *sense* of b.

Now the most popular exemplification of this solution is connected with the sentence *morning star = evening star*: the mode of presentation of Venus is distinct in the case of *morning star* and in the case of *evening star*: the two expressions express distinct *senses*.

If we take into account the above example only, we are tempted to say that the Fregean sense is an 'intension', as construed later on by possible-world semanticists. Church, for example,[14]calls the senses of such expressions like 'morning star', 'the present King of France' etc. *individual concepts,* for Tichý[15] they would be *individual offices*. As for intensions, see Ch.2; here we only intuitively describe the idea of senses as intensions. Suppose that the sense of 'morning star' is an intension: this means that this expression is connected (*via* 'expressing') with a function which associates any particular possible world (and time point) with at most one object (individual). Frege's idea could be then explicated as follows: such a function expressed by 'morning star' is distinct from such a function expressed by 'evening star'; it is logically thinkable that an individual 'occupies the office' of being the morning star and that another (if

any) individual 'occupies the office' of being the evening star. This idea is intuitive: if somebody understands the expression 'morning star' or 'evening star', then it is because of the fact that both expressions are 'hidden descriptions' rather than neutral 'labels': is a celestial body the clearest one in the morning? in the evening? Indeed, to understand those expressions you do not need to know that one and the same object (viz.Venus) fulfils both criteria.

Unfortunately, this idea is wrong. The first example adduced by Frege in his [Frege 1892] is not the morning star/evening star example: his motivation for introducing the notion of *sense* is given by the following consideration:[16]

*Let **a,b,c** be the lines connecting the vertices of a triangle with the midpoints of the opposite sides. The point of intersection of **a** and **b** is the same as the point of intersection of **b** and **c**. So we have different designations for the same point, and these names ('point of intersection of **a** and **b**', 'point of intersection of **b** and **c**') likewise indicate the mode of presentation, and hence the statement contains actual knowledge.*

In this example we cannot apply intensions ('individual concepts' etc.); the point of intersection of a and b is not a function the values of which would depend on empirical facts (as in the case of intensions); the same holds of the points of intersection of b and c. That the resulting point is the same in both cases is not a contingent fact (as in the case of Venus) - it is a necessary mathematical (geometrical) fact. Yet something - called by Frege 'sense' and characterized as 'the mode of presentation' - is present in this example too, and it is this something what distinguishes the two expressions. Let the point of intersection of a and b be represented by P_{ab}, and let P_{bc} be the analogous representation . Let the 'resulting point' be A. Surely the sentence

$P_{ab} = P_{bc}$

says something fully different from

A = A

(similarly as the sentence

$3 + 2 = 7 - 2$

says something fully different from

$5 = 5$).

A precise explication of Frege's sense will be formulated later (see, e.g., Ch.3)[17]. Now whatever this sense is, it should be already now clear that it is closely connected with the normal (i.e., non-Fregean) use of the word 'meaning'. Frege's sense was meant to be *the* semantic characteristics of an expression; obviously, it is highly relevant for LANL. Let us, therefore, forget Frege's meaning (= *Bedeutung*) and use the word 'meaning' promiscue with the word 'sense'.

Explaining semantic foundations of logic Church[18] essentially reproduces Fregean views with one important (although seemingly only verbal) exception: since the sense of an expression E is a mode of presentation of an object denoted by E and since every (meaningful) expression expresses a sense, we can - according to Church - state the following equation:

the sense of an expression E = a concept of the object denoted by E[19]

Later (5.1.4) we will see in more details that this equation means a radical refusal of Frege's notion of concept: whereas Frege's concept (*Begriff*) is construed as the object denoted by a predicate[20] (so that neither descriptions - contrary to Bolzano - nor sentences - similarly as for Bolzano - are connected with concepts), Church changes this rather traditional explication and *associates every meaningful expression with a concept.*

The 'formal structure' of Church's scheme is very similar to the 'formal structure' of our scheme (S). What remains to be explained are two points:

1. What kind of entity is Church's *denotation*?
2. What kind of entity is Church's *sense* (*concept*)?

   The first point is probably clear: Church's *denotation* is simply Frege's *Bedeutung*. We have already shown that the relation that links expressions with 'referents' (better than with 'references') is an empirical relation, so that one of the fundamental distinctions dividing (S) from the Church's scheme consists in our refusal to construe objects as Frege-Church denotations.

   To elucidate the second point we again have to recall that Church essentially reproduces Frege's conception. Church only tries to explain the Fregean idea of sense. In [Church 1956] not much is made for making this idea more precise.[21] Instead, we must appreciate Church's "departure from Frege's terminology" (as Church says in his book) as regards using the term 'concept'. This departure is important not as a terminological change only, but as an attempt to formulate a far more intuitive explication than the traditional one has been. Church's concept is construed more generally than Frege's (and even Bolzano's) concept; see 5.1.

2. First order objects

The aim of our study is to show how a new theory of concept - based on TIL - can be built up. To fulfil this intention we have to interpret the scheme (S). In the present chapter we begin to realize this interpretation. For some methodological reasons we proceed bottom up, so that we start with what we have called *first order objects*, viz. intensions and extensions.

2.1 Intensions, extensions I.

The problem of distinguishing between intensional and extensional entities arose due to a phenomenon called *failure of substitutivity*[1].

The principle of substitutivity is a consequence of the (Fregean?) principle of compositionality: if the meaning of a compound expression E depends exclusively on the meanings of the subexpressions of E, then, of course, substituting a subexpression E' of E by an expression E'' that possesses the same meaning as E' cannot change the meaning of E. Now already Frege discovered that *some contexts seem not to obey this principle.*

Although this story is well-known, we can adduce some examples.

E1 *It is necessary that 3 is an odd number.*

If we accept (as we do not!) that the meaning of a sentence is its truth-value then substituting for the subordinate clause in E1 another true sentence (*London is an English town*) should not change the 'meaning' (=the truth-value) of E1, but it does.

(No remedy is obtained if the meaning of a sentence is vaguely called *thought* (*Gedanke*)[2]: which sentence might share this 'thought' with the subordinate clause? Indeed, the futility of this 'solution' is due to the vagueness of the word. We will see that if a suitable explication is given, the problem vanishes.)

E2 *It is forbidden to smoke in a railway station.*

Here what is forbidden is having a property (smoking in a railway station). If properties were dealt with as classes then the principle of substitutivity would be again violated: imagine that the class of those people who are just smoking in a railway station is identical with the class of those people who are just walking in the railway station. Then the meaning of E2 surely changes if the substitution is performed:

E2' *It is forbidden to walk in a railway station.*

Further examples:

E3 *Charles knows that London is bigger than Oxford.*

The story is in principle the same as in E1 (unless Charles is omniscient).

Similarly:

E4 *Charles asserts that London is bigger than Oxford*

Furthe*r* cases:

E5 *It is getting warmer because the wind dispersed the clouds.*
E6 *It is highly probable that there are no animals on the surface of Venus.*
(Suppose that you will win 1 000 000 $; try to substitute.)

Our examples represent following 'kinds of context': *modal* (E1), *deontic* (E2), *epistemic* (E3), *assertion* (E4), *causal* (E5), *probabilistic* (E6). The list is, of course, far from being exhaustive; besides, the whole contextual approach to intensionality is branded by circularity[3] and eclecticism[4].

The contextualistic approach is characteristic already of Frege's conception. One and the same sentence possesses one meaning (viz. a truth-value) in one kind of context ('direct contexts') and a distinct one (viz. a 'thought') in another one ('indirect contexts', called by more recent authors 'oblique' or 'opaque' contexts).

Later we will see that the contextual derivation of the category *intension* (and saying 'intensional contexts' instead of 'indirect contexts') ignores some essential distinctions between particular kinds of context; so, e.g., modal contexts essentially differ from epistemic contexts.[5]

TIL is a *transparent,,* i.e., *anti-contextualistic* system. Meaning of any unambiguous (disambiguated) expression is independent of context. Thus a sentence denotes an object independently of whether it is embedded in a direct, modal, or epistemic etc. context. The object (if any) denoted by a sentence can be a truth-value or a proposition, but this is given by the character of the sentence iself rather than by the kind of

the respective context. Also, expressions of modality concern what the subordinate clause denotes whereas epistemic verbs concern the concepts of what is denoted by this clause, which is fully compatible with the assumption that a sentence denotes independently of a context.[6]

Although we refuse contextualism of the above kind, we admit that the phenomenon of intensionality has been discovered due to the fact that 'normal', extensional principles seemed not to work in some kinds of context ('paradox of analysis', 'paradox of omniscience'). But they only *seemed* not to work; this 'failure of substitutivity' (Aho) can be explained in another way than by distinguishing 'extensional' and 'intensional' contexts: *what if the logico-semantic analysis of the parts of 'intensional contexts' is wrong independently of any context?*

Our scheme (S) and our later analyses based on it will show that this is really the case. Therefore, we will try to distinguish intensions and extensions independently of particular kinds of context.

Let us adduce some examples of expressions (without any context) and try to find out the reason why some of them denote what we would like to call *intensions.*

E7a	*Aristotle*	**E7b**	*the teacher of Alexander the Great*
E8a	*prime number*	**E8b**	*town*
E9a	*is identical with*	**E9b**	*is more intelligent than*
E10a	*3 + 5 = 8*	**E10b**	*the morning star is the evening star*
E11a	*nine*	**E11b**	*the number of the big planets of the Solar system*

We claim that the left side expressions denote extensions and the right side expressions denote intensions.[7]

Thus E7a denotes an individual (see 2.3; we let aside the problem of proper names - maybe that *Aristotle* is not a name ('label') of an individual: it might be construed as a hidden description[8], but here we need a maybe simplified illustration). E7b, however, denotes an *individual role ('office')*[9]: E7a would be a '*rigid designator*' for Kripke contrary to E7b. If we accept that *Aristotle* is a label of an individual, then the individual is unambiguously determined by that label, independently of any empirical fact; on the other hand, which individual played the role of the teacher of Alexander the Great is surely dependent on empirical facts - on the state of the world at the given time points. E7b cannot, therefore, denote a definite individual (if denoting is an *a priori* relation, as it should be, of course). In other words, the fact that the given individual (Aristotle) was the teacher of Alexander the Great is not logically necessary: it is a contingent fact. All information contained in E7b concerns the role, office; it does not concern the bearer of the role.

Consider E8ab: E8a clearly denotes a set, a class[10] of numbers. Does E8b - belonging to the same grammatical category - also denote a set, a class?

Suppose that it does. Any set is unambiguously determined by its members. Which members belong to the 'set of towns'?

Two points refute our hypothesis. First, which object is/is not a town is not given by the meaning of the word 'town': it is dependent on the given state of the world, i.e., on empirical facts. This dependence can be called *modal dependence* (here: *of the membership*), since it is logically possible that other objects

could be towns than those which are actually towns. Second, even following the actual states-of-affairs we can see that which object is a town changes with time: some objects become towns, some other cease being a town (remember Carthago!). This kind of dependence can be called *temporal dependence (of membership)*.

(An analogous pair of dependences holds for the case E7b.) Thus it is not a definite set what is denoted by 'town'. The entities denoted by such expressions (empirical common nouns, adjectives, intransitive verbs), i.e., entities which can be sets/classes dependently on world and time, are standardly called *properties.*

A similar case is illustrated by E9ab . ''is identical with' denotes a relation[11], here a set of ordered pairs where the first member of the pair is the same object as the second. Which pairs belong to this relation is independent of any empirical fact. Sometimes such relations are called *relations-in-extension* . As for E9b, the same modal and temporal dependence of the membership as above can be stated: which pairs of objects are such that the first object is more intelligent than the second, depends on empirical facts and changes with time. The entities denoted by such expressions, i.e., entities which can be relations-in-extension dependently on empirical facts, are often called *relations-in-intension.*

(E8 and E9 can be handled as follows: relations-in-extension are sets/classes of tuples, relations-in-intension are properties of tuples.)

E10ab are sentences. There is no other object that could be thought of as being denoted by E10a than the truth-value TRUE. Further, that this truth-value is TRUE rather than FALSE is, of course, totally independent of empirical facts; no modal or temporal dependence of the truth-value is thinkable. On the other hand, both these dependences hold for E10b: the modal dependence follows from the fact that E10b is not an analytical sentence, so that any truth-value is logically possible, and the actual one (it is TRUE, I hope) has been associated with it due to an astronomical discovery (and it could not have been logically proved). As regards the temporal dependence, there were surely time points when E10b did not possess any truth-value at all (there was no such thing that could be called 'morning star' or 'evening star').

The entities which can be - dependently on empirical facts - true or false (or lack any truth-value) are standardly called *propositions.*

Finally let us compare E11a and E11b. E11a clearly denotes a number. (Let numbers by whatever: they certainly differ from the expressions that denote them.) Again, modal and temporal dependence can be stated for E11b: it denotes only the role that must be played by a number to be the number of the big planets of the Solar system. The number 9 only *happens* to play this role.[12] Also, there were some time points when this number was 0.

The entities that can be a number dependently on empirical facts will be called *magnitudes* here.[13]

Now we can sum up our examples; we use a table.

extension	intension
individual (E7a)	individual role ('office') (E7b)
set/class (E8a)	property (E8b)

relation-in-extension (E9a)	relation-in-intension (E9b)
truth-value (E10a)	proposition (E10b)
number (E11a)	magnitude (E11b)

In this way we can characterize (later on: define) intensions independently of any context. *It does not hold, for example, that the sentence E10a denotes a truth-value in one context and a proposition in another one* (with an irrelevant exception mentioned in [6]); *it does not hold that E8b can denote a property in one context and a set in another one*[14], etc.

So which criterion will distinguish intensions from extensions in this pre-theoretical stage?

A brief inspection of our examples will show that there is a good intuitive criterion. E8a - E11a are mathematical, i.e., non-empirical expressions, whereas E7b - E11b are clearly empirical expressions. Let us further assume that proper names like *Aristotle* - if construed as 'labels' - are also non-empirical expressions.[15]

Our criterion is then:

Intensions are denoted by empirical expressions, extensions are denoted by non-empirical (especially, mathematical) expressions.

Now our explications of particular kinds of intensions shared (or could share) the following form:

The entities which can be X dependently on empirical facts...

where X has been an individual, a set/class, a relation-in-extension, a truth-value, a number (and any other thinkable kind of object). If, however, an entity E *can be* X *dependently on* Y, then the most natural way of construing E is to say that E is a *function* (a *mapping*) the arguments of which are members of Y and the values are members of X.[16] (Bealer[17] has chosen the obviously only rational alternative: to construe intensions as primitive entities.)

So we could say that intensions should be construed as *functions from the set of empirical facts*. Yet the notion of empirical facts is in itself a rather vague notion, so that another explication will be necessary. This will follow in 2.3. Now we will exploit the functional character of our approach and define (first) a general form of any *simple hierarchy of types*.

2.2 Simple types

The history of the theory (or better: theories) of types is well-known.[18] Its fruitfulness by far transcends the original purpose, i.e., blocking set-theoretical paradoxes; this has been discovered especially by A.Church, who has shown[19] that a marriage between types and λ-calculus can serve to solving interesting problems and that the child - the typed λ-calculus - is the best tool for dealing with functions. It was, first of all, Richard Montague[20], who exploited this fact for building up a system of LANL. There have been and will be several opportunities of taking a critical standpoint as regards Montague's approach. One of the points - maybe not the most important one - can be formulated just now. Montague defines the simple hierarchy of

types in two steps (which is, however, not specific for him - this 'formalist' method is one accepted nearly without exceptions): the first step consists in (inductively) defining well-formed type-expressions, the second step standardly 'interprets' these expressions by associating them with set-theoretical objects (besides, the *s* is construed as an improper symbol, i.e., what is interpreted is always the whole expression of the form ($s{\rightarrow}a$), where *a* is a type-expression[21]).

As Tichý states[22], the advantage of this approach is dubious. TIL, as an 'objectual' system, does not define language and interpretation separately. 'Looking through' the necessary symbolic expressions we speak directly about what is encoded by them.[23] Hence our definition of *simple types* is a straightforward one.

Def 1   Let B be a collection of pairwise disjoint non-empty classes (called 'base').

   a) Each member of B is *a type over B*.

   b) Let α, β_1,...,β_n be *types over B*. Then the set of all (partial) functions with domain (a subset of) β_1 x...x β_n and range (a subset of) α, denoted by ($\alpha\beta_1...\beta_n$), is *a type over B*.

   c) *A type over B* is only what obeys points a) and b).

We do not accept Schoenfinkel's reduction of functions to monadic functions, for Tichý has proved[24] that this reduction does not work, if we take into account partial functions.

The phrase 'over B' will be omitted wherever no misunderstanding can arise.

Notice that functions (as mappings) are *'flat objects'* - this is a most important point in the present theory. By 'flatness' we understand the fact that a function is, properly speaking, a set of n-tuples, where the (*n*-1)-tuples are arguments and the *n*th members values. One and the same function can be, however, given in infinitely many ways which are structure-sensitive (i.e., not flat).[25] On the linguistic level this can be easily seen: distinct expressions determine one and the same function; consider, e.g.,

$2x + 4$

and

$2(x + 1) + 2$.

In Ch.3 we will see that this 'linguistic level' is a secondary one.

From Def 1 it follows that any type is a set/class. Types according to a) (*atomic types*) are the 'basic' classes, types according to b) are sets of functions. This fact justifies the following definition:

Def 2   Let α be a type. Any member of α will be called an α-*object.*

It follows from Def 2 that, for any type α, α-objects are 'flat'. (This holds for simple types only.)

Some instructive examples will be useful.

Examples 1

Let B be the collection of two sets: let o be the set of truth-values, {**T**,**F**}, and τ be the set of real numbers. The type of unary truth functions will be (oo), of binary truth functions (ooo), etc. Arithmetical operations: addition, subtraction, multiplication, division are ($\tau\tau\tau$)-objects, binary numerical relations (-in-extension, of course) like > are (o$\tau\tau$)-objects. Natural numbers, positive/negative integers, prime numbers etc. are (oτ)-objects.

A generalization: Since classes/relations-in-extension can be construed as characteristic functions, we can identify classes of α-objects with $(o\alpha)$-objects, and relations-in-extension between β_1-,...,β_n-objects with $(o\beta_1...\beta_n)$-objects. Classes of classes of α-objects will be $(o(o\alpha))$-objects (etc.).

In general, functions taken into account here are *partial*, i.e., they associate every argument with *at most* one value (so that total functions, returning exactly one value, are a special case of partial functions). Even in such simple cases as those from Examples 1 it is obvious that partiality is an important feature of functions. Division, for example, is a 'typically partial' (i.e., non-total) function: wherever the argument is a pair $\langle x,0 \rangle$ (with x a real number), there is no value for it. Also, the function denoted by the expression 'the greatest' (in the area of real numbers) is non-total: its type is $(\tau(o\tau))$, since if a class of real numbers - i.e., an $(o\tau)$-object - contains the greatest number the above function returns this very number; otherwise, however, it returns nothing. (So that no number is denoted, e.g., by the expression 'the greatest prime number'.)[26]

The base B from Examples 1 does not enable us to define intensions as α-objects for some type α (over B). We cannot distinguish between sets and properties, relations-in-extension and relations-in-intension, truth-values and propositions, numbers and magnitudes, etc. A plausible hypothesis is then that the possibility of defining intensions as α-objects is dependent on which atomic types (members of the base) are chosen. Our problem can be, therefore, formulated as follows:

> (P) Which atomic types must be members of a base B so that intensions could be defined as α-objects over B?

Let us recall that whenever a kind of intension has been introduced two kinds of dependence have been stated: modal dependence and temporal dependence. Very informally speaking we could characterize the two dependences as follows:

> *modal dependence:* it is thinkable (or: possible) that the value of the given intension would differ from the 'actual' one;

> *temporal dependence:* the value of the given intension can change.

As soon as we make these two dependences more precise we will be able to solve our problem.

2.3 Possible worlds

Again, the history of possible-world semantics is well-known.[27] What is important for us just here is the construal of possible worlds specific for TIL; there are many distinct approaches to this category,[28] and since the distinctions may be relevant as regards possible solutions of semantic problems, we have to offer a rather precise characteristics of our conception.[29] First we positively define this conception and then we mention some important distinctions as regards other approaches.

The term 'possible world' introduced by Leibniz[30] can be easily misinterpreted due to the metaphoric character of the word 'world'. *World* is here not a collection of things (together with their properties and relations): it would be fully unjustified and irrational to suppose that there could be more than one world in this sense. A better approximation is obtained if 'world' is interpreted in the spirit of Wittgenstein[31] as denoting a collection of (possible, consistently thinkable) *facts*. The table I am observing is a thing, an

object; as an object it cannot be somehow 'pluralized'. On the other hand, *that there is a table* I am observing is no thing, no object in this sense:[32] it is a *fact*, and since there is no doubt that this fact is contingent, we are naturally allowed to suppose that another fact is thinkable (possible). Thus we could say (very informally) that *a possible world is a collection of thinkable facts;* one of such collections is the real, '*actual' world.*

This explication is still unsatisfactory. True, any interpretation of the term 'possible world' will possess a pre-theoretical character, but more must be said to make such an interpretation definite enough.

First, let us make explicit a rather trivial statement: Any possible world is a *consistent* set of facts. (In a possible world London is bigger than Oxford, in another world it is smaller than Oxford, but no possible world contains both these facts.) Further, in a very clear sense a possible world is a *maximum* such set of facts.[33]

Two further points are relevant on the present level of analysis. Since the category *possible world* is based on the notion of (logical) possibility, it is highly important to be aware of the *objective character of possibilities.* Let us consider any contingent event, say, my tossing a coin. Suppose I get head. Obviously, I could have got tails. This non-actualized possibility differs from the actual one just by 'non-actualizedness', not by being subjective (or 'less objective'?): this can be seen, e.g., from the simple fact that the number of possibilities is not arbitrary: if I toss a coin, there are just two of them. So *possible worlds are objective.* (There is no place here for a futile restarting of a discussion with nominalists: nobody has succeeded up to now in convincing them that the 'nominalistic paradise' is only an old and well objectionable illusion. Their refusal to accept objective abstract entities determines, of course, their refusal of our conception from the very beginning.)

The second point has fundamental consequences: it explains the last part of our intuitions (I) (see *Introduction*). Unless we confine ourselves to some little interesting 'artificial' or 'experimental' examples, i.e., if we are in earnest with the task of logically analysing *natural* language(s), *we never can say which of the possible worlds is the actual one.[34]* An intuitive reason is rather simple: If a possible world is a collection of possible facts, then the actual world is the collection of actual facts. To know which of the possible worlds is the actual one means, therefore, to know all actual facts. So unless we are omniscient we cannot identify the actual world. (We will see that the type of the actual world is not the type of possible worlds.)

Now we go to a deeper level of analysis. What kind of object (in the broadest sense) is a *fact*?[35] Consider a simple fact

that Warsaw is the capital of Poland.

Whether this fact does or does not obtain is dependent on possible worlds and on the respective time point in the given possible world: it is thinkable that Warsaw is not the capital of Poland, i.e., in other words, there are possible worlds where Warsaw is not the capital of Poland - this is what we have called *modal dependence* - and even in the actual world (and many other possible worlds) there were time points where Warsaw was not the capital of Poland - *temporal dependence.*

A brief inspection of this result shows that any fact (or at least any empirical fact) can be construed as a proposition: it can be true or false dependently on possible worlds and time points. Now we can get a little nearer to the solution of our problem (P). We already know that our base should contain atomic types corresponding to our intuitive notions of (the set of) possible worlds and (the set of) time points. The latter is achieved in TIL *via* an innocuous simplification. Let us assume that time points make up continuum and that time is linear.[36] Then - at least from the viewpoint of logical tractability - nothing happens if τ (the set of real numbers) at the same time serves as the set of time points. It remains to make the notion of possible world still more clear and to investigate whether there are some objects that could not be after all type-theoretically classified or constructed by means of the three resulting atomic types, viz. o, τ, ω, where ω would be the type (= the set) of possible worlds.

Let us return to our fact (proposition) about Warsaw. Consider the term

the capital of Poland.

Surely, what is denoted by it, cannot be Warsaw, otherwise our proposition/fact would be necessary, whereas it brings an empirical information. (The same case as with the proposition *that the morning star is the evening star.)* Thus what is denoted by the term is an 'individual role/office'. Such a role can be played only by some kind of simple objects called in TIL *individuals*.[37] Now the missing atomic type for our base is the type (=the set) of individuals, let it be ι. More must be said, however, about this most problematic type.

TIL construes individuals in a purely anti-essentialist style. Individuals are simple material entities possessing no non-trivial property necessarily. (As for detailed argumentation see the first study mentioned in [37].) Thus it is not necessary for a lump of matter called 'Venus' to be a planet; the same individual could have any distinct property, which can be formulated saying that there are possible worlds and time points where Venus is no planet, being instead something fully different. Also, the individual I am just observing only happens to be a table - I must observe it to state that it is a table rather than a chair or a dog. Among the arguments for this 'ontological nudism' (Hintikka) one of the strongest is the following one: Investigating any individual for its properties we must know *which* individual is tested, otherwise there is nothing to which the respective properties could be ascribed. But to know *which* individual it is we cannot apply experience (an infinite regress!) - the latter is needed only for deciding *what sort of* individual it is, i.e., which properties it possesses. So individuals are given *a priori*, they are really 'naked', not in the sense that they might lack every non-trivial property - they are always given together with some such properties - but that they only *happen* to be bearers of such properties. As for the claim that individuals are simple we refer to the second study mentioned in [37].

There are many - mostly philosophical - problems about individuals. This theme has been frequently [38] investigated and it is closely connected with the semantic problem of proper names.[39] We already have seen in 2.1 that saying *Aristotle* we cannot be sure that it is a particular individual what is denoted by this term. We have used the phrase *hidden description* to cover such cases (this approach is compatible with the theory offered in [Tichý 1988] and obviously incompatible with Kripke's 'historical' explanation.) In many cases the phrase *hidden description* can be applied without any problem: so anybody should admit that 'morning star', although possessing the form of a proper name rather than the form of a Russellian
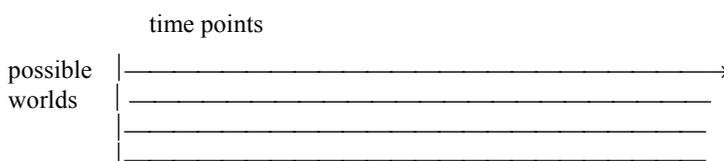
descriptive phrase, all the same exhibits the essential features of the latter: people understand that expression without necessarily knowing that it is Venus what plays the role in question, and without being somehow acquainted with Venus.[40]
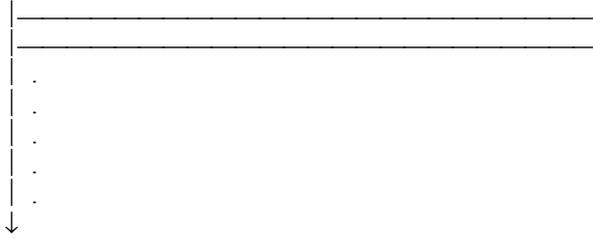
Superficially viewed there are some counterintuitive consequences of the present construal of individuals or at least of the claim that 'bare individuals' are what is denoted by proper names. Being 'bare' the individuals are in a sense eternal: there is always an individual which at some time points in some possible worlds plays the role of Aristotle (in other worlds it can play a fully distinct role) and which after some time gets the property *being dead Aristotle* and begins to play another role. Such consequences are surely hard to swallow, but an essentialist conception leads to much worse troubles. Since our study is not primarily about the problems with individuals we just state that we hold the anti-essentialist conception of individuals, as defined in TIL. Besides, we admit that the only way how to become acquainted with individuals might be ostension and that all proper names might be 'hidden descriptions'; for some didactic reasons we will, however, exploit in our examples of analyses some commonly used proper names and suppose that they denote individuals.

One technically important consequence of our construal of individuals is that ι, the set of individuals (='universe') is shared by all possible worlds. *There are no 'possible individuals'*, and, furthermore, we do not need Hintikka's 'world-lines'.[41]

(We can imagine that a kind of 'Pegasus-argument' could be adduced to prove that we must take into account 'possible individuals': in some possible worlds, we are said, there is an individual Pegasus, in other possible worlds - including the actual one - there is no such individual. This objection is easily refutable as soon as we accept that Pegasus is an individual role/office rather than an individual. Then it simply holds that in some possible worlds an individual does and in the other ones no individual does play this role (i.e., 'pegasize') - which is compatible with the assumption of one common universe for all possible worlds.)[42]

Returning to our problem of an apt base we can recapitulate that the base $\{o,\iota,\tau,\omega\}$ could serve our purpose. A concluding remark will make the idea of possible worlds still clearer. Some pre-theoretical intuitions are necessary. They can be formulated *via* Tichý's idea of *intensional base*.[43] Briefly reproduced, this idea is based on the following assumption: Every natural language has at its disposal a collection of some distinguishing features that enable us (in principle) to say whether an object does or does not exhibit a given feature.[44] At any time point there is some objective distribution of these features (which are, essentially, pre-theoretical ancestors of properties) over objects. The set of all such possible distributions during a temporal span is, of course, *a priori*. What is not *a priori,* is which of these possibilities has been realized. Possible worlds - as a pre-theoretical notion - are just the mentioned distributions during time, the realized distribution can be called *the actual world*. A simplified picture models the relation between possible worlds and time:

```
          time points

possible  |———————————————————————————————————————————→
worlds    |————————————————————————————————————————
          |——————————————————————————————————————
          |——————————————————————————————————————
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|——————————————————————————————
|——————————————————————————————
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|   .
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There are more systems that use the term 'possible world'. Now, after we have explained our interpretation, we only add some remarks about specific features of it with respect to other theories.

    a)   For TIL, possible worlds are not non-interpreted parameters, 'indices' that would play only an 'instrumental role'.[45]

    b)   They are objective and not relativized to particular speakers of the given language. They are not 'epistemic possible worlds'.[46]

    c)   Kripke's 'accessibility relations'[47] are not part of our conception. They have been introduced in order to make possible semantic interpretation of modal systems S1 etc., but the result can be hardly called semantics: only formal properties of those 'accessibility relations' are taken into account, and the resulting relativism cannot offer a satisfactory explication of logical modalities.[48]

A more detailed comparative study would surely discover some further distinctions but this is no comparative study.

2.4 Intensions, extensions II.

In the preceding section we have proposed the following base as one serving the purpose of type-theoretically distinguishing extensions and intensions: $\{o,\iota,\tau,\omega\}$. These abstract types have to be interpreted.[49] The type o is interpreted as the set $\{\mathbf{T},\mathbf{F}\}$, viz. {True, False}. As for τ, we have stipulated the convention according to which τ is the type of real numbers and, at the same time, of time points. The type ι is the universe (members: individuals) and ω is the set of possible worlds (relative to particular languages).

Now imagine a function which associates the particular time points with at most one truth-value. Any such function will be called a *chronology of truth-values*. Similarly, we can define *chronologies of individuals, of classes of individuals, of numbers, of classes of classes of individuals, of n-ary relations between individuals,* etc. ad infinitum. Clearly, any chronology is an $(\alpha\tau)$-object, where α is an arbitrary type. Now we can define functions which associate any possible world W with some chronology specific for W. The type of any such function will be $((\alpha\tau)\omega)$, which will be abbreviated here as $\alpha_{\tau\omega}$. Obviously, the members of α will be dependent on time points and possible worlds (temporal and modal dependence). Thus one of the possible definitions of intensions[50] will be:

Def 3 *Intensions* are $\alpha_{\tau\omega}$-objects.

Now we need following definitions:

Def 4 *First order objects* are α-objects, where α is a simple type (i.e., a type according to Def 1.

Def 5 *Extensions* are first order objects which are not intensions.

Def 6 *Non-trivial intensions* are intensions the values of which are distinct in at least two
possible worlds.

Informally we can state the following claim:

Claim 1 *Any empirical expression [51] denotes a non-trivial intension.[52]*

Arguments: An expression will be called 'empirical' if our decision whether this expression can be correctly used about some object in the given state-of-affairs depends on the state of the world (in other words: if experience is needed). If it denoted an extension or a trivial intension (i.e., such an intension whose value would be constant over worlds and times), the above decision would not need experience. -

Now we will exemplify our definitions and exploit for this purpose our examples E7 - E11 (2.1).

Expression	Denoted object	Type
Aristotle	the individual Aristotle[53]	ι
the teacher of Alexander the Great	an individual role / office	$\iota_{\tau\omega}$
prime number	the set of prime numbers	$(o\tau)$
town	a property	$(o\iota)_{\tau\omega}$
is identical with	the relation of identity	$(o\alpha\alpha)$[54]
is more intelligent than	a relation-in-intension	$(o\iota\iota)_{\tau\omega}$
3 + 5 = 8	the truth-value T	o
the morning star is the evening star	a proposition	$o_{\tau\omega}$
nine	the number 9	τ
the number of the big planets ...	a magnitude	$\tau_{\tau\omega}$

These examples lead us to a general table (only some important kinds of extension/intension are adduced).

Extensions	Type	Intensions	Type
truth-values	o	propositions	$o_{\tau\omega}$
individuals	ι	individual roles	$\iota_{\tau\omega}$
classes of α-objects	$(o\alpha)$	properties of α-objects	$(o\alpha)_{\tau\omega}$
relations-in-extension	$(o\beta_1...\beta_n)$	relations-in-intension	$(o\beta_1...\beta_n)_{\tau\omega}$
numbers	τ	magnitudes	$\tau_{\tau\omega}$

One point concerning TIL and all similar (possible-world-) explications of intensions is extremely important: such intensional systems preserve the principle of extensionality; they are in a well-defined sense *extensionalistic*.[55] For such systems it does no more hold that *intensionality is failure of substitutivity* (Aho), at least it does not hold whenever the problem of substitutivity concerns empirical expressions. To illustrate this claim we can again return to some examples from 2.1. Consider E2. Here substitutivity concerns the name of a property (*smoking in a railway station*); since properties are functions, i.e., set-theoretical objects, the principle of substitutivity is not violated if the name of the given *property* is substituted for by another name of *the same property* (of the same function); *the sameness of properties, not the sameness of classes is what counts*, and nothing prevents us from making such substitution - one name of a function is replaced by another name of the same function. (Here the expression 'to smoke in a

railway station' could be replaced by some expression like 'to use tobacco in a railway station' or by another such expression: the denoted property must be the same function from possible worlds as that denoted by 'to smoke in a railway station'. Similarly, in the case of E3: the subordinate clause ('London is bigger than Oxford'), denoting a proposition, can be replaced by the sentence 'Oxford is smaller than London', since the denoted proposition ('truth-conditions') is the same.[56]

Thus it would seem that this *extensional treatment of intensions* could save the principle of compositionality. This would really hold, if a) *meaning* were identical with what the given expression *denotes*, and if b) empirical expressions denoted (non-trivial) intensions. Yet whereas b) should be obviously accepted (see Claim 1) the point a) is by far not as clear (by the way, it is incompatible with (I) and (S)). In fact, a) can be rather easily shown to be wrong. Consider the sentence

E 12 Charles knows that 3 is a prime number.

If we accepted a), then we would be bound to say that the meaning of the subordinate clause is either the truth-value **T** or the constant true proposition. In both cases, replacing this clause in E12 by any other true mathematical sentence we either violate the principle of substitutivity, or suppose that Charles is omniscient (*paradox of omniscience*).[57] Thus a) cannot be accepted. We will see that even cases like E3 can be deprived of their seeming obviousness. Assuming this we formulate the following claim:

Claim 2 *First order objects cannot be identified with meanings.*

Concluding remarks.

1. There are, of course, $(\alpha\omega)$-objects where α is not $(\beta\tau)$ for any β. Among such 'pseudo-intensions' a most interesting one is *the actual world*: similarly as the 'President of USA' is not the name of Bill Clinton but a description denoting an individual role, i.e., an $\iota_{\tau\omega}$-object, the expression 'the actual world' denotes only a role/office : it is a description; we can never know which of the possible worlds is the actual one. Thus it is a function which associates any possible world with this very possible world - it is an identical function.[58] (Let a possible world W be given. Which possible world is the actual one in W? Clearly, W itself. Therefore, saying, e.g., *London is actually bigger than Oxford* we communicate no other information than by saying *London is bigger than Oxford.*[59]) The actual world is, therefore, not an ω-object but an $(\omega\omega)$-object.

2. Up to now our examples covered what could be called *intensions of 1st degree,* i.e., $\alpha_{\tau\omega}$-objects, where α was a type of extension. A wrong impression could arise as if values of intensions in the particular possible worlds necessarily were extensions. We adduce an example of an intension of 2nd degree: Consider the expression

E 13 the favourite proposition of Albert Einstein .

The denoted object is clearly an intension, i.e., an $\alpha_{\tau\omega}$-object. This time, however, α is no type of an extension: the α-object (if any, don't forget partiality!) which is dependent on world and time is always a proposition, i.e., an $o_{\tau\omega}$-object. Thus the object denoted by E13 is an $(o_{\tau\omega})_{\tau\omega}$-object (an intension of 2nd degree). Intensions of nth degree are easily definable *via* induction.

3. Some important extensions are connected with the s.c. *logical words*. Especially, this concerns *truth-functions, quantifiers, identity, singularizer* (denoted by logical connectives, quantifiers (as expressions), identity sign, descriptive operators, respectively). We have already stated the type(s) of identity. Now the type of unary truth-functions is (oo) , of binary truth-functions (ooo), of quantifiers[60] $\forall, \exists$ (o(oα)) (with α an arbitrary type), of singularizers ('descriptive operators') ι (α(oα)). See Examples in Ch.4.

3. Constructions

3.1 Basic intuitions

In Introduction and in 2.4, particularly in (I), and Claim 2, we have formulated the following *hypothesis:*

(H) First order objects cannot be meanings (concepts), for they are 'flat', unlike meanings, which have to be structured.

In this general form, (H) is no special discovery. Cresswell[1] , for example, has introduced the term '*structured meanings'*, and long time before Carnap[2] grasped the impossibility of applying his *method of intensions and extensions* to propositional attitudes, and tried to solve the problem by means of '*intensional isomorphism'*[3]. In the area of computer science Girard[4] has formulated the problem as follows: considering the equality

   27 x 37 = 999

   he says:

   *... This equality makes sense in the mainstream of mathematics by saying that the two sides **denote** the same integer and that x is a **function** in the Cantorian sense of a graph.*

   This is the denotational aspect, which is undoubtedly correct, but it misses the essential point:

   *There is a finite **computation** process which shows that the denotations are equal. It is an abuse... to say that 27 x 37 **equals** 999, since if the two things were **the same** then we would never feel the need to state their equality. Concretely we ask a **question**, 27 x 37, and get an **answer**, 999. The two expressions have different **senses** and we must **do** something (make a proof or a calculation, or at least look in an encyclopedia) to show that these two **senses** have the same **denotation.***

Letting aside a terminological misunderstanding in the last sentence we can say that the above consideration rather precisely formulates the necessity of interpreting *sense* as a structured 'process' (here: a calculation).

In Tichý's book[5] we find a similar consideration:

   *If the term '(2 x 2) - 3' is not diagrammatic of anything, in other words, if the numbers and functions mentioned in this term do not themselves combine into any whole, then the term is the only thing which holds them together. The numbers and functions hang from it like Christmas decorations from a branch. The term, the linguistic expression, thus becomes more than a way of **referring** to independently specifiable subject matter: it becomes **constitutive** of it. An arithmetical finding must, on this approach, be construed as a finding **about** a linguistic expression. ... But since an expression is*

always part of a particular notational system, our theorist must construe the arithmetician as being concerned specifically with a definite notation.[6]

The common idea of both quotations can be formulated as follows:

Structuredness is not a privilege of linguistic expressions. The semantics of a compound expression E is not reducible to the semantics of particular simple subexpressions of E.[7]

Moreover, the structured entities encoded by linguistic expressions are objective, i.e., *independent of particular languages.*

The general idea may be clear; the concrete problem is: how should be these abstract structured entities defined so that they could be logically dealt with?

P.Tichý[8] has introduced and in all his publications used a new notion that makes it possible to solve this problem; he called these entities *constructions* and offered an inductive definition of them. A slightly modified version of this definition is the subject matter of the present chapter.

Before this definition is given (3.2 - 3.5) some important intuitive commentaries will be necessary.

First, our problem might seem to be unsolvable: if constructions are independent of particular languages and if at the same time we want to logically deal with them, can we introduce a symbolic apparatus without violating the independence of any language? After all, any symbolic apparatus is a kind of language!

To give a correct answer to this objection is of key importance for understanding our conception. So let us begin at length.

Any rational theory of language accepts the following trivial principle:

*A (linguistic) expression E is always **about** an object distinct from E.*
The autonymous use of expressions is only apparently an exception. We can write s*oon is an adverb* but this is only an abbreviation. Correctly written we have *'soon' is an adverb* - here it is already clear that the expression *'soon'* is about the distinct expression *soon*.

Now introducing a symbolic apparatus for handling constructions we introduce, no doubt, an *artificial* language. Yet using this language we speak about the entities which the expressions of that artificial language are about. (The formalist view would require that first a formal language were given and then an interpretation proposed. To this see our commentary in 2.2.) So just as saying *Venus is a planet* we speak about Venus, the individual, and not about the respective expression, we can write (see 3.6)

$[\lambda x \, [^0 > x \,^0 0]]$ *constructs the class of positive numbers*

and relate the predicate ('constructs the class...') to the construction itself, not to the expression

$[\lambda x \, [^0 > x \,^0 0]]$.

(On the other hand, if we want to speak about such artificial expressions rather than about the constructions depicted by them, we can do it, for example as follows:

$'[\lambda x \, [^0 > \,^0 0]]'$ *contains two pairs of brackets*)

Second, after constructions are defined, we will have at our disposal a tool of distinguishing between a (target) object and the way it is constructed.[9] So the semantics of an arithmetical expression, say, 2+3, will be able to distinguish between the 'flat' object 5 and a structured construction $[^0+ \ ^02 \ ^03]$ which is *one of the ways how to construct* 5. In this sense we can say that the theory of constructions *transcends the standard set-theoretical approach.*

Third, it is useful to be aware of *the abstract character of constructions.* Being abstract means not to be spatio-temporal. For example numbers are abstract, since we cannot ask where or when a number is. The same holds of constructions. We can adduce an analogy:

A computer *program* is abstract. It is not identical with its record (as a list of symbolic expressions) neither with its realization (as a process going on during a time span in a particular computer). From this viewpoint we can say that any programmer, when writing down a program, *discovers* this abstract program. A *construction* can be construed as an *abstract procedure* the outcome of which is - in the better case - an object (a first order one or - as we will see - a higher order one). An abstract procedure is *abstract* because it is not identical with the expression depicting it neither with its realization; it is a *procedure*, because particular parts ('steps', or even 'intellectual steps'[10]) of it "do not get lost", are distinguishable.

Fourth, to be able to describe constructions by finite means we need some supply of *basic constructions* ('building blocks') that would enable us to create all 'useful' or 'desirable' constructions from the basic ones. The general considerations that would justify such a choice of basic constructions have not been explicitly formulated in TIL, so let us do it here.

The *'atomic' constructions* construct objects *via* a 'valuation' over a given type. They are called *variables* (see 3.2).[11]

Objects can be given 'immediately'. This possibility becomes a necessity from the viewpoint of finite methodology: some ways of constructing objects have to be unanalyzable unless we admit *regressus ad infinitum*. Just like we work with *primitive terms* on the linguistic level, we need *primitive concepts* on the abstract ('ontological') level; we will see in Ch.5 and 6 that primitive concepts are generated by 'primitive constructions': these are called *trivializations* (see 3.3.).

Our approach to LANL is functional (Frege's heritage)[12]. Therefore, one kind of 'intellectual step' badly needed is the construction consisting in *applying the given function to an argument*. This procedure is well-known from λ-calculi;[13] it is often called *application* but in TIL the name *composition* has been introduced, since 'application' can be interpreted as the *outcome* of applying a function to an argument, whereas composition contains all the steps, i.e., the function as well as the argument and the (possible) parts of the latter. Composition is defined in 3.4.

Yet to apply a function to its arguments presupposes that the function is at our disposal. The two up to now explained ways of constructing a function, viz. variables and trivialization, do not suffice: a function can be constructed from more simple elements, as it is well-known from the school examples such as

$(y =)$ $2x^2 + 1$.

Thus a construction of functions by means of variables is needed. Its 'linguistic analogy' is again the well-known *abstraction* in λ-calculi. The construction that corresponds in TIL to abstraction is called *closure* and is defined in 3.5.

Is it everything we need?

In his book Tichý introduces two further kinds of construction: *execution* and *double execution*[14] (an infinite hierarchy of 'executions' could be inductively defined). I do not feel that these kinds of construction would be indispensable for LANL; they will not be defined here.

Well, would we perhaps need some other kinds, ones that have not been introduced in TIL? Hard to say. We will base our theory of concepts on the above characterized kinds of construction. This is, however, not to say that the theory of constructions is not an open-ended theory. In its present form, however, it has proved competent for solving many semantic problems.[15]

The inductive definition of constructions (Def 7) will proceed in the next five sections. In the last section of the present chapter some informal examples will suggest the applicability of the notion of construction in LANL.

3.2 Variables

Variables as constructions are construed similarly as in Tarski's *Wahrheitsbegriff*[16]: for every type α there are denumerably many (α-)variables at our disposal; valuations are total functions that associate each variable with one object of the respective type. One distinction is, however, essential. Whereas not only Tarski but practically every standard logician consider variables to be letters, characters, this 'linguistic' conception is unacceptable by TIL: constructions are *language independent* entities; hence if variables are a kind of construction they cannot be letters, characters. The letters standardly used for variables (like *x, y,...,p,q,...*) are conceived to be *names of variables* here.[17] Thus we can say that, e.g., for the type τ we will use variables $x_1, x_2,...$, but this will mean that 'x_1', 'x_2',... are names of numerical variables. In the case of τ, there are infinitely many 'τ-sequences' at our disposal, i.e., infinite sequences of the members of τ, viz. numbers; the same holds of any type α. The *k*-th variable (denoted by 'x_k') constructs the *k*-th member of such an α-sequence. It is just *valuation*, which submits a definite α-sequence, so that any variable is, properly speaking, an *incomplete construction*: it constructs an object *dependently on valuation*. We say, therefore, that variables *v-construct* objects, where *v, v'*, etc. are parameters of valuations.

In our case, i.e., with base = $\{o,\iota,\tau,\omega\}$, there are infinitely *many* types over this base at our disposal; a valuation can submit not only infinitely many α-sequences for the given type α, but an infinite array

$$X^1_1, X^1_2, X^1_3,...$$
$$X^2_1, X^2_2, X^2_3,...$$
$$X^3_1, X^3_2, X^3_3,...$$
$$.$$
$$.$$
$$.$$

where X_j^i is the *j*-th sequence over the type *i*; thus infinitely many columns correspond to there being infinitely many (infinite) α-sequences, and infinitely many rows correspond to there being infinitely many types.

For example let x_3 be a τ-variable (i.e., a variable *v*-constructing τ-objects, or: ranging over τ) and let *v* submit the τ-sequence

0, 0, 3.2, 5, 0, ...

(the other members of the array are irrelevant, of course). Then we say that x_3 *v*-constructs 3.2.

Naturally, we will not determine in advance which names for which α-variables we will use - it is impossible to do. Yet whenever a misunderstanding could arise we will say which type is the range of the given variable.

Notice that *technically* the variables *qua* constructions behave exactly so as the variables *qua* letters in the standard systems. On the other hand, due to the above 'objectual' approach to variables, the whole theory of constructions is consistently objectual: *no construction is a linguistic expression.*[18]

Def 7a Variables are (atomic*) constructions*.

3.3 Trivialization

Def 7b Let X be any object or *construction.* ^{0}X is a *construction* called *trivialization.* It constructs X without any change.

If X is a first order object, ^{0}X will be called a *first order trivialization.* The higher order trivializations will be explained and fruitfully exploited in Ch.4.

In general, objects are not constructions.[19] The latter have been introduced to enable us to distinguish between objects and the ways they are constructed. This distinction should be preserved even in the case of 'immediate' constructions. Thus in the world of abstract entities trivialization simply serves this purpose.

There are, however, some 'bridges' between our abstract realm of constructions and the world dealt with by epistemology. Just as numbers are abstract whereas counting is a mental process using numbers, just as concepts are abstract whereas possessing,[20] learning etc. concepts are psychologically and epistemologically interesting processes, so trivialization is an abstract construction whereas 'possessing trivialization' could be conceived of as an 'immediate identification': notice that if X is constructed by ^{0}X then it means that X is identified without the use of any other construction. A simplified example can illustrate this fact by means of a psychological 'counterpart':[21] A child is able to identify circularity without exploiting other geometrical constructions like POINT, EQUIDISTANT, etc. What the child 'possesses' is just the trivialization ^{0}Circle.

The importance of trivialization will be obvious as soon as ramified hierarchy of types is used.

Examples 2

^{0}Moon constructs the individual Moon. Don't forget that *using* the word 'Moon' we speak about the respective individual and not about the expression itself. Thus 'Moon' in $^{\prime 0}$Moon' denotes the individual Moon, and ^{0}Moon is (or: $^{\prime 0}$Moon' denotes) the immediate construction of the Moon. Therefore, if, e.g., 'catastrophe' and 'disaster' denote one and the same object (a property of events), then we say that ^{0}catastrophe and ^{0}disaster are *one and the same construction.*

Let x_1 be (the first) τ-variable. We have seen that it v-constructs real numbers; $^0x_1$, on the other hand, constructs (i.e., v-constructs for all valuations v, i.e., independently of valuations) just the variable x_1. Notice that the question *Which type $\alpha`$ is such that x_1 is an α-construction (i.e., that x_1 v-constructs α-objects) ?* gets the simple answer: τ. The similar question concerning $^0x_1$ cannot be answered within the simple theory of types: whereas x_1 v-constructs τ-objects, so that it is a τ-construction, $^0x_1$ constructs the variable x_1, but variables are no α-objects where α would be a simple type. Our question will be answered as soon as ramified hierarchy (Ch.4) is introduced.

Further examples will be adduced after some other kinds of construction are defined.

3.4 Composition

Def 7c Let X be a *construction* that v-constructs a function F (type $(\alpha\beta_1...\beta_n)$) and
      let $X_1,...,X_n$ be *constructions* that v-construct β_1-,...,β_n-objects $b_1,...,b_n$,
      respectively. If F is defined on $<b_1,...,b_n>$, then the *construction*
      $[XX_1...X_n]$, called *composition*, v-constructs the value of F on that
      tuple. Otherwise it v-constructs nothing: it is *v-improper.*

(From this definition it follows that if any of the constructions $X,X_1,...,X_n$ is v-improper, the whole composition is v-improper, too.)

Examples 3

^{0}: constructs the standard function called *division* (type $(\tau\tau\tau)$). The composition

(c1) $[^0: {}^04\ {}^02]$

constructs 2; unlike this outcome it contains particular compounds ^{0}: , ^{04}, ^{02}. The composition

(c2) $[^0+ {}^01\ {}^01]$

also constructs 2 but it is a distinct construction.[22] We would like to state the fact that both these constructions are equal in the sense that they construct one and the same object; applying identity we get

(c3) $[^0= [^0: {}^04\ {}^02]\ [^0+ {}^01\ {}^01]]$

where the type of $=$ is $(o\tau\tau)$: really, the pair $<2,2>$ constructed by (c1) and (c2) belongs to the relation $=$. On the other hand, we cannot as yet express the fact that the two above constructions of 2 are *not identical*. Better to say, we can write down

(c4) $[^0\neg\ [^0= {}^0[^0: {}^04\ {}^02]\ {}^0[^0+ {}^01\ {}^01]]]$

but then the type of $=$ is undetermined within our simple theory of types: the constructions whose distinctness is claimed by (c4) are not type-theoretically determined; so we have to wait till Ch.4 defines the ramified hierarchy.

What does the construction

(c5) $[^0\!: \ ^0\!5 \ x_1]$

v-construct $(x_1$ being a τ-variable)? The answer is simple: it v-constructs the outcome of dividing 5 by what is v-constructed by x_1. If x_1 v-constructs 2, (c5) v-constructs 2.5. If x_1 v-constructs 0, then (c5) is v-improper.

Let V be the ι-object Venus. Let Pl be the $(o\iota)_{\tau\omega}$-object (property) *being a planet. Let w, t* be ω-, τ-variables, respectively. The construction

(c6) $[[[^0\text{Pl} \ w] \ t] \ ^0\text{V}]$

v-constructs **T** or **F** dependently on whether w and t do or do not v-construct such a possible world and time point where Venus is a member of the class which is the value of the property Pl in the v-constructed possible world and time point.

Abbreviations: If X is an $\alpha_{\tau\omega}$-object and w, t as above, we will write $^0\text{X}_{\text{wt}}$ instead of

$[[^0\text{X} \ w] \ t]]$. Thus (c6) can be written

(c6') $[^0\text{Pl}_{\text{wt}} \ ^0\text{V}]$.

Further, instead of writing *X is an α-object* we write *X/α,* instead of *the construction C v-constructs α-objects* (i.e., *C* is an 'α-construction') we write *C* ... α.

An example of an improper (= v-improper for all valuations v) construction: Let $\text{Gr}/(\tau(o\tau))$ be the (partial) function which, when applied to a class of numbers one of which is the greatest one in that class, returns this very number; otherwise, it is undefined. The class $\text{Pr}/(o\tau)$ of prime numbers is an infinite class, hence it contains no 'greatest member'. Therefore, the construction

$[^0\text{Gr} \ ^0\text{Pr}]$

constructs nothing; it is improper.

The composition is the only kind of construction[23] that can be (v-)improper.

3.5 Closure

Def 7d   Let $x_1,...,x_m$ be arbitrary pairwise distinct β_1-,...,β_m-variables and X an

         α-*construction*. Then $[\lambda x_1...x_m X]$ is an $(\alpha\beta_1...\beta_m)$-*construction* called

         closure. It v-constructs the following function *F*: Let the tuple

         $<b_1,...,b_m>$ of $\beta_1,...,\beta_m$-objects, respectively, be an argument of *F*. Let

         v' associate with $x_1,...,x_m$ the respective members of the above tuple and be

         otherwise identical with *v*. Then the value of *F* on that tuple is the (α-)object

         v'-constructed by X; if X is *v'*-improper, then *F* is undefined on $<b_1,...,b_m>$.

As in the case of composition, this definition is inspired by (typed) λ-*calculi*. Only that a closure is *no expression*; as the other constructions, it is an 'abstract procedure'. Closures produce functions. Whereas

functions as mappings are 'flat', closures are structured: they might be construed as 'instructions' how to 'create' a function. [24] The arguments of this function are the m-tuples of objects v-constructed by $x_1,...,x_m$, the values are given by X, the 'body' of the functional construction. The parts of X are 'lost': the resulting function is only a table with the m-tuples (arguments) on the left side and the values (α-objects) on the right side.

Examples 4
Let v associate τ-variables x_1, x_2, x_3 with 1, 5, 8, respectively.

$[\lambda x_1 x_2 \; [^0\colon \; x_1 x_3]]$

v-constructs the function (type $(\tau\tau\tau)$) that associates every pair $<j,k>$ of numbers with the outcome of dividing j by 8. (So that the value of this function is, e.g., 1.5 for every pair $<12, k>$, k any number.)

Some special functions contructed by a closure:

an *identical* (numeric) *function* $(x_1 ... \tau)$:

$[\lambda x_1 \; x_1]$

(If the outmost brackets are omitted, as we will often do, we get $\lambda x_1 x_1$)

A *constant* (numeric) function:

$\lambda x_1 \; {}^0 3$

(constructs 3 for every argument).

Obviously, which object is constructed by a construction depends only on variables which are - intuitively - *free*. The standard definition of free and bound variables must, however, be modified, for *trivialization also binds variables*.

Def 8 Let C be a construction and x a variable.

Let C contain at least one occurrence of x.

a) If C is x, x *is free in* C.

b) If C is ${}^0 X$, x *is o-bound in* C.

c) If C is $[X X_1 ... X_n]$, x *is free in* C iff at least one occurrence of x *is free in* X or $X_1,...$ or X_n.

d) If C is $[\lambda x_1 ... x_m \; X]$, then x *is free in* C iff it is distinct from $x_1,...,x_m$ and *is free in* X. The variables $x_1,...,x_m$ *are λ-bound in* C iff they are not *o-bound in* X.

e) The variable x *is free or o- (λ-)bound in* C only due to a) - d).

It is useful (if not necessary) to distinguish between *o-boundness* and *λ-boundness*. If two constructions differ only due to a correct (i.e., collisionless) renaming of λ-bound variables, then they are *equivalent* in that they (v-) construct one and the same object (or are both (v-)improper). On the other hand, if two constructions differ only due to a renaming of o-bound variables they do not (v-)construct the same object.

For example

(c7) $\lambda x_1 \; [^0 \rangle \; x_1 \; {}^0 0]$

constructs the class of positive numbers and

(c8) $\lambda x_2 \; [^0 \rangle \; x_2 \; {}^0 0]$

constructs the same class. (See the *α-rule* in the λ-calculi.) On the other hand,

(c9) $^0[\lambda x_1 \, [^0\rangle \; x_1 \; {}^00]]$

constructs (c7) and

(c10) $^0[\lambda x_2 \, [^0\rangle \; x_2 \; {}^00]]$

constructs (c8) ; since (c7) and (c8) are *distinct* (although equivalent) constructions, (c9) is *not equivalent* to (c10). (Notice that x_1 and x_2 are o-bound, not λ-bound.)

Now let us return to (c6); there the variables *w, t* are free, and therefore various truth-values are *v*-constructed. As soon as both these variables become bound a single object is constructed. If o-boundness is tried nothing interesting arises: the construction

$^0[\; ^0Pl_{wt} \; {}^0V]$

constructs (c6). (Here, as well as in the case of (c9), (c10), we perform something not quite legal: the simple theory of types is not closed w.r.t. trivializing constructions.) But try to λ-bind *w,t*. We get, e.g.,

(c11) $[\lambda w \lambda t \, [^0Pl_{wt} \; {}^0V]]$

Checking type-theoretically (c11) we obtain gradually :

$Pl/ (o\iota)_{\tau\omega}$, $Pl_{wt...} (o\iota)$, V/ι, $[^0Pl_{wt} \; {}^0V]...o$, $\lambda t \, [^0Pl_{wt} \; {}^0V] \, ... \, (o\tau)$, $(c11)...o_{\tau\omega}$.

We see that (c11) constructs a *proposition.* This proposition is true in those possible worlds at those time points where the individual Venus belongs to the class which is in those possible 'worlds-times' the value of the property *being a planet.* Is this not the maximum what can be expected from a logical analysis of the sentence *Venus is a planet.?*

3.6 Concluding remarks and examples

The importance of the category *construction* for solving the numerous problems of semantics for natural language will be clearer in Ch.7. Here only a global, 'philosophical' point will be mentioned; we adduce a rather long quotation where this point is precisely formulated and where also *logic* is construed in a 'non-standard' way (if we take into account some recent trends). [25]

> *Logic is the study of logical objects (individuals, truth-values, possible worlds, propositions, classes, properties, relations, and the like) and of **ways such objects can be constructed from other such objects.** The logician makes it his business to explain, for example, how Bill, the individual, and walkerhood, the property, combine to yield or construct the proposition that Bill walks, and how walkerhood combines with some other object(s) to yield or construct the proposition that everything walks. The point of investigating logical constructions of objects is two-fold. In the first place, the nature of such constructions often guarantees noteworthy properties or relationships between the objects generated by those constructions. For instance, the two constructions mentioned above assure that the proposition generated by the former is weaker than (i.e., is implied by) the proposition constructed by the latter. In the second place, **logical constructions can be assigned to linguistic expressions as their analyses.** For example, the former construction will serve as the logical analysis of the sentence "Bill walks" and the latter as the logical analysis of "Everything walks". Provided*

that those analyses are correct, the aforementioned relationship between the constructions legitimizes an argument from "Everything walks" to "Bill walks". (Emphasis mine.-P.M.)

(By the way - since this topic is not the main topic of the present study - the above construal of logic is in good accordance with Bealer's conception[26]; logic is not reducible to a set of 'formal tools': it is not the case that there is no logic without axiomatic systems; that if a system of knowledge is not finitely axiomatizable then it is uninteresting from the logical viewpoint; and, of course, playing with symbols deprived of any meaning or investigating into formal/algebraic properties of particular systems of axioms (with even 'logical constants' being deprived of a definite meaning) may be interesting from the, e.g., algebraic point of view but should not be called *logic*.)

Now it is the point suggesting the connection of *constructions* with *expressions* which is the most important one in the above quotation. This problem - we could call it *constructions and meaning* - is far more complicated than it could seem on the base of that quotation. The last task which Tichý attempted to perform before his death consisted just in solving this problem for (an essential fragment of?) English.[27] This task can be performed only *via* an intensive cooperation of logicians, linguists and programmers. Tichý's idea - explicitly formulated in one of his papers[28] and having to be realized in the fragment of his last work - can be articulated as follows:

To realize the abstractly claimed but never realized Choomsky's proposal to build up a set of rules that would make it possible to produce pairs *<expression, meaning>* we cannot proceed like Montague, i.e., define a logical artificial language, ensure rules for translating expressions of a natural language into that 'logical language' and solve the problem of functioning the latter instead of solving the problem of functioning the former.

The idea (of *cracking the natural language code*, i.e., of the direct, for the given natural language specific derivation of 'Chomskian pairs') is hard to realize; the respective work is time-consuming, and this study cannot replace what has to be done in order to perform the task. Besides, the topic is distinct - it is not a particular natural language what should be analyzed here; instead, a general framework of such analyses (moreover, based on a theory of concepts) will be defined here. Since, however, some characteristic examples can clarify our principles we will adduce simplified cases of analyses; no background detailed systematic theory of such analyses e.g. for English is at our disposal. One general principle can be stated: *In general, a meaning of an expression has to be structured.* So - as we have emphasized in Claim 2 - no first order object can be a candidate for meaning. Later (Ch.7) meanings will be in a sense identified with concepts. Now we will show an intuitive method of associating expressions with constructions; the latter surely satisfy the requirement of structuredness.

Examples 5

Offering his explication of the *de dicto - de re* distinction[29] Tichý shows that the contextualist view according to which the expression *The American President* possesses two distinct meanings depending on whether it occurs in a *de dicto* or in a *de re* context can (and should) be replaced by a transparent analysis according to which this expression has one and the same meaning in both kinds of context. The illustrating contexts are

(1)   *The American President is a Democrat. ,*

(2)   *The American President is electible.*

Our intuitive method of analysing (1) and (2) can be illustrated as follows:

a) Let us make a *type-theoretical analysis* of particular simple subexpressions of (1) and (2).
We have seen that the description *The American President* cannot denote a definite individual. The object denoted by it is - independently of 'world-time' - an individual office/role. This means that the type of the object denoted by *The (present) American President* is $\iota_{\tau\omega}$. The expression *(is a) Democrat* (i.e., *is a member of the Democratic Party*) obviously denotes a property of individuals: an individual can have/lack this property depending on worlds-times. Hence the object denoted by this expression is an $(o\iota)_{\tau\omega}$-object. Finally, the expression *(is) electible* also denotes a property, but this time the property cannot concern individuals: it is an office what can be/not be electible. Recalling that the properties of α-objects are $(o\alpha)_{\tau\omega}$-objects we see that the object denoted by *electible* is an $(o\iota_{\tau\omega})_{\tau\omega}$-object.

b) *Synthesizing.*

Similarly as in the simple 'Bill-walks-example' in our above quotation the problem to be solved can be formulated as follows: (1), as well as (2) is an empirical sentence. Hence the objects denoted by these sentences are propositions, i.e., $o_{\tau\omega}$-objects. The parts of the respective construction have to be constructions of A (the American President), type $\iota_{\tau\omega}$, D (Democrat), type $(o\iota)_{\tau\omega}$, E (electible), type $(o\iota_{\tau\omega})_{\tau\omega}$. Thus our questions are:

How to construct an $o_{\tau\omega}$-object from a $\iota_{\tau\omega}$- and an $(o\iota)_{\tau\omega}$-object? (case (1)), and How to construct an $o_{\tau\omega}$-object from a $\iota_{\tau\omega}$- and an $(o\iota_{\tau\omega})_{\tau\omega}$-object? (case (2)). (Our building stones being variables, trivialization, composition, closure.)

Let us formulate the intuitions connected with (1) and (2). Both these sentences ascribe a property to an object. (An intuition which can be supported by linguistic input data.) (1) ascribes a property of individuals to *whoever occupies the office* A, (2) ascribes a property of individual offices just to the office A. Thus (1) cannot be about a definite individual (e.g., Bill Clinton) but it is about the office itself, saying that its bearer (*whoever it is*) has the property D. We cannot write

$[^0D_{wt}\,^0A]$

since $^0D_{wt}$ *v*-constructs a class of individuals (of Democrats in the world-time *v*-constructed by *w, t*) whereas ^0A constructs an office rather than an individual. If, however, $^0A_{wt}$ replaces ^0A, we get

$[^0D_{wt}\,^0A_{wt}]$

which *v*-constructs a truth-value **T/F** depending on whether the individual occupying A in the world-time *v*-constructed by *w,t* is or is not a member of the class that is the value of the property D in that very world-time. (2) is also about the office A, but this time the property is ascribed directly to this office. Hence we have

$[^0E_{wt}\,^0A]$

which *v*-constructs **T/F** depending on whether the office A (as a whole) is or is not electible in the world-time *v*-constructed by *w,t*. (Notice that non-electibility of A is nothing logically impossible.)

Now it remains to bind (λ-bind!) *w,t* so that a proposition is constructed. We get

(1') $\lambda w \, \lambda t \, [^0D_{wt} \, ^0A_{wt}]$,

(2') $\lambda w \, \lambda t \, [^0E_{wt} \, ^0A]$

Already this intuitively formulated example shows that constructions can be used to semantically analyse expressions. Moreover, if *constructions* replaced *concept* in (S) some interesting consequences could be detected: Consider the sentence (1). According to (S) it denotes a proposition (a first order object). Now we can see that one of the ways how to *identify* this proposition is *to construct it* so as it is realized in (1'). To get our proposition (= truth-conditions for (1)) we have to 'abstract' over worlds-times the composition consisting in the application of the property *being a Democrat* (to a world and time point and then) to the role of *the American President* applied to the respective world and time point. Thus the above constructions (an analogous consideration concerns (2')) could be good *explicates* of Frege's *Sinn*. Further we can see that - in accordance with Claim 2 - *constructions cannot be first order objects:* they are not α-objects where α would be a simple type (see Def 4). Also - which is connected with the preceding statement - *constructions are not 'flat'* : they are not reducible to a set, for the components of them are well distinguishable; for another example illustrating this let Pr/(oτ) be the set of primes, Odd/(oτ) the set of odd numbers, ∧/(ooo) conjunction, ⟩/(oττ) the grater-than relation on numbers, $x... \tau$. Then

(3) $\lambda x \, [^0\wedge \, [^0Pr \, x][^0\rangle \, x \, ^02]]$

is a construction distinct from

(4) $\lambda x \, [^0\wedge \, [^0Pr \, x][^0Odd \, x]]$

but both (3) and (4) construct one and the same set.

(A 'standard' variant of stating this simple fact is a 'linguistic' one: (3) and (4) are taken to be expressions which determine one and the same set - they are equivalently 'interpreted'. To this 'linguistic surrogate' see 3.1, especially the first quotation from Tichý's book.)

Now a confrontation with our scheme (S) can be made. Consider the English expressions

(5) *the primes greater than 2,*

(6) *the odd primes.*

(3) can be said to be a construction which *generates* the concept *represented* by (5), and the respective set is then *denoted* by (5) and *identified* by the concept generated by (3). (Analogically as for (4) and (6).) What remains to be explained is the relation *generate* holding between constructions and concepts: see Ch.5.

Remark: Using constructions for analysing expressions we are immediately confronted with the following problem: Consider the construction

(3') $\lambda y \, [^0\wedge \, [^0Pr \, y][^0\rangle \, y \, ^02]]$,

where $y ... \tau$. Is it important to decide which of (3),(3') will be connected with the analysis of (5)? (Our decision would be bound to choose one of infinitely many constructions that differ only by containing distinct λ-bound variables.) This problem will be solved in 5.3.

4. Ramified hierarchy

Confronting Def 1 (*simple types*), Def 4 (*1st order objects*), and Def 7 (*constructions*), we can state

Claim 3   *The class of first order objects is not closed w.r.t. constructions.*

Proof: According to Def 4 first order objects are α-objects where α is a simple type. Yet if X is a construction then ^{0}X constructs this very construction and Def 1 cannot determine its type (constructions are neither basic[1] nor functions, for they are not 'flat').

So unless we take into account only first order trivializations (see 3.3) we cannot construct only first order objects.

On the other hand, we can imagine that constructions themselves can play the role of objects in the broader sense, i.e., as something what is spoken about. A convincing example may be the analysis of the sentence

(1) *Charles calculates 2 + 3.*

Our type-theoretical analysis results in the following problem:

Let Charles be (for simplicity's sake) a ι-object. The types of 2,3,+ are clear. Yet which type can be ascribed to calculating?

We can refuse two following options:

a)   Calculating relates individuals with expressions.

b)   Calculating relates individuals with numbers.

As for a), calculating is independent of a specific notation. It is, e.g., completely irrelevant whether Charles calculates 2+3 or two plus three. We should expect that calculating concerns what the expressions mean rather than the expressions themselves.

As for b), calculating 2+3 is surely distinct from calculating, e.g., 7-2 or $\sqrt{25}$. Thus calculating does not concern the outcome of a procedure: it concerns the procedure itself. But then what remains seems to be only that calculating links individuals with constructions.[2] Hence the type of calculating must be $(o\iota?)_{\tau\omega}$, where ? stands for the type of the construction $[^0+\ {}^02\ {}^03]$.

This example (as well as indefinitely many of the same or even of a distinct kind) illustrates the necessity of extending our hierarchy of types so as to be able to logically deal with constructions *qua* objects in a broader sense. More technically, this means that *a transition from simple types to a ramified hierarchy is necessary.*

In what follows a slightly modified version of Tichý's definition[3] of a ramified hierarchy of types is given. For our purposes suppose that the base B (Def 1) is fixed and consists of the types o,ι,τ,ω.

Informally, the definition has three parts: First, *types of order 1* are defined (T_1), second, *constructions of order n* are defined (C_n), third, *types of order n+1* are defined.

Notice that being a construction of order n is not the same as being an object of a type of order n; moreover, it is always distinct.

Def 9

T_1

Simple types are *types of order 1*.

C$_n$

Let α be a type of order n.

   a) Any variable that *v*-constructs α-objects is *a construction of order n*.

   b) Let X be an α-object. Then ^0X is *a construction of order n*.

   c) Let $X_0, X_1, ..., X_m$ be constructions of order n. Then $[X_0X_1...X_m]$ is *a construction of order n*.

   d) Let $x_1,...,x_m, X$ be constructions of order n. Then $[\lambda x_1...x_mX]$ is *a construction of order n*.

T$_{n+1}$

Let $*_n$ be the collection of all *constructions of order n*.

   1. $*_n$ and every type of order n is *a type of order n+1*.

   2. Let $\alpha,\beta_1,...,\beta_m$ be *types of order n+1*. Then $(\alpha\beta_1...\beta_m)$, i.e., the collection of partial functions from $\beta_1 \times ... \times \beta_m$ into α, is *a type of order n+1*.

   3. Nothing is *a type of order n+1* unless it is determined by T_{n+1} 1.,2.

Comments.

The formulation in T_{n+1} "every type of order *n* is a type of order *n+1*" makes it possible to ascribe types to complex constructions the components of which are constructions of distinct orders. Let us demonstrate it on our example (1):

Type-theoretical analysis: C(harles)/ ι, Cal(culate)/ $(o\iota*_1)_{\tau\omega}$, $2/\tau$, $3/\tau$, $+/(\tau\tau\tau)$, $w...\omega$, $t...\tau$ (but, of course, $w/*_1$, $t/*_1$). We get

(1') $\lambda w\, \lambda t\, [^0Cal_{wt}\, ^0C\, ^0[^0+\, ^02\, ^03]]$

Now primarily, (1') is not type-theoretically homogeneous:

According to T_{n+1}: since o,ι,τ,ω are types of order 1 and $*_1$ is a type of order 2, we must exploit the formulation in T_{n+1}, i.e., admit that also o,ι,τ,ω are types of order 2, so that the type of Cal is - according to T_{n+1} 2 - of order 2. So ^0Cal, as well as $^0Cal_{wt}$, is a construction of order 2 (= a member of $*_2$). Now $^0+/*_1$, $^02/*_1$, $^03/*_1$, so that also $[^0+\, ^02\, ^03]$ is a member of $*_1$ (see C$_n$ c). Since this means that the *type* of this construction is of order 2, $^0[^0+\, ^02\, ^03]$ is a construction of order 2. But since the type of C (i.e., ι) is of order 1, ^0C is a construction of order 1! So - exploiting again T_{n+1} 1 - we admit that the type of C is also of the order 2, hence ^0C is a construction of order 2. Repeating this cumulative method for *w, t* we make (1') ("secondarily") type-theoretically homogeneous and we can claim that it is a construction of order 2 (and its type is of order 3).

On the basis of Def 9 we can define what has been called in (S) "higher order objects":

Def 10 *Higher order objects* are α-objects, where the order of α is greater than 1.

Notice that neither constructions, nor functions whose values or arguments contain constructions are first order objects.

When saying - as we do in our scheme (S) - that expressions may denote also higher order objects we mean that *via* concepts we can also talk about such objects (in the broadest sense) as are constructions, classes of constructions, etc. What is important is, however, that the higher order objects are of two kinds: they may

be constructions/concepts - then they are 'quasi-objects' (see Introduction) - or they are 'normal' 'flat' objects: set-theoretical objects, i.e., essentially, functions with quasi-objects as arguments or values (see propositional attitudes below).

Most trivial (and little interesting) examples are of the following kind $(x_1 \ldots \tau)$:

(2) $[^0+\ {}^07\ x_1]$ *contains the variable* x_1

Type-theoretical analysis: Con(tain)/ $(o*_1*_1)$ (a relation holding between a construction of order 1 and a τ-variable if the former contains the latter)[4], $x_1/\ *_1$. (The rest is obvious.) In (2) the construction $[^0+\ {}^07\ x_1]$ *is spoken about:* it plays the role of an object (in the broadest sense). Thus we have

(2') $[^0\text{Con}\ {}^0[^0+\ {}^07\ x_1]\ {}^0x_1]$.

Notice that if, e.g., x_1 replaced ${}^0x_1$ in (2'), we would get a type-theoretical nonsense which would be paralleled by an epistemic nonsense: the result (2'') of this replacement would 'claim' that the construction $[^0+\ {}^07\ x_1]$ contains the number v-constructed by x_1. Besides, whereas (2) is a full-blooded true sentence, (2'') would not construct any truth-value even if Con'/ $(o*_1\tau)$ replaced Con: x_1 would be *free* in (2'').

There are much more interesting cases where we speak about higher order objects. One of the most classical ones is the case of *propositional* and *notional attitudes.* We have already adduced an example of a notional attitude in (1): calculating is one of such attitudes. As for propositional attitudes, we here only suggest an approximation to the final solution of the problem of *belief sentences.*[5]

Consider E3 from 2.1, i.e.,

(3) *Charles knows that London is bigger than Oxford.*

It is obvious that *knowing* (as well as *believing, doubting,* etc.) cannot be a relation between individuals and *truth-values.* (This is what Frege was bound to admit already in 1892.[6]) That it cannot be a relation between individuals and *sentences* (as Quine was and obviously still is convinced) has already been convincingly shown[7]. Let us try to test the next hypothesis[8], viz. that *knowing* etc. relate individuals with *propositions* (i.e., with $o_{\tau\omega}$-objects). This idea is *prima facie* attractive: a *propositional* attitude should concern *propositions* because what I believe, know etc. about is a *state-of-affairs*, best modelled just as a proposition. Using this hypothesis we get: Kn(ow)/ $(o\iota o_{\tau\omega})_{\tau\omega}$, L/$\iota$, O/$\iota$ (both for simplicity's sake), B(igger than)/ $(o\iota\iota)_{\tau\omega}$. So that we get

(3') $\lambda w\lambda t\ [^0\text{Kn}_{wt}\ {}^0\text{C}\ [\lambda w\lambda t\ [^0\text{B}_{wt}\ {}^0\text{L}\ {}^0\text{O}]]]$.

It seems now that our intuition is satisfied. To ascertain ourselves, however, we can ask whether our analysis would validate the following intuition: If *knowing* etc. link individuals with propositions, then the truth conditions of (3) will not change if the clause *London is bigger than Oxford* is replaced by another sentence denoting the same proposition, e.g., by the sentence

(4) *Oxford is smaller than London.*

Let Sm/ $(o\iota\iota)_{\tau\omega}$ be the relation *smaller than.* We get

(4') $\lambda w\lambda t\ [^0\text{Sm}_{wt}\ {}^0\text{O}\ {}^0\text{L}]$.

Clearly, if the construction following in (3') after ${}^0\text{C}$ were replaced by (4') the constructed proposition would be the same. And of course we are ready to say: whoever knows that London is bigger than Oxford knows also that Oxford is smaller than London (and *vice versa).*

But unfortunately neither this last hypothesis is plausible. Also Montague, who has explicitly formulated it, suspected that it could be endangered.[9] We can prove its untenability in two points:

First, consider the sentence

(5) *Charles knows that 2+3 = 5.*

If the type of Kn were $(\text{o}\iota\text{o}_{\tau\omega})_{\tau\omega}$ (according to our hypothesis) then (5) would be analyzed as follows:

(5') $\lambda w \lambda t\ [^0\text{Kn}_{wt}\ ^0\text{C}\ [\lambda w \lambda t\ [^0= [^0+\ ^02\ ^03]\ ^05]]].$

Then, however, nothing should change if the constant proposition constructed by

(6) $\lambda w \lambda t\ [^0= [^0+\ ^02\ ^03]\ ^05]]$

were denoted by another (subordinate) clause than "2 + 3 = 5" if both sentences denoted the same proposition. Yet which proposition is denoted by (6)? Obviously, it is such a function which with every pair <possible world, time point> associates **T**; now every (true) mathematical sentence can be said to denote either **T** or the above constant proposition (see the remark [6] in Ch.2 and Claim 1). Hence any true mathematical sentence could be substituted for "2 + 3 = 5" in (5) without changing the truth conditions (and, therefore, the respective truth-value) of (5). A well-known *paradox of omniscience.*

Second, one could object that the above counterexample concerns only the case when the subordinate clause is an analytic sentence, i.e., a sentence denoting a constant (true or false) proposition; if the subordinate clause is an empirical sentence then - we could be said - the situation is distinct: in such case (see our example (3)) what is known, believed etc. is the state-of-affairs referred to by the subordinate clause, i.e., just the function which distinguishes truth-values dependently on worlds-times, i.e., a proposition. This objection necessarily leads to a dualism: for mathematical (in general, analytic) subordinate clauses the type of propositional attitudes differs from the type of propositional attitudes for empirical clauses.[10] Yet it can be shown that the case *empirical subordinate clauses* is by far not as clear as it could seem to be on the basis of (3). A trivial (logical) fact is that *any proposition can be constructed in infinitely many ways*. On the linguistic level this can be easily seen when we are aware of (theoretically) infinitely many *equivalent transformations* of one and the same sentence. The transformed sentences can become more and more complex so that it is very natural to suppose that somebody who, e.g., believes the sentence A does no more believe the (L-)equivalent sentence B if B is sufficiently complex. To adduce only a very simple example, we can replace the clause *London is bigger than Oxford* in (3) by the equivalent sentence

(7) *If London is not bigger than Oxford then it is not true that if London is not bigger than Oxford then Oxford is at least as big as London.*

Many people who know that London is bigger than Oxford cannot be said to know also that (7) is true. What is the moral of this point? Clearly, our propositional attitudes do not concern immediately propositions: primarily, they are '*constructional*' attitudes (similarly as 'notional'attitudes, see (1)). Surely, the construction underlying (7) is distinct from - and, of course, more complex than - $\lambda w \lambda t\ [^0\text{B}_{wt}\ ^0\text{L}\ ^0\text{O}].$

So it seems that types of propositional attitudes are $(\text{o}\iota*_n)_{\tau\omega}$ for $n \geq 1$ (most frequently used for $n = 1$) rather than $(\text{o}\iota\text{o}_{\tau\omega})_{\tau\omega}.$

(This result is still only an approximation: in the next chapter we will see that the above attitudes concern *concepts* rather than particular constructions, but the argument that we need ramified hierarchy is also supported by such a correction, for also concepts are higher order objects - see Chap.5.)

The ramified hierarchy makes it possible to speak about constructions, to *mention* them. Thus we can observe that for example describing or deriving some properties/classes of constructions we do not need

'metalanguage'. So whereas the 'linguistic' approach entertained by metamathematics[11] formulates its results so that they concern *expressions* of a formal language, and must therefore use a metalanguage, the objectual approach can exploit constructions of a higher order to speak about constructions of a lower order. A trivial example has already been given, see (2), (2'). For another example consider the following one:

(8) *Some constructions do not contain variables.*

To analyse (8) we must be from the very beginning aware of the limitations (to be compensated in one way or another) which our approach shares with all type-theoretical conceptions: properly speaking, unless some specific devices are defined or a still more general type hierarchy is used[12], (8) cannot be analysed within the above framework. What can be done is to analyse sentences given by the scheme

(9^n) *Some constructions of order n do not contain variables of order 1,...,n.*

Let *n* be 1. We get $(c,c'...*_1$, i.e., $c,c'/*_2$, Con(tain)$/ (o*_1*_1)$, Var$/ (o*_1)$, $\exists/ (o(o*_1))$) (existential quantifier, the class of non-empty classes of - here - constructions of order 1), $\neg/(oo)$.

(9^1) $[^0\exists \lambda c [^0\neg [^0\exists \lambda c' [^0\wedge [^0\text{Var } c'][^0\text{Con } c \, c']]]]]$.

Using standard abbreviations ($\exists c...$ instead of $[^0\exists \lambda c ...]$, infix notation) we can write

$(9^{1'})$ $[\exists c \,^0\neg \exists c' [[^0\text{Var } c'] \,^0\wedge [^0\text{Con } c \, c']]]$.

As for a logically more interesting application of the ramified hierarchy, the old problem of quantification into 'intensional contexts' can be correctly solved on the objectual level if higher order types are exploited.[13] In this case it is especially relevant, since the only formally correct attempt at analyzing inferences of the form

X *believes that* $\Phi_{[...A...]}$

$\exists x$ X *believes that* $\Phi_{[...x...]}$

has been made by Kaplan[14], who, however, was obliged to circumvent the troubles with distinct character of the variable *x* in the subordinate clause and outside of it by *quantifying over expressions* (substitutional quantification). Thus this solution is a 'sententialist' one, which can be effectively criticized. No *objectual* solution is, however, possible if only first order objects are admitted.[15]

5. Concept

5.1 Basic intuitions. Historical comments

The term *concept* is vague and ambiguous; at the same time, it is frequently used, in special intellectual contexts as well as in everyday discourse. Obviously there are some traditional intuitions connected with this term, intuitions which, on the one hand, are rather global (so that various distinct interpretations can be observed in various texts and/or at various times), on the other hand, however, in a sense important, so that they were many times critically investigated and made even some thinkers produce theories of concepts.[1] The present study will not offer a systematic survey of various theories: instead, it offers a new *explication*; indeed, our problem is the problem of optimal explication in Carnap's sense.[2] So we will try to give such an explication which could be compatible with most cases of using the term in such contexts where it is

not redundant, i.e., replaceable by another term (like *property, general term, representation,* etc.).
Therefore, wherever some other explication is criticized, it is because of this redundancy or because of
leaving some important contexts of use unclear. From this viewpoint the scheme (S) (Ch. 0) plays an
important role, for the category *concept* is not redundant there, and it should be helpful for understanding
most kinds of context.[3]

In the following paragraphs we have to say some comments to some past and contemporary conceptions.
These comments will be brief and they are not intended to be exhaustive analyses. The proper
explications begin with the section 5.2.

First of all, we will recapitulate which intuitions from (I) and (S) are of importance for our explication.

5.1.1 Concepts are non-mental entities

One of the most controversial questions concerning the nature of concepts is the question whether
concepts are *objective, or mental*. (In other words, are the concepts *discovered, or invented*?) A
philosopher answers this question dependently on his more general attitudes. In particular, it is unthinkable
that a nominalist would admit the objective character of concepts. So we are not surprised when we see that
among the 'objectivists' there are such philosophers as Plato, Aristotle, (in a sense) Augustin, Descartes,
Leibniz, (in a sense) Kant, Bolzano, Frege, Russell, Moore, whereas nominalists, British empiricists are
adherents of 'mentalism'. A simple correlation between the attitude to concepts and the general
philosophical viewpoint does not hold, of course: for example, it is not the case that every realist is an
'anti-mentalist' - see Geach's interpretation of Aquinas[4].

Sometimes the objectivist thesis is formulated in terms of *'existence'*. Do the concepts *exist*? This is a
rather misleading formulation, since *to exist* is mostly interpreted as *to be in space and time*. No objectivist
assumes, of course, that concepts are in space and time (even the great objectivist B.Bolzano does not
hesitate to claim that concepts have "no reality", see 5.1.5). What is important is that concepts are objects
which, according to G.E.Moore, *though not in space or time or dependent on mind, are real or have being
rather than existence.*[5] This view is equivalent with saying that we *discover* rather than *invent* concepts.

A frequently used argument against this view can be formulated as follows: You say - we are told - that,
e.g., the concept of a train always existed, i.e., even when there were no people. Is it not completely
absurd? The objectivists' answer is simple: Abstract entities, like numbers, qualities, and also concepts are
nowhere in space - this will be admitted by everybody, so that we are never asked *"Where is the concept of
a train?"* Why should be any question concerning *when* w.r.t. concepts less futile? Concepts, being
abstract entities, are neither in space, nor in time. Saying that we discover concepts means that they are
objective, unlike images, representations (*Vorstellungen*). The ways the objects can be identified are
independent of our minds, just as, e.g., the probability of an event.

There is still another argument against using such phrases like *concepts exist*. It has been shown [6] that
existence (not in the sense of existential quantifier) is a property of *intensions*. For example, if A is an
empirical definite description then to say "A *exists*" means to claim that the respective $\iota_{\tau\omega}$-object is
occupied (has a value) in the given world-time. If A is a name of an empirical property then such a
sentence means that in the given world-time a non-empty class is the value of this property, etc. However,
there is no rational possibility of construing the property *to be a concept* as being an intension. The

sentence *Concepts exist* is not an empirical sentence. Indeed, we could logically analyse this sentence as follows:

$\exists x\ C(x)$,

where 'C' would be interpreted as the class of concepts, but this 'solution' answers only what Carnap would call an *internal question*: therefore, it is entirely irrelevant for answering the question whether concepts are 'real', objective.

Thus *we will not apply the existence predicate to concepts.* Instead, we accept the objectivistic way of speaking and claim that *concepts are objective, i.e., not created, invented by our mind, but discovered with the aim to use them for the purpose of our orientation in the world. What is of mental character are the processes of 'concept acquisition', of using concepts etc.*

Remarks:

According to E.E.Smith[7] our position could be classified as a *metaphysical* (in contrast to *epistemological*) account of concepts:

   ...only a metaphysical account provides 'identity' conditions for concepts, i.e., conditions for deciding whether two concepts are the same or different.

Therefore, we can understand

   some of the philosophical complaints..., claims like '...concepts fall outside the domain of psychological processes'...

According to Smith, this does not "spell the end to a psychological (epistemological) approach to concepts":

   ...the notion of <u>sameness of concepts</u>, which is what psychologists presumably are unable to offer, may not be essential for elucidating many aspects of concept use.

We can agree, of course: our anti-psychologism does not deny that there is a domain of pragmatics/epistemology/psychology where empirical research of *concept use* or *concept acquisition* is highly interesting; on the other hand, this fact does not refute the non-mental character of the *concepts themselves*. Can a 'metaphysical', non-empirical theory of concepts influence this empirical research? Our everyday experience seems to answer this question positively: does mathematics, which is a non-empirical discipline *par excellence*, not influence empirical research?

Comparing the standpoint of Generative Semantics with the standpoint of "Truth-functional Semantics" Jackendoff[8] characterizes his "Conceptual Semantics" as such in which

   a level of mental representation called conceptual structure is seen as the form in which speakers encode their construal of the world

and tries to avoid a conflict between these two semantics:

   One is about the way the world <u>is,</u> and the other is about the way we <u>grasp</u> the world.

The question remains, of course, whether this *grasping* is a mental process of "discovering" objective concepts, or whether it is a subjective, in its 'creativity' fully free process. Some Jackendoff's formulations in his dispute with Fodor seem to signalize that Jackendoff would not subscribe to realism. Nevertheless, his "Conceptual Semantics" is compatible with the claim that "grasping is discovering", as we would put it.

We would use the following formulation: *It is not the case that forming a concept we can hope that it will identify any object <u>we wanted to identify</u>.* (An 'anti-Humpty-Dumpty' claim.) If we, e.g., want to identify

the property **being a whale** and form the concept **THE GREATEST FISH**, we see that the concept does not behave according to our wishes. This is, among other things, why we claim that *concepts are objective.*

5.1.2 Concepts are extra-linguistic entities

Concepts have been frequently identified with *universals*. Nominalists, refusing the objective character of universals, refused the objective character of concepts. Letting aside the mistaken identification of concepts with general concepts we can state that using language involves using general terms like the names of properties and relations. If concepts are to be abandoned, if *universalia sunt flatus vocis*, then what remains are linguistic expressions. We can then either directly identify concepts with general terms or linguistic definitions[9] or consider concepts as being "significations" of general terms[10] . As regards the first possibility, its choice makes it impossible to explain translatability: indeed, what does it mean that an expression *A* is a translation of an expression *B*? Clearly, *A* and *B* must have something in common. We could say, for example, that *A* denotes, or signifies, or expresses *the same concept as B* or has *the same meaning as B*. (Quine's objections[11] will be dealt with in Ch.7.) Thus if concepts (meanings) are not linguistic expressions, we have got a good starting point for possible further explanations. If, on the other hand, concepts (meanings) are rejected as extra-linguistic entities, we have got nothing to offer for the phenomenon of translatability (or, perhaps, a *regressus ad infinitum*). Such a 'semantic nihilism' does exist and its proposal is:

Do not look for abstract a priori entities: observe instead the actual verbal behavior, and where you were used to say that an expression <u>means</u> an (abstract) entity, describe instead the ways the expressions is <u>used.</u>

To accept this proposal, i.e., to delete the realm of concepts, means to be left with only two areas relevant for explaining and/or analysing linguistic phenomena: on the one hand the 'pure syntax', on the other hand the world of 'linguistic behavior' or 'linguistic events'. That the attempts to explain linguistic phenomena on the basis of 'pure syntax' necessarily break down has been sufficiently clearly shown by Tichý[12]. As regards the 'pragmatic area', it is a very interesting and important area, but it simply cannot compensate the loss of the 'semantic area': within the pragmatic area we are no longer careful about obeying the principle of compositionality, we solve only empirical problems and begin to believe that the only non-empirical problems about language are those connected with 'pure syntax'. We also begin to believe that the well-known semantic puzzles (including the Liar paradox) are pseudo-problems. Among the consequences of this semantic nihilism there are extreme contextualism (which should evidently fill the gap left after the loss of the principle of compositionality) and therewith connected relativism. So our thesis about the extra-linguistic character of concepts should be rejected only by those who would be happy with the above consequences.

(The suggested reduction of semantics to pragmatics can be, of course, connected with the late Wittgenstein. Hintikka would dissent[13] , but it seems to me that the 'late Wittgenstein trend' is not compatible with Hintikka's game-theoretical interpretation.)

As for the Millean 'signification' theory, it does not bring the desired notional parsimony: the category *concept* would be simply replaced by the category *signification*.

Remark:

Our thesis is supported, e.g., by Gödel and Russell. As Hao Wang reports[14], Gödel said to him in 1971:

> *If you use language to define combinations of concepts replaced by combinations of symbols, the latter are completely unimportant. Symbols only help us to fix and remember abstract things: in order to identify concepts, we associate them with certain symbols. All primitive evidence of logic is, when you investigate it, always of concepts; symbols have nothing to do with it. Seeing complicated symbols is easier; they are easier to handle. One can overview more symbols. We remember a complicated concept by means of a symbol denoting it.*

A very similar conception can be found already in Russell[15]:

> *I had thought of language as transparent - that is to say, as a medium which could be employed without our paying attention to it... I have never been able to feel any sympathy with those who treat language as an autonomous province. The essential thing about language is that is has a meaning - i.e., that it is related to something other than itself, which, in general, is non-linguistic.*

Since nobody proved the necessity of abandoning the 'semantic area', I am free not to accept the 'anti-semantic' (or perhaps 'pseudo-semantic') trend. To sum up:

Languages are systems of encoding (fixing) concepts. Semantic problems are not reducible to pragmatic problems. There is not only linguistic syntax on the one hand and either the things which we talk about or the world of linguistic behavior on the other hand. Concepts are abstract entities distinct from linguistic expressions. They can be attached to linguistic expressions as their meanings.

5.1.3 Concepts identify objects

The claims formulated in 5.1.1 and 5.1.2 show that we construe concepts as being abstract objects, unlike, e.g., mental representations or linguistic expressions.[16] Yet to state this is to state very little. There are more kinds of abstract entities: numbers, for example; but numbers are mostly construed as entities distinct from concepts. There is a specific feature due to which an abstract entity can be called *concept*. If a concept can serve as a link between an expression E and the object (if any) denoted by E (see (S) for this) then its role obviously consists in determining, or *identifying* the object.[17] There can be four cases of such a *conceptual identification*:

First:

> The identification *breaks down*. This is the case of *strictly empty concepts* (see 5.5).
> Example: THE GREATEST PRIME[18] .

Second:

> An *extension* is identified. The case of *non-empirical first order concepts*.
> Examples: any mathematical concepts. Also including *quasi-empty concepts* like A ROUND SQUARE (see 5.5).

Third:

> An *intension* is identified. The case of *empirical first order concepts*.
> Examples: PLANET, THE NUMBER OF PLANETS, THE KING OF FRANCE, TALLER THAN.

Fourth:

A *higher order object* is identified.   The case of *higher order concepts.*

Examples: THE CONCEPT OF THE KING OF FRANCE, BELIEVE

(see Ch.4).

The assumption that concepts identify objects is supported by the use of such phrases as *the concept of God, the/a concept of justice, the/a concept of horse*, etc.

Now what is most important when we compare the identifying role of concepts and the scheme (S) with the 'standard' semantic conceptions is the following fact:

An empirical (first order) concept never identifies the value of the respective intension in the actual world.

(We will see that our apparatus renders such identification impossible.)

We can, of course, ask why this is a necessary feature of our conception. Our answer can be formulated as a **fundamental epistemological hypothesis (FEH)**:

FEH

Our knowledge of the world is mediated by concepts (which are connected with expressions as their meanings). The outcome of applying empirical concepts to reality is, however, determined by two factors: a) by the objects (if any) identified by the respective concepts, and b) by the state-of-affairs. Whereas b) is given empirically, a) must be a priori: otherwise the concepts, whose connection with expressions is surely relatively fixed, would already solve our empirical knowledge for any state-of-affairs; no empirical methods would be necessary. Besides, the confrontation of non-changing concepts with changing reality would result in an unsolvable enigma.

Our explication (5.2 ...) will be compatible with FEH.

So we have:

Concept	identifies	does *not* identify
PLANET	a property (type $o\iota_{\tau\omega}$)	the class {Mercury, Venus,...} (type $o\iota$)
THE NUMBER OF PLANETS	a magnitude (type $\tau_{\tau\omega}$)	a number (9) (type τ)
THE KING OF FRANCE	an individual office/role (type $\iota_{\tau\omega}$)	an individual (type ι)
TALLER THAN	a relation-in-intension (type $(o\iota\iota)_{\tau\omega}$)	a relation-in-extension (type $o\iota\iota$)

To illustrate FEH more concretely, let us adduce the following example:

Accepting Church's scheme (see 2.1), according to which *any* kind of expression is connected with ('expresses') a concept[19], we can consider the concept **C** connected with (expressed by, represented by) the empirical sentence

(1)   *Warsaw is the capital of Poland.*

Letting aside the 'modal variability' let us take into account the 'temporal variability' only. It is well-known that what (1) says was not always the case. The *a priori* character of the conceptual identification given by **C** consists in the fact that *independently of the given state-of-affairs* **C** *identifies a proposition, i.e., an* $o_{\tau\omega}$-*object;* what *is* dependent on (is co-determined by) the given state-of-affairs is the *truth-value* of this proposition. But this is why we need *two* things: a) *identification of a proposition,* b) *verification of a proposition.* The job a) is what a concept does; it is only a tool serving our orientation in the world. Using a metaphor we can say that *empirical concepts 'pose questions' to the world and that the answers are given just by the respective state-of-affairs.* The mentioned 'questions' are the 'outcomes' of

empirical concepts, i.e., they are intensions ($\alpha_{\tau\omega}$-objects, α any type), and the 'answers' are the values of these intensions in the actual world at the given time point.

Other examples: the concept THE PRESIDENT OF THE USA 'asks' (the reality) who (if any) occupies the individual office - an $\iota_{\tau\omega}$-object - identified by it. Dependently on time the actual world 'answers': G.Washington,..., B.Lincoln,...,F.D.Roosevelt,...,J.F.Kennedy,...,B.Clinton,... .What changes are presidents, what does not is the above empirical concept and its 'outcome', the intension (here: individual office). (Compare therewith the concept THE KING OF FRANCE, where the concept produces such an $\iota_{\tau\omega}$-object that, e.g., at the present time points the reality cannot answer by offering an individual. Therefore, the proposition denoted by the 'classical' sentence

(2)   *The King of France is bald.*

and identified by the respective concept asks (nowadays) a futile question - no answer - i.e., 'yes' or 'no' - is given. This is how 'truth-value gaps' come into being in the case of empirical concepts.)

Or take the concept THE NUMBER OF THE GREAT PLANETS OF THE SOLAR SYSTEM. Here it also should be clear that this concept is a 'question-posing tool' distinct from the answer (the number 9). And certainly we would not say that a necessary condition of our understanding the expression

(3)   *the number of the great planets of the Solar system.*

should be the knowledge of the 'answer', i.e., 9. Only the fact that we associate (3) with the above concept (THE NUMBER...) is the sufficient and necessary condition of our understanding (3). (By the way, we perfectly understand the expression

(4)   *the number of planets*

where nobody knows the 'answer' . We need the concept THE NUMBER OF PLANETS to be able to 'ask'; we do *not* need it as something which would be a (redundant) name of a definite number.)

Let us now add some comments to the case of *non-empirical (first order) concepts.* Here our metaphor ('asking questions') must be modified. If non-empirical concepts were construed as 'posing questions to the reality' then we would be bound to say that these questions are futile (but not in the same way as in the case of 'truth-value gaps'): the 'answers' would be the same in any state-of-affairs; in other words, they would be independent of empirical facts. The concept THE EVEN PRIME identifies 2 under whatever thinkable conditions. All the same, the three levels from the scheme (S) are clearly distinguishable: the expression *the even prime*, the concept THE EVEN PRIME, and the object 2.

The distinction between the empirical and non-empirical concepts can be seen from another vantage point:

> *The only case when extensions are the conceptually identified objects is the case of non-empirical concepts.*

We have to explain the distinction between the *strictly empty concepts* and such non-empirical concepts which could be called *quasi-empty concepts.* Let our paradigmatic examples be

(5)   THE GREATEST PRIME (a strictly empty concept)

and

(6)   EVEN PRIMES GREATER THAN 2.

Whereas (5) cannot identify any object (for there is no such object that would 'fall under' (5)), (6) *does* identify an object, viz. the empty class of numbers. Classes, including the empty classes, are objects. There is, however, no 'empty object', 'empty number', for example. This distinction should be especially clear as soon as we are aware of our identification of classes with characteristic functions. The function which

would associate a number with **T** iff it is even, a prime, and greater than 2 (see (6)) is the function which for any (numerical) argument returns **F**. Are we ready to say that such a function is not a (mathematical) object?

Remark: Our identification of classes with characteristic functions solves the problem whether there is only one empty class, or whether there are many empty classes. That the latter is the right answer could have been expected: type-theoretical thinking is incompatible with the former answer. So the empty class of numbers is distinct from the empty class of, say, individuals or properties, etc.: the respective characteristic functions are distinguished by their domains (numbers, individuals, properties,...), and since functions with distinct domains are distinct, our claim is justified.

Returning to the case of empirical (first order) concepts we have to add a remark concerning what we could (and will) call *empirically empty concepts*.[20]. We often say that an empirical concept is empty. A frequently occurring misinterpretation can be observed: consider the following examples of empirically empty concepts:

(7)   THE PRESENT KING OF FRANCE

(8)   (THE) PEGASUS

(9)   GOBLIN .

The mentioned misinterpretation consists in saying that a) (7) and (8) identify nothing, and b) (9) identifies a/the empty class. Yet we have stated that empirical concepts never identify an extension. So if a) is considered to be true then it means that we assume that (7) and (8) (as 'singular concepts') should identify an individual but no individual is just now the occupant of the role of (7), and no individual 'pegasizes'. As for (9), we obviously assume that it should identify a class (for (9) is a 'general concept'). Remembering FEH we can, however, reject these assumptions. The concepts (7) and (8) identify an individual *role/office* which is simply either now (case(7)) or even always (case(8)) unoccupied in the actual world. The concept (9) identifies a property which is instantiated by an empty class in the actual world: the claim that goblins exist is (probably) false but it is no contradiction. This conception is highly intuitive. For admit for a moment that, e.g., the concept GOBLIN identifies an empty class. Naturally, the concept HOBBIT would - under this assumption - also identify (the same) empty class. But then GOBLIN would be if not identical with so at least equivalent (in an easily definable sense) to HOBBIT. Every reader of Tolkien would only laugh.

Now let us explain somehow more thoroughly the case of *higher order concepts*. The example THE CONCEPT OF X, where X may be THE PRESENT KING OF FRANCE or, say, A HORSE, should be clear; we will return to this kind of examples in 5.1.6. Why, however, should be BELIEVE a higher order concept? The answer can be found in Ch.4, where an 'approximately true' claim is defended: propositional attitudes are $(\omega *_n)_{\tau\omega}$-objects. Having a look at Def 9, point T_{n+1}b), we can state that the type of such objects is always higher than 1, i.e., that these attitudes are higher order objects (Def 10).

For the - still to be made - explication of 'concept' the following consideration is of key importance: Accepting - as we do - that the role of concepts consists in identifying objects we have to suppose that the character of concepts has something in common with what is usually called *procedure.* Yet this common feature reduces to the fact that both concepts and procedures are *structured*, i.e., that, in general, both contain some 'steps', without which we cannot say that the given entity is a concept/procedure. At the same time, the word 'procedure' can be - and frequently is - used in such a sense that is incompatible

with our claim that concepts are abstract: a procedure is often construed as a time-consuming *process,* i.e., as a spatio-temporal entity. Therefore, if comparing concepts with procedures we must be aware of this possible distinction. Hence let us recall the notion of *abstract procedures*, as it was introduced and explained in 3.1; we can then say that *concepts are* (a kind of) *abstract procedures,* which will lead us to 5.2 - 5.4.

5.1.4 Historical examples

Our explicative task is now in a stage which can be described as follows:

a)   An *apparatus* involving *ramified hierarchy of types* and definition of *constructions;*

b)   an anti-contextualistic conception of *intensions* vs. *extensions*;

c)   a list of *intuitions* based on (I) and (S) from Ch.0.

Now - if Carnap's requirements concerning explication should be satisfied - a 'historical support' could be expected, namely a more or less detailed overview of conceptions of concepts that have been formulated by our philosophical and logical predecessors and some contemporary thinkers.

It is, however, obvious that the present study must have made a choice: it would be either a detailed historical monograph, or else an attempt to formulate a rather new theory with some references to such conceptions which seem to be relevant for such a theory. It is this latter approach that has been chosen. So there will be no historical analyses concerning, e.g., Plato's theory of ideas, Aristotle's theory of definitions, medieval controversy realism vs. nominalism, Mill's conceptual nihilism and his & German psychologism, etc. Many competent pages on these topics have been written[21], and we do not wish to engage ourselves in the problems of historical interpretation. So only the following points will be - more or less briefly - mentioned: the s.c. *traditional doctrine,* further *Bolzano, Frege-Church, the Fins (Kauppi, Palomäki), Bealer;* a 'summary' will prepare the transition to 5.2, where the explication proper begins.

5.1.4.1 Traditional doctrine

As A.N.Prior says[22], we can always find in the history of logic contrapositions of 'old' and 'new' logic. The term *traditional logic*, as "inherited from the 16th and 17th century" is, however, used when essential distinction w.r.t. the contemporary ('mathematical', 'symbolic') logic has to be emphasized. (From this viewpoint it would not be adequate to classify, e.g., Leibniz´s logic with traditional logic(s).) Under the great influence of the Port-Royal logic a didactically remarkable system has been developed where the main chapters have been *Concept, Judgment, Inference.* So all the 'traditional textbooks of logic' (or perhaps 'textbooks of traditional logic') - let them be voluminous German compendia[23] or modest textbooks for high schools - contain these three chapters.

Let us compare the structure of these books with the structure of some contemporary (text)books. One of the most striking distinctions relevant for our theme is that whereas the topic contained in the traditional chapters *Judgment* and *Inference* has been essentially modified and explained in the modern textbooks (truth functions, predicates, quantifiers, theory of models, proof theory, etc.), the theme *concept* simply disappeared; at most a chapter *Definition* can be found.

We must ask: Why the chapter *Concept* has not been 'inherited' in some way?

There are surely more factors which can explain this 'gap'. We will adduce only some of them, which are probably the most important ones.

First, concepts have been defined in a most unsatisfactory way.

On the one hand, they have been construed as something which 'reflects' 'essential features' of objects; but a) this means that they have been considered to be *mental entities*, so that the chapter *Concept* should belong under the heading *Psychology*, rather than *Logic*; this was, of course, fully compatible with the long lasting tendency to *psychologism*[24] but is absolutely unacceptable in the modern logic; b) the theory of *essential features* is likewise untenable: A concept identifies (in the better case) an object$_1$; the theory of essential features presupposes that this object may be distinct from that object$_2$ which *should have been identified* by the concept, because the object$_1$ does not have some 'essential features' of the object$_2$. But then our concept is in some way 'wrong' and it should be corrected. This standpoint mixes up two problems: that of *which object is identified by the given concept*, and that of *whether we need the concept in the given context*. We cannot assume that a concept is 'right' or 'wrong': any concept simply determines some (at most one) object. A concept may be empty or quasi-empty or only empirically empty (see 5.1.3) but this does not justify the classification of concepts into 'the right ones' and 'the wrong ones'.[25]

To adduce an example, let us consider the concept (A) TRIANGLE. The traditional logician would say that it 'reflects' essential features of triangles. But which features of triangles are not essential? We would be probably said that *having one angle right* is one such feature, since a geometrical figure need not have a right angle to be a triangle, or simply because a triangle need not have a right angle. Our reply is: *having one angle right* is not an essential or inessential feature of triangles - it is no feature of triangles at all. The object identified by the concept TRIANGLE is the set of all triangles, and this object has got just those features which are given by this concept, e.g., having members which have three angles, three sides, etc. One of such features is *having at most one right angle*, but no feature can be construed as *having one right angle.*

On the other hand, there is a long tradition of identifying concepts with predicates, or with some kind of meaning of predicates[26]; this means that concepts are construed as *general concepts*, which can be seen to be counterintuitive when confronted with such expressions like 'the concept of the least prime', 'the concept of the contemporary President of USA', etc. Surely, the term *concept* is "one of the most equivocal" terms[27] but this Kantian-Fregean reduction of concepts to general concepts is a strikingly narrow construal. See also 5.1.4.2 and 5.1.4.3.

Second, all what could be 'attractive' on the traditional conception from the viewpoint of the mathematically oriented modern logic concerns the theory of 'extensions' (or 'denotations', etendues) and 'intensions' (or 'connotations', comprehensions) of concepts.[28] But this topic is mathematically primitive and can be reduced to trivial set-theoretical operations, which has been performed by Boole, Jevons, Schroeder et alii. Also, both the conceptions of 'extensions' - the Aristotelian (*per notiones, secundum ideas*) and the scholastic one (*per exempla, secundum individua*) - are connected with some troubles. As for the Aristotelian conception, according to which the extension of a concept consists of its 'subconcepts', it is ambiguous, since there are, in general, various criteria of classification. So, for example, the extension of the concept (A) LIVING BEING could consist of the concepts (A) PLANT, (AN) ANIMAL, (A) MAN, or, however, of the concepts of all the kinds (according to some criterion) of plants, of animals, etc., or (*ad*

absurdum) of the concepts (A) LIVING BEING WITHOUT LEGS, (A) LIVING BEING WITH TWO LEGS, (A) LIVING BEING WITH MORE THAN TWO LEGS, etc.etc. This sort of trouble could be 'solved' under some 'absolutistic' assumptions; a fix classification taking into account only 'essential' properties is such an assumption,[29] which, however, is unacceptable in the modern logic and philosophy. As for the scholastic conception, the extension of empirical concepts cannot be a definite set because of the modal and temporal variability (this can be seen from 2.1). Besides, the extension of higher order concepts has never been taken into account.

Summing up, the prevailing psychologistic way of defining concepts together with the meagre results that could be mathematically interesting are probably the most important factors which caused that modern textbooks of logic have lost any interest in handling the topic *concept.*

There is a further moment disqualifying the traditional approach to concepts: defining *intension* of a concept as the sum of its 'superconcepts' (*Oberbegriffe, Attribute*) means that the intension (in this sense) is *de facto* reduced to extensions (cf. Euler's circles) and that we are left in dark as regards the question *What is a concept itself?*, i.e., what is a distinction between a concept and its 'intension'. (This question is brilliantly answered by Bolzano, who has abandoned the traditional scheme; see the next section.) We will return to this problem in 5.1.4.5.

5.1.4.2 Bolzano

Bolzano cannot be classified with traditional logicians. As regards his theory of concepts, he has not accepted the traditional careless way of 'defining' concepts (he, of course, could not have accepted any mentalistic, psychologistic definition); instead, he has worked out a systematic remarkable theory of concepts,[30] based on his realistic conception of *'sentences in themselves'* (*Saetze an sich*) : these are construed as abstract entities which are what is common to 'real' sentences (i.e., those which are written, spoken or thought); now those parts of a real sentence which are not sentences have their abstract counterparts in '*images (representations) in themselves'* (*Vorstellungen an sich*), the most important of which are *concepts* (*Begriffe*).

So we can say that we share our philosophical intuitions with Bolzano: his concepts are non-mental extra-linguistic entities. Further, they can be said to identify objects or to be empty[31] (NB the empirical emptiness - see 5.1.3 - is distinguished from the non-empirical case).

The most interesting feature of Bolzano's theory of concepts is, however, his anti-traditional doctrine about *intension and extension* (*Inhalt und Umfang*) of a concept. Let us recapitulate the *traditional* definition of a concept. Imagine concepts - whatever they are - $C_1,...,C_n$; let their 'extensions' be represented by *n* circles. Let the extension of a concept C be the intersection of these circles. The sum of $C_1,...,C_n$ is then said to be the intension of C. We can call this conception a *conjunctive* one, since we can write (in the monadic case, easily generalizable to the *n*-adic case)

$C(x) = C_1(x) \wedge ...\wedge C_n(x)$,

or perhaps

$\forall wt (C_{1wt}(x) \wedge...\wedge C_{n\,wt}(x))$.

The 'intension' of C is defined *via* extensions, and the top wisdom of this traditional doctrine has got the name *The Law of inverse proportion of extension and intension*: the greater is *n*, the smaller is the extension of C and *vice versa* (which is a most trivial consequence of this conjunctive conception). Bolzano has not accepted the conjunctive conception.. His definition of *the intension of a concept*[32] can be paraphrased as follows (exploiting a contemporary vocabulary):

Let an expression E represent a concept C. Then the intension (Inhalt) of C is the sum of all (simple) parts of C, i.e., of all (simple) concepts $C_1,...,C_n$ which are represented by the meaningful parts of E.

(Notice that this definition is, in general, incompatible with the traditional one.)

Since the 'intension' is only the 'sum' of parts, and since the 'conjunctive' way is no more the only way of holding these parts together Bolzano has to distinguish between 'intension' and, say 'structure', which he does when he speaks about the way in which these parts of *Inhalt* are connected. [32] *So the possibility of precisely distinguishing between 'intension' of a concept and the concept itself is realised by Bolzano in an ingenious manner.* **The concept is a structured entity**; one and the same *Inhalt* can be shared by more concepts. Bolzano's examples are: A LEARNED SON OF A NON-LEARNED FATHER vs. A NON-LEARNED SON OF A LEARNED FATHER and 5^3 vs. 3^5. These examples show, at the same time, that the traditional scheme is broken and that there is no way back. Already the first example is a most important one, for its analysability by Bolzano is not even imaginable in the traditional doctrine, within which, e.g., the first concept would allow at most for a 'conjunctive' analysis leading to the concepts A LEARNED (MAN) and A SON OF A NON-LEARNED FATHER. As for the second example, both concepts are singular - for traditionalists (including Kant) they would be no concepts at all.[33]

In his par. 120 of *Wissenschaftslehre* Bolzano criticizes the above mentioned "law of reverse proportion of extension and intension". True, he could not have refuted this simple 'law', for 'intension' and 'extension' have been traditionally defined in a fully distinct way than it has been done by him, see above. If, however, we accept Bolzano's definitions, then, of course, there is no way of even formulating such a law.[34] His well-known example - the extension of A MAN WHO UNDERSTANDS ALL EUROPEAN LANGUAGES is smaller than that of A MAN WHO UNDERSTANDS ALL LIVE EUROPEAN LANGUAGES - is, moreover, very interesting, for both these concepts are unanalysable for traditional logic, which blocks such simple inferences like

   X understands all European languages.

   Latin is an European language.

   ∴ *X understands Latin*

or, still better, the proof of the statement that everybody who falls under the first concept falls also under the second one (and not *vice versa*).

Summing up: *Bolzano has formulated a nearly gapless realistic (Platonist) theory of concepts, a fully anti-psychologistic one, and he became a pioneer of the modern theories of structured meaning*[35].

5.1.4.3 Frege- Church

In Ch.1 we have already mentioned that Frege construed concepts as being denoted by (unary) predicates.[36] An explicit (not-just-a) definition is: *a concept is a function whose value is always a truth-value.*[37]

(According to this, *n*-ary predicates should also denote concepts, but in this case, Frege speaks about 'relations'; this is not important here, however.)

There are three points that must be mentioned in the connection with this Frege's explanation; all of them are critical.

First, functions the values of which are truth-values can be seen to be 'characteristic functions' of *classes*. To identify empirical concepts with classes means, however, to forget the temporal and modal variability of the class-membership relation (see 2.1). It is completely absurd to suppose that the concept (A) CAT would be a class of things - we already know that neither the object denoted by the expression *a cat* can be such a class; by the way, if our 'objects' are approximately what Frege calls *Bedeutungen* and if he introduces the category *Sinn* as that "what is grasped when one understands a name"[38], then it is not quite clear why concepts should be just *Bedeutungen*, since concepts should serve rather to determine these *Bedeutungen* (which Church recognized quite clearly).

Second, if Fregean concepts are, properly speaking, classes, then the traditional (Kantian) prejudice is supported, viz. that concepts are always *general* concepts.

Third, Frege's polemic with B.Kerry in his *On Concept and Object* misses its point. If a concept is a function (associating its arguments with truth-values) and if functions are 'unsaturated' [39] then, of course, "':the words *the concepts 'horse'* do designate an object, but on that very account they do not designate a concept" [40], but *what is the object designated by 'the concept of 'horse' '* ? Frege's assumption can be reconstructed as follows (paraphrasing his footnote †): By the very act of explicitly calling it a concept, we deprive it of this property. If, however, the respective concept is an *unsaturated* entity (being a function) then something like a 'free variable' is present. It is completely unclear how the 'act of explicitly calling it (i.e., the concept) 'a concept' ' could get us rid of this free variable.

These three points make Frege's explication of concepts doubtful. Besides, Bolzano's theory is incomparably more contemporary than Frege's.

Moreover, Frege accepts - although using a very precise style - the traditional 'conjunctive' (see 5.1.4.1, 5.1.4.2) theory of concepts.[41]

On the other hand, Frege's notion of *sense* as the *mode of presentation* of an object (of a *Bedeutung*) can inspire us incomparably more than Frege's problematic notion of concept. If a concept should identify an object then the 'sense' would be just the way to do it. And therefore, it is the Fregean sense rather than the Fregean concept (*Begriff*) what could be exploited when explicating the term *concept*. Frege himself never took the opportunity to build up a theory of concepts based on his category *sense*. It was, however, Church who recognized that this way sense→concept is a promising way, and who has identified *the sense of an expression E* with *a concept of the object denoted by E* (see Ch.1).

Church has been a realist like Frege.[42] He has generalized the notion of concept and defended the anti-nominalistic position. On the other hand, we can interpret 'his' concepts as something like intensions:[43] indeed, if concepts were intensions (see Def 3) then we could say with Church that, e.g., PEGASUS were a concept of no actual thing, but as soon as we admit that *concepts are another level than that of objects* we solve the problem as follows: the concept PEGASUS identifies just the respective intension (an $\alpha_{\tau\omega}$-object); the latter *happens* to be empty (better: to lack a value) in the actual world - so the above concept is 'empirically empty' (see 5.1.3) rather than (simply) empty.

The last remark to be stated concerns a place in Frege[44] where he combats ":the view that, e.g., 2+5 and 3+4 are equal but not the same" and where he argues that such a view means a *confusion of form and content, sign and thing signified*. These Frege's considerations can explain why he defended his class-like nature of concepts and why he never really defined his *Sinn*; furthermore, quoting the above formulation we must again admire his more than fifty years distant predecessor Bolzano. Bolzano would understand that if we distinguish 2+5 from 3+4 then it is not necessarily because of comparing *expressions*: he would associate these expressions with distinct   (extra-linguistic !) concepts.

[From our viewpoint it is obvious that we have three levels: the expression itself, the respective concept (or at least a construction), and the object identified by it. To illustrate this view, let us have

2+5,   3+4   (expressions),

$[^0+\ ^02\ ^05]$, $[^0+\ ^03\ ^04]$ (constructions),

7   7   (object).

We see that the expressions, as well as constructions, are distinct, and that the only 'sameness' can be stated on the level of objects.]

We can only wonder why Frege did not say : $2 + 5$ and $3 + 4$ have got the same *Bedeutung* but their *senses* are distinct.

5.1.4.4 Bealer

Bealer's book[45] is a remarkable attempt to define concepts so that an intuitive solution of many highly important semantic problems (such as, e.g., the 'paradox of analysis') could be obtained. Some of Bealer's starting points are essential from our viewpoint so that we will use relevant quotations with some comments.

Bealer states that

> ...there have been two fundamentally different conceptions of properties, relations, and propositions. On the first conception intensional entities are considered to be identical if and only if they are <u>necessarily equivalent</u>... On the second conception...each definable intensional entity is such that, when it is defined completely, it has a <u>unique, non-circular definition</u>. (p.2)

To compare both   conceptions Bealer adduces the following example:

> (c) *x is a trilateral iff x is a closed plane figure having three sides.*
>
> (d) *x is a trilateral iff x is a closed plane figure having three angles.*
>
> On the first conception both (c) and (d) count as correct definitions since they both express necessary truths. On the second conception... (d) does not count as a correct definition; only (c) does. (p.3)

Setting aside Bealer's argument justifying the (c) -definition we can state already here that Bealer has characterized - in a somewhat unusual way - an important distinction: one which divides intensions in the sense of Def 3 from such entities whose identity is given due to their not-negligible structure.[46]

Another important therewith connected idea is formulated on p.4:

The first conception... is ideally suited for treating the modalities - necessity, possibility, impossibility, contingency, etc. However, it has proved to be of little value in the treatment of intentional matters - belief, desire, perception, decision, assertion, etc. ... The second conception, on the other hand, while ideally suited for the treatment of intentional matters, has only complicated the treatment of the modalities.

Let us compare modalities with such intentional entities as beliefs. We can see that Bealer's distinction corresponds to our opposing *intensions as first order objects* to *higher order objects*. Indeed, while modalities (for example, logical possibility) can be construed as classes of propositions ($(oo_{\tau\omega})$-objects), i.e., as 'flat' objects, we have seen (see Ch.4) that such 'propositional attitudes' as *believing* admit of a fine-grained analysis only if they are construed as higher-order objects. Bealer's intensions in the sense of 'the first conception' are *qualities* (i.e., properties[47]), *connections* (i.e., our relations-in-intension) and *conditions* (i.e., our propositions). 'The other intensional entities' are called by Bealer *concepts* and *thoughts* (p.10). Returning now more concretely to his general statement (p.2) Bealer says:

...qualities, connections, and conditions are identical if and only if they are necessarily equivalent. However, though necessary equivalence is a necessary condition for the identity of concepts and thoughts, it is not a sufficient condition. (p.10)

Now there is, according to Bealer, a correspondence between 'conception 1 intensions' and 'conception 2 intensions' :

there are certain fundamental logical operations such that, when qualities and connections are combined by means of them, what we get are concepts and thoughts; all concepts and thoughts are obtained from qualities and connections (plus perhaps subjects of singular predications) by means of these fundamental thought-building operations. (p.11)

Parallel to these 'thought-building operations' the s.c. 'condition-building operations' are introduced; a logical synthesis is attempted at in Bealer's Ch.8. What can be said fully informally is perhaps that a thought is given by just one 'logical tree' (using thought-building operations) whereas there are infinitely many such trees using condition-building operations which determine one and the same condition. Summing up with Bealer:

Qualities, connections and conditions are the intensional entities that pertain to the world. Thoughts and concepts are those that pertain to thinking. And qualities, connections, conditions, thoughts, and concepts are all the intensional entities there are. (p.185)

There are several points that can be said to be shared by Bealer and our conception. Let some of them be spoken out.

Bealer's theory is a *realistic* ('anti-representationalistic') theory. So is ours.

Bealer is aware of the necessity to distinguish between the *coarse-grained* conception 1 intensions and the *fine-grained* conception 2 intensions. So am I (using, however, another conceptual basis and, therefore, another terminology).

Finally, Bealer's conception leads (among other things) to the conclusion (see the last above quotation) that the 'conception 1 intensions' (approximately our Def 3-intensions) "pertain to the

world" (we would say: they are what our expressions are *about*), whereas the 'conception 2 intensions' (approximately our higher order objects) "pertain to thinking" (we would say: they pertain to the 'realm of *meanings*').

It looks like Bealer somehow registers the same problems and distinctions as our conception does; only he explains them in another way, using other conceptual tools. So there are also 'points of disagreement':

Bealer's theory is a 1st order theory (even in the sense of 'being based on 1st order predicate logic'); we use a ramified hierarchy. Further, he avoids possible worlds (so that, for example, properties are unanalysable primitives); our approach is based on the conviction that possibilities are objective and the idea of possible worlds is highly helpful (if not indispensable) in solving the problems of LANL. Bealer does not know 'constructions' and he tries to show that problems like 'paradox of analysis' can be solved without, but his solution is objectionable.[48]) Finally, Bealer's concepts are general concepts only.

Bealer adduces arguments supporting his 1st order choice and his avoiding possible worlds. Let me be permitted not to react to them here: the reason is simple - the present study does not intend to criticize the competing approaches, and the possible discussion would be too long. It will be better, therefore, to show by presenting a positive conception that his arguments do not apply.

5.1.4.5 Kauppi

Raili Kauppi´s monograph[49] did probably the best what can be done with the traditional doctrine. Kauppi accepts essential assumptions of the doctrine: concepts are universals, the content ('intension') of a concept C consists of those concepts which are ('intensionally') contained in C; the relation of *intensional containment* is an undefined (primitive) relation to be used in constructing conceptual systems. Undefined, as it is within the system of axioms and definitions, it gets a pre-theoretical explication derivable from the informal explication of the distinction between concepts and classes[50]: every class is connected with a fixed universe of discourse, whereas the 'extension' of a concept is dependent on the choice of such a universe. So the 'applicability' of a concept changes with various definitions of its 'intension'.[51]

Kauppi is aware of the great problem with the traditional theories of concepts, namely of determining the distinction between (general) concepts and classes. Her solution differs from what we would like to see: according to her, the concept (A) MAN possesses distinct extensions dependently on whether the definition "which determines its intension" is a standard biological definition, or. e.g., that famous definition that construes men as rational beings; we would rather accept the view that there are *two concepts* here (although connected with one and the same expression). In this connection we state - a little surprised - that Kauppi ignores the category of possible worlds, and so she has got no key to the very instructive distinction between empirical and non-empirical concepts.[52]

The core of Kauppi's monograph consists in introducing many definitions and axioms concerning 'intensional' properties and relations of concepts; while the apparatus used is fully contemporary and

every sentence is intelligible and precise, the ideas are traditional[53] and the distinction *concept x class* is not satisfactorily explained. Since her concepts are universals and the notion of 'intension' (content) is the conjunctive one, so that the 'law of reverse proportion of extension and intension' holds, the nearly symmetrical connection between 'intensional' and extensional relations of concepts enables Kauppi to repeat - indeed, on much higher and much more elaborated level - the claims stating the 'dual' character of these relations (Section 22). This fact only reveals that what the traditional construal of 'intension' did is actually a very simple (and only in details refinable) statement that having concepts $C_1, \dots, C_n, C_{n+1}$ and assuming an extensional relation *falling under*, say, *f*, it holds

$\lambda x\,(f(x,C_1) \wedge \dots \wedge f(x,C_n) \wedge f(x,C_{n+1})) \subset \lambda x\,(f(x,C_1) \wedge \dots \wedge f(x,C_n))$,

where the left side of the inclusion constructs the extension of a concept whose 'intension' consists of $C_1, \dots, C_{n+1}$ and the right side constructs the extension of the concept whose 'intension' is a subclass of the left-side 'intension'.

One of the Kauppi's followers, Jari Palomaki, has written a monograph[54] which in a way sums up some attempts at a 'theory of concepts' (which should clarify *what* a concept is) as well as at a 'concept theory' (which 'formally represents' concepts). Interesting as it is, his study cannot break the limitations given by the extensionalist 'denotational semantics' [55] implied by the traditional doctrine. Schock's approach[56] referred to in details by Palomaki is symptomatic for those logicians who have not accepted Bolzano's challenge. What is namely characteristic for them is a tendency to *axiomatize* from the very beginning our vague intuitions *without any deeper analysis of the terms used in the extra-logical axioms.* So, for example, we ignore any type-theoretical considerations, let variables simply 'range over concepts', introduce an *ad hoc* symbolism, etc. So it happens that we get Schock's 'definition' D2: *t* int = *t*, which should express the claim that "the intension of a concept is defined as being the concept itself" [57]; actually, such a 'definition' expresses only a kind of despair. Another such dubious result can be found in Schock's definition D5, where 'attributes' are defined as "concepts which are not sets" [58] The thereby introduced constant *att* is then supposed to be sufficiently clear.

The whole 'absolutely axiomatic' approach is defective mainly because it relies on 'implicit definitions', which, however, do not define what they are supposed to[59].

5.1.4.7 Summing up: Set-theoretical approach?

One important point seems to follow from the above - very fragmentary, indeed - overview of some approaches to the theory of concepts (and to the 'concept theories' as well). Only Bolzano did not construe concepts as set-theoretical entities; Bealer can be perhaps interpreted in various ways, but as for the other theories, the prevailing approach can be globally characterized as follows: concepts are universals, i.e., they can be predicated of particulars, since they are results of generalizations that fix what some particulars have in common. Since classes/sets contain - in general - more particulars, concepts can be either construed as a kind of classes/sets, or at least as entities that are somehow 'similar' to classes/sets. (We are, of course, never said what kind of similarity we should expect.) So

the best way how to handle concepts logically is to exploit a theory of sets and explore properties and relations of the extensions of concepts (extensions always seem to be classes). As for contents ('intensions') of concepts, their properties and relations are simply 'dual' counterparts of the properties and relations of the extensions of concepts. Since the 'intensions' are sums of concepts, they are unstructured. If we all the same speak - as we inconsistently do - about the 'parts' of the content/'intension' of a concept C we construe them as such concepts that the intersection of their extensions is equal to the extension of C.

It can be shown that consequences of this set-theoretical approach are deeply counterintuitive at least from the viewpoint of our asking why we intuitively need a notion of concept as something distinct from the notion of class.

First, it is difficult to explain the 'class character' of concepts if the given concept is empirical (if it determines, e.g., an empirical property): unless we distinguish in a logically acceptable way between *classes* and *properties*, our theory of concepts will be gappy.

But **second**, even if a mathematical concept is given, the class theory of concepts will be strange enough not to be accepted: which distinction could be found between the *concept* (A) PRIME NUMBER and the *class* of prime numbers?

Third, what would be done with *individual concepts* which, of course, cannot be predicated of anything but whose presence is fully compatible with our primary intuitions concerning concepts?

Fourth,- and this is most important - to assume that concepts are set-theoretical entities implies that if two expressions denote one and the same set-theoretical object then both are connected with one and the same concept ('L-equivalence suffices'); thus we would not have *two distinct, although equivalent concepts* (AN) EQUILATERAL TRIANGLE , (AN) EQUIANGULAR TRIANGLE, but *one and the same concept*. This is intuitively at least suspicious (which inspired Bealer to his theory). Let us comment those approaches which accept the idea of distinguishing between extensions and intensions (in our sense) and which would probably identify concepts with intensions in the sense of Def 3 (or in a similar sense; see Montague's approach). Even then we would say that such an approach is a set-theoretical one: intensions in the sense of Def 3 are essentially again sets. As mappings from worlds-times they can be represented as sets of *n*-tuples the first two members of which would be worlds and times, the following *n*-3 members would be the remaining 'arguments' (they also can be, of course, sets/relations) and the *n*-th member would be the value (if any) of that intension in the given world-time. And examples can be adduced that show the same kind of counterintuitiveness as the above example: consider, e.g., the concepts NOT TO BE AT MOST AS TALL AS and (TO BE) TALLER THAN, which are somehow connected with one and the same relation-in-intension (type $(o\iota\iota)_{\tau\omega}$).

Intermezzo: Gödel vs. intuitionism.

Kurt Gödel has been a realist. The Hilbertian formalism has not been accepted by him [60] - the mathematical reasoning cannot be reduced to working with chains of symbols. According to his

conception of language, expressions of a language fix abstract entities (concepts)[61] ,and all (cognitive) importance of language is given by this fact.

But what is a concept according to Gödel?

An at least partial answer to this question can be found in Gödel's remarks to Russell's mathematical logic[62] . Here Gödel confronts his conception with the intuitionistic conception where the 'notion' is determined by a way of defining, or constructing, an object. Gödel does not identify what he thinks is a *concept* with this 'constructivist' *notion*, and construes concepts as properties of (mathematical) objects: where two or more intuitionistic notions (definitions, constructions) are present there can be just one Gödel's concept - as something what these notions have in common.[63]

As this standpoint is formulated, it looks like a defence of realism/Platonism against nominalism (Gödel explicitly speaks about intuitionism as about a kind of nominalism). Surely, this is connected with the rather psychologistic interpretation of intuitionistic constructions: if they are free creations of our mind, no realist would accept them. An other question would be whether Gödel's attitude to constructions and concepts would change if constructions were considered to be objective (ideal) entities (as it is realized in TIL and, of course, in the present monograph). Actually, however, Gödel's characteristics of concepts must be classified among the set-theoretical conceptions.

What remains to be said?

We have seen that any set-theoretical conception of concepts contradicts in some or other way our intuitions. To find a way out we first specify the sense in which we use the term *set-theoretical*, then we will show that a *non-set-theoretical* approach which obeys the standards of logical preciseness has already been applied by Bolzano, and finally we will recollect some results obtained in the preceding chapters (Ch.4, Ch.5) to show that very efficient tools for using non-set-theoretical approach on the contemporary level are already at our disposal.

1. Set-theoretical vs. non-set-theoretical

It would seem as if a logician who doubts that the set-theoretical approach is universal (in logic, at least) rejected thereby all the progress made by the contemporary logic. It would seem that problematizing the set-theoretical approach we defend the old good intuitive traditional logic and voluntarily abandon the means successfully used by the modern logic.

Well, it only seems to be the consequence of our 'heretic' doubts. Logic is unfounded without semantics - actually, logic is an analytical semantics supported by syntactical ('technical') means. The theory of truth and denotation is inseparable from logic; the central logical notion of consequence is not well founded without the notion of interpretation. Everything what *elementary* logic can say is closely connected with denoting *first order objects*, which are best construed as set-theoretical objects. But *denotation* is not all what there is. The questions of *meanings/concepts* are much more fine-grained than the questions of denotation. Now what I mean by 'set-theoretical approach' (to concepts) is that *coarse-grained approach* which is sufficient for denotation but breaks down when meanings are to be analysed. We have already stated (Claim 2, hypothesis H) that the first order objects cannot be meanings. Now the set-theoretical approach can capture only first order objects[64], and from among higher order objects only functions (cf.Ch.4, propositional attitudes); i.e., only objects, not quasi-

objects. If A is an α-object, α a type other than $*_n$, then the set-theoretical ('denotational') semantics of various expressions denoting A is entirely insensitive to distinct ways in which A is presented by these expressions. **The set-theoretical approach ignores - and can't help ignoring - the semantic distinction between the expressions "3 + 2" and, say, "√ 25": it can 'see' only expressions and denoted objects - the level of meanings/concepts cannot be taken into account (it cannot even be registered) by it.**

(This can be seen already in the elementary parts of logic: interpretation is in the '1st order logic' defined so that, e.g., $P(t_1,...,t_n)$ denotes True iff etc.: what is structured are only expressions - their semantics leads to unstructured set-theoretical objects (here to a truth-value, in the case of P alone it would be an n-ary relation, etc.). Thus every first order object can be denoted by infinitely many distinct expressions. The meanings of expressions are either identified with their denotations, or are not even mentioned.

Or: we can observe that *realism* in semantics is mostly automatically connected with denotational aspects: according to Dummett, for example, to be a realist means to associate sentences with truth-conditions (our propositions!) rather than with 'verification procedures'. Let these procedures be anything, Dummett's 'non-realist semantics' shows that its author cannot imagine that a fine-grained semantics of meanings could be formulated within a realist frame. See [Putnam 1983a, 439].

2. Bolzano

In 5.1.4.2 we have characterized Bolzano´s conception of concepts.[65] Already from this fragmentary characteristics it should be clear that this conception transcends the limits given by the set-theoretical approach. (We have to be aware of the fact that what has been defined above as a set-theoretical approach had been used intuitively hundreds years before the contemporary set theory began to be formulated.) Bolzano´s concepts are not 'flat'- their structure is an inseparable part of them. Also, they are situated exactly on the level of meanings - when using the more contemporary terminology - and are distinguished from objects (if any) which they are 'about'. So (AN) EQUIANGULAR TRIANGLE and (AN) EQUILATERAL TRIANGLE are distinct concepts of one and the same object (set).

Still one feature of his doctrine should be emphasized: the parts of the given concept can be parts of other concepts - what distinguishes one concept from other concepts is, in general, not the content (the 'sum' of these parts) but the way the parts of the content are connected. And this way is obviously irreducible to the traditional conjunctive cumulation of features ('Merkmale'). (By the way, it could seem that these traditional features could be construed as some 'parts' of the structure of the traditional concepts, but in the traditional doctrine these 'parts' play only the role of a ladder that is thrown away as soon as the proper concept theory is exposed.) Thus it is not just an anachronism to say that Bolzano opens the way to a general theory of constructions.

3. Constructions as non-set-theoretical entities

The most characteristic feature of constructions, as they have been defined by Tichý (see Def 7), is that their parts are not 'amalgamated' as they are in the entity constructed: they share this feature ('no part is lost') with linguistic expressions. This supports Gödel´s thought that the linguistic expressions *fix* abstract entities 'like concepts'; it would be even more adequate to claim this 'structural similarity' if concepts were construed as non-set-theoretical entities (i.e., not as they have been construed by Gödel - see Intermezzo). This observation makes up a bridge to the following section.

*5.2 Concept**

5.2.1 Concepts and concepts[*]

Let us compare what has been said about constructions with our intuitions from 5.1.1 - 5.1.3:

a) *Concepts are non-mental entities.*

The same holds of constructions. As 'abstract procedures' consisting of variables, trivializations, compositions, and closures they are, of course, - just as Bolzano´s *Saetze an sich* and *Begriffe* - independent of our minds. We can 'possess' them just as we can be said to 'possess' concepts[66] but whatever is 'possessed' is distinct from the 'possessor'. Constructions - as well as concepts - do not 'disappear' when they are not possessed or used: our mental processes (images etc.) exist only there and then where and when they are realized by some subject. It is absurd to claim this fact about constructions.

b) *Concepts are extra-linguistic entities.*

If constructions were linguistic entities then, first, there would be no need at all to define the extra category of constructions. And, of course, there is nothing in Def 7 what could make us suspect that constructions are expressions of a language. We have already refuted the possible misunderstanding arising from mixing up 'use' and 'mention' , and consisting in the claim that using a symbolic record of a construction we therefore admit the linguistic character of constructions: *a record of a construction is not the construction itself,* just as the word 'town' is not the property *being a town*. The whole idea of constructions is based on our conviction that *abstract procedures are only recorded, fixed, encoded by linguistic expressions*. Hence this point is also shared by concepts and constructions.

c) *Concepts identify objects.*

The idea of *identifying objects* is intuitively very similar to the idea of *constructing objects*. We already know that constructions (*v*-)construct objects or are (*v*-)improper. A distinction can be, however, stated: saying that a concept identifies an object (or nothing) we do not suppose that there is some parameter that would determine what is identified. BEING GREATER (THAN) (say, between individuals) is a concept that identifies an $(o\iota\iota)_{\tau\omega}$-object, BEING GREATER THAN EARTH is a concept that identifies a property, an $(o\iota)_{\tau\omega}$-object. But 'BEING GREATER THAN *x*' , *x* ranging over individuals, is nothing what could be called a concept: the property that could be identified is dependent on the valuation.

So we define:

Def 11 A construction C is called *a closed construction* iff C does not contain any free variable. (Cf. Def 8.)

Now we can see an essential analogy between concepts and *closed* constructions. This analogy, together with the points a) and b), justifies the following definition which serves as *a preliminary explication* of the term 'concept':

Def 12 *A concept[*] (of order n)* is a closed construction (of order *n*).

In 5.1.3 we have distinguished four cases of conceptual identification. Exploiting now Def 12 we show that the same classification applies when 'concepts∗' and 'constructing' are used.

First: *The identification breaks down.* (THE GREATEST PRIME)

Let $G / (\tau(o\tau))$ be the function that associates every class of numbers with that member of it which is the greatest one; if the given class does not contain the greatest member, G is undefined. Let $Pr/ (o\tau)$ be the class of prime numbers. The (closed) construction of order 1 (= the concept[*] of order 1) $[^0G \ ^0Pr]$ is an improper construction.

Def 13 A concept[*] C is *a strictly empty concept[*]* iff C is an improper construction.

Second: *An extension is identified.*

Any concept[*] of order *n* that constructs an extension is this case.

Notice that an empty class is also an extension, so that we cannot claim that, e.g., the concept BEING ODD AND (AT THE SAME TIME) EVEN is strictly empty. Let $Od/ (o\tau)$, $Ev/ (o\tau)$ be the class of odd (even) numbers, respectively. Then, of course, the concept[*]

$$\lambda x [^0\wedge [^0Od \ x] [^0Ev \ x]]$$

constructs an empty class.

Def 14 A concept[*] C is *a quasi-empty concept[*]* iff it constructs an empty class/relation.

Third: *An intension is identified.*

Consider, e.g., the concept THE NUMBER OF PLANETS. Let $No/ (\tau(o\iota))$ be the function that associates any finite class of individuals with its cardinal number, and is undefined on infinite classes. Let $Pl/ (o\iota)_{\tau\omega}$ be the property *being a planet*. Then the following concept* corresponds to THE NUMBER OF PLANETS:

$$\lambda w \lambda t [^0No \ ^0Pl_{wt}]$$

Notice that our principle according to which empirical concepts never identify the value of the respective intension in the actual world is now supported by our concept*: unless we are omniscient there is no way how our concept[*] could construct this 'actual value'. (Omniscience would mean that we could apply the above construction to the actual world.[67])

The case of such concepts as THE PRESENT KING OF FRANCE or GOBLIN can be dealt with in terms of the following definition:

> **Def 15** Let W, T be a possible world, a time point, respectively. Let C be a concept[*] that constructs an $\alpha_{\tau\omega}$-object A. C is *a concept[*] empty w.r.t. W,T* iff A is not defined in W at T, or its value in W at T is an empty class/relation.

The concepts[*]

$\lambda w \lambda t\ [^0K_{wt}\ ^0F]$, ^0Go correspond to these two cases. (For the sake of simplicity, K/ $(\iota\iota)_{\tau\omega}$, F/ ι, Go/ $(o\iota)_{\tau\omega}$.)

Fourth: *A higher order object is identified.*

So whereas HORSE identifies a property of individuals, i.e., a first order object, THE CONCEPT OF HORSE identifies the concept HORSE. In terms of concepts* this can be stated as follows:

Let Ho/ $(o\iota)_{\tau\omega}$ be the above property. Then ^0Ho is a concept* of this property and ^{oo}Ho is a concept* that identifies ^0Ho.

For another example, we have seen in Ch.4 that believing (Bel) is a relation of the type $(o\iota*_n)_{\tau\omega}$ (say, $(o\iota*_1)_{\tau\omega}$ here), i.e., that it is a second order (in general, $(n+1)$the order) object. Thus ^0Bel is a concept* constructing a higher order object.

5.2.2 Extension of a concept*

Now we begin to look for some intuitive analogies with the traditional intuitions concerning concepts. We have seen that the notions of 'extension' and of 'intension' of a concept played an essential role in the traditional doctrine. (Needless to say that the terms 'extension (of)' and 'intension (of)' are used in this context in another sense than that attached to these terms by Def 5 and Def 3.) We have also criticized the traditional doctrine for some ambiguities and for its poor content also w.r.t. the notions of 'extension of' and 'intension of' a concept.

Now a rather precise notion of extension of a concept will be introduced.

Def 16 *The extension of a concept* C *is the object constructed by C.*

Def 17 Let C be a concept* that constructs an $\alpha_{\tau\omega}$-object A. *The extension of C w.r.t. a world W and a time point T is the value of A in W at T.*

Some *examples*:

The concept THE GREATEST PRIME, and therefore the concept* $[^0G\ ^0Pr]$ do not identify (construct) any object. So they do not have any extension.

The extension of the concept* (x ranges over τ)

$\iota x\ [^0\wedge\ [^0Even\ x][^0Pr\ x]]$

is the number 2. The extension of the concept*

$\lambda x\ [^0\wedge\ [^0Even\ x][^0Pr\ x]]$

is the singleton $\{2\}$.

The extension of the concept*

$\lambda xy\ [\iota z\ [^0=\ z\ [^0{:}\ x\ y]]]$

is the function (operation) *dividing* (in the set of real numbers).

The extension of

$\lambda xy\ [^0\wedge\ [^0>\ x\ y][^0\leq\ x\ y]]$

is the empty relation (over the set of real numbers).

The extension of the concept*

$\lambda w \lambda t\ [^0Highest_{wt}\ ^0Mount_{wt}]$,

where Highest/ $(\iota(o\iota))_{\tau\omega}$ associates every world-time with a function which for every class K of individuals selects the highest member of K, and Mount/ $(o\iota)_{\tau\omega}$ is the property *being a mountain*, is the individual role ('office') - i.e., an $\iota_{\tau\omega}$-object - to be played ('occupied') by an individual to be the highest mountain. The extension of this concept[*] w.r.t. the world W at the time point T is that individual which is the highest mountain in W at T. (Notice that if in W there are no mountains at T, then the above concept[*] does not have any extension w.r.t. W,T, i.e., it is empty w.r.t.W,T.)

The extension of the concept[*]

$\lambda w \lambda t \lambda x\ [^0\wedge\ [^0Pl_{wt}x][^0\neg\ [^0Aster_{wt}x]]]$,

where Pl/ $(o\iota)_{\tau\omega}$ is the property *being a planet in the (our) Solar system*, and Aster/ $(o\iota)_{\tau\omega}$ is the property *being an asteroid*, is the property *being a 'big' planet (in our Solar system)* . The extension of this concept[*] w.r.t. the actual world at the present time points is (probably) the set {Mercury, Venus, Earth, Mars,..., Pluto}.

Our splitting of the notion of extension (of) , as realized by Def 16 and Def 17, makes it possible to distinguish between the rather simple case of non-empirical concepts and the more complicated case of empirical concepts. Since the traditional doctrine has ignored the distinction between semantic analyses of empirical and non-empirical expressions, it never needed two definitions.

The following examples concern the 'empirically empty concepts', here 'concepts[*] empty w.r.t. W,T' (Def 15).

The extension of the concept[*]

$\lambda w \lambda t\ [^0K_{wt}{}^0F]$ (see above in 5.2.1)

is the respective individual role. There is no extension of it w.r.t. the actual world at the present time points.

The extension of the concept[*] ^0Go is the property *being a goblin*, whereas its extension w.r.t. the actual world (probably always) is the empty class of individuals.

Our definitions of 'extension of' are not only more adequate as compared with the traditional doctrine: they also take care of those cases which have been ignored by Bolzano, Frege, and, properly speaking, by all the 'standard', or 'mainstream' semantics.

Remark 1: Be aware of the fact that only the 'extension of' as defined in Def 16 is given *a priori*. Which (if any) extension is connected with the given concept[*] w.r.t. the given world-time (Def 17) is, of course, co-determined by the given world-time. To find out this ('empirical') extension is an empirical problem (i.e., not a semantical one).

Remark 2: Some general statements look like claims about *relations between extensions of concepts (concepts[*]) w.r.t. W,T*. Sometimes these claims are empirical (hypotheses). Cf. the sentence

All swans are white.

Here we claim that the extension of the concept SWAN w.r.t. W,T is (at least if W is actual) a subclass of the extension of the concept (concept[*]) WHITE w.r.t. W,T. Let Sw, Wh be the respective $(o\iota)_{\tau\omega}$-objects. Our analysis is ($\forall, \exists$ will be written 'standardly', i.e., instead of $[^0\forall\lambda x\ X]$ we will write simply $\forall x\ X$):

$\lambda w \lambda t \; \forall x \; [^0\supset [^0Sw_{wt}x][^0Wh_{wt}x]]$.

This concept[*] constructs a proposition the truth-value of which in the actual world proved to be false at (at least) some time points. There is nothing logically impossible with the assumption that the mentioned extensions could be related in the above way. So the respective proposition is an *empirical* proposition.

There are other cases, where the claim about the first extension's w.r.t. W,T being included in the second extension w.r.t. W,T is an *a priori* (analytic) claim. Consider

All mammals are vertebrates.

Let M, V be $(o\iota)_{\tau\omega}$-objects (the properties *being a mammal, being a vertebrate*, respectively). Our analysis is then

$\lambda w \lambda t \; \forall x \; [^0\supset [^0M_{wt}x][^0V_{wt}x]]$.

Here the concepts[*] ^0M and ^0V are not independent: there is no possible world where a mammal would not be a vertebrate. Thus the proposition constructed is the proposition TRUE, i.e., the proposition that takes the value True in every world-time.

The situation is a specific one if 'empirically empty' concepts[*] are considered. Compare two sentences:

(1) *Every dragon has wings.*

(2) *Every dragon is a manager.*

If both these sentences claimed that the extension of the concept DRAGON w.r.t. W,T is a subclass of the extension of the concept WINGED (MANAGER) w.r.t. W,T, then a fine distinction between them would disappear on the level of analysis: Let Drag, Win, Man be the three respective properties. Our analyses could be:

(1') $\lambda w \lambda t \; \forall x \; [^0\supset [^0Drag_{wt}x][^0Win_{wt}x]]$,

(2') $\lambda w \lambda t \; \forall x \; [^0\supset [^0Drag_{wt}x][^0Man_{wt}x]]$.

Clearly, both constructions construct a proposition which is true in such worlds-times in which there are no dragons, in other words, where the extension of the concept[*] ^0Drag is the empty class of individuals. But surely we feel that this is very strange to say of the sentence (proposition) (2) whereas the proposition denoted by (1) is true in a somehow natural way. This intuitive distinction is still more striking if we try to paraphrase both sentences as follows:

(3) *A dragon is winged.*

(4) *A dragon is a manager.*

Now we would protest if a logician tried to convince us that (4) is true, whereas the truth of (3) remains plausible.

The reason is that the truth of (2) derives from the 'degenerative' case of inclusion ('the empty class is a subclass of any class of the same type'), whereas the truth of (1) and (3) is due to some kind of necessary link between the respective concepts. Indeed, we can analyse (1) and (2) assuming that what is claimed by these statements is better expressed by (3) and (4):

A relation (-in-extension) between properties (say, of individuals) can be defined: two properties A,B, constructed by C_A, C_B, are linked by this relation ([^{0}Req $C_A C_B$]) iff whatever possesses B, necessarily (in every world-time) possesses the property A. We will call this relation *requisite relation* (Req/ $(o(o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$[68].

Then our analysis of (1) (and (3)) will be:

(3') [^{0}Req ^{0}Win ^{0}Drag],

and (4) gets the analysis

(4') [^{0}Req ^{0}Man ^{0}Drag].

Now we can see that a) (1), (3) , and (4) are analytic (non-empirical) sentences (whereas (1') and (2') resulted from our assumption that (1) and (2) are empirical claims),

b) (1) and (3) are true, and (4) - if it is construed rather as a claim distinct from (2) than as its paraphrase only - is false. Thus not every all-sentence is a claim about inclusion between extensions.

(By the way, the sentence

All mammals are vertebrates

could also be analysed in this way. We would get

[^{0}Req ^{0}V ^{0}M].)

5.2.3 Intension of a concept[*]

Essentially we construe 'intensions' of concepts[*] in the Bolzanian way. Before we define the 'intensions of' (or: contents of) let us define a most important class of concepts:

Def 18 Let X be either an object which is not a construction, or a variable (of any type). C is *a simple concept*[*] iff C is ^{0}X.

Claim 4 *No simple concept[*] is strictly empty*. [69]

Proof: Obvious.

In other words, a simple concept[*] *immediately* constructs an object or a variable.

Examples of simple concepts[*]:

Let *x* be a variable ranging over (= *v*-constructing the members of) some type. Let 2 be the number two, Mount be the property *being a mountain*, Catastrophe/Disaster be the property of events (i.e., of a kind of propositions) *being a catastrophe*. The following concepts[*] are simple:

^0x ... constructs (identifies) the variable *x*;

^{0}2 ... constructs the number two;

^{0}Mount ... constructs the property *being a mountain*;

^{0}Catastrophe (or: ^{0}Disaster) ... constructs the property *being a catastrophe*.

(The objective, extra-linguistic character of constructions causes that ^{0}Catastrophe is *the same construction* as ^{0}Disaster: what is recorded behind '^{0}' is not a word but - *via* a word - an object. Assuming that in English 'disaster' and 'catastrophe' denote one and the same object (property) we are justified to speak about *the same* construction rather than about, say, an equivalent construction only. See 3.3.)

Later on - in Ch.6 and Ch.7 - we will appreciate the role of simple concepts. Here we need them for defining 'intensions of' , but it is very important to understand this notion. Therefore, some further comments are useful.

If a simple concept* constructs an object then it happens without any other concept*'s 'help'. Thus $^0 2$ identifies 2 immediately whereas the following concepts*, being not simple, do it 'by means' of other concepts*:

Let Suc/ $(\tau\tau)$ ('successor') be the well-known function which associates every natural number n with the number $n + 1$, and is undefined on the other numbers. The remaining symbols are obvious.

a) $[^0 Suc\ [^0 Suc\ ^0 0]]$,

b) $\iota x\ [^0 \wedge\ [^0 Even\ x][^0 Pr\ x]]$,

c) $[^0\!\!:\ ^0 16\ ^0 8]$.

In a), the (simple) concepts* $^0 Suc$ and $^0 0$ are needed, similarly in b) and c).

Remark: We are dealing with 'pure' constructions and concepts*. From this viewpoint it is thinkable that, e.g., the class of primes is identified 'immediately', i.e., without the 'help' of such concepts as DIVIDE, NATURAL NUMBERS, etc. Similarly - see c) - it is thinkable that 16 and 8 are identified 'immediately' , whereas 2 is identified by means of $^0 16$ and $^0 8$. The seemingly counterintuitive character of such considerations is given by our tacit assumption that we attach the concepts[(*)] to some linguistic expressions. The relation between concepts and expressions will be, however, dealt with till in Ch.7.

Summing up, simple concepts* immediately construct non-constructions or variables. But we must not identify non-constructions with first order objects.

Claim 5 *Every first order object is a non-construction but not vice versa..*

Proof: The first part is obvious (see Def 9). A counterexample proving 'not vice versa' : *Believe* is not a first order object (being of the type $(o\iota *_n)_{\tau\omega}$, but it is a relation-in-intension, so it is no construction either.

Therefore, eg.. , $^0 Believe$ is a simple concept*.

Still one auxiliary (but in general very important) definition is needed:

Def 19

    Let C be a construction.

    I, C is *a subconstruction of* C.

    II, Let C be $^0 X$. If X is a construction then X is *a subconstruction of* C.

    III, Let C be $[XX_1... X_n]$. Then $X, X_1,...,X_n$ are *subconstructions of* C.

    IV, Let C be $[\lambda x_1...x_n X]$. Then X is *a subconstruction of* C.

    V, If A is *a subconstruction of* B , and B is *a subconstruction of* C, then A is *a subconstruction of* C.

    VI, Anything is *a subconstruction of* C only due to I -V.

Now we can define 'intension/content' of a concept*:

Def 20 The *intension/content of a concept$^♣$ C* is the set of all simple concepts$^♣$ that are subconstructions of C.

Claim 6 *The content of a simple concept$^♣$ is a singleton.*

<u>Proof</u>: The only subconstruction-concept* of a simple concept* is this concept* itself.

It is easy to see that Def 20 is a Bolzanian definition. So if he says that the content of 3^5 is the same as the content of 5^3, then our Def 20 makes it possible to agree on a 'more formal' level: Let Exp/ $(\tau\tau\tau)$ be the function that associates any pair $<m,n>$ of numbers with the number m^n. Then the two Bolzanian concepts* are $[^0\mathrm{Exp}\ ^03\ ^05]$ and $[^0\mathrm{Exp}\ ^05\ ^03]$. Exploiting Def 20 we obtain the content of both these concepts* as the same set $\{^0\mathrm{Exp},\ ^03,\ ^05\}$.

Returning to our examples a), b), c) we get the following 'intensions/contents of' :

a') $\{^0\mathrm{Suc},\ ^00\}$,

b') $\{^0|,\ ^0\wedge,\ ^0\mathrm{Even},\ ^0\mathrm{Pr}\}$,

c') $\{^0:,\ ^016,\ ^08\}$.

(Bolzano would see that the content of a) is at the same time the content of all concepts* that construct natural numbers in this Peano-like way. Similarly, the content of c) is also the content of the concept* $[^0:\ ^08\ ^016]$.)

Our (and Bolzano's) notion of 'content of' is, of course, radically distinct from the traditional one. Adding to the content of a given concept* new elements does not, in general, diminish the extension of this concept*. This can be shown on indefinitely many examples including the Bolzanian one (*ein gelehrter Sohn...*). To take a more simple example, the extension of c) is the number 2, but consider the concept*

d) $\lambda x\ [^0\vee\ [^0=x\ [^0:\ ^016\ ^08]][^0=x[^0:\ ^015\ ^05]]]]$

The content of d) is

$\{\ ^0:,\ ^016,\ ^08\}\cup\{\ ^0\vee,\ ^0=,\ ^015,\ ^05\}$

but the extension of d) is $\{2,3\}$.

Let us consider a more complicated case:

Let Ch(arles) and Peter be ι-objects, Young/ $(\mathrm{o}\iota\iota)_{\tau\omega}$ be the relation *being younger (than)*, Wif/ $(\iota\iota)_{\tau\omega}$ be the wife-function. The content of the concept*

$\lambda w\lambda t\ [^0\mathrm{Bel(ieve)}_{wt}{}^0\mathrm{Ch}\ ^0[^0\mathrm{Young}_{wt}\ [^0\mathrm{Wif}_{wt}\ ^0\mathrm{Ch}]\ ^0\mathrm{Peter}]]$,

which constructs the proposition that *Charles believes that Peter's wife is younger than Peter* is - due to the transitivity of the subconstruction-relation (Def 19) - $\{^0\mathrm{Bel},\ ^0\mathrm{Ch},\ ^0\mathrm{Peter},\ ^0\mathrm{Young},\ ^0\mathrm{Wif}\}$.

This conception of content makes it possible to see the logical structure of such concepts which are simple, non-analysable from the traditional viewpoint. So for example consider the concept COLOR(S) HAD BY ALL TREES.

Since this concept (containing not the concept COLOR but the concept COLOR OF) is not analysable from the traditional viewpoint, the simple inference

Green is a color had by all trees.

(All) oaks are trees.

∴ *Green is a color had by oaks.*

is impossible. Assuming that the above concept is - according to tradition - simple we get the following 'formalization' of the above premises (in the style of predicate logic):

Col (Green)

$\forall x$ (Oak(x) ⊃ Tree(x)),

so that we cannot obtain the intuitively correct conclusion.

In terms of concepts[*] we get:

Col/ $((o(o\iota)_{\tau\omega})\iota)_{\tau\omega}$ (a function that associates with every world-time a function defined on individuals and returning the set of colors - i.e., of a kind of properties - had by that individual), Gr,Tree,Oak/ $(o\iota)_{\tau\omega}$, $x...(o\iota)_{\tau\omega}$, $y...\iota$;

COLORS HAD BY ALL TREES:

$\lambda w \lambda t \lambda x \, \forall y \, [^0{\supset} \, [^0\text{Tree}_{wt} \, y][[^0\text{Col}_{wt} \, y]x]]$

Premisses:

$(\lambda w \lambda t) \, \forall y \, [^0{\supset} \, [^0\text{Tree}_{wt} \, y][[^0\text{Col}_{wt} \, y] \, ^0\text{Gr}]]$

$(\lambda w \lambda t) \, \forall y \, [^0{\supset} \, [^0\text{Oak}_{wt} \, y][^0\text{Tree}_{wt} y]]$

By transitivity of implication, rules of introduction and of elimination of $\forall$ we immediately derive

$(\lambda w \lambda t) \, \forall y \, [^0{\supset} \, [^0\text{Oak}_{wt} y][[^0\text{Col}_{wt} y] \, ^0\text{Gr}]]$,

i.e., our conclusion.

Remark: It seems that this comparison of two construals of 'content/intension of' supports the view according to which logic is not a calculus or a set of calculi but an 'open-ended theory'.[70] Whether the conclusion is or is not derivable from premises is given not only by the rules of inference but also by our ability to find the finest possible logical structure underlying the premises. So what we have called LANL is a necessary part of the science called 'logic'. Our theory of concepts will be therefore connected with LANL in Ch.7, and a kind of digression in this respect can be sometimes observed even in the present chapter, including the last example. The importance of a concept theory for logic becomes especially clear when we accept the following claim: While the empirical sciences **use** concepts in order to increase our knowledge of the reality, logic is engaged in studying properties and relations of **concepts themselves.**

5.3 Quasi-identity

The concepts[*] adduced in the preceding section under a), b), c) exemplify an equivalence relation between concepts[*].

Def 21 *Concepts[*] C_1, C_2 are equivalent* iff they construct one and the same object or are both strictly empty.

This definition implies that the following concepts[*] are *not* pairwise equivalent ($x...\tau$, $y...\tau$, $z...(o\tau)$, Card/ $(\tau(o\tau))$):

d) $\lambda x \, [^0{\wedge} \, [^0{\wedge} \, [^0{>} \, x \, ^02][^0\text{Pr(ime)} \, x]][^0\text{Even} \, x]]$,

e) $\lambda xy \ [^0\wedge \ [^0> x \ y][^0\leq x \ y]]$,

f) $\lambda z \ [^0\wedge \ [^0> [^0\text{Card } z] \ ^01][^0\text{Empty } z]]$.

One could object: all these concepts* construct the empty class, so Def 21 says that they are all pairwise equivalent, since they construct one and the same object. This objection is wrong. It would hold only if there were just one empty class. Our approach is, however, a type-theoretical one, and classes/sets are construed as characteristic functions. So the objects constructed by d), e), f) are, respectively, $(o\tau)$-, $(o\tau\tau)$-, and $(o(o\tau))$-objects. Being empty classes they are functions associating, respectively, (real) numbers, pairs of numbers, and classes of numbers with the value FALSE. Since they are functions *differing in arguments* (although not in values) they are simply *distinct functions*, and so they represent *distinct empty classes*. Therefore the   respective concepts* are not equivalent.

Returning to the examples a), b), c) we state that the ways those concepts* construct the number 2 are essentially distinct procedures. In a) we apply twice the successor function (starting with 0 as the argument), in b) we find that unique number which is even and also a prime, in c) we apply the dividing function to the pair <16, 8>. What is typical for such a kind of equivalence is that two *equivalent concepts* differ in at least one member of their contents*. Now we can ask whether this fact holds universally, i.e., for all cases of equivalent concepts*.

Our answer is negative. In the following (Def 24) we define an important subclass of the set of pairs of equivalent concepts*. Two auxiliary definitions (Def 22, Def 23) will be necessary.

Def 22   Let C_1, C_2 be concepts*, and let C_2 arise from C_1 by correctly (i.e., 'collisionless') renaming n λ-bound variables, $n\geq0$. Then C_1 is *α-equivalent to* C_2.

Claim 7 *α-equivalence is reflexive, symmetric, and transitive.*

Proof: Symmetry and transitivity are obvious. Reflexivity is guaranteed by the fact that n may equal 0.

As for examples, take concepts*

$\lambda x_1 \ [^0> x_1 \ ^00]$,

$\lambda x_2 \ [^0> x_2 \ ^00]$,

.
.
.

They are α-equivalent and they share the same content. This can be said about all α-equivalent pairs (a trivial consequence of Def 22].

Let Cont/ $(o*_1*_1)$ be the relation which holds between a construction C of order 1 and a variable x of order 1 iff C contains x. The concepts*

$[^0\text{Cont } ^0[\forall x[^0> x \ y]] \ ^0y]$,

$[^0\text{Cont } ^0[\forall z \ [^0> z \ y]] \ ^0y]$,

$[^0\text{Cont } ^0[\forall x \ [^0> x \ z]] \ ^0z]$

are pairwise equivalent (all construct the value True), but they are *not pairwise α-equivalent*: none of the variables x, y, z is λ-bound (see Def 8).

Claim 8   *All pairs of α-equivalent concepts* are equivalent but not vice versa.*

(A trivial consequence of Def 21, Def 22, and the preceding examples.)

Abbreviation: Let α, $\beta_1,...,\beta_n$ be various (not necessarily distinct) types. Instead of $(\alpha\beta_1...\beta_n)$ we will write $\alpha\beta_n$. Let $x_1,...,x_n$ be variables ranging over the types $\beta_1,...,\beta_n$, respectively, and let X be a function of the type $(\alpha\beta_n)$. Instead of $\lambda x_1...x_n$ $[Xx_1...x_n]$ we will write $\lambda x_n[Xx_n.]$.

Def 23 Let C be a simple concept* constructing an $(((\alpha\beta_{m1})\gamma_{m2})...\delta_{mn})$-object, and let $x_{mn},...,y_{m2},z_{m1}$ be variables ranging over $\delta_{mn},...,\gamma_{m2},\beta_{m1}$, respectively. Then C is *β-equivalent to* each of the concepts* $\lambda x_{mn}[Cx_{mn}]$, ..., $\lambda x_{mn+}...\lambda y_{m2}[[Cx_{mn}]...y_{m2}]$, $\lambda x_{mn}...\lambda y_{m2}\lambda z_{m1}[[[Cx_{mn}]...y_{m2}]\,z_{m1}]$, and all these concepts* are pairwise *β-equivalent*; besides, every concept* is *β-equivalent to* itself.

Trivial consequences :

Claim 9 *β-equivalence is reflexive, symmetric, and transitive.*

Claim 10 *All pairs of β-equivalent concepts* are equivalent but not vice versa.*

(Proof: The 'vice versa' case is obvious. That β-equivalent pairs are equivalent follows from Def 7d.]

Def 23 shows that, for example, the following concepts* are pairwise β-equivalent ($\text{Bel}/(\text{o}\iota*_1)_{\tau\omega}$, $x...\iota,\ c...*_1$):

^{0}Bel, λw [^{0}Bel w], $\lambda w\lambda t$ ^{0}Bel$_{wt,}$, $\lambda w\lambda t$ λxc [^{0}Bel$_{wt}x$ c]

Again we see that the content of all these three concepts* is the same (which holds of all β-equivalent pairs).

Let us now compare the cases of α- or β-equivalence with the cases where equivalent concepts* are neither α- or β-equivalent.

Claim 11 *Concepts* that are α- or β-equivalent share their content.*

We *did not* prove that it holds

The contents of C_1 and C_2 are distinct if C_1 and C_2 are equivalent but not α- or β-equivalent.

Now we add some more informal comments.

Consider the following pairs of concepts* (the ranges of variables are obvious):

g1)   [^{0}]x [^{0}= x [^{0}Exp ^{0}2 ^{0}3]]],

g2)   [$^0\sqrt{}$ ^{0}64],

h1) λx [^{0}> x ^{0}0],

h2) λy [^{0}> y ^{0}0],

i1) ^{0}Cat

i2) $\lambda w\lambda t$ ^{0}Cat$_{wt.}$

The concept* g1) is equivalent but neither α- nor β-equivalent to the concept* g2). Intuitively, the abstract procedures - which a concept should be - represented by g1) and g2) are entirely distinct: to get the number 8 by determining which unique number equals 2^3 is obviously another procedure than that which computes the result of calculating the square root of 64.

The concept* h1) is α-equivalent to the concept* h2). Which distinction between the respective procedures could be stated? To determine the class of positive numbers by testing whether the given

number is or is not greater than 0 is obviously a procedure which is not (essentially? at all?) influenced by which variable is bound by λ. No distinction can be stated.

The concept* i1) is β-equivalent to the concept* i2). To determine (immediately) the property *being a cat* means to be able to say in any possible world W at any time point T whether a given individual does or does not belong to the class which is the value of this property in W at T.[71] Inspecting, however, i1) and i2) we can see that they both reflect this procedure. A more simple example can show it for a non-empirical concept* : The concept* $\lambda xy\ [^0> x\ y]$ is β-equivalent to $^0>$; indeed, the abstract procedure that (immediately) identifies the relation > is the same procedure as that which consists in verifying $[^0>x\ y]$ for any pair of numbers.

The above considerations result in distinguishing a special subclass of the class of pairs of equivalent concepts*:

Def 24 Concepts* C_1 and C_2 are *quasi-identical* iff they are α- or β-equivalent.

Quasi-identity between concepts* of order n will be denoted $Quid^n$. The type of $Quid^n$ is obviously $(o*_n*_n)$.[72]

A trivial consequence of Def 24:

Claim 12 *For any n, $Quid^n$ is reflexive, symmetric, and transitive..*

Def 24 makes up a bridge to the next section, which is central in the present chapter.

5.4 Concepts

Let us begin with an interesting observation. We have said that an expression (of a natural language) represents a concept (letting aside equivocations). Assuming that expressions 'encode' concepts we naturally expect that they are sensitive to the structure of the given concept, so that two *distinct - even if equivalent - concepts will be encoded by distinct expressions*, since distinct equivalent concepts must differ by their structures.

Now if concepts were concepts* then our hypothesis would be confirmed by all pairs of equivalent but not quasi-identical concepts*. Indeed, having such equivalent concepts* as

$[^0Suc\ [^0Suc\ ^00]]$

and

$[^0|x\ [^0\wedge\ [^0Even\ x][^0Prime\ x]]]$

we 'encode' them distinctly taking into account their distinct components; we would use the expressions *the successor of the successor of zero* and *the only number such that it is even and prime*. (Maybe that in some cases it would not be as obvious: compare, e.g.,

$^0\angle$

and

$\lambda yx\ [^0>\ x\ y]$;

even here, however, we should use the expression *being smaller* in the first case and *being greater* in the second one, saying, for example, *being smaller (than)* and *being such pairs of numbers that the second number is greater than the first one*.)

As soon, however, as we consider Quid-related concepts[*] we can see that languages cannot distinguish between them.

In the case of α-equivalence (take, e.g., the concepts[*] $\lambda x_1 [{}^0{>} x_1 \, {}^00]$, $\lambda x_2 [{}^0{>} x_2 {}^00]$, ...) the reason is obvious: natural languages do not use names for bound variables. But this holds also for the case of β-equivalence: no language - if not artificially enriched by expressions describing logical structures - will find distinct expressions for ${}^0Cat$, $\lambda w [{}^0Cat \, w]$, $\lambda w \lambda t {}^0Cat_{wt}$. *Being a cat* is sufficient.

So the time comes *to revise our hypothesis that concepts[*] are what we mean by concepts.*

Metaphorically, *abstract procedures do not take care of bound variables.*

Our revision is rather simple. The relations $Quid^n$, $n{\geq}1$, being equivalence relations (see Claim 12), induce, for every concept[*], an equivalence class. Concepts can be construed as these equivalence classes.

Def 25 Let c,d be variables ranging over $*_n$ for some n. *A concept of order n is the function constructed by* $\lambda c \lambda d [{}^0Quid^n \, c \, d]$.

In other words, concept of order n associates every open construction of order n with an empty class, and every concept[*] with the class of those concepts[*] which are Quid-related with it. So we can say that every concept[*] of order n, i.e., every closed construction of order n, *generates* a concept of the same order.

(We could be tempted to define the class *concept generated by a concept[*] C* as the class of constructions which are equivalent to C and have the same content (Def 20). A simple counterexample: let C be ${}^0\neg$. The concept[*] C'

$\lambda p [{}^0\neg [{}^0\neg [{}^0\neg p]]]$

is equivalent to C, and the content of both concepts[*] is the same, viz. $\{{}^0\neg\}$, but C and C' belong to distinct concepts.)

Examples:

The concept[*] ${}^0Cat$ generates the concept

$\{ {}^0Cat, \lambda w [{}^0Cat \, w], \lambda w \lambda t \, {}^0Cat_{wt} , \lambda w_1 [{}^0Cat \, w_1], \lambda w_1 t \, {}^0Cat_{w1t}, \lambda w \lambda t_1 \, {}^0Cat_{wt1}, \lambda w_1 \lambda t_1 \, {}^0Cat_{w1t1}, \lambda w_2 [{}^0Cat \, w_2], ... \}$.

The concept[*] ${}^0{>}$ generates the concept

$\{ {}^0{>}, \lambda x_1 x_2 [{}^0{>} x_1 \, x_2], \lambda x_2 x_1 [{}^0{>} x_2 x_1], \lambda x_1 x_3 [{}^0{>} x_1 \, x_3], ...\}$.

The concept[*] $\lambda x_1 x_2 [{}^0{>} x_2 \, x_1]$ generates the concept

$\{ \lambda x_1 x_2 [{}^0{>} x_2 \, x_1], \lambda x_2 x_1 [{}^0{>} x_1 \, x_2], \lambda x_1 x_3 [{}^0{>} x_3 x_1], ... \}$.

The concept[*] ^{00} generates the singleton

$\{{}^00\}$.

The concept[*] ${}^0[{}^0{+} x \, y]$ generates the singleton

$\{ {}^0[{}^0+ \ x \ y]\}$.

We can immediately see that *if a concept* identifies an α-object with α an atomic type (i.e.,

$o, \iota, \tau, \omega, *_1, *_2,...$), then it generates a concept-singleton. Otherwise , the generated concept is an infinite

set.

Intermezzo: Propositional attitudes revisited.

In Ch.4 we have argued that propositional attitudes (*believe, know, doubt, suppose,...*) are, actually,

constructional attitudes, i.e., $(o\iota*_n)_{\tau\omega}$-objects. Now we will deduce from this assumption a claim

which will be an argument for a refinement of this assumption.

Claim 13   *Let Bel/ $(o\iota*_n)_{\tau\omega}$ be a propositional attitude; let C_1 and C_2 be distinct equivalent*

constructions (type $*_n$) that construct either a truth-value or a proposition. Then it does not hold that

from the premisses stating the equivalence and distinction of C_1 and C_2 , and the premiss

<div align="center">

X believes that C_1

</div>

('believes' as any instance of Bel) the conclusion

<div align="center">

X believes that C_2

</div>

can be deduced.

<u>Proof:</u> Our premisses represent the following concepts*:

(1) $[{}^0= C_1 \ C_2]$ (equivalent constructions),

(2) $[{}^0\neg \ [{}^0= {}^0C_1 \ {}^0C_2]]$ (distinct constructions)

(3) $\lambda w \lambda t \ [\ {}^0Bel_{wt} \ {}^0X \ {}^0C_1]$.

Clearly, the second premiss blocks the transition to the conclusion

$\lambda w \lambda t \ [\ {}^0Bel_{wt} \ {}^0X \ {}^0C_2]$,

since for replacing ${}^0C_1$ by ${}^0C_2$ the second premiss would be bound to be

$[{}^0= {}^0C_1 \ {}^0C_2]$;

the first premiss is entirely irrelevant.

Now we can present indefinitely many examples supporting this impossibility of any non-trivial

deduction from the premisses of the above kind. The case where C_1, C_2 construct a truth-value is

especially obvious. It is immediately clear that if Charles believes etc. that $1+1 = 2$ then it does not

follow that he believes etc. that $2^3 = 8$ (in the case of *knowing* instead of *believing* we obtain the

paradox of omniscience).

(Indeed, our premisses are

$[{}^0=_1 \ [{}^0=_2 \ [{}^0+ \ {}^01 \ {}^01] \ {}^02] \ [{}^0=_2 \ [{}^0Exp \ {}^02 \ {}^03] \ {}^08]]$,

where $=_1/$ (ooo), $=_2/$ (o$\tau\tau$),

$[{}^0\neg \ [{}^0=_3 \ {}^0[{}^0=_2 \ [{}^0+ \ {}^01 \ {}^01] \ {}^02] \ {}^0[{}^0=_2 \ [{}^0Exp \ {}^02 \ {}^03] \ {}^08]]]$,

where $=_3/$ (o$*_1*_1$), and

$\lambda w \lambda t \ [{}^0Bel_{wt} {}^0Ch \ {}^0[{}^0=_2 \ [{}^0+ \ {}^01 \ {}^01] \ {}^02]]$.)

If C_1, C_2 construct a proposition then again it is sufficient to refer to the premiss stating that C_1 and C_2 are distinct; it can seem that believing that London is larger than Oxford implies believing that Oxford is smaller than London: but this is by far not necessary (see Ch. 4).

But we can present such examples that will problematize our type-theoretical assumption about propositional attitudes.

So there is something suspicious with the following application of Claim 13:

Let Charles believe that there are even primes. We have

$[^0=_1 [\exists x_1[^0\wedge [^0\text{Even } x_1][^0\text{Prime } x_1]]][\exists x_2[^0\wedge [^0\text{Even } x_2][^0\text{Prime } x_2]]]]$,

$[^0\neg [^0=_3 {}^0[\exists x_1 [^0\wedge[^0\text{Even } x_1][^0\text{Prime } x_1]]] {}^0[\exists x_2 [^0\wedge[^0\text{Even } x_2][^0\text{Prime } x_2]]]]]$,

$\lambda w\lambda t [^0\text{Bel}_{wt} {}^0\text{Ch} {}^0[\exists x_1[....]...].$

But then, as we know (Claim 13, Proof), we cannot infer that Charles' believing concerns the concept[*] $[\exists x_2 [...]...]$, which is absurd, of course. (Analogous examples can be easily constructed for the case where C_1, C_2 construct propositions.)

Now why is our last example absurd unlike the example with Charles' knowing that $1+1 = 2$ vs. knowing that $2^3 = 8$? In other words, why is the construction $[^0= [^0\text{Exp } {}^02 {}^03] {}^08]$ 'more distinct' from the construction $[^0= [^0+{}^01 {}^01] {}^02]$ than the construction

$[\exists x_1 [^0\wedge [^0\text{Even } x_1][^0\text{Prime } x_1]]]$

from

$[\exists x_2 [^0\wedge [^0\text{Even } x_2][^0\text{Prime } x_2]]]$?

The reason should be clear already: the last two constructions are α-equivalent, therefore, quasi-identical, therefore, *members of one and the same concept*.

So we could say that believing etc. is, properly speaking, a relation linking individuals with *concepts*. Since, however, such 'intentionalities' like propositional attitudes cannot concern set-theoretical entities, and concepts - as sets - are set-theoretical entities (indeed, a strange kind of them, see the end of the present section), we would be unable to solve the problem of substitution: any concept - being a set - can be constructed in infinitely many ways, which would cause in most cases X's not believing etc. what X certainly believes etc.. So let instead the type of propositional attitudes remain $(\omicron\iota*_n)_{\tau\omega}$, and let us add an 'unless-sentence' to the Claim 13:

Claim 13' *...(Claim 13: can be deduced) unless C_1 and C_2 are members of one and the same concept: in this case the conclusion is correct.*

The <u>Proof</u> of the 'unless-part' exploits a further premiss:

(4) $[^0\text{Quid } {}^0C_1 {}^0C_2]$

which makes (2) irrelevant, for (4) specifies the kind of distinctness of C_1 and C_2, and since this kind does not change the abstract procedure connected with C_1, and a propositional attitude does not care of bound variables, the distinction between C_1 and C_2 can (and must) be neglected.

Abbreviation: Let C be a concept[*] of order n. The concept generated by C will be denoted by <u>C</u>. Clearly, '<u>C</u>' is an abbreviation for '$\lambda d [^0\text{Quid } {}^0C d]$' ($d$ ranging over $*_n$).

Our definitions of 'extension of' and 'intension of' a concept*, as well as some other definitions, have to be generalized so as to apply to *concepts.*

Def 26 Let $\underline{C}$ be a concept (of order *n*). The *extension of* $\underline{C}$ is the extension of C..

Def 27 The *extension of* $\underline{C}$ *w.r.t. a world W and time point T* is the extension of C w.r.t. a world W and the time point T.

Def 28 $\underline{C}$ is *a simple concept* iff it contains a simple concept*.

Examples: The concept (*c* ranging over $*_1$)

$\lambda c[^0\text{Quid } c \ ^0[\lambda xy \ [^0{>} x\text{y}]]]$, i.e., $\underline{\lambda xy \ [^0{>} x \, y]}$,

is simple because it contains the simple concept* $^0{>}$. The concept

$\underline{\lambda xy \ [^0{>} y \, x]}$ is not simple since none of its members is simple. ($^0{\angle}$ is not a member of it!)

Def 29 *The intension/content of* $\underline{C}$ is the intension/content of C.

(Remember that according to Def 20 and Def 25 the intension/content of all members of $\underline{C}$ is the same.)

Def 30 The concept $\underline{C}$ is *strictly empty, quasi-empty, empty w.r.t. a world W and time point T* iff C is, respectively, strictly empty, quasi-empty, empty w.r.t. a world W and time point T.

Now we can formulate an easy generalization of Claim 4:

Claim 4' *No simple concept is strictly empty.*

Def 31 Concept $\underline{C}$ *identifies an object A* iff C constructs A.

Def 32 Concepts $\underline{C}_1$, $\underline{C}_2$ are *equivalent* iff they identify the same object or are both strictly empty.

Def 33 A concept $\underline{C}$ is *an empirical concept* iff it identifies a non-trivial intension (see Def 6).

Claim 14 *There are concepts which are not simple.*

Proof: By example: $\underline{\lambda xy \ [^0{>} y \, x]}$ does not contain any simple concept*.

Now we have to react to a serious possible criticism. It could be said that our definition of concepts is incompatible with what has been promised on an intuitive level. Indeed, we have rejected the set-theoretical approach to analysing the level of concepts. We have said that set-theoretical entities like sets, relations, functions are 'flat', that the 'steps' determining such entities are 'lost' in the latter whereas the level of meanings/concepts must contain 'structured entities' - structured in the sense that the components ('steps') of them must be taken into account and that distinct structures mean distinct entities independently of whether these entities do or do not determine (=identify) one and the same object. This proclamation seems to be incompatible with our construal of concepts as *sets*: or are not sets typical 'flat' set-theoretical entities?

Well, this objection is really a serious objection, and it requires a serious reaction. Perhaps what follows will clarify this point.

Indeed, concepts have been defined as sets of a kind; what is, however, most important is that the members of a concept are constructions (concepts*). Not only that: any concept is *unambiguously determined* by any of its members; any closed construction unambiguously determines the concept generated by it; every property of a concept $\underline{C}$ can be 'computed' from some properties of the

construction/concept* C. So we could say that the 'flat' character of C is compensated by the structured character of its members.

The most important reason why concepts - even as sets - are to be construed as quasi-objects rather than objects can be formulated as follows:

If concepts are 'abstract procedures' - as we suppose - then we can say that *no member of a concept,* i.e., no concept* belonging to a concept, *determines a procedure distinct from that one determined by any other member of that concept.* Indeed, both α- and β-equivalent concepts* are connected with one and the same abstract procedure: this has been already stated in the beginning of the present section. The seemingly similar cases do no more determine one and the same procedure. Consider the following case: Let C_1, C_2 be constructions constructing two $o_{\tau\omega}$-objects (=propositions). One could suppose that the concepts*

$\lambda w \lambda t \, [^0\wedge \, C_{1wt} \, C_{2wt}]$

and

$\lambda w \lambda t \, [^0\wedge \, C_{2wt} C_{1wt}]$

belong to one and the same concept. Yet the two procedures are distinct: the first one takes the two concepts* in the order C_1, C_2, the second one takes first C_2 and second C_1.

In a sense, we can even say that any concept* is a representative of a concept, and that in our concept theory we could work with concepts* only. There is a problem, however, which can be dealt with just *via* exploiting the distinction between concepts* and concepts: the problem of *using* vs. *mentioning* concepts.

5.5 Using and mentioning concepts

The phenomenon 'using vs. mentioning *expressions*' is well-known; it is, however, not identical with 'using vs. mentioning *concepts*'. To see it consider the English expressions

a)   *Cats are carnivorous.*

b)   *'Cats' is the plural of 'Cat'.*

In a), we speak about the property denoted by 'Cat' (saying that *being carnivorous* is a requisite of the property *being a cat*, see 5.2.2, Remark 2); in b) we speak about the expression 'Cats' itself. So we can say that the expression 'Cats' is used in a) and mentioned in b). (But the expression ' 'Cat' ' is used in b), of course.)

 Yet speaking about something we use concepts. So the concept CAT is used in a), and the concept THE EXPRESSION 'CAT' is *used* in b). The concept CAT cannot be mentioned in b), since b) does not say anything about this *concept.*

Before we define using and mentioning concepts let us analyze the following sentences:

c)   *Pussy is a cat.*

d)   *Cats are carnivorous.*

e)   *The concept CAT is a zoological concept.*

f) $\lambda w \lambda t \, ^0Cat_{wt}$ *contains the variable w.*

Types: Pussy/ ι, Cat/ $(o\iota)_{\tau\omega}$, Carn(ivorous)/ $(o\iota)_{\tau\omega}$, Zool(ogical concept)/ $(o(o*_1))$ (a class of concepts of order 1), Cont/ $(o*_1*_1)$, $c...*_1$.

c') $\lambda w \lambda t\ [^0\mathrm{Cat}_{wt}\,^0\mathrm{Pussy}]$,

d') $[^0\mathrm{Req}\ ^0\mathrm{Carn}\ ^0\mathrm{Cat}]$

e') $[^0\mathrm{Zool}\ \lambda c\ [^0\mathrm{Quid}\ c\ ^{00}\mathrm{Cat}]]$ $(= [^0\mathrm{Zool}\ \underline{\mathrm{Cat}}]\ )$,

f') $[^0\mathrm{Cont}\ ^0[\lambda w \lambda t\ ^0\mathrm{Cat}_{wt}]\ ^0 w]$.

Our analyses are based on the following considerations:

Using a concept $\underline{C}$ we speak about the object identified by this concept. According to Def 31, this is to say that we speak about the object constructed by C. Hence *using a concept $\underline{C}$ we can represent it by any member of $\underline{C}$.* This is the case of c') and   d') where the concept $\underline{\mathrm{Cat}}$, being used, is represented by $^0\mathrm{Cat}$, which is only a member of $\underline{\mathrm{Cat}}$.

Remark: The obvious distinction between c') and d') concerns the way we use the **property** *(being a cat)*: in c') this property is used - the case of *de re supposition*[73]: Pussy is a member of the class which is the value of the property *being a cat* in the given world-time - the sentence c) is true or false dependently on the particular population of cats in a world-time; in d'), however, we speak about the 'whole' property - *de dicto supposition* - either it is true that a necessary condition for *being a cat* is *being carnivorous,* or not. A particular population of cats in a world-time is irrelevant here. This distinction is well visible in our constructions: the *de re* occurrence of $^0\mathrm{Cat}$ is connected with the 'index' $_{wt}$, unlike the *de dicto* occurrence.

In e' we speak *about the concept CAT;* CAT is mentioned there. Therefore, it must be represented by a concept that *identifies the concept CAT.* Such a concept is, however, given by Def 25: this concept - $\lambda c\ [^0\mathrm{Quid}\ c\ ^{00}\mathrm{Cat}]$ - is *used* here, and therefore it is represented by a member only, namely by the *construction* $\lambda c\ [^0\mathrm{Quid}\ c\ ^{00}\mathrm{Cat}]$.

(Remember that the *concept* $\lambda c\ [^0\mathrm{Quid}\ c\ ^{00}\mathrm{Cat}]$ is a concept of order 2, containing, for example, concepts* $\lambda c_1[^0\mathrm{Quid}\ c_1\ ^{00}\mathrm{Cat}]$, $\lambda c_2[^0\mathrm{Quid}\ c_2\ ^{00}\mathrm{Cat}]$, etc.)

Summing up:

Def 34 Let D be a construction. The concept $\underline{C}$ is *used in* D wherever a member of $\underline{C}$ occurs in D not as a part of a trivialization. It is *mentioned in* D wherever its construction occurs in D.

Thus in c'), as well as in d'), the concept $\underline{\mathrm{Cat}}$ is used: one of its members, viz. $^0\mathrm{Cat}$, occurs in c') and d') (not within a trivialization). In e') the concept $\underline{\mathrm{Cat}}$ is mentioned: its construction occurs in e'). Now what about f')? It could seem that $\underline{\mathrm{Cat}}$ is used here, since it is not mentioned, and one of its members - $^0\mathrm{Cat}$ - occurs there. But this time $^0\mathrm{Cat}$ is a component of a trivialization, so that Def 34 does not apply. So we must say - in accordance with our intuition - that the concept $\underline{\mathrm{Cat}}$ is *neither used nor mentioned in f').* Indeed, here the only entity which is dealt with is a particular construction; we do not 'speak' about cats nor about the concept CAT.

Understanding the distinction between using and mentioning concepts can explain some 'strange phenomena' with sentences that contain expressions representing strictly empty concepts.[74] Consider sentences

g) *The greatest prime number is odd.*

h) *The greatest prime number is an empty concept.*

i) *There is no greatest prime number.*

Types: Gr/ $(\tau(o\tau))$, Pr/ $(o\tau)$, Od/ $(o\tau)$, Em$^{(1)}$/ $(o(o*_1))$

g') $[^0\text{Od} \ [^0\text{Gr} \ ^0\text{Pr}]]$ (the concept is used)

h') $[^0\text{Em} \ \underline{[^0\text{Gr} \ ^0\text{Pr}]}]$ (the concept is mentioned)

i') $[^0\neg \ [\exists x \ [^0= x \ [^0\text{Gr} \ ^0\text{Pr}]]]]$ (the concept is used)

Intuitively, we would say that g) lacks any truth-value, whereas h) and i) are true. Our analyses g')-i') support (or even: verify) these intuitions. g') is an improper composition, h') constructs True; to see that also i') constructs True, remember that $\exists$, due to its type, is used so that '$\exists x$ X' abbreviates '$[^0\exists \lambda x \ X]$': the class whose non-emptiness is asserted by $\exists$ is a function which is everywhere undefined, so it is not non-empty, and its negation must be true.

Our question is then: *Under which conditions is a sentence that contains an expression representing a strictly empty concept without any truth-value?*

Our partial answer can be derived from our paradigmatic examples:

There is no reason for lacking a truth-value if the strictly empty concept is *mentioned*: in this case what is spoken about is not the object which should be identified by the concept but the concept itself, i.e., a class of constructions: *no improper construction is **used** in this case.*

As for the case where the strictly empty concept is *used*, two subcases must be distinguished. First, if something is predicated about the *object* which should be identified by the concept (case g),g')) then, of course, no truth-value is constructed, since, properly speaking, no predication takes place. Second: Something is predicated about the *class* of the objects to be identified by the concept (case i),i')). This class is not a 'standard class': *its characteristic function is undefined on all arguments*. We can, therefore, say that it is non-empty (this claim is false) or not non-empty (this claim is true) ; in both cases we get a truth-value.

Remark: Notice that there is no standard way at our disposal in which a (natural language) sentence would inform us that it is a *concept* what is spoken about. If, for example, in h) the expression *the greatest prime number* stood between quotes it would mean that it is the *expression* what is spoken about ('mentioning expressions'); but h) is about a concept.

Now we can comment the well-known Frege's remark made in his polemic with Kerry[75], where Frege argues that a concept name cannot stand in the place of grammatical subject. Frege used two criteria for distinguishing concepts from objects. One of them consists in the assumption that concepts - unlike objects - can be predicated (they are 'general concepts'). The other criterion is just the grammatical one - concepts cannot be 'denoted' by a grammatical subject. Frege assumed that the second criterion is a consequence of the first one, which is, however, not the case. Even if only 'general concepts' were admitted their 'ability' to be predicated would not be lost if they were *mentioned* in the position of a grammatical subject: they would remain to be concepts.

Yet there is a much deeper reason why Frege rejected the possibility of concept names' standing in the position of subject. This reason is given by Frege's construal of concepts as functions, i.e., as 'unsaturated' entities.[76] A sentence cannot contain an 'unsaturated entity': if a concept 'stands in the position' of a predicate (i.e., the concept name stands in this position) it becomes to be 'saturated' by the subject of the sentence. In the subject position, however, it would not be saturated.

Frege's 'horse-example' does not prove his thesis as soon as his conception of concepts is changed so as we have done it: his example is structurally the same as our example e), where the concept CAT is mentioned: this concept remains to be a concept even in the 'subject position', and, of course, no problem with 'saturated vs. non-saturated' arises: concepts are no functions for us, and, besides, also functions are not 'non-saturated', because their constructions do not contain free variables.

All the same, we could find a 'rational core' in Frege's considerations. The subject position, which causes - according to Frege - the change concept → object, can be analysed in terms of our conception as follows: in the subject position the given concept is mentioned: what is spoken about is the whole $(\text{o}*_n)$-object. But as a set it is a set-theoretical object (with some features of quasi-objects, i.e., constructions), and this circumstance can be interpreted so that whereas *using* concepts requires dealing with constructions (generating the concepts in question) *mentioning* (subject position !) deals with sets of constructions, i.e., with set-theoretical *objects*. Thus whereas using concepts deals with *quasi-objects*, their mentioning needs (set-theoretical) *objects*.

6. Conceptual systems
6.1 Simple concepts and 'primitive concepts'

An impression could arise that there is a unique set of simple concepts (Def 28), and that all 'remaining' concepts can be 'derived' from the members of this set. One could even 'refer to classics' to show that this is a kind of common conviction.[1]

Everything is possible in the 'realm' of ideal entities. If, however, a 'bridge' between this realm and a language (primarily) and language use (secondarily) is to be built it proves much more useful to consider various sets of simple concepts and investigate which further concepts can be found on the basis of such sets.

This idea leads us to defining *conceptual systems*.

Def 35 Let $\underline{C}_1,...,\underline{C}_m$ be simple concepts of orders $k_i \leq n$. Let $\underline{C}_{m+1},...$ be all concepts distinct from $\underline{C}_1,...,\underline{C}_m$ such that the components (subconstructions) of the constructions C_{m+i}, $i>0$, are only members of $\{C_1,...,C_m\}$ and variables ranging over those types that are composed of types given by $C_1,...,C_m$. The set

$$\{\underline{C}_1,...,\underline{C}_m\} \cup \{\underline{C}_{m+1},...\}$$

will be called *a conceptual system of order n* (CS_i^n); the set $\{\underline{C}_1,...,\underline{C}_m\}$ is the set of *primitive concepts* (of CS_i, $\text{P}(\text{CS}_i)$), the second set is the set of *derived concepts* (of CS_i), $\text{D}(\text{CS}_i)$).

Remark: The concepts $\underline{C}_{m+1},...$ are, of course, of any order $\geq n$.

So any primitive concept of some conceptual system is a simple concept, and any simple concept can be a primitive concept of a conceptual system. Theoretically, a conceptual system containing infinitely many primitive concepts is thinkable (for example, a system whose primitive concepts are $\{^0 0\}$, $\{^0 1\}$, $\{^0 2\}$, ...)

Examples:

Let CS_1 be

$\{\,{}^0\underline{\neg},\,{}^0\underline{\vee}\} \cup \{\lambda p\,[{}^0\underline{\neg}\,[{}^0\underline{\neg}\,p]],\,\lambda pq\,[{}^0\underline{\neg}\,[{}^0\underline{\vee}\,[[{}^0\underline{\neg}\,p][{}^0\underline{\neg}\,q]]]],\,\lambda pq\,[{}^0\underline{\vee}\,[{}^0\underline{\neg}\,p]\,q],...,{}^{0\,0}\underline{\neg}p,...\}.$

CS_1 is a conceptual system of order 1; notice that among the derived concepts is, e.g., the concept[*] ${}^{0\,0}\underline{\neg}p$, i.e., a concept of order 2. Higher and higher order concepts arise by simply 'cumulating' trivializations.

CS_1 contains, among others, the concepts necessary for defining the (classical) propositional logic.

Let CS_2 be

$\{^0\underline{Suc},\,{}^0\underline{0}\} \cup \{\,[{}^0\underline{Suc}\,{}^0\underline{0}],\,[{}^0\underline{Suc}\,[{}^0\underline{Suc}\,{}^0\underline{0}]],\,...\}.$

SC_2 contains, among others, the concepts of all natural numbers.

Let SC_3 be

$P(CS_2) \cup \{^0\underline{=},\,{}^0\underline{+},\,{}^0\underline{-},\,{}^0\underline{\cdot},\,{}^0\underline{:},\,{}^0\underline{N}\} \cup \{\lambda xy\,[{}^0\underline{:}\,y\,x],\,...\,\}.$

CS_3 contains some basic concepts used in the arithmetic of natural numbers.

Let CS_4 be (where $P(\,CS'_1)$ contains some further concepts so that new variables of types ι $(x,...)$ and $(o\iota...\iota)$ $(P^i,...)$ are at our disposal):

$P(CS'_1) \cup \{^0\underline{\forall}{}^1\} \cup \{\,\lambda P^1\lambda x\,[P^1x],...,\lambda P^n\lambda x_1...x_n[P^nx_1...x_n],\,...\}.$

CS_4 contains, among other things, some basic concepts used in the 'first order predicate logic'.

Let CS_5 be

$\{^0\underline{\wedge},\,{}^0\underline{\vee}\} \cup \{\,\lambda pq\,[{}^0\underline{\wedge}\,[[{}^0\underline{\vee}\,p\,q]\,q],\,...\,\}.$

CS_5 contains some concepts used in the (classical) propositional logic.

Let CS_6 be

$P(CS_3) \cup P(CS_4) \cup \{\,...\,[{}^0\underline{\forall}P\,[{}^0\underline{\vee}\,[{}^0\underline{\vee}\,[{}^0\underline{\neg}\,[{}^0P^00]]][{}^0\underline{\neg}\,[{}^0\underline{\forall}x\,[{}^0\underline{\vee}\,[{}^0\underline{\neg}\,[{}^0Px]][{}^0P[{}^0\underline{Suc}\,x]]]]]]]]$
$\qquad\qquad\qquad [{}^0\underline{\forall}x\,[{}^0Px]]],\,...\}$

Some concepts important for arithmetic of natural numbers and not contained in CS_3 are contained in CS_6. (The above adduced member of $D(CS_6)$ is the concept represented by the Axiom of Induction.)

Let CS_7 be

$P(CS_1) \cup P(CS_5) \cup \{\,...\,\lambda pq\,[{}^0\underline{\neg}\,[{}^0\underline{\wedge}\,[{}^0\underline{\neg}\,p][{}^0\underline{\neg}\,q]]],\,...\,\}.$

CS_7 does the same job as CS_1.

Now we adduce only some definitional schemes of special kinds of conceptual systems.

Def 36 Let CS_i be conceptual systems such that $P(CS_i)$ contains some empirical concepts. We call such CS_i *empirical conceptual systems*.

Def 37   Let CS_j be conceptual systems such as to contain a) 'logical concepts', i.e., concepts that make it possible to build up at least the 1^{st} order predicate logic, and b) some mathematical and/or empirical concepts. We call such conceptual systems *normal conceptual systems.*

If we considered bridges connecting the abstract 'realm' of conceptual systems with real scientific systems we would agree that the latter are based on normal conceptual systems, and that such empirical conceptual systems which are not normal live their poor life in that abstract realm only.

6.2   Some properties and relations of conceptual systems

Def 38 A conceptual system *identifies* an object /quasi-object A iff some member of it identifies A.

Def 39   The set of those objects (not quasi-objects!) that are identified by a conceptual system CS_i will be called *the area of* CS_i .

Def 40 A conceptual system CS_i is (*strongly) weaker than* a conceptual system CS_j iff the area of CS_i is a (proper) subset of the area of CS_j .

Def 41 A conceptual system CS_i is *equivalent to* a conceptual system CS_j iff the area of CS_i is identical with the area of CS_j .

Def 42 A conceptual system CS_i is *a (proper) part of* the conceptual system CS_j iff $P(CS_i)$ is a (proper) subset of $P(CS_j)$.

Def 43 A concept $\underline{C}_1$ is *dependent on* a concept $\underline{C}_2$ iff a subconcept* of C_2 is a subconcept* of C_1. Otherwise $\underline{C}_1$ is *independent of* C_2. A conceptual system CS_i is *independent* iff no member $\underline{C}$ of $P(CS_i)$ identifies the same object as a member of $D(CS_i)$ independent of $\underline{C}$.

Examples:

$CS_1$is strongly weaker than CS_4, CS_6.

CS_1 is a part of CS_4, CS_6, CS_7.

CS_5 is strongly weaker than CS_1, CS_7.

CS_1 is equivalent to CS_7.

CS_1 is independent: it is true, $^0\underline{\neg}$ identifies the same object as, e.g., $\lambda p\ [^0\neg\ [^0\neg\ [^0\neg p]]]$ but the latter is not independent of $^0\underline{\neg}$.

CS_7 is not independent: $^9\underline{\wedge}$ identifies the same object as $\lambda pq\ [^0\neg\ [^0\vee\ [^0\neg p][^0\neg q]]]$.

Claim 15 *The set $D(CS_i)$ is unambiguously determined by the set $P(CS_9)$.*

(A direct consequence of Def 35.)

Claim 16 *If $P(CS_i) \subseteq\ P(CS_j)$ then $D(CS_i) \subseteq D(CS_j)$.*

(A direct consequence of Claim 15.)

Claim 17   *If CS_i is a proper part of CS_j, and CS_j is independent then CS_i is strongly weaker than CS_j.*

Proof: Let CS_i be a proper part of CS_j. According to Def 42, $P(CS_j)$ contains a concept $\underline{C}$ not contained in $P(CS_i)$. If CS_j is independent then $\underline{C}$ does not identify any object which is identified by the members of $D(CS_j)$ independent of $\underline{C}$, and, therefore, it does not identify any object identified by

the members of $D(CS_i)$ (Claim 16). Since $\underline{C}$ is simple it cannot be strictly empty (Claim 4'), and it identifies a 'new' object, i.e., an object outside the area of CS_i), QED.

So CS_1 is not strongly weaker than CS_7, although it is a proper part of it: CS_7 is not independent. The converse of Claim 17 does not hold. CS_i may be strongly weaker than CS_j even not being a proper part of it (cf CS_5 and CS_1).

Remark: We surely see the analogy between a conceptual system and a system of terms in an axiomatic system. $P(CS_i)$ corresponds to the set of 'primitive terms', $D(CS_i)$ corresponds to the set of terms 'definable' *via* primitive terms. We will return to this point later (Ch.8); now it is also important to see the distinctions: Concepts are not linguistic expressions (they are 'abstract procedures'); so the conceptual systems are independent of any axiomatic system. As for the connection of conceptual systems with language, see Ch.7.

We could consider the question of possible *orderings* of conceptual systems. For obvious reasons we can replace conceptual systems by isomorph systems of concepts*, i.e., where a conceptual system CS_i contains a primitive concept $\underline{C}$ the isomorph system CS^* contains C. Any ordering of conceptual systems is derivable from an ordering of the systems of concepts*. Further, due to the fact stated in Claim 15 we can consider *only orderings of the sets of primitive concepts**.

An ordering based on *contents* of concepts (concepts*) is thinkable; the ordering relation is then obviously the *part-of relation* (Def 42). There is nothing especially interesting on this kind of ordering; yet if we take into account independent systems only, then such an ordering is at the same time an ordering partially based on *areas* of conceptual systems (cf Claim 17): if CS_i is a part of CS_j then the area of CS_i is included in the area of CS_j. On the other hand, an ordering based primarily on areas is not identical with the preceding one: in the resulting lattice a conceptual system CS_i precedes CS_j iff CS_j is weaker than CS_j, which does not mean that CS_i is a part of CS_j. So, for example, CS_5 will precede CS_1 whereas in the first case CS_5 will not precede CS_1.

What is called *conceptual system* by Kauppi[2] is only a special case of conceptual systems: it is based on the traditional idea of general concepts making up the content of 'less general concepts' (see 5.1.4.1). In our terms, we could imagine such a system as containing the most general primitive concepts ($P(CS_i)$), and synthesizing the less general ones in $D(CS_i)$ by means of conjunctions/intersections.

(Let a simplifying example be

$P(CS^*) = \{{}^0Dog, {}^0Black, {}^0Wild, ... (at least {}^0\wedge)\}$,

$D(CS^*) = \{\lambda w\lambda t\lambda x \,[{}^0\wedge\,[{}^0Dog_{wt}\,x][{}^0Black_{wt}\,x]], \; \lambda w\lambda t\lambda x \,[{}^0\wedge\,[{}^0Dog_{wt}\,x][{}^0Wild_{wt}\,x]],$

$\qquad\qquad \lambda w\lambda t\lambda x \,[{}^0\wedge\,[{}^0Wild_{wt}\,x][{}^0Black_{wt}\,x]],$

$\qquad\qquad \lambda w\lambda t\lambda x \,[{}^0\wedge\,[{}^0\wedge\,[{}^0Dog_{wt}\,x][{}^0Black_{wt}\,x]][{}^0Wild_{wt}\,x]], ...\}$

with possible enlargements due to exploiting negation, etc.)

Intermezzo: The problem of independence between concepts.

 At first sight it could seem that if a concept $\underline{C_1}$ is independent of the concept $\underline{C_2}$ (Def 43) then $\underline{C_1}$ cannot be equivalent to $\underline{C_2}$. Yet a brief inspection shows that things are more complicated. Compare, for example, two following concepts $(D/ (o\tau\tau) - [^0D\ x\ y] \dots y$ divides x, Card $/ (\tau\ (o\tau)) \dots$ cardinality of number sets):

a) $^0\underline{Prime}$

b) $[\ \lambda x\ [^0{=}\ [^0Card\ [\lambda y\ [^0D\ x\ y]]]\ ^02]]$.

Clearly, neither of the concepts a),b) is dependent on the other, but both procedures identify one and the same set of numbers, so that they are equivalent in the sense of Def 32.

Any logician who has read the present study till the immediately preceding lines will undoubtedly ask the following question:

Which logical procedure(s) could verify the claim that, e.g., a) and b) are equivalent concepts?

The question is fully justified. No such question can arise in the following case:

We surely can state that the concepts

c) $\lambda x\ [^0{\wedge}\ [^0Odd\ x][^0Prime\ x]]$

d) $\lambda x\ [^0{\wedge}\ [^0Prime\ x][^0Odd\ x]]$

are equivalent (notice that they are mutually dependent)[3]; the well-known logical rules make it possible to easily derive this result. It seems, however, that in the cases such as a), b) there is no correct logical rule which could prove this equivalence.

From the 'practical' point of view, some relevant commentary will be formulated in Ch.7 (7.3). Theoretically, however, if we accept our definitions, then we can see that a) identifies the same object as b) does; the strange feeling connected with this pair of concepts is due to the unusual construction called *trivialization*: trivializations simply construct the objects/constructions which are denoted by what follows ^{0}.

In the cases similar to a) and b) no other logical 'rule' can be used than Def 32 (and, of course, Def 7).

(The problem of equivalence of *expressions*, i.e., in our case, the problem of justifying our use of the expression *a prime* in the same sense as the expression *a number having just two divisors*, is a distinct problem, which transcends the abstract level of concepts. See Ch.7.)

Dealing with the abstract entities like concepts we can pose a question which seems to be rational outside this abstract 'realm' but which can be construed so that it becomes meaningful inside this realm: I mean the question of *'conceptual priority'* : can we say that a concept is 'prior' to another concept? Or, in other words, can we say that an object is *'conceptually prior'* to another object?[4] In general, to say that some A is prior to some B means that we presuppose some necessary link leading from A to B but not *vice versa*: we will exemplify this kind of link by reminding of the *requisite* relation (see 5.2.2). We have seen that, for example, being a vertebrate is a requisite of being a mammal: there is no mammal which would not be a vertebrate (a necessary link) but there are vertebrates which are not mammals. So we can say that the *property* being a vertebrate is *prior to the property* being a mammal. But by transfer we can say: *The concept A VERTEBRATE is prior to the*

concept A MAMMAL. But then a new question arises: *If a concept A is prior to a concept B, is it possible to have B among the primitive concepts of a(n)(independent) conceptual system, and A among the derived concepts of the same system?*

At first sight, the plausible answer seems to be: No, it is impossible; for how could a concept A that is prior to a concept B be derived from B?

This argumentation is wrong. We will demonstrate the possibility of such an arrangement of our concepts that will show that our answer must be: *Yes, it is possible.*

So let our (normal!) conceptual system CS be:

$P(CS)= \{^0\underline{Mammal}, {}^0\underline{Bird}, {}^0\underline{Fish}, {}^0\underline{Reptile}, {}^0\underline{Amphibian, ...}\}$.

Now observe the set D(CS): one of its members (CS is normal!) is

$\lambda w\lambda t\lambda x\, [^0\vee\, [^0\underline{[^0Mammal_{wt}\, x]}][^0\vee\, [^0\underline{Bird_{wt}x}]...[^0\underline{Amphibian_{wt}x}]...]]$.

But this concept identifies just the property *being a vertebrate*. (True, it is only a concept *equivalent to* the concept $^0\underline{Vertebrate}$ but it is also prior to the concept $^0\underline{Mammal}$.)[5]

7. Languages and conceptual systems

7.1 Meaning vs. pragmatic meaning

In the preceding chapters we tried to show that our Platonist intuitions (formulated in Introduction) can be made more specific. With some few exceptions we confined ourselves to abstract (ideal) entities (constructions, concepts, etc.). The application of our construal of meanings/concepts to real languages can proceed in two steps: first, the connection between *expressions and concepts* should be clarified, second, some consequences of our conception for real *use of expressions* should be studied or at least suggested. First of all, however, the distinction between the two last mentioned points has to be explained.

The history of the great misunderstanding which accompanies most 'post-analytic' theories begins with Quine,[1] whose more recent summarizing study[2] essentially (with more details) repeats the ideas contained in his first attack aiming at the very essence of LANL. It seems now - in the light of Quine's philosophy (together with the work of the late Wittgenstein[3]) - as if any return to such abstract entities like meanings[4] or intensions were impossible, and as if any attempt at such a returning to abstract entities were hopelessly old-fashioned. *Use instead of meaning* seems to be the modern (or: post-modern?) solution to all problems that have been formulated by the naive semanticists.

The misunderstanding I am talking about consists in this very 'shift' from meaning to use. Needless to say that since Quine's classical formulations[5] there have been written thousands of pages where Quine's arguments for this shift are either further elaborated or simply quoted as obvious truths.

The present study offers a (Platonist) theory of concepts; its task differs from the task of a thorough and systematic criticism of the Quinean conception. Speaking however about the role of concepts as (potential) meanings of expressions we have to formulate some principal critical comments as regards the above misunderstanding.

Why do we call the requirement of shifting meaning to use a misunderstanding?

One of the clearest 'pragmatic confessions' made by Quine and his followers is:

> *If we are allergic to meanings as such, we can speak directly of* **utterances** *as significant or insignificant, and as synonymous or heteronomous one with another.*[6]

Now what the semanticists, logicians, and some philosophers of language hoped to define is the *meaning of an expression*; it is an *expression* of a language what can be connected with meaning. Exptressions are *abstract entities*: we cannot ask where or when an expression 'takes place'. *Using an expression* is an *event*: a *speech act, linguistic act, utterance.* The misunderstanding I try here to explain is based on the assumption that what is intuitively construed as a *meaning of an expression* has to be *not complemented* but *replaced* by an attribute of *utterances*, i.e., of spatio-temporal entities. So the Quinean conception is based on a *category mistake.* This mistake is furthermore accompanied by the conviction that a theory of meaning would be *incompatible* with the theory of *use*; a consequence thereof is that *the job made by semanticists should be passed over to pragmatics.*

Properly speaking this means that *abstraction should be replaced by empirical generalization.* Let me adduce an analogy. No physical body falls in 'normal circumstances' so as to obey the gravitational law. Registering times, heights, weights of falling objects, etc., we can get an empirical theory, never a 'law of physics': to get the latter we have to *abstract*; the result of this abstraction is, of course, an *ideal entity*, let it be a function or a construction of this function. A Quinean could call this ideal entity an 'obscure one', and be content with concrete events: but to calculate the time of falling of a concrete object (to predict this time, that is) we need *the law*, even if corrections due to the particular empirical circumstances must be done. So let expressions be 'vehicles' of meaning; we can understand what an utterance of an expression 'means' in the given situation, *if we know the meaning of the expression*, even if some corrections due to the situation must be done.

So our point is that *the distinction between semantics and pragmatics is an important distinction,* and that *semantics* - the denotational one as well as the theory of meaning - *is not reducible to pragmatics.* One of the best formulations of the mentioned distinction can be found in the classical articles *Languages and Language* by D.Lewis [Lewis 1985] and *Pragmatics* by Stalnaker[7]. Setting aside that part of pragmatics which analyses types of speech[8] we can state - as Stalnaker does - that (the other part of) pragmatics concerns

> *the features of the speech context which help determine which proposition is expressed by a given sentence* (p.383).

So the problems with indexicals (for instance: pronouns, but in a sense also proper names) are problems solvable only if the situation ('context') of the given utterance are taken into account. Yet to claim that if no situation of utterance, no context is given nothing remains, i.e., that only some nonsensical sound or sequence of characters is what there is when the expression is studied *in abstracto* is an incredible simplification. The fact that this reductionist view - even if not as radically formulated - is wide-spread can be at least partially explained by the fact that the nominalistic misuse of Occam's razor is also wide-spread[9]. A behavioristic illusion that all what there is can be construed as physical particular objects and that any kind of *universalia* is at least suspicious is what can explain the 'postanalytic' mistrust of meanings as 'things'[10]. The conflict between this scepticism and our view

is too deep to be overcome. Neither do I believe that a convinced nominalist can ever change her opinion 'by force of arguments'.

In connection with Stalnaker's excellent analysis of the s.c. Donnellan's Problem (attributive vs. referential meaning)[11] we now return to our scheme (S) (see Introduction) and to our claim that empirical expressions never denote the values of intensions in the actual world.

There are two ways leading from an empirical expression to the 'actual objects'. One of them is unambiguously given by the expression[12] *(as an abstract entity) and the state of the world. The other way is given by the expression, the state of the world, and the situation of the utterance of the expression.*

To illustrate this we adduce some examples.

1. Let us consider the expression *town*. We have said that such expressions denote - *via* constructions/concepts - properties,. here a property of individuals (an $(o\iota)_{\tau\omega}$-object). The value of this property in the actual world at the given time point (= 'in the actual world-time') is a class (of the 'actual towns'). Using the expression *town* we *denote* the property *being a town* and *refer to* that class. At any time point this class is unambiguously determined[12] just by this expression and the given state ot the world.

2. Compare therewith the expressions *here, the man over there (who...),* etc. The place referred to by *here*, as well as the individual referred to by *the man over there...* are determined by the expression, the state of the world, *and* the situation in which the given expression is uttered. (Notice that within our conception the expression *now* does not belong to this category: it denotes a $(\tau\tau)$-object, viz. the identical function on time points. *Independently of any utterance of this expression,* the time point referred to is given simply by the given time point. In the case of *here,* the dependence on the place of the utterance is essential.)

A contribution to the theory of proper (personal) names: consider the expressions *Bill, Clinton, Bill Clinton.* Properly speaking, any of these names is 'utterance dependent': the expression determines the individual not only due to the state of the world but also due to the situation of the utterance. There are thousands of people whose name is *Bill, Clinton*, or even *Bill Clinton*, and whom I refer to is given by the context; so I can refer to the same person saying *Bill* and saying *Clinton*: in the first case my presupposition is that my partners in the discourse know whom I mean (the person with the name *Bill* whom my partners know is one person only), similar but distinct assumptions are connected with other situations. Some proper names seem to bear another , 'utterance independent' character[13] : *Mount Everest, the Moon,* etc. are organic parts of our vocabularies, and wherever, whenever and by whomever they are uttered they refer to one and the same object (setting aside some anomalous cases - nicknames, etc.). In this respect they share a property with mathematical names : they simply denote extensions, and there is no good sense in distinguishing their denotation from 'reference'. (But then they denote directly extensions, without the intermediary role of intensions; a somewhat strange consequence thereof would be, of course, that those names are *not empirical expressions* - see Claim 1.) The other proper names (personal names) share an essential feature with

indexicals, being utterance dependent.Now compare two kinds of empirical sentences, exemplified by

a) *Some mammals are carnivorous*

b) *This hat is too exclusive.*

The proposition denoted by a) is not logically necessary; a) is an empirical sentence. Therefore, it is true in some worlds-times and false in others. In the actual world it is true; a) can be said to *refer to* Truth. (For Frege this would be the *denotation* relation, *Bezeichnen*.) This reference is given only by the state of the world (at the given time points). The sentence b) cannot be said to refer to a truth-value only due to the state of the world: within *the same world-time* it can be true in some 'utterance situations' and false in others. This is so because of the occurrence of the indexical *this*. Without having 'input data' from the utterance situation we cannot offer any construction underlying b).

Here a question arises: If *meanings* should be construed as *concepts/concepts*[*], then expressions like indexicals or expressions containing them would be *meaningless* (see Def 12). Yet everybody will say (if she knows English) that she can understand sentences like b). So if we consider 'pragmatically anchored' expressions, we can ask : How come that expressions to which we are unable to attach a concept are all the same intelligible? In other words, *what is the meaning of pragmatically anchored expressions?* Not assuming that a fully satisfactory explication can be already given[14] we only suggest one global hypothesis apparently compatible with our whole conception.

So why do we understand, e.g., the sentence b), even when we do not know the circumstances of the pragmatic context, and, moreover, are not able to construct the respective proposition?

Let us compare the case of the sentence b) with another, more simple case: why do we understand such expressions like $x + 3$, $x > 3$, etc., even if we do not know, which number or truth-value, etc. they are about? (And we do understand them, which is proved by our ability to use such expressions, to distinguish one such expression from another, perform some rational manipulations with them, etc.) This time it is not very difficult to answer the question. Our understanding the above expressions can be explained by the fact that the underlying constructions are clear: $[^0+ \ x \ ^03 \ ]$,

$[^0> x \ ^03]$, etc. We cannot say, of course, that the meanings of such expressions are concepts: the above constructions are *not closed*. Yet the free variables make it possible to semantically 'work' with the respective constructions, since the types which the variables range over are (sometimes even explicitly) given.

Now some definitions will be helpful.

Def 44 *A pragmatically anchored expression* is an expression that contains indexicals, demonstratives[15] or (personal) proper names.

Def 45 *The meaning of a pragmatically anchored expression E* is the (open) construction which is the result of the logical analysis of E, where indexicals, demonstratives, and proper names are represented by the (alphabetically first) variables ranging over the respective types.

Def 46 *The pragmatic meaning of a pragmatically anchored expression E in the situation S* is the concept which is generated by the construction that arises from the meaning of E so that the free variables contained in the latter are replaced by the respective objects determined by S.

Def 47 *The pragmatic denotation of a pragmatically anchored expression E in the situation S is the*
extension/intension identified by the pragmatic meaning of E.

Def 48 *The WT-reference of a pragmatically anchored expression E in the situation S is*

a)   the object denoted by E in S if the pragmatic denotation of E is an extension,

b)   the WT-value of the intension pragmatically denoted by E in S, otherwise.

Examples:

Suppose that a place is a set of individuals. (Other type-theoretical assumptions are thinkable, of
course, but the choice is irrelevant here).

 The <u>meaning</u> of the expression *here* is the variable x_1 which ranges over (οι).

The <u>pragmatic meaning</u> of *here* in the situation where the speaker means a particular room is the
concept $\lambda w \lambda t \ A_{wt}$, where A constructs the property *being (in) the given room*.

The <u>pragmatic denotation</u> of *here* in the above situation is just the property *being (in) the given room.*

Finally, the <u>reference</u> of *here* in the given situation in W at T is the set of all points/individuals which
is the value of the above property in W at T.

Similarly, consider the sentence

(1) *I am hungry.*

The <u>meaning</u> of (1) is the construction $\lambda w \lambda t \ [^0\text{Hungry}_{wt} \ x_1]$, where x_1 ranges over individuals.

The <u>pragmatic meaning</u> of (1) in the situation where (1) is uttered by Albert Einstein (suppose that
Einstein behaves like *the Moon*) is the concept $\lambda w \lambda t \ [^0\text{Hungry}_{wt} \, {}^0\text{Einstein}]$.

The <u>pragmatic denotation </u> of (1) in the above situation is the proposition *that Albert Einstein is*
hungry.

The <u>reference</u> of (1) in the above situation in the world- time where the speaker Einstein is hungry is
True.

All these definitions (they are rather 'characteristics') and examples should only suggest that the
realistic assumption that there are such ideal entities like meanings is compatible with theories of
linguistic behavior. No systematic theory of pragmatics has been articulated here, of course.[16]

Now let E be *a not-pragmatically anchored expression which does not contain free variables.*

Def 49 *The meaning of E is the concept generated by any construction (concept[*]) that is the result of*
the logical analysis of E.

(The concept in question can be said to be *represented by* E.)

Def 50 *The denotation of E is the entity (if any) identified by the meaning of E.*

Def 51 *The reference of E in W at T is*

a)   the denotation of E if E is a non-empirical expression;

b)   the value of the denotation of E in W at T otherwise.

Examples:

The <u>*meaning*</u> of the expression *town* is ^{0}<u>Town </u> (see, however, 7.3).

The <u>denotation</u> of *town* is the property *being a town.*

The <u>reference</u> of *town* in the actual world-time is the set of all 'actual towns (now)'.

The <u>meaning</u> of the sentence

(2) *Some mammals are carnivorous.*

is the concept

(2') $\lambda w \lambda t\ [\exists x\ [^0\wedge\ [^0Mamm_{wt}x][^0Carniv_{wt}x]]]$.

The <u>denotation</u> of (2) is the proposition identified by (2'), i.e., the proposition *that some mammals are carnivorous.*

The <u>reference</u> of (2) in the actual world-time is the truth-value True.

If E contains free variables the respective definitions will be analogous, with open constructions as meanings, and with valuation-dependent denotations.

We will see that some oversimplifications are present in our explications. Nevertheless, after some corrections are made in 7.2 and 7.3 we can say that Quine's 'problem of circularity' in defining analyticity, synonymy, and meaning can be solved. The only presupposition necessary for this solution is to admit that abstract entities are a part (*sui generis*) of the world. Then, of course, concepts, meanings, etc. are not more 'obscure entities' than, say, numbers.

This presupposition is not shared by Quineans, for the maximum concession made by them as regards realism consists in admitting that there are sets: even such set-theoretical entities like properties are not admitted.[17] To require, moreover, that also such 'strange entities' like constructions were acknowledged would be too provocative a requirement. Yet without this requirement a (probably rather poor) denotational semantics is thinkable but - as I believe - *no rational theory of meaning is possible:* **the level of meaning is indistinguishable from the level of denotation** ('level of reference' , for some authors) **unless the principal distinction between an entity and the way this entity is constructed is defined.**[18] One can accept or not Tichý's definition of constructions, but this definition is at least one of viable ways how to define the above distinction.

Remark: As for our strict distinguishing between semantics and pragmatics, this is an important point motivating the impossibility of our accepting 'relevance logics'.[19] If it seems that such formulas as A ⊃ (B⊃ A) or A ⊃ (B⊃B) do not function as tautologies then it is clear that it is not semantics/logic what should be corrected: it is the *use* of expressions of the above form what is at least strange. To try to 'correct' such formulas on the semantic level means that we have to 'reinterpret' at least the sign ⊃. But doing it - as the relevantists do - we must let variables range over 'persons', and speak about a 'commitment relation' whose logical character is dubious: *to be committed* can be either an empirical relation (which would destroy the impression that relevantists do logic), or there is 'something external' which causes the necessary character of it - I am committed to infer B from A if B is a logical consequence of A, so that the term *being committed to* denotes a superfluous variant of the 'normal' logical inference relation. Mixing up a psychological analysis with a logical analysis is the essence of the 'relevantist revolution' against classical logic.

7.2 Expressions and Concepts

7.2.1 Homonymy, Ambiguity

From now on let us be confined to the non-pragmatically anchored expressions.

One of the necessary corrections to be made in the preceding section concerns Def 49 - 51.

Talking about *the* meaning etc. of an expression is obviously an oversimplification. Let us define two properties of expressions which must be taken into account when using the above definitions.

Def 52 Let E be a simple expression. E is *homonymous* iff it represents more than one concept.

Def 53 Let E be a complex expression. E is *ambiguous* iff it represents more than one concept.

Examples:

The simple expression *bank* represents at least two concepts. Consider

(3) *The bank of Thames is nice.*

(4) *Some banks get bankrupt.*

One popular thesis of contextualists could be formulated as follows:

The meaning of a homonymous expression is determined by a context.

I would not say that the thesis is adequate. I would rather say that *bank* represents all 'its' concepts in any context, i.e., independently of a context. What is determined by a context is only the *choice* of one of these concepts. Sometimes even the context does not determine this choice. Consider the following case:

(5) *Albert Einstein has been an idealist.*

Now even the context of (5) does not determine 'in which sense' Einstein has been an idealist.[20]

As for Def 53, the problem of 'various readings' is the linguistic variant of the problem of ambiguity. Consider the classical

(6) *Flying planes can be dangerous.*

Let Pl/ $(o\iota)_{\tau\omega}$ be the property *being a plane*,

  Fl/ $(o\iota)_{\tau\omega}$ the property *to fly,*

  Fl'/ $(o\iota\iota)_{\tau\omega}$ the relation(-in-intension) *to fly (an object),*

  D/ $(o\iota)_{\tau\omega}$ the property of individuals *to be dangerous,*

  D'/ $(oo_{\tau\omega})_{\tau\omega}$ the property of events ([propositions) *to be dangerous,*

  Pos/ $(oo_{\tau\omega})_{\tau\omega}$ the property of events (propositions) *to be possible* (not *logically possible* !).

  $x,y...\iota.$

A very imprecise attempt[21] at offering the concepts represented by (6) could be:

(6') $\lambda w\lambda t\ [^0Pos_{wt}\lambda w\lambda t\ [\exists x\ [^0\wedge\ [^0D_{wt}x][^0\wedge\ [^0Fl_{wt}x][^0Pl_{wt}x]]]]]$,

(6'') $\lambda w\lambda t\ [^0Pos_{wt}\lambda w\lambda t\ [^0D'_{wt}\ \lambda w\lambda t\ [\exists xy\ [^0\wedge\ [^0Fl'_{wt}xy][^0Pl_{wt}y]]]]]$.

Any context which 'disambiguates' (6) in the sense of showing the preferable choice extends beyond the particular sentence (6); but what is especially interesting in this case is that the proposition identified by (6') is obviously the same proposition as that identified by (6''). Actually, the proposition identified by (6') is true/false in exactly those worlds-times where the proposition identified by (6'') is true/false. So a following question can be formulated:

Can an expression be taken to be homonymous/ambiguous if the concepts it expresses identify one and the same object?

As far as I know this question has been never raised. There are two reasons thereof. First, denotation and meaning/concept have not been distinguished precisely enough. Second, even if some suspicion about this distinction existed, the possibility of simultaneously representing more concepts and denoting one entity has not been taken into account.

In a sense, the above question is rather a terminological question. All the same, the decision can be based on some very natural intuitions. Let us consider (6). Our intuition is that if the above question were answered negatively then we could not understand why (6) is conceived by linguists to be worthy of 'disambiguation'. Moreover, translating (6) into other languages (German, Czech,...) results in two distinct sentences. So our answer to the above question is positive.

Remark: Our example, viz. (6), is an example of ambiguity. Similar examples concerning homonymy can be found if a theory of abbreviations is accepted, see 7.3.

From the viewpoint of our theory of concepts, the distinction between homonymy and ambiguity can be characterized as follows: A homonymous expression is simple; so the fact that it represents more concepts can be explained by the fact that more 'direct associations' of such an expression with a concept had been realized by a linguistic convention. On the other hand, ambiguity comes into being due to some ambiguities in the *grammar/ syntax* of the given language.

Remark: The role of *grammar* from the viewpoint of LANL consists in stating rules that make it possible to apply constructions.[22] So the distinctions between particular languages are given not only by vocabularies but, importantly, by grammars. The constructions (and, therefore, concepts) represented by the expressions of a language L may be in general the same as those represented by a distinct language L', but the way from (complex) expressions of L to constructions/concepts differs from that way from expressions of L' to (the same) constructions/concepts. For a simple example: where the English speaker says

(7) *XY is hungry.*

The Czech speaker says

(8) *XY má hlad.,*

which could be 'verbatim' translated as (a non-grammatical sentence)

- *XY has hunger.*

So whereas in English a copula connects the subject noun with an adjective, in Czech a verb 'má' ('has') connects the subject noun with a substantive 'hlad' ('hunger'). Yet from the logical point of view, the underlying construction is the same: we apply a property (first to worlds-times, and thereafter) to an individual, getting

$\lambda w \lambda t \ [^0Hung_{wt} \ ^0XY].$

7.2.2 Synonymy, Equivalence, Coincidence

Our distinguishing between the *conceptual and objectual level* enables us to make more precise the category *synonymy*, and to explicate other - really or seemingly similar - relations of expressions.

Def 54 Expressions E_1, E_2 are *synonymous* iff they represent one and the same concept.

Def 55 Expressions E_1, E_2 are *equivalent* iff they denote one and the same object or represent two strictly empty concepts. They are *weakly equivalent* iff they are equivalent and not synonymous.

Def 56 Expressions E_1, E_2 are *coincident in W at T* iff they denote two intensions whose value in W at T happens to be the same object.

The corresponding schemes are (E_1,E_2 ...expressions, C, C_1, C_2...constructions/concepts, O...object, I_1,I_2...intensions):

Synonymous expressions: *Weakly equivalent e.:*   *Coincident expressions:*

E_1 E_2 E_1 E_2 E_1 E_2

C C_1 C_2 C_1 C_2

O I_1 I_2

O (contingently)
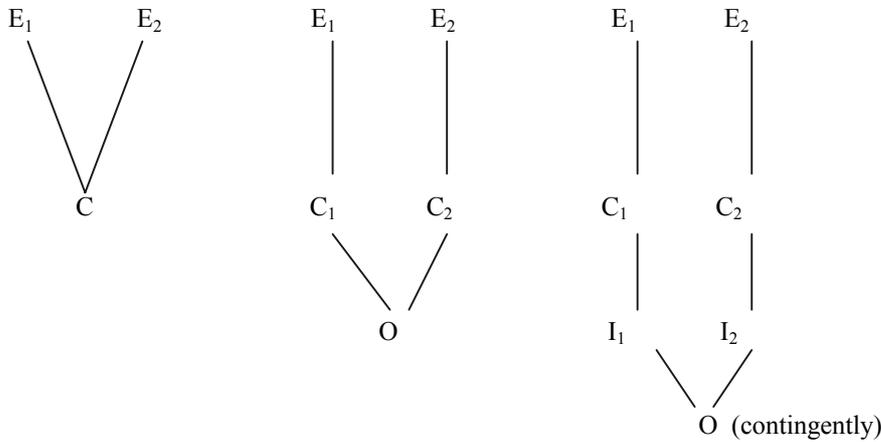
Now synonymous, (weakly) equivalent, and coincident expressions may be again simple or complex. Let us adduce some examples.

It is rather difficult to find in the given language pairs of simple synonymous expressions. Setting aside the problem of definitions (see Ch.8) only few pairs of strictly synonymous simple expressions can be found. We already had some opportunity to offer the following example (within another context): Let *catastrophe* and *disaster* be such a pair. Theoretically, one and the same simple concept can be associated with both these expression, let it be <u>^{0}catastrophe</u>, or <u>^{0}disaster</u> - we already mentioned that both '^{0}catastrophe' and '^{0}disaster' denote one and the same construction. In the case of complex expressions a general claim can be formulated: *Every pair of purely syntactical variants of an expression is a pair of synonymous expressions.* Indeed, constructions are not sensitive to purely syntactical distinctions. To adduce an example, consider the sentences

(9) *XY believes that his wife is clever.*

(10) *XY believes his wife to be clever.*

An intuition-based analysis of (9) (and of (10) as well) leads to the concept (B/ $(\omega*_1)_{\tau\omega}$, Cl/ $(\omega)_{\tau\omega}$, Wife/ $(\omega)_{\tau\omega}$):

$\lambda w \lambda t \ [^0B_{wt}{}^0XY \ ^0[^0Cl_{wt} \ [^0Wife_{wt}{}^0XY]]]$.

No abstract procedure would distinguish (9) from (10)[23]: this is the case of distinct 'syntactical tools' for expressing one and the same meaning (representing one and same concept).

Looking for examples of weak equivalence we can see that in the case of simple expressions we are confronted with a problem similar to that of independence of concepts (see the latest Intermezzo). If the expression

(11) *a prime number*

represented the simple concept

^0Prime

and if our analysis of the expression

(12) *the numbers having exactly two divisors*

leads (intuitively) to the concept

$\lambda x \ [^0= \ [^0Card \ [\lambda y \ [^0Dxy]]]^02]$

then, of course, we must say that (11) and (12) are weakly equivalent expressions. Yet we will see that this is not the only possible construal of the semantics of such pairs of expressions (see 7.3).

As for the weak equivalence of complex expressions there are plenty of immediately obvious examples at our disposal. We adduce three of them.

(13) *the square root of nine*; *one plus two*.

(14) *being a doctor or an ingenieur*; *not being anything other than a doctor or an ingenieur*.

(15) *The Earth is bigger than Venus*; *Venus is smaller than the Earth*.

Clearly, the above pairs are pairs of expressions that represent distinct concepts:

(13') $[^0\sqrt{} \ ^09]$; $[^0+ \ ^01 \ ^02]$

(14') $\lambda w \lambda t \lambda x \ [^0\vee \ [^0Doc_{wt}x][^0Ing_{wt}x]]$; $\lambda w \lambda t \lambda x \ [^0\neg \ [^0\neg \ [^0\vee \ [^0Doc_{wt}x][^0Ing_{wt}x]]]]$

(15') $\lambda w \lambda t \ [^0Larg_{wt}{}^0E \ ^0V]$; $\lambda w \lambda t \ [^0Smal_{wt}{}^0V \ ^0E]$.

(By the way, saying that two sentences express the same *fact* does not mean that they are synonymous; this is clear from our analysis. Putnam only states this distinction without clarifying it, when he says:

> *The same fact can be expressed either by saing that the electron is a wave with a definite wavelength λ or by saying that the electron is a particle with a sharp momentum **p** and an indeterminate position. What 'same fact' comes to here is, I admit, obscure. Obviously what is **not** being claimed is **synonymy** of **sentences**. It would be absurd to claim that the **sentencce** 'there is an electron-wave with the wavelength λ' is **synonymous** with the **sentence** 'there is a particle electron with the momentum h/λ and a totally indeterminate position'.* [Putnam 1983, 298].)

Now it is *coincidence* what has been systematically mixed up with equivalence or even synonymy. The following pairs are pairs of coincident expressions w.r.t. the actual world-time:

(16) *Morning star*; *Evening star*

(17) *the capital of Poland*; *the largest town in Poland*

(18) *the President of USA*; *the husband of Hillary Clinton*[24].

In each of (16)-(18) two distinct intensions ('roles', 'offices', $\iota_{\tau\omega}$-objects) are denoted; the fact that their value in the actual world-time is the same (celestial body, town, man) is contingent.

The following example shows that to construe pairs of coincident expressions as pairs of equivalent expressions is especially unnatural a construal if the expressions are sentences:

(19) *Some people are criminals. Jupiter is the biggest planet in the Solar system.*

Clearly, both these sentences represent distinct concepts (so they are not synonymous) and denote distinct propositions (so they are not equivalent either); but their truth-value in the actual world (now) is the same. 'Equivalent' would these sentences be, of course, if we accepted the Fregean claim that what is denoted by sentences is 'their' truth-value.

In general, if the level of meanings-concepts were not precisely distinguished from the level of objects (denotations) the distinction between synonymy and equivalence would not be definable. And if we did not strictly defend the principle according to which an (empirical) expression denotes only a non-trivial intension (never the value of it in the actual world) we could not make distinction between equivalence and coincidence, which would lead to absurd consequences: it is surely highly counterintuitive not to register the analytic character of A = B if A is equivalent to B, and the synthetic (empirical) character of A=B if A is only coincident with B; compare

(20) *black and wild dogs = wild and black dogs*

(or any true mathematical equality) with

(21) *the capital of Poland = the largest town in Poland.*

The equality (20) is logically true unlike (21).

Another example: if *morning star* were equivalent to *evening star* then the analysis of

(22) *morning star = evening star*

would lead to

(22') $[^0= \ ^0\text{Mo} \ ^0\text{Ev}]$.

We have seen, however, that the adequate analysis results in

(22'') $\lambda w\lambda t[^0= \ [\lambda w\lambda t \ ^0\text{Mo}_{wt}]_{wt} \ [\lambda w\lambda t \ ^0\text{Ev}_{wt}]_{wt}]$

(which can be recorded as

$\lambda w\lambda t \ [^0= \ ^0\text{Mo}_{wt} \ ^0\text{Ev}_{wt}]$).

The result (22') does not seem to be adequate: true, it is rather linguists who should decide whether (22) is a linguistic convention that allows for naming Venus promiscue *morning star* and *evening star:* in this case these expressions would be not only equivalent but just synonymous. Yet Frege's problem became an interesting one due to the assumption that (22) states an astronomical discovery, rather than a linguistic convention. Then, however, these expressions are only coincident, and (22'') - as the adequate analysis - shows the contingent character of the equality (22).

7.2.3 Vagueness: a short remark

So expressions - unless they contain free variables or are pragmatically anchored (Def 44) - represent concepts, and concepts - unless they are strictly empty - identify objects. Yet there is an old problem with this identification. The Greeks formulated some puzzles concerning *definiteness* of the objects

denoted *via* concepts by some expressions. The puzzles (*'heap', 'bald'*) formulate a well-known question: *where does lie the borderline which separates A-objects fromn non-A-objects?* The puzzles adduced some cases where this borderline is obviously indeterminate (' for which number of sand corns a heap 'ceases to be' a heap?' etc.). It has been shown much later[25] that, theoretically, such puzzles can be formulated for all empirical expressions. So not only *heap* but also *chair* (as shown by Black) can be seen (a little unexpectedly) to be *vague*.

The history of the problem of vagueness is rather rich[26] , and in some respects interesting results have been achieved; especially this concerns the s.c. *fuzzy logics*. Our remark concerns a specific problem: *how to harmonize our conception of concepts with the unpleasant fact of vagueness?*

Here we will only suggest a line of analysis that could lead to the desired harmonization. No systematic theory can be expected here.

First we try to answer the question whether vagueness is a property of expressions, of concepts, or of objects themselves.

It is difficult to imagine in which sense an object itself could be vague. Let the property *being a heap* be such an object. Containing 10^6 sand corns may somebody make admit that a heap is present. Somebody other will claim that this is still not a heap (in the sense that the object containing 10^6 sand corns does or does not possess the property *being a heap*). Is, however, this difference of opinions a good argument for saying that this *property itself* is not determinate enough? We have construed properties (of individuals, to be generalized, of course) as functions that associate every pair <W,T> with a class of individuals. Let us compare two such functions: H_1, H_2. H_1 differs from H_2 in that it associates the given pair <W,T> with the class of all such objects that consist of some kind of corns and contain at least $10^6 +1$ corns, whereas H_2 'takes into account' already those objects that contain 10^6 corns. We would say in this case that H_1 and H_2 are *two distinct properties*, and that the English word *heap* covers, therefore, at least two distinct properties. Now we can imagine thousands of such properties H_i differing by the number of corns admitted as the cardinal number of those sets which are 'taken into account' by the given H_i. If, moreover, the cardinalities differ 'systematically' (the cardinal numbers increase by one, and the minimal cardinality is a positive number) then the set of properties H_i corresponds in some essential aspects to the set of 'possible heaps'. The expression *heap* will then be *homonymous* in a special way: it will cover a whole set of concepts each of which will identify one of H_i.

We also can only suggest the direction in which a further explication would proceed: the concepts associated with particular properties, for us most probably ${}^0\underline{H_1}$, ${}^0\underline{H_2}$, ..., would be attached to the expression (say, *heap*), and one feature which distinguishes this kind of homonymy from the other kinds could be formulated as follows: The concepts associated with the given expression are in a sense *similar*: no analogy with *bank, idealist,* etc. If A does not admit that the given object is a heap while for B it is (since A connects with *heap* another concept, 'another ${}^0\underline{H_i}$',than B) A will all the same admit that the controversial object is similar to a heap.

If we dare to generalize this example we get a somewhat strange result: There are what is called 'vague expressions', but the property *vagueness* can be reduced to the specific sort of homonymy

characterized above. Further: there is no such property which could explicate vagueness and which would be a property of concepts.

Claim 18 *If to be vague means to be indeterminate then no concept is a source of vagueness.*

<u>Proof:</u> It is sufficient to prove this claim for concepts[*].

I,   Simple concepts[*] are not vague: where X is an object or a construction no indeterminacy can arise due to trivialization - ^0X constructs just X (Def 7b). If X itself were in some sense 'vague' then trivialization would only preserve (rather than create) this 'objectual' indeterminacy.

II, If X, $X_1,...,X_n$ are constructions then neither $[XX_1...X_n]$ nor $[\lambda x_1...x_nX]$ can give rise to vagueness where $X,X_1,...,X_n$ are not already vague (Def 7c,d).

What is interesting on vagueness is that the borderline which defines the 'area of indeterminacy' is fuzzy.[27] This fuzziness can be explained if our conception of vagueness is accepted: whereas there are many ('similar') concepts which can be associated with an expression to make it '*mildly homonymous'* (= vague, see the example with $\underline{H}_i$) the *choice* made by a user of the given language is unpredictable, and, moreover, in most cases that concept which has been chosen cannot be identified (even if the criteria are numerically expressible, as it has been the case in our example). Yet according to our conception a concept has been always chosen, and only the fact that the particular concepts are very similar *inter se,* together with the fact that there are nearly so many 'sublanguages' (see 7.4) as the users of the given language cause the impression that the given expression possesses an enigmatic property '*vagueness'* rather than that it is homonymous.[28]

7.3 Simple concepts and simple expressions

Up to now, analysing an expression we have proceeded as follows: a) Whenever a subexpression was a simple (one-word-)expression we attached a simple concept[(*)] to it, and then b) we applied various kinds of construction according to our intuitive understanding of the relations between the grammatical and logical structure of the given expression.

This last point is connected with a special, already mentioned problem of replacing our intuitions by a systematic theory, which is by far not a trivial task[29]. Here the point a) will be commented.

Let us consider our frequently occurring examples with the expression *(a) prime (number).* Analysing, eg. the sentence

(23) *There is a (real) number which is even and prime.*

we get - using our method -

(23') $\exists x\ [^0\wedge\ [^0\text{Even } x][^0\text{Prime } x]]$

Here *number* cannot be represented in another way than *via* variables (τ is a basic type); *and* and *there is* denote $\wedge$ and $\exists$, respectively (and represent the respective - simple? - concepts), and *even* and *prime*, being simple expressions, have been supposed to represent simple concepts.

Before we show more precisely the oversimplifying character of the above analysis let us discuss the problem of justifying the questions satisfying the following scheme:

(24) *Is the concept X a simple concept?*

The particular instances of (24) may still have distinct forms. Consider four cases:

(24) a) *Is the concept ^{0}Prime a simple concept?*

    b) *Is the concept PRIME a simple concept?*

    c) *Is the concept associated with the expression ' prime' a simple concept?*

    d) *Is the concept of prime numbers a simple concept?*

The case a) is trivial. According to Def 28 there is no problem here.

As for the case b), it may be identified with the case c). It is these cases where an interesting answer can be expected.

In the case d), it seems that the question is futile since the definite article here is not justified: the way the phrase *being a/the concept of* is used makes us assume that a definite object is talked about after 'of' . But if A is a definite object (here: the set of primes) then there are infinitely many concepts that identify A. So a racionalization of the case d) will be

    d') *Is there a concept of prime numbers which is simple?*

The answer to it is also trivial: we always can find such a concept which identifies the given object and is simple. (Here it is ^{0}Prime.)

So let us consider the case c) (= its variant b)). The same suspicion about the definite article as in the case d) can be justified: namely, can we be sure that a simple expression of the given language represents just one concept, NB the simple one?

Take any simple expression for conjunction, let it be *and*, or $\wedge$. Can we say that the only concept that is represented by such an expression is $^0\wedge$? Remember the conceptual systems CS_1 and CS_5 from Ch.6. *The* concept of conjunction *in* CS_5 is a simple one; in CS_1 there is no simple concept of conjunction, but a complex concept of it is a member of $D(CS_1)$. Which of these two concepts is represented by, say, *and* ? The same question can be raised as regards the expression *prime*. As already stated, in the pure theory nothing prevents us from considering such conceptual systems which contain (among their primitive concepts) the simple concept ^{0}Prime. But, of course, there are other conceptual systems, with such primitive concepts like DIVIDE, CARDINALITY, etc., where the concept PRIME is a complex one, i.e., a member of the set of derived concepts.

The 'realm' of ideal, language independent entities is one thing, the problem of meanings (of the natural language expressions) is another one.

Therefore, even if our study is primarily a logical, i.e., *a priori* analysis, we have to suggest some 'bridges' between the area of abstract rntities (especially concepts[*] and concepts) and the phenomenon of language, where also other moments than the logical ones must be brought into play. From this wider and more complex viewpoint we will certainly agree that understanding the expression *prime* is obviously connected with such conceptual systems where the concept PRIME is among the complex, derived concepts. The reason thereof can be explained in various ways.

First, we can repeat here our *epistemological* comment concerning trivialization (see 3.3): if this kind of construction has the 'immediate identification' as its epistemological counterpart then, of course, it would be a dubious assumption to construe understanding of *prime* as an 'immediate' identification of the set of primes. *To immediately identify means to identify without using any other concept*. But to be

able to say of any number whether it is or not a prime without knowing such procedures as, e.g., dividing is to possess a kind of some mysterious intuition, which does not correspond to our typical way of understanding expressions.

Second, the most natural explanation of our ability to understand *prime* consists in referring to the indisputable fact that this expression has been introduced into English as an *abbreviation* (see Ch.8), so that the procedure connected with this expression is by far not a simple, 'immediately identifying' procedure but rather that one which we have been taught as we have learnt 'the right side of (the respective) definition'.

Now our languages are - at any 'ripe' stage of their development - full of such abbreviations. (See Def 60 in 8.3.) If an expression is a simple one it does not mean, therefore, that it represents a simple concept.

So we could ask:

If some simple expressions do not represent simple concepts, does it mean that - in a given language - there are two disjoint classes of simple expressions - the class of 'semantically simple expressions', and the class of 'abbreviations'?

This question is characteristic of the complex area which could be called *Analyse of Natural Language*; LANL is only a part of it, and it is certainly not logic alone which could be competent for giving an adequate answer. All the same, let us exploit, on the one hand, our conceptual frame within which we construe meanings of (not-pragmatically anchored) expressions as concepts, concepts as abstract identifying procedures generated by constructions, and simple concepts according to Def 18 and Def 28, and, on the other hand, our intuitions, viz. the intuitions of an attentive language user.

Perhaps we know enough to say that the above (interrogatively expressed) hypothesis is highly counterintuitive. If there really were a class of 'semantically simple expressions', say, **Si**, separate from the class of 'abbreviations', then linguistic dictionaries would not be compiled so as they really are: every complex expression occurring in the 'right side' of particular (simple) entries would contain just the members of **Si**, and no phenomenon known as 'definitional circularity' (necessarily occurring in any dictionary whatsoever) could be stated. Actually, we always find expressions occurring in the right sides ('explicans') also among those ones which occur in the left sides ('explicandum'); but if the expressions in the right sides were composed just of semantically simple expressions, why should the latter occur in the left sides, which should be only abbreviations? Either is *minute* an abbreviation, but then no such entry as *hour = 60 minutes* should occur in the dictionary (which is, however, a very frequent, rather innocent case of 'circularity'), or it is a semantically simple expression, and then why should it occur in the left side?

So the hypothesis that the vocabulary of a language could be divided into two disjoint classes according to the above criterion is untenable. But then it means that one and the same simple expression can be 'weakly' homonymous in the sense that it denotes a single object but represents (at least) two distinct (equivalent) concepts.[30]

This strange situation calls for some further explanation.

7.4 'Sublanguages'

It would be difficult to define what we feel that is *'the standard language'* (within a given natural language). Languages are 'living organisms', and even if we do not take into account their development (LANL works on a 'synchronic' basis) we must see that their homogeneity - important as it is from the viewpoint of their integrative role - is not absolute. Within a given natural language L there are many language systems (definable from various standpoints) the distinctions of which are not as great as to classify them outside L. The criteria distinguishing such *'sublanguages'* (of L) are various. Let us mention some of them.

a) 'Languages' of *professional groups'*. In general, we do not say that she who does not understand the scientific jargons of, say, physics, mathematics, psychology, biology, etc. simply does not understand her native language. All the same, some easily constructible logical puzzles can be solved when this case is construed as the case of a partial unfamiliarity with the given L.[31]

b) 'Languages' of *social* or even *ethnic* groups. This is the case of slangs and dialects.

c) 'Languages' of *metaphoric* and *idiomatic* expressions (cf. 'grass widow').

The specific features distinguishing 'sublanguages' from the 'standard language' concern, for the most part, the lexical aspect of L. Sometimes, however, we can observe even some grammatical 'aberrations' (especially in b). Anyway, standard dictionaries are able to fix some specific expressions (including idioms) of particular sublanguages.

The existence of sublanguages of the mentioned kind can explain some cases of homonymy. Yet more must be said to explain 1) the *'weak homonymy'* mentioned in the end of the preceding section, and 2) the phenomenon of *vagueness*.

Ad 1). It certainly happens that one learns to understand a simple expression first so that she immediately identifies the entity denoted, and afterwards so that she connects the same expression with a complex procedure (using other concepts) given by the 'definiens' (or 'explicans') - see Ch.8. (Sometimes this is connected with 'immediate vagueness' first and removing this vagueness afterwards.) To adduce examples: Children may be said to understand the expressions *a circle* or *the Moon* due to their ability to immediately identify the circularity (the set of circles) or the role the Moon plays[32]. Later on, they are taught the exact criteria using other concepts (DIAMETER, DISTANCE, SATELLITE, etc.) Letting aside the factor of original vagueness we have got a scheme explaining the 'weak homonymy'. So, e.g., the entry *circle* in a dictionary (left side) represents the abbreviation: we learn what a circle is in terms of other concepts represented in the right side of the entry. If, however, the expression *circle* occurs in the same dictionary in the right side it does not mean that it again represents an abbreviation: the respective entry can be understood even if only 'immediate knowledge' of the denoted set is assumed.

Ad 2) Let us briefly recapitulate our construal of vagueness (see 7.2.3). An expression which is standardly called *vague* is, actually, 'mildly homonymous': it represents many concepts each of which identifies an entity that is in a sense similar to the other members of this 'family of concepts'. Now the users of the given language L differ *inter se*, among other things, by connecting with particular 'vague expressions' distinct members of the respective 'families of concepts'. What 'looks like red', is a heap,

is a chair, etc. for the user A does not 'look like red', is not a heap (but, e.g., is a chair, too) for the user B, etc., etc.

It would be highly unnatural to say that the 'language' spoken by those users of L who connect all empirical expressions with one and the same concept were a 'sublanguage of L'. In a very broad sense, however, this remark about vagueness belongs into this section. Such 'quasi-sublanguages' would be defined by 'the same degree of membership' ascribed to the identified entities by the users of these quasi-sublanguages.

Intermezzo: Kripke's puzzle about belief.

Another problem which can be classified with a broad interpretation of the term *sublanguages* is the problem of (semantics of) *idiolects.* There are groups of language users or even particular language users who connect some expressions with concepts that are distinct from those ones which are connected with these expressions by the 'standard interpretation'. An example: somebody may understand the expression *morning star* as follows: it is the *star* which is the brightest one in the morning sky. In this case the respective user connects with the above expression a role which is, actually, occupied by another individual than Venus. All the same, this user is probably able to recognize Venus as the bearer of this role but this is possible only due to the fact that the expression *star* 'means' for her the property *being a(ny) celestial body.* So we can say that the 'standard meaning' of *star* is not the same as that one which the given user associates with it.

One aspect of the well-known Kripke's puzzle about belief[33] is connected just with the 'problem of idiolects'. We will show how our conception works when such a problem is formulated.

Let us briefly recapitulate the story known as *Kripke's puzzle.*

A Frenchman Pierre has got in France such an information (mediated by some pictures, etc.) about London that he *sincerely assents* with the sentence

Londres est jolie.

Kripke formulates a '*principle of disquotation'* (let us set aside the other principle, the *principle of translation*, which is not important for us now) according to which we can correctly infer from Pierre's assent that he believes that London is pretty[34]. Later it happens that Pierre moves to London, learns a fragment of English, and since he lives in a highly inattractive part of London he *sincerely assents* to the sentence

London is not pretty.,

which means that - according to the principle of disquotation - that he belives that London is not pretty (which implies that he does not believe that London is pretty). Kripke's question is:

> *Does Pierre, or does he not believe that London (not the city satisfying such and such description but London) is pretty?*

This question - to be answered later - indicates that Kripke *de facto* (not admitting it) *rejects his own principle of disquotation,* and rightly so, since this principle is false, not taking into account the possible distinctions between an idiolect and the 'standard interpretation'. If A assents to a sentence S (sincerely, of course), and if the concept[*] represented by S contains, e.g., the construction ^0X, where ^0X is represented by the expression E (occurring in S), then her assent can be interpreted as believing

'what S says' *only if* A associates with E its 'standard meaning', say, ^{0}X. As soon as A uses an 'idiolect' where E is construed as meaning ^{0}Y, X ≠ Y, the above assent to S *does not justify our conclusion that A believes S.*

This is out first point, which also explains why the story is in principle possible, i.e., not inconsistent. But before we answer Kripke's question (the answer is trivial and could be given just now) we will ask a much more interesting question *what Pierre means when he uses the terms 'Londres', 'London'.* Here we can react to a discussion between Garrett and Putnam[35] which formulates some problems relevant from our viewpoint. Garrett, adducing an example with a child that writes the sentence *Plato was here* on the town fences, writes (p.279, c.d.):

> [w]e cannot refer to anything in speech or in thought and so we cannot have a belief about anything unless we have an identifying knowledge of that thing

and formulates the following principle (p.281):

> [w]e entirely abandon the notion of having a belief about a bare object without any qualification whatever.

This claim is rejected by Putnam (p.415-416):

> If we assume that belief is usually <u>not</u> under a description (contrary to Garrett),...then any description which succeeds in disambiguating the belief being described in the puzzle context will be alright. But to give content to Garrett's view that all belief is <u>really</u> under a description, we would need to know what the 'real' description is. But that won't be easy.

The above controversy is a kind of generalization of Kripke's problem. We can sum up:

> <u>Either</u> our beliefs concern the way the 'bare' objects are identified, and then distinct 'descriptions' of the same objects make us decide which of such competing 'descriptions' is the adequate one (and whether there is any), <u>or</u> our beliefs concern directly the 'bare' objects, and then we can choose any 'successive' description, because all of such 'disambiguating' descriptions are coextensive.

Finally, two relevant subproblems are:

I.   *Do our beliefs concern 'bare objects', or the way they are identified?*

II. *When can we say that two 'ways of identification' (or two 'descriptions') are coextensive?*

Now we will answer I. and II. (confining ourselves to empirical expressions, of course), and thereafter we will come back to Kripke's story.

Ad I.:

We already know - being equipped with our scheme (S), whose justification has been pursuited in the preceding chapters - that what can be at most identified by an expression *via* its meaning is an intension. So we must ask*: What is a 'bare object'*, mentioned in the above controversy? Since that discussion has been inspired by Kripke's puzzle we have to accept that the 'bare object' here is (the individual) London. There are many ways how London can be identified. If, however, we take into account linguistic means only then there are two kinds of the ways how to get London. One of them is direct and connected with the problem of proper names: if the name *London* is supposed to be a 'label' of the concrete object (Kripke would speak about a 'rigid designator') then it functions as a 'direct

way'. On the other hand, the whole story offered by Kripke excludes this possibility - it would lack any sense. Pierre is informed about something due to some identifying procedures he acquired *via* some lecture, pictures, etc. (case *Londres*) or *via* some personal experience (case *London*). We are not informed about the character of these procedures but for our analysis it is sufficient to suppose that it is a *concept* what is associated by Pierre (within his idiolect) with *Londres* and with *London*; for the sake of simplicity we assume that there are two concepts here, say, ^{0}Londres, and ^{0}London. If *Londres* and *London* really were 'proper proper names' then our concepts would identify 'directly' the respective objects - but which ones? Since this is an assumption that is absurd from the viewpoint of Kripke's story we have to suppose that Londres, as well as London are *intensions* given by some 'hidden descriptions'. The pretty town which is associated by Pierre with *Londres* may be the value of Londres/ $\iota_{\tau\omega}$ in some possible worlds (a 'fairy tale town'), the ugly town associated with *London* is the value of London/ $\iota_{\tau\omega}$ in such worlds-times where Pierre's generalization of his fragmentary experience holds. (Notice again that to make sense of Kripke's story we are bound not to take into account the axiological (and vague) character of *pretty*.) In any case, the object whose concept occurs in the belief that

$\lambda w\lambda t\ [^0\text{Pretty}_{wt}\ ^0\text{Londres}_{wt}]$

or

$\lambda w\lambda t\ [^0\neg\ [^0\text{Pretty}_{wt}\ ^0\text{London}_{wt}]]$

is not determined by the concept itself - *the state of the world at the given time point co-determines this object (or its absence).*

So we see that Garrett rightly doubts that 'bare objects' could be identified by our beliefs; at least in the case of empirical expressions (including 'hidden descriptions') believing etc. concerns 'determiners' rather than 'determinees' [36]. (This is obvious in such cases as when somebody believes, for example, that Santa Claus exists.[37] This belief is no nonsense claiming that an individual which does not 'exist' really exists: it concerns the 'office', 'role', viz. an $\iota_{\tau\omega}$-object, and the believer is convinced that this 'office' is occupied.)

Putnam's reaction is essentially connected with his conviction that there is always a set of descriptions which 'succeed in disambiguating' the belief and which are coextensive in that they determine one and the same ('bare') object. Obviously, no co-determination given by the state of the world is supposed. So we come to the question

II.

Consider some examples of seemingly coextensive descriptions:

(25) a) *the capital of Poland*

　　 b) *the largest town in Poland*

(26) a) *my acquaintance called Ortcutt*

　　 b) *the tallest spy* 　　　　　　(Quine)

(Here we do not take into account the fact that the description a) is a pragmatically anchored expression (Def 44).)

Inspecting these examples we can see that the so-called *coextensiveness* of descriptions, if we admit that it is not an empty category, is not a purely semantical category. Semantically, two concepts would be coextensive if they identified one and the same object (see, e.g., Def 16). The concepts represented by 25 a), b), as well as those represented by 26 a),b), are, of course not coextensive in this sense. To see this for 25 we state that neither it is necessary that the capital of Poland were, at the same time, Poland's largest town, nor is it an 'eternal fact' even within the actual world. As for 26, if Quine's acquaintance called Ortcutt really is the tallest spy, we cannot see why it should be so necessarily; besides, also here no 'eternality' even in the actual world can be supposed.

In which rational sense can we claim that the cases a) and b) are coextensive? Clearly, 25 a), b), as well as the artificial assumption 26 a), b) are what has been defined as *coincident expressions* (Def 56): the respective concepts identify intensions (here: $\iota_{\tau\omega}$-objects) whose value *happens to be* the same in the actual world. This interpretation of coextensiveness can be tolerated, even if there is something strange on such a concept: to call two names of functions coextensive if they coincide in the value on one argument is unthinkable unless this argument is the actual world - nobody will call sine and cosine coextensive .

But if the coextensiveness in this sense bears a contingent character then Putnam's belief that all 'successful' descriptions identify one and the same 'bare object' is only an illusion.

So we come to the conclusion that what Pierre means by *Londres* and *London* when he formulates his belief is a respective intension ('individual office') given by a 'hidden description'. To justify his belief (in the actual world) an empirical test is necessary. But very probably no such test has been performed by Pierre - Kripke does not say anything about it.

Then our answer to Kripke's question is easy:

> *Pierre's concepts (i.e., the concepts he associates with Londres and London) do not identify the object which is identified by the concepts associated with these terms by a 'standard interpretation'. The question what is Pierre's belief concerning the 'real London' cannot be answered, since Kripke does not tell us whether Pierre does, or does not have an expression of his own that would be associated with the standard meaning of the terms, and if so, whether he does, or does not believe that the object satisfying this meaning in the actual world is pretty. Briefly: lack of 'input data'.*

8. Definitions

8.1 Tradition

It is not the purpose of this section to repeat again what has been already stated - in so many works - about the traditional doctrine(s) of definitions. Any good textbook of the history of logic, as well as any good Encyclopedia will do.[1] Further, some contemporary views as regards the traditional notions (nominal vs. real definitions, etc.) can be found in the Robinson's book[2] or, e.g., in the well-known monograph by Cohen and Nagel[3].

Here we would like to briefly sum up some most frequent traditional views and globally characterize the shift - as regards definitions - performed by the contemporary logic.

Traditionally - up to now - definitions are construed as linguistic *expressions*. If they possess the explicit equation form A = B, then its left side, called *definiendum,* is a - mostly simple - expression whose meaning should be explained by the right side expression, called *definiens*; this is - or should be - a complex expression consisting of such expressions only the meanings of which are 'known'. This traditional 'explicitly equational' definition has been introduced and studied by Aristotle, and only few modifications of the Aristotelian doctrine can be found in the traditional textbooks. Even the 'rules' of defining, known from Τοπικα, have been always reproduced.[4] What became a topic of controversial views was the way how to epistemologically interpret already this particular form of definition. Not to repeat what has been in details said about it in the historical overviews we at least mention the key global distinction between two conceptions the first of which is characteristic of the 'old' traditional views, and the second of the 'modern' views connected with positivist philosophy and contemporary logic (not only; also some 'historical predecessors' like Hobbes, Pascal, et alii, can be mentioned here).

The first conception can be called *essentialist*[5]; it is this conception which caused Popper's hating definitions[6]. According to it, definitions (of the explicit form) are read 'from left to right': when using a definiendum we need an information about the 'essence' of the entity denoted by it. So speaking about men, justice, triangles, etc. we can obtain an information about the essence of men, justice, triangles, etc. *via* the definiens of the respective definitions. (We could find many typical examples of this view in the Socratian way of discourse, known from Plato's dialogues.) The radical version of this conception means that the definiens brings a new information about that entity whose name has been used: the definitions in this sense would be informative, therefore, perhaps[7], contingent Needless to say that this version of the 'left-to-right' conception, being untenable from the epistemological viewpoint, is no more defended; a more or less strong criticism of it comes from philosophers already in the beginning of New age. There is, however, an 'innocuous' interpretation of the 'left-to-right' reading: if the definiendum is an expression whose meaning is already given (within the respective language) then our definition becomes an 'analytical' definition, which is a 'normal' descriptive sentence that only reproduces (identically or at least equivalently) that definiens which has determined the meaning of the definiendum. Since such a 'definition' - or rather 'pseudo-definition' - is a descriptive sentence it can be, indeed, true or false; the cases where a definition is 'wrong', mistaken, as they are adduced in the traditional textbooks ('too narrow, too broad' definitions, etc.) concern , therefore, just 'analytic definitions'.

The other, 'non-essentialist' conceptions of definition are 'prescriptive' or 'linguistic'[8] - the distinction is not important here. What is important is that the 'right-to-left' reading is what characterizes definitions now. A classical formulation can be found in Russell-Whitehead (*Principia Mathematica VII*, 77, quoted also by Abelson):

> *A definition is a declaration that a certain newly introduced symbol or combination of symbols is*
> *to mean the same as a certain combination of symbols of which the meaning is already known*

Two points should be adduced as comments to this quotation.

a) Clearly, definition becomes, from this viewpoint, an *abbreviation.* It is an imperative, convention, which can be neither true nor false. No meaning of the definiendum is *discovered* (as it was the case in the essentialist theories): the definiendum *gets a meaning* in virtue of a *fiat.* (This interpretation is acceptable even for Popper.)

b) What does a definition prescribe? Obviously, it is the following imperative:

Let the symbol(s) adduced in the left side be associated with the same meaning as is that one possessed by the expression in the right side.

It would be very strange therefore if the following claim did not obtain a 'general consensus':

Definitions are not definable without (some) notion of meaning; the notion of definition is a semantic notion.

All the same, some nominalists seem not to agree: if they distinguish between definitions in formal systems and definitions in the interpreted systems (Quine, Goodman) then it remains only to say as Abelson says:[9]

...from a purely formal standpoint, there is no such thing as definition at all.

Indeed, what syntactic feature would characterize definitions if no semantics is given? Even if the form $A = B_{[C1,...,Cn]}$ were such a distinguishing feature, and even when the identity (or equivalence) sign were given the 'intended interpretation', even then only a syntactic form of sentences possessing the form of identity (equivalence) would be determined. Moreover, neither our explicit definitions can be said to have a fixed syntactic form.[10]

Further, let meaning be what has been introduced by the definitions in 7.1, i.e., let meanings be concepts*/ concepts. Then a definition associates the definiendum with a procedure which identifies (at best) an object; if this object is an intension then the value of it in the actual world is *not* determined by the meaning - *an expression does not possess various meanings in various possible worlds.* So even if it is

not always clear whether in traditional accounts a given definition is assumed to be contingent, to apply only to 'actual objects', or whether they are thought of as being of a more necessary nature, applicable in 'all possible worlds' [11]

in our conception this question is unambiguously answered as above.[12]

Those problems of definition that are developed in the contemporary logic concern definability in formal/interpreted systems (in theories) and in models. Some theoretical questions from this area can be perhaps illuminated by a general theory of concepts/definitions but in general the connection between concepts and definitions should be primarily analysed without taking into account formal systems: definability in the latter is a notion interesting because of the fact that the (s.c. 'extra-logical') constants of the system admit various distinct interpretations; we could say that they admit to be associated with various distinct objects. The semantics of this approach reduces to a denotational semantics, the conceptual level (in our sense) is missing. There is a parallel between the set of primitive constants of a formal system, as well as between introducing a new 'constant' *via* defining 'it' by the means of complex expressions, and saying - as we will do in 8.2 - that a member of the set

of derived concepts (of the given conceptual system) defines an object not identifiable by the particular members of the respective set of primitive concepts, and such parallels could be inspiring for a secondary introduction of the 'conceptual level' into the semantics of formal systems.

Let us now comment the traditional classification of definitions into '*nominal definitions*' and '*real definitions*'. In one respect we can state that introducing the category of 'nominal definitions' (as establishing in one or another way the meaning of a symbol/expression) has been an obvious reaction to the essentialist view (letting the 'real definitions' - analysing the idea represented by the definiendum - play the role ascribed to definitions by the essentialists); on the other hand the distinction is neither very sharp[13] nor very clear. If 'real definitions' were not just identical with definitions in the essentialist sense, then the contraposition of real and nominal definitions is not intelligible. Maybe that some consequences of what will be said in the following sections will illuminate this problem.

8.2 Definability of objects in a conceptual system

We can return to the dialogue presented in Introduction.. Can we really say that what is defined are *objects?*

To positively answer this question we only need to recapitulate some principles that characterize our approach:

1. Concepts - as (sets of) abstract identification procedures - are objective, i.e., they can be attached to expressions as their meanings (Def 29) but otherwise are independent of language. (A 'Bolzanian' principle.)

2. Concepts identify objects (in the sense of Def 31).

3. No simple concept is a member of D(CS) for some conceptual system CS (Def 35).

Now the following definition answers our question:

Def 57 Let S be a conceptual system (of some order). Let C̲ be a member of D(S), and let C̲ identify the object A. If A is not identified by a member of P(S) then C̲ *defines* A.

Examples:

 Some simple cases can be presented if we return to the conceptual systems adduced in 6.1. So the second member of $D(CS_1)$ defines conjunction, the third member implication; the first member of $D(CS_2)$ defines the number 1; the 'inverse division' is defined by the adduced member of $D(CS_3)$; the adduced member of $D(CS_6)$ defines the truth-value **T** (which is denoted by the axiom of induction), etc.

Remark: The last example is all but not intuitive, which is the case of all arithmetical concepts attached to arithmetical sentences. This is obviously one motive for not accepting our general scheme (S) in the case of mathematical expressions[14]: semantics of them reduces for Tichý to denoting (not representing or expressing!) constructions; what is constructed by them is not relevant for semantics of mathematics. Admitting that at least in the case of mathematical sentences this standpoint is far more intuitive than ours I all the same let (S) unchanged. The two reasons thereof are:

a) The purpose of Tichý's emphasis on constructions as on what mathematics is about can be attained by a 'shift of importance': not what is denoted but what is represented is important in mathematics.

b) (A positive argument:) The area of semantics/logical analysis is the whole area of *a priori* relations. Since what mathematical expressions denote according to (S) is denoted *a priori* (as the result of constructing) - as well as the intensions are denoted *a priori* in the case of empirical expressions - I believe that even in the case of mathematical expressions the denoting relation belongs to the area of semantics.

By the way, Def 57 is completely intuitive if the mathematical expression is *not* a sentence: imagine a conceptual system S, where a member of D(S) is a concept identifying the class of prime numbers. No conflict with our intuitions arises if we say that this concept *defines* the class of prime numbers (in S).

So any object can be either immediately identified, or defined: what is the case is relative to conceptual systems.

Remark: Don't forget that using Def 57 we are moving in the 'realm' of ideal (abstract) procedures whose objective character is presupposed by the intuitions characteristic of our approach. Indeed, for a nominalist this definition and the considerations like the above lack any sense. We repeat again that presenting our conception we do not intend to continue the old polemics realism-vs.-nominalism; if nominalists succeed in explaining the nature of numbers or computer programs (without confusing numbers with, say, numerals, and programs with their record or realisation) then much of what has been said here can be accepted even by them.

We can formulate a rather banal ('corollary-like') definition

Def 58 An object A is *definable in* the conceptual system S iff some member of S defines A.

So the *area* of a conceptual system S (Def 39) is the set of objects immediately identified or definable in S.

Exploiting Def 57 we can formulate a trivial claim which, however, can be of some interest if we remember Aristotle's ᶜορισμοσ:

Claim 19 *Every not strictly empty complex concept defines some object in some conceptual system.*

A slight modification of this claim can be realised if the term *definition* is used in a sense inspired by Def 57:

Claim 19' *Every not strictly empty complex concept is a definition in some conceptual system.*

Finally the following modification can be conceived of as a modern interpretation of Aristotle's *horismos* in the sense of a concept=definition:

Claim 19'' *Any definition is a not strictly empty complex concept.*

Notice that this use of the term *definition* is by far not the standard one. Yet it is not certain whether the first Greek ideas concerning concepts and defining were not rather friendly to this conception.[15]

One of the most striking distinctions between the above and the standard one is, of course, that there is no 'definiendum' and 'definiens' here. Indeed, 'definiendum' as well as 'definiens' are linguistic expressions, and there are none in the 'realm of concepts and conceptual systems'.

8.3 Explicit definitions in a 'sublanguage'

Now, when the semantic character of definitions (see 8.1) has been supported by showing (Def 57, Claim 19'') that we can define an 'ontological counterpart' of 'linguistic definitions', and when, at the same time, the idea of sublanguages is intelligible (see 7.4), we can consider the case of 'linguistic definitions', beginning with 'equational definitions' as expressions of a sublanguage having the form

(**DEF**) *Definiendum = Definiens*.

The connection between 'definitions' according to Claim 19'' and 'linguistic definitions' of the form (DEF) can be modelled as follows:

Def 59 Let CS be a conceptual system based on $P(CS) = \{\underline{C}_1,...,\underline{C}_n\}$.

A *language* L_{CS} is a language satisfying following conditions:

a) There are simple expressions in L_{CS} that represent $\underline{C}_1,...,\underline{C}_n$.

b) If E, $E_1,...,E_m$ are expressions that represent constructions X/ $(\alpha\beta_1...\beta_m)$, $X_1/\beta_1,$ ...,X_m/β_m, respectively, and if $[XX_{1...m}]$ is a composition, then there is always a grammatical rule (or a set of such rules) of L_{CS} that makes it possible to create an expression $E_{[XX1...Xm]}$ that represents this composition.

c) If E_X represents the construction X then there is always a grammatical rule (or a set of such rules) that makes it possible to create an expression E_λ that represents an abstraction $[\lambda x_1...x_m X]$.

A language L_{CS} is a language which may not contain any 'linguistic definition'. An artificial hierarchy of languages each of which adds some new simple expressions *via* 'linguistic definitions' can be defined as follows:

Def 60 i) Let L_{CSD0} be $L_{CS.}$

ii) Let L_{CSDi}, $i > 0$, result from L_{CSDi-1} by adding a set D_i of simple expressions $SE_{i1,...}SE_{ik}$, $k > 0$, together with expressions interpreted as true sentences

$(D_{i1)}$ $SE_{i1} = \Phi_{i1}$

.

.

.

(D_{ik}) $SE_{ik} = \Phi_{ik}$

where $\Phi_{i1},...,\Phi_{ik}$ contain only expressions occurring in L_{CSDi-1}. The expressions of the form $(D_{i1}),...,(D_{ik})$ can be called *equational definitions* expressed by the language L_{CSDi}, $SE_{i1},...,SE_{ik}$ are *definienda,* $\Phi_{i1},...,\Phi_{ik}$ are *definiens* expressed by L_{CSDi}.

Clearly, the definienda expressed by a language L_{CSDi} may become parts of $\Phi_{j1},...,\Phi_{jk}$ for $j > i$. So what has been introduced by definition 'on level *i'* becomes a 'self-evident' part of a language 'on level *j*' : thus the well-known chain of definitions arises according to the scheme:

"What does A mean?"

"A is $\Phi(...B...)$."

"Well, but what does B mean?"

"B is $\Phi'(...C...)$."

"O.K. But what is C?"

Etc. etc.

The hierarchy of languages L_{CSDi} represents one possible kind of linguistic changes ('development'); obviously, languages change by far not in this schematic way - 'shifts' of meanings are a phenomenon too frequently occurring during of a development of a language, which is not caught by our scheme. Yet an empirical research of real developmental phenomena of a given language is not the topic of the present study. Our modest task here consists in *illuminating the connection between concepts (conceptual systems) and expressions* in terms of our conception. We can see at least that Def 60 determines *an essential connection between a language and a conceptual system*; all languages belonging to the family L_{CSDi} are based on one and the same conceptual system (CS). And taking into account the way these languages are determined by Def 59 and Def 60 we can see immediately that equational definitions are *non-creative* in the following sense:

Claim 20 Let **the linguistic area of a language L_{CS}** be the area of CS (Def 39). For j > i, the linguistic area of L_{CSDi} = the linguistic area of L_{CSDj}.[16]

<u>Proof</u> is extremely simple. For any $i \geq 0$, the linguistic area of L_{CSDi} is *ex definitione* the area of CS, since by adding (linguistic equational) definitions (and new simple expressions) we do not change the underlying conceptual system.

Def 60 excludes in a most natural way the danger of *circular definitions*: the definiens never contains an expression specific for that level on which the definiens is used as definiens.

Now in the case of s.c. *analytical definitions* (see 8.1) we can speak about 'wrong' and 'right' definitions. The wrong ones can be too 'wide' or too 'narrow' or 'crossing'. Can we formulate analytical definitions (we know that they are no genuine definitions, of course) in our family of languages L_{CSDi} ?

Of course we can. Let

(D) $Se_{im} = \Phi_{im}$

be a linguistic definition according to Def 60. Let

(AN) $Se_{im} = \Psi$

be an equality in L_{CSDi}. There are three possibilities of what kind of expression Ψ is:

a) Ψ is Φ_{im}. Then (AN) can be construed as just the definition (D), but if (AN) is *used in a discourse* (pragmatic dimension!) it may be used as a right analytical definition, simply reproducing (D).[17]

b) Ψ is weakly equivalent to Φ_{im} (Def 55). Then we can say that (AN) is a right analytical definition.

c) Ψ is not equivalent to Φ_{im}. Then (AN) is a wrong (inadequate) analytical definition.

Intermezzo: Simple expressions and analyticity.

Let us compare the notion *logically true* with the notion *analytically true.*

The standard definitions are:

(**LT**)　　A sentence is *logically true* iff it is true in all interpretations that do not differ as for what is associated with 'logical terms' (logical connectives, quantifiers, identity).

(**AT**)　　A sentence is *analytically true* iff it is true in virtue of its meaning only.

A frequent modification of (AT) is

(**AT'**)　　A sentence is *analytically true* iff it is true in all possible worlds.

The difference between (LT) and (AT) or (AT') is what makes logicians a little nervous. Is logic competent to analyse such analytically true sentences which are not logically true?

It has been Quine who had articulated the negative answer to this question, and to rid logic of this task he started a campaigne[18] which had to persuade us that any analyticity transgressing logical truth is wrongly defined because undefinable because essentially vague. Up to the present day whoever dares to suppose that there *is* something like the difference between analytically true and empirically true sentences is suspect: he/she probably did not read that and that; *old-fashioned* is the right expression. It should be already clear by now that the conception offered in the present study should make it possible to save notions like *meaning, analyticity, concept,* etc., for logic, i.e., to say on the abstract level of logical semantics everything what can be said here before to make surely interesting excursions into pragmatics. On the present occasion we will react to the paradigmatic example studied by Quine in his classical *Two dogmas....*

Observe the sentence

(**Bach**)　*Any bachelor is unmarried.*

(Bach) is intuitively taken to be an analytically true sentence. Indeed, understanding the particular expressions making up (Bach) we do not need any empirical support to state that it is true. So what is the problem?

The problem consists in *finding a logical justification, a logical procedure which would validate our intuition* - cf a similar problem in the Intermezzo in 6.2.

Quine knew that a way leading to such a logical justification must exploit the notion of definition. Yet his philosophy of language did not enable him to go this way - his criticism of the 'definitoric way out' is well-known.

We will try to use our approach to clarify the analytic character of (Bach).

The expression *bachelor* is a simple expression. Theoretically, we could construe this expression as representing the simple concept $^0\underline{bachelor}$ (see also 6.2, Intermezzo), i.e., as being an expression of such a language L_{CS} whose underlying conceptual system contains this concept among its primitive concepts. (Adducing our intuitive examples of analyses we have had no other choice than to let simple expressions represent simple concepts - 'didactical reasons', of course.) Yet having introduced Def 60 we can exploit the other - much more plausible - possibility, namely, to suppose that the (sub)language containing the word *bachelor* is some L_{CSDj} for $j > 0$, and that a linguistic definition

(Bach j)　*(a) bachelor = an unmarried man*

is a part of this language. Suppose, further, that for some $i < j$, the definiens of (Bach j) represents the following construction/concept :

(Con Bach *i*) $\lambda w \lambda t \lambda x \, [^0 \wedge \, [^0 \neg \, [^0 \text{Mar}_{wt} \, x]][^0 \text{Man}_{wt} x]]]$.

Now what is the semantics of (Bach *j*)? It cannot be anything other than associating (Con Bach *i*) with the expression *(a) bachelor* as its meaning. *(a) bachelor* becomes an abbreviation for the definiens so that it represents - beginning with the level *j* - the construction/concept (Con Bach *i*). The trouble with the logical support of the intuition that (Bach) is analytically true is caused by the fact that - on our construal - the semantics of *(a) bachelor* ceased to be transparent due to its abbreviational character. Indeed, if (Bach) were analysed as

$\lambda w \lambda t \, [^0 \forall x \, [^0 \supset \, [^0 \text{Bachelor}_{wt} \, x][^0 \neg \, [^0 \text{Mar}_{wt} x]]]$

no 'logical way' of detecting the analytic character of (Bach) could be found. Accepting, however, our semantics of (Bach *j*) we must analyse (Bach) as follows:

$\lambda w \lambda t \, [^0 \forall x \, [^0 \supset \, [^0 \wedge \, [^0 \neg \, [^0 \text{Mar}_{wt} x]][^0 \text{Man}_{wt} x]][^0 \neg \, [^0 \text{Mar}_{wt} x]]]]$

and then, of course, logic says: (Bach) is logically (and, for that reason, analytically) true.

In one respect, this solution is no great discovery; we have seen that Quine supposed that he could get an answer mentioning definitions as tools of regaining the transparency of the semantics of analytically (not logically) true sentences. Yet our conception, showing linguistic definitions as being anchored by complex concepts that define objects (Claim 19), is immune against Quine's criticism: it does not pretend that it serves to investigating the real process of linguistic changes (from the viewpoint of such an investigation such phenomena as introducing definitions are, of course, contingent) - it is simply a semantic explanation of analyticity; no empirical questions are touched here.

Quine was right in 1936 when he construed definitions as merely transforming truths instead of generating them[20]. (He could not have solved the puzzle contained in Carroll's *What the Tortoise said to Achilles*, of course, since trying to explain the 'conventional truths' of logic by looking for infinitely many cases of using various logical items is not the way to explain it; the problems of meaning of 'logical words' are not problems to be solved by behavioristic methods.) But in Quine's later works his aversion to semantics and semantic treatment of, e.g., definitions has been articulated much more explicitly. So in his paper *Vagaries of Definition* [21] he attacks the notion of *real definitions*: he interprets this notion in the Aristotelian sense:

> A **real** definition ... gives the essence of the kind of thing defined. This defining property is part of the essence of each thing of the kind.

We have seen, however, that no essence in the Aristotelian sense is needed to explain the 'real' (i.e., not only verbal) character of definitions.

For Quine, who believes that even such set-theoretical entities as "properties, classes, kinds, and numbers...were conceived in sin", and who, moreover, does not even suspect that there could be such abstract entities as constructions, the only way how to deal with semantic problems is to sell them for "speech dispositions and the behavioral psychology of language learning". Needless to emphasize that the questions of pure semantics, so important w.r.t. logical tractability, are here confused with empirical questions of learning. What Quine has done with semantics and realism never has been a refutation, only a refusal.

8.4 Expressive power

This is a very short section containing, properly speaking, only one definition, which is, moreover, unnecessary:

Def 61 Let L$_{CS}$ be a language in the sense of Def 59. *The expressive power of L$_{CS}$ is the area of CS.*

So the linguistic area of L$_{CS}$ is another name for the expressive power of L$_{CS}$.

Clearly, a partial ordering of (sub)languages as for their expressive power is easily definable, being induced by inclusion of linguistic areas.

8.5 'Implicit definitions'

With the rise of 'formal axiomatics' a following opinion, based on some Hilbert's formulations, became a part of the formalistic dogma, despite the deep polemical remarks made by Frege:[22]

> *The 'extra-logical constants' occurring in a formal ('uninterpreted') axiomatic system do not possess beforehand given meanings but the axiomatic system determines their meanings 'implicitly': it plays the role of an 'implicit definition' of these constants.*

Setting aside the frequently used formulation 'defining constants', i.e., 'defining an expression', which is unacceptable from our viewpoint, we cannot but state that Frege has been right in his refusal of this conception.

If the 'constants' occurring in the given formal system do not represent concepts 'beforehand', and if the system determines - *via* its various interpretations - various meanings of these 'constants' then the latter do not bear the character of constants. So what they represent are not concepts but variables, i.e., open constructions.

As Tichý rightly states, a formal system is, however, connected with a concept. The procedure of obtaining this concept is sufficiently clearly described in [Tichý 1988], although the term 'concept' is not used there. To reformulate it in our terms we proceed as follows:

> *Let a formal system FS consist of axioms $Ax_1,...,Ax_n$, containing 'extra-logical constants $k_1,...,k_m$ of types $\alpha_1,...,\alpha_m$, respectively.[23]. Let $k_1,...,k_m$ be construed as being names of variables, say, $x_1,...,x_m$, respectively, ranging over $\alpha_1,...,\alpha_m$, respectively. Let $Ax'_1,...,Ax'_n$ be universal generalizations of $Ax_1,...,Ax_n$. Any Ax'_i can be associated with a construction, say, C_i, containing at most $x_1,...,x_m$ as free variables. The concept given by FS is then generated by*
>
> *$[\lambda x_1...x_m[^0\wedge C_1[^0\wedge...[^0\wedge C_{n-1}C_n]...]]]$.*

To adduce an example, consider the axioms that are sometimes viewed as 'implicitly defining' 0 and the successor function:

Ax$_1$ $\neg\, x = 0$

Ax$_2$ $x' = y' \supset x = y$

Ax$_3$ $(A(0) \wedge \forall x\,(A(x) \supset A(x'))) \supset \forall x\, A(x)$

There are two 'extra-logical constants' here: 0, associated with the type τ, and ' , associated with $(\tau\tau)$. The 'A' in Ax$_3$ is a free variable 'of order 2' ; by replacing Ax$_3$ with its universal generalization we get the 2nd order version (the categorical one); otherwise no concept would be obtained.

So we get

Ax'$_1$ $\forall x \neg x' = 0$

Ax'$_2$ $\forall xy\,(x' = y' \supset x = y)$

Ax$_3$ $\forall A\,((A(0) \wedge \forall x\,(A(x) \supset A(x'))) \supset \forall x\,A(x))$

Our concept is then generated by $(z \ldots \tau, s \ldots (\tau\tau))$:

$\lambda zs\,[^0\wedge\,[\forall x\,[^0\neg\,[^0=\,[sx]\,z]]][^0\wedge\,[\forall xy\,[^0\supset\,[^0=\,[sx][sy]][^0=\,x\,y]]][\forall A\,[^0\supset\,[^0\wedge[[Az][\forall x\,[^0\supset$

$[Ax][A[sx]]]]][\forall x[Ax]]]]]]$

which constructs an $(o\tau(\tau\tau))$-object, i.e., a relation between such numbers and numerical (monadic) functions that make the construction behind λzs construct TRUE.

In our terms we would say that the conjunction of the above axioms represents the above concept*/concept, and denotes the above relation. No other concept is present, nothing is 'implicitly defined'.

If, on the other hand, the axioms in our example are construed as quasi-euclidean axioms, i.e., as true sentences expressing some properties of zero and of the successor function, then they do not define 0 and Suc (implicitly or explicitly either), since these objects are already supposed to be conceptually given (either as ^{0}0 and ^{0}Suc, or in some other way.)

Comparing formal and quasi-euclidean approach to axiomatics we have to make a brief comment to possible conceptual interpretations of *recursive definitions*. We explain our point on a simple paradigmatic example of defining a *primitive recursive function,* say, adding.

The well-known definition of adding in terms of Peano's arithmetics is:

(+) i) $x + 0 = x$

   ii) $(x + y') = (x + y)'$.

The quasi-euclidean interpretation is simple. We already possess the concepts of adding and of the successor function, and (+) is *no definition at all.* It is only a pair of claims expressing some 'properties' of adding. In our terms we get two constructions, each of them constructs **True**[24]:

(+') i') $[\forall x\,[^0=\,[^0+\,x\,^0 0]\,x]]$

   ii') $[\forall xy\,[^0=\,[^0+\,[^0Sy]][^0S\,[^0+\,x\,y]]]]$.

The formalist view could try to construe (+) as an implicit definition of adding; a thinkable compromise would be that the successor function - like 'the logical words' - would already possess a fix interpretation. Then we get $(f\ldots(\tau\tau\tau); x,y$ must range over natural numbers only - this condition can be explicitly formulated but (+'') would become too complicated):

(+'') $\rceil f[\forall xy\,[^0\wedge\,[^0=\,[fx\,^0 0]\,x]\,[^0=\,[fx\,[^0Sy]]\,[^0S\,[fxy]]]]]$.

We can realize a genuinely formalist view, where even the successor is not identified 'beforehand'. Then we get a concept generalized by $(s\ldots(\tau\tau), f\ldots(\tau\tau\tau))$

(+''') λsf [...],

where the rest in the brackets is as in (+''), only s (as a variable) stands instead of ^0S.

The particular pairs $\langle s_i, f_i \rangle$ that are members of the relation identified by (+''') (type $(o(\tau\tau)(\tau\tau\tau))$)

are what is standardly named 'models' of the given system (if we take into account the whole Peano

axiomatization). With (+''') as a part of the conceptual system underlying Peano's arithmetics we can

also find such pairs $\langle s_i, f_i \rangle$ which correspond to the s.c. *non-standard models*.

The preceding consideration can be generalized:

If what is actually defined by formal systems are 'higher order' relations then what is called 'models'

of such systems are simply members of these relations. What is called 'constants' are (higher order)

variables, and what is called 'interpretation' is a valuation which associates the variables in the open

construction following the λ with objects of the respective types.

8.6 *Explication*

Carnap[25] has been the first who has named and characterized an important and most frequently

practiced linguistic activity which is often confused with defining. He speaks about *explications*, which

have some features in common with ('synthetic') definitions and share some other features with

analytic definitions.

Let L_{CSDi} be a language ('level i') as defined in Def 60. Let Se_{im} be a definiendum, i.e., Se_{im} is an

expression newly introduced on level i but 'normally used' on the possible levels $j > i$.

Now it may happen that Φ_{im} is a vague ('mildly homonymous', see 7.2.3) expression or that the

denoted object (property, relation, etc.) shows some features similar to another object which has been

defined in CS, but there is no simple expression in L_{CSDk}, $i \le k \le j$, which would denote this 'another

object'.

In the first case, Φ_{im} is replaced by Φ'_{im} which denotes some object that is one of the objects 'given

vaguely' by Φ_{im}: Se_{im} becomes homonymous, one of its meanings is now given by Φ'_{im}: we get rid of

vagueness. To explicate in this sense means to make (more) precise. As a hypothetical example might

serve the term 'science', if we compare the phase where its definiens is rather vague, with the phase

where the criteria of 'being scientific' become (more) definite.

In the second case the term denoting an object is 'borrowed' to serve for denoting another, in some

respect similar object (this similarity being often felt as justifying a metaphor). Thus the expression

'energy' is associated with a specific concept in physics in virtue of some connotations connected

with this term in its original sense.

Clearly the two cases need not be disjoint. That they may overlap we can see just on the example with

'energy', which is rather vague in its original sense.

Anyway, we can state that the phenomenon of explication is well accessible to the kind of analysis

stemming from our construal of concepts and definitions.

Remark: If we take into account some 'communicational exchange' between sublanguages based on

distinct (maybe overlapping) conceptual systems then, of course, the case of 'borrowing terms' is not

bound to concern one L_{CSDi} only - just the example with 'energy' is probably the case where a 'sublanguage of physics' borrows a term from a distinct sublanguage.

9. Concluding remarks

9.1 Concepts and objects

In general, we have identified *objects* with *set-theoretical entities.* Here we have to recapitulate some principles of this identification and to stress some points *via* particular remarks.

1. What it is to be a set-theoretical entity? We have enumerated particular cases:

a) Members of atomic types of order 1, i.e., truth-values, individuals, real numbers/time points, possible worlds are set-theoretical entities.

b) Any functions, i.e., members of types $(\alpha\beta_1...\beta_n)$, are set-theoretical entities.

Yet then we must answer the question: Why just the entitis under a), b) are to be called set-theoretical entities? In other words, which criterion results just in the above enumeration?

It is not a trivial task to offer a satisfactory answer. As for a detailed exposition I refer to Tichý's most interesting and important Vienna paper[1] which can inspire logicians (unless they reduce logic to a formal calculus or algebra) to make a principal progress in the area of logical analysis of natural language. The core of the mentioned paper can help us to justify the following answer to the above question:

Set-theoretical entities are simple.

This criterion justifies nearly immediately the point a). Some problems can arise with individuals and possible worlds.

As for individuals, the seeming structuredness of individuals ("Doesn't this table consist of parts?") can be shown to concern (various) *ways of constructing* one and the same individual. (Therefore, which parts belong to the given individual is always ambiguous: Which parts our table consists of depends on the criterion of analysis.) A thorough argumentation in this respect can be found in the Vienna paper by Tichý.

As for possible worlds we sometimes can speak about the 'structures' of particular possible worlds[2]. This is only a metaphoric license: we know that possible worlds are (in our conception) sets of facts, not of things plus relations. Besides, any explication of possible worlds is a pre-theoretical one; within our theory they are unstructured entities (*v*-constructed by variables like *w*).

Let us consider the point b): why can we say that functions are simple? Here we recapitulate the distinction between the 'old' and the contemporary notion of function. The 'old' functions were 'prescriptions of procedures', of 'calculation'; the principle of extensionality could not be applied to them. Today, functions are just mappings: *no 'parts' relevant for one or another way of constructing them can be distinguished on the mapping in question*. Mappings are just sets of various types; the principle of extensionality holds for them, and this is the sense in which they are simple.

2. Concepts and concepts[*].

If *objects* are construed as set-theoretical 'simples' one expects that - contrasting therewith - *concepts* are not set-theoretical entities.

Yet we have seen already that only *concepts*[*] satisfy this expectation. Concepts, being $(o*_n)$-objects, are set-theoretical entities. True, they are objects *sui generis* - they differ from the other $(o*_n)$-objects by the essential fact that every member of $(o*_n)$ not only constructs the same object as any other member but is at the same time a representative of the same (abstract) procedure as any other member. Yet concepts are, after all, sets.

Let us recapitulate, however, our way of finding constructions attached to an expression. Any concept used 'in' the given expression has been represented as any member of it, i.e., as a *construction*; since using a concept C̲ means to be related to the object (if any) identified by it, and any member C of C̲ identifies one and the same object. So *using a concept* is, properly speaking, using a *concept**, i.e., a non-set-theoretical entity. Only where a concept is *mentioned* the respective $(o*_n)$-object is constructed. (We have already suggested that this could be interpreted as a kind of 'rehabilitation' of Frege's claim that a concept in the 'subject position' becomes an 'object', even if 'concept' (*Begriff*) is construed by Frege in a fully distinct way than ours.)

Thus speaking about concepts we can simply 'forget' that they are collections of (closed) constructions (=concepts*), and confirm our intuition that they are *ways* of obtaining entities; as such they are not reducible to 'indifferent' set-theoretical entities, since their parts are inseparable components of them. 'Hyperintensionality' is the result; the essential distinction between modal and intentional contexts (clearly formulated by Bealer) is explained: whereas modal predicates like   and ◊ concern what is constructed, i.e., propositions, the attitude predicates are related to the respective constructions themselves - see the type-theoretical distinction: $(oo_{\tau\omega})$ for modalities, $(o\iota*_n)_{\tau\omega}$ for attitudes (inten*t*ionalities).

So the main hero featuring in our story about concepts is (Tichý's) *construction*: the only kind of entities not being set-theoretical.

One can ask: How is it possible that the notion of construction, so central for solving so many semantic problems, has not been introduced before Tichý?

I cannot help repeating here what has been already explained in 3.1, esp. footnote 6: where E is an expression composed of subexpressions $E_1,...,E_n$ according to the grammar of the given language the standard ('denotational') semantics determines the denotations of $E_1,...,E_n$, as well as of E, but what guarantees that the denotation of E is determined by the denotations of $E_1,...,E_n$ is only the *grammar* of the language (Sluga!), not the sense/meaning of E. Separating the sense/meaning of E from the linguistic level we get just something like constructions: they are composed of the denotations of $E_1,...,E_n$ but it is this *way of giving together* the particular denotations to obtain the resulting denotation what defines a construction (= a concept* in the case of expressions without free variables). It is difficult not to remember Bolzano's distinguishing between a concept and the content of this concept. After Bolzano nobody - perhaps with the exception of Cresswell's 'structured

meanings' - dared to say explicitly that it is not the structure of an *expression* what is primary, that the structuredness can be - primarily! - an attribute of abstract (ideal) entities.

9.2 Concepts and meanings

The present theory tries to rehabilitate the category **concept** as a logical category. By now it is surely clear that our construal of this category contributes, at the same time, to solving the central problem of semantics, viz. the problem of clarifying the category **meaning**, Quine's *b ê te noire*. One of the motives of rejecting this category by Quineans falls off now: one can no more claim that meaning is an 'obscure entity' which cannot be logically represented and handled. The explications given by Def 45, Def 46, Def 49 show that meanings can be construed as **constructions** or **concepts**.

We can be dissatisfied with the way constructions have been defined; we can propose an alternative definition which would perhaps 'cover more cases'; our inspiration could be, e.g., combinatory logic instead of λ-calculus; yet all such proposals will share the main feature: they will result in non-set-theoretical entities (in 'complexes') representable and logically tractable, and, furthermore, recursively distinguishable from set-theoretical entities.

True, the way how to associate particular expressions of a language with such constructions that should represent their meanings remains to be defined in a systematic manner. All the same, even our intuitive attempts at connecting expressions with constructions should support our claim that in principle such a semantic analysis based on the category **construction** is possible. (The thinkable objection that meanings in our conception are again some expressions has been refuted in 3.1.)

The Quinean term 'obscure entity' could be, however, interpreted in a distinct manner. An entity is obscure not (only?) because of the impossibility to logically represent it but because it is not a 'nominalistic entity': for Quine, every abstract entity (with the only exception of sets) is obscure. So when constructions are offered as meanings the price to be paid is to neglect the 'Occam's razor', this favourite tool of nominalists, since constructions are typical abstract entities (being 'abstract procedures').

To react to this kind of objections means to repeat again and again: while Occam warns against *multiplying* entities (NB *'praeter necessitatem'*!) Menger ('Mengers's comb') warns against *omitting* entities (praeter necessitatem, indeed). So before one proves that a notion is not necessary one should not omit it. To omit abstract entities like properties, relations, meanings

(also: numbers!) means to get rid of most important and well tested means of understanding and explaining the world.. I cannot imagine the world which would lack, e.g., properties. I admit that the nominalist - who regards the above abstract entities as having been 'conceived in sin' (Quine!) - cannot imagine 'the realm of abstract entities' because the inhabitants of it, being time- and spaceless, can hardly be 'imagined' as 'communicating' with the ordinary inhabitants of all four dimensions. Neither our realistic nor Quineans' nominalistic intuitions can be proved or disproved.[3] So there remain only two ways to show that my choice is better: first, to show convincingly that my opponents' intuitions lead to such consequences which even (s)he would not like to accept, second, to show that

the class of problems solvable by my opponents is a proper subclass of the class of problems solvable by me.

We will now attempt to demonstrate that at least some problems solvable by means of the category **construction** are not solvable without that category.

9.2.1 Intentionality: the general case

We have seen in Ch.4 that intentional relations ('propositional attitudes', 'notional attitudes') , unless analysed in terms of 'constructional attitudes' - type $(\text{οι}*_n)_{\text{τω}}$ - represent a stumbling-block for semantic analysis. To summarize the reasons thereof we state:

Let an 'attitudinal' sentence possess the form

X Bs Y

where *B* is an attitudinal verb (*believe, know, calculate...*) and *Y* is a sentence in the case of propositional attitudes, and a term (name, description) in the case of notional attitudes. Then there are only two alternatives to our conception (not taking into account some mentalistic interpretations involving 'neural states' etc.):

    i)   Attitudes link individuals with what is denoted by the expression *Y* (truth-values for mathematical sentences, propositions otherwise in the case of propositional attitudes, individuals, numbers, individual roles etc. in the case of notional attitudes).

    ii)   Attitudes link individuals with the expression *Y* itself.

As for ii), we have already referred to Tichý's and Schiffer's criticism of sententialism (which, among other things, concerns also the 'paratactic' form of sententialism invented by Davidson). In my opinion, their arguments are sufficiently convincing, and I do not see why to repeat them. Therefore, let us consider the alternative i) and summarize the reasons for its refusal.

In general, the objects denoted are set-theoretical entities, and as Zalta[4] rightly states,

> *[A]lthough sets may be useful for describing certain structural relationships, they are not the kind thing that would help us to understand the nature of presentation. There is nothing about a set in virtue of which it may be said to present something to us.*

This point is essential. The 'indifferent' character of set-theoretical entities ('simplicity') does not make it possible to use them as **intentional** objects. As Zalta says in the same place about attitude relations,

> *[D]e dicto reports link the subject of the attitude directly with **a way of** conceiving the propositional object.* (My emphasis.)

(True, Zalta - as most semanticists - assumes that attitudes can be construed also as '*de re*' reports, i.e., as direct links to the proposition(al object). This can be perhaps admitted - only in the case of empirical propositional/notional objects - but even this is not certain: if the respective construction ('the way of conceiving') is sufficiently complex then the *de re* interpretation results in our claim that, e.g., *X* does believe *Y*, whereas our question addressed to *X* can be answered "No, I cannot believe something like that". In any case, from our viewpoint there would be no *de dicto* vs. *de re*

interpretation; there would be *two relations* instead, one of them an $(\omega*_n)_{\tau\omega}$-object, i.e., an *intentional* object, the other one an $(\omega\omega_{\tau\omega})_{\tau\omega}$-object, I would call it a *quasi-intentional* object.)

Now our result is:

The sententialist approach to intentionalities is 'too much fine-grained', and it can be shown that its consequences are unacceptable.

The 'denotational' view is coarse-grained, and paradoxes like 'paradox of analysis' and 'paradox of omniscience' are necessary consequences of it.

The only kind of entity standing between expressions and objects denoted are **constructions**. *Thus it is only the 'constructional' view which can be applied to the analysis of intentionalities.*

There remains one kind of objection against my pretension to prove the inevitability of using the category *construction* in solving the problems connected with intentionalities: one can say - you quote, e.g., Bealer and Zalta as authors well aware of the insufficiency of the denotational approach, but just these authors handle the mentioned problems in another way, without using the category *construction*. So does from your criticism of points i), ii) really follow that the only way out must be based on constructions?

As for Zalta, see the next section. Here I will comment once more 'the case Bealer'.

Two points only (and I also refer to 5.1.4.4):

First, Bealer's solution of the paradox of analysis is objectionable - see the note 48 to Ch.5. Also, it is not clear how Bealer would handle the analysis of attitudes to mathematical constructions (e.g., the notional attitude 'calculate').

Second, Bealer's distinction 'thought-building operations' vs. 'condition-building operations' is not definite enough. What Bealer tries to show *via* axiomatization shows at most some features of 'behavior' of both the kinds of operation. The category *construction* enables us, however, to recognize particular tokens of constructions.

9.2.2 'Zalta's case': <u>Existential</u> and existential generalization, Substitution, 'Strong extensionality

Among those works which did not give in to some 'post-analytic' relativist trends there is the remarkable study about the 'metaphysics of intentionality' by E.N.Zalta[4]. Zalta concentrates on four kinds of 'puzzle'which demonstrate the insufficiency of extensionalist views. The four kinds are:

i) *Existential generalization:* a rule according to which a sentence '...D...' infers the sentence 'Some existing thing is such that...'. This rule fails in such cases as 'Sherlock Holmes still inspires modern detectives.'

Remark: Zalta construes 'existing' as meaning simply 'having a location in space' (like Bolzano or, e.g., Meinong). See p.21.

This is a rather strange rule; its failure can be easily explained if we admit such a predicate as 'existing' in Zalta's sense. Much more interesting is the well-known rule of

ii) *existential generalization* according to which from '...D...' we can infer '...something...'. The rule fails (better: 'appears to fail') in cases like 'Ralph believes that the tallest spy is a spy' where it seems that the generalization is unjustified if 'Ralph has no idea who the tallest spy is'.

Before we formulate our standpoint to the way Zalta tries to solve his puzzles we show the way of solving them which is given by our conception.

If Ralph really believes that the tallest spy is a spy but only in virtue of logic, without knowing which individual the tallest spy is then his belief cannot concern a definite individual but an individual role, an $\iota_{\tau\omega}$-object (*whoever plays the role of the tallest spy...*) - so a part of the construction that constructs the 'believed proposition' is the construction of that individual role (in the supposition *de re*). Since variables ranging over constructions of the given order are at our disposal we can construct an existential generalization over one of such variables.

Let us demonstrate this procedure on the adduced particular example:

Let ^0R construct Ralph, ^0B believing; let S(py) be the respective property (an $(o\iota)_{\tau\omega}$-object), Tal/ $(\iota(o\iota))_{\tau\omega}$ be an intension which associates with every world-time a function that selects the tallest individual (if any) from any class of individuals. Further, let $Sub^n/(*_n*_n*_n*_n)$ be the function which, applied to <a construction C, a variable V, a construction C'> returns the construction which is the result of (correctly) substituting C for V in C'.[5] Finally, let $c/*_2$ be a variable ranging over $*_1$. Our response to the problem with existential generalization is that there is no failure here. From the proposition constructed by

$$\lambda w\lambda t\ [^0B_{wt}\ ^0R\ ^0[\lambda w\lambda t[^0S_{wt}\ [\lambda w\lambda t\ [^0Tal_{wt}\ ^0S_{wt}]]_{wt}]]]$$

there follows the proposition constructed by

$$\lambda w\lambda t\ [\exists c\ [^0B_{wt}\ ^0R\ ^0[^0Sub\ c\ ^0c\ ^0[\lambda w\lambda t\ [^0S_{wt}c_{wt}]]]]]^6.$$

iii) *Substitutivity*, according to which we can derive '...D'...' (by substituting D' for at least one occurrence of D) if D is identical to D' (Leibniz's principle). The two counterexamples adduced by Zalta are:

a) From

It is necessary that the teacher of Aristotle is a teacher

and

The teacher of Aristotle is Plato

it does not follow

It is necessary that Plato is a teacher.

b) From

Susie believes that Mark Twain wrote 'Huckleberry Finn'

and

Mark Twain is Samuel Clemens

it does not follow

Susie believes that Samuel Clemens wrote 'Huckleberry Finn'' .

The case a) is solvable already within our denotational semantics: briefly - the necessary character of the first proposition means that the predication of BEING A TEACHER of the teacher of Aristotle holds in *all worlds-times*. It is easy to show *via* our constructions that the conclusion is blocked

therefore. The case b) is more interesting: here we need the apparatus of Tichý's ramified hierarchy (see Ch. 4). Let us use this apparatus to represent the premisses:

$\lambda w \lambda t \ [^0B_{wt} {}^0S \ ^0[\lambda w \lambda t \ [^0Wrt_{wt} {}^0MT \ ^0HF]]]$

$[^0= \ ^0MT \ ^0SC]$

The conclusion seems to be correct since the second premiss states the identity of Mark Twain to Samuel Clemens independently of possible worlds-times.

Now if - as we have assumed and justified - the belief relation links individuals with constructions then the second premiss, stating identity of persons constructed by ^0MT and by ^0SC does not justify the substitution of ^0SC for ^0MT, since an identity concerning individuals is not an identity of constructions (which alone could justify such a substitution).

Remark: In 3.3 we have however stated that for such trivializations like ^{0}catastrophe and ^{0}disaster it holds that they are one and the same construction: a 'direct' identification of this property. If we are ready to admit that even direct identifications of one and the same object can be distinct constructions then our solution of the b) case is simple (see above). Otherwise (i.e., if ^0MT and ^0SC were one and the same construction) a complication arises, similar to that one which we meet if we try to solve the well-known Mates' puzzle of 'chewing' vs. 'masticulating'. Then we assume that a meta-language must be analysed so that such *expressions* of the object language as 'Mark Twain', 'Samuel Clemens' become parts of the universe of our meta-language. The conclusion is then blocked because the above names behave like 'individual offices' - the fact that MT and SC concern one and the same individual is contingent from the viewpoint of our metalanguage.

iv) *Strong Extensionality* is the most interesting case. According to this principle, if a *necessary* equivalence $\forall x \ (Fx \equiv Gx)$ is stated then identity $F = G$ can be derived. A colorful counterexample: It surely holds that necessarily, all and only brown and colorless dogs are barbers who shave just those who don't shave themselves; but it does not hold that being a brown and colorless dog is the same thing as being a barber who shaves just those who don't shave themselves.

This looks like a very convincing counterexample. Nonetheless, it is only a misleading example. The underlying false consideration can be formulated as follows:

Let BCD be the construction of the property *a brown colorless dog*, and RB ('Russell's barber') be the construction of the property *a barber who shaves just those who don't shave themselves.* Both the properties constructed by these constructions are necessarily empty, but to say that they are identical (i.e., that the construction

$[^0= BCD \ RB]$

constructs TRUE is counterintuitive.

In our system this last identity simply holds. If properties are set-theoretical objects determined by their members only then, of course, BCD and RB construct one and the same property. So our problem consists in explaining the reason of the counterintuitive character of this claim.

Solution: Our aversion to refusing the intuitive claim that the mentioned properties are distinct stems from our unclear notions of properties and concepts. Actually, *what is in the above example distinct*

are the respective concepts (or: constructions). The way the empty property is constructed by BCD differs from the way it is constructed by RB. Thus our doubts about the validity of identifying what BCD and RB construct are based on the following *erroneous claim:*

(*)   *In semantics, only what is denoted counts.*

This reduction of semantics to the denotational semantics (see also [Girard 1990]) is what underlies Bar-Hillel's erroneous criticism of Bolzano (see the note 33 to Ch.5).

We have shown that our conception can solve Zalta's main puzzles in virtue of our using the notion CONSTRUCTION. Yet we have promised more: we have claimed that the solution is impossible without this notion, whereas Zalta, not using this notion, does, of course, offer his solutions.

My conviction that Zalta's solutions are at least not as satisfactory as mine is based on the fact that Zalta's main tool - the notion of *abstract (A-) objects* - is not as definite as the notion of constructions. A-objects are introduced in terms of 'encoding' vs.'exemplifying' (Meinongian inspiration), where the semantics of 'encoding' determines the 'behavior' of A-objects rather than defining them (see p.45 of Zalta's book). All the same, the structuredness connected with A-objects reveals something like an anticipation of constructions. So Zalta's solutions are good as far as A-objects can be interpreted as constructions. There is a way to do it when exploiting Zalta's construal of relations as of 'Russellian' structured entities.

An important distinction of Zalta's conception from ours can be seen in Zalta's view that what is *denoted* is structured: see p.209, where Montague's conception is rightly criticized and where Zalta says:

> *The denotations of necessarily equivalent sentences ... have a difference in structure that mirrors the difference in structure of the sentences.*

Another important place is p.183, where the insufficiency of set-theoretical semantics is perfectly characterized:

> *Although sets may be useful for describing certain structural relationships, they are not the kind of thing that would help us to understand the nature of presentation. There is nothing about a set in virtue of which it may be said to present something to us.*

In my opinion, an explicit introduction of constructions could not have been expected from Zalta, for his approach is based on *relations* rather than on *functions*. The following Intermezzo explains this idea.

Intermezzo: Relations vs. functions.

 Our theory of concepts, as well as Tichý's transparent intensional logic (TIL), is based on a *functional approach*. Theories based on predicate logics are essentially based on *relations*. Seemingly this is a fully unimportant distinction, for there is a mutual convertibility of relations and functions: any *n*-place relation can be viewed as a *n*-adic function (type $(o\beta_1...\beta_n)$), and any *n*-adic function is a *n*- or (*n*+1)-place relation. Yet a methodological asymmetry *relations vs. functions* can be stated. Starting with functions we can define a most important operation - one of our *constructions* - namely the *application of a function to its arguments.* This operation is one of the ways which produce from the set-theoretical object called here (and by Bolzano) the *content* (*Inhalt*) of the given concept the

concept itself: the content of the concepts $[^0: {}^03\ {}^02]$ and $[^0: {}^02\ {}^03]$ is the same, viz. $\{^0:, {}^03, {}^02\}$, but the concepts are distinct in virtue of this 'invisible' operation of applying a function to a pair of numbers <3,2>, <2,3>, respectively. The predicate logic (say, the '1ˢᵗ order' one) does not know the construction 'composition' (neither any other), for there is no way of deriving this operation from the relational view. Therefore, the semantics of predicate logics is not satisfactory: wherever we have semantically self-contained functors whose semantics would produce the semantics of the whole expression in virtue of the application of the respective functions to their arguments the predicate logic 'circumvents' this fact by depriving the respective symbols of their real semantic character and by calling them 'improper symbols', interpreting not them but merely a respective context. A typical *denotational semantics* arises which ignores meanings and knows only denotata, i.e., individuals or truth-values. Thus unary and binary connectives cannot be interpreted as (oo)-, (ooo)-objects, and quantifiers as (o(oι))-objects. So let P,Q be interpreted as $\{a,b\}$, $\{a,b,c\}$, respectively. The so interpreted formula

$\forall x\ (\mathrm{P}(x) \supset \mathrm{Q}(x))$

gets inductively interpretation TRUE. The way to this value goes over the 'grammatical' properties of the artificial language of the predicate logic. The respective construction, which *is* the definite way to TRUE, is

$[\forall \lambda x\ [^0{\supset}\ [^0\mathrm{P}x][^0\mathrm{Q}x]]]$:

here the particular components of the 'content', i.e., of $\{^0\forall, {}^0{\supset}, {}^0\mathrm{P}, {}^0\mathrm{Q}\}$, are forced (*via* composition, and closure) to make up a structured complex no more reducible to a set, and explicable as one of the ways to construct TRUE, or, from a distinct vantage point, as the meaning/sense of this interpreted formula. Compositionality is preserved, of course, which is impossible within the standard 'relational' approach.[7]

We could ask now: *Is there really no way to constructions from the relational approach?* After all, Cresswell's tuples should capture structuredness, and they are a kind of set-theoretical objects. Here I cannot than refer to [Tichý 1994], pp.74-80, where a critical standpoint to Cresswell's tuples is argumented. The main general argument is expressed on p.78, where Tichý, analysing the Cresswellian triple <-, 9, 5>, which should represent the complex of subtracting 5 from 9, says:

> *A convention is needed which* **interprets** *the triple as a proxy for the construction of applying the first component...to the other two as arguments. ... The triple merely enumerates the objects of which the construction is composed; it does nor combine those objects into the construction. It is an ersatz of the construction.*

So it is these 'invisible operations' like application (composition), abstraction (closure), and trivialization which can - together with the variables - produce complex entities. And it is the functional approach which - unlike the relational one - leads to this result. Therefore, it is λ-calculus rather than the predicate logics what has inspired TIL.

Notes
Chapter 0

1 Tichý was exasperated by some aspects of 'linguistic turn'; he has formulated his reasons thereof in [Tichý 1988]. In his Vienna lecture he has expressed his optimistic opinion that 'linguistic turn' is no more a dominant trend. See [Tichý 1995].

2 Tichý's aversion to non-classical logics was very radical. As we can read in his book he even connected them with the well observable growth of relativistic and sometimes irrationalist trends in the 'post-analytic' philosophy; in this respect he did not hesitate to accuse especially paraconsistent logics of a potential moral support of totalitarian régimes.

3. See, e.g., [Materna 1984], [Materna 1992], [Materna 1995], [Materna 1997].

4. See Wang's testimony in [Wang 1993].

5 See Ch.1, where the importance of the respective note formulated in [Church 1956],p. 6, is stressed.

6 There is an apt metaphor in [Tichý 1988], where Tichý speaks about the distinction between an itinerary and the target place of it. The places (towns or alike) which define an itinerary are, of course, parts of it, and they are, of course, no parts of the target place. Analogically, the 'intellectual steps' leading to an object (from the viewpoint of semantics) are not parts of the latter, but they define the respective 'ways' to the object. Needless to emphasize that there are infinitely many ways leading to one and the same object.

7 We can state that all Quine's works - from his [Quine 1953] till [Quine 1990] consistently realize Quine's behavioristic and holistic *idée fixe* about the possibility of replacing the semantic abstraction leading to the notion of meaning by a pragmatic theory of stimulus-reaction as explaining the semantic phenomena. We will return to Quine more often later.

8 I mean especially [Wittgenstein 1958]; there are thousands of possible quotations documenting Wittgenstein's illusion that the processes of learning and using language are the only subject matter of any reasonable theory of language.

9 It is characteristic of Quine's frequent attempts to exploit this seeming circularity (first formulated in [Quine 1953] for his own replacement of semantics by pragmatics that while he tries to analyse the notions of synonymy and analyticity, the 'third' (or 'first'?) member of the 'circle', the notion of *meaning*, is for him anathema (as an 'obscure entity') from the very beginning.

10   This is very convincingly shown in [Tichý 1992].

11   Some interesting criteria of the scientific value of a logic-semantic theory are adduced in [Bealer 1982]. The ability to solve the problems adduced there is, of course, one of the most important criteria. A misleading interpretation of this criterion would be, however,if particular *ad hoc* 'solutions' to particular problems were offered. These solutions have to follow from one conception. Another example of the strategy of solving particular problems by the means of one consistent conception can be found in [Zalta 1988].

12   A (not exhausting) outline of the history can be found in [Weitz 1988]. Particular pieces of information are, of course, contained in some general historical works about logic, as well as in good Encyclopedias.

13   See [Tichý 1988], p.14.

14   As I see it, if the idea of constructions had been understood and accepted, TIL would have become a most important part of the 'mainstream' in (philosophical) logic.

15   [Tichý 1988], p.66.

16   See [Cresswell 1985].

17   I mean [Materna 1995a]. If I am allowed to boast a little, the book has got a general positive acceptance among our readers. Also the 'official' reviews have stated that there are inspirative ideas contained in it.

Chapter 1

1   In [Church 1956].

2   Frege's conception, known especially from [Frege 1891], [Frege 1892], [Frege 1892a], and from many places in [Frege 1971], is a 'modern' version of the traditional set-theoretical conception. See 5.1.4.3.

3   As for more details, especially for a comparison with Bolzano, see 5.1.4.

4   It is of no use to reproduce here the list of all articles and monographs referring to this problem. Dummett's [1981, 1981a] are perhaps the most frequently quoted sources; some of his interpretations are critically met in [Tichý 1988], which I think is - without pretending to be a historical analysis - the deepest insight into the fundamental questions connected with Frege's problem. In a concentrated

form, one of the most important objections to the way Frege himself had tried to solve this problem is contained in [Tichý 1992], where 'Frege's Thesis', according to which "the meaning of a term" is "not ... what is connected with it by linguistic fiat, but rather the object which is presented thereby" is subjected to a principal criticism.

5   This fact makes the 'situation semantics' (Barwise) rather a part of pragmatics: Be a situation defined (explicated) in an arbitrary way, it always presupposes a speech-act rather than an expression only. On the other hand, selling possible worlds for situations is obviously connected with the right suspicion that possible worlds, as non-structured entities, are too a coarse-grained tool for explaining, e.g., intentional relations. See [Barwise,Perry 1983].

6   This *a priori* is connected with the fundamental assumption characteristic of semantics: that the language (or: a temporal stage of it) is *given*, and that, therefore, a linguistic convention has already determined the meanings of particular expressions (*via* lexicon and grammar). If, on the other hand, a language is viewed as a (social) phenomenon to be studied as for its properties and relations, then we do not entertain semantics, and then, of course, we have to state that the fact that particular expressions mean just what they mean is fully contingent.

7   Paradoxically, it is Quine himself who has made this clear *via* his thought experiments proving 'indetermionacy of translation'. See, e.g., [Quine 1960]. Unfortunately, it seems that Quine does not draw the same consequences from this indeterminacy thesis as I do here.

8   One of the first papers working with these 'indices' is Montague's [1974].

9   Tichý uses the term 'individual office'; see his book. Even in the German translation of one of his drafts in [Tichý 1987] we find the term 'Amt'.

10   It should be translated as *meaning* (see [Frege 1988]). We will see that Frege's *Bedeutung* will be redefined and called 'object (denoted)' here. The English 'meaning' rather corresponds to the Fregean *Sinn*; no 'official' distinction between 'sense' and 'meaning' can be stated. The term 'reference' could be used as a pragmatic term - see 7.1.

11   Church's 'slingshot argument', reproduced briefly in his [1956], p.25, is, of course, wrong. This is clear as soon as we understand Tichý's refutation of Frege's Thesis. To replace 'the number of Waverley novels' by 'the number of counties in Utah' is simply *not* an equivalent transformation: that this number is the same is a matter of contingent facts. 29 is not the object *denoted* by these two expressions - it is their *reference* for example in the actual world-time. No substitution works.

12   See [Frege 1971], p.26: here we have concisely and unambiguously:

Der Begriff ist...eine funktion eines Argumentes, deren Wert immer ein Wahrheitswert ist.
(At the same time, we observe that only one-place predicates can count as concepts.)

13   A more detailed analysis of this criterion from the viewpoint of TIL can be found in [Materna 1995a].

14   Church's terminology in this respect is introduced in [Church 1956].

15   See his book, where, however, the term *(ι-)determiner* is used promiscue. Where α is an arbitrary type, the entities which are usually called *intensions* (with values in α) are what Tichý calls *α-determiners* (or *α-offices*). From this viewpoint, for example properties of individuals are (οι)-determiners, propositions are o-determiners, etc.

16   From the English translation, i.e., from [Frege 1988], p.57.

17   An impatient Reader can contemplate at least what Tichý writes on pp.98,99 in his book, and compare his scheme with our (S).

18   In [1956] but also in [1985], where Church emphasizes some modifications.

19   This equation is based on Church's formulations:
   *Of the sense we say that it **determines** the denotation, or **is a concept** of the denotation.*
   ([1956], p.6), and
   ...anything which is capable of being the sense of some name in some language, actual or possible, is a concept
   ([1985], p.41)
This second quotation makes it clear that for Church - as well as for, say, Bolzano and Frege - concepts are independent of a particular language: they are only *potential* or *possible* senses of expressions.

20   Better: by a monadic predicate - see Note 12.

 21   More can be found in [Church 1985]. In [Tichý 1988] Church's conception is critically analysed in Ch.10.

Chapter 2

1   See [Aho 1994], p.234.

2   [Frege 1918].

3   [Tichý 1986], p.256.

4   [Bealer 1982], p.31.

5   This distinction is convincingly characterized in [Bealer 1982]. See 5.1.4.4.

6   In a sense mathematical sentences (and, in general, expressions) are (pseudo-)ambiguous. They may be construed either as denoting truth-values, or as denoting 'constant' propositions. It will be later clear that this fact does not violate the above principle. In any case, this (pseudo-)ambiguity is not context dependent.

7   Unlike Montague, we do not conceive of intensions/extensions as if they were intensions *of* expressions/extensions *of* expressions. We do not say, e.g., that E10a *has* an intension and an extension; instead we say in this case that E10a denotes an extension (and E10b denotes an intension).

8   See [Kripke 1979] and [Tichý 1988], p.261-270.

9   See [Tichý 1987].

10   Our 'naive' notion of *set* is independent of a particular axiomatization. Therefore, no distinction between sets and classes is assumed. Tichý uses - among others - the neutral term 'collection'.

11   Here we do not take into account the type-theoretical polymorphism obviously connected with identity. For our purposes here it is irrelevant.

12   It seems that many authors do not see this fundamental semantic distinction between a) and b), and pose therefore a 'problem', for example: If it is necessary that 9 is odd (which surely is necessary), and if E11b = 9, why does it not hold that E11b is necessarily odd? To this Quine's favourite example we will return later.

13   [Tichý 1988].

14   Therefore, our attitude to Montague's philosophy will be a strongly critical one.

15   If this assumption is justified our contrasting E7a and E7b works. Otherwise we are probably bound to admit that any individual is (linguistically) given only *via* a (hidden or explicit) description. See 2.3.

16   This principle is formulated in [Janssen 1986], p.29.

17   [Bealer 1982].

18   See, e.g., Quine's introductory comments to [Russell 1908 (1969)]. Quine's and Russell's opinion that no ramified hierarchy is necessary due to the 'principle of reducibility' can be opposed by simply showing that indeed this principle is "natural and obvious" ([Tichý 1988], 226), but that, at the same time, entities of higher order are necessary. See Ch.4.

19   As for Church's exploitation of types for semantics see especially [Church 1951] and the 'rectified version' thereof in [Tichý 1988]. See also [Church 1940].

20   See [Montague 1974].

21   In the s.c. 'two-sorted' systems *s* gets a self-contained interpretation (see [Gallin 1975]). In this general sense TIL could be classified with two-sorted systems.

22   See especially [Tichý 1994].

23   More will be said in 3.1.

24   [Tichý 1982], 52-53.

25   Cf. the distinction between *function* in the old sense, and *function* in the contemporary sense. See [Tichý 1988], 2-4.

26   See 5.5. for some interesting consequences.

27   A good piece of history is contained in [Gamut 1991].

28   See, e.g., [Stalnaker 1985], [Stalnaker 1986], [Lewis 1995], [Peregrin 1995].

29   Such a characteristics can be found in [Tichý 1988], 177-185. I have never read such a thorough exposition and such a convincing argumentation as there. The present text only briefly summarizes and comments what has been said in the mentioned section of Tichý's book.

30   See [Wiener 1951]

31   [Wittgenstein 1922] . A similar conception can be derived (using some modifications) from Hintikka's works. Among the numerous Hintikka's writings see, e.g., [Hintikka 1969], [Hintikka 1975].

32   Indeed, if *object* is understood in a general sense (see our scheme S) then even facts are objects *sui generis.*

33   These two points are what Hintikka's conception is based upon. See, however, [Hintikka 1975].

34   This has been the first time shown in [Tichý 1972].

35   We adduce a most simple and intuitive explication of *fact*. More complicated explications are possible - see [Kolář,Materna 1993].

36   ...and 'absolute': no deep considerations which try to apply theory of relativity or the 'discrete time' hypothesis will be taken into account here. Even if somebody attempts to do something like that nothing except partial modifications of the present theory will be achieved. This is not to say that it would be uninteresting.

37   The most important places in TIL concerning individuals are [Tichý 1987] and [Tichý 1995].

38   For only one famous example see the classical philosophico-linguistic study [Strawson 1959].

39   See, e.g., [Kripke 1979a] and a critical comment in [Tichý 1988], p. 261-270.

40   Kripke would say, of course, that *Venus* - unlike *the morning star* - is a 'rigid designator' (see [Kripke 1979]). His distinguishing between rigid and non-rigid designators is, however, incompatible with TIL: see [Tichý 1986], [Tichý 1994].

41   For example [Hintikka 1972].

42   If we insisted on the hypothesis that *Pegasus* denotes an individual then a contradiction would be provable. For let P be the individual constant representing *Pegasus*. The true empirical claim

<center>*Pegasus does not exist*</center>

gets the 1st order analysis

$$\neg \exists x \, (x = P)$$

which, together with the tautology

$$\exists x \, (x = P)$$

(derivable from $P = P$) creates a contradiction.

Indeed, existence can be distinguished here from the existential quantifier (as Meinong and Zalta would do - see [Zalta 1988]) - but even in this case we have to construe Pegasus as a role only unless we want to get an absurd Meinongian universe. (Zalta is not convinced that the Meinongian universe is that absurd; he, however, could hardly reject the well-known criticism of Meinong - see also [Tichý 1987] - if he did not define existence in another way than it is usually conceived of in semantics.) See Ch.9.

43   [Tichý 1988], 199.

44   This reminds us of Carnap's *states of description* in [Carnap 1947], which, however, have been defined on the linguistic level only.

45   Neither can be possible worlds identified with models of formal systems.

46   See Hintikka's notion of *epistemically possible worlds* in [Hintikka 1975], [Hintikka 1975a].

47   [Kripke 1963].

48   [Tichý 1988], 278-279.

49   [Tichý 1988], 194-200.

50   Mathematicians often use the term *intension* on a fully different sense. Later we will see that their *intension* could correspond to our *construction*.

51   By *expression* we mean *expression of a natural language.*

52   Here we do not take into account indexicals. Indexicals (and still other expressions) connect expressions with pragmatic moments. Let a speaker and a situation be given: then the respective indexicals can be replaced by non-indexical determiners. This is not to say that a theory of indexicals should be uninteresting. See, e.g., [Almog,Perry, Wettstein 1989]. See also 7.1.

53   Unless *Aristotle* is a hidden description.

54   The polymorph character of identity is obvious; α is an arbitrary type.

55   This is thoroughly explained in [Tichý 1978].

56   Carnap would say that the mentioned sentences are L-equivalent. [Carnap 1947].

57   This example shows, in general, that intentional relations cannot concern simply intensions. We have seen already that Carnap has registered this important fact. That intensions cannot be *meanings* can be recognized even without referring to intentionalities - see [Lewis 1972].

58   See [Tichý 1972].

59   Another support for our view that we cannot 'discover' the actual value of an intension by some linguistic trick. A nice example of an illusion of this kind can be found in Kaplan's *dthat*. (See [Almog, Perry, Wettstein 1989]. Kaplan's illusion can be formulated as follows: *To make some function do what it cannot do it suffices to define a 'miraculous operator' which achieves this goal 'con forza'.*

Using a parody, it is as if we were not content with the brute fact that dividing by zero does not return any number, and tried to 'correct' this fact by defining an 'operator', say, *Divz*, which, applied to any number *n*, should result in returning some definite number as the outcome of dividing *n* by zero. A definite description denotes (*via* a respective concept) an $\iota_{\tau\omega}$-object. The possible value of this function in the actual world(-time) is nothing what semantics could determine: it cannot do it since semantics cannot determine which of the possible worlds is the actual one. So Kaplan's *dthat* tries to do something what semantics alone cannot do and what is done by the state of the world or, epistemologically, by experience.

60   We do not take into account Henkin's and Hintikka's 'branching quantifiers' here.

Chapter 3

1   See [Cresswell 1985].

2   [Carnap 1947].

3   In [Tichý 1988], 8-9, this Carnap's notion is critically analysed.

4   [Girard 1990].

5   [Tichý 1988], 7.

6   That such a 'linguistic' conception of meaning is really taken as being standard can be seen, e.g., in a very explicit formulation in [Sluga 1986].

7   For Bolzano this idea was self-evident. See [Bolzano 1837].

8   In most of his works. See especially [Tichý 1986, 1988, 1992, 1995].

9   [Tichý 1988], 1.

10 [Tichý 1988].

11   Doing without variables is not in principle impossible. Such an alternative theory of constructions would be obviously inspired by Curry's combinatory logic (see, e.g., [Curry 1958]) rather than by λ-calculi.

12 See Ch.9, Intermezzo ('Relations and functions').

13 See, e.g., [Barendregt 1981].

14 [Tichý 1988], 63-64.

15 See the nearly complete bibliography of P.Tichý in the journal *From the Logical Point of View* III, 2 (1994), Institute of Philosophy of the Academy of Sciences of Czech Repuiblic, Prague.

16   [Tarski 1956].

17 [Tichý 1988], 56-62.

18 For obvious reasons, as regards the atomic types of our base only (beginnings of) o-sequences and τ-sequences can be explicitly exemplified. This does not mean, of course, that ι- and ω-sequences do not exist. Only 'technical reasons' prevent us from explicitly illustrating such sequences.

19 In the 1st phase of TIL (i.e., before [Tichý 1988]) objects were conceived of as constructions *sui generis*. This proved, later on, to be not very intuitive (see the main text), and, moreover, not satisfactory with regards to the transition to the ramified hierarchy.

20 Interesting 'bridges' connecting concepts and 'possessing concepts' have been built in [Peacocke 1992].

21 A similar example in another context adduces G.Bealer (in a draft).

22 Bolzano would speak about distinct concepts of 2. See [Bolzano 1837].

23   (unless we take into account Tichý's 'executions').

24   See [Tichý 1988], 2-3.

25   [Tichý 1978], 275.

26   [Bealer 1982].

27   [Tichý 1994].

28   [Tichý 1992].

29   [Tichý 1978a].

Chapter 4

1   In Def 1 the base is construed as consisting of classes of any objects. Yet as soon as we have determined our specific base as $\{o,\iota,\tau,\omega\}$, there is no possibility to ascribe types to constructions.

2   Neither this is the right solution (see Ch.5) but it approximates the right one in a satisfactory manner.

3   [Tichý 1988], 66.

4   There are, of course, infinitely many relations that are called *containment* in an analogical sense. This class is given by the type scheme $(o *_n *_n)$.

5   As for the history of this problem see [Lenzen 1978], [Aho 1994].

6   [Frege 1892 a].

7   In [Tichý 1988], 12-14, [Schiffer 1987], [Bealer 1982].

8   Explicitly, e.g., in [Thomason 1974].

9   In [Thomason 1974], [Materna 1997].

10   The author defended this dualism in [Materna 1984].

11   For a paradigmatic example see the classical [Kleene 1952] and Tichý's critical commentary in [Tichý 1988], 92-97.

12   See [Duží 1995].

13   See [Materna 1997]. The aversion of many logicians to higher order types is, of course, well-known. Here the philosophical 'parsimony' ([Tichý 1995]) characteristic of the contemporary nominalists is guilty.

14   See [Almog,Perry,Wettstein 1989].

15   Bealer's attempt in [Bealer 1982] seems to refute this claim, but see [Tichý 1988], 133, 285. The fact that our approach is a type-theoretical one and that the presented hierarchy is ramified causes that we can hardly compare our theory with some other surely interesting theories. This concerns not only Bealer but also [Zalta 1988] or [Parsons 1980]. As for Zalta, some comparison is made in Ch.9. As regards Parsons, I think that his attempt to solve the problems with 'nonexistent objects' is too artificial (and unnecessarily complicated) in virtue of his not exploiting the type-theoretical approach, so that he has to introduce the not very clear distinction 'nuclear' vs. 'extranuclear' predicates; besides, he ignores the distinction 'extension' vs. 'intension' .

Chapter 5

1   To some points in this 'historical respect' we will return later. See at least [Weitz 1988], [Peacocke 1992], [Schock 1969], [Kauppi 1967], [Palomaki 1994], and many special studies. Nevertheless, we can read statements like

> *Weitz was astonished to discover that while practically every modern philosopher talks about concepts, and almost all contemporary students of philosophy take for granted that some use of concepts and some theory of concepts is present in every philosophical system, hardly anyone had paid serious attention to a number of key questions.*

(Marvin Fox in his Foreword to [Weitz 1988].)

2   [Carnap 1947].

3   It can be interesting to test our explication against Bealer's requirements in [Bealer 1982]. Some of his criteria I do not respect, however; for example I cannot confine my analyses to first order.

4   See [Weitz 1988], 54-55.

5   [Weitz 1988], 214.

6   [Tichý 1979]. Being spatio-temporal is another property that is sometimes identified with existence (probably Bolzano, explicitly Zalta).

7   [Smith 1989].

8   [Jackendoff 1989].

9   Hobbes, according to [Weitz 1988], 108.

10   Mill in [Mill 1865].

11   Quine's first important attack came in [Quine 1953].

12   Very often, but see especially [Tichý 1988], 12-14.

13   [Hintikka 1979], esp. 18-19.

14   [Hao Wang 1993].

15   [Russell 1959]

16   Expressions are also abstract, of course, but particular occurrences of expressions ('tokens') are not abstract (which obviously is the source of nominalist illusions). On the other hand, there is nothing like 'occurrence of a concept'.

17   See a good formulation in [Bogdan 1989], 17:
    What is the business of concepts? To pick up relevant and useful properties of the environment.

18   Independently of the definition of concepts we will use capitals to mention particular concepts.

19   There are exceptions, of course, especially a) interjections, b) expressions playing a purely syntactic role in the given language.

20   Among other authors especially Bolzano has explicitly (and w.r.t. the year 1837 very clearly) distinguished between non-empirically and empirically empty concepts.

21   See [Weitz 1988].

22   [Edwards 1967].

23   For example a relatively valuable textbook [Ziehen 1920]. Notice the psychologistic features especially in the chapters concerning concepts.

24   Not only Germans; see also [Mill 1878].

25   Notice that propositions - 1st order objects - can be true or false, but a concept of a proposition is neither true nor false.

26   See, e.g., [Edwards 1967], [Ritter 1971]. This tradition has been petrified makinly by I.Kant, and it has been explicitly criticized in [Bolzano 1837]. See also 5.1.4.2.

27   See for example [Edwards 1967], where P.L.Heath in the chapter *Concept* says, besides, that the notion of concept is

   a passkey through the labyrinths represented by the theory of meaning, the theory of thinking, and the theory of being. Logic, epistemology, and metaphysics have all accordingly found use for it in one capacity or another.

28   See A.N.Prior , *Traditional Logic*, in [Edwards 1967], or R.Kauppi in [Ritter 1971]. The history of these categories can be prolonged to Aristotle, of course.

29   Cf. Porphyrius' tree.

30   In [Bolzano 1837].

31   *Ibidem*, §66.

32   *Ibidem*, p.244.

33   Bolzano's 'structure-friendly' construal of concepts is his original contribution to the theory of concepts. Unfortunately, neither this part of his doctrine has been appreciated or even understood up to our days. This is best seen from the following example: In [Bolzano 1837], §148, Bolzano distinguishes between the concept, say, TRIANGLE$_1$, as defined in terms of having three sides, and the concept, say, TRIANGLE$_2$, as defined in terms of having the sum of its angles equal to 2R. Now Bar-Hillel (a famous logician!) says about Bolzano's reasoning:

> [i]ts uncritical acceptance may lead to strange, even contradictory formulations. ...the two occurrences of the word 'triangle'...though differently defined, express both the property Triangle, have the property Triangle as their intension, so that the property Triangle is different from the property Triangle. [Bar-Hillel 1950].

If Bolzano used our terminology he would, of course, reply along the following lines:

> I do not speak about the **property** (being a) triangle: I speak about the **concepts** TRIANGLE$_1$, TRIANGLE$_2$: these are mutually distinct, for - taking into account the respective definitions - they possess distinct structures.

34   Bolzano's criticism is, therefore, unfair. He works with other notions than the criticized law.

35   As for *Saetze an sich* this has been explicitly stated in [Sebestik 1992], 124.

36   This is documented by every Frege's work where Frege speaks about concepts.

37   In [Frege 1892] Frege says that this explanation is not meant as a proper definition.

38   See [Church 1956], 6.

39   To the problem of 'non-saturatedness' of functions see the excellent analysis in [Tichý 1988].

40   See not only the well-known study [Frege 1891] but also [Frege 1971].

41   This follows from his construal of concepts as what is denoted by a predicate.

42   [Church 1956], 4.

43   *Ibidem,*, e.g. 7.

44   [Frege 1988], p.22

45   [Bealer 1982].

46 Some 'friendly' guessings in this respect can be ascribed to Carnap and Church
('intensional isomorphism'), Cresswell ('hyperintensionality', 'structured meanings'), and perhaps
some others. An exact system of definitions can be found only in TIL.

47 ...but not the 'Cambridge properties', i.e., non-natural, artificial, logically intractable properties
like *grue*, etc.

48 One of the objections (the most general one) is formulated in [Tichý 1988], p.144:
[b]y putting (double) inverted commas around oblique occurrences of 'the author of Waverley' we
remove the ambiguity, but only at the cost of notationally obliterating the logical relation they
bear to the regular occurrences of the phrase. And, needless to say, replacing the inverted
commas with some other notational device (like Montague's hat, Grossman's diamond, or
*Bealer's brackets) can make little difference. (*Emphasis mine.)

49 [Kauppi 1967].

50 *Ibidem,* 9-10.

51 *Ibidem,* 31-32.

52 This is a little surprising: we know that Kauppi has been an expert as regards Leibniz's
philosophy.

53 This does not mean that Kauppi's analysis does not add new (and often even surprising) results to
what has been standardly taught. Cf. her results concerning the (intensional) product, sum , negation,
etc.

54 [Palomäki 1994].

55 The term '*denotational semantics'* is used in this sense in [Girard 1990].

56 [Schock 1969].

57 See [Palomäki 1994], 63.

58 *Ibidem.*

59 See [Tichý 1988], Section 50 ('Formal axiomatics').

60   A very conspicuous description of the respective history is in [Weibel,KÖhler 1986].

61   [Wang 1993]; especially 43-44.

62   [GÖdel 1990]. Godel construes here concepts as "*the properties and relations of things existing independently of our definitions and constructions*". (p.128) About the distinction between the intuitionistic 'notion' and what he names 'concept' Godel says:

   [a]ny two different definitions of the form α(x) = φ(x) can be assumed to define two different notions α in the constructivistic sense.

It is interesting that just this connection with 'definability' has led G.Bealer to construing concepts as mirroring the fine-grained structure of definitions, whereas for Godel the concept is independent of the way the given object is defined; so Godel's concept is a set-theoretical object.

63   *Ibidem.*

64   We repeat that first order objects are what has been defined in Def 4; so it should be clear that, e.g., a set of relations between sets of individuals - for example   an $(o(o(o\iota)(o\iota)(o\iota)))$-object - is, from this point of view, still a first order object.

65   See also [Sebestik 1992].

66   [Peacocke 1992].

67   [Tichý 1972, 1978].

68   As for the term 'requisite', see [Tichý 1979].

69   For Bolzano, empty concepts arise in virtue of combining various concepts ([Bolzano 1837], §66.

70   [Bealer 1982]; Bealer says also (referring to his own conception of logic):
   [t]he conception of logic that emerges is very far indeed from that of Aristotle, for whom logic is primarily a tool; instead, it is more like that of Plato, for whom logic is akin to reason itself. (p.221).

71   The problems with vagueness (fuzziness) will be briefly mentioned later, see 7.2.3.

72   In [Materna 1992] *Quid*[n] included also 'γ-equivalence' that linked C with

$$^0]x\ [^0= x\ C]$$

but this notion is incompatible with an important generalization of Claim 11:

the content of the above concept[*] contains concepts[*] [0]] and [0]=, not included in the content of C.

73   See [Tichý 1978].

74   See [Duží, Materna 1995]

75   In [Frege 1892].

76   A thorough analysis of the problem of 'unsaturatedness' can be found in [Tichý 1988].

Chapter 6

1   See [Martin 1967], esp. 25 ...

2   [Kauppi 1967].

3   Needless to say that not all pairs of mutually dependent concepts are pairs of *equivalent* concepts. Trivial counterexamples can be easily found.

4   This term is used and explained in [Schlesinger 1991]

5   An analogous, although a little more complicated argumentation could be applied to Schlesinger's example.

Chapter 7

1   [Quine 1953].

2   [Quine 1990].

3   For example [Wittgenstein 1958].

4   [Quine 1990].

5   See [Quine 1953], esp. *On What There Is*, *Two Dogmas of Empiricismus, Meaning in Linguistics.*

6   *On What There Is,* 12.

7   [Lewis 1985], [Stalnaker 1972]

8   In [Svoboda, Materna, Pala 1976, 1979] we have called this part characterized by Stalnaker *internal pragmatics.*

9   [Tichý 1995].

10   See [Schiffer 1987]

11   See [Donellan 1985], [Stalnaker 1972]

12   Here we set aside the phenomena of homonymy and vagueness; See 7.2.1, 7.2.3.

13   See [Kripke 1979]

14   How many articles and even books have been written that have tried to give a general answer to the question *What are the things called meanings*? See*, e.g.* [Schiffer 1987], [Dummett 1993], [Kripke 1963], etc..Yet not too far went the authors from the classical Quine. See also [Almog, Perry, Wettstein 1989].

15   See [Almog, Perry, Wettstein 1989]-

16   There have been many attempts at formulating a kind of a theory. The most inspiring ones are, probably, [Montague 1974], [Almog, Perry, Wettstein 1989]. Some points shared by our brief characteristics and Kaplan's theory could be found.

17   The well-known Quine's argumentation against modal logics and intensions has used, e.g., the example of the incorrect inference from the premisses about *the number of planets*, and some analytic fact, for example, 9 > 7. If the nominalistic aversion to possible worlds and like did not have worked the famous example would have been easily solved even by Quine.

18   The attempts to explicate meanings mostly ignore this point or even conflate meaning with denotation. Implicitly is this most important distinction derivable from Bealer's onception in [Bealer 1982]; for example Dummett in [Dummett 1993] or D.Lewis in [Lewis 1972] seem not to see this point, Montague in [Montague 1974] explicitly realizes the denotational semantics. Paradoxically, it is the non-philosopher Girard, who shows in a most clear way in [Girard 1990] that the denotational semantics is unsatisfactory.

19   [Lance, Kremer 1996].

20   See [Fowler, Fowler 1956].

21 See [Tichý, Oddie 1982]. '*can*' is, of course, no logical modality.

22 Cf. Gödel's idea of language as the means of '*fixing concepts*,' [Wang 1993].

23 We do not see why 'that' should play a self-contained semantic role.

24 Let us imagine that this name is accompanied - 'in Dummett's spirit' - with some identification ('hidden description'?), or else let us forget for the time being about the pragmatic character of personal names.

25 In {Black 1949].

26 See an interesting study [Kubinski 1958].

27 This is not taken into account in [Kubinski 1958].

28 The question *Are borderlines parts of meaning?* dealt with in [Sörensen 1991 could be answered in a sense positively with the above approach.

29 A remarkable fragment of Tichý's attempt to solve this problem for English is his [Tichý 1994].

30 It seems that Dummett, explaining the distinction between a 'molecular' and a holistic view of language, would connect the former with the acceptance of not just two disjoint classes of expressions but of some hierarchy of expressions given by an asymmetric (in contradistinction to holismus) dependence between them, which presupposes that there is just one 'line' given by successive defining ("verbal explanations") so that some expressions are 'more primitive' than others. Clearly, this is not what is claimed in our theory. See [Dummett 1993], p.44.

31 These are 'Mates-like' puzzles. See the classical [Mates 1950].

32 MOON identifies an individual role (an $\iota_{\tau\omega}$-object). Here we do not consider the possibility of construing *the moon* as a proper name.

33 [Kripke 1979].

34 We set aside the fact that we could doubt that there were a proposition here, since *'pretty'* is not a descriptive term.

35   A more thorough discussion is contained in [Materna 1995].

36   See, e.g., [Tichý 1988].

37   [Tichý 1978], [Materna 1997].

Chapter 8

1   Among recommendable articles Abelson's paper in [Edwards 1967] is a good analytical survey of the topic.

2   [Robinson 1965].

3   Some characteristics of it, as well as of Robinson's book, as for the problem of 'real definitions', can be found in [Rantala 1991].

4   Cf. Rules 1 - 6 in Abelson's Encyclopedia paper.

5   *Ibidem.*

6   See Popper's critics of Aristotle in [Popper 1966].

7   It depends on the way we define *information.* Are, e.g., mathematical theorems informative?

8   Terms used by Abelson in his Encyclopedia paper.

9   *Ibidem,* p.320.

10   See [Rantala 1991], 140.

11 *Ibidem.*

12   This problem has been tackled from a highly interesting viewpoint by Hintikka in [Hintikka 1991]; the distinction between definability and identifiability is an essential distinction. Hintikka's notion of identifiability is, of course, distinct from ours.

13   This is stated by Cohen and Nagel, as Rantala emphasizes in [Rantala 1991],137.

14 That constructions are for Tichý what is *denoted* rather than *expressed* or *represented* is explicitly said in [Tichý 1988]; also from the most important [Tichý 1995] it is clear.

15 Such an interpretation of Greek *dicta* would mean that the idea of the set-theoretical character of concepts should not be necessarily connected with the Greek ακμη. The later 'standardization' of the Greek logic might have misinterpreted (because simplified) some original ideas.

16 A language is not a theory. Therefore, our intuition connected with the non-creativity of definitions is more general than that one which is defined for theories (see, e.g., [Rantala 1991], 150; if, however, we speak (very naturally) about a *conservative extension of a language*, then the similarity of our notion of non-creativity is conspicuous: the standard notion is connected with the notion of a *conservative extension of a theory.*

17 In this sense the notion of analytic definition is at least partly a pragmatic notion. See [Rantala 1991], 140.

18 Lycan in [Fetzer, Shatz, Schlesinger 1991].

19 [Quine 1953].

20 See his *Truth by Convention* in [Benacerraf,Putnam 1983].

21 [Quine 1976],p.51

22 See [Tichý 1988] for a brilliant discussion.

23 Associating types with expressions means that the objects denoted or *v*-denoted by these expressions belong to these types.

24 The variables may range over τ; for the values distinct from natural numbers S is undefined, so we simply get no truth-value.

25 [Carnap 1947].

Chapter 9

1 [Tichý 1995].

2 [Oddie 1986].

3   As a malicious realist I should quote a nice refutation of nominalism in [Bealer 1993], but I only refer to it.

4   [Zalta 1988].

5   See [Tichý 1988], 75.

6   Naturally, it is impossible to quantify over c without the mediating role of Sub, since c in the following subconstruction is o-bound. The same trick has been used in [Materna 1997].

7   A special problem arises as soon as we consider Henkin's and Hintikka's 'branching quantifiers'. Which semantics of such quantifiers (construed as 'self-contained' semantic units) our conception can offer I cannot say without a deeper **analysis**, but it is surely a highly interesting problem.

List of Definitions

ef 1   *(simple) types*

Def 2   *α-objects*

Def 3   *intensions*

Def 4   *first order objects*

Def 5   *extensions*

Def 6   *non-trivial intensions*

Def 7   *constructions*

    a)   variables

    b)   trivialization

    c)   composition

    d)   closure

Def 8   *free, o-bound, λ-bound variables*

Def 9   *ramified hierarchy (types of order n)*

Def 10   *higher order objects*

Def 11   *closed constructions*

Def 12   *a concept* of order n*

Def 13   *a strictly empty concept**

Def 14   *a quasi-empty concept**

Def 15   *a concept* empty w.r.t.W,T*

Def 16   *the extension of a concept**

Def 17   *the extension of a concept* w.r.t.W,T*

Def 18   *a simple concept**

Def 19 *a subconstruction of a construction*

Def 20 *the intension/content of a concept[*]*

Def 21 *equivalence of concepts[*]*

Def 22 *α-equivalence of concepts[*]*

Def 23 *β-equivalence of concepts[*]*

Def 24 *quasi-identity (Quid)*

Def 25 *concept of order n*

Def 26 *the extension of a concept*

Def 27 *the extension of a concept w.r.t.W,T*

Def 28 *simple concept*

Def 29 *the intension/content of a concept*

Def 30 *a strictly empty, quasi-empty, empty w.r.t.W,T concept*

Def 31 *a concept identifies an object*

Def 32 *equivalence of concepts*

Def 33 *an empirical concept*

Def 34 *a concept is used, mentioned*

Def 35 *a conceptual system of order n; primitive and derived concepts*

Def 36 *empirical conceptual systems*

Def 37 *normal conceptual systems*

Def 38 *a conceptual system identifies an object*

Def 39 *the area of a conceptual system*

Def 40 *(strongly) weaker (conceptual systems)*

Def 41 *equivalence of conceptual systems*

Def 42 *a (proper) part of a conceptual system*

Def 43 *independent conceptual systems*

Def 44 *a pragmatically anchored expression*

Def 45 *the meaning of a pragmatically anchored expresion*

Def 46 *the pragmatic meaning of a pragmatically anchored expression in the situation S*

Def 47 *the pragmatic denotation of a pragmatically anchored expression in the situation S*

Def 48 *the WT-reference of a pragmatically anchored expression in the situation S*

Def 49 *the meaning of an expression*

Def 50 *the denotation of an expression*

Def 51 *the reference of an expression in W at T*

Def 52 *a homonymous expression*

Def 53 *an ambiguous expression*

Def 54 *synonymy of expressions*

Def 55 *equivalence, weak equivalence of expressions*

Def 56 *coincidence of expressions*

Def 57 *(concept) defines (an object) in (a conceptual system)*

Def 58 *definability (in conceptual systems)*

Def 59 *language L_CS*

Def 60 *equational definitions*

Def 61 *the expressive power of L_CS*

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