Presupposition, Admittance and Karttunen Calculus

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Abstract: Classic works define presuppositions of a sentence *S* as conclusions that follow from both *S* and its negation. Other studies focus on the necessary conditions for admitting *S* as true or false, assuming that those conditions converge with *S*'s presuppositions. Here we study this assumption in three systems: asymmetric Kleene truth tables, Heim's admittance-based theory, and a new propositional calculus inspired by Karttunen's entailment-based approach. Common versions of the Kleene and Heim systems are known to be semantically congruent, and we show that they identify presuppositions with admittance conditions. By contrast, it is proved that the proposed *Karttunen calculus* distinguishes the two notions. This aspect of the Karttunen calculus avoids the "proviso problem" for the Kleene/Heim approaches: the generation of presuppositions that appear to be too weak.

Keywords: presupposition, admittance, propositional logic, Kleene truth tables, three valued logic, proviso problem

1 Introduction

Presuppositions may disappear when the expression that triggers them is embedded in a complex sentence. For instance, the term "*the king of France*" famously presupposes that France is a monarchy, but the sentence "if France has a king, *the king of France* must be living at the Élysée Palace" does not. In such cases, we say that the presupposition "France is a monarchy" does not *project*. Karttunen (1973, 1974) analyzed presupposition projection and the lack thereof using rules that draw on entailment relations between logical forms. Peters (1979) suggested to emulate Karttunen's proposals using a truth-functional analysis that employs an asymmetric version of the Strong

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Kleene tables. In this three-valued semantics, a *presupposition* of a sentence S is classically defined as a proposition that follows from both S and its negation (van Fraassen, 1971). The Kleene-Peters analysis has been opposed to the "dynamic" approach in (Heim, 1983; Stalnaker, 1978), which defines a presupposition of a sentence S as a proposition that is entailed by all contexts that *admit* S, i.e., make S true or false.

This paper first shows that the Heim-Stalnaker account derives the same consequence relation as the Kleene-Peters system. In both systems, a close relation is rendered between classic presuppositions and admittance conditions: a proposition is a logically *strongest* presupposition of a sentence S if and only if it is a *weakest* admittance condition of S. This property has linguistically undesirable ramifications, known as the *proviso problem* (Winter, 2019 and references therein). For example, in the sentence "if Sue is busy, her spouse is away", the Kleene-Peters and Heim-Stalnaker analyses expect an unintuitive presupposition: "Sue is married if she is busy". We propose a solution of the proviso problem that generalizes Karttunen's rules into a so-called *Karttunen calculus*. This calculus derives the same admittance conditions as in the Kleene/Heim system. However, the presuppositions that are derived in the Karttunen calculus may be stronger than those admittance conditions, which allows the system to avoid the proviso problem.

The Karttunen calculus is similar to the Kleene-Peters system in relying on *left-determinant* values of binary operators for defining presupposition projection. Like the Heim-Stalnaker system, it uses local contexts for satisfying presuppositions. However, unlike the Kleene/Heim systems, the calculus relies on entailment between propositional formulas as in Karttunen's work, rather than on implication or set inclusion between their denotations. Contrary to Peters' claims, this makes context a non-redundant element of the Karttunen calculus, indeed of Karttunen's (1974) original proposal.

Section 2 introduces the notions of *left-determinant value* and *projection calculus* and illustrates their use for presenting the Kleene-Peters tables. Section 3 shows that the Heim-Stalnaker semantics leads to the same equivalence and entailment relations as those tables. Section 4 shows that the Kleene/Heim system conflates strongest presuppositions with weakest admittance conditions. It is conjectured that this conflation is inadequate for describing natural language and leads to the proviso problem. Section 5 introduces the *Karttunen calculus* and shows that it distinguishes a sentence's strongest presuppositions from its, possibly weaker, weakest admittance conditions, thus avoiding the proviso problem. Section 6 concludes. For proofs and further technical notes see (Winter, 2021).

2 Kleene truth tables and projection calculi

The Strong Kleene truth tables are one of the earliest logical treatments of presupposition. While these tables are symmetric, presupposition projection is often not (Mandelkern, Zehr, Romoli, & Schwarz, 2020). In view of this fact, Peters (1979) proposed the tables in Figure 1, where '1', '0' and '$*$' stand for *true*, *false* and *undefined*, respectively. These trivalent *Kleene-Peters* (KP) tables asymmetrically extend the standard bivalent tables. A bivalent conjunction (disjunction/implication) is false (true/true) when the lefthand operand is false (true/false, respectively). This property is preserved in the KP truth tables, also when the righthand operand is undefined. However, when the lefthand operand is undefined, the result is undefined with no respect to the value of the righthand operand.

α	$\neg\alpha$
0	1
1	0
$*$	$*$

$\alpha \wedge \beta$	0	1	$*$
0	0	0	0
1	0	1	$*$
$*$	$*$	$*$	$*$

$\alpha \vee \beta$	0	1	$*$
0	0	1	$*$
1	1	1	1
$*$	$*$	$*$	$*$

$\alpha \rightarrow \beta$	0	1	$*$
0	1	1	1
1	0	1	$*$
$*$	$*$	$*$	$*$

Figure 1: The Kleene-Peters (KP) truth tables

Presuppositional and assertive elements of English sentences are analyzed as *bivalent*, and are expressed using a standard *propositional language*: a closure of a non-empty set of constants C under the propositional operators $\neg$, $\wedge$, $\vee$ and $\rightarrow$. When the constants in C are arbitrary we assume that they are assigned a bivalent interpretation, and refer to the propositional language over C as 'L_2'. English sentences are analyzed as simple *trivalent* propositions, which are represented as pairs of formulas from L_2: a presuppositional part and an assertive content. Such pairs are denoted $(\alpha\!:\!\beta)$ and are interpreted in $\{0, 1, *\}$ using Blamey's (1986) *transplication* operator, which is defined below:

Definition 1 (transplication) *For any bivalent interpretation* $[\![\cdot]\!]^{bi}$ *of* L_2*, we extend the interpretation* $[\![\cdot]\!]^{bi}$ *of* L_2 *into an interpretation of* $L_2 \times L_2$ *by defining, for any* $\alpha, \beta \in L_2$*:*

$$[\![(\alpha\!:\!\beta)]\!]^{bi} = \begin{cases} [\![\beta]\!]^{bi} & [\![\alpha]\!]^{bi} = 1 \\ * & [\![\alpha]\!]^{bi} = 0 \end{cases}$$

Complex trivalent formulas are obtained using Definition 2 below:

Definition 2 (L_3) *Given a propositional language L_2 over arbitrary constants, the language L_3 is a propositional language over $L_2 \times L_2$.*

One way to analyze presupposition projection is by defining the trivalent denotation of complex L_3 formulas for any bivalent interpretation of L_2. Definition 3 below uses the KP truth tables to extend the trivalent interpretation of $L_2 \times L_2$ in Definition 1 into a trivalent interpretation of L_3:

Definition 3 (KP interpretation of L_3) *Let $[\![\cdot]\!]^{bi}$ be a bivalent interpretation of L_2, which is extended to $L_2 \times L_2$ as in Definition 1. For any formula $\kappa \in L_3$, the* KP interpretation *of κ is denoted $[\![\kappa]\!]^{KP}$ and is defined as follows:*

$$[\![\kappa]\!]^{KP} = \begin{cases} [\![\kappa]\!]^{bi} & \kappa \in L_2 \times L_2 \\ [\![\neg]\!]^{KP}([\![\varphi]\!]^{KP}) & \kappa = \neg\varphi \\ [\![\mathsf{op}]\!]^{KP}([\![\varphi]\!]^{KP}, [\![\psi]\!]^{KP}) & \kappa = \varphi\,\mathsf{op}\,\psi \end{cases}$$

where $[\![\varphi]\!]^{KP}$ and $[\![\psi]\!]^{KP}$ are inductively defined, and negation and the binary operator 'op' are interpreted using the KP tables (Fig. 1)

It is useful to note that the corresponding equivalence relation ($\overset{\text{KP}}{\equiv}$) over L_3 satisfies the following standard equivalences, for any $\varphi, \psi \in L_3$:

Fact 1 $\varphi \vee \psi \overset{KP}{\equiv} \neg((\neg\varphi) \wedge \neg\psi) \qquad \varphi \to \psi \overset{KP}{\equiv} (\neg\varphi) \vee \psi$

The following example illustrates how KP semantics is used for analyzing presupposition projection:

Example 1 Sentences S1 and S2 below are represented as L_3 formulas:

S1 = *if Sue is married her spouse is away* = $(\top{:}\alpha_1) \to (\beta{:}\gamma)$

S2 = *if Sue is busy her spouse is away* = $(\top{:}\alpha_2) \to (\beta{:}\gamma)$

where α_1="Sue is married", α_2="Sue is busy", β="Sue has a spouse", and γ="Sue has a spouse who is away" are bivalent propositions. We now observe the following KP equivalence:

$$(\top{:}\alpha) \to (\beta{:}\gamma) \quad \equiv \quad (\alpha \to \beta{:}\alpha \to \gamma)$$

While $\alpha_1 \to \beta$ is tautological, $\alpha_2 \to \beta$ is not. Thus, according to the KP semantics, the presupposition of sentence S1 is expected to be patently true, in agreement with linguistic judgements, where S1 shows no presupposition. By contrast, S2 is analyzed as presupposing "if Sue is busy, she has a spouse", which is weaker than the presupposition that ordinary speakers report ("Sue has a spouse"). This incongruence between theory and speaker judgements illustrates the *proviso problem* (Karttunen, 1973, p. 188; Geurts, 1996).

An alternative way of analyzing presupposition projection is by rewriting any L_3 formula κ into a formula $(\kappa_1 : \kappa_2)$ in $L_2 \times L_2$, where the bivalent formula κ_1 is viewed as κ's strongest presupposition and κ_2 is viewed as κ's assertive context. We refer to this technique as a *projection calculus*.

The *Weak Kleene* (WK) tables let a propositional formula be interpreted as '$*$' if any of its sub-formulas is interpreted as '$*$'. Thus, the WK tables trivially "project" all presuppositions of κ's sub-formulas by letting κ_1 be their conjunction. This is modelled by the following projection calculus:

Definition 4 (WK calculus) *For any formula κ in L_3, let $WK(\kappa)$ be the formula in $L_2 \times L_2$ that is inductively defined as follows:*

$$WK(\kappa) = \begin{cases} \kappa & \kappa = (\kappa_1 : \kappa_2) \\ (\varphi_1 : \neg\varphi_2) & \kappa = \neg\varphi \\ (\varphi_1 \wedge \psi_1 : \varphi_2 \, \mathsf{op}^{bi} \, \psi_2) & \kappa = \varphi \, \mathsf{op} \, \psi \end{cases}$$

where: - op^{bi} is the bivalent propositional operator corresponding to op
- $\kappa_1, \kappa_2 \in L_2$, and inductively: $(\varphi_1 : \varphi_2) = WK(\varphi)$ and $(\psi_1 : \psi_2) = WK(\psi)$

A similar rewriting technique describes the KP tables (Figure 1). We first assign a unary operator 'LDV_{op}' (*left determinant value*) to any bivalent binary operator op. This is defined below:

Definition 5 (left determinant value) *For any binary operator* op*, the corresponding unary operator specifying the* left determinant value*(*s*) of* op *is defined as follows for any $\alpha \in L_2$:*

$$\text{LDV}_{op}(\alpha) = (\alpha \, \mathsf{op} \, \bot \leftrightarrow \alpha \, \mathsf{op} \, \top).$$

Thus, we have: $\text{LDV}_{\wedge}(\alpha) = \text{LDV}_{\rightarrow}(\alpha) \equiv \neg\alpha$ and $\text{LDV}_{\vee}(\alpha) \equiv \alpha$.

Using the LDV operator, we define the *KP calculus* as follows:

Definition 6 (KP calculus) *For any formula κ in L_3, let $KP(\kappa)$ be the formula in $L_2 \times L_2$ that is inductively defined as follows:*

$$KP(\kappa) = \begin{cases} \kappa & \kappa = (\kappa_1 : \kappa_2) \\ (\varphi_1 : \neg\varphi_2) & \kappa = \neg\varphi \\ WK((\varphi_1 : \varphi_2) \, \mathsf{op} \, ((\psi_1 \vee \text{LDV}_{op}(\varphi_2)) : \psi_2)) & \kappa = \varphi \, \mathsf{op} \, \psi \end{cases}$$

where $\kappa_1, \kappa_2 \in L_2$, and inductively: $(\varphi_1 : \varphi_2) = KP(\varphi)$ and $(\psi_1 : \psi_2) = KP(\psi)$

By Definitions 4, 5 and 6 we have:

$$\begin{aligned} KP(\varphi \wedge \psi) &\equiv (\varphi_1 \wedge (\psi_1 \vee \neg\varphi_2) : \varphi_2 \wedge \psi_2) \\ KP(\varphi \vee \psi) &\equiv (\varphi_1 \wedge (\psi_1 \vee \varphi_2) : \varphi_2 \vee \psi_2) \\ KP(\varphi \rightarrow \psi) &\equiv (\varphi_1 \wedge (\psi_1 \vee \neg\varphi_2) : \varphi_2 \rightarrow \psi_2) \end{aligned}$$

Definition 6 of the KP calculus is *sound* with respect to KP interpretations:

Fact 2 *For any formula $\kappa \in L_3$ and KP interpretation:* $[\![KP(\kappa)]\!]^{KP} = [\![\kappa]\!]^{KP}$.

Example 2 $KP((\top : \alpha) \rightarrow (\beta : \gamma)) \equiv (\top \wedge (\beta \vee \neg\alpha) : \alpha \rightarrow \gamma)$, which is equivalent to $(\alpha \rightarrow \beta : \alpha \rightarrow \gamma)$ as in Example 1.

3 Heim-Stalnaker semantics

Heim (1983) analyzes presupposition projection in terms of a sentence's admittance by a given context. Following Stalnaker (1978), Heim defines a context as a set of possible worlds, which *admits* a sentence S if it is contained in the set of possible worlds where S's presuppositions hold. A sentence S is analyzed using a pair $\langle A, B \rangle$, where A and B are the sets of possible worlds denoted by S's presupposition and S's assertive content, respectively. Such pairs are used to update the context. Propositional connectives modify the updates induced by their operand(s). This view of presupposition projection seems quite different from the Kleene tables in traditional three-valued logic. However, following Peters (1979), this section shows that in terms of the entailment and equivalence relations they describe over formulas in L_3, the *Heim-Stalnaker* (HS) semantics and the KP truth tables are congruent.

3.1 Heim-Stalnaker semantics – language and interpretation

When representing a sentence's semantic import as its *context change potential* (CCP), it is convenient to use the following propositional language:

Definition 7 $L_{CCP} \stackrel{def}{=} L_2 \cup \{ \chi[\kappa] : \chi \in L_{CCP}$ *and* $\kappa \in L_3 \}$

Thus, any L_{CCP} formula is made of a context formula in L_2 and a (possibly empty) sequence of formulas in L_3.

Example 3 Given $C, \alpha, \beta, \gamma \in L_2$, the following are all L_{CCP} formulas:
$C, \; C[(\alpha : \beta)], \; (C[(\alpha : \beta)])[(\top : \gamma) \vee (\alpha : \beta)]$.

Adding disjunction to Heim's system, we get the following canonical semantics of L_{CCP} (Nouwen, Brasoveanu, van Eijck, & Visser, 2016; Rothschild, 2011):

Definition 8 (HS interpretation of L_{CCP}) *Let $W \neq \varnothing$ be an arbitrary set of possible worlds, and let $[\![\cdot]\!]^M_W$ (in short: '$[\![\cdot]\!]^M$') be a* modal *interpretation of L_2, which assigns any constant $p \in L_2$ a set $[\![p]\!] \subseteq W$, and interprets any complex L_2 formula using the set-theoretical operators corresponding to the propositional connectives. An* HS interpretation *over W is a function $[\![\cdot]\!]^{HS}_W$ (in short: '$[\![\cdot]\!]^{HS}$' or '$[\![\cdot]\!]$') from L_{CCP} to $\wp(W) \cup \{*\}$ that inductively extends $[\![\cdot]\!]^M_W$ to any formula $\chi \in L_{CCP}$. This is defined as follows:*

- *For any $\chi \in L_2$, we define: $[\![\chi]\!] = [\![\chi]\!]^M$.*
- *For any $\chi = \mu[\kappa] \in L_{CCP} \setminus L_2$:*

(a) If $[\![\mu]\!] = *$, we define: $[\![\mu[\kappa]]\!] = *$.

(b) If $[\![\mu]\!] \neq *$ and $\kappa = (\kappa_1 : \kappa_2) \in L_2 \times L_2$, we define:

$$[\![\mu[(\kappa_1 : \kappa_2)]]\!] = \begin{cases} [\![\mu]\!] \cap [\![\kappa_2]\!] & \textit{if } [\![\mu]\!] \subseteq [\![\kappa_1]\!] \\ * & \textit{otherwise} \end{cases}$$

(c) If $[\![\mu]\!] \neq *$ and $\kappa \in L_3 \setminus (L_2 \times L_2)$, we define inductively:

$$[\![\,\mu[\neg\varphi]\,]\!] = \begin{cases} [\![\mu]\!] \setminus [\![\mu[\varphi]]\!] & \textit{if } [\![\mu[\varphi]]\!] \neq * \\ * & \textit{otherwise} \end{cases}$$

$$[\![\,\mu[\varphi \wedge \psi]\,]\!] = [\![(\mu[\varphi])[\psi]]\!]$$

$$[\![\,\mu[\varphi \vee \psi]\,]\!] = \begin{cases} [\![\mu[\varphi]]\!] \cup [\![(\mu[\neg\varphi])[\psi]]\!] & \textit{if } [\![\mu[\varphi]]\!] \neq * \\ & \textit{and } [\![(\mu[\neg\varphi])[\psi]]\!] \neq * \\ * & \textit{otherwise} \end{cases}$$

$$[\![\,\mu[\varphi \rightarrow \psi]\,]\!] = \begin{cases} [\![\mu[\neg\varphi]]\!] \cup [\![(\mu[\varphi])[\psi]]\!] & \textit{if } [\![\mu[\neg\varphi]]\!] \neq * \\ & \textit{and } [\![(\mu[\varphi])[\psi]]\!] \neq * \\ * & \textit{otherwise} \end{cases}$$

Similarly to KP connectives (Fact 1), the corresponding *HS equivalence* relation ($\overset{\text{HS}}{\equiv}$) over L_{CCP} satisfies, for any $\chi \in L_{CCP}$ and $\varphi, \psi \in L_3$:

Fact 3 $\chi[\varphi \vee \psi] \overset{HS}{\equiv} \chi[\neg((\neg\varphi) \wedge \neg\psi)]$ $\qquad$ $\chi[\varphi \rightarrow \psi] \overset{HS}{\equiv} \chi[(\neg\varphi) \vee \psi]$

For the *proof* of Fact 3 see (Winter, 2021, Appendix A).

In HS semantics, the analysis of presupposition projection in sentences S1 and S2 from Example 1 goes as follows:

Example 4 Sentences S1 and S2 below are represented as L_{CCP} formulas:

S1 = *if Sue is married her spouse is away* = $C[(\top:\alpha_1)\to(\beta:\gamma)]$

S2 = *if Sue is busy her spouse is away* = $C[(\top:\alpha_2)\to(\beta:\gamma)]$

where C is arbitrary, and $\alpha_1, \alpha_2, \beta$ and γ are as in Example 1. We observe that under HS interpretations:

$$C[(\top:\alpha)\to(\beta:\gamma)] \quad \equiv \quad C[(\alpha\to\beta:\alpha\to\gamma)]$$

Thus, for any context C and interpretation $[\![\cdot]\!]^{HS}$, the formula $\kappa = (\top:\alpha)\to(\beta:\gamma)$ is well-defined relative to C (i.e., has a non-'$*$' interpretation) *iff* the set $[\![C]\!]^M$ is contained in $[\![\alpha\to\beta]\!]^M$. Accordingly, and similarly to KP semantics (Example 1), the proposition $\alpha\to\beta$ is viewed as κ's presupposition.

3.2 HS semantics and KP semantics

Any HS interpretation over a set of possible worlds $W \neq \varnothing$ has a modal interpretation of L_2 at its basis. Such a modal interpretation corresponds with a family F of bivalent interpretations of L_2 that is indexed by W. Thus, a modal interpretation of L_2 gives rise to a family of KP interpretations of L_3. In this section we show that any HS interpretation can be represented as such a family of KP interpretations. First, for any family of bivalent interpretations of L_2 we define an alternative semantics of L_{CCP} that directly employs the KP semantics of L_3. Definition 9 specifies this *KP-based interpretation*:

Definition 9 *Given a set $W\neq\varnothing$, let $F=[\![\cdot]\!]_i^{bi}|_{i\in W}$ be a family of bivalent interpretations of L_2. For any $i\in W$, let $[\![\cdot]\!]_i^{KP}$ be the KP interpretation of L_3 corresponding to $[\![\cdot]\!]_i^{bi}$. A* KP-based interpretation of L_{CCP} relative to F *is a function $[\![\cdot]\!]_W^{KP}$ from L_{CCP} to $\wp(W)\cup\{*\}$ that is inductively defined as follows for any $\chi\in L_{CCP}$:*

<u>*If $\chi=\alpha$*</u>*, s.t. $\alpha\in L_2$:*

$$[\![\alpha]\!]_W^{KP} = \{i\in W : [\![\alpha]\!]_i^{bi} = 1\}$$

<u>*If $\chi=\mu[\kappa]$*</u>*, s.t. $\mu\in L_{CCP}$ and $\kappa\in L_3$:*

$$[\![\mu[\kappa]]\!]_W^{KP} = \begin{cases} [\![\mu]\!]_W^{KP}\cap\{i\in W : [\![\kappa]\!]_i^{KP}=1\} & [\![\mu]\!]_W^{KP}\neq * \textit{ and } \\ & \quad [\![\mu]\!]_W^{KP}\subseteq\{i\in W : [\![\kappa]\!]_i^{KP}\neq *\} \\ * & \textit{otherwise} \end{cases}$$

In the standard Definition 8 of HS interpretations, a complex formula $\mu[\kappa]$ is interpreted by updating the context μ inductively using sub-formulas of κ. By contrast, in KP-based interpretations, $\mu[\kappa]$ is interpreted using the KP semantics of κ in L_3, through the given family F of bivalent interpretations. Theorem 1 shows that this way of interpreting L_{CCP} using the KP tables covers all HS interpretations. As summarized in (Winter, 2021, Appendix B), this theorem makes the same point as the main property proved by Peters (1979).

Theorem 1 *Let* $[\![\cdot]\!]^{HS}_W$ *be an HS interpretation of* L_{CCP} *for some* $W \neq \emptyset$*. For any* $i \in W$*, let* $[\![\cdot]\!]^{bi}_i$ *be the bivalent interpretation of* L_2 *s.t. for any* $\alpha \in L_2$*:* $[\![\alpha]\!]^{bi}_i = 1$ *iff* $i \in [\![\alpha]\!]^{HS}$*. Let* $[\![\cdot]\!]^{KP}_W$ *be the KP-based interpretation of* L_{CCP} *relative to the family* $F = [\![\cdot]\!]^{bi}_i |_{i\in W}$*. Then for any* $\chi \in L_{CCP}$ *we have:*

$$[\![\chi]\!]^{HS}_W = [\![\chi]\!]^{KP}_W.$$

The *proof* of Theorem 1 in (Winter, 2021, Appendix C) is by induction on the structure of χ for the subset of L_{CCP} involving only negation and conjunction. This proof is directly applicable to disjunction and implication due to the standard Facts 1 and 3 under KP and HS interpretations.

Using Theorem 1, we now show that HS semantics is congruent with KP semantics in two senses. First, we show the soundness of a so-called *HS calculus*, which uses the KP calculus to rewrite any formula $\chi \in L_{CCP}$ as a maximally simple formula in L_{CCP}. Second, we use the HS calculus to show that entailment and equivalence relations over L_3 that are naturally induced by the HS semantics are identical to those induced by KP semantics.

3.3 HS calculus

Relying on Theorem 1, we first show that the KP calculus can be used to simplify any L_{CCP} formula while preserving its HS semantics:

Corollary 1 *For any* $\chi \in L_{CCP}$ *and* $\kappa \in L_3$*:* $\chi[\kappa] \overset{HS}{\equiv} \chi[KP(\kappa)]$.

For the *proof* see (Winter, 2021, Appendix D).

On the basis of Corollary 1, we show that the KP calculus extends into a sound method of rewriting L_{CCP} formulas into equivalent, maximally simple CCP formulas. Rewriting in this *HS calculus* is defined below:

Definition 10 (HS calculus) *Let* $min(L_{CCP})$ *be the following set of* minimal L_{CCP} *formulas:*

$$min(L_{CCP}) \;=\; L_2 \;\cup\; \{\, C[(\kappa_1 {:} \kappa_2)] \;:\; C, \kappa_1, \kappa_2 \in L_2 \,\}.$$

For any formula χ *in* L_{CCP}*, we define HS*(χ) *as the formula in* $min(L_{CCP})$

that is inductively defined as follows:

For any $\chi = C \in L_2$*:*

$HS(C) = C.$

For any $\chi = C[\kappa] \in L_{CCP}$ *where* $C \in L_2$ *and* $\kappa \in L_3$*:*

$HS(C[\kappa]) = C[KP(\kappa)].$

For any $\chi = (\mu[\varphi])[\kappa]$ *where* $\mu \in L_{CCP}$ *and* $\varphi, \kappa \in L_3$*, inductively:*

$HS((\mu[\varphi])[\kappa]) = HS(\mu[\varphi \wedge \kappa]).$

This calculus maps any simple L_2 formula in L_{CCP} to itself. Any formula $C[\kappa]$ where $C \in L_2$ is mapped to $C[KP(\kappa)]$, where $KP(\kappa)$ is the $L_2 \times L_2$ formula obtained from κ in the KP calculus. More complex L_{CCP} formulas are of the form $(\ldots((C[\varphi_1])[\varphi_2])\ldots)[\varphi_n]$, where $\varphi_1, \varphi_2, \ldots, \varphi_n \in L_3$. Such formulas are "flattened" to the form $C[\varphi_1 \wedge \varphi_2 \wedge \ldots \wedge \varphi_n]$, which is inductively simplified using the KP calculus.

Example 5 $HS(C[(\varphi_1{:}\varphi_2)])[(\psi_1{:}\psi_2)]) = HS(C[(\varphi_1{:}\varphi_2)\wedge(\psi_1{:}\psi_2)])$
$= C[KP((\varphi_1{:}\varphi_2)\wedge(\psi_1{:}\psi_2))] = C[(\varphi_1\wedge(\psi_1\vee\neg\varphi_2){:}\varphi_2\wedge\psi_2)]$

This HS calculus is *sound* with respect to HS interpretations of L_{CCP} formulas:

Corollary 2 *For any formula* $\chi \in L_{CCP}$ *and HS interpretation:*

$[\![HS(\chi)]\!]^{HS} = [\![\chi]\!]^{HS}.$

The *proof* for formulas $C[\kappa]$ where $C \in L_2$ follows from Corollary 1. For other formulas of the form $\mu[\kappa]$, Corollary 2 is proved inductively, relying on its proof for μ. See (Winter, 2021, Appendix E) for details.

3.4 KP/HS entailment and KP/HS equivalence

KP interpretations naturally specify an equivalence relation ($\overset{\mathrm{KP}}{\equiv}$) over L_3. As for *entailment* over L_3, we standardly employ the following "Tarskian" definition in trivalent semantics (van Fraassen, 1971):

Definition 11 (trivalent entailment) *Let* $\mathcal{C}$ *be a class of trivalent interpretations mapping a language* L *to* $\{0, 1, *\}$*. For any two formulas* $\varphi, \psi \in L$*, we denote* $\varphi \overset{\mathcal{C}}{\Rightarrow} \psi$ *if for every interpretation* $[\![\cdot]\!] \in \mathcal{C}$*:*

if $[\![\varphi]\!] = 1$ *then* $[\![\psi]\!] = 1.$

When $\mathcal{C}$ in Definition 11 is the class of KP interpretations of L_3, we obtain a relation of *KP-entailment* ($\overset{KP}{\Rightarrow}$). We note that Definition 11 distinguishes bidirectional entailment from equivalence: trivalent propositions may agree on the interpretations that make them *true* without necessarily agreeing on the interpretations that make them *false*.

HS semantics is defined over the language L_{CCP}, hence specifies an equivalence relation ($\overset{\text{HS}}{\equiv}$) over that language. To allow comparing HS and KP semantics, this relation is extended to an equivalence relation over L_3:

Definition 12 (HS equivalence over L_3) *For any two formulas $\varphi, \psi \in L_3$, we denote $\varphi \overset{HS}{\equiv} \psi$ iff for every $\chi \in L_{CCP}$: $\chi[\varphi] \overset{HS}{\equiv} \chi[\psi]$.*

For any two L_3 formulas φ and ψ, we also define *HS entailment*, by requiring that whenever φ leaves a context intact, so does ψ. Formally:

Definition 13 (HS entailment over L_3) *For any two formulas $\varphi, \psi \in L_3$, we denote $\varphi \overset{HS}{\Rightarrow} \psi$ iff for every $\chi \in L_{CCP}$ and HS interpretation:*

if $[\![\chi[\varphi]]\!]^{HS} = [\![\chi]\!]^{HS}$ then $[\![\chi[\psi]]\!]^{HS} = [\![\chi]\!]^{HS}$.

The claim below follows from the soundness of HS calculus (Corollary 2):

Corollary 3 *For any two formulas $\varphi, \psi \in L_3$:*

(i) $\varphi \overset{KP}{\equiv} \psi$ iff $\varphi \overset{HS}{\equiv} \psi$ *(ii) $\varphi \overset{KP}{\Rightarrow} \psi$ iff $\varphi \overset{HS}{\Rightarrow} \psi$*

The *proof* of Corollary 3 is in (Winter, 2021, Appendix F).

The KP/HS entailment relation is *monotonic*, in the following sense:

Fact 4 *For all $\varphi, \psi, \kappa \in L_3$: if $\varphi \overset{KP}{\Rightarrow} \psi$ then $\kappa \wedge \varphi \overset{KP}{\Rightarrow} \psi$.*

This fact is related to the proviso problem, discussed in the following section.

4 Admittance vs. presupposition

In the HS semantics of L_{CCP}, *admittance* of a proposition by a context is defined as follows:

Definition 14 (HS-admittance) *We say that a context $C \in L_2$* HS-admits *a formula $\kappa \in L_3$ if $[\![C[\kappa]]\!]^{HS} \neq \star$ for all HS interpretations $[\![\cdot]\!]^{HS}$.*

A parallel notion is defined over L_3 using the KP semantics. We first introduce the following general notation:

Notation. Given a projection calculus Ω *mapping* L_3 *to* $L_2 \times L_2$*, for any formula* $\kappa \in L_3$ *we denote:*

$\Omega(\kappa) = (\alpha_\kappa^\Omega : \beta_\kappa^\Omega)$*, where* $\alpha_\kappa^\Omega, \beta_\kappa^\Omega \in L_2$.

In KP semantics we define admittance by first observing the following fact:

Fact 5 *For any* $C \in L_2$ *and* $\kappa \in L_3$*:* C *HS-admits* κ *iff* $\alpha^{KP}_{(\top:C)\wedge\kappa} \equiv \top$.

Thus, C admits κ in HS semantics *iff* the KP calculus rewrites the conjunction $(\top : C) \wedge \kappa$ into a pair $(\alpha : \beta)$ where α is a tautology. By soundness of KP calculus, this means that no KP interpretation makes the formula $(\top : C) \wedge \kappa$ *undefined* ('$*$'). When this condition holds we say that C *KP-admits* κ.

Presuppositions are standardly defined using entailment:

Definition 15 (presupposition) *Given an entailment relation* $\overset{\mathcal{C}}{\Rightarrow}$ *over* L_3*, we say that* $\kappa \in L_3$ $\mathcal{C}$-presupposes $\beta \in L_2$ *if* $\kappa \overset{\mathcal{C}}{\Rightarrow} (\top : \beta)$ *and* $\neg\kappa \overset{\mathcal{C}}{\Rightarrow} (\top : \beta)$.

Due to the convergence of the entailment relations in KP and HS semantics (Corollary 3), KP-presupposition and HS-presupposition converge as well.

Furthermore, in KP/HS semantics, the logically *weakest admitting context* and *strongest presupposition* converge for any formula $\kappa \in L_3$. This is shown by the following theorem:

Theorem 2 *For* $\kappa \in L_3$*, let* C *be a weakest formula in* L_2 *that KP-admits* κ*, and let* β *be a strongest KP-presupposition of* κ *in* L_2*. Then* $C \equiv \beta \equiv \alpha_\kappa^{KP}$.

Standardly, we here say that $\alpha \in L_2$ is a weakest (strongest) formula in L_2 with a property Π if any $\alpha' \in L_2$ that has the property Π and satisfies $\alpha \Rightarrow \alpha'$ (respectively: $\alpha' \Rightarrow \alpha$) satisfies $\alpha' \equiv \alpha$. The *proof* of Theorem 2 is in (Winter, 2021, Appendix G).

Theorem 2 is closely related to the *proviso problem* for KP/HS semantics (Example 1). To highlight this, we propose the following empirical conjecture about English, which stands in opposition to Theorem 2:

Conjecture 1 *There exists an English sentence S that is admitted by a context C such that C is logically weaker than any strongest presupposition of S.*

Example 6 Sentences S3 and S4 below are represented as L_3 formulas:

S3 = *if Sue visited Dan, his beard annoyed her* = $(\top : \alpha) \rightarrow (\beta : \gamma)$

S4 = *if Sue visited Dan, he had grown a beard before she arrived*
= $(\top : \alpha) \rightarrow (\top : \beta')$

Where α="Sue visited Dan", β= "Dan had a beard", β'= "Dan had grown a beard before Sue arrived" and γ= "Dan had a beard that annoyed Sue".

Substantiating Conjecture 1, we make the following empirical claims:

(a) Sentence *S3* presupposes that Dan had a beard.

(b) The conjunction *S4 and S3* does not presuppose that Dan had a beard. Furthermore, *S4 and S3* has no non-tautological presupposition.

Claim (b) is consistent with the expectation of KP/HS-semantics that the weakest admittance condition of S3 is S5 below, which is entailed by S4:

S5 = *if Sue visited Dan, he had a beard* = $(\top{:}\alpha)\rightarrow(\top{:}\beta)$

However, claim (a) is inconsistent with the expectation of KP/HS-semantics that S5 is also the strongest presupposition of S3.

5 The Karttunen calculus

Conjecture 1 as illustrated in Example 6 suggests that Theorem 2 is problematic for using KP/HS semantics as a model of presupposition projection in English. To solve this problem, we propose an alternative projection calculus called the *Karttunen* (K) *calculus*. Like the KP calculus, the K-calculus maps any L_3 formula to a formula in $L_2 \times L_2$. However, unlike the KP calculus, the K-calculus does not emerge from any straightforward trivalent semantics. Rather, as in (Karttunen, 1973, 1974), the K-calculus takes *entailment* between bivalent formulas (or "logical forms") as the key to admitting a sentence by way of satisfying its presuppositions.

At the basis of the mechanism lie two assumptions: (i) a context $C \in L_2$ *admits* a simple L_3 formula $(\kappa_1 : \kappa_2)$ iff C *entails* κ_1 in bivalent logic; (ii) in binary constructions $\varphi\,\mathsf{op}\,\psi$, the assertive content of φ updates the context of ψ's evaluation using the LDV operator. The reliance on entailment in (i) prevents a direct interpretation of L_3 according to the K-calculus. Rather, L_3 formulas need to first be transformed into formulas in $L_2 \times L_2$ before they can be semantically interpreted. This representational analysis of presupposition projection follows Karttunen's reliance on logical forms, but it squarely aligns with the truth-functional practice of involving left-determinant values as the key to presupposition projection and admittance, as in Kleene-Peters semantics (Winter, 2019). Unlike HS semantics, where contexts are arguably redundant due to the operational equivalence with KP semantics (see Peters 1979 and Corollary 3 above), the K-calculus uses L_2 formulas non-redundantly for recording *local contexts*. These local contexts are not denotations like sets of possible worlds as in HS semantics but L_2 formulas (or "logical forms") as in (Karttunen, 1974).

Formally, the K-calculus maps any L_3 formula to a formula in $L_2 \times L_2$ using a bivalent *context* $C \in L_2$, which is assumed to be tautological in the base case. This is specified in Definition 16 below:

Definition 16 (K-calculus) *For any formula $C[\kappa]$ in L_{CCP} where $C \in L_2$, let $K(C[\kappa])$ be the formula in L_3 that is inductively defined as follows:*
If $\kappa = (\kappa_1 : \kappa_2) \in L_2 \times L_2$:

$$K(C[(\kappa_1 : \kappa_2)]) = \begin{cases} (\top : \kappa_2) & C \Rightarrow \kappa_1 \\ (\kappa_1 : \kappa_2) & \textit{otherwise} \end{cases}$$

If $\kappa \in L_3 \setminus (L_2 \times L_2)$:

$$K(C[\kappa]) = \begin{cases} (\varphi_1 : \neg\varphi_2) & \kappa = \neg\varphi \\ WK(\, (\varphi_1 : \varphi_2) \,\mathsf{op}\, K((C \wedge \varphi_1 \wedge \neg \text{LDV}_{op}(\varphi_2))[\psi])\,) & \kappa = \varphi \,\mathsf{op}\, \psi \end{cases}$$

$$\textit{where inductively: } (\varphi_1 : \varphi_2) = K(C[\varphi])$$

For any $\kappa \in L_3$ (without any given C), we abbreviate:

$$K(\kappa) = K(\top[\kappa]).$$

By definition of the WK calculus and the LDV_{op} operator we now have:

$$K(C[\varphi \wedge \psi]) = WK((\varphi_1 : \varphi_2) \wedge K((C \wedge \varphi_1 \wedge \varphi_2)[\psi]))$$
$$K(C[\varphi \vee \psi]) = WK((\varphi_1 : \varphi_2) \vee K((C \wedge \varphi_1 \wedge \neg\varphi_2)[\psi]))$$
$$K(C[\varphi \to \psi]) = WK((\varphi_1 : \varphi_2) \to K((C \wedge \varphi_1 \wedge \varphi_2)[\psi]))$$

According to Definition 16, in binary constructions both the presuppositional content and the (negation of) the assertive content of the lefthand operand are accommodated into the context of the righthand operand. This is useful in sentences like *if Sue stopped smoking, then Dan knows that Sue stopped smoking*. This sentence inherits the presupposition of the antecedent ("Sue used to smoke"), but not the presupposition of the consequent ("Sue used to smoke and doesn't smoke now"). According to the K-calculus, this happens due to the accommodation of the whole antecedent (both presupposition and assertive content) into the context of the consequent. This local context of the consequent entails its presupposition, hence that presupposition is not projected.

Let us now consider the application of the K-calculus to the analysis of sentences S3 and S5 from Example 6:

Example 7 For S3 and S5 from Example 6, we denote, respectively: $\eta = (\top : \alpha) \to (\beta : \gamma)$, and $\theta = (\top : \alpha) \to (\top : \beta')$.

Since $K(\top[(\top{:}\alpha)] = (\top{:}\alpha))$, and since $\alpha \not\Rightarrow \beta$, we conclude:
$K(\eta) = K((\top{:}\alpha) \to (\beta{:}\gamma)) = K(\top[(\top{:}\alpha) \to (\beta{:}\gamma)])$
$\quad = WK((\top{:}\alpha) \to K((\top \wedge \top \wedge \alpha)[(\beta{:}\gamma)])) = WK((\top{:}\alpha) \to (\beta{:}\gamma)) = (\beta{:}\alpha \to \gamma)$
$K(\theta \wedge \eta) = \ldots = WK((\top{:}\alpha{\to}\beta') \wedge K((\alpha{\to}\beta')[(\top{:}\alpha) \to (\beta{:}\gamma)]))$
$\quad = WK((\top{:}\alpha{\to}\beta') \wedge WK((\top{:}\alpha) \to K(((\alpha{\to}\beta') \wedge \alpha)[(\beta{:}\gamma)])))$, since $\beta' \Rightarrow \beta$:
$\quad = WK((\top{:}\alpha{\to}\beta') \wedge WK((\top{:}\alpha) \to (\top{:}\gamma))) = \ldots = (\top{:}\alpha{\to}(\beta' \wedge \gamma))$

Unlike the KP/HS semantics, these derivations are consistent with claims (a) and (b) in Example 6. They show that β is the strongest K-presupposition of η, but the bivalent proposition $\alpha \to \beta'$ (=$K(\theta)$'s assertive content) K-admits η although it does not logically entail that presupposition. Thus, K-admittance and KP/HS-admittance converge in this case, although the K-presupposition is stronger than its KP/HS correlate.

More generally, we claim that weakest admittance conditions in the K-calculus are the same as in the KP/HS-calculus, for all L_3 formulas. By contrast, presuppositions in the K-calculus are at least as strong as those of the KP/HS-calculus, but they may also be properly stronger as in Example 7. For this comparison between calculi, we first define the necessary semantic notions in the K-calculus. Definition 17 below *K-interprets* any $\kappa \in L_3$ by rewriting it into $K(\kappa)$ — an $L_2 \times L_2$ formula interpreted by transplication under any bivalent interpretation (Definition 1):

Definition 17 (K-interpretation of L_3) *Let $[\![\cdot]\!]^{bi}$ be a bivalent interpretation of L_2, and let κ be an L_3 formula. The* Karttunen *(K)* interpretation *of κ is defined by $[\![\kappa]\!]^{K} = [\![K(\kappa)]\!]^{bi}$.*

Using K-interpretations, we define *K-equivalence* ($\overset{K}{\equiv}$), *K-entailment* ($\overset{K}{\Rightarrow}$) and *K-presupposition*, similarly to KP semantics. It is useful to note that similarly to KP/HS semantics (Facts 1 and 3), K-interpretations satisfy the following standard equivalences, for any $\varphi, \psi \in L_3$:

Fact 6 $\quad \varphi \vee \psi \overset{K}{\equiv} \neg((\neg\varphi) \wedge \neg\psi) \qquad \varphi \to \psi \overset{K}{\equiv} (\neg\varphi) \vee \psi$

The *proof* in (Winter, 2021, Appendix H) simply applies the K-calculus.

Unlike entailment in KP/HS semantics (Fact 4), K-entailment is *not monotonic*. This is illustrated by Example 7, where η K-entails $(\top{:}\beta)$ but $\theta \wedge \eta$ does not.

K-admittance of $\kappa \in L_3$ by a context $C \in L_2$ is defined, similarly to KP-admittance, as follows:

Definition 18 (K-admittance) *We say that a context $C \in L_2$* K-admits *a formula $\kappa \in L_3$ if $\alpha^{K}_{(\top:C)\wedge\kappa} \equiv \top$.*

By Definition 17, this boils down to requiring that no K-interpretation assigns the formula $(\top : C) \wedge \kappa$ an *undefined* value ('$*$').

We now observe the following general fact about the K-calculus:

Theorem 3 *For any $\kappa \in L_3$, let C be a weakest formula in L_2 that K-admits κ, and let α be a strongest K-presupposition of κ in L_2. Then we have:*

$$\alpha \equiv \alpha_\kappa^K,\ \alpha_\kappa^K \Rightarrow C, \text{ and } C \equiv \alpha_\kappa^{KP}.$$

In words: the strongest *K-presupposition* of κ is directly obtained in the K-calculus as α_κ^K. This K-presupposition entails any weakest context that *K-admits* κ, although it is not necessarily entailed by it (see Example 7). Rather, any weakest context that admits κ in the K-calculus is equivalent to any weakest context that KP-admits κ. See (Winter, 2021, Appendix I) for a *proof* of Theorem 3.

6 Conclusions

The Kleene-Peters (KP) and the Heim-Stalnaker (HS) systems are at the basis of many on-going attempts to describe presupposition projection. The proviso problem threatens these attempts. Following Peters (1979), this paper has argued that the KP system and the HS systems are logically congruent. However, against Peters' claim that his system adequately mimics the proposal in (Karttunen, 1974), we have developed the so-called K-calculus. This system maintains Karttunen's aim of avoiding the proviso problem by distinguishing presuppositions from admittance conditions. Thus, the K-calculus is also distinguished from the KP/HS systems. The proviso problem for these proposals is argued to result from these systems' conflation of *strongest presuppositions* with *weakest admittance conditions*. Both systems rely on a truth-functional account, where the semantic value of a sentence's presupposition is fully determined by the base language's bivalent interpretation. By contrast, the K-calculus relies, following Karttunen, on bivalent *entailment* as the basis for presupposition projection. This system is conjectured to be empirically more adequate than the KP/HS semantics. Notwithstanding, similarly to the KP tables, the K-calculus relies on *left determinant values*, and like the HS semantics, it uses local contexts operationally in its account of presupposition projection. Furthermore, the admittance conditions that the K-calculus derives are the same as in those two systems.

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