Conjunction, Type Ambiguity, and Wide Scope "Or"

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1. Generalized conjunction and type ambiguity. We want to explore some consequences of assuming that all kinds of conjoined constituents are directly generated syntactically (i.e. no syntactic conjunction reduction). We begin by reviewing in this section the central argument of [11] to the effect that the most natural unified cross-categorial interpretation for and and or provides evidence for revising Montague's rules for assigning semantic types to syntactic categories in such a way that most syntactic categories will have a family of associated semantic types rather than a single type. In section 2 we investigate a puzzling class of counter-examples to our account, involving wide-scope or in opaque contexts.

Our semantics for cross-categorial and and or, like that of [12] and [4], is built on the type of theory of Montague's PTQ ([10]).

(1) Recursive definition of conjoinable type
  (i) t is a conjoinable type
  (ii) if b is a conjoinable type, then for any a, <a,b> is a conjoinable type.

(All the types of PTQ are conjoinable types except for e and <s,e>; see [9] and [11] for a discussion of an e-based ("group" reading) and.) Starting from the standard $\wedge$ and $\vee$ operations for type t, we can recursively define the corresponding generalized operations for all the conjoinable types by exploiting the observation that any operation defined on the range (codomain) of a class of functions can be "lifted" to apply to the functions themselves, as in the following.

(2) Recursive definition of (generalized) $\sqcap$ and $\sqcup$.
  (i) In D_t, $\sqcap$ and $\sqcup$ are equivalent to $\wedge$ and $\vee$ respectively (defined by standard truth tables.)
  (ii) Let f,g be arbitrary functions in $D_{<a,b>}$. Then $f \sqcap g$ is that function in $D_{<a,b>}$ which maps any element x of D_a onto the element $f(x) \sqcap g(x)$ of D_b. (Similarly for $\sqcup$)

$$f \sqcap g : x \rightarrow f(x) \sqcap g(x)$$
$$f \sqcup g : x \rightarrow f(x) \sqcup g(x)$$

As a consequence of definition (2), the following holds in the resulting intensional logic, where α and β are of any conjoinable type <a,b>.

(3) $\alpha \sqcap \beta = \lambda z[\alpha(z) \sqcup \beta(z)]$, where z is of type a.

Now assume that (2) is correct, and also that the type of transitive verbs is as in PTQ,[2] <type (T), type (IV)>. Then the interpretation of $[TV_1$ and $TV_2]$ should be as in (4) (we use $\wedge$,$\vee$ when conjuncts are of type t):

(4) $\lambda\mathcal{P}\ \lambda x[TV_1'(\mathcal{P})(x) \wedge TV_2'(\mathcal{P})(x)]$.

This translation gives the wrong result for (5) and (6), with extensional TV's, but the right result for (7), with intensional TV's and (8), with an intensional and an extensional TV.

(5) John caught and ate a fish. (≠ caught a fish and ate a fish)

(6) John hugged and kissed three women.

(7) John wants and needs two secretaries.

(8) John needed and bought a new coat.

If the type of TV were <e, type (IV)>, then (2) would predict that the interpretation of [TV_1 and TV_2] should be as in (9):

(9) $\lambda y\lambda x[TV_1'(y)(x) \wedge TV_2'(y)(x)]$

This translation gives the right result for (5) and (6), the wrong result[3] for (7) and (8). This suggests (see also [1] and [2]) that we give up Montague's strategy of putting all items of a given syntactic category into the highest type needed for any of them and adopt the strategy exemplified for TV's in (10) instead.

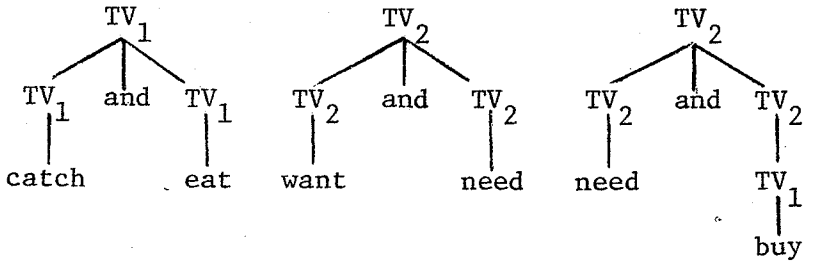
(10) Type ambiguity principles
- (i) Each verb is entered in the lexicon in its minimal type.
- (ii) A phrasal rule promotes lower-type TV's to higher-type homonyms.
- (iii) Processing strategy: interpret all expressions at the lowest type possible; in particular, interpret conjoined expressions at the lowest type they both have.

In particular, extensional TV's will be lexically of type <e,<e,t>>, and intensional TV's of type <type(T), <e,t>>. But an extensional TV like <u>buy</u> will have, by (10ii), a predictable[4] second translation with type <type (T), <e,t>>:

(11) buy" = $\lambda\mathcal{P}\ \lambda x[\mathcal{P}(\lambda y[buy'(y)(x)])]$

The effect of (10iii) is schematically illustrated in (12), using TV_1 to represent 'TV with lower-type translation' and TV_2 for 'TV with higher-type translation'.

(12)



These principles correctly predict the interpretations of all of (5)-(8). Parallel arguments are given in [11] for extensional

and intensional intransitive verb phrases, and we also propose that term phrases be assigned to type e where possible (e.g. proper nouns and pronouns), with Montague's higher term phrase type as the minimal type for quantified term phrases; e-type terms would also have predictable higher-type translations (their PTQ translations).

Formally, it appears that we have just achieved a uniform conjunction interpretation by giving up a uniform category-to-type assignment.[5] But the resulting overall system has potential advantages both for processing and for language acquisition. The intuitively simpler cases now have simpler interpretations, and acquisition could proceed from simpler types to higher types by accretion, without major restructuring.

II. Wide Scope Or

Sentence (13a) has a reading not predicted by the treatment sketched in section I.

(13) a. Mary is looking for a maid or a cook.

b. $\lambda P V x[maid'(x) \wedge [^{\vee}P](x)] \sqcup \lambda P V x[cook'(x) \wedge [^{\vee}P](x)]$

c. $\lambda P[Vx[maid'(x) \wedge [^{\vee}P](x)] \vee Vx[cook'(x) \wedge [^{\vee}P](x)]]$

$= \lambda P V x[(maid'(x) \vee cook'(x)) \wedge [^{\vee}P](x)]$

d. $look\text{-}for'(^{\wedge}\lambda P V x[(maid'(x) \vee cook'(x)) \wedge [^{\vee}P](x)])(m)$

e. $Vx[(maid'(x) \vee cook'(x)) \wedge look\text{-}for'(^{\wedge}\lambda P[^{\vee}P](x))(m)]$

The phrase a maid or a cook receives the translation (13b), which by virtue of (3) is equivalent to (13c). This phrase may be quantified in (yielding the de re reading (13e)) or not quantified in (yielding the de dicto reading (13d)). In the predicted de dicto reading, finding a maid and finding a cook would both satisfy Mary (this follows from the lexical semantics for look-for' suggested by meaning postulate (9) of Montague [10]). However, (13a) has a second de dicto reading, suggested by the continuation "... but I don't know which". The reading in question, which we call the wide scope or reading, is equivalent to the reading of 'Mary is looking for a maid or looking for a cook', where the object is de dicto in both conjuncts.

It is instructive to compare the wide-scope or reading of (13a) to (14). According to section I, John and Bill have translations of type e. Since this is not a conjoinable type, conjunction must be preceded by promotion to the type $\langle\langle s,\langle e,t\rangle\rangle,t\rangle$; this is effected by the rule (15). Conjunction at the highest type level yields the translation (16).

(14) Mary kicked John or Bill.

(15) If α is a T with translation α' of type e, then α has a second translation $\alpha'' = \lambda P[{}^{\vee}P](\alpha')$, where P is a variable of type <s,<e,t>>.

(16) a. $\lambda P[{}^{\vee}P](j) \sqcup \lambda P[{}^{\vee}P](b) = \lambda P[[{}^{\vee}P](j) \vee [{}^{\vee}P](b)]$

b. $\lambda\mathcal{P}\lambda x\,{}^{\vee}\mathcal{P}({}^{\wedge}\lambda y\ kick'(y)(x))$

c. $\lambda x[kick'(j)(x) \vee kick'(b)(x)] = kick'(j) \sqcup kick'(b)$

According to the proposal of section I, kick' has minimal type <e,<e,t>>, so it can not combine directly with (16a). However, its higher-type counterpart (16b) (cf. (10ii)) can combine with (16a), yielding (together with some logic) (16c), which can in turn combine with m, the translation of Mary.

Notice that conjunction of the arguments of kick' at a higher type level has the effect of distributing the function kick' over its conjoined arguments. By imitating the type-promotion rule (15), we can obtain the same result for the phrase look for a maid or a cook.

(17) If α has a translation α' then α has a second translation $\alpha'' = \lambda\Phi[[{}^{\vee}\Phi]({}^{\wedge}\alpha')]$, where Φ is a variable of type <s,<type (α'),t>>

(18) a. $\lambda\Phi[[{}^{\vee}\Phi]({}^{\wedge}a\text{-}maid')] \sqcup \lambda\Phi[{}^{\vee}\Phi]({}^{\wedge}a\text{-}cook')$
$= \lambda\Phi[{}^{\vee}\Phi({}^{\wedge}a\text{-}maid') \vee {}^{\vee}\Phi({}^{\wedge}a\text{-}cook')]$

b. $look\text{-}for'(\mathcal{P})(m)$

c. $\lambda\Phi[{}^{\vee}\Phi({}^{\wedge}a\text{-}maid') \vee {}^{\vee}\Phi({}^{\vee}a\text{-}cook'](^{\wedge}\lambda\mathcal{P}look\text{-}for'(\mathcal{P})(m)$
$= look\text{-}for'({}^{\wedge}a\text{-}maid')(m) \vee look\text{-}for'({}^{\wedge}a\text{-}cook')(m)$

(17) is the general version of (15). We apply (17) to a-cook' and a-maid' and conjoin the results to get (18a). Rather than promoting look-for' to a yet higher type level TV_3, we suggest that (18a) be quantified into the expression (18b), which has type t, yielding the desired result (18c) (cf. rule T14 of PTQ).

This is the type promotion/quantifying in solution to the wide scope or problem. Before dismissing it, let us mention an advantage. The analysis predicts that, like quantifiers, or-phrases suffer multiple scope ambiguities in sentences with several intensional operators. This seems correct; in (19), the conjoined verb phrase can take scope between say and believe (meaning that John is uncertain about which property was attributed to Mary), as well as above believe (meaning that the speaker is uncertain).

(18) John believes that Bill said that Mary was drinking or playing video games.

We are aware of two defects in the type-promotion/quantifying-in proposal. First, the analysis predicts that there are wide-scope

readings for phrases conjoined by and. In many cases, it is impossible to check this prediction. For instance, the (normal) de dicto readings of (20a) and (20b) do not differ in truth conditions.

(20) a. Mary is looking for a maid and a cook.

b. Mary is looking for a maid and looking for a cook.

(21) a. Bill hopes that someone will hire a maid and a cook.

b. Bill hopes that someone will hire a maid and hopes that someone will hire a cook.

c. Bill hopes that someone will hire a maid or a cook.

But in cases where the truth conditions are different, there clearly is no wide-scope and reading. (21a) has no de dicto reading equivalent to the de dicto reading of (21b); (someone should be read as subordinate in scope to hope. Note that there is a wide-scope de dicto reading for (21c), again suggested by the condition "... but I don't know which".)

Second, wide-scope or participates in the curious anaphoric processes associated with indefinite noun-phrases, processes notoriously unamenable to a description via quantifier scope. Just as a quantifier scope treatment of (22a) would require converting an existential quantifier to a universal one (every donkey is such that if Pedro owns it, he beats it), the quantifier scope treatment of one reading of (22b) requires that or be replaced by and (swimming and dancing are properties P such that if Mary has P, then Sue has P).

(22) a. If Pedro owns a donkey, he beats it.

b. If Mary is swimming or dancing, then Sue is.

These facts suggest that the appropriate model for a treatment of wide-scope or is not quantifier scope, but a theory of indefinite noun phrases. Accordingly, we will now describe a way of extending the theory of indefinites proposed in Kamp [6] and Heim [5] to cover the wide-scope or data. The following exposition, which is most parallel to Heim's, is not intended as an adequate or complete description of their proposals.

According to Heim and Kamp, indefinites have no quantificational force of their own. Instead, their translations contain free variables which can be bound by quantifiers. We will take the translation of a syntactic phrase to be an ordered pair consisting of an intensional logic translation and a set of variables (the selection store) available for binding. This is illustrated in the derivation below.

(23) (i) $[\text{a man}]_3 \rightarrow \lambda P[\text{man}'(x_3) \wedge P\{x_3\}], \{x_3\}$

(ii) $[\text{a donkey}]_4 \rightarrow \lambda P[\text{donkey}'(x_4) \wedge P\{x_4\}], \{x_4\}$

(iii) own a donkey$_4$ $\rightarrow$ $\lambda x[\text{donkey}'(x_4) \wedge \text{own}(x,x_4)]$, $\{x_4\}$

(iv) a man owns a donkey $\rightarrow$
$\text{man}'(x_3) \wedge \text{donkey}'(x_4) \wedge \text{own}'(x_3,x_4)$, $\{x_3,x_4\}$

(v) he$_3$ beats it$_4$ $\rightarrow$ $\text{beat}'(x_3,x_4)$, $\emptyset$

(vi) if a man owns a donkey, he beats it $\rightarrow$
$[\text{man}'(x_3) \wedge \text{donkey}'(x_4) \wedge \text{own}'(x_3,x_4)] \rightarrow \text{beat}'(x_3,x_4)$, $\{x_3,x_4\}$

(vii) always, if a man$_3$ owns a donkey$_4$ he$_3$ beats it$_4$ $\rightarrow$
$\forall \{x_3,x_4\}[[\text{man}'(x_3) \wedge \text{donkey}'(x_4) \wedge \text{own}'(x_3,x_4)] \rightarrow \text{beat}'(x_3,x_4)]$, $\emptyset$

The translations of a man and a donkey involve no quantifiers. Instead, a variable (x_3 in (i)) is included in the selection store of the translation. As indicated in (iv), when constituents combine, the selection store of the result is the union of the selection stores of the parts. Only indefinites introduce indices into the selection store. Thus, while the denotation in (v) contains free variables, it has an empty selection store.

In the final step of the derivation, the adverb of quantification always binds the variables in the selection store. Heim, following Lewis [8], describes always as an 'unselective' quantifier which can bind any number of variables. Notice that the expression following $\forall$ in (vii) is a set; the intended semantics for formulas like this is given by (24). We take the domain of an assignment function to be the set of variable names.

(24) $\forall$ Sϕ is true in a world w with respect to an assignment function g iff for all g' such that for all variables v, v $\notin$ S $\rightarrow$ g(v) = g'(v), ϕ is true in w with respect to g'.

Kamp's and Heim's theories of indefinites provide an account of the properties that distinguish indefinites from quantifier phrases like every man, no man on the one hand and from proper names and other referring expressions like this book on the other hand. On their treatment, the semantic contribution of an indefinite NP like a donkey consists of two parts: a variable x_i (or a higher-type pronoun meaning, $\lambda P[P(x_i)]$) and an open proposition donkey'(x_i), attached at a level corresponding to the scope of the indefinite. (In example (23), we introduced these together, corresponding to a PTQ 'direct insertion' derivation.) The variable may be bound by some 'unselective quantifier' like always, or by the interpretation rules associated with modals, negation, propositional-attitude verbs, etc., elements which have typically been regarded as having scope but not commonly regarded as variable-binders. If the variable is not bound by any such operator internal

to the sentence, it is in effect bound by a discourse-level existential quantifier. Thus indefinites, like quantifier phrases and unlike proper names, do have relevant scope, but unlike quantifier phrases, they do not have variable-binders as part of their meanings. The apparent 'referential' interpretation of indefinites, Heim argues, can be explained by the possibility of leaving the introduced variable free at the sentence level, to be bound at the discourse level; the scope-island escaping properties of indefinites are captured on her approach by restricting scope-island constraints to genuine quantifier phrases.[6]

The logical parallelism between disjunction and existential quantification is well-known; disjunction is tantamount to existential quantification over an explicitly given finite domain. We believe that the exceptional behavior of or in English noted above can be accounted for in much the same way that Kamp and Heim account for the discrepancies between English indefinites and what would be predicted for straightforward existential quantifier phrases. Without giving details of all the relevant rules, we illustrate the extension of their treatment of indefinites to disjunction with examples.

(25) If John lost a watch or a compass, Mary found it.

With the PTQ-translation of a watch or a compass, we can only bind the pronoun it via the non-preferred reading which assigns maximal scope to the disjoined phrase. To make the more natural 'donkey-sentence' reading available, we let disjunctions, like indefinites, introduce a single new free variable (corresponding to the introduction of a single discourse referent in Kamp's system or a single 'file card' in Heim's) together with a disjunctive condition on that variable.

(26) (i) a watch or a compass →
$\lambda P[P(x_5) \wedge [watch'(x_5) \vee compass'(x_5)]$, $\{x_5\}$
("direct insertion reading")

(ii) - (iv) analogous to (23)(iii)-(v)

(v) if John lost a watch or a compass, Mary found it →
$\forall \{x_5\}[[[watch'(x_5) \vee compass'(x_5)] \wedge lost'(j,x_5)] \rightarrow found'(m,x_5)]$, $\emptyset$

(For the wide scope reading of (25), the condition "$watch'(x_5) \vee compass'(x_5)$" would be attached at the top sentence level, with just the pronoun meaning $\lambda P[P(x_5)]$ introduced at stage (i). This can be accomplished either by an analog of "quantifying in" as in PTQ or by an analog of the Cooper storage mechanism.)

One difference between indefinites and disjunction is that indefinites are restricted to noun phrases while disjunction is cross-categorial. Generalizing this treatment of disjunction cross-categorially does not overgenerate in the way that the quantifying-in solution did, because this treatment does not apply to and-conjunction (for the natural reason that a conjunctive specification of a single variable as equalling both of the two distinct individuals, properties, or the like would generally be contradictory[7].) We illustrate the extension with a derivation of (22b), repeated below; again we concentrate on the 'donkey-sentence' reading, with narrow-scope or.

(22b) If Mary is swimming or dancing, then Sue is.

(27) (i) swimming or dancing →
$\lambda x[P_7(x) \wedge [P_7 = \hat{}\text{swim}' \vee P_7 = \hat{}\text{dance}']]$, $\{P_7\}$

(ii) Sue is → $P_7(s)$, $\emptyset$

(iii) if Mary is swimming or dancing, then Sue is →
$[[P_7(m) \wedge [P_7 = \hat{}\text{swim}' \vee P_7 = \hat{}\text{dance}']] \rightarrow P_7(s)]$, $\{P_7\}$

(iv) (Always,) if Mary is swimming or dancing, then Sue is →
$\forall\{P_7\}[[P_7(m) \wedge [P_7 = \hat{}\text{swim}' \vee P_7 = \hat{}\text{dance}']] \rightarrow P_7(s)]$, $\emptyset$

To return to the wide-scope or cases with which we introduce this section, consider again sentence (13).

(13) Mary is looking for a maid or a cook.

The problematical reading was the one on which each disjunct was construed de dicto, but the or had wide scope ("... but I don't know which".) Since the disjunction rule applies to all types, we can apply it in this case to an intensional term-phrase variable, with the disjunctive specification of the variable brought in at the sentence level. The variable is bound by the general discourse-level existential closure rule.

(28) $\exists\{\mathcal{P}_2\}[\text{look-for}'(m,\mathcal{P}_2) \wedge [\mathcal{P}_2 = \hat{}\text{a-maid}' \vee \mathcal{P}_2 = \hat{}\text{a-cook}']]$, $\emptyset$

The same treatment will yield all the readings of sentence (18) above; the most deeply embedded clause will contain a VP-variable P_3, and the disjunctive condition $[P_3 = \hat{}\text{be-drinking}' \vee P_3 = \hat{}\text{be-playing-video-games}']$ may be attached at any clause level. Since each propositional attitude verb acts as an operator which can bind any variable free with its scope (simultaneously removing it from the selection store), the disjunction can be interpreted as the speaker's ignorance, or John's ignorance, or as part of what Bill said.

Since both Kamp's and Heim's theories involve a variety of departures from PTQ, considerable work remains to turn these sketchy remarks into an explicit set of rules in either framework. But assuming that this solution does work as uniformly as it appears to, it provides further support both for the generalized cross-categorial treatment of conjunction and disjunction and for a non-quantificational treatment of indefinites and disjunction.

Footnotes

1. This paper is an outgrowth of [11], and the acknowledgements there to Gazdar, Keenan and Faltz, Cooper, Dowty, Bach, Chao, Frazier, Higgins, Hajičova, Link, Sgall, and Weisler apply here as well. The problem treated in section 2 was found by the second author; the solution was found by the first author. The first was supported in part by NSF grant BNS80-14326 and in part by a University Fellowship; the second author was supported by a Faculty Fellowship and is grateful to the Linguistics Department for an accompanying teaching release.

2. For expository purposes we present oversimplified types, omitting s's. A more detailed treatment is given in [11].

3. Sentences (7) and (8) are probably ambiguous; translation (9) allows only the less natural "quantified in" reading of the object, and disallows the more natural intensional interpretation, whereas translation (4) plus the usual scope ambiguity allows both.

4. The principles for predicting the higher-type translation are given in [11].

5. The treatment in [7] preserves a uniform category-to-type assignment, but uses a homomorphism approach for some conjunctions and the equivalent of (2) for others. We compare our system with theirs in [11].

6. We are inclined to agree with Heim that, contrary to the claim of Fodor and Sag [3], indefinites can escape from scope islands to higher domains not just to the topmost ("referential") position. The facts seem to be the same for disjunction; compare (ii) with Heim's (i).

(i) Each teacher overheard the rumor that a student of his that he had thought very highly of had been called before the dean.

(ii) Every Englishman always cherished the conviction that his King or his Queen was noble and pure.

(Escape to positions in the immediate scope of a verb of thinking or or saying seems easier than escape to other non-topmost domains; we are not certain of the full range of the phenomenon.)

7. Not always literally contradictory. We may need an elaboration of the non-coindexing constraint to explain the impossibility of using a single 'discourse referent' in conjunctions to generate examples like (i).

(i) When Cicero and Tully spoke, he was eloquent.

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