Quantification over alternative intensions

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Abstract In footnote 13 on p. 85f. of his dissertation, Mats Rooth (Rooth (1985)) addresses certain peculiarities of his treatment of *only* as a quantifier over propositions. The current note elaborates on that footnote to conclude that the lack of adequacy of this approach to quantification is more severe than previously thought. Section 1 presents a gap in the alternative[s] semantics treatment of *only*. In Section 2 an attempt is made to close it by way of meaning postulates to eliminate 'degenerate' models (Rooth's term) in which extensions do not vary enough across Logical Space. In view of the lack of feasibility and systematicity of that approach, Section 3 explores a more principled, yet ultimately futile, strategy for determining 'realistic' models (Rooth's term) that reflect the extensional variation offered by Model Space as a whole. Section 4 points out the limitations any such repair encounters when it comes to sentences with non-contingent at-issue contents. Section 5 briefly discusses a variant of the interpretation of *only* as a quantifier over propositional alternatives and how it fares with respect to the problems addressed in the previous sections.

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According to one version of alternative semantics, focus-sensitive operators like *only* quantify over propositions rather than individuals. [1] As a case in point, a sentence like:

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1 Cf. (Rooth 1985: ch. III), where a cross-categorial approach to focus-sensitive particles is developed. The slightly more general treatment of alternative quantification in terms of predicate intensions ((Rooth 1985: ch. II); Rooth (1992)) is plagued by the same problems but will not be addressed before Section 5. Certain aspects of both approaches, such as context dependence, domain restrictions, compositionality, and multiple or wide foci, will be neglected in the following, since they are irrelevant for the comparison with ordinary quantification; only the conventional implicature triggered by *only* will make a short appearance in Section 4.

(1) Only Mary is asleep.

comes out as expressing: [2]

(2) $(\forall p)\ [[\ ^{\vee}p \wedge (\exists x)\ p =\ ^{\wedge}\mathbf{S}(x)] \rightarrow p = {}^{\wedge}\mathbf{S}(\mathbf{m})]$

rather than the more straightforward predicate logic formalisation:

(3) $(\forall x)[\mathbf{S}(x) \rightarrow x = \mathbf{m}]$

Mats Rooth (*ibid.*) observed that, given the standard modelling of propositions as regions in Logical Space, this analysis requires a certain abundance of possible worlds: (2) only comes out as equivalent to (3) if (4) holds, i.e., if the proposition denoted by '$^{\wedge}\mathbf{S}(\mathbf{m})$' – that Mary is asleep – differs from the proposition denoted by '$^{\wedge}\mathbf{S}(x)$' – that x is asleep – for any x other than Mary:[3]

(4) $(\forall x)[x \neq \mathbf{m} \rightarrow {}^{\wedge}\mathbf{S}(x) \neq {}^{\wedge}\mathbf{S}(\mathbf{m})]$

Generalising (4) from (1) by abstracting from Mary, the equivalence of (2) and (3) turns out to impose a certain degree of granularity on the propositions expressed by simple predications:

(5) $(\forall x)(\forall y)[x \neq y \rightarrow {}^{\wedge}\mathbf{S}(x) \neq {}^{\wedge}\mathbf{S}(y)]$

The possible worlds setting as such does not guarantee the truth of (5). In particular, (5) does not hold in models in which the number n of individuals by far exceeds the number m of worlds in that $n > 2^m$: for some x and y the propositions denoted by '$^{\wedge}\mathbf{S}(x)$' and '$^{\wedge}\mathbf{S}(y)$' would have to coincide, since there are only 2^m propositions to begin with. Similarly, if $n > 2$ and the intension of **S** happens to be rigid, the propositions denoted by '$^{\wedge}\mathbf{S}(x)$' and '$^{\wedge}\mathbf{S}(y)$' coincide for at least two x and y in the (constant) extension of **S**. In order to guarantee (4), then, such 'degenerate' models (Rooth, *ibid.*) would have to be discarded by semantic theory. This could be achieved by way of meaning postulates (or similar constraints) to the effect that the lexical predicates must satisfy (5) in lieu of **S**. However, the above reasoning extends way beyond the realm of lexical intransitives. Thus, e.g., in order for the interpretations *(b)*

2 Apart from the self-explanatory (and eliminable) terms for n-tuples of individuals, the intensional type logic formulae used here are largely in line with Montague's notation (Montague (1970)). In particular, '$^{\wedge}$' abstracts over the (implicit) world parameter, and '$^{\vee}$' indicates application to the world of evaluation; thus if φ is a truth-valuable formula, '$^{\wedge}\varphi$' stands for the proposition expressed by φ; and if π stands for a proposition, the formula '$^{\vee}\pi$' says that π is true (of the world at stake).

3 It is understood that the individual constants corresponding to proper names (like **m**, which is supposed to translate *Mary*) are rigid designators in the sense of Kripke (1972); this assumption is standardly being taken care of by meaning postulates like '$(\exists x)\ \Box\ \mathbf{m} = x$'.

of the sentences *(a)* under (6)–(9) to come out right, the corresponding assumptions *(c)* turn out to be crucial: [4]

(6) a. John only meets MARY.
b. $(\forall p)[[{}^{\vee}p \wedge (\exists x)\ p = {}^{\wedge}\mathbf{M}(\mathbf{j},x)] \rightarrow p = {}^{\wedge}\mathbf{M(j,m)}]$
c. $(\forall x_1)(\forall x_2)(\forall y_1)(\forall y_2)[(x_1,x_2) \neq (y_1,y_2) \rightarrow {}^{\wedge}\mathbf{M}(x_1,x_2) \neq {}^{\wedge}\mathbf{M}(y_1,y_2)]$

(7) a. John only introduces MARY to Sue.
b. $(\forall p)[[{}^{\vee}p \wedge (\exists x)\ p = {}^{\wedge}\mathbf{I}(\mathbf{j},x,\mathbf{s})] \wedge \rightarrow p = {}^{\wedge}\mathbf{I(j,m,s)}]$
c. $(\forall x_1)(\forall x_2)(\forall x_3)(\forall y_1)(\forall y_2)(\forall y_3)[(x_1,x_2,x_3) \neq (y_1,y_2,y_3) \rightarrow {}^{\wedge}\mathbf{I}(x_1,x_2,x_3) \neq {}^{\wedge}\mathbf{I}(y_1,y_2,y_3)]$

(8) a. Only Mary is both drunk and asleep.
b. $(\forall p)[[{}^{\vee}p \wedge (\exists x)\ p = {}^{\wedge}[\mathbf{D}(x) \wedge \mathbf{S}(x)]] \rightarrow p = {}^{\wedge}[\mathbf{D}(\mathbf{m}) \wedge \mathbf{S}(\mathbf{m})]]$
c. $(\forall x)(\forall y)[x \neq y \rightarrow {}^{\wedge}[\mathbf{D}(x) \wedge \mathbf{S}(x)] \neq {}^{\wedge}[\mathbf{D}(y) \wedge \mathbf{S}(y)]]$

(9) a. John only knows that MARY knows that Harry introduces Bill to Sue.
b. $(\forall p)[[{}^{\vee}p \wedge (\exists x)\ p = {}^{\wedge}\mathbf{K}(\mathbf{j},{}^{\wedge}\mathbf{K}(x,{}^{\wedge}\mathbf{I}(\mathbf{h},\mathbf{b},\mathbf{s})))] \rightarrow p = {}^{\wedge}\mathbf{K}(\mathbf{j},{}^{\wedge}\mathbf{K}(\mathbf{m},{}^{\wedge}\mathbf{I}(\mathbf{h},\mathbf{b},\mathbf{s})))]$
c. $(\forall x_1)\ldots(\forall x_5)(\forall y_1)\ldots(\forall y_5)[(x_1,\ldots,x_5) \neq (y_1,\ldots,y_5) \rightarrow {}^{\wedge}\mathbf{K}(x_1,{}^{\wedge}\mathbf{K}(x_2,{}^{\wedge}\mathbf{I}(x_3,x_4,x_5))) \neq {}^{\wedge}\mathbf{K}(y_1,{}^{\wedge}\mathbf{K}(y_2,{}^{\wedge}\mathbf{I}(y3,y4,y5)))]$

(6) and (7) illustrate that an adequate generalisation of (5) would have to cover lexical verbs of higher valencies, like the binary and ternary predicates **M**[*eet*] and **I**[*ntroduce*]. (8) and (9) indicate that the rôle of the predicate **S** in (5) can be played by non-lexical (*n*-ary) predicates like:

(10) $\lambda x.[\mathbf{D}(x) \wedge \mathbf{S}(x)]$

(11) $\lambda x_5.\lambda x_4.\lambda x_3.\lambda x_2.\lambda x_1.\mathbf{K}(x_1,{}^{\wedge}\mathbf{K}(x_2,{}^{\wedge}\mathbf{I}(x_3,x_4,x_5)))$

the general pattern being:[5]

(12) $(\forall \overrightarrow{x})(\forall \overrightarrow{y})[\overrightarrow{x} \neq \overrightarrow{y} \rightarrow [{}^{\wedge}R\overrightarrow{\{x\}}] \neq [{}^{\wedge}R\overrightarrow{\{y\}}]]$

where '$\overrightarrow{x}$' and '$\overrightarrow{y}$' range over *n*-tuples of individuals (for fixed but arbitrary *n*).

4 To see this, as in the case of (1), variants of the (*a*)-sentences with different names must be considered.

5 The curly brackets, which indicate application of the extension of a property, are again part of the intensional logic of Montague (1970), where they reflect a certain asymmetry between non-logical constants (like **S**) and variables (like *R*).

2

Closer inspection of the counter-examples to (5) reveals that there is something wrong with them for independent reasons: such 'degenerate' models according to which Logical Space is severely limited, ought be done away with anyway. What distinguishes such models from more 'realistic' (Rooth, *ibid.*) contenders, is their austerity: rather than varying freely across Logical Space, they restrict the range of possible extensions of **S**[*leep*] in that it does not cover every set (of individuals). This defect could be remedied in a natural way by demanding a maximal degree of extensional variation:

(13) $(\forall X)\Diamond[\mathbf{S} = X]$

where 'X' ranges over arbitrary sets of individuals. It is readily seen that (13) implies (5) (though not *vice versa*).[6] In the case of **M** and **I**, too, it may seem natural to derive the pertinent requirements (6-c) and (7-c) from some more general principle of variation, analogous to (13). Arguably, however, in these cases the variation is conceptually bounded. In particular, it would seem that the extension of **M** needs to be irreflexive; for it does not make (literal) sense for anyone to meet him- or herself. Similarly, one may want to exclude from the possible extensions of **I** any triples the third components of which coincide with either of the other two; for no one can introduce anyone (or anything) to him- or herself – on either reading of that clause.[7] Further, possibly more clear-cut cases along these lines may be found in comparatives and degree verbs like *exceed*. Given these considerations, it appears dubious that (13) should be postulated for lexical predicates across the board. If anything, the range of possible extensions needs to be determined predicate by predicate. On top of (13) we may thus have:

(14) $(\forall R)[[\neg(\exists x)R(x,x)] \rightarrow \Diamond[\mathbf{M} = R]]$

(15) $(\forall S)[[\neg(\exists x)(\exists y)[S(x,y,x) \vee S(x,y,y)]] \rightarrow \Diamond[\mathbf{I} = S]]$

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6 If (13) holds, then for any individual x there is a world at which the extension of **S** is the singleton $\{x\}$, thus excluding any individual y distinct from x and satisfying (5). On the other hand if, e.g., the extension of **S** could be any singleton but nothing else, (5) would hold, but (13) would not. – (13) strengthens, and is in the spirit of the notion of *lexical freedom* as defined in Keenan (1987) (methinks), which implies that for any X, there is some lexical predicate **P** satisfying '$\Diamond[\mathbf{P} = X]$'.

7 To the extent that time travel into the past is a coherent concept, it might seem that persons could meet, be introduced to, or introduce others to, themselves. Arguably, however, such scenarios involve multiple copies or representations of one person – and thus more than one individual – rather than one person with more than one body. Similar things could be said about encountering and acquainting representations of persons in mirrors or on TV. – I am indebted to Peter Smith for valuable discussion and sharing his native intuitions relating to the meaning and use of *to meet*.

where '*R*' and '*S*' respectively range over binary and ternary relations, and so on, for any pertinent lexical predicate. However, this cannot be the whole story. For conjoined possibility does not imply compossibility, and so while both **D**[*runk*] and **S** may have an unlimited range, (5) and its **D**-variant do not exclude models according to which they are contradictories and thus the extension of (10) is necessarily empty. To ensure (8-c), the variation postulates would thus have to cover the compound predicate in (10):

(16) $(\forall X) \Diamond [\lambda x.[\mathbf{D}(x) \wedge \mathbf{S}(x) = X]]$

Similarly, for (9-c), one would need:

(17) $(\forall T) \Diamond [[\lambda x_5.\lambda x_4.\lambda x_3.\lambda x_2.\lambda x_1.\ \mathbf{K}(x_1,{}^{\wedge}\mathbf{K}(x_2,{}^{\wedge}\mathbf{I}(x_3,x_4,x_5)))] = T]$

with '*T*' ranging over five-place relations among individuals. One might hope to derive (17) from some general principle of extensional variation for **K** plus its veridicality:

(18) $(\forall x)(\forall p)[\mathbf{K}(x,p) \rightarrow {}^{\vee}p]$

Alas, at least the most obvious candidate for such a variation principle turns out to be inconsistent, due to a variant of Kaplan's paradox:[8]

(19) $(\forall A)[[\neg(\exists x)(\exists p)[A(x,p) \wedge \neg \Diamond {}^{\vee}p]] \rightarrow \Diamond[\mathbf{K} = A]]$

where '*A*' ranges over propositional attitudes, i.e., relations between individuals and sets of possible worlds. If (19) were true (in a model), then for any non-empty proposition *p*, the (Schönfinkeled) relation U × $\{p\}$ would be the extension of **K** at some world w_p, where *U* is the domain of individuals. Hence there would have to be a function *f* assigning to each non-empty proposition *p* a world w_p of which *p* is true and the sole proposition **K**nown by anyone (and also known by everyone); but since for distinct *p* and *q*, the extensions of **K** at w_p and w_q differ, *f* would be an injective mapping from all non-empty propositions to Logical Space – which is impossible for cardinality reasons. It thus seems safer to take (17) as an independent principle, along with (13)–(16).

Apart from idiosyncratic limitations due to lexical meanings as in (14)–(16), further restrictions on extensional variation ensue from interdependencies of lexical meanings. Thus, e.g., while unlimited variation seems to be desirable for the intersection of logically independent predicates like **D** and **S**, constraints would have to

8 Kaplan (1995); see also Kripke (2011). One may hope that a natural variation principle ensues from some independently motivated solution to Kaplan's paradox. However, this is not necessarily so; for example, merely restricting (19) along the lines of (Lewis 1986: 104ff.) would only shift the burden of characterising variation to the realm of thinkable propositions.

be imposed on the possible extensions when it comes to Boolean combinations of, say, **S** and **A**[*wake*], **K** and **B**[*elieve*], etc. In fact, the range of extensional variation of complex predicates seems to be largely delimited by independent meaning postulates that the variation principles would have to somehow take into account – and it is not clear how this could be done in a systematic way.

3

Formulating extensional variation postulates for ever more complex predicates appears neither feasible nor particularly insightful. Thus a more principled way to go about ought to be found; and, indeed, the examples of the previous section suggest a direction the search may take. For the kind of extensional variation that would secure the equivalence of (2) and (3) in a natural way, cannot only be found across the worlds of Logical Space but also across different models. In fact, even if the extension of a predicate does not vary sufficiently *within* a given ('degenerate') model, its extensions in *other* models ('degenerate' or not) may still display a maximal range of variation. More specifically, if D and W are arbitrary domains of individuals and possible worlds, the following holds:

(20) For any $X \subseteq D$, there is a $\mathscr{K}_0$ - model $\mathfrak{M} = (D, W, F_{\mathfrak{M}})$ and a world $w \in W$ such that:
•$F_{\mathfrak{M}}(\mathbf{S})(w) = X$

where $\mathscr{K}_0$ is the class of all models of intensional type logic. While (13) grants the predicate **S** a maximal amount of extensional variation across Logical Space, (20) records that the same variation can be observed across Model Space. Yet the latter variation does not depend on (13): $\mathscr{K}_0$ also contains models that do not satisfy (13) at every world – which means that they do not satisfy (13) at any world.[9]

In a similar vein, the extensions of **M** and **I** vary wildly across Model Space – even in the absence of meaning postulates (14) and (15):

(21) a. For any $R \subseteq D^2$ there is a $\mathscr{K}_0$-model $\mathfrak{M} = (D, W, F_{\mathfrak{M}})$ and a world $w \in W$ such that:
•$F_{\mathfrak{M}}(\mathbf{M})(w) = R$.
b. For any $S \subseteq D^3$ there is a $\mathscr{K}_0$-model $\mathfrak{M} = (D, W, F_{\mathfrak{M}})$ and a world $w \in W$ such that:
•$F_{\mathfrak{M}}(\mathbf{I})(w) = S$.

9 This is so because (13) does not make implicit reference to a world of evaluation, i.e., the formula is *modally closed* in the sense of (Gallin 1975: 14).

As a consequence, the variation noted in (21) abides if Model Space is restricted by irreflexivity postulates for **M** and **I**:

(22) a. $\neg(\exists x)\mathbf{M}(x,x)$
b. $\neg(\exists x)(\exists y)[\mathbf{I}(x,y,x) \vee \mathbf{I}(x,y,y)]$

In other words, if $\mathscr{K}_1$ is the class of models satisfying (22), then (23) holds:

(23) a. For any irreflexive $R \subseteq D^2$ there's is a $\mathscr{K}_1$-model $\mathfrak{M} = (D, W, F_{\mathfrak{M}})$ and a world $w \in W$ such that:
•$F_{\mathfrak{M}}(\mathbf{M})(w) = R$.
b. For any irreflexive$_3$ $S \subseteq D^3$ there's is a $\mathscr{K}_1$-model $\mathfrak{M} = (D, W, F_{\mathfrak{M}})$ and a world $w \in W$ such that:
•$F_{\mathfrak{M}}(\mathbf{I})(w) = S$.

where *irreflexivity*$_3$ is the property attributed to (the extension of) **I** in (22-b). Moreover, it is readily shown (not here, though) that the unlimited variation does not stop at compound predicates like (10) and (11), whose extensions vary freely across Model Space, independently of any trans-world variation principles like (16) and (17).

It is important to realise that cross-model variation cannot make up for a lack of cross-world variation.[10] Yet it is the latter that leads to a lack of intensions in general, and propositions in particular, which again may have dramatic consequences such as the non-equivalence of (2) and (3). Since propositions, and intensions in general, are constructed by abstracting from worlds rather than models, the varying extensions of the latter are inaccessible to them. However, one may still hope that some, 'realistic' models would capture enough of the cross-model variation within their Logical Spaces. More concretely, given a class $\mathscr{K}$ of models, a model $\mathfrak{M}^*$ may be said to *reflect* $\mathscr{K}$ in that for any $\mathfrak{M} \in \mathscr{K}$ and any $\mathfrak{M}$-world w there is an $\mathfrak{M}^*$-world w^* in which the same (type-logical) sentences are true:

(24) $\{\varphi | \mathfrak{M} \models_w \varphi\} = \{\varphi | \mathfrak{M}^* \models_{w*} \varphi\}$[11]

In particular, a model that reflects a class of models across which the extension of a (definable) predicate varies *ad libitum*, makes any combination of attributions of that predicate true at at least one of its worlds. Hence, e.g., any set Σ of sentences of the form '**S(n)**', where '**n**' is an individual constant, will have to be true at some world w_Σ. And if pertinent postulates guarantee the distinctness of sufficiently many constants, it would seem that the equivalence of (2) and (3) ought to emerge.

10 See also (Zimmermann 2011: 798f.) for this point.
11 '$\models_w$' relates (type-logical) models $\mathfrak{M}$ to the *sentences* (closed, truth-valuable formulae) that are true at world w (in $\mathfrak{M}$'s Logical Space).

It is therefore natural to look for Rooth's 'realistic' models among the ones that reflect the class $\mathscr{K}^*$ of models satisfying all pertinent meaning postulates, including irreflexivity assumptions like (22) but excluding the variation principles (13)–(17); after all, the intended effect of the latter shows in the variation across $\mathscr{K}^*$ and should thus be reflected in any $\mathfrak{M}^*$ satisfying (24). Alas, this search strategy is futile. To see this, one may consider a random contingency:

(25) Mary is asleep.

Clearly, $\mathscr{K}_0$ contains models $\mathfrak{M}_0$ and $\mathfrak{M}_1$ according to which (25) expresses a contradiction and a possibility, respectively:

(26) a. $\mathfrak{M}_0 \not\models_w \mathbf{S}(\mathbf{m})$, for any $\mathfrak{M}_0$-world w;
b. $\mathfrak{M}_1 \models_{w'} \mathbf{S}(\mathbf{m})$, for some $\mathfrak{M}_1$-world w'.

With the help of the unrestricted possibility operator '$\Diamond$', (26) may be expressed as a statement about arbitrary $\mathfrak{M}_0$- and $\mathfrak{M}_1$-worlds w and w': [12]

(27) a. $\mathfrak{M}_0 \models_w \neg\Diamond\mathbf{S}(\mathbf{m})$
b. $\mathfrak{M}_1 \models_{w'} \Diamond\mathbf{S}(\mathbf{m})$

In the (assumed) absence of any lexical postulates restricting the extensional range of the constants '**S**' and '**m**', $\mathscr{K}^*$, too, ought to contain models $\mathfrak{M}_0$ and $\mathfrak{M}_1$ satisfying (26) and (27). Hence a 'realistic' model $\mathfrak{M}^*$ that reflects $\mathscr{K}^*$ would have to contain worlds w_0 and w_1 that reflect arbitrary $\mathfrak{M}_0$- and $\mathfrak{M}_1$-worlds w and w':

(28) a. $\mathfrak{M}^* \models_{w_0} \neg\Diamond\mathbf{S}(\mathbf{m})$
b. $\mathfrak{M}^* \not\models_{w_1} \Diamond\mathbf{S}(\mathbf{m})$

But the formulae in (28-a) are modally closed and so the truth of neither depends on the particular choice of worlds w_0 and w_1; hence (29) would have to hold for all w^* in $\mathfrak{M}^*$'s Logical Space:

(29) $\mathfrak{M}^* \models_{w^*} [[\neg\Diamond\mathbf{S}(\mathbf{m})] \wedge \Diamond\mathbf{S}(\mathbf{m})]$

... which cannot be.[13] So the strategy of narrowing down the class of all models to the more realistic ones in one fell swoop, leads to a contradiction.

12 Again, the two formulae under scrutiny are modally closed. – Without loss of generality, one may actually assume that $\mathfrak{M}_0$ and $\mathfrak{M}_1$ share their Logical Space and hence that w = w'; this is so because Model Space is closed under arbitrary isomorphisms.

13 The situation is vaguely reminiscent of modal logic: in the canonical model of the logic of universal accessibility (S5), accessibility is an equivalence relation but not universal; cf. (Hughes & Cresswell 1996: 118).

One may try and escape this conclusion by excluding modal formulae like the ones in (27) from the reflection requirement (24). Again, it is not obvious how this could be done in a sensible way. In particular, the above reasoning does not depend on the fact that the formulae under scrutiny are modally closed; a conjunction of either with some contingent sentence would have done just as well. Moreover, while it is crucial that the formulae involve intensionality, i.e., abstraction from particular worlds, restricting (24) to purely extensional φ would make the reflection property unattractively weak. Thus, e.g., in order for (9-a) and its focus variants to come out right, a 'realistic' model ought to reflect the extensional range of the property in (11) and thus make arbitrary combinations of attribution of it true, as long as they obey the relevant postulates (like the veridicality of **K**). However, these attributions are not extensional in that they involve the propositional attitude **K**[nowledge].

4

Since it does not seem to be possible to formulate general principles of extensional variation that would allow to derive them, the relevant instances of (12) –here repeated as (30) – would have to be assumed as meaning postulates in their own right:

(30) $$(\forall \overrightarrow{x})(\forall \overrightarrow{y})[\overrightarrow{x} \neq \overrightarrow{y} \rightarrow [{}^\wedge R\{\overrightarrow{x}\}] \neq [{}^\wedge R\{\overrightarrow{y}\}]]$$ [= (12)]

The problem is to define these instances. To begin with, (30) cannot be assumed for arbitrary (n-place) relations R; for its universal closure (31) turns out to be inconsistent:

(31) $$(\forall R)(\forall \overrightarrow{x})(\forall \overrightarrow{y})[\overrightarrow{x} \neq \overrightarrow{y} \rightarrow [{}^\wedge R\{\overrightarrow{x}\}] \neq [{}^\wedge R\{\overrightarrow{y}\}]]$$

Among others, certain trivial properties such as self-identity, or being identical with some fixed n-tuple, contradict (31). Hence (31) ought to be suitably relativised by some (2nd order) property $\wp$, if only for consistence:

(32) $$(\forall R)[\wp(R) \rightarrow (\forall \overrightarrow{x})(\forall \overrightarrow{y})[\overrightarrow{x} \neq \overrightarrow{y} \rightarrow {}^\wedge R\{\overrightarrow{x}\} \neq {}^\wedge R\{\overrightarrow{y}\}]]$$

It is not clear how $\wp$ should be specified so as to imply all relevant instances of (30), including (5) and (6-c)–(9-c). The problem is not to filter out those R that contradict (30): (30) is satisfied by precisely the relations that never conflate the propositions that two distinct n-tuples stand in them. Hence in (32), $\wp$ cannot be the property $\wp^*$ of being consistent with (30): given any relation R, either $\wp^*(R)$ holds or its negation does; and R is inconsistent with (30) precisely in the latter case. So the properties that are consistent with (30) are the ones that satisfy (30), which means that equating $\wp$ with $\wp^*$ would turn (32) into the tautology that any R that satisfies (30), satisfies

(30). The problem, then, is to find a property $\wp$ shared by relations like **S, L, I**, (10), and (11) so that (30) would make sure that they also have $\wp^*$.

One may thus ask what the relations mentioned have in common. For one thing, they are all definable in terms of the lexical predicates featuring in indirect interpretation. However, $\wp$ must not be identified with definability in **S, L, I, K**, etc.; for not all definable relations R can be assumed to satisfy (30): as already indicated, at least some of them, like self-identity (which is definable in these constants, even without them), should be excluded for logical reasons.[14]

In order to define a suitable restriction $\wp$, one may try to characterise those relations that are definable from the logical representations of sentences like (1) and (6-a)–(9-a). In principle, this could be done in terms of an indirect interpretation algorithm that takes (the LFs of) natural language sentences to their translations. Yet again, this method of defining $\wp$ would overshoot unless at least tautologies and contradictions are exempted from it. In fact, Mats Rooth (*ibid.*) already observed that the strategy of mimicking individual by propositional quantification reaches its limits when it comes to rigid intensions:[15]

(33) Only three is an odd number .

(33) is clearly false, and so is a straightforward predicate logic formalisation of it:

(34) $(\forall x)[\mathbf{O}(x) \rightarrow x = \mathbf{3}]$

However, (33) comes out true on an analysis of *only* as a quantifier over propositions:

(35) $(\forall p)[[{}^{\vee}p \wedge (\exists x)p = {}^{\wedge}\mathbf{O}(x)] \rightarrow p = {}^{\wedge}\mathbf{O}(\mathbf{3})]$

The reason for this failure lies in a peculiarity of the predicate **O**[*dd number*], whose extension is taken to not vary across Logical Space.[16] Hence, in particular, any proposition of the form '${}^{\wedge}\mathbf{O}(x)$' will coincide with either the empty set or all of Logical Space. So the latter is the only true proposition of that form, thus verifying (35). Unlike in the case of (1) and (2), though, there is no escape from the conclusion that (35) is an inadequate formalisation of the truth conditions of (33), which, again, seem aptly captured by (34). One may therefore doubt that the failure to adequately capture the truth-conditions of (33) should be adduced to the notorious lack of fine-grainedness of possible world semantics, as Rooth (*loc. cit.*) seems to suggest: after

14 Actually, self-identity does satisfy (30) in totally 'degenerate' models with just one individual.

15 For expository reasons, Rooth (*ibid.*) uses a slightly more involved example to prove the point: *Nine is only the square of THREE*.

16 As in the case of proper names (cf. fn. 3 above), this would have to be guaranteed by a specific meaning postulate, e.g., '$(\exists X)\,\Box\,\mathbf{O} = X$'.

all, the adequate formalisation (34), too, can be expressed in that very framework. Moreover, the inadequacies due to rigid intensions may lead to contingencies:

(36) Only Mary is one of John and Mary and exactly as tall as either one.

Clearly, under the assumption (!) that *John* and *Mary* are (rigid) names of different persons, (36) ought to come out as a contradiction. And again it does if the sentence is analysed in terms of quantification over individuals:

(37) $(\forall x)[[x = \mathbf{j} \vee x = \mathbf{m}] \wedge (\forall y)[[[y = \mathbf{j} \vee y = \mathbf{m}] \rightarrow \mathbf{h}(x) = \mathbf{h}(y)] \rightarrow x = \mathbf{m}]]$

(37) implies (by Universal Instantiation):

- $[[\mathbf{j} = \mathbf{j} \vee \mathbf{j} = \mathbf{m}] \wedge (\forall y)[[[y = \mathbf{j} \vee y = \mathbf{m}] \rightarrow \mathbf{h}(\mathbf{j}) = \mathbf{h}(y)] \rightarrow \mathbf{j} = \mathbf{m}]]$

which, given the validity of its left conjunct is equivalent to:

- $(\forall y)[[[y = \mathbf{j} \vee y = \mathbf{m}] \rightarrow \mathbf{h}(\mathbf{j}) = \mathbf{h}(y)] \rightarrow \mathbf{j} = \mathbf{m}]$

which in turn implies (by Universal Instantiation):

- $[[[\mathbf{j} = \mathbf{j} \vee \mathbf{j} = \mathbf{m}] \rightarrow \mathbf{h}(\mathbf{j}) = \mathbf{h}(\mathbf{j})] \rightarrow \mathbf{j} = \mathbf{m}]$

which, given the validity of its antecedent is equivalent to:

- $\mathbf{j} = \mathbf{m}$

which is contradictory, by assumption (!). On the other hand, given a (hardly objectionable) analysis of the predicate *be one of John and Mary* as rigidly denoting a set of two individuals, alternative semantics has (37) come out as:

(38) $(\forall p)[[{}^{\vee}p \wedge (\exists x)p = {}^{\wedge}[[x = \mathbf{j} \vee x = \mathbf{m}] \wedge (\forall y)[[y = \mathbf{j} \vee y = \mathbf{m}] \rightarrow \mathbf{h}(x) = \mathbf{h}(y)]]] \rightarrow p = {}^{\wedge}[[\mathbf{m} = \mathbf{j} \vee \mathbf{m} = \mathbf{m}] \wedge (\forall y)[[y = \mathbf{j} \vee y = \mathbf{m}] \rightarrow \mathbf{h}(\mathbf{m}) = \mathbf{h}(y)]]]$

Closer inspection reveals that (38) is logically valid. To see this, one may observe that (38) can be equivalently rewritten as the tautology (39), where $p_{\mathbf{j}\approx\mathbf{m}}$ abbreviates '${}^{\wedge}[\mathbf{h}(\mathbf{j}) = \mathbf{h}(\mathbf{m})]$':

(39) $(\forall p)[[{}^{\vee}p \wedge p = p_{\mathbf{j}\approx\mathbf{m}}] \rightarrow p = p_{\mathbf{j}\approx\mathbf{m}}]$

The details of the reformulation are kindly left to the reader. As in the case of (33), then, the alternative treatment misanalyses a contradiction, (36), as valid. However, there is a difference that comes to the fore once *full [conventional] content* is taken into consideration, i.e., the combination of ordinary (truth-conditional, assertoric, at-issue) content and conventional implicature. In the above formalisations the latter has been suppressed in the interest of readability.[17] In the case of sentences containing *only*, it can be identified with the (ordinary) content of the sentence without *only*. As a case in point, the predicate logic and alternative formalisations of the full

17 Rooth's analysis ((Rooth 1985: 120ff.)) takes both assertions and implicatures into account.

conventional content of (1) come out as (40) and (41), respectively, where the second line gives a logically equivalent (‘≡’) reformulation of the first:

(40) $[(\forall p)[[{}^{\vee}p \wedge (\exists x)p = {}^{\wedge}\mathbf{S}(x)] \rightarrow p = {}^{\wedge}\mathbf{S}(\mathbf{m})] \wedge \mathbf{S}(\mathbf{m})]$
≡ $[(\forall p)[[{}^{\vee}p \wedge (\exists x)p = {}^{\wedge}\mathbf{S}(x)] \leftrightarrow p = {}^{\wedge}\mathbf{S}(\mathbf{m})]$

(41) $[(\forall x)[\mathbf{S}(x) \rightarrow x = \mathbf{m}] \wedge \mathbf{S}(\mathbf{m})]$
≡ $(\forall x)[\mathbf{S}(x) \leftrightarrow x = \mathbf{m}]$

By the same token, the full content of (33) ought to be (42), as predicted by standard predicate logic formalisation, but comes out as (43) in alternative semantics:

(42) $(\forall x)[\mathbf{O}(x) \leftrightarrow x = \mathbf{3}]$

(43) $(\forall p)[[{}^{\vee}p \wedge (\exists x)\ p = {}^{\wedge}\mathbf{O}(x)] \leftrightarrow p = {}^{\wedge}\mathbf{O}(\mathbf{3})]$

Since the ordinary content (34) of (33) is already contradictory, so is the stronger full content (42). On the other hand, (43) only adds the uninformative conjunct that 3 is odd to the trivial ordinary content (35) of (33), and is thus also trivially true. So on both analyses of (33), ordinary content and full content coincide. This may be taken to confirm the suspicion that (33) is an exceptional or even neurotic case, somewhat beyond the proper area of application of possible worlds semantics.

However, things stand differently with (36), which also proved to come out as expressing a necessarily true assertoric content (39), instead of the (metaphysical) impossibility (37) it does express. And again, according to predicate logic formalisation, its full content is the same as its ordinary content. However, if added to the alleged trivial ordinary content (39) of (36), the conventional implicature triggered by *only* has an effect on the full content: according to the alternative analysis, the full content of (36) is that of its conventional implicature, i.e., the proposition that Mary is [one of John and Mary and] as tall as [either Mary or] John:

(44) $[(\forall p)[[{}^{\vee}p \wedge p = p_{\mathbf{j}\approx\mathbf{m}}] \rightarrow p = p_{\mathbf{j}\approx\mathbf{m}}] \wedge$
$[[\mathbf{m} = \mathbf{j} \vee \mathbf{m} = \mathbf{m}] \wedge (\forall y)[[y = \mathbf{j} \vee y = \mathbf{m}] \rightarrow \mathbf{h}(\mathbf{m}) = \mathbf{h}(y)]]]$
≡ $[(\forall p)[[{}^{\vee}p \wedge p = p_{\mathbf{j}\approx\mathbf{m}}] \rightarrow p = p_{\mathbf{j}\approx\mathbf{m}}] \wedge \mathbf{h}(\mathbf{m}) = \mathbf{h}(\mathbf{j})]$
≡ $\mathbf{h}(\mathbf{m}) = \mathbf{h}(\mathbf{j})$

The fact that contradictory sentences are misanalysed as expressing contingent propositions can hardly be blamed on the notorious lack of fine-grainedness of possible worlds semantics and may instead be taken as indication against the analysis of *only* as a propositional quantifier.

5

The upshot of the above reasoning is that quantification over individuals cannot be fully captured by quantification over propositions. However, in many applications of alternative semantics starting with (Rooth 1985: ch. II), (alternative) propositions give way to (alternative) intensions in general. As a case in point, instead of analysing (45-a) in terms of alternatives to its conventional implicature, as in (6-b) above (and repeated below), alternatives to the predicate intension may be quantified over to achieve a similar effect, as in (45-c):[18]

(45) a. John only meets MARY. [= (6-a)]
b. $(\forall p)[[\,^{\vee}p \wedge (\exists y)\ p = {}^{\wedge}\mathbf{M}(\mathbf{j},y)] \rightarrow p = {}^{\wedge}\mathbf{M}(\mathbf{j},\mathbf{m})]$ [≈ (6-b)]
c. $(\forall P)[[\ P\{\mathbf{j}\} \wedge (\exists y)\ P\ =\ \hat{x}\,\mathbf{M}(x,y)] \rightarrow P =\ \hat{x}\,\mathbf{M}(x,\mathbf{m})]]$

where 'P' ranges over properties, i.e., possible intensions of unary predicates. One may wonder whether (45-c) fares better than (45-b) with respect to the problems discussed above. To begin with, it should be noted that the two formulae are not equivalent. For although (45-c) implies (45-b), this implication does not reverse:

c⇒b: Given a true proposition p_0 denoted by '$^{\wedge}\ \mathbf{M}(\mathbf{j},y_0)$' (for some fixed y_0), it needs to be shown that p_0 is also the proposition denoted by '$^{\wedge}\ \mathbf{M}(\mathbf{j},\mathbf{m})$'. In view of the logical equivalence:[19]

- '$^{\wedge}\ \mathbf{M}(\mathbf{j},y_0)$' ≡ '$[\hat{x}\,\mathbf{M}(x,y_0)]\{\mathbf{j}\}$'

  p_0 is of the form '$^{\wedge}\ P_0\{\mathbf{j}\}$' (where P_0 is the property denoted by '$\hat{x}\,[\mathbf{M}(x,y_0)]$'). Thus, since p_0 is true (by assumption), '$P_0\{\mathbf{j}\}$' holds. It thus follows by (45-c) that '$\hat{x}\ \mathbf{M}(x,y_0)$' and '$\hat{x}\ \mathbf{M}(x,\mathbf{m})$' denote the same property – and thus '$^{\wedge}\ \mathbf{M}(\mathbf{j},y_0)$' and '$^{\wedge}\ \mathbf{M}(\mathbf{j},\mathbf{m})$' the same proposition p_0, given the above equivalence and:

- '$^{\wedge}\ \mathbf{M}(\mathbf{j},\mathbf{m})$' ≡ '$^{\wedge}[\hat{x}\,\mathbf{M}(x,\mathbf{m})]\{\mathbf{j}\}$'

b⇏c: One may consider a (totally 'degenerate') model with one world w and four individuals h, j, m, and s that interprets the individual constants $\mathbf{h},\mathbf{j},\mathbf{m}$, and $\mathbf{s}$ in the obvious way and assigns to $\mathbf{M}$ the set consisting of the following five pairs

- $(h,j),(h,s),(j,h),(j,m),(m,h)$

  as its extension (at w). Then (45-b) is trivially true (of w): '$\mathbf{M}(\mathbf{j},\mathbf{m})$' expresses the only true proposition $\{w\}$, which is thus of the form '$^{\wedge}\ \mathbf{M}(\mathbf{j},y)$', as required by (45-a). However, (45-c) is false (of w): '$\hat{x}\,\mathbf{M}(x,\mathbf{h})$' and '$\hat{x}\,\mathbf{M}(x,\mathbf{m})$' denote distinct properties: m lacks the latter (in w), but j has both – i.e., *two*

18 '$\hat{x}\ \varphi$' abbreviates '$^{\wedge}\lambda x.\ \varphi$', thus denoting the intension of a λ-abstracted predicate.
19 See, e.g., (Gallin 1975: ch. 1), for the relevant laws.

properties of the form '$\hat{x}\,\mathbf{M}(x,y)$' and not just the one denoted by '$\hat{x}\,\mathbf{M}(x,\mathbf{m})$', as (45-c) would have it.

Despite an increase in granularity brought out by this argument, the analysis (45-c) still cannot escape the worries and objections previously raised against quantification over propositions. In fact, a closer look at the model considered in the preceding paragraph reveals that properties are almost as indiscriminate as propositions: though (45-a) does come out false (as it should), the analogous (46-a) comes out true (though it shouldn't) if analysed as (46-b):

(46) a. Harry only meets SUE.
b. $(\forall P)[[\,P\{\mathbf{h}\} \wedge (\exists y)\ P\ =\ \hat{x}\,\mathbf{M}(x,y)] \rightarrow P =\ \hat{x}\,\mathbf{M}(x,\mathbf{s})]$

The reason for this again lies in a lack of fine-grainedness: '$\hat{x}\,\mathbf{M}(x,\mathbf{j})$' and '$\hat{x}\,\mathbf{M}(x,\mathbf{s})$' denote the same property, whose only possible extension is $\{h\}$, and this is the only property of the form '$\hat{x}\,\mathbf{M}(x,y)$' that h has (at w), just as (46-b) requires. So (46-b) is true even though, according to the only world of the model described, the referent of *Harry* stands in the relation denoted by $\mathbf{M}$ to more than one individual. Needless to say, even for the above 'degenerate' model, a predicate logic formalisation taking *only* as quantifying over individuals would get the truth values of both (45-a) and (46-a) right.

The example illustrates that the more general approach involving predicate intensions inherits the deficiencies of propositional quantification. This may come as a surprise in view of the adequacy of the predicate logic formalisation, according to which *only* is a quantifier over individuals and thus ought to be less discriminative than a quantifier over alternative properties. However, the finesse of the latter is lost on the individuals to be quantified over as the sample analysis (45-c) indicates and, more generally, a survey of the three quantifiers in question makes clear:[20]

(47) a. $\lambda y.\ \lambda Q.\ (\forall z)\ [Q\{z\} \rightarrow z\ =\ y]$
b. $\lambda p.\ \lambda \mathscr{A}.\ (\forall q)\ [[^{\vee}q \wedge \mathscr{A}(q)] \rightarrow q\ =\ p]$

20 (47-a) is a special case of a folklore analysis of *only* as a binary (non-conservative) quantifier; cf. (Geach 1980: 207ff.) for an early source. – (47-b) is Rooth's propositional quantifier analysis (Rooth 1985: 120), with some minor (and mostly notational) changes. In particular, Rooth's Russellian format has been Frege-Churched (in the sense of Kaplan (1975)); the variable ranging over the set of alternatives has been λ-bound (and renamed from 'C' to'$\mathscr{A}$'); and the conventional implicature has been omitted, in line with the analyses in Section 1. – (47-c) is adapted from Rooth's *domain selection* analysis of *only* (cf. p. 44). This 'direct' account of *only* as a quantifier over properties must not be confused with the type-shifted version derived from (47-b) in the cross-categorial account of (Rooth 1985: ch. III), which reads (on p. 121, again glossing over matters of notation and type regimentation):
(*) $\lambda x.\ \lambda P.\ \lambda \mathscr{A}.\ (\forall p)[[^{\vee}p \wedge\ \mathscr{A}(p)] \rightarrow p =^{\wedge}\ P\{x\}]$
Obviously, (*) boils down to the propositional quantifier analysis scrutinized in Sections 1–4 above.

c. $\lambda x.\, \lambda P.\, \lambda \mathscr{S}.\ (\forall S)\ [[S\{x\} \wedge \mathscr{S}(S)] \rightarrow S\ =\ P]$

The root of the problems encountered with the propositional quantifier analysis (47-b) of *only* can be brought out by the contrast with the individual quantifier formalisation (47-a) and the way the two are supposed to relate to their linguistic environment:

- The quantifier in (47-a) applies to (the referent of) the focussed element y_0 associated with *only* and (the intension of) the predicate Q_0 expressed by the rest of the sentence (minus *only*); the latter may be obtained by a standard λ-abstraction mechanism.[21] Taking (45-a) as a model, y_0 would be the bearer of the name *Mary*, and Q_0 the property denoted by '$\hat{z}\ \mathbf{M}(\mathbf{j},z)$'. Thus analysed, *only* imposes the assumedly correct truth condition on the resulting sentence, viz., that the singleton $\{y_0\}$ cover the extension of Q_0.
- The quantifier in (47-b), on the other hand, applies to the proposition $p_0\ = \{w|y_0 \in Q_0(w)\}$ expressed by the whole sentence (minus *only*) and the set $\mathscr{A}$ of its alternatives (including p_0 itself):
  - $\mathscr{A} = \{q|(\exists z)\ q = \{w|z \in Q_0(w)\}\}$

  In the above example (45-a), p_0 would thus be the proposition denoted by '${}^\wedge\mathbf{M}(\mathbf{j},\mathbf{m})$', and $\mathscr{A}$ would be the set of propositions of the form '${}^\wedge\mathbf{M}(\mathbf{j},z)$'. As observed in Section 1, the adequate truth conditions are not guaranteed to be expressible in terms of these two ingredients. The reason is that, in general, the 'natural' correspondence f between the domain of individuals and the alternative propositions in $\mathscr{A}$ defined by:
  - $f(z) = \{w|z \in Q_0(w)\}$, for any individual z,

  need not be injective (one-one). If it happens to be so (in a given model), then y_0 and Q_0 can be identified in terms of p_0:
  - $y_0 = f^{-1}(p_0)$
  - $Q_0(w) = \{z|w \in f(z)\}$

  …for any world w. The first equation holds because $f(y_0) = \{w|y_0 \in Q_0(w)\} = p_0$, and f is injective. The second equation holds by the definition of f (and very basic set theory). Given these identifications, the condition imposed by the quantifier in (47-a) obtains just in case the set $\mathscr{A}$ contains at most p_0 – as the quantifier in (47-b) would indeed have it. For if ("$\Rightarrow$"), at some world w_0, (47-a) applies to y_0 and Q_0 and $w_0 \in q_0 \in \mathscr{A}$ (for some given q_0), then $q_0 = \{w|z_0 \in Q_0(w)\}$, for some z_0, i.e., $q_0 = f(z_0)$. But then, since $w_0 \in q_0$, $z_0 \in Q_0(w_0)$, and thus: $z_0 = y_0$, by (47-a), and $q_0 = f(z_0) = f(y_0) = p_0$. – And if ("$\Leftarrow$"), (47-b) applies to p_0 and $\mathscr{A}$, then given any $z_0 \in Q_0(w_0) : w_0 \in f(z_0) \in \mathscr{A}$. Hence by (47-b), $f(z_0) = p_0 = f(y_0)$,

21 Cf. (Heim & Kratzer 1998: 96).

i.e., $z_0 = y_0$, since f is injective. – On the other hand, if f is not injective, the condition in (47-b) will be insufficient, as readers may verify for themselves.

- The quantifier in (47-c) applies to (the referent of) the subject x_0, (the intension of) the (clause) predicate P_0, and the set $\mathscr{S}$ of its alternatives:
  - $\mathscr{S} = \{S|(\exists z)(\forall w)S(w) = R_0(w)(z)\}$

  where R_0 is constructed from the predicate (minus the focussed element) by a λ-abstraction device, in analogy to the construction of Q_0 from the whole sentence; in particular, for any world $w : P_0(w) = R_0(w)(y_0)$. In the case of (45-a), R_0 would coincide with the relation expressed by **M**. And the three arguments would be: the referent of the name *John* (= x_0), the property denoted by '$\hat{x}\,\mathbf{M}(x,\mathbf{m})$' (= P_0), and the set of properties of the form '$\hat{x}\,\mathbf{M}(x,z)$' (= $\mathscr{S}$). Now the reason why these three ingredients do not suffice to guarantee the expressibility of adequate truth conditions comes to the fore: as in (47-b), but in contrast to (47-a), none of the arguments of the quantifier in (47-c) matches the focussed element y_0. So y_0 and Q_0 would have to be reconstructed from x_0, P_0, and $\mathscr{S}$ to get the truth conditions right. And as in the previous case, whether this can be done depends on whether a certain 'natural' mapping g from the domain of individuals to the alternative properties in $\mathscr{S}$ is injective, to wit:
  - $g(z)(w) = R_0(w)(z)$, for any individual z and world w

  If g is one-one, then again x_0 and Q_0 can be identified in terms of x_0 and P_0:
  - $y_0 = g^{-1}(P_0)$
  - $Q_0(w) = \{z|x_0 \in g(z)(w)\}[= \{z|x_0 \in R_0(w)(z)\}]$

  ... for any world w. The first equation holds because, for any $w : g(y_0)(w) = R_0(w)(y_0) = P_0(w)$, and hence $g(y_0) = P_0$ – and g is injective. The second equation holds due to the definition of g and because $Q_0(w) = \{z|x_0 \in R_0(w)(z)\}$, since R_0 derives by abstracting from the focussed element y_0 in $q_0 = \{w|x_0 \in R_0(w)(y_0)\}$. As expected, given these identifications, the condition in (47-c) holds just in case the truth condition of the quantifier in (47-a) is met. For if ("$\Rightarrow$") (47-a) applies to y_0 and Q_0 at some world w_0 and $x_0 \in S_0(w_0)$ (where $S_0 \in \mathscr{A}$), then for some z_0 and any world w : $S_0(w) = R_0(w)(z_0) = g(z_0)(w)$, by the definitions of $\mathscr{A}$ and g; in other words: $S_0 = g(z_0)$. But then, since $x_0 \in S_0(w_0) : z_0 \in Q_0(w_0)$, given the above characterisation of Q_0. Hence according to (47-a), $z_0 = x_0$, and so: $S_0 = g(z_0) = g(x_0) = P_0$, given the above characterisation of x_0. – And if ("$\Leftarrow$") (47-c) applies to x_0, P_0, and $\mathscr{S}$ (at some world w_0), then given some $z_0 \in Q_0(w_0) : x_0 \in g(z_0)(w_0)$, by the characterisation of Q_0. Moreover, since $g(z_0) \in \mathscr{S}$ (by the definition of $\mathscr{S}$), (47-c) applies to $g(z_0) = P_0$, and thus $z_0 = y_0$, given that g is injective. – Again, a closer inspection of the argument would reveal that the injectivity of g turns out to be crucial for the equivalence just established.

So despite its more complex type and finer granularity, the quantifier in (47-c) does not attain the adequacy of the truth conditions imposed by (47-a). Loosely speaking, the reason is that its extra argument is wasted on the subject instead of relating directly to the focussed element; the above reasoning brings this out in that the injective function required for the equivalence between (47-c) and (47-a) does not concern the subject argument. Of course, this does not make the quantifier in (47-c) as such inadequate, but only the way it is matched with its linguistic environment, following the alternative semantics approach. In fact, if applied to the focussed element y_0, the property Q_0 of abstracting from it, and the set of all properties that nothing but y_0 has, the quantifier (47-c) would impose the truth conditions of the full content; but then this procedure would not be in line with the alternative semantics architecture.

6

The superiority of the predicate logic formalisation of *only* does not seem to hold across the board. As Mats Rooth (2016) pointed out in a comment on the previous sections, there are cases in which the alternative quantifier analysis appears to fare better, his example being a dialogue between zoo manager *A* and his assistant *B* about two tigers:

(48)
A: I only advise co-housing FLUFFY with a distinct one of Fluffy and Buffy.
B: Really? I assume you also advise co-housing BUFFY with a distinct one of Fluffy and Buffy.
A: That's the same thing. To co-house Buffy with a distinct one of Fluffy and Buffy is to co-house Fluffy with a distinct one of Fluffy and Buffy.

Contrasting it with similar reactions to sentences like (36), Rooth judges "*A*'s followup reasonable, and not just pragmatically" [ibid.]. I agree, given the following chain of equivalences, which exploits the (presumed) symmetry of (the irreflexive kernel of) the binary relation expressed by being **C***o-housed with by PRO*:[22]

(49) $\Phi(\mathbf{b})$
$\equiv$ $(\exists y)[\mathbf{C}(\mathbf{b},y) \wedge [y=\mathbf{f} \vee y=\mathbf{b}]]$
$\equiv$ $(\exists y)[\mathbf{C}(\mathbf{b},y) \wedge y=\mathbf{f}]$
$\equiv$ $\mathbf{C}(\mathbf{b},\mathbf{f})$

22 The *irreflexive kernel* of a binary relation R consists of the members (x,y) of R, where $x \neq y$. In the case at hand, it thus corresponds to the distinctness condition repeatedly mentioned in (48).

$$
\begin{array}{ll}
\equiv & \mathbf{C}(\mathbf{f},\mathbf{b}) \\
\equiv & (\exists y)[\mathbf{C}(\mathbf{f},y) \;\wedge\; y=\mathbf{b}] \\
\equiv & (\exists y)[\mathbf{C}(\mathbf{f},y) \;\wedge\; [y=\mathbf{f} \;\vee\; y=\mathbf{b}]] \\
\equiv & \Phi(\mathbf{f})
\end{array}
$$

where 'Φ' abbreviates '$\lambda x.\ (\exists y)[\mathbf{C}(x,y) \;\wedge\; [y=\mathbf{b} \;\vee\; y=\mathbf{f}]]$'. So *A*'s rejoinder is fully justified. However, a predicate-logic formalisation of his original utterance would support *B*'s objection:

$$
\begin{array}{ll}
(50) & (\forall x)[\mathbf{A}(\mathbf{a},{}^{\wedge}(\exists \mathbf{y})[\mathbf{C}(x,y) \;\wedge\; [y=\mathbf{b} \;\vee\; y=\mathbf{f}]]) \rightarrow x=\mathbf{f}] \\
\equiv & (\forall x)[\mathbf{A}(\mathbf{a},{}^{\wedge}\Phi(x)) \;\rightarrow\; x=\mathbf{f}]
\end{array}
$$

Given (49), (50) can only be vacuously true: if anything satisfies the antecedent, then both Fluffy and Buffy do; in particular, (50) is inconsistent with the implicature that Φ applies to Fluffy. This is clearly inadequate. The contradiction can be avoided by a straightforward alternative-based construal of *A*'s initial utterance:

$$
(51) \qquad (\forall p)[[{}^{\vee}p \wedge (\exists x)p = {}^{\wedge}\mathbf{A}(\mathbf{a},{}^{\wedge}\Phi(x))] \rightarrow p \;=\; {}^{\wedge}\mathbf{A}(\mathbf{a},{}^{\wedge}\Phi(\mathbf{f}))]
$$

Even if Φ is satisfied by Fluffy and thus Buffy, there may still be only one true proposition of the form '$\mathbf{A}(\mathbf{a},{}^{\wedge}\Phi(x))$'. So it seems that, in this case, quantification over alternative propositions fares better than the predicate logic account. I agree, but I still think that this cannot be the whole story about (48), and for two reasons.

The first reason why (51) is problematic, takes us back to the previous sections: depending on the particular model of intensional logic, the truth conditions may turn out to be inadequate. Thus, e.g., if $\mathbf{C}$ received an empty extension, then '${}^{\wedge}\Phi(\mathbf{f})$' would express a contradiction; hence even if *A* had advised co-housing penguin Duffy with either Buffy or Fluffy, such a model would still have (51) come out true on the basis of '${}^{\wedge}\Phi(\mathbf{f})$' and '${}^{\wedge}\Phi(\mathbf{d})$' denoting the same, contradictory, proposition. Similarly, even if $\mathbf{C}$ (and thus Φ) had a less degenerate intension, '${}^{\wedge}\mathbf{A}(\mathbf{a},{}^{\wedge}\Phi(\mathbf{b}))$' and '${}^{\wedge}\mathbf{A}(\mathbf{a},{}^{\wedge}\Phi(\mathbf{f}))$' could still denote the same proposition, due to a lack of variation in the extension of $\mathbf{A}$. In other words, (51) still suffers from the general indeterminacy discussed above.

In any case, I dare say, (51) is not *quite* what *A*'s response in (48) is about anyway. Rather, *A* argues that the proposition denoted by '${}^{\wedge}\Phi(\mathbf{f})$' is *the same thing* as the one denoted by '${}^{\wedge}\Phi(\mathbf{b})$', thus saving his original statement against *B*'s objection that two pieces of advice had been given. So what is at stake here is the number of (certain) propositions *to which A* stands in the relation expressed by $\mathbf{A}$ – and not the number of (true) propositions expressing *that A* stands in the relation expressed by $\mathbf{A}$ to these propositions. More specifically, it is not the number of true propositions of the form '${}^{\wedge}\mathbf{A}(\mathbf{a},{}^{\wedge}\Phi(x))$' that *A* and *B* argue about, as (51) would have it, but the number of propositions of the form '$^\wedge\Phi(x)$' such that '$^\wedge\mathbf{A}(\mathbf{a},^\wedge\Phi(x))$' is true. And what *A* said that this latter number is (at most) 1:

(52) $(\forall q)[[\mathbf{A}(\mathbf{a},q)\wedge(\exists x)q={}^\wedge\Phi(x)] \rightarrow q={}^\wedge\Phi(\mathbf{f})]$

I think (52) captures the truth conditions of *A*'s utterance more adequately than either of (50) and (51). And though (52) implies (51), the implication does not reverse[23] – at least not without further assumptions about 'realistic' models. Still, quantification over propositional alternatives may be said to deal better with [the first sentence of] (48) than the clearly inadequate predicate logic formalisation (50). (51) comes close to (52); in fact, if it were not for the 'degenerate' models, it almost gets there. Yet to the extent that (52) is the true logical form of *A*'s first utterance in (48), close is not close enough. And even though (51) bears some superficial similarities with (52), the fact that the latter does not quantify over true propositions makes it unlikely that it can be obtained by adapting the alternative interpretation strategy. Lifting this restriction, (52) may be read as a quantifier over alternatives. Replacing universal by restricted quantification brings this out more clearly:

(53) $(\forall q:(\exists x)q=^\wedge\Phi(x))[A(a,q) \rightarrow q=^\wedge\Phi(\mathbf{f})]$

However, (53) is a far cry from the interpretive strategy of the alternative semantics treatment of *only*, which is committed to quantifying over true alternative propositions. In fact, (53) is closer to the predicate logic formalisation in that it directly quantifies over the alternative denotations of constituents, which in the case at hand happen to be propositions rather than individuals. In any case, neither interpretive strategy seems to be able to cope with these cases without further ado. I will leave it at that.

7

So is the alternative semantics analysis of *only* doomed to assign inadequate truth conditions? Maybe not. Part of the trouble is the identification of propositions with sets of possible worlds. As Mats Rooth (*op. cit.*) already pointed out, going ' "more intensional" ' – along the lines of property theory[24] – might be an option. Alternatively one may stick to the possible worlds framework but incorporate (parts

23 (52) $\Rightarrow$ (51): If (*materialiter* speaking) $p_0={}^\wedge\mathbf{A}(\mathbf{a},^\wedge\Phi(x_0))$ is true, then $q_0:={}^\wedge\Phi(x_0)$ satisfies the antecedent of (52) and so, if (52) holds: $q_0={}^\wedge\Phi(\mathbf{f})$, and thus $p_0={}^\wedge\mathbf{A}(\mathbf{a},q_0)={}^\wedge\mathbf{A}(\mathbf{a},^\wedge\Phi(\mathbf{f}))$. – (51) $\not\Rightarrow$ (52): If **C** has a non-empty extension and the extension of **A** necessarily coincides with all pairs of the form $(a,^\wedge\Phi(y))$, where **a** denotes *a* and *y* is one of the individuals denoted by **f** or **d**, then (51) holds, because the propositions expressed by '$^\wedge\mathbf{A}(\mathbf{a},^\wedge\Phi(\mathbf{f}))$' and '$^\wedge\mathbf{A}(\mathbf{a},^\wedge\Phi(\mathbf{d}))$' coincide. On the other hand, (52) is false, given that **C** is not empty.

24 Bealer (1982). Rooth quotes Chierchia (1984) in this connection.

of) a competing semantic approach to focus: the theory of structured propositions and meanings,[25] which offers precisely the degree of granularity needed to capture the truth conditions of ordinary quantification over individuals. In any case it would seem that life in the alternative paradise that Rooth has created does not come for free.

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25 I am indebted to Irene Heim (p.c., June 2014) for bringing up this suggestion. See Stechow (1991) and Krifka (2001) for expositions and comparison of the frameworks. In fact, the structured alternatives employed by (Onea 2013: ch. 8) come close to this kind of hybrid architecture.

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