# Free choice and presuppositional exhaustification

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**Abstract** Sentences such as *Olivia can take Logic or Algebra* ('$\Diamond\vee$-sentences') are typically assigned the 'Free Choice' (FC) reading that Olivia can take Logic and can take Algebra. Given a standard semantics for modals and disjunction, such FC readings are not predicted from the surface form of $\Diamond\vee$-sentences. An influential approach treats FC readings as a kind of scalar enrichment generated by a covert exhaustification operator. This approach can also account for the 'dual prohibition' readings of $\neg\Diamond\vee$-sentences like *Olivia can't take Logic or Algebra* via general principles of implicature cancellation in downward entailing environments. Marty & Romoli (2020) and Romoli & Santorio (2019) examine the projection and filtering behavior of embedded $\Diamond\vee$ and $\neg\Diamond\vee$-sentences, focusing on two kinds of cases which challenge this influential approach. First, $\Diamond\vee$-sentences under negative factives: e.g., *Noah is unaware that Olivia can take Logic or Algebra*, which in its most salient reading presupposes that Olivia has free choice and yet attributes to Noah ignorance not just concerning whether Olivia has free choice but also whether she can take even one of the classes. Second, $\neg\Diamond\vee$-sentences embedded under disjunction: e.g., *Either Maria can't study in Tokyo or Boston, or she is the first in our family who can study in Japan and the second who can study in the States*, which in its most salient reading filters out the FC presupposition of the second disjunct while the first disjunct gets the standard dual prohibition reading. These sentences present a serious challenge to extant accounts of FC. In this paper, we present a novel exhaustification-based account of FC that issues in a uniform solution to Marty, Romoli and Santorio's puzzles concerning the presuppositional and filtering behavior of embedded $\Diamond\vee$, $\neg\Diamond\vee$, and related FC sentences. Our account builds on the proposal—advanced in Bassi et al. (2019) and Del Pinal (2021) as a general theory of scalar implicatures—that covert exhaustification is a presupposition trigger such that the prejacent forms the assertive content while any excludable (or includable) alternatives are incorporated at the non-at issue, presuppositional level.

**Keywords:** free choice, scalar implicatures, exhaustification, presuppositions, presupposed free choice, filtering free choice, accommodation, pragmatics. **Words:** 12,046

## Contents



## 1 Introduction

According to increasingly popular 'Grammatical' theories, the scalar implicatures (SIs) of a sentence $\phi$ are the result of adjoining to $\phi$ a covert exhaustification operator, **exh**, which outputs as its assertive content both $\phi$ and the negation of each excludable alternative of $\phi$ (see Chierchia et al. 2012, a.o.):

(1) Given $\phi$, a set of excludable alternatives $Excl(\phi)$, and utterance context $c$, a use of $\mathbf{exh}(\phi)$ in $c$ has as its assertive content the conjunction of $\phi$ with the negation of each member of $Excl(\phi)$ that is relevant in $c$.[1]

Grammatical theories differ on how to precisely define the set of excludable alternatives, *Excl*, and also on their assumptions about the distribution of **exh**. In this paper, we propose a high level constraint on Grammatical theories which we argue greatly

1 For simplicity, we represent the common ground at a context $c$ as a set of worlds compatible with all the propositions mutually believed by the participants in $c$.

improves their empirical coverage. The basic idea is that covert exhaustification—like similar overt operators such as *even* and *only*—is a presupposition trigger. According to standard 'flat' formulations like (1), the entire output of **exh**, i.e., both the prejacent and the negated excludable alternatives, goes into the assertive content. In contrast, we will argue that while the prejacent is indeed part of the assertive content, the negated excludable alternatives are part of the non-at issue, presuppositional content. We also show that this perspective can incorporate the recent hypothesis that covert exhaustification has not just an 'exclusion' but also an 'inclusion' of alternatives function (Bar-Lev & Fox 2020), and substantially improves its predictions in various important cases.

To motivate our proposal that exhaustification is a presupposition trigger, we will focus on a range of puzzles involving the free choice (FC) readings of certain sentences. These readings have played a critical role in the argument for **exh**-based (Grammatical) accounts of SIs over both neo-Gricean (or more broadly 'pragmatic') and Lexicalist (also called 'Semantic') accounts (Zimmerman 2000, Fox 2007). More recently, FC readings have also been at the center of debates amongst specific Grammatical theories (Chierchia 2013, Bar-Lev & Fox 2020). To set the stage, it will be useful to recap why FC inferences have had such impact on the development of theories of SIs.

First, sentences with disjunction in the scope of a possibility modal, such as (2), have a FC reading which can't be captured based solely on their surface form, given standard accounts of those modals and connectives. In contrast, this FC reading can be derived via recursive exhaustification, a kind of operation that is expected on a **exh**-based theory, but harder to capture with a genuine pragmatic account of SIs. To be sure, recent Grammatical accounts formulate **exh** so that FC readings can be derived without the need of recursive exhaustification (Bar-Lev & Fox 2020). Still, the revised notion of **exh** doesn't have any (obvious) pragmatic analogues, and has the property—distinctive of syntactically 'real' covert operators—that it can be inserted in embedded, non-asserted environments.

(2) Maria can study in Tokyo or Boston.

a. $\rightsquigarrow$ *Maria can study in Tokyo*
$\rightsquigarrow$ *Maria can study in Boston*

Secondly, FC readings, like SIs in general, tend to be cancelled in downward entailing (DE) environments. This is illustrated by the ddefault 'dual prohibition' reading of (3), which conveys not merely the negation of FC, but the stronger claim that Maria can't study in either one of Tokyo or Boston. The preference for this reading of (3) is hard to explain for standard 'Lexicalist' accounts of the FC reading of (2), which instead of appealing to SIs, 'hard-wire' such readings via a non-standard semantics for possibility modals and/or disjunction. In contrast, in **exh**-based accounts, the cancellation of FC readings in DE environments is captured by appealing to a general preference for parses with strong meanings. Roughly, when a parse with **exh** occurs in an environment that leads to a weaker meaning relative to a corresponding parse without **exh**, the latter parse is treated as the default, non-marked option.

(3) Maria can't study in Tokyo or Boston.

a. $\rightsquigarrow$ *Maria can't study in Tokyo*
$\rightsquigarrow$ *and Maria can't study in Boston*

Finally, **exh**-based accounts of FC readings of sentences with disjunction under a possibility modal can be extended to structurally analogous FC readings observed in other kinds of configurations, including sentences with disjunctions under other existential quantifiers, not just a possibility modals, sentences with conjunctions under the scope of negation and universal quantifiers, and indefinites under universal quantifiers (Fox 2007, Chierchia 2013, Meyer 2020).

Despite its initial success, recent work suggests that standard **exh**-based Grammatical accounts of FC have important shortcomings. The challenge which will be the main focus of this paper, due to Marty & Romoli (2020) and Romoli & Santorio (2019), concerns the way in which sentences like (2) and (3) behave when embedded in two kinds of environments.

Consider first an occurrence of (2) embedded under a negative factive, as in (4) (Marty & Romoli 2020). In its most natural reading, (4) presupposes that Maria has free choice. This is what we would predict—given the factive presupposition of *unaware*—if the embedded sentence is parsed with (recursive) **exh**, i.e., parallel to the parse that supports the FC reading of (2). Yet this assumption generates the wrong prediction for the content of Sam's beliefs, on this natural reading: (4) conveys that what Sam doesn't believe is that Maria can study in either city, not just that Sam doesn't believe that Maria has free choice.

(4) Sam is unaware that Maria can study in Tokyo or Boston.

a. $\rightsquigarrow$ *Maria can study in Tokyo*
$\rightsquigarrow$ *Maria can study Boston*

b. $\rightsquigarrow$ *Sam doesn't believe that Maria can study in Tokyo*
$\rightsquigarrow$ *Sam doesn't believe that Maria can study in Boston*

Consider next an embedded occurrence of (3) in a sentence like (5) (Romoli & Santorio 2019). While the nuclear scope/consequent of *unless* in (5) (= *Maria is the first … that can study in Japan and the second … that can study in the US*) presupposes that Maria can study in Japan and in the US, (5) doesn't seem to inherit that FC presupposition, as captured in (5a). The reason for that, it is natural to think, is that the FC presupposition is somehow filtered out by (negation of) the restrictor of *unless* in (5) (= *Maria can't study in Tokyo or Boston*). Yet the embedded *Maria can't study in Tokyo or Boston* in the restrictor has the dual prohibition reading, which calls for a parse without (recursive) exhaustification under negation. Yet given such a parse, the (negated) restrictor wouldn't help filter out the FC presupposition triggered in the scope/consequent of *unless*.

(5) Unless Maria can't study in Tokyo or Boston, she is the first in our family that can study in Japan and the second that can study in the US.

a. $\not\rightsquigarrow$ *Maria can study in Japan*
$\not\rightsquigarrow$ *Maria can study in the US*

As Marty & Romoli (2020) and Romoli & Santorio (2019) show, standard **exh**-based account of the FC reading of sentences like (2) and the dual prohibition reading of sentences like (3) don't seem to derive, in a principled way, the target readings for sentences like (4) and (5). This problem extends to (the standard version) of the recent proposal that exhaustification involves not just the assertion of the prejacent and the negated excludable alternatives, but also the assertion of any formal alternatives of the prejacent that can be included without inconsistency (Bar-Lev & Fox 2020). Call the challenge of giving a principled account of sentences like (2)-(5), the 'presupposed & filtering FC puzzle'.

There have been two kinds of responses to parts of the presupposed & filtering FC puzzle. Grammatical theorists have proposed that exhaustification applies in analogous ways to both the assertive and the presuppositional content of the prejacent (Gajewski & Sharvit 2012, Marty & Romoli 2020). While such accounts can deal with part of the puzzle, we will argue that they don't provide a general solution to the challenge, i.e., they can't explain (2)-(5) taken together. The other response has been to revise Lexicalist accounts, i.e., accounts which tweak or abandon the standard semantics for possibility modals and/or disjunction (Starr 2016, Willer 2017, Goldstein 2018). These accounts also fail to provide a fully general solution to the puzzle. The main reason for this, emphasized in Marty & Romoli (2020) and Romoli & Santorio (2019), is that the puzzle can be restated with (negative) FC readings of sentences with conjunction under negation and a universal quantifier, as in *Maria isn't required to study in Tokyo and Boston*. Such negative FC readings present a problem for both traditional and remodeled Lexicalist accounts. In short, neither standard nor these recent accounts of FC provide a principled, general solution to the presupposed & filtering FC puzzle.

The goal of this paper is to show that, once we revise the Grammatical approach to FC in terms of our hypothesis that exhaustification is presupposition trigger, we get a uniform solution to the presupposed & filtering FC puzzle. Specifically, we propose to replace **exh** with **pex**: as a first approximation, **pex** asserts its prejacent and presupposes the negation of each of its excludable alternatives (Bassi et al. 2019, Del Pinal 2021):

(6) Given $\phi$, a set of excludable alternatives, $Excl(\phi)$, and utterance context $c$, an assertion of $\mathbf{pex}(\phi)$ in $c$ presupposes the negation of each member of $Excl(\phi)$ that is relevant in $c$ and asserts $\phi$.

Relative to how it structures presupposed vs. assertive content, **pex** is the mirror image of the overt exhaustifier *only*, given the standard presuppositional account of the latter (Horn 1969): roughly, **pex** presupposes what *only* asserts and asserts what *only* presupposes. The basic proposal in (6) is compatible with different accounts of how, precisely, the set of excludable alternatives, *Excl*, is determined. In addition, we will also show that it can extended, with additional empirical payoffs, so as to incorporate Bar-Lev & Fox hypothesis that exhaustification has the function of not only excluding but also including certain alternatives. Concerning its distribution, we follow Magri (2009) and assume that propositional clauses are parsed with **pex** by default. In this implementation, the role of a relevance parameter $R$ is crucial: for familiar cases in which an SI is 'cancelled' are handled by showing that the alternative responsible for the potential SI is not in $R$. On this approach, a simple scalar sentence like (7) will be parsed by default as in (7a). Assuming its only excludable alternative is the 'all'-alternative, the output of (7a) is as in (7c).

(7) Some students passed the exam.

a. **pex** [some students passed]

b. $Excl(\textit{some students passed}) = \textit{All students passed}$

c. $[\![(7a)]\!] = \begin{cases} \textbf{ps: } \neg\text{all students passed} \vee \neg R(\text{all students passed}) \\ \textbf{asserts: } \text{some students passed} \end{cases}$

It is increasingly appreciated that there are different notions, and probably different kinds, of 'non-at issue' and 'presuppositional' content (Tonhauser et al. 2013). So what specific properties are we attributing to the SIs triggered by **pex**? In terms of their global constraints on the common ground, we assume that if the presuppositions triggered by **pex** are at least consistent with the common ground, they are globally accommodated by default. When the SIs triggered by **pex** appear in embedded positions, they behave, in terms of their projection and accommodation properties, like standard presuppositions. Indeed, we will argue that our account should be paired with the standard assumption that local accommodation of embedded presuppositions has strict licensing conditions.

Although simple examples like (7) are not useful for distinguishing **pex** and **exh**, in Bassi et al. (2019) and Del Pinal (2021) we show that our **pex**-based theory solves a number of open empirical challenges faced by current **exh**-based and neo-Gricean theories of SIs. These cases include the oddness-inducing SIs studied by Magri (2009, 2011), as well as various puzzles about the way SIs project from embedded environments, including in *some*-under-*some* sentences and in various kinds of DE positions. Based on those results, we argued that Grammatical approaches to SIs should incorporate the hypothesis that covert exhaustification is a presupposition trigger, roughly along the lines of **pex**. This paper substantially extends our argument by developing a novel **pex**-based Grammatical theory of FC. In §2, we present and defend a **pex**-based theory of basic FC and dual prohibition sentences such as (2) and(3). We then show, in §3-4, that our theory issues in a uniquely attractive account of presupposed FC cases like (4) and filtering FC cases like (5).

## 2 Free choice: the basic effect

### 2.1 Standard accounts of free choice

Sentences with a possibility modal over disjunction, such as (8), give rise to the 'conjunctive' inference that both disjuncts are possible. These 'free choice' (FC) inferences don't follow from standard assumptions about the semantics of existential modals and disjunction.

(8) Maria is allowed to eat cake or ice-cream. $\Diamond(C \vee IC) \Leftrightarrow (\Diamond C \vee \Diamond IC)$

a. $\rightsquigarrow$ *Maria is allowed to eat cake*
$\rightsquigarrow$ *Maria is allowed to eat ice-cream* $\Diamond C \wedge \Diamond IC$

Some authors try to encode this FC effect directly into the target lexical entries for possibility modals and/or disjunction. Early versions of this 'Lexicalist' approach had trouble capturing the complementary observation that the FC effect disappears in downward-entailing (DE) environments, as illustrated by the 'dual prohibition' (DP) reading of (9).

(9) Maria is not allowed to eat cake or ice-cream. $\neg\Diamond(C \vee IC)$

a. $\rightsquigarrow$ *Maria is not allowed to eat cake*
$\rightsquigarrow$ *Maria is not allowed to eat ice-cream* $\neg\Diamond(C \vee IC) \Leftrightarrow \neg\Diamond C \wedge \neg\Diamond IC$

Suppose that (8), based on its surface form, semantically entails FC, namely, $\Diamond C \wedge \Diamond IC$. When (8) is embedded under negation, as in (9), one would then expect a reading that corresponds to the negation of FC—i.e., $\neg(\Diamond C \wedge \Diamond IC) \Leftrightarrow \neg\Diamond C \vee \neg\Diamond IC$—which is weaker than the observed dual prohibition reading. From now on, we refer to sentences of the surface form of (8) as '$\Diamond\vee$-sentences' and those of the form of (9) as '$\neg\Diamond\vee$-sentences'. When useful, we also add the prefix 'FC' (free choice) or 'DP' (dual prohibition) to clarify which target readings we are analyzing.

In contrast to standard Lexicalist accounts, the pattern in (8)-(9) can be easily explained by accounts, such as Fox (2007), which treat FC readings as a kind of scalar enrichment. Fox's influential account builds on Grammatical theories of SIs, and derives FC readings via recursive application of **exh**. Given suitable alternatives at each **exh** site, Fox established the following result:

(10) $\textbf{exh}[\textbf{exh}[\Diamond(p \vee q)]] = \textbf{exh}^*[\Diamond(p \vee q)] = (\Diamond p \leftrightarrow \Diamond q) \wedge \Diamond(p \vee q) = \Diamond p \wedge \Diamond q$

In addition, SIs in general tend to disappear in DE environments. According to Fox, this is due to an economy constraint which says, roughly, that parses with **exh** are dispreferred when they lead to a weaker meaning relative to a corresponding parse without **exh**—which is precisely what happens in DE environments. This constraint can be lifted when inserting **exh** is required to prevent inconsistency or if its insertion is marked in specific ways. Given this account, we can derive the target pattern via the parse in (11a) for the FC reading and the one in (12a) for the dual prohibition reading:

(11) Maria is allowed to eat cake or ice-cream.

a. $\textbf{exh}^*[\Diamond[C \vee IC]]$ $\Leftrightarrow \Diamond C \wedge \Diamond IC$

(12) Maria is not allowed to eat cake or ice-cream.

a. $\neg\Diamond[C \vee IC]$ $\Leftrightarrow \neg\Diamond C \wedge \neg\Diamond IC$

### 2.2 Presuppositional exhaustification-based account of free choice

Our working hypothesis, recall, is that the covert exhaustification, **pex**, divides its output into an assertive and a non-at issue/presupposed component. This hypothesis can be used to reformulated the standard Grammatical view that the main function of exhaustification, other than asserting the prejacent, is to negate any excludable, relevant alternatives. For concreteness, we follow Fox (2007) and assume that the excludable alternatives are selected from the set of 'innocently excludable' ones, defined as follows:

(13) Innocently Excludable alternatives of $\phi$:

a. Take all maximal sets of alternatives of the prejacent $\phi$ that can be assigned 'false' consistently when conjoined with $\phi$.

b. Those alternatives that are members in all such sets form the set of the 'innocently excludable' (IE) alternatives of $\phi$.

**pex**, then, will assert the prejacent and presuppose the negation of each IE alternative that is relevant. As mentioned earlier, our approach can be used to reformulate the hypothesis that exhaustification also has the function of including certain alternatives. We focus for now on reformulating the standard view with just IE, partly because it is better understood, and partly because, for most of the cases we consider (but see §4.3), a purely IE based **pex** operator and one with both an IE and inclusion function make equivalent predictions (but see §4.3).[2] Due to this decision, our account of FC is based on recursive application of covert exhaustification, as in Fox (2007) and similar accounts. However, compared to standard Grammatical accounts, the use of **pex** may require a subtly different parse for deriving the FC readings, depending on independent assumptions about presupposition projection in general. More importantly, **pex** generates FC readings that, while having the same overall entailments as predicted by standard Grammatical accounts, structure them into a unique array of presupposed and assertive components. Let us examine these points in turn by going over a **pex**-based derivation of the FC reading of simple $\Diamond\vee$-sentences and the dual prohibition reading of simple $\neg\Diamond\vee$-sentences.

To begin with the derivation of the FC reading of $\Diamond\vee$-sentences, consider first the parse in (14a), which is the one used in Fox (2007) except with **pex** in place of **exh**.[3] The problem when we compute $\mathbf{pex}_1$, in terms of getting the FC reading, is that—unlike accounts with **exh**—we can't innocently exclude the disjunctive alternatives of its prejacent, $\mathbf{pex}_2[\Diamond[p\vee q]]$, i.e., we can't exclude $\mathbf{pex}_2[\Diamond p]$ and $\mathbf{pex}_2[\Diamond q]$. For due to the projection of presuppositions under negation, captured in (14c)-(14d), these alternatives can't jointly be part of any maximal subset of (negated) alternatives that is consistent with $\mathbf{pex}_2[\Diamond[p\vee q]]$, as shown in (14f).[4] This result holds independently of whether $\mathbf{pex}_2[\Diamond[p\vee q]] = \Diamond[p\vee q]$, as I assumed for simplicity in (14e), or relevance at a context is such that $\mathbf{pex}_2[\Diamond[p\vee q]] = \Diamond[p\vee q]_{\neg\Diamond[p\wedge q]}$. Now, although we don't get the FC reading on this parse, $\mathbf{pex}_1$ might not be entirely vacuous, since it could still negate the $\mathbf{pex}_2[\Diamond[p\wedge q]]$ alternative. Yet this readings for $\Diamond\vee$-sentences—namely, $\Diamond[p\vee q]_{\neg\Diamond[p\wedge q]}$—could also be obtained

2 A version of **pex** with an inclusion function does allow for a simpler, non-recursive derivation of basic FC readings. However, incorporating an inclusion function into a **pex**-like operator requires various non-trivial choices—and independently of any advantage this kind of operator may have specifically for FC puzzles, its predictions for other kinds of SIs would have to be validated. Still, we will propose and defend a version of **pex** with an inclusion function in §4.3), where we discuss cases of filtering FC with $\Box\wedge$-sentences.

3 We use subscripts to formulas, as in $p_q$ to indicate that $q$ is a presupposition of $p$. In addition, for recursive presuppositions, where subscripts of subscripts may be confusing, we use fractions to indicate, in the numerator, the assertive content and, in the denominator, the presuppositional content of a proposition (see Harbour 2014).

4 It is easy to check that the result in (14f) guarantees that no other viable notion of exclusion would generate the FC inferences via negation of $\mathbf{pex}_2[\Diamond p]$ and $\mathbf{pex}_2[\Diamond p]$.

via the structurally simpler parse $\mathbf{pex}[\Diamond[p \vee q]]$. It follows that the parse in (14a) would be ruled out by most reasonable principles of structural economy.

(14)
a. $\mathbf{pex}_1[\mathbf{pex}_2[\Diamond[p \vee q]]]$

b. $\text{Alt}(\mathbf{pex}_2[\Diamond[p \vee q]]) = \left\{ \begin{array}{l} \mathbf{pex}_2[\Diamond[p \vee q]], \\ \mathbf{pex}_2[\Diamond p], \mathbf{pex}_2[\Diamond q], \\ \mathbf{pex}_2[\Diamond[p \wedge q]] \end{array} \right\}$

c. $\neg\,\mathbf{pex}_2[\Diamond p] = \neg[\Diamond p_{\neg\Diamond q}] = \neg\Diamond p_{\neg\Diamond q}$

d. $\neg\,\mathbf{pex}_2[\Diamond q] = \neg[\Diamond q_{\neg\Diamond p}] = \neg\Diamond q_{\neg\Diamond p}$

e. $\mathbf{pex}_2[\Diamond[p \vee q]] = \Diamond[p \vee q]$

f. $\Diamond[p \vee q] \wedge \neg\Diamond p_{\neg\Diamond q} \wedge \neg\Diamond q_{\neg\Diamond p} = \bot$

Interestingly, the FC readings of $\Diamond\vee$-sentences can be derived, in a **pex**-based system, by adopting a slightly different parse.[5] Consider the parse in (15a). This parse is licensed because local accommodation—here implemented in terms of an ACC operator such that $\text{ACC}(p_q) = q \wedge p$—is introduced so as to rescue a parse that would otherwise be defective, as we just established for (14a). The parse in (15a), we can easily show, generates the FC reading of $\Diamond\vee$-sentences. Due to the effect of the embedded ACC, negation of the disjunctive alternatives for the prejacent of $\mathbf{pex}_1$ now has the effect of generating the homogeneity proposition $\Diamond p \leftrightarrow \Diamond q$, as shown in (15d). Adding that homogeneity proposition as a presupposition to the prejacent, $\Diamond$[p $\vee$q], guarantees the FC entailments, as shown in (15f).

(15)
a. $\mathbf{pex}_1[\text{ACC}[\mathbf{pex}_2[\Diamond[p \vee q]]]$

b. $\text{Alt}(\text{ACC}[\mathbf{pex}_2[\Diamond[p \vee q]]]) = \left\{ \begin{array}{l} \text{ACC}[\mathbf{pex}_2[\Diamond[p \vee q]]], \\ \text{ACC}[\mathbf{pex}_2[\Diamond p]], \text{ACC}[\mathbf{pex}_2[\Diamond q]], \\ \text{ACC}[\mathbf{pex}_2[\Diamond[p \wedge q]]] \end{array} \right.$

c. $[\![(15a)]\!] = \left\{ \begin{array}{l} \textbf{ps: } \neg\text{ACC}[\mathbf{pex}_2[\Diamond p]] \wedge \neg\text{ACC}[\mathbf{pex}_2[\Diamond q]] \\ \textbf{asserts: } \text{ACC}[\mathbf{pex}_2[\Diamond[p \vee q]]] \end{array} \right.$

d. $\neg\text{ACC}[\mathbf{pex}_2[\Diamond p]] \wedge \neg\text{ACC}[\mathbf{pex}_2[\Diamond q]]$
$= \neg[\neg\Diamond q \wedge \Diamond p] \wedge \neg[\neg\Diamond p \wedge \Diamond q]$
$= [\Diamond q \vee \neg\Diamond p] \wedge [\Diamond p \vee \neg\Diamond q] = [\Diamond p \rightarrow \Diamond q] \wedge [\Diamond q \rightarrow \Diamond p]$
$= \Diamond p \leftrightarrow \Diamond q$

e. $\text{ACC}[\mathbf{pex}_2[\Diamond[p \vee q]]] = \Diamond[p \vee q]$

f. $[\![(15a)]\!] = \Diamond[p \vee q]_{\Diamond p \leftrightarrow \Diamond q}$

We should mention two things about presupposed and assertive content of (15a), as captured in (15c). First, we get the good result that, in the **asserts** part of (15c), $\mathbf{pex}_2$

5 In appendix A, we explore a second strategy for deriving FC readings which maintains the classic parse with recursive exhaustification, yet uses different assumptions concerning presupposition projection based on an extension of a Strong Kleene semantics to cases of recursive presuppositions.

can't innocently exclude the $\Diamond p$ nor the $\Diamond q$ alternatives to its prejacent $\Diamond[p \vee q]$: for since $\neg\Diamond p$ and $\neg\Diamond q$ can't both be consistently conjoined with $\Diamond[p \vee q]$, they can't jointly be part of any maximal subset of (negated) alternatives that is consistent with $\Diamond[p \vee q]$. Secondly, when calculating the **ps** content triggered by $\mathbf{pex}_1$, so far we have ignored the conjunctive alternative $\textsc{acc}[\mathbf{pex}_2[\Diamond[p \wedge q]]]$. Yet if we are interested in cases in which that alternative is relevant, we immediately get the result that it is excludable, and ultimately resolves to adding $\neg\Diamond[p \wedge q]$ to the **ps** part of (15c).[6] In short, we have derived the following results:

(16) If $\textsc{acc}[\mathbf{pex}_2[\Diamond[p \wedge q]]] \notin Alt \cap R$ for $\mathbf{pex}_1$:

a. $\mathbf{pex}_1[\textsc{acc}[\mathbf{pex}_2[\Diamond[p \vee q]]]] = \mathbf{pex}^*[\Diamond[p \vee q]] = \begin{cases} \textbf{ps: } \Diamond p \leftrightarrow \Diamond q \\ \textbf{asserts: } \Diamond[p \vee q] \end{cases}$

(17) If $\textsc{acc}[\mathbf{pex}_2[\Diamond[p \wedge q]]] \in Alt \cap R$ for $\mathbf{pex}_1$:

a. $\mathbf{pex}_1[\textsc{acc}[\mathbf{pex}_2[\Diamond[p \vee q]]]] = \mathbf{pex}^*[\Diamond[p \vee q]] = \begin{cases} \textbf{ps: } \Diamond p \leftrightarrow \Diamond q \wedge \neg\Diamond[p \wedge q] \\ \textbf{asserts: } \Diamond[p \vee q] \end{cases}$

This captures the observation, due to Simons (2005a), that $\Diamond\vee$-sentences like (11) have two subtly different FC readings, one in which Maria can choose cake, can choose ice-cream, and also can have both, and one in which she can choose between cake and ice-cream but cannot have both. The first reading is captured by (16a) and the second by (17a). Now, the difference between (16a) and (17a) is irrelevant for the cases and readings we focus on in this paper, so from here on we focus for simplicity on the FC reading in (16a).

Using our **pex**-based account, let us next derive the target FC reading for basic $\Diamond\vee$-sentences, such as (11), and the dual prohibition reading for basic $\neg\Diamond\vee$-sentences, such as (12). To begin with (11), repeated in (18), it follows based on our previous result that the parse in (18a) generates the FC reading. The key difference, compared to standard **exh**-based accounts, is that the entailments are now divided into a presupposed part, $\Diamond C \leftrightarrow \Diamond IC$, and an assertive part, $\Diamond[C \vee IC]$, such

6 To be clear, $\neg\textsc{acc}[\mathbf{pex}_2[\Diamond[p \wedge q]]]$ resolves to $\neg\Diamond[p \wedge q]$ assuming here that the target focus for $\mathbf{pex}_2$ doesn't include the modal operator. It is important to preemptively add that, to predict the correct behavior for embedded $\Diamond\vee$-sentences in the presupposed & filtering FC puzzles, deriving $\neg\Diamond[p \wedge q]$ as part of the presuppositional content, as we have done here, is desirable (precisely why will become clear later). But couldn't $\neg\Diamond[p \wedge q]$ be added when we compute the output of $\mathbf{pex}_2$ in the **asserts** part of (15c), which due to its being embedded under an ACC operator, would mean that $\neg\Diamond[p \wedge q]$ ends up as part of the assertive content? Are we just stipulating in which structural position the $\neg\Diamond[p \wedge q]$ is introduced? No, for our assumption follows from independent principles: $\neg\Diamond[p \wedge q]$ isn't generated twice when computing recursive exhaustification because that would be redundant and inefficient, and it is generated when computing $\mathbf{pex}_1[\textsc{acc}[\mathbf{pex}_2[\Diamond[p \vee q]]]$ rather than $\textsc{acc}[\mathbf{pex}_2[\Diamond[p \vee q]]$ because that option triggers stronger presuppositions (so is preferred given a standard 'Maximize Presuppositions!' principle).

that for any context $c$ which entails the former, updating it with the latter results in a context $c'$ that entails $\Diamond C \wedge \Diamond IC$. Bracketing for now whether the homogeneity presupposition is attested, we get the desired FC inferences for basic $\Diamond\vee$-sentences.

(18) Maria is allowed to eat cake or ice-cream.

a. $\mathbf{pex}[\textsc{acc}[\mathbf{pex}[\Diamond[C \vee IC]]]]$ $\qquad = \mathbf{pex}^*[\Diamond[C \vee IC]]$

b. $[\![(18a)]\!] = \Diamond[C \vee IC]_{\Diamond C \leftrightarrow \Diamond IC}$ $\qquad \models \Diamond C \wedge \Diamond IC$

Consider next $\neg\Diamond\vee$-sentences, such as (12), repeated in (19) below. Dual prohibition readings are derived differently in our **pex**-based account compared to in standard **exh**-based accounts. On Fox's account, recall, the dual prohibition reading for (19) follows from economy. When a parse with **exh** leads to an overall weakening of meaning compared to a parallel parse without **exh**, the latter parse is preferred. In cases like (19), this procedure selects a parse without **exh**$^*$ under negation. The situation changes somewhat when we move to a **pex**-based account of SIs. Since we take parses with **pex** as the default, we predict that the preferred or unmarked parse for (19) is as in (19a). Yet due to the presuppositional structure of the FC effect, the negated part only directly affects the assertive output of **pex**$^*$, as captured in (19b). This gets us precisely the dual prohibition inference, a result that parallels the one obtained in **exh**-based accounts. Yet there is also an important difference: as in the FC readings of $\Diamond\vee$-sentences, this **pex**-based account also predicts a homogeneity presupposition for the dual prohibition reading of $\neg\Diamond\vee$-sentences.

(19) Maria is not allowed to eat cake or ice-cream.

a. $\neg[\mathbf{pex}^*[\Diamond[C \vee IC]]$

b. $[\![(19a)]\!] = \neg[\Diamond[C \vee IC]_{\Diamond C \leftrightarrow \Diamond IC}] = \neg\Diamond[C \vee IC]_{\Diamond C \leftrightarrow \Diamond IC}$

In §3-4, we will argue that the unique components of our **pex**-based account of FC—in particular, the uniform underlying parses and presupposed vs. assertive structuring of the FC reading of $\Diamond\vee$-sentences and dual prohibition (DP) reading of $\neg\Diamond\vee$-sentences—resolve the problems presented by the presupposed & filtering FC puzzle to **exh**-based theories. Yet before delving into the presupposed & filtering FC puzzle, it is worth reviewing some recent independent evidence for the unique components of our **pex**-based theory of FC.

Consider first the uniform parses used by our **pex**-based to derive FC readings of $\Diamond\vee$-sentences and DP readings $\neg\Diamond\vee$ sentences. This is part of a more general result, discussed in Bassi et al. (2019), where **exh**-based accounts make use of different representations depending on monotonicity, while **pex** allows for more uniform representations. This leads, in turn, to subtly different predictions in certain cases where scalar items appear under DE and non-monotonic operators. In the case of FC readings, Gotzner, Romoli & Santorio (2020) report data relevant to this pattern.

Gotzner et al. (2020) show experimentally that (20) has a reading where one student has free choice, while all other students have dual prohibition. They call this the 'all others dual prohibition' reading, and show that it isn't predicted by standard **exh**-based theories of FC.

(20) Exactly one student can take Spanish or Calculus.

In contrast, our **pex**-account captures the all-other-dual-prohibition reading by means of the LF in (21), which matches the LFs which we used in (18) and (19) above ($\text{ONE}!$ represents the quantifier *exactly one* such that $\text{ONE}!(P)(Q) = 1 \Leftrightarrow |P \cap Q| = 1$):

(21) $\text{ONE}!(\llbracket student \rrbracket)\ \lambda x\ \mathbf{pex}^*[\Diamond[s(x) \vee c(x)]]$

The scope of the non-monotonic quantifier in (22) presupposes, for an assignment dependent variable over individuals $x$, $\Diamond s(x) \leftrightarrow \Diamond c(x)$. Given standard assumptions about presupposition projection from the scope of quantifiers (Chemla 2009a, Fox 2013, Mayr & Sauerland 2015), we then expect that (22) inherits as a presupposition the homogeneity proposition universally quantified over all relevant students. That is, for each student $x$, $x$ can take both or neither of Spanish and Calculus. In addition, from the assertive part, we have that only one student is allowed to take both Spanish and Calculus, so it follows that all the others can't take either one. This captures exactly the target 'all others dual prohibition' interpretation.

Moving next to the potential homogeneity presupposition for $\Diamond\vee$ and $\neg\Diamond\vee$-sentences, Tieu, Romoli & Bill (2019) report relevant experimental evidence. Recall: although both **pex** and **exh**-based accounts predict the FC reading of $\Diamond\vee$-sentences and the dual prohibition reading $\neg\Diamond\vee$-sentences, only the **pex**-based account predicts that both readings also come with a non-trivial homogeneity presupposition. Specifically, the **pex**-based account predicts that the FC reading of (18) and the dual prohibition reading of (19) *both* presuppose $\Diamond C \leftrightarrow \Diamond IC$, i.e., that Maria is allowed to eat ice cream if and only if she is allowed to eat cake. Suppose that (18) and (19) are evaluated against a situation that is explicitly inconsistent with that homogeneity presupposition, e.g., a situation $s_1$ which entails that Maria can have cake but not ice-cream. The account based on **exh** predicts that, in $s_1$, (18) should be judged as true but having a false implicature, whereas (19) should be judged as straightforwardly false. In contrast, our **pex**-based account predicts that, in $s_1$, (18) and (19) should both result in presupposition failure. Accordingly, only **pex** predicts that (18) and (19) should elicit parallel and high ratings with respect to a reasonable measure of presupposition failure. The experiments reported by Tieu et al. (2019) confirm the latter prediction. Although Tieu et al. present their results as a challenge to Grammatical accounts in general, their results are as expected given our **pex**-based Grammatical account of SIs and FC.

## 3 Free choice under (negative) factives

### 3.1 The challenge

The first part of the presupposed & filtering FC puzzle, due to Marty & Romoli (2020), concerns the behavior of FC when embedded under certain factive attitude verbs. Let us focus on the pattern in (22)-(24). We begin with a simple example of a FC-$\Diamond\vee$-sentence, in (22), and of DP-$\neg\Diamond\vee$-sentence, in (23). The challenge is to have an account of those basic cases that can simultaneously capture the default reading of (24), which presupposes FC, as captured in (24a), yet asserts that Noah doesn't believe Olivia can take either one of Logic or Algebra, as captured in (24b).

(22) Olivia is allowed to take Logic or Algebra.

a. $\rightsquigarrow$ *Olivia can take Logic*
$\rightsquigarrow$*Olivia can take Algebra*

(23) Olivia isn't allowed to take Logic or Algebra.

a. $\rightsquigarrow$ *Olivia can't take Logic*
$\rightsquigarrow$ *Olivia can't take Algebra*

(24) Noah is unaware that Olivia is allowed to take Logic or Algebra.

a. $\rightsquigarrow$ *Olivia can take Logic*
$\rightsquigarrow$*Olivia can take Algebra*

b. $\rightsquigarrow$ *Noah doesn't believe that Olivia can take Logic*
$\rightsquigarrow$ *Noah doesn't believe that Olivia can take Algebra*

The pattern in (22)-(24) poses a serious problem for standard, **exh**-based accounts of FC. Recall that, according to Fox (2007) and similar accounts, the DP reading of $\neg\Diamond\vee$-sentences is obtained by applying an economy constraint which prevents **exh** from being inserted in positions where it leads to an overall weakening of meaning. On this kind of view, the parse in (25a) supports the FC reading of (22), and the one in (25b) supports the DP reading of (23):

(25) a. $\textbf{exh}^*[\Diamond[L\vee A]]$ $\qquad \Leftrightarrow \Diamond L\wedge\Diamond A$

b. $\neg[\Diamond[L\vee A]]$ $\qquad \Leftrightarrow \neg\Diamond L\wedge\neg\Diamond A$

It is easy to see why (24) is problematic for this **exh**-based account. While (24) presupposes FC—namely, that Olivia can take Logic and can take Algebra—what Noah doesn't believe is that Olivia can take either class (as opposed to that Olivia doesn't have FC, which is compatible with believing that she can take, say, Logic but not Algebra, or vice-versa). Assume an interpretation for the target propositional attribution in (26), according to which '$x$ is unaware that $p$' presupposes that $p$ and asserts that it is not the case that $x$ believes that $p$:

(26) $[\![x \text{ is unaware that } p]\!] = \neg B_x(p)_p$

To try to capture the default reading of (24), there are two natural candidate parses, presented in (27) and (28). If we parse (24) as in (27), with **exh**$^*$ over the embedded *Olivia is allowed to take Logic or Algebra*, we predict the correct presuppositions for the entire sentence. For the factivity of *unaware* will guarantee that the FC content projects, as captured in the **ps** part of (27a). However, for the assertive part we predict that Noah doesn't believe that Olivia has FC, as captured in the **asserts** part of (27a). Yet the target reading, recall, is that Noah doesn't believe Olivia can take even one of the classes.

(27) Noah is unaware [**exh**$^*[\Diamond(L \vee A)]]$

a. $[\![(27)]\!] = \begin{cases} \textbf{ps: exh}^*[\Diamond(L \vee A) = \Diamond L \wedge \Diamond A \\ \textbf{asserts: } \neg B_N(\textbf{exh}^*[\Diamond(L \vee A)]) = \neg B_N(\Diamond L \wedge \Diamond A) \end{cases}$

If we parse (24) as in (28), where the embedded *Olivia is allowed to take Logic or Algebra* appears without **exh**$^*$, we predict the correct content for Noah's doxastic state, namely, that he doesn't believe Olivia can take even one of Logic or Algebra, as captured in the **asserts** part of (28a). However, s captured in the **ps** part of (28a), we also derive a presupposition that is too weak, namely, that Olivia is allowed to take at least one class (which is compatible with Olivia not being allowed to take the other), whereas what we wanted was to get the entailment that Olivia has FC.

(28) Noah is unaware $[\Diamond(L \vee A)]$

a. $[\![(28)]\!] = \begin{cases} \textbf{ps: } \Diamond(L \vee A) \\ \textbf{asserts: } \neg B_N(\Diamond(L \vee A)) \end{cases}$

Importantly, the standard **exh**-based account of FC is not the only Grammatical account of SIs that has trouble deriving the default readings of sentences with presupposed FC under negative factives such as (24). As Marty & Romoli (2020) show, (24) also presents a serious challenge to the innocent exclusion + innocent inclusion-based theory of Bar-Lev & Fox (2017, 2020), and to the account with multidimensional exhaustification developed by Gajewski & Sharvit (2012).

### 3.2 A solution based on presuppositional exhaustification

We now show that the pattern in (22)-(24) can be directly captured given our **pex**-based Grammatical account of the FC reading of $\Diamond\vee$-sentences and the dual prohibition (DP) reading of $\neg\Diamond\vee$-sentences. In §2.2 we showed that the parse in (29a) supports the FC reading of (22), and the one in (29b) supports the DP reading of (23):

(29) a. $\mathbf{pex}[\textsc{acc}[\mathbf{pex}[\Diamond(L\vee A)]]] = \mathbf{pex}^*[\Diamond(L\vee A)] \Leftrightarrow \Diamond(L\vee A)_{\Diamond L\leftrightarrow\Diamond A}$

b. $\neg[\mathbf{pex}[\textsc{acc}[\mathbf{pex}[\Diamond(L\vee A)]]]] = \neg[\mathbf{pex}^*[\Diamond(L\vee A)]] \Leftrightarrow \neg\Diamond(L\vee A)_{\Diamond L\leftrightarrow\Diamond A}$

Since, on this account, covert exhaustification is a presupposition trigger, to deal with sentences like (24) we will also need an independent account of presupposition projection from the complement of belief operators. Two well-known options, each defended on independent grounds, are (30a) and (30b):

(30) a. Heim (1992)/Schlenker:
$B_x(p'_p) = B_x(p')_{B_x(p)}$

b. Geurts (1999) (DRT):
$B_x(p'_p) = B_x(p')_p$

(24) is repeated in (31). Since the target reading presupposes the FC reading of the embedded *Olivia is allowed to take Logic or Algebra*, it is natural to consider, given our **pex**-based account, the parse in (31a) (ignoring for simplicity a matrix **pex** which could associate with *Olivia* or *Noah*, among other options which are not relevant here):

(31) Noah is unaware that Olivia is allowed take Logic or Algebra.

a. Noah is unaware $[\mathbf{pex}^*[\Diamond(L\vee A)]]$

b. $[\![(31a)]\!] = \begin{cases} \textbf{ps: } \mathbf{pex}^*[\Diamond(L\vee A)] \\ \textbf{asserts: } \neg B_N(\mathbf{pex}^*[\Diamond(L\vee A)]) \end{cases}$

c. $\mathbf{pex}^*[\Diamond(L\vee A)] = \Diamond(L\vee A)_{\Diamond L\leftrightarrow\Diamond A}$

d. $\neg B_N(\mathbf{pex}^*[\Diamond(L\vee A)]) =$
$\neg[B_N(\Diamond(L\vee A)_{\Diamond L\leftrightarrow\Diamond A})] =$ (given Heim's (30a))
$\neg[B_N(\Diamond(L\vee A))_{B_N(\Diamond L\leftrightarrow\Diamond A)}] =$ (ps projects under $\neg$)
$\neg B_N(\Diamond(L\vee A))_{B_N(\Diamond L\leftrightarrow\Diamond A)}$

Based on the **ps** part of (31b), and given the equivalence in (31c), we get the target result that (31) presupposes that Olivia is allowed to take Logic and is allowed to take Algebra. This positive result is not surprising. We next need to check whether the **asserts** part of (31b), $\neg B_N(\mathbf{pex}^*[\Diamond(L\vee A)])$, captures the correct content for Noah's doxastic state. Given the Heim/Schlenker assumption for how belief operators interact with presupposed content in their complements, we derive the target result, as shown in (31d). What Noah doesn't believe, on this account, is that Olivia can take either one of Logic or Algebra (as opposed to Noah not believing merely that she has free choice, which, again, is compatible with Noah believing that Olivia can take Algebra but not Logic, or vice-versa). As a result, the parse in (31a) issues in the correct predictions for sentences like (31) concerning both their overall presuppositions and the content of their doxastic attribution.

Now, as captured by the last equivalence of (31d), we also derived the additional presupposition that Noah believes the homogeneity proposition that if Olivia can take either one of the classes, she can take the other one. This additional presupposition is perhaps unattested, or hard to detect, in cases specifically like (24). Yet we can avoid that prediction, without affecting the rest of the derivation, including the other desired predictions, by adopting, instead of the Heim/Schlenker rule in (30a), the DRT rule in (30b) concerning how presuppositions project from the complement of belief operators. In this case, we get that $\neg[B_N(\Diamond(L \vee A)_{\Diamond L \leftrightarrow \Diamond A})] \Leftrightarrow \neg B_N(\Diamond(L \vee A))_{\Diamond L \leftrightarrow \Diamond A}$, since the derivation in (31d) should be replaced with the following one:

(31) d′. $\neg B_N(\mathbf{pex}^*[\Diamond(L \vee A)]) =$
$\neg[B_N(\Diamond(L \vee A)_{\Diamond L \leftrightarrow \Diamond A})] =$ (given DRT's (30b))
$\neg[B_N(\Diamond(L \vee A))_{\Diamond L \leftrightarrow \Diamond A}] =$ (ps projects under $\neg$)
$\neg B_N(\Diamond(L \vee A))_{\Diamond L \leftrightarrow \Diamond A}$

The homogeneity presupposition $\Diamond L \leftrightarrow \Diamond A$ doesn't result in additional presuppositional constraints to (31), since it is entailed by the presuppositions triggered by the factivity of *unaware*, derived in (31c). So in this case the DRT-based route captures exactly the target reading singled out by Marty & Romoli (2020). For our purposes, however, we can remain neutral between these different views concerning the interaction between belief operators and the projection behavior of presuppositions triggered in their complement.[7]

### 3.3 Extension to presupposed negative free choice

As Marty & Romoli (2020) point out, the phenomenon of presupposed FC extends to other types of embedded FC sentences, not just $\Diamond\vee$-sentences. This is important because recent Lexicalist accounts can explain the puzzle in the standard version illustrated by patterns like (22)-(24), but have trouble with simple variations with embedded 'negative' FC sentences.

An example of a negative FC sentence is (32), which consist of a conjunction under negation and a universal quantifier. In one of its salient readings, (32) suggests that Olivia is allowed to not take Logic and is also allowed to not take Algebra, as captured in (32a). Experimental evidence that this reading is in general available is presented in Marty, Romoli, Sudo & Breheny (2021). From now on, we refer to

7 Indeed, consider simpler examples of $\Diamond\vee$-sentences embedded under belief and knowledge operators, such as *Noah believes that Olivia is allowed to take Logic or Algebra*, focusing on the reading which attributes to Noah the belief that Olivia has free choice vis-a-vis taking Logic or Algebra. It is easy to check that, given our **pex**-based account, a simple route to that reading is via a parse with embedded $\mathbf{pex}^*$ immediately over the $\Diamond\vee$-sentence and using the Heim/Schlenker assumption in (30a). Noah is then represented as believing both $\Diamond(L \vee A)$ and $\Diamond L \leftrightarrow \Diamond A$.

sentences of the surface form of (32) as '$\neg\Box\wedge$-sentences', and when needed use a 'FC' prefix to signal the target reading.

(32) Olivia is not required to take Logic and Algebra.

a. $\rightsquigarrow$ *Olivia is allowed to not take Logic*
$\rightsquigarrow$ *Olivia is allowed to not take Algebra*

Lexicalist accounts are not designed to handle these case, and it is hard to see how they can be modified to handle them without negatively affecting other predictions. Yet as Marty & Romoli (2020) emphasize, the puzzle of presupposed FC under (negative) factives is also observed in cases with embedded $\neg\Box\wedge$-sentences. Consider (33), which in one of its most salient reading presupposes that Olivia is allowed to not take Logic and is also allowed to not take Algebra ('negative FC'), as captured in (33a), and asserts that Noah doesn't believe that Olivia can take either one (and not just that Olivia doesn't have negative free choice), as captured in (33b):

(33) Noah is unaware that Olivia is not required to take Logic and Algebra.

a. $\rightsquigarrow$ *Olivia is not required to take Logic* [$= \Diamond\neg L$]
$\rightsquigarrow$ *Olivia is not required to take Algebra* [$= \Diamond\neg A$]

b. $\rightsquigarrow$ $\neg$*Noah believes Olivia is allowed to not take Logic*
$\rightsquigarrow$ $\neg$*Noah believes that Olivia is allowed to not take Algebra*

In short, Lexicalist accounts can handle one variation of the puzzle of presupposed FC under (negative) factives, with embedded $\Diamond\vee$-sentences. Yet it is hard to see how they can be extended to a different variation, with embedded $\neg\Box\wedge$-sentences as in (33), which seem to call a parallel treatment. As is well known, standard **exh**-based Grammatical theories do predict the negative FC reading of $\neg\Box\wedge$-sentences such as (32). Yet as Marty & Romoli (2020) show, they also fail to predict the target reading of (33), for reasons which parallel their difficulties in dealing with $\Diamond\vee$-sentences embedded under negative factives (see §3.1 above). In contrast, we will now show that our **pex**-based account can be easily extended to this variation of the puzzle. To do this, we first sketch the derivation of the negative FC reading for basic $\neg\Box\wedge$-sentences like (32). We then show how that result, when combined with the same assumptions used in §3.2, issues in the correct predictions for $\neg\Box\wedge$-sentences embedded under (negative) factives, such as (33).

The derivation of the negative FC reading of $\neg\Box\wedge$-sentences is based on the parse in (34a), which is structurally analogous to the one used to derive the FC reading of $\Diamond\vee$-sentences (see §2.2). The interpretation of (34a) is presented in (34c). Given the equivalences in (34d) for the **ps** part and in (34e) for the **asserts** part, (34c) reduces to (34f), i.e., to $\Diamond\neg p \vee \Diamond\neg q_{\Diamond\neg p \leftrightarrow \Diamond\neg q}$, which entails the target negative FC conjuncts, $\Diamond\neg p$ and $\Diamond\neg q$.[8]

(34) a. $\mathbf{pex}_1[\textsc{acc}[\mathbf{pex}_2[\neg\Box[p \wedge q]]]$

b. $\text{Alt}(\textsc{acc}[\mathbf{pex}_2[\neg\Box[p \wedge q]]]) = \left\{ \begin{array}{l} \textsc{acc}[\mathbf{pex}_2[\neg\Box[p \wedge q]]], \\ \textsc{acc}[\mathbf{pex}_2[\neg\Box p]],\ \textsc{acc}[\mathbf{pex}_2[\neg\Box q]], \\ \textsc{acc}[\mathbf{pex}_2[\neg\Box[p \vee q]]] \end{array} \right.$

c. $[\![(34a)]\!] = \left\{ \begin{array}{l} \textbf{ps: } \neg\textsc{acc}[\mathbf{pex}_2[\neg\Box p]] \wedge \neg\textsc{acc}[\mathbf{pex}_2[\neg\Box p]] \\ \textbf{asserts: } \textsc{acc}[\mathbf{pex}_2[\neg\Box[p \wedge q]]] \end{array} \right.$

d. $\neg\textsc{acc}[\mathbf{pex}_2[\neg\Box p]] \wedge \neg\textsc{acc}[\mathbf{pex}_2[\neg\Box q]]$
$= \neg[\neg\Box p \wedge \Box q] \wedge \neg[\neg\Box q \wedge \Box p]$
$= [\Box p \vee \neg\Box q] \wedge [\Box q \vee \neg\Box p]$
$= [\neg\Diamond\neg p \vee \Diamond\neg q] \wedge [\neg\Diamond\neg q \vee \Diamond\neg p]$
$= [\Diamond\neg p \rightarrow \Diamond\neg q] \wedge [\Diamond\neg q \rightarrow \Diamond\neg p]$
$= \Diamond\neg p \leftrightarrow \Diamond\neg q$

e. $\textsc{acc}[\mathbf{pex}_2[\neg\Box[p \wedge q]]] = \neg\Box[p \wedge q]$
$= \Diamond\neg p \vee \Diamond\neg q$

f. $[\![(34a)]\!] = \Diamond\neg p \vee \Diamond\neg q_{\Diamond\neg p \leftrightarrow \Diamond\neg q}$

As before, we abbreviate the derivation in (34) as in (35):

(35) $\mathbf{pex}_1[\textsc{acc}[\mathbf{pex}_2[\neg\Box[p \wedge q]]] = \mathbf{pex}^*[\neg\Box[p \wedge q]] = \Diamond\neg p \vee \Diamond\neg q_{\Diamond\neg p \leftrightarrow \Diamond\neg q}$

Building on that result, we can now move to the more complex cases of $\neg\Box\wedge$-sentences embedded under negatives factives such as (33), repeated in (36). Consider the parse in (36a), which is structurally analogous to the one we used to deal with $\Diamond\vee$-sentences under (negative) factives. As captured in (36b)-(36c), this parse issues in the target prediction that (36) presupposes negative FC. This takes care of the first Marty & Romoli's desiderata, captured in (33a). We next need to check the predictions for Noah's doxastic state. Recall that, on the target reading of (36), Noah doesn't believe that Olivia is allowed to not take Logic, and also doesn't believe that Olivia is allowed to not take Algebra. That doxastic state is different from one in which what Noah doesn't believe is that Olivia doesn't have the (negative) FC to not take either class. The latter, but not the former, is compatible with Noah believing that Olivia is required to take Logic but not Algebra (or vice versa). This is precisely

8 As in the derivation of the FC reading of $\Diamond\vee$-sentences, we are again treating as non-relevant—mainly for simplicity but also because it makes no difference to the target reading—the alternatives with disjunction for both $\mathbf{pex}_1$ and $\mathbf{pex}_2$. As before, this simplifying assumption can be easily dropped. For reasons similar to those covered in the derivation of the FC reading of $\Diamond\vee$-sentences, the disjunctive alternative should be added at the level of $\mathbf{pex}_1$. This would generate a reading for (32) which entails, as before, that Olivia is allowed to not take Logic and is allowed to not take Algebra, but also that Olivia is required to take at least one of Logic or Algebra.

the result we derive for the **asserts** part, captured in (36b), given the equivalences in (36d). As before, although the **asserts** part triggers the additional presupposition $\Diamond\neg p \leftrightarrow \Diamond\neg q$, that homogeneity presupposition is already entailed by the **ps** part, as captured in (36c), so doesn't strengthen the overall presuppositions of (36).

(36) Noah is unaware that Olivia is not required to take Logic and Algebra

a. Noah is unaware $\mathbf{pex}^*[\neg\Box[L \wedge A]]$

b. $[\![(36a)]\!] = \begin{cases} \textbf{ps: } \mathbf{pex}^*[\neg\Box[L \wedge A]] \\ \textbf{asserts: } \neg B_N(\mathbf{pex}^*[\neg\Box[L \wedge A]]) \end{cases}$

c. $\mathbf{pex}^*[\neg\Box[L \wedge A]] = \Diamond\neg L \vee \Diamond\neg A_{\Diamond\neg L \leftrightarrow \Diamond\neg A}$

d. $\neg B_N(\mathbf{pex}^*[\neg\Box[L \wedge A]]) =$
$\neg[B_N(\Diamond\neg L \vee \Diamond\neg A_{\Diamond\neg L \leftrightarrow \Diamond\neg A})] =$ (given DRT's (30b))
$\neg[B_N(\Diamond\neg L \vee \Diamond\neg A)_{\Diamond\neg L \leftrightarrow \Diamond\neg A}] =$ (ps projects under $\neg$)
$\neg B_N(\Diamond\neg L \vee \Diamond\neg A)_{\Diamond\neg L \leftrightarrow \Diamond\neg A}$

### 3.4 Summary

We have examined the first part of the presupposed & filtering FC puzzle, which focuses on the behavior of $\Diamond\vee$ and $\neg\Box\wedge$-sentences embedded under (negative) factives. These sentences, taken together, present a serious challenge to many theories of FC, including influential **exh**-based theories and revised Lexicalist theories. In contrast, we have shown that our **pex**-based theory of FC issues in a uniform solution to this part of the puzzle. These results are summarized in Table 1 below. As we note there, the new Grammatical theory developed in Marty & Romoli (2020)—which uses a multidimensional exhaustification operator with innocent exlusion and inclusion—can arguably also deal with these cases of presupposed FC under negative factives. However, that theory doesn't help with the filtering FC part of the puzzle, to which we now turn.

## 4 Filtering free choice

### 4.1 The challenge

The second part of the presupposed & filtering FC puzzle, due to Romoli & Santorio (2019), concerns an intricate pattern of presupposition projection and filtering in certain complex sentences that involve FC effects. The basic pattern is presented in (37)-(39), where the key case is (39). Recall that (37) has a FC reading, which tends to disappear under negation, as in the dual prohibition (DP) reading of (38). With that in mind, consider the most salient reading of (39). The first main disjunct (*Maria can't study in Tokyo or Boston*) has the DP reading. In addition, although the second main disjunct (*she is the first/second in our family who can go study in*

| **Accounts** | **FC-**$\Diamond\vee$ | **DP-**$\neg\Diamond\vee$ | $\neg K > \Diamond\vee$ | $\neg K > \neg\Box\wedge$ |
|---|---|---|---|---|
| Lexicalist old | ✓ | ✗ | ✗ | ✗ |
| Lexicalist new | ✓✓ | ✓✓ | ✓ | ✗ |
| $\mathbf{exh}^{IE/IE+II}$ | ✓ | ✓ | ✗ | ✗ |
| $\mathbf{exh}^{IE}$+MD | ✓ | ✓ | ✗ | ✗ |
| $\mathbf{exh}^{IE+II}$+MD | ✓ | ✓ | ✓ | ✓ |
| **pex** | ✓✓ | ✓✓ | ✓ | ✓ |

**Table 1** Predictions of theories of FC concerning presupposed FC under (negative) factives. Theories get double marks for predicting results concerning the definedness conditions of the FC reading for $\Diamond\vee$-sentences and dual prohibition (DP) readings of $\neg\Diamond\vee$-sentences, as reported by Tieu et al. (2019). We use '$\neg K > P$' to represent sentences where $P$ is embedded under a negative factive. References for theories: Lexicalist old: Zimmerman (2000) and Simons (2005b). Lexicalist new: Willer (2017), Goldstein (2018), Aloni (2018), Rothschild & Yablo (2018). Grammatical with $\mathbf{exh}^{IE/IE+II}$: Fox (2007), Chierchia (2013), Bar-Lev & Fox (2020). Grammatical with $\mathbf{exh}^{IE}$+MD: Gajewski & Sharvit (2012), Spector & Sudo (2017). Grammatical with $\mathbf{exh}^{IE+II}$+MD: Marty & Romoli (2020).

*Japan/States*) triggers the FC presupposition that Maria can study in Japan and in the States, (39) as a whole doesn't seem to inherit that presupposition, as captured in (39a). Intuitively, that is because the negation of the first main disjunct somehow filters out that FC presupposition.

(37) Maria can go study in Tokyo or Boston.

a. $\rightsquigarrow$ *Maria can study in Tokyo*
$\rightsquigarrow$ *Maria can study in Boston*

(38) Maria can't go study in Tokyo or Boston.

a. $\rightsquigarrow$ *Maria can't study in Tokyo*
$\rightsquigarrow$ *Maria can't study in Boston*

(39) Either Maria can't go study in Tokyo or Boston, or she is the first in our family who can go study in Japan and the second who can go study in the States.

a. $\not\rightsquigarrow$ *Maria can study in Japan*
$\not\rightsquigarrow$ *Maria can study in the States*

To appreciate the challenge presented by this pattern, it will be helpful to introduce some background conventions and assumptions about presupposition projection. Following Romoli & Santorio (2019), we schematically represent the target example in (39) as in (40), where $A^+$ asymmetrically entails $A$, $B^+$ asymmetrically entails $B$ and $C_{\Diamond A \wedge \Diamond B}$ says that $C$ is asserted while $\Diamond A \wedge \Diamond B$ is presupposed (for readability, we represent the assertive part of the conjunction in the second main disjunct with just $C$).

(40) Either $\neg\Diamond(A^+ \vee B^+) \vee C_{\Diamond A \wedge \Diamond B}$ $\quad\not\rightsquigarrow \Diamond A; \not\rightsquigarrow \Diamond B$

We assume that disjunctions with a presupposition in the second disjunct, $p \vee q_r$, project a conditional presupposition, as in (41) (Heim 1982, Chierchia 1995, Beaver 2001), from which it follows that the presupposition $r$ is filtered if $\neg p \models r$. That explains why a sentence like *Either Maria didn't study in Tokyo, or she is the first in our school who studied in Japan* doesn't presuppose that Maria studied in Japan. We also assume that presuppositions triggered in the first disjunct tend to project unconditionally, as in (42) (since $\neg p_r \rightarrow q$ presupposes $r$, e.g., *if John doesn't find out that Mary came, she will feel sad* presupposes that Mary came).[9]

(41) $p \vee q_r = \begin{cases} \textbf{ps: } \neg p \rightarrow r \\ \textbf{asserts: } p \vee q \end{cases}$

(42) $p_r \vee q = \begin{cases} \textbf{ps: } r \\ \textbf{asserts: } p \vee q \end{cases}$

With these conventions and assumptions in place, we can now see why the pattern in (37)-(39) presents a problem for **exh**-based accounts of FC (Romoli & Santorio 2019). On the one hand, to get the dual prohibition reading of the first main disjunct of (39) (*Maria can't go study in Tokyo or Boston*), we need a parse for that disjunct without **exh**$^*$ under negation, as in (43a). Yet given that parse, we predict an incorrect presupposition, captured in the **ps** part of (43b). For as shown in (43c), the conditional presupposition in the **ps** part doesn't filter out $\Diamond A$ and $\Diamond B$ (i.e., that Maria can study in Japan and in the States), since $\Diamond(A^+ \vee B^+) \not\models \Diamond A \wedge \Diamond B$.

(43)
a. $\neg\Diamond(A^+ \vee B^+) \vee C_{\Diamond A \wedge \Diamond B}$

b. $[\![(43a)]\!] = \begin{cases} \textbf{ps: } \neg\neg\Diamond(A^+ \vee B^+) \rightarrow (\Diamond A \wedge \Diamond B) \\ \textbf{asserts: } \neg\Diamond(A^+ \vee B^+) \vee C \end{cases}$

c. $\neg\neg\Diamond(A^+ \vee B^+) \rightarrow (\Diamond A \wedge \Diamond B)$
$= \Diamond(A^+ \vee B^+) \rightarrow (\Diamond A \wedge \Diamond B)$
$= (\Diamond A^+ \rightarrow (\Diamond A \wedge \Diamond B)) \wedge (\Diamond B^+ \rightarrow (\Diamond A \wedge \Diamond B))$
(by dist. of $\Diamond$ over $\vee$; simp. of disj. ant.)

9 The projection rules in (41) and (42) are obviously not the only candidates. We selected those rules because, on the one hand, they are fairly standard and, on the other hand, because, as Romoli & Santorio (2019) show, other candidate projections rules not only don't help but actually worsen the predictions of **exh**-based theories for sentences like (39).

On the other hand, consider a parse for (39) where we introduce $\mathbf{exh}^*$ under negation in the first main disjunct, as in (44a). As captured in the **ps** part of (44b), and given the simple equivalence in (44c), this would correctly filter out the presupposition, triggered in the second main disjunct, that Maria can study in Japan and in the States. However, we now loose the target dual prohibition reading for the first disjunct, *Maria can't go study in Tokyo or Boston*, and get instead the unattested and weaker ('negation of FC') reading, as captured in the **asserts** part of (44b).

(44) a. $\neg\mathbf{exh}^*[\Diamond(A^+ \vee B^+)] \vee C_{\Diamond A \wedge \Diamond B}$

b. $[\![(44a)]\!] = \begin{cases} \textbf{ps: } \neg\neg\mathbf{exh}^*[\Diamond(A^+ \vee B^+) \to (\Diamond A \wedge \Diamond B)] \\ \textbf{asserts: } \neg\mathbf{exh}^*[\Diamond(A^+ \vee B^+)] \vee C \end{cases}$

c. $\neg\neg\mathbf{exh}^*[\Diamond A^+ \vee \Diamond B^+] \to (\Diamond A \wedge \Diamond B)$
$= (\Diamond A^+ \wedge \Diamond B^+) \to (\Diamond A \wedge \Diamond B)$

## 4.2 A solution based on presuppositional exhaustification

In contrast to **exh**-based accounts, our **pex**-based account of FC directly predicts the target readings for all of (37)-(39). The key difference between these accounts, recall, is that while **exh**-based accounts assign a flat, fully assertive structure to the FC readings $\Diamond\vee$-sentences, as captured in (45), our **pex**-based account derives FC readings via the interaction between its presupposed and assertive content, as captured in (46):

(45) $\mathbf{exh}^*[\Diamond(p \vee q)] = (\Diamond p \leftrightarrow \Diamond q) \wedge \Diamond(p \vee q)$ $\qquad \models \Diamond p \wedge \Diamond q$

(46) $\mathbf{pex}^*[\Diamond(p \vee q)] = \Diamond(p \vee q)_{\Diamond p \leftrightarrow \Diamond q}$ $\qquad \models \Diamond p \wedge \Diamond q$

Both accounts predict the FC reading for $\Diamond\vee$-sentences like (37) and the DP reading $\neg\Diamond\vee$-sentences like (38). Yet due to its structuring of the FC interpretation in terms of presupposed vs. asserted content, only the **pex**-based account can derive both readings using the same underlying parse, i.e., $\mathbf{pex}^*[\Diamond(A^+ \vee B^+)]$ for (37) and $\neg\,\mathbf{pex}^*[\Diamond(A^+ \vee B^+)]$ for (38) (see §2.2 for details). This unique result will play a crucial role when trying to derive both the DP reading for the first disjunct of (39) (= *Maria can't go study in Tokyo or Boston*), and the normal FC reading for the negation of that first disjunct to determine whether the presuppositions of the second disjunct are filtered out (i.e., to determine whether 'Maria can study in Japan and in the States' is filtered out).

Sentence (39)—the representative filtering FC sentence—is repeated in (47). Given our **pex**-based theory, the parse in (47a) is not only a natural choice but, as we now show, also generates the target reading. Applying (41) to (47a) to derive its presupposition, we get the prediction that (47) presupposes $\mathbf{pex}^*[\Diamond(A^+ \vee B^+)] \to (\Diamond A \wedge \Diamond B)$, as shown in the **ps** part in (47b). The antecedent of this conditional is just an FC interpretation of a $\Diamond\vee$-sentence, as shown in (47c), which entails $\Diamond A^+ \wedge \Diamond B^+$ and thus correctly filters out $\Diamond A$ and $\Diamond B$. Consider next the content of the first main disjunct *Maria can't go study in Tokyo or Boston*, which is parsed as $\neg\,\textbf{pex}^*[\Diamond(A^+ \vee B^+)$. As shown in (47d), due to the presupposed vs. assertive structure generated by **pex***, the homogeneity presupposition projects from under negation. As a result, the negation applies directly to $\Diamond(A^+ \vee B^+)$, which results in the target DP reading of *Maria can't go study in Tokyo or Boston*.[10]

(47) Either Maria can't go study in Tokyo or Boston, or she is the first in our family who can go study in Japan and the second who can go study in the States.

a. $\neg\,\textbf{pex}^*[\Diamond(A^+ \vee B^+)] \vee C_{\Diamond A \wedge \Diamond B}$

b. $[\![\text{(47a)}]\!] = \begin{cases} \textbf{ps: pex}^*[\Diamond(A^+ \vee B^+)] \rightarrow (\Diamond A \wedge \Diamond B) \\ \textbf{asserts: } \neg\,\textbf{pex}^*[\Diamond(A^+ \vee B^+)] \vee C \end{cases}$

c. $\textbf{pex}^*[\Diamond(A^+ \vee B^+)] \rightarrow (\Diamond A \wedge \Diamond B) =$
$\Diamond(A^+ \vee B^+)_{\Diamond A^+ \leftrightarrow \Diamond B^+} \rightarrow (\Diamond A \wedge \Diamond B)$

d. $\neg\,\textbf{pex}^*[\Diamond(A^+ \vee B^+)] =$
$\neg[\Diamond(A^+ \vee B^+)_{\Diamond A^+ \leftrightarrow \Diamond B^+}] =$ (ps projects under $\neg$)
$\neg\Diamond(A^+ \vee B^+)_{\Diamond A^+ \leftrightarrow \Diamond B^+}$

Summing up, unlike **exh**-based accounts of FC, our **pex**-based account resolves the filtering FC puzzle captured by the pattern in (37)-(39). Specifically, our **pex**-based account predicts that (39)/(47) doesn't presuppose that Maria can study in Japan and in the States, while simultaneously assigning to the embedded *Maria can't go study in Tokyo or Boston* the desired DP (dual prohibition) reading. We have shown that those predictions follow directly from our original **pex**-based account of simple FC-$\Diamond\vee$-sentences like (37) and DP-$\neg\Diamond\vee$-sentences like (38), given reasonable assumptions about presupposition projection and filtering in disjunctions. That satisfies the main desiderata, laid out by Romoli & Santorio (2019), to capture the default reading of sentences like (39)/(47). To be clear, our account does predict that those sentences have a non-trivial homogeneity presupposition (in this specific case, $\Diamond A^+ \leftrightarrow \Diamond B^+$).[11] We discuss that prediction in §4.4 below.

10 The first disjunct of the main disjunction, $\neg\,\textbf{pex}^*[\Diamond(A^+ \vee B^+)$, also presupposes $\Diamond A^+ \leftrightarrow \Diamond B^+$, as shown in (47d). Assuming (42), we would then predict that this homogeneity presupposition should be inherited by (47). For discussion of this additional entailment, see §4.4 below.

11 Recall that our **pex**-based account predicts analogous homogeneity presuppositions for the FC reading $\Diamond\vee$-sentences and the DP reading of $\neg\Diamond\vee$-sentences. As we argued in §2.2, the results reported by Tieu et al. (2019) support those predictions.

### 4.3 Extension to filtering negative free choice

The filtering FC puzzle is also observed in variants with embedded FC conjunctions, illustrated in (48)-(50) below. Recall that, in one of its salient readings, (49) has the negative FC entailment that Maria is allowed to not study in Japan and is also allowed to not study in the States. Now, notice that while the second main disjunct of (50) (*[Maria is] the first in her family who is not required to go to Tokyo and the second who's not required to go to Boston*) presupposes the negative FC proposition that Maria is allowed to not go to Tokyo and is also allowed to not go to Boston, that proposition doesn't project as a presupposition of (50), as captured in (50a). Intuitively, that is because the negative FC proposition triggered by the second disjunct is entailed and hence filtered out by the (negative FC reading of) the negation of the first disjunct of (50).

(48) Maria is required to study in Japan and the States.

a. $\rightsquigarrow$ *Maria is required to study in Japan*
$\rightsquigarrow$ *Maria is required to study in the States*

(49) Maria is not required to study in Japan and the States.

a. $\rightsquigarrow$ *Maria is allowed to not study in Japan*
$\rightsquigarrow$ *Maria is allowed to not study in the States*

(50) Either Maria is required to go to Japan and the States, or she's the first in her family who is not required to go to Tokyo and the second who's not required to go to Boston.

a. $\not\rightsquigarrow$ *Maria is allowed to not go to Tokyo*
$\not\rightsquigarrow$ *Maria is allowed to not go to Boston*

Following Romoli & Santorio (2019), let us schematically represent the target reading of (50)—our example of filtering negative FC—as in (51), where $A^+$ asymmetrically entails $A$ and $B^+$ asymmetrically entails $B$:

(51) Either $\Box(A \wedge B) \vee C_{\neg\Box A^+ \wedge \neg\Box B^+}$ $\qquad \not\rightsquigarrow \neg\Box A^+$, $\not\rightsquigarrow \neg\Box B^+$

Based on the projection rule for disjunctions in (41), the presupposition triggered in the second main disjunct of (50)/(51), $\neg\Box A^+ \wedge \neg\Box B^+$, is filtered out if it is entailed by the negation of the first main disjunct, $\neg\Box(A \wedge B)$. Accordingly, given an LF which closely matches the surface form of (51), we would not predict the target filtering effect, since obviously $\neg\Box(A \wedge B) \not\models \neg\Box A^+ \wedge \neg\Box B^+$.

As Romoli & Santorio (2019) emphasize, this version of the filtering FC puzzle is interesting in part because, while recent Lexicalist accounts (e.g., Goldstein 2018) predict the main desiderata for the previous filtering FC pattern in (37)-(39), they can't be extended to the analogous pattern in (48)-(50). Ultimately, the source of the problem for these theories is the same as that posed by the examples of presupposed negative FC under (negative) factives (see §3.3): Lexicalist accounts do not predict a negative FC readings for $\neg\Box\wedge$-sentences such as (49). We just said that, for sentences like (50), the presuppositions triggered in the second main disjunct are filtered out if they are entailed by the negation of the first disjunct. Accordingly, we would get the target filtering effect if we could derive a negative FC reading given $\neg$*Maria is required to go to Japan and the States*—yet the point is that recent Lexicalist accounts can't derive negative FC readings for $\neg\Box\wedge$-sentences.

A **pex**-based approach to FC, in contrast, can capture patterns like (48)-(50). Two options seem initially promising. The first is conservative with respect to the operations of **pex**, but requires a non-trivial syntactic stipulation which may have some independent motivation. The second option—our preferred one—doesn't require any additional syntactic stipulations, but enriches the computations of **pex** in a way which helps with the filtering negative FC cases while preserving our previous analyses with **pex**.

Before discussing these two options, it is worth clarifying why (50) is so tricky even for our **pex**-based account. Given the commitments and results of our account, the most natural parse to consider is (52a). Yet notice that in this case the **pex**$^*$ in the first disjunct is vacuous, i.e., there are no excludable alternatives that can be negated and added as presuppositions. The overall effect is as if we had a parse without **pex**$^*$ in the first disjunct. As a result, the $\neg\Box A^+$ and $\neg\Box B^+$ presuppositions of the second disjunct are not filtered out, since $\neg\,\mathbf{pex}^*[\Box(A\wedge B)]$ entails neither $\neg\Box A^+$ nor $\neg\Box B^+$, as captured in (52c).

(52) a. $\mathbf{pex}^*[\Box(A\wedge B)]\vee C_{\neg\Box A^+\wedge\neg\Box B^+}$

b. $[\![(52a)]\!] = \begin{cases} \textbf{ps: } \neg\,\mathbf{pex}^*[\Box(A\wedge B)] \to (\neg\Box A^+\wedge\neg\Box B^+) \\ \textbf{asserts: } \Box(A\wedge B)\vee C \end{cases}$

c. $\neg\,\mathbf{pex}^*[\Box(A\wedge B)] \to (\neg\Box A^+\wedge\neg\Box B^+)$
$= \neg[\Box(A\wedge B)] \to (\neg\Box A^+\wedge\neg\Box B^+)$
$= (\neg\Box A\vee\neg\Box B) \to (\neg\Box A^+\wedge\neg\Box B^+)$

It is worth noting that the same difficulty obtains if we use a parse like (52a) except with standard **exh**$^*$ instead of **pex**$^*$. For it is easy to check that, for analogous reasons, $\neg\mathbf{exh}^*[\Box(A\wedge B)]$ entails neither $\neg\Box A^+$ nor $\neg\Box B^+$.

From this analysis of the difficulty, one may extract an observation which motivates the first, syntactic proposal for a **pex**-based analysis of (50). Note that to filter out the presuppositions of the second disjunct of (50), we want to somehow get—when calculating the presuppositions of (50)—**pex**$^*$ to scope over the negation of the first disjunct, since $\mathbf{pex}^*[\neg\Box[A \wedge B]]$ entails both $\neg\Box A^+$ and $\neg\Box B^+$. Can we get that scoping effect via some kind of syntactic operation or alternative parse?

To develop a unified trivalent account of anaphora and presupposition projection, Rothschild (2017) proposes a mechanism for the free insertion of redundant material at logical form. This helps with examples like (53), which are felicitous although they seem to lack a suitable constituent that can serve as the antecedent of the pronoun *it*. For if (53) is parsed as in (53a)—where the underlined part marks the freely inserted redundant conjunct—we can explain why there is a suitable antecedent for that pronoun at logical form.

(53) There isn't a bathroom here, or it's under the stairs. (Partee 2004)

a. There isn't a bathroom here, or <u>there is a bathroom $\wedge$</u> it's under the stairs

For our purposes, we need a licensing condition for adding redundant material along the lines of (54) (see Romoli & Santorio 2019):

(54) ADDING REDUNDANT CONJUNCTIONS (ARC). If a sentence $A$ contains clauses $C$ and $B$, you may replace any instance of $B$ with $C \wedge B$ or with $\neg C \wedge B$ if the resulting sentence is S-equivalent to $[\![A]\!]$.

a. $\phi$ is 'S-equivalent' to $\psi$ iff they have the same truth-value at any world in which they are both defined.

Given ARC, (50) can be parsed as in (55a). We can then get the target reading by applying $\mathbf{pex}^*$ to the redundant, negated conjunct, as in (55b). A standard assumption about conjunctions is that presuppositions triggered in the second conjunct are filtered out if they are entailed by the first conjunct. As we showed in §3.3, $\mathbf{pex}^*[\neg\Box(A \wedge B)]$ generates the negative FC entailment, namely, $\Diamond\neg A \wedge \Diamond\neg B$, which entails, and thus filters out, $\neg\Box A^+$ and also $\neg\Box B^+$ (i.e., that Mary is allowed to not go to Japan and allowed to not go to the US entails that she is not required to go to Tokyo and also not required to go to Boston).

(55)
a. $\Box(A \wedge B) \vee \underline{\neg\Box(A \wedge B) \wedge} \; C_{\neg\Box A^+ \wedge \neg\Box B^+}$
b. $\Box(A \wedge B) \vee \underline{\mathbf{pex}^*[\neg\Box(A \wedge B)] \wedge} \; C_{\neg\Box A^+ \wedge \neg\Box B^+}$

This first **pex** compatible account of filtering negative FC, although perhaps not entirely indefensible, has several shortcomings. On the one hand, it depends on the fate of ARC, which is at this point a convenient syntactic stipulation whose independent justification remains doubtful.[12] On the other hand, the ARC is not

12 For further discussion of ARC and the filtering FC puzzle see Romoli & Santorio (2019). Also Elliott (2020) presents a promising alternative account of the data used to originally motivate the ARC.

uniquely useful to **pex**-based accounts of FC. As Romoli & Santorio (2019) point out, and can be easily checked by replacing **pex**$^*$ with **exh**$^*$ in (55b), it also helps standard **exh**-based theories deal with the filtering FC puzzles.[13]

The second **pex**-based approach to filtering negative FC is more promising. So far, we have reformulated the Grammatical approach to SIs assuming that covert exhaustification has the sole function of excluding (relevant) alternatives to the prejacent. Yet Bar-Lev & Fox (2020) argue that exhaustification also includes alternatives, so long as their inclusion is consistent with the conjunction of the prejacent and the negation of any excludable alternatives.

(56) Innocently Includable alternatives of the prejacent $\phi$:

a. Take all maximal sets of alternatives of $\phi$ that can be assigned 'true' consistently with $\phi$ and the falsity of all IE alternatives of $\phi$.

b. The set of alternatives that are members in all such sets form the set of 'innocently includable' (II) alternatives of $\phi$ (note: for our purposes, we will substract the prejacent $\phi$ from this set).

Bar-Lev & Fox's proposal is that exhaustifiation of $\phi$ asserts the falsity of its IE alternatives and the truth of its II alternatives. From our perspective, how should an exhaustification operator with II be formulated, i.e., when conceived of as a presupposition trigger? Here's a conservative extension of our basic approach. We continue to take the prejacent $\phi$ as part of the assertive content and the negation of any IE alternatives as part of the presupposed content. In addition, the II alternatives will be part of the presupposed content, but are incorporated as follows. Instead of simply including each II alternative as part of the presupposed content—which superficially might seem like the more direct implementation of Bar-Lev & Fox (2020)'s original proposal—we include the subtly weaker homogeneity proposition that the II alternatives have the same truth value. Call this version of presuppositional exhaustification '**pex**$^{IE+II}$'. To justify its descriptive adequacy, we now show that **pex**$^{IE+II}$ preserves our previous good results with **pex** and in addition issues in a straightforward solution to the filtering negative FC challenge.

What is the effect of **pex**$^{IE+II}$ for basic cases of SIs and FC sentences? For many ordinary examples of SIs, **pex**$^{IE+II}$ outputs the same result as our original **pex**. For example, assuming the only non-trivial relevant alternative of *some students smoke* ($= \exists$) is *all students smoke* ($=\forall$), $[\![\textbf{pex}^{IE+II}(\exists)]\!] = [\![\textbf{pex}(\exists)]\!] = \exists_{\neg\forall}$, since in this case there are no II alternatives. This is a good result because it guarantees that

13 Yet we should emphasize that, even if coupled with the ARC, **exh**-based theories cannot account for the full presupposed & filtering FC puzzle. For the ARC doesn't help **exh**-based theories—at least in any obvious way—with the presupposed FC under negative factives puzzles discussed in §3. So Grammatical theorists who accept ARC still have an empirical incentive to switch to a **pex**-based account of FC.

moving from **pex** to $\textbf{pex}^{IE+II}$ will not negatively affect the empirical advantages of adopting a **pex**-based account of such basic (non-FC) SIs, esp., concerning how they project and the conditions under which they are locally accommodated when they appear in embedded environments (see Bassi et al. 2019, Del Pinal 2021). Consider next FC readings of $\Diamond\vee$-sentences. As shown in (57a)-(57d), $\textbf{pex}^{IE+II}$ simplifies their derivation, since we no longer need recursive exhaustification, but it ultimately outputs the same result as **pex**:

(57) a. $\textbf{pex}^{IE+II}[\Diamond[p \vee q]$

b. $\text{Alt}(\textbf{pex}^{IE+II}[\Diamond[p \vee q]) = \{\Diamond[p \vee q], \Diamond p, \Diamond q, \Diamond[p \wedge q]\}$

c. $\text{IE}(\Diamond[p \vee q]) = \{\Diamond[p \wedge q]\}$

$\text{II}(\Diamond[p \vee q]) = \{\Diamond p, \Diamond q\}$

d. $[\![(57a)]\!] = \begin{cases} \textbf{ps: } (\Diamond p \leftrightarrow \Diamond q) \wedge \neg\Diamond(p \wedge q) \\ \textbf{asserts: } \Diamond(p \vee q) \end{cases}$

Given the prejacent $\Diamond(p \vee q)$, and its formal alternatives in (57b), the only IE alternative is $\Diamond(p \wedge q)$. Yet notice that the disjunctive alternatives, $\Diamond p$ and $\Diamond q$, are in II, since they can be simultaneously and consistently conjoined with the prejacent and the negation of the IE alternatives, i.e., with $\Diamond(p \vee q) \wedge \neg\Diamond(p \wedge q)$. Given our formulation of $\textbf{pex}^{IE+II}$, this means that we should add the homogeneity presupposition that those II alternatives get the same truth-value, as captured in (57d). At this point, it is easy to check that, for FC readings of $\Diamond\vee$-sentences, the account based on a standard flat entry for **exh** with IE+II, our original account with recursive application of **pex** with just IE, and the current account based on **pex** with IE+II predict the same overall entailments, as captured in (58)-(59) (for simplicity, let us assume for now that the $\Diamond(p \wedge q)$ alternative is not relevant):

(58) $[\![\textbf{exh}^{IE+II}[\Diamond(p \vee q)]]\!] = \Diamond(p \vee q) \wedge \Diamond p \wedge \Diamond q$

(59) $[\![\textbf{pex}^{IE+II}[\Diamond(p \vee q)]]\!] = [\![\textbf{pex}^{*}[\Diamond(p \vee q)]]\!] = \Diamond(p \vee q)_{\Diamond p \leftrightarrow \Diamond q}$

Yet although they have the same overall entailments, we have seen that, relative to the presupposed and filtering FC puzzles with $\Diamond\vee$ and $\neg\Diamond\vee$-sentences, theories that output uniformly assertive ('flat') interpretations as in (58) face problems not faced by those with interpretations that have a presuppositional and assertive structure as in (59). This is also a good result because it guarantees that, when we move from **pex** to $\textbf{pex}^{IE+II}$, we preserve our previous good results for the presupposed and filtering FC puzzles with $\Diamond\vee$ and $\neg\Diamond\vee$-sentences.[14]

14 At this point, we can clarify why we proposed that $\textbf{pex}^{IE+II}$ should presuppose homogeneity over—and not simply the conjunction of—the II-alternatives. The main reason is purely descriptive. While presupposing the truth of each II alternative seems superficially closer to Bar-Lev & Fox's proposal, it makes incorrect predictions for the original cases of filtering FC with $\neg\Diamond\vee$-sentences, given their

When we move to cases like (50), however, $\mathbf{pex}^{IE+II}$—our presuppositional exhaustification operator with innocent inclusion—has an empirical advantage over the simpler, solely IE-based **pex**. Consider the parse in (60a), which is structurally like the one in (52a), except that $\mathbf{pex}^{IE+II}$ is substituted for $\mathbf{pex}^*$. As in the parallel case with **pex**, $\mathbf{pex}^{IE+II}[\Box(A \wedge B)]$ doesn't exclude anything. However, $\Box A$ and $\Box B$ are in the set of II alternatives, since taken together they can obviously be consistently conjoined with the conjunction of the prejacent and the negation of any excludable alternatives. It then follows that $\mathbf{pex}^{IE+II}[\Box(A \wedge B)] = [\Box(A \wedge B)]_{\Box A \leftrightarrow \Box B}$. The key observation, then, is that when determining the the presupposition of (60a), shown in (60b), the homogeneity presupposition will project out of the negation of the antecedent, as shown in (60c).

(60) a. $\mathbf{pex}^{IE+II}[\Box(A \wedge B)] \vee C_{\neg\Box A^+ \wedge \neg\Box B^+}$

b. $[\![(52a)]\!] = \begin{cases} \textbf{ps: } \neg\,\mathbf{pex}^{IE+II}[\Box(A \wedge B)] \rightarrow (\neg\Box A^+ \wedge \neg\Box B^+) \\ \textbf{asserts: } \Box(A \wedge B) \vee C \end{cases}$

c. $\neg\,\mathbf{pex}^{IE+II}[\Box(A \wedge B)] \rightarrow (\neg\Box A^+ \wedge \neg\Box B^+)$
$= \neg[\Box(A \wedge B)]_{\Box A \leftrightarrow \Box B} \rightarrow (\neg\Box A^+ \wedge \neg\Box B^+)$
$= (\neg\Box A \vee \neg\Box B)_{\Box A \leftrightarrow \Box B} \rightarrow (\neg\Box A^+ \wedge \neg\Box B^+)$

It is easy to check that the antecedent of (60c) entails the consequent, since $(\neg\Box A \vee \neg\Box B)_{\Box A \leftrightarrow \Box B} \models \neg\Box A \wedge \neg\Box B$, and $\neg\Box A \wedge \neg\Box B \models \neg\Box A^+ \wedge \neg\Box B^+$ (i.e., that Maria is not required to study in Japan and not required to Study in the States entails that she is not required to study in Tokyo and not required to study in Boston). It follows that the $\neg\Box A^+ \wedge \neg\Box B^+$ presupposition triggered in the second disjunct of (60a) is filtered out (by the negation of the first main disjunct) hence is not inherited as presupposition of (60a) as a whole. This precisely the target result.

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DP reading. The basic puzzle is repeated in (ia). Recall that, to filter out the presuppositions triggered in the second disjunct, we need a parse as in (ia):

(i) Either Maria can't go study in Tokyo or Boston, or she is the first in our family who can go study in Japan and the second who can go study in the States.

a. $\neg\,\mathbf{pex}^{IE+II}[\Diamond(A^+ \vee B^+)] \vee C_{\Diamond A \wedge \Diamond B}$

Now assume the superficially direct implementation, according to which $\mathbf{pex}^{IE+II}$ presupposes the conjunction of the II alternatives. This would predict that $\mathbf{pex}^{IE+II}[\Diamond(A \vee B)]$ presupposes $\Diamond A \wedge \Diamond B$. That conjunctive presupposition would then project from under negation, and, as a result, we wouldn't get the target DP reading for the first main disjunct, since $\neg\,\mathbf{pex}^{IE+II}\,\Diamond(A \vee B)$ would entail $\Diamond A \wedge \Diamond B$, contrary to fact (local accommodation from under the negation would also not help, since we would then only get the negation of FC reading, which is will too weak). What about a parse without $\mathbf{pex}^{IE+II}$ under negation in the first main disjunct? In this case we wouldn't filter out the $\Diamond A \wedge \Diamond B$ presupposition triggered in the second main disjunct, since obviously $\neg\neg\Diamond(A^+ \vee B^+) \not\models \Diamond A \wedge \Diamond B$. This is one reason why, when modelling exhaustification as a presupposition trigger, the II-part of the presuppositions should be weaker than simply the conjunction of the II alternatives.

It is important to note that the result that $\mathbf{pex}^{IE+II}$, when applied to $\Box\wedge$-sentences, triggers a homogeneity presupposition helps solve various additional puzzles for extant theories of SIs and FC. Consider the pattern in (61)-(62) with universal and negative universal FC from Chemla (2009b), which Bar-Lev & Fox (2020) use to motivate their proposal that exhaustification has an inclusion function.

(61) Every student is allowed to eat cake or ice cream.

a. $\rightsquigarrow$ *Every student is allowed to eat cake* $\forall x \Diamond Cx$
$\rightsquigarrow$ *Every student is allowed to ice cream* $\forall x \Diamond ICx$

(62) No student is required to solve problem A and problem B.

a. $\rightsquigarrow$ *No student is required to solve problem A* $\neg\exists x \Box Ax$
$\rightsquigarrow$ *No student is required to solve problem B* $\neg\exists x \Box Bx$

The problem for standard **exh**-based theories is this. The target FC reading of (61) can be derived from a parse with embedded $\mathbf{exh}^*$ (recursive **exh**) over $\Diamond(Cx \vee ICx)$. However, a parallel parse for (62) with embedded $\mathbf{exh}^*$ doesn't predict its target reading: for $\mathbf{exh}^*[\Box(Ax \wedge Bx)]$ is vacuous, since $\mathbf{exh}[\Box(Ax \wedge Bx)]$ has no IE alternatives. In contrast, given a $\mathbf{pex}^{IE+II}$-based account, we can directly derive the target FC reading for both (61) and (62) based on the following structurally analogous LFs with embedded $\mathbf{pex}^{IE+II}$:

(63) a. $\text{EVERY}(\llbracket student \rrbracket)\ \lambda x.\ \mathbf{pex}^{IE+II}[\Diamond(Cx \vee ICx)]$
b. $\text{NO}(\llbracket student \rrbracket)\ \lambda x.\ \mathbf{pex}^{IE+II}[\Box(Ax \wedge Bx)]$

Consider first (61) given the LF in (63a). $\mathbf{pex}^{IE+II}[\Diamond(Cx \vee ICx)]$ triggers the homogeneity presupposition $\Diamond Cx \leftrightarrow \Diamond ICx$ in the scope of *every student*. Assuming as before that the presupposition in the scope of the quantifier projects universally, when we combine it with the assertive content we get the target FC inference:

$$\forall x \in \llbracket student \rrbracket (\Diamond Cx \leftrightarrow \Diamond ICx) \wedge \forall x \in \llbracket student \rrbracket \Diamond (Cx \vee ICx) \\ \models \forall x \in \llbracket student \rrbracket \Diamond Cx \wedge \forall x \in \llbracket student \rrbracket \Diamond ICx$$

Consider next (62) given the LF in (63b). $\mathbf{pex}^{IE+II}[\Box(Ax \wedge Bx)]$ triggers the homogeneity presupposition $\Box Ax \leftrightarrow \Box Bx$ in the scope of *no student*. Assuming again universal projection of that presupposition, when combined with the assertive content we get the target (negative) FC inference:

$$\forall x \in \llbracket student \rrbracket (\Box Ax \leftrightarrow \Box Bx) \wedge \neg\exists x \in \llbracket student \rrbracket \Box (Ax \wedge Bx) \\ \models \neg\exists x \in \llbracket student \rrbracket \Box Ax \wedge \neg\exists x \in \llbracket student \rrbracket \Box Bx$$

This kind of result supports our proposal that we should implement the hypothesis that exhaustification has both an exclusion and an inclusion function using a $\mathbf{pex}^{IE+II}$-like operator For not only do we resolve the presupposed and filtering FC puzzles, which we don't with a flat $\mathbf{exh}^{IE+II}$ operator, but we can also account for the core cases that motivated the adoption of $\mathbf{exh}^{IE+II}$ in the first place.[15]

Summing up, we have seen that filtering negative FC, in cases like (50), poses a serious problem to current theories of FC. Our solution begins with the observation that our core proposal—namely, that Grammatical accounts of SIs should conceive of covert exhaustification as a presupposition trigger—can incorporate the recent hypothesis, due to Bar-Lev & Fox (2020), that exhaustification has the function of not only excluding but also including certain alternatives of the prejacent. We presented one concrete implementation of that hypothesis, in terms of the $\mathbf{pex}^{IE+II}$ operator, and showed, first, that it preserves our previous good results in terms of **pex**, and, secondly, that it straightforwardly accounts for the target reading of the filtering negative FC sentences.

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15 Bar-Lev & Fox (2020) do try to show that there must be a non-local/embedded derivation of the FC reading even cases like (61). The data they use to support this view, illustrated in (i), consists of positive universal FC sentences like (61) licensing a VP ellipsis where the elided material is in a DE environment (and gets the negative FC reading):

(i) Every student in section A is allowed to eat cake or ice cream on their birthday. Weirdly, no student in section B is ~~allowed to eat cake or ice cream on their birthday~~.

a. $\rightsquigarrow$ *Every girl is allowed to eat cake*
$\rightsquigarrow$ *Every girl is allowed to eat ice cream*

b. $\rightsquigarrow$ *No boy is allowed to eat cake*
$\rightsquigarrow$ *No boy is allowed to eat ice cream*

Suppose that, in cases like this, the material embedded under *every girl* has to be part of the parallelism domain for ellipsis. Still, the target reading can be straightforwardly derived using embedded $\mathbf{pex}^{IE+II}$. At this point, it is easy to check that, using the same structure in the scope of the quantifiers with embedded $\mathbf{pex}^{IE+II}$—i.e., $\mathbf{pex}^{IE+II}[\Diamond(Cx \vee ICx)]$—we predict the FC reading for the first sentence and the DP reading for the second sentence with the ellided VP, depending on how the homogeneity presupposition interacts with the corresponding quantificational claim:

(ii) $$\forall x \in [\![student\ in\ A]\!](\Diamond Cx \leftrightarrow \Diamond ICx) \wedge \forall x \in [\![student\ in\ A]\!]\Diamond(Cx \vee ICx) \models \forall x \in [\![student\ in\ A]\!]\Diamond Cx \wedge \forall x \in [\![student\ in\ A]\!]\Diamond ICx$$

(iii) $$\forall x \in [\![student\ in\ A]\!](\Diamond Cx \leftrightarrow \Diamond ICx) \wedge \neg\exists x \in [\![student\ in\ A]\!]\Diamond(Cx \vee ICx) \models \neg\exists x \in [\![student\ in\ A]\!]\Diamond Cx \wedge \neg\exists x \in [\![student\ in\ A]\!]\Diamond ICx$$

### 4.4 Homogeneity in enemy territory

Before concluding our discussion of the filtering FC puzzle, we would like to address a potential objection against our account. Both the objection and our eventual response are independent of whether we use **pex** with just IE or with IE+II, so for simplicity we will frame this discussion using the former. Recall that our **pex**-based account of filtering FC examples such as (64) uses the parse in (64a) to predict the target reading: i.e., a dual prohibition reading for the first main disjunct and filtering of the presupposition of the second main disjunct. Now, as we showed in (47a)-(47d), the parse in (64a) also predicts that (64) presupposes the homogeneity proposition $\Diamond A^+ \leftrightarrow \Diamond B^+$ (= Maria can study in Tokyo iff she can study in Boston).

(64) Either Maria can't go study in Tokyo or Boston, or she is the first our family who can study in Japan (and the second who can study in the States).

a. $\neg\,\mathbf{pex}^*[\Diamond(A^+ \vee B^+)] \vee C_{\Diamond A \wedge \Diamond B}$

Romoli & Santorio (2019) argue that accounts which predict that homogeneity presupposition may face an independent problem. Namely, that it seems that a speaker $S$ can felicitously assert (64)—with the target reading—in a context that makes explicit that $S$ doesn't believe the homogeneity proposition. This is illustrated by examples like the following:

(65) Maria applied to Tokyo or Boston. I have no idea whether she was admitted to only one, both, or neither, but ...

a. Either she can't go study in Tokyo or Boston, or she is the first in our family who can study in Japan and the second who can study in the States.

Can our **pex**-based account handle these cases? As we said at the outset (see also Bassi et al. 2019, Del Pinal 2021), the presuppositions triggered by **pex** tend to be globally accommodated when they are consistent with the common ground. A non-collapsed common ground which entails, say, the information in (65) can be consistently updated with additional information so as to entail the homogeneity proposition that Maria can study in either both or neither of Tokyo and Boston. So why not appeal, as in other superficially similar cases, to global accommodation? The problem is that, in (65), $S$ explicitly acknowledges ignorance concerning the homogeneity proposition; and as the example is intended to be framed, $S$ doesn't, in the middle of the discourse, acquire or remember any relevant new information. Accordingly, interlocutors can't reasonably globally accommodate the homogeneity proposition when they process (65a): for they would then represent $S$ as simultaneously agnostic and believing in the homogeneity proposition.[16]

What we seem to need, then, is to somehow block the projection of the homogeneity proposition from out of the first main disjunct of (65a), without thereby affecting other components of the target reading. Could we just apply local accommodation (ACC) over the first main disjunct? As Romoli & Santorio (2019) point out, this doesn't solve the problem. To see why, consider again the parse, repeated in (66a), which supports the target predictions (in neutral contexts) for our original filtering FC examples with dual prohibition $\neg\Diamond\vee$-sentences in the first disjunct. Next, attach ACC to the first disjunct, as in (66b), to block the projection of homogeneity. The problem now is that, when we calculate the presupposition for the entire sentence based on that parse, we no longer filter out $\Diamond A$ and $\Diamond B$, as can be easily verified from the **ps** part of (66c).

(66)
a. $\neg\,\mathbf{pex}^{*}[\Diamond(A^{+}\vee B^{+})]\vee C_{\Diamond A\wedge\Diamond B}$

b. $\textsc{acc}[\neg\,\mathbf{pex}^{*}[\Diamond(A^{+}\vee B^{+})]]\vee C_{\Diamond A\wedge\Diamond B}$
$= [(\Diamond A^{+}\leftrightarrow\Diamond B^{+})\wedge\neg\Diamond(A^{+}\vee B^{+})]\vee C_{\Diamond A\wedge\Diamond B}$

c. $[\![(66b)]\!] = \begin{cases} \textbf{ps: } \neg[(\Diamond A^{+}\leftrightarrow\Diamond B^{+})\wedge\neg\Diamond(A^{+}\vee B^{+})]\rightarrow\Diamond A\wedge\Diamond B \\ \textbf{asserts: } [(\Diamond A^{+}\leftrightarrow\Diamond B^{+})\wedge\neg\Diamond(A^{+}\vee B^{+})]\vee C \end{cases}$

Could we go for a 'direct' solution and apply ACC over each disjunct, as in (67)?

(67) $\textsc{acc}_1[\neg\,\mathbf{pex}^{*}[\Diamond(A^{+}\vee B^{+})]]\vee\textsc{acc}_2[C_{\Diamond A\wedge\Diamond B}]$

Focusing on (67), note that $\textsc{Acc}_1$ cancels the projection of the homogeneity proposition from the first main disjunct, without altering its desired dual prohibition reading. In addition, $\textsc{Acc}_2$ cancels the projection of $\Diamond A$ and $\Diamond B$ from out of the second disjunct. Accordingly, the parse in (67) captures the two core components of the target reading of (65a). The key question, then, is this: Under what conditions, if any, do independently motivated licensing conditions permit the insertion of ACC so as to generate parses like (67)?

Many theorists hold that local accommodation has strict licensing conditions. A standard hypothesis is that ACC is only licensed when it is marked with specific intonation patterns and/or the corresponding parse without ACC would result in incoherent or defective contents. This approach will work for our purposes, if understood as including the condition that avoidance of inter-sentential, speaker-anchored incoherence licenses ACC. Indeed, the idea that ACC can be used to avoid not just intra but also inter-sentential incoherence is needed to apply the local

16 For similar reasons, we can't plausibly use matrix level ACC to block (65a) from presupposing the homogeneity proposition.

accommodation-based account (of the coherence) of sentences like (68a) to parallel discourses like (68b), as seems natural:

(68) a. The king of France isn't bald, since there is no king of France!
b. The kind of France isn't bald. For there is no king of France!

Based on those standard licensing conditions for ACC, we can show that the parse in (67) is licensed when a speaker *S* asserts (65a) after (65)—or more generally, when (65a) is evaluated relative to a common ground which entails that *S* doesn't believe the homogeneity proposition. As we showed earlier, $\text{ACC}_1$ is required to avoid attributing to *S* the incoherent attitude of being both agnostic towards and believing in the homogeneity proposition $\Diamond A^+ \leftrightarrow \Diamond B^+$. $\text{ACC}_2$ is required to avoid uncharitably attributing to *S* a basic incapacity to draw the implications of *S*'s own doxastic states. For without $\text{ACC}_2$, *S* would be represented as holding both of the following beliefs:

($B_1$) Maria can study in Japan and in the States.

($B_2$) (Only) if Maria can study in Tokyo and in Boston, she is the first in her family who can study in Japan and the second who can study in the States.[17]

Given $B_1$, it is hard to see why *S* would believe $B_2$. For given that Maria can study in Japan and the States, under what conditions could it follow that it is only if she can study specifically in Tokyo and Boston that she will be the first/second in her family that is allowed to study in Japan/the States?

- Option 1: we can try to reconcile $B_1$ and $B_2$ by interpreting them as implying—given the background that *S* believes that no one else in Maria's family prior to her can study in Japan, only one other person in her family prior to her can study in the States—that Maria can study in Tokyo and Boston, and hence is the first in her family who can study in Japan and second who can study in the States. An obvious problem with this analysis is that it clashes with the prior assertion that *S* is agnostic concerning whether Maria can study in Tokyo, Boston, both or neither.

17 Note that the conditional belief attributed to *S* is likely exhaustive (i.e., read as an *only if* conditional), because the main disjunction in sentences like (65a) is usually interpreted as exclusive disjunction. Indeed, this kind of enrichment is systematically triggered in configurations of the form *either* ... *or* and ... *or else*.... This can be captured by adding a matrix **pex** to the parse in (67), as in (i), and assuming that it is associated with the main $\vee$:

(i) $\mathbf{pex}[\text{ACC}_1[\neg\mathbf{pex}^*[\Diamond(A^+ \vee B^+)]] \vee \text{ACC}_2[C_{\Diamond A \wedge \Diamond B}]]$

- Option 2: *S* believes that Maria can study in Japan and in the States, and is agnostic about whether Maria can study in Tokyo or Boston. *S* also believes that only if Maria can study in Tokyo, but not elsewhere in Japan, and in the Boston, but not elsewhere in the States, will she then count as the first/second in her family who can study in Japan/States. The strange belief attributed to *S* here is this: that whether anyone in her family before her was allowed to study in Japan/States depends on whether she is now allowed to study specifically in Tokyo/Boston.

These results suggest that, in contexts that entail that *S* is agnostic with respect to the homogeneity proposition, both ACC operators in (67) are licensed to avoid attributing to *S* incoherent or strange beliefs. Importantly, this result is compatible with holding that parallel ACC operators over each of the main disjuncts are not licensed in, say, out of the blue assertions of sentences like (64), or more specifically, when the common ground is at least compatible with (hence allows for global accommodation of) the homogeneity presupposition.[18]

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18 In light of our previous argument, one might ask: why not deal with filtering FC sentences, in general, by appealing to the presence of ACC over the second main disjunct, as in (ia)-(iia)?

(i) Either Maria can't go study in Tokyo or Boston, or she is the first in our family who can go study in Japan (and the second who can study in the States).

a. $\neg\Diamond(A^+ \vee B^+) \vee \text{ACC}[C_{\Diamond A \wedge \Diamond B}]$

(ii) Either Maria is required to go to Japan and the States, or she's the first in her family who is not required to go to Tokyo (and the second who's not required to go to Boston).

a. $\Box(A \wedge B) \vee \text{ACC}[C_{\neg\Box A^+ \wedge \neg\Box B^+}]$

This proposal would have the desired effect of cancelling, in each case, the presuppositions triggered in the second main disjunct. It would also have the advantage of being independent of specific accounts of FC. Romoli & Santorio (2019) consider and reject this proposal. The problem, they argue, is that it seems predict no difference between the presuppositions of, say, (i) and (iii). For the presupposition triggered in the second disjunct of (iii) could then also be suspended by applying ACC, as in (iiia). Yet unlike (i), (iii) does seem to presuppose that Maria can study in Japan.

(iii) Either Maria's brother can go to study in Tokyo, or Maria is the first in her family who can go study in Japan.

a. $\Diamond D \vee \text{ACC}[E_{\Diamond A}]$

A proponent of this generalized ACC-based approach may try to respond as follows (continue to assume that, in each case, the main *or* is interpreted exclusively). There is a difference between (i) and (iii) which explains why ACC is licensed (or reliably inserted) in (i) but not in (iii). Namely, that cancelling the presupposition of the second main disjunct is needed to avoid attributing to *S* a weird belief in (i) but not in (iii). For suppose we don't cancel the target presuppositions (via insertion of ACC) in either case:

### 4.5 Summary

In this section, we have examined the second part of the presupposed & filtering FC puzzle, which concerns the FC filtering effects of embedded $\neg\Diamond\vee$ and $\Box\wedge$-sentences such as (39) and (50). These cases present a serious challenge to various **exh**-based theories, including standard IE and IE+II versions and also enhanced systems with multidimensional exhaustification. These cases are also problematic for recent Lexicalist theories, which lacking an account of negative FC conjunctions, can only deal with half the puzzle, just as in case of the presupposed FC part of the puzzle. We argued that, in contrast, our **pex**-based theory issues in a promising account of the filtering FC puzzle. Although filtering (negative) FC with $\Box\wedge$-sentences poses special challenges, we showed that our **pex**-based theory can directly deal with these cases once **pex** is formulated so as to have not just an IE but also an II function. We proposed a specific implementation of the hypothesis that **pex** has an inclusion function, and argued that it amounts to a principled extension because it preserves the good predictions of our original **pex** with IE-based theory of SIs. These results are summarized in Table 2 below.

## 5 Conclusion

FC phenomena present various challenges to semantic and pragmatic theories. In this paper, we focused on recent observations, due to Marty & Romoli (2020) and

- (iii) would presuppose that Maria can study in Japan, and assert that if her brother can't go to Tokyo, she is the first one in her family that can study in Japan. There is nothing problematic about attributing those contents to *S*—e.g., *S* might be unsure whether Maria's brother can study in Tokyo, but believe that if he can, he was allowed to attend before Maria.
- (i), in contrast, would entail that *S* believes ($B_1$) that Maria can study in Japan/States, and also ($B_2$) that (only) if she can study in Tokyo/Boston is she the first/second in her family who can study in Japan/USA. At first glance, it seems strange that *S* could hold $B_1$ and $B_2$ together. For given that *S* believes Maria can study in Japan/States, why would *S* conditionalize (exclusively) the consequent of $B_2$ on that specific antecedent?

This argument for a generalized ACC approach to filtering FC has a problem in its analysis of cases like (i), which springs from the observation that they actually issue in different licensing predictions than similar cases but which appear specifically in contexts like (64). The difference is that, when the common ground is compatible with *S* believing the homogeneity proposition, there is nothing incoherent about resolving the potential tension in $B_1$ and $B_2$ by e.g. attributing to *S* the belief that Maria can study in Tokyo and in Boston, and so is the first/second in her family who can study in the Japan/the States. Granted, expressing that content via (i) seems like gratuitously indirect, but the point is just that ACC is not needed to avoid attributing to *S* an incoherence or highly implausible set of beliefs. Despite this shortcoming, this objection does help emphasize that, ultimately, the filtering FC puzzle can be taken as challenge specifically to theories of FC to the extent that one also accepts that ACC has relatively strict licensing conditions.

| **Accounts** | **FC-$\neg\Box\wedge$** | **filt. FC w. DP-$\neg\Diamond\vee$** | **filt. FC w. FC-$\Box\wedge$** |
|---|---|---|---|
| Lexicalist old | ✗ | ✗ | ✗ |
| Lexicalist new | ✗ | ✓ | ✗ |
| **exh**$^{IE/IE+II}$ | ✓ | ✗ | ✗ |
| **exh**$^{IE}$+MD | ✓ | ✗ | ✗ |
| **exh**$^{IE+II}$+MD | ✓ | ✗ | ✗ |
| **pex** | ✓ | ✓ | ✓ |

**Table 2** Predictions of theories of FC concerning filtering FC with embedded $\neg\Diamond\wedge$ and $\Box\vee$-sentences. To complete the results reported in Table 1, we include here predictions concerning the negative FC reading of simple $\neg\Box\wedge$-sentences, as well as the two main cases of filtering FC. The **pex** operator whose predictions are summarized here has officially both an exclusion and an inclusion function, but note that this doesn't affect any of the results reported in Table 1 with a purely exlusion based **pex**. References for theories: Lexicalist old: Zimmerman (2000) and Simons (2005b). Lexicalist new: Willer (2017), Goldstein (2018), Aloni (2018), Rothschild & Yablo (2018). Grammatical with **exh**$^{IE/IE+II}$: Fox (2007), Chierchia (2013), Bar-Lev & Fox (2020). Grammatical with **exh**$^{IE}$+MD: Gajewski & Sharvit (2012), Spector & Sudo (2017). Grammatical with **exh**$^{IE+II}$+MD: Marty & Romoli (2020).

Romoli & Santorio (2019), concerning how $\Diamond\vee$, $\neg\Diamond\vee$, $\Box\wedge$ and $\neg\Box\wedge$ sentences, such as those in (69), behave with respect to their projection and filtering properties when embedded in environments such as those in (70)-(71). The 'presupposed & filtering FC puzzle' is the challenge of giving an account of the (negative) FC reading of sentences like (69a), (69c), and (69d), and of the dual prohibition reading of sentences like (69b) that—when combined with independently justified assumptions about presupposition projection, accommodation and filtering—explains their complex embedded behavior in and the default reading of (70a)-(70b) and (71a)-(71b):

(69) **FC reading of $\Diamond\vee$ and $\Box\wedge$-sentences, negative FC reading of $\neg\Box\wedge$-sentences, and DP reading of $\neg\Diamond\vee$-sentences** (for availability evidence, see Tieu, Romoli & Bill 2019, Marty, Romoli, Sudo & Breheny 2021):

a. Maria can study in Tokyo or Boston.
b. Maria can't study in Tokyo or Boston.
c. Olivia is required to take Logic and Algebra.
d. Olivia is not required to take Logic and Algebra.

(70) **FC and negative FC under (negative) factives** (Marty & Romoli 2020):

a. Noah is unaware that Olivia can take Logic or Algebra.
   $\rightsquigarrow$ *Olivia can take Logic*
   $\rightsquigarrow$ *Olivia can take Algebra*

$\rightsquigarrow$ *¬Noah believes that Olivia can take Logic*
$\rightsquigarrow$ *¬Noah believes that Olivia can take Algebra*

b. Pete is unaware that Olivia is not required to take Logic and Algebra.
$\rightsquigarrow$ *Olivia is not required to take Logic*
$\rightsquigarrow$ *Olivia is not required to take Algebra*
$\rightsquigarrow$ *¬Noah believes Olivia is allowed to not take Logic*
$\rightsquigarrow$ *¬Noah believes that Olivia is allowed to not take Algebra*

(71) **Filtering FC and negative FC conjunctions** (Romoli & Santorio 2019):

a. Either Maria can't go study in Tokyo or Boston, or she is the first in our family who can go study in Japan and the second who can go study in the States.
$\not\rightsquigarrow$ *Maria can study in Japan*
$\not\rightsquigarrow$ *Maria can study in the States*

b. Either Maria is required to go to Japan and the States, or she's the first in her family who is not required to go to Tokyo and the second who's not required to go to Boston.
$\not\rightsquigarrow$ *Maria is allowed to not study in Tokyo*
$\not\rightsquigarrow$ *Maria is allowed to not study in Boston*

We have seen that some accounts of FC can deal with parts of the presupposed & filtering FC puzzle. Yet, as far as we know, no previous account can deal with the full pattern in (69)-(71). We have proposed a novel Grammatical-account of FC, formulated in terms of an independently motivated exhaustification operator, **pex**, which is a presupposition trigger with respect to any innocently excludable or includable alternatives to the prejacent. We argued that our **pex**-based account issues in a uniform solution to the presupposed & filtering FC puzzle, a solution that is descriptively superior to the one provided by both standard **exh**-based Grammatical accounts (Fox 2007, Bar-Lev & Fox 2020) and their more complex multi-dimensional variants (Gajewski & Sharvit 2012, Marty & Romoli 2020). In addition, although our **pex**-based account shares some predictions with recent Lexicalist accounts of FC (Willer 2017, Goldstein 2018), it has the substantial comparative advantage that it can be extended to variations of the puzzle with negative FC conjunctions.

## A Recursive pex and Free Choice in a Strong Kleene Semantics

This appendix shows that **pex** allows the derivation of a free choice inference via recursive exhaustification without use of accommodation but using a Strong Kleene semantics to calculate presupposition projection.

**Calculus for presuppositions** Assume that $D_{st}$ is the domain of propositions, i.e. total functions from possible worlds to truth values. We adopt the fraction notation for partial functions Harbour (2014) proposes:

(1) $$\frac{a}{p} = \lambda w \in \{w \in D_s \mid p(w)\}\ .\ a(w) \text{ for } a, p \in D_{st}$$

Also assume $\mathbf{1} = \lambda w \in D_s\ .\ 1$, $\mathbf{0} = \lambda w \in D_s\ .\ 0$ and $\mathbf{\#} = \lambda w \in \emptyset\ .\ 0$. Then these corollaries hold as Harbour notes:[19]

(2) $$\frac{a}{\mathbf{1}} = a \text{ and } \frac{a}{\mathbf{0}} = \mathbf{\#} \text{ for any } a$$

**Strong Kleene semantics** A number of recent works argue that the Strong Kleene semantics gives rise to an attractive account of presupposition projection (Fox (2013), George (2008), and others). The basic intuition of the Strong Kleene semantics is that a third truth value # represents an indeterminate truth value. If # occurs in the argument of a logical operator, the result is therefore predictable from inspecting the results of replacing # with 0 or 1. If both replacements result in the same truth value $b$, the indeterminacy of # is irrelevant, and $b$ is the result of the whole. Otherwise though the indeterminacy remains and # is the result. For example, $1 \blacktriangle \# = \#$ and $0 \blacktriangle \# = 0$ where $\blacktriangle$ represents conjunction. Generally conjunction $\blacktriangle$ and disjunction $\blacktriangledown$ amount to the following:

(3) $$\frac{a}{a'} \blacktriangle \frac{b}{b'} = \frac{a \wedge b}{(a' \wedge b') \vee (a' \wedge \neg a) \vee (b' \wedge \neg b)}$$

(4) $$\frac{a}{a'} \blacktriangledown \frac{b}{b'} = \frac{a \vee b}{(a' \wedge b') \vee (a' \wedge a) \vee (b' \wedge b)}$$

**Recursive presupposition and Strong Kleene** Via the recursive application of **pex**, our analysis gives rise to cases of double fractions, where the assertion or the presupposition of a complex term has presuppositions of it own. The first case is easy: When the assertion has a presupposition, we want to just conjoin that presupposition with the presupposition of the more complex term.

(5) $$\frac{\frac{a}{b}}{c} = \frac{a}{b \wedge c} \text{ for } a, b, c \in D_{st}$$

The second case is more difficult: What if the presupposition itself has a presupposition, i.e. $\frac{b}{b'}$ in the denominator. When $b'$ is not satisfied, this intuitively

19 Note that one of us (Sauerland 2005) used a fraction notation for presuppositions but with opposite roles for numerator and denominator. Harbour's corollaries make his proposal more intuitive, which is why we adopt it.

corresponds to uncertainty of whether the result of the complex term is uncertain. We assume that the Strong Kleene interpretation is to use the value of $a$ unless we are sure the presupposition is false. I.e. we assume that only the worlds where $\frac{b}{b'}$ is false are not in the domain of the double fraction. For the other worlds, the truth value of the double fraction is defined as that of $a$:[20]

(6) $$\frac{a}{\frac{b}{b'}} = \frac{a}{b \vee \neg b'} = \frac{a}{b' \rightarrow b}$$

**Free Choice** Consider now the basic case of Free Choice below, where we assume for simplicity that conjunctive alternatives are not active. In other words, we assume that $C_1 = \{\Diamond(A \vee B), \Diamond A, \Diamond B\}$ and $C_2 = \{\mathbf{pex}_{C_1}\ \Diamond(A \vee B), \mathbf{pex}_{C_1}\ \Diamond A, \mathbf{pex}_{C_1}\ \Diamond B\}$:

(7) $$\mathbf{pex}_{C_2}\ \mathbf{pex}_{C_1}\ \Diamond(A \vee B)$$

The elements of $C_2$ are the following propositions (here and in the following we disregard the conjunctive alternative for simplicity):

(8) $$\mathbf{pex}_{C_1}\ \Diamond A \quad = \quad \frac{\Diamond A}{\neg \Diamond B}$$

(9) $$\mathbf{pex}_{C_1}\ \Diamond B \quad = \quad \frac{\Diamond B}{\neg \Diamond A}$$

The fact that there are two exclusions, (8) and (9), results in another theoretical choice. In a Strong Kleene system, we must combine multiple exclusions with $\blacktriangle$ instead of $\wedge$, but $\blacktriangle$ exhibits more sensitivity to its method of application. Consider first the result when the exclusions are combined by Strong Kleene conjunction in the presupposition. As the following computation shows, this would maximize the exclusion and result in a contradictory function similar to the result in the main text (specifically, the presupposition and assertion contradict each other, and hence the result is a contradictory proposition with a presupposition):

20 Assumption 6 has non-trivial implications for other examples with two presupposition triggers in a single sentence. It is beyond the present scope to evaluate these in detail.

$$(10)\quad \mathbf{pex}_{C_2}\ \mathbf{pex}_{C_1}\ \Diamond(A \vee B) = \frac{\Diamond(A \vee B)}{\neg\frac{\Diamond A}{\neg\Diamond B} \blacktriangle \neg\frac{\Diamond B}{\neg\Diamond A}}$$

$$(11)\quad = \frac{\Diamond(A \vee B)}{\frac{\neg\Diamond A}{\neg\Diamond B} \blacktriangle \frac{\neg\Diamond B}{\neg\Diamond A}}$$

$$(12)\quad = \frac{\Diamond(A \vee B)}{\frac{\neg\Diamond A \wedge \neg\Diamond B}{(\neg\Diamond A \wedge \neg\Diamond B) \vee (\neg\Diamond A \vee \Diamond B) \vee (\Diamond A \vee \neg\Diamond B)}}$$

$$(13)\quad = \frac{\Diamond(A \vee B)}{\frac{\neg\Diamond A \wedge \neg\Diamond B}{(\neg\Diamond A \wedge \neg\Diamond B) \vee (\neg\Diamond A \vee \Diamond B) \vee (\Diamond A \vee \neg\Diamond B)}}$$

$$(14)\quad = \frac{\Diamond(A \vee B)}{(\neg\Diamond A \wedge \neg\Diamond B) \vee (\neg\Diamond A \vee \Diamond B) \vee (\Diamond A \vee \neg\Diamond B)}$$

$$(15)\quad = \frac{0}{(\neg\Diamond A \wedge \neg\Diamond B) \vee (\neg\Diamond A \vee \Diamond B) \vee (\Diamond A \vee \neg\Diamond B)}$$

A second option is to apply Strong Kleene conjunction at the assertion level, i.e. $\mathbf{pex}\,\phi = \frac{\phi}{\neg\psi_1} \blacktriangle \frac{\phi}{\neg\psi_2} \blacktriangle \cdots$ for exclusions $\psi_1, \psi_2, \ldots$. This application of $\blacktriangle$ minimizes the exclusion, and therefore seems intuitively preferable given the Strong Kleene approach. The following computation shows this method yields a homogeneity presupposition:

$$(16)\quad \mathbf{pex}_{C_2}\ \mathbf{pex}_{C_1}\ \Diamond(A \vee B) = \frac{\Diamond(A \vee B)}{\neg\frac{\Diamond A}{\neg\Diamond B}} \blacktriangle \frac{\Diamond(A \vee B)}{\neg\frac{\Diamond B}{\neg\Diamond A}}$$

$$(17)\quad = \frac{\Diamond(A \vee B)}{\Diamond B \rightarrow \Diamond A} \blacktriangle \frac{\Diamond(A \vee B)}{\Diamond A \rightarrow \Diamond B}$$

$$(18)\quad = \frac{\Diamond(A \vee B)}{(\Diamond B \leftrightarrow \Diamond A) \vee (\Diamond A \wedge \neg\Diamond(A \vee B)) \vee (\Diamond B \wedge \neg\Diamond(A \vee B))}$$

$$(19)\quad = \frac{\Diamond(A \vee B)}{\Diamond B \leftrightarrow \Diamond A}$$

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