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0. *Fox & Hackl (2006): The universal density of measurement*

It is discovered (Krifka 1999) that comparative measures are denied scalar implicatures bare measures license:

9 September 1965,

(1) a. Sandy Koufax pitched 113 glorious pitches.
b. Sandy Koufax pitched a glorious 113 pitches.

(2) (1) $\Vdash$ Sandy Koufax did not pitch 114 glorious pitches.

(3) a. Sandy Koufax pitched 103 glorious minutes.
b. Sandy Koufax pitched a glorious 103 minutes.

(4) (3) $\Vdash$ Sandy Koufax did not pitch 104 glorious minutes.

(5) Sandy Koufax pitched more than 112 glorious pitches.

(6) (5) $\nVdash$ Sandy Koufax did not pitch more than 113 glorious pitches.

(7) Sandy Koufax pitched more than 102 glorious minutes.

(8) (7) $\nVdash$ Sandy Koufax did not pitch more than103 glorious minutes.

Being said to have pitched 113 glorious pitches implicates not having pitched more, not 114 ((1) $\Vdash$(2)). Being said to have pitched 103 glorious minutes implicates not more, not 104 minutes ((3) $\Vdash$(4)). But, a comparative measure does not implicate not more than the more mentioned. That is, being said to have pitched more than 112 glorious pitches does not implicate that not more than 113 were pitched ((5) $\nVdash$(6)). Being said to have pitched more than 102 minutes does not implicate that it wasn't more than 103 minutes ((7) $\nVdash$(8)).

For Fox & Hackl (2006), dense measurements, (3) and (7), afford two routes between bare *103 minutes* and comparative *more than 102 minutes*. For the first, consider that if *p* is more informative than what has been said it is implicated that *p* is not known to the speaker who would have otherwise said it. A scalar implicature that *not p* derives from both an *ignorance* implicature that it is *not known* that *p* and prior assumption that it *is* known whether *p* or *not p*. Then, *not p* is the one that must be known, the scalar implicature. For a dense measurement, (7)'s *ignorance* implicatures are that for every real number $\varepsilon > 0$, it is not known that Koufax pitched more than $102+\varepsilon$ minutes, since as much would have been said if as much were known. But, the ignorance implicatures cannot become the *scalar* implicatures, among which is (8), that for every real number $\varepsilon > 0$, Koufax did not pitch more than $102+\varepsilon$ minutes. As Fox & Hackl (2006: 542) remark, the class of these scalar implicatures is inconsistent with (7)'s assertion that for some $\varepsilon > 0$, Koufax indeed pitched $102+\varepsilon$ minutes. Under pain of contradiction, the prior assumption is rejected. For any real number $\varepsilon > 0$, it remains unknown whether Koufax pitched $102+\varepsilon$ minutes or not. The ignorance implicatures that come with *more than 102 minutes* thus do not issue it scalar implicatures. In contrast, reasoning as above, the ignorance implicatures that come with *103 minutes* do issue in scalar implicatures that amount to Koufax pitching 103 minutes and no more than 103 minutes.

Fox & Hackl (2006: 559) favor a second route between bare and comparative measures. It derives all scalar implicatures from an original implicature that refers by singular definite description to a degree or number. That is,

(9) (3) $\Vdash$ The largest *m* such that Sandy Koufax pitched *m* minutes is 103.
(10) (9) $\Vdash$ (4).

But,

(11) (7) $\nVdash$ The largest *m* such that Sandy Koufax pitched more than *m* minutes is 102.
(12) (11) $\nVdash$ (8)

Since there isn't a largest amount of time that Koufax pitched more than, the failure of singular reference in (11) is said to deprive the comparative measure in (7) of all scalar implicatures.

The difference *via* either route between bare and comparative dense measures trusts the density of time to provide to the comparative measure an open interval within which there is no first moment. But, the difference persists into the discrete, with pitching *113 pitches* implicating not pitching 114 pitches ((1) $\Vdash$(2)) and pitching *more than 112 pitches* not implicating not pitching more ((5) $\nVdash$(6)). So, the *Universal Density of Measurement* (Fox & Hackl 2006: 542) declares the discrete dense:

(13) The Universal Density of Measurement (UDM): measurement scales needed for natural language semantics are always dense.

Given the UDM, (5) comes with ignorance implicatures that for every real number $\varepsilon > 0$, it is not known that Koufax pitched more than $112+\varepsilon$ pitches. As above, the ignorance implicatures cannot become their scalar counterparts that for every real number $\varepsilon > 0$, Koufax did not pitch more than $112+\varepsilon$ pitches, as these are inconsistent with (5)'s assertion that for some $\varepsilon > 0$, Koufax pitched $112+\varepsilon$ pitches. Of course, (5) isn't really asserted in ignorance of whether Koufax pitched 112.314 pitches, false nonsense known so. The realistic ignorance implicatures are those that for every *natural* number $\varepsilon > 0$, it is not known that Koufax pitched more than $112+\varepsilon$ pitches. But, then the corresponding scalar implicatures and (5) would be consistent and implicate that Koufax pitched exactly 113, which (5) plainly does not implicate—hence, UDM. The UDM is equally necessary deriving scalar implicatures *via* singular reference to a number or degree:

(14) (5) $\nVdash$ The largest *p* such that Sandy Koufax pitched more than *p* pitches is 112.
(15) (14) $\nVdash$ (6)

The largest *natural* number such that Koufax pitched more than that many pitches is indeed 112, having pitched 113. But, there is no largest real number that 113 is more than. The definite description in (14) fails to refer and scalar implicature for (5) thus said to fail with it, according to the UDM.

An utterance's meaning fixed *in excelsis*, pragmatics or practical reasoning draws out the implicatures, implications, and nuances of what has just been said. It is not practical reasoning to let Koufax pitch 112.314 pitches. Even for a formal pragmatics clanging without much reason (*v.* Sauerland (2012)), an utterances' effect on conversation has always been a contest among what was said and what else might have been said instead, pondering their meanings. Under the Universal Density of Measurement, the semantic processing of *113 pitches* pauses blinded to discrete pitches and sees instead pitch blur so dense it demands measurement by real numbers. Scalar implicatures are registered or rejected, shielded from meanings true to what baseball's reports report.

Is there a less violent, *boring* pragmatics for bare and comparative measures that leaves semantics intact and receives from it meanings whole and complete to do whatever is desired to implicate other meanings?

In his perfect game 9 September 1965,
(16) # Sandy Koufax pitched 62 curveballs and 40.314 fastballs.
(17) # Sandy Koufax pitched more than 61 curveballs and more than 40.314 fastballs.

What is that understanding of (16)-(17) that concurrently understands their implicatures and balks at the nonsense? As much as an official pitch count at 62 or more than 61 is pending, it is known in advance that pitches do not vanish a π^{th} the way to home plate. Indicting (16)-(17) is the understanding that they presume possible what is known to be impossible and known to be known impossible. Understanding their implicatures, according to the UDM, celebrates *not* knowing that the known impossible is so known. Is there instead a pragmatics that affords understanding these utterances the constant, consistent cognitive state and unwavering, common knowledge that Koufax did not and could not have pitched 112.314 pitches that perfect game?

1. *Universal equative and comparative implicature*

Absent the Universal Density of Measurement, a common explanation still beckons for bare and comparative measures discrete and dense. At a glance, it seems to be something about *more*. Its syntax and semantics unify the dense and the discrete wherever the word occurs (Higginbotham 1994, Hackl 2001, 2002, 2009, Wellwood 2019). In these minimal pairs, something about it somehow derives a contrast between equative and comparative not only in meaning but also in implicature.[1] The contrast reaches sentences that do not mention numbers. It's all about *more*, not about the numbers, enclosing (1)-(8) in a non-numeric paradigm (43)-(50).[2]

More in 'x_0 is more than x_1' expresses a relation between empirical magnitudes. When magnitudes and numbers are playing nicest together, it could seem otherwise, in 'surrogative reasoning', reasoning about magnitudes reasoning about numbers:

(18) "In ['surrogative reasoning' Swoyer (1991)], we reason indirectly about relations R_i among [magnitudes] in the represented empirical domain X by reasoning directly about the surrogative relations S_i among [numbers] in the representational domain Y. Thus, we can reason surrogatively about mass relations among physical objects by reasoning about relations among numerical representatives of their masses, concluding, for example, from the fact that one object has a mass represented by the number 2 on the kg scale (i.e., has a mass of 2 kg), that a second object has a mass represented by the number 3 on that scale, and that a third object has a mass represented by the number 5 on that scale, and in conjunction with such arithmetic truths as 2+3=5 and 2<3<5, that the third object has a greater mass than either of the other two, that the third has the same mass as the combined mass of the first two objects, and so on. The fact that [measure function μ] respects the relations Ri guarantees that these empirical relations will be represented in the homomorphic representation and furthermore that truth-preserving inferences defined over the surrogative relations Si will entail true conclusions about the relations Ri for which they are surrogates. Arguably, the *entire* point and purpose of homomorphic representation is to enable such surrogative reasoning. That we typically overlook the surrogative character of such reasoning testifies to the seamless nature of the representation of X in Y that underpins it: we think of the magnitudes *themselves* as intrinsically numerical, and not simply as sets of *non*-numerical properties that are structured in a manner that permits their numerical representation. We think of the magnitudes as intrinsically numerical precisely because their homomorphic representation in the real numbers offers the only practicable way of thinking and reasoning reliably about the magnitudes. How else, for example, are we to worry about our weights, the effects of our diets, except in numerical terms!" (Matthews 2007: 134f.)

When magnitudes and numbers play nicest together, there is, for example, a measure μ in kilograms, as in (18), so that x_0 is more massive than x_1, or mass x_0 is more than mass x_1,[3] just in case $\mu(x_0)$ is greater than $\mu(x_1)$:

(19) $x_0 \succ x_1 \leftrightarrow \mu(x_0) > \mu(x_1)$

"x_0 is more than x_1"

Reasoning surrogatively, one might then mistake the thought that the round weight is, or weighs, more than the square weight ("$x_0 \succ x_1$") for the equivalent thought that the round weight's kilogram weight is greater than the square weight's kilogram weight ("$\mu(x_0) > \mu(x_1)$"). The Representational Theory of Measurement (Krantz et al. 1971, Suppes *et al.* 1989, Luce et al. 1990 [*FM* I-III]) "studies the nature of qualitative aspects of the empirical world that can be usefully recoded, in some reasonably unique fashion, in terms of familiar mathematical structures, most commonly the real numbers under their ordinary ordering and operations" (Luce & Suppes 2004). "A key insight is that the empirically significant aspects of a given mathematical structure and operations are those that *mirror* relevant relations among the empirical objects or quantities measured—the 'bigger than' among numbers mirroring the 'longer than' among lengths or distances" (Tal 2017). When magnitudes and numbers aren't playing so nice together, it is to be discovered what about the magnitudes can still be mirrored with what numbers and what numerical operations if any:

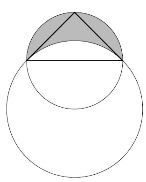
(20) Butter is yellower than milk, a daffodil is yellower than butter, and a daffodil is more yellower than butter than butter is yellower than milk (after Fechner 1860/1966).
(21) A paper cut is more painful than a mosquito bite, and childbirth is much more more painful than a paper cut than a paper cut is more painful than a mosquito bite.
(22) After KGB special spy school, Natasha Fatale was more American than Boris Badenov but not enough for deep cover. (*The Rocky and Bullwinkle Show* 1959-1964, *The Americans* 2013-2018).

Comparative and equative language by itself ('x_0 is more than x_1', 'x_0 is as much as x_1') merely reports (Sassoon 2010) on the empirical relation between magnitudes, determined pairwise, on, say, a balance comparing masses in (18) or for (20)-(22), in perception (with the lives of secret agents in the balance). The empirical relation is prior to whether the conditions for metrization in three volumes [*FM* I-III] are satisfied enough for it to then make sense for numeric language to be applied to magnitudes of mass, color, pain, or Americanism:

(23) A daffodil is *h* degrees in hue, *s* degrees in saturation, and *l* degrees in luminosity closer to cardinal yellow than milk is.
(24) ? A daffodil is twice as yellow as milk.
(25) ? A paper cut is a couple ouches more painful than a mosquito bite.
(26) ? A paper cut is twice the pain of a mosquito bite.
(27) After KGB special spy school, Natasha Fatale was three-skills more American than Boris Badenov.
(28) ? After KGB special spy school, Natasha Fatale was twice as American as Boris Badenov.

The paradigmatic examples of measurement have been numerical since the nineteenth century. Kneeling to Lord Kelvin's (1891) dictum (cited in van Fraassen 2008: 158)—no numbers, no science—and parrying the slander that measurement in psychophysics (*v.*, *e.g.* (20), (23)) might not equal measurement in the natural sciences, Stevens (1946), a behaviorist, moved the goal posts whining it's scientific measurement enough to assign numbers. But, not all is numerical.[4] Eudoxus of Cnidus (390-337 BCE) introduced "non-quantified mathematical magnitude to measure continuous geometrical entities such as lines, angles, areas and volumes, [in order to] avoid the use of irrational numbers [$\sqrt{2}$ and π]" (Eudoxus of Cnidus, *Wikipedia*, Irrational Number, *Wikipedia*) to represent the proportion between incommensurable magnitudes such as the lengths of hypotenuse and side in a right triangle or circumference and diameter in a circle. Measuring a temple—columns and pediments—for a scaled model on another site, a series of measurements—caliper, compass and straight edge— has outcomes that do not represent magnitudes in the language of real numbers but in a *mixed* language of primitive, geometric magnitudes—lengths, areas, volumes—, a relation of proportion between them (ratio), and natural numbers. Hippocrates of Chios (470-410 BCE) was the first to have *measured* a curved area, proving that the area of a lune of Hippocrates (in grey) was equal to the subtended right triangle, without numeric assignment of an (irrational) number to that area (Wigderson 2019):

(29)

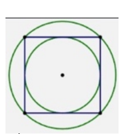


That is, the mathematical structure into which qualitative aspects of the empirical world are recoded is itself geometric, mixing non-numerical primitives and rational numbers, *i.e.*, natural numbers and their ratios.[5]

More, in meaning and implicature, works the same whatever the language of measurement, as measurement with Irrational Rulers illustrates, once constructed.

The square of side s and perimeter $4s$ is circumscribed with circumference $\pi\sqrt{2}s$ and inscribed with circumference of πs inscribed with perimeter $2\sqrt{2}s$ (not shown):

(30)



(31) $2\sqrt{2}s < \pi s < 4s < \pi\sqrt{2}s$
$CSs1 < Cs1 < Ss1 < SCs1$

Think of these four lengths, two perimeters and two circumferences, as four notes in a tetrachord within the s^{th}-octave centered on $s1$. Higher octaves are generated doubling, tripling, quadrupling, *etc*., the reference length s, and lower octaves halving, thirding, quartering it, and so on:

(32) $[\forall n{:}\mathbf{N}n](\Sigma 2\sqrt{2}s^{-n}$ & $\Sigma\pi s^{-n}$ & $\Sigma 4s^{-n}$ & $\Sigma\pi\sqrt{2}s^{-n}$ & $\Sigma 2\sqrt{2}ns$ & $\Sigma 4ns$ & $\Sigma\pi ns$ & $\Sigma\pi\sqrt{2}ns)$
& $\forall\Sigma'($
$[\forall n{:}\mathbf{N}n](\Sigma' 2\sqrt{2}s^{-n}$ & $\Sigma'\pi s^{-n}$ & $\Sigma' 4s^{-n}$ & $\Sigma'\pi\sqrt{2}s^{-n}$ & $\Sigma' 2\sqrt{2}ns$ & $\Sigma' 4ns$ & $\Sigma'\pi ns$ & $\Sigma'\pi\sqrt{2}ns)$
$\rightarrow \forall\sigma(\Sigma\sigma \rightarrow \Sigma'\sigma))$
"Σ is the scale of all tetrachord notes, & nothing else."

The resultant scale Σ of tetrachord-notes across all octaves is not in just intonation, which demands that the proportion between any two notes be rational, *i.e.*, a ratio of natural numbers. No two notes within the same tetrachord are in just intonation, *v.* (31). Across the scale, only notes in the same pitch class, *i.e.*, with the same tonal color, are justly tuned to one other. This scale is a hot mess of irrationality and highly prized for it. The only rational numbers represented are those represented by a perimeter $4sn$ or by a pair of lengths from the same pitch class, <CS*sk*,CS*sl*>, <C*sk*,C*sl*>, <S*sk*,S*sl*>, or <SC*sk*,SC*sl*>, all representing $^k/_l$. The fun fact about these irrational numbers is that they are also constructible numbers (*v.* Constructible Number, *Wikipedia.*): given length s, the perimeters and circumferences of its tetrachord (*v.* (31)) are constructible with only drafting compass and unmarked straight-edge, magnitude à magnitude, without intervention from numeric calculation, with compass and straight-edge then straightening into straight lengths any constructed perimeters and circumferences. An Irrational Ruler is a straight-edge that for a finite number of octaves marks off in color-coded lengths the straightened perimeters and circumferences of the tetrachords in the selected octaves. On an irrational ruler, the yellow lengths correspond to rational numbers and the blue, green, and red lengths to irrational numbers (*v.* (31)). Classical engineers surely had many for re-scaling temples for different sites— calipers to measure the diameter of a column, and an irrational ruler to measure out the length of rope doubling the doubled circumference, for the rope to subsequently form the form into which concrete is poured for the doubled column. What else to do if π hasn't been invented yet?[6]

To draw out *more*, an irrational ruler is aligned to a column of mercury to calibrate its height to irrational lengths in an irrational thermometer measuring the temperature of a mystery gas in a sealed chamber. Prior to their metrization, nature correlates some empirical magnitudes— the amounts, volumes, pressures and temperatures of gases— with other empirical magnitudes, the heights of mercury columns in sealed vacuums, as reflected in the expansion and contraction of the latter in the experimental apparatus of Amonton, Guy-Lussac, Charles, Boyle and Avogadro establishing the Ideal Gas Law. In (33)-(36), *that* is demonstrative to a blue, green, or red length on the irrational ruler, which is demonstrative to *that* height of a column of mercury, or to *that* volume of mercury, or to *that* amount of mercury in its present condition in a sealed vacuum. Note that any metrization, *e.g.*, 1mm = 1°K, ends in irrational numbers for both the length and temperature *that* demonstrates. The mystery gas is maintained under a constant pressure and volume, so that according to the Ideal Gas Law, its temperature correlates directly with its amount, even if its pressure and volume are otherwise unknown. Sentences (33)-(36) are equative and comparative, dense and discrete:

(33) a. (A cloud of) *that* much mystery gas is inside.
b. (A cloud of) as much mystery gas as *that* is inside.
c. (A cloud of) as much mystery gas is inside as there is of a gas at *that* temperature under its unknown constant pressure and volume.

(34) a. (A cloud of) more than *that* much mystery gas is inside.
b. (A cloud of) more mystery gas than *that* is inside.
c. (A cloud of) more mystery gas is inside than there is of a gas at *that* temperature under its unknown constant pressure and volume.

(35) a. (A mass of) *that* many mystery molecules are inside.
b. (A mass of) as many mystery molecules as *that* are inside.
c. (A mass of) as many mystery molecules are inside as there are in a gas at *that* temperature under the unknown constant pressure and volume.

(36) a. (A mass of) more than *that* many mystery molecules are inside.
b. (A mass of) more mystery molecules than *that* are inside.
c. (A mass of) more mystery molecules are inside than there are in a gas at *that* temperature under the unknown constant pressure and volume.

Since the number of molecules is a natural number, there must be that same number of molecules in many a gas with temperatures that merely approach the temperature *that* irrational length π*s*1 measures—gases with the same number of molecules at temperatures the thermometer calibrates to lengths measured with rational numbers shy of π. Yet, since it is unknown how small the mystery molecule or how large the amount of gas, it is unknown how slight a deviation in temperature amounts to a molecule more or less. It only compounds error to refer in (35)-(36) to any temperature other than that of *that* length π*s*1. Moreover, economy of expression is no excuse for a bad guess substituting a long approximate numeral for 'that' in (35)-(36) or for 'π' in 'π*s*1° K' in a metrized version of (35)-(36). Often enough, sentences (33)-(36) with their direct reference to magnitudes (or, metrized using the *descriptive* constant 'π') are the best report of what they report. Even when there is a simple, familiar metrization relating empirical magnitudes and numbers, measurement and measurement report may be dissociated from any direct reference to number. Pill counts correlate with their weight, which automated pill dispensers count on. At PhunHouse Pharma, a game show, prescriptions prescribe pills of different size and weight. Contestants are presented a heap of pills of a single prescription. They are asked to balance the heap against a large set of irrational weights, number-stamped in order of their weight, all cylinders to mask their origin in √2- or π- weight or volume (*v.* (31)). After a run of trial measurements, the game is a memory task about a story line that distinguished some prescriptions as *per diem*. Contestants answered variously, in which *that* points to the 113th weight:

(37) (A heap of) that many blue pills that balanced *that* were dispensed to last that many days.
⊩ Not more blue pills than balance the next weight (№ 114) were dispensed to last as many days.

(38) More blue pills than balanced *that* were dispensed to last that many days.
⊮ Not more blue pills than balance the next weight (№ 114) were dispensed to last as many days.

The measurement measures the unknown pill count by its position in an ordering of weights of unknown weight, without in any contestant's mind a numeric thought about pill count or a weight's weight, and yet the answers sustain the contrast in implicature between equative and comparative language.

The metrization of the Ideal Gas Law is among the firstborn poster children for the Representational Theory of Measurement [*FM* I-III]. Its numbers are good, as are the automated pill dispensers', even if one doesn't know what they are, talking freely and accurately without them about the magnitudes they measure, as in (33)-(36) and in (37)-(38). The distance between a surrogative language of number and the equative and comparative language is even plainer when the numbers are not so good or not known so good. Direct apprehension of equative and comparative relations between the empirical magnitudes themselves suffices for accurate judgments about an empirical structure that perhaps has only a fractured mirror in a remote mathematical structure (*v.* (23)-(28)):

(39) A pain the pain of a mild headache persisted.
⊩ No migraine abortive is indicated.
(40) A pain more than the pain of a mild headache persisted.
⊮ No migraine abortive is indicated.

(41) Undercover agents as American as Boris Badenov are on it.
As much of an American as Boris Badenov is on it.
⊩ The mission is doomed.
(42) Undercover agents more American than Boris Badenov are on it.
More of an American than Boris Badenov is on it.
⊮ The mission is doomed.

In a clinical setting, (39) implicates that the patient suffered no worse a pain than that of a mild headache, for which only aspirin is indicated, which (40) does not implicate. Boris Badenov cannot pass for an American, but better agents do: (41) implicates that no one better is on the mission, dooming it, which (42) does not. The meaning and implicature of the equative and comparative language in (41)-(42) is plain. Only later might spy-craft and the darker arts of educational and industrial psychology quantify Americanism in "meaningful" measurement that is not pseudo-scientific (Falmagne & Narens 1983, Narens 1985, Narens & Luce 1986). In (39)-(42) and in (33)-(36), there is as yet no reasoning surrogatively in a numeric language but directly in an equative and comparative language ('x_0 is more than x_1', '$x_0 \succ x_1$') about empirical magnitudes, as Sassoon (2010) insists. The essential problem of Krifka-Fox-Hackl implicature, generalizing and distilling (1)-(8), is that there are empirical magnitudes q discrete and dense about which equative but not comparative language implicates not more than q:

(Krifka-Fox-Hackl) *For some* q, $q \succ$ *that*:
(43) a. *That* many are thus.
b. (A mess of) as many as *that* are thus.
(44) (43) $\Vdash$ Not more than q are thus.

(45) a. *That* much is thus.
b. (A mess of) as much as *that* is thus.
(46) (45) $\Vdash$ Not more than q is thus.

(47) More than *that* are thus.
(48) (47) $\nVdash$ Not more than q are thus.

(49) More than *that* is thus.
(50) (49) $\nVdash$ Not more than q is thus.

And how naked with nothing more for logical form than:

For some q, $q \succ$ *that*:
(51) $[\exists x : x \approx \textit{that}]$ thus(x)
(52) (51) $\Vdash \neg[\exists x : x \succ q]$ thus(x)

(53) $[\exists x : x \succ \textit{that}]$ thus(x)
(54) (53) $\nVdash \neg[\exists x : x \succ q]$ thus(x)

Is there yet a suitably boring pragmatics that affords a uniform explanation for the contrast in implicature between equative and comparative language, discrete or dense, numeric or non-numeric?

2. *Error*: *the universal delicacy of measurement.*
Implicature, (44) or (46), is inference under conditions of ignorance: first to an *ignorance* implicature, that it is not known that more than q be thus, and then from not knowing to knowing not— knowing that not more than q be thus, a *scalar* implicature so-called despite its unmetrized examples in the previous section. Recall how (44) unfolds. It has been said, (43), that *that* many are thus. Since, it would have been more informative to have said that more than q are thus, it is implicated that the speaker eager to be as

informative as can be does *not* know that more than q are thus. This ignorance implicature graduates to a scalar implicature only under the further assumption that the speaker *does* know *whether* more than q or not more than q are thus— the *known whether-or-not* implicature. That is, it is known that more than q are thus or known that not more than q are thus. It only then follows, given the ignorance implicature, that the known alternative is that *not* more than q are thus, the scalar implicature (44). However it goes, the essential problem of Krifka-Fox-Hackl implicature is that all this somehow goes awry when (47) rather than (43) is the thing said under conditions of ignorance. Despite the import of ignorance for measurement, nothing is said about it in describing either the empirical structure, the relation among magnitudes ('$x_0 \succ x_1$'), or a measurement structure ('$\mu(x_0) > \mu(x_1)$').[7]

Behind the veil of (43)-(54), error is the marriage of measurement and ignorance. Every tinker, tailor, baker, and candlestick maker measuring twice to cut once knows that to measure is to err, to an extent contingent on the measurement set-up $\mathfrak{C}$:

(55) $\text{measure}_{\mathfrak{C}}(e) \rightarrow \text{error}(e)$[8]

The set-up comprises instrument and protocol and anything else in the context for measurement event e that may compound its error. Each measurement event suffers from imprecision in the set-up, from flawed design or execution of the protocol, and from randomness in the ambient conditions[9]—a crooked balance, humidity, a seismic flutter, a teary eye. Some error is intrinsic to the apparatus; but the error in *a* measurement is *ad hoc* to the event, compounded from both the apparatus' error and anything else that might have gone wrong. Error afflicts the measurements that report the empirical relations between magnitudes, prior to metrization, errant too. Unless the balance, the rest of the set-up, and measurement events are without error, there are magnitudes, *this* and *that*, such that it is not measured and thus not known that *this* is more than *that*, or that *this* is as much as *that*, or that *that* is more than *this*. Sometimes there are yellows the yellowest of which cannot be discerned, pains that are not noticeably different, and eyecharts alike enough in clarity to obscure the optical difference in the lenses that refract them.

If every measurement suffers a compound error, it has erred by just so much, estimated to fall within a *margin of error.*[10] A balance measured that *these* salts are as many grains or as much salt as *those* salts, *to a pinch more or less*. If it is measured at e that x is as much as q, $x \approx q$, measurement e shrouds q in a margin of error between magnitudes, q_0 and q_1, $q_1 \succcurlyeq q \succcurlyeq q_0$.[11] Despite the measurement outcome that x is as much as q, $x \approx q$, for any magnitude m within e's margin of error, $q_1 \succcurlyeq m \succcurlyeq q_0$, it is not known as of e that x is not as much as m, $\neg$ Known that $\neg\, x \approx m$. Measurement e's error is its failure to discern what it measures, x, among the magnitudes in its margin of error. Error and ignorance are intimates. What could it mean to know of a margin of error without knowing that it declares unknown as of this measurement where within it the true value lies among the values it is not already known not to be? That is, e is no new evidence that x is not m for any m within e's margin of error.

It comes with understanding 'measure$_{\mathfrak{C}}(e)$' that to measure is to err (55), with a margin of error:

(56) $\text{measure}_{\mathfrak{C}}(e) \rightarrow \exists q_0 \exists q_1 \forall m(\text{margin-of-error}_e(m) \leftrightarrow q_1 \succcurlyeq m \succcurlyeq q_0)$

Call now the empirical target a *quantity* (*v.* n. 3), reserving *magnitude* for continuous, dense quantities— lengths, areas, volumes, weights, times, velocities, temperatures, probabilities, etc.—and *numerosity* for discrete quantities.[12] *A* measurement e is measurement of *a* quantity x:

(57) $\text{measure}_{\mathfrak{C}}(e) \rightarrow \exists x(\text{quantity}_{\mathfrak{C}}(e, x) \ \& \ \forall y(\text{quantity}_{\mathfrak{C}}(e, y) \rightarrow x = y))$

Let 'Known$_{\mathfrak{C}}$' be what is known at $\mathfrak{C}$ prior to measurement and 'Known$_{\mathfrak{C}+e}$', what is known as of measurement e. Measurement does not relieve prior ignorance about values within its margin of error:

(58) (*Original Error*)
$(\text{measure}_{\mathfrak{C}}(e) \ \& \ \text{quantity}_{\mathfrak{C}}(e, x) \ \& \ \text{margin-of-error}_e(m)) \rightarrow$
$(\neg \text{Known}_{\mathfrak{C}} \vdash \neg\, x \approx m) \leftrightarrow (\neg \text{Known}_{\mathfrak{C}+e} \vdash \neg\, x \approx m)$

At its best, when the language of measurement is surrogative and numeric, a numeric version of *Original Error* (63) follows from (59)-(60), and symmetric margins of error take on the familiar form of a $[\pm\varepsilon]$-deviation, (62):

(59) $\mu(x_0) = \mu(x_1) \rightarrow x_0 \approx x_1$
$\mu(x_0) > \mu(x_1) \rightarrow x_0 \succ x_1$

(60) $\text{Known}_{\mathfrak{C}} \vdash (\mu(x_0) = \mu(x_1) \rightarrow x_0 \approx x_1)$
$\text{Known}_{\mathfrak{C}} \vdash (\mu(x_0) > \mu(x_1) \rightarrow x_0 \succ x_1)$

(61) $\text{margin-of-}\mu\text{-error}_e(n) \leftrightarrow \exists q(\text{margin-of-error}_e(q) \ \& \ \mu(q) = n)$
(62) $\text{margin-of-}[\pm\varepsilon]\text{-error}_e(n) \leftrightarrow (\text{margin-of-}\mu\text{-error}_e(n) \ \&$
$\exists m \forall k(\text{margin-of-}\mu\text{-error}_e(k) \leftrightarrow m+\varepsilon > k > m-\varepsilon))$

(63) (*Original* μ*-Error*)
$(\text{measure}_{\mathfrak{C}}(e) \ \& \ \text{quantity}_{\mathfrak{C}}(e, x) \ \& \ \text{margin-of-}\mu\text{-error}_e(n)) \rightarrow$
$(\neg \text{Known}_{\mathfrak{C}} \vdash \neg\, \mu(x) = n) \leftrightarrow (\neg \text{Known}_{\mathfrak{C}+e} \vdash \neg\, \mu(x) = n)$

Original Error (58) and its corollary *Original* μ*-Error* (63) codify a fraction of what is second nature, like original sin, to measurement's apprentices, tinker, tailor, baker and candlestick maker. *Original Error* is an *entailment*—that measurement e affords no relief from ignorance about values within the margin of error— exactly what "margin of error" means if it means anything at all. It is an absolute limit on *e's* information.[13]

Measurement meets not the natural world except in error, which a certain first science officer never neglects, never saying anything *is* so much, only that it *measures* (or *counts*) so:

Time-traveling aboard the USS Enterprise (NCC-1701) to Los Angeles, *9 September 1965*,

Kirk: Spock, so how much has Koufax pitched?

Spock: Captain, it appears that…

(64) Sandy Koufax pitched pitches measure[-'d /-ing]/counted (at) 113.
$\Vdash$ Sandy Koufax pitched 113 pitches.

(65) (64) $\Vdash$ Sandy Koufax did not pitch 114 pitches.

(66) Sandy Koufax pitched time on the mound measure[-'d /-ing] 103 minutes.
$\Vdash$ Sandy Koufax pitched 103 minutes.

(67) (66) $\Vdash$ Sandy Koufax did not pitch 104 minutes.

It retraces the scalar implicature, as it goes in (2) or in (44), to go from Spock's caution that *some* pitches *measured* 113 to Kirk's conviction that they *were* 113 and not more, adding steps licensed by the premise that Spock's measurement is accurate:[14]

(68) (*Accuracy*) Let '$\text{Measure}_{\mathfrak{c}+e}$' be statements φ of the outcome of measurement$_{\mathfrak{c}}$ e:

$$(\text{Measure}_{\mathfrak{c}+e} \vdash \varphi) \rightarrow (\text{Known}_{\mathfrak{c}+e} \vdash \varphi)$$

Given Spock's exactitude, it is presumed that he reports measurements only if they are accurate, and thus if he says (64) it is inferred from (68) that he knows that some 113 pitches were pitched. As resolute as anyone to be informative, Spock also implicates in saying (64) that he is not hiding better, more accurate measurements counting more pitches than attested to—a *best-measurement* implicature—had he better measurements, he would have had, by (68), something more informative to say. Despite the detour through Spock's use of the word 'measure', (64) also first implicates that Spock does *not* know of 114 pitches. As before, this ignorance implicature then graduates to a scalar implicature that there weren't 114, only under a *known whether-or-not* implicature, derived here from a *measured whether-or-not* implicature. That is, Kirk presumes that Spock's measurement and measurement set-up would have detected—it would have known— whether or not there were 114 pitches. Since it did not—Spock would have said so—, there were not. How else does Kirk find the answer to his question, which we do too, in Spock's aloof, yet presumed cooperative, reply (64) (*cf.* n. 14)?

Spock's aloof language affords words and the occasion for the first science officer to be explicit about measurement's margin of error, too, and for us to witness its effect on implicature. Language explicit in its mention imitates Krifka-Fox-Hackl implicature in language for which the pragmatics is as *boring* as the reasoning to the contradiction behind 'but' in:

(69) a. Parent: You eaten? Child: Yes, candy corn. Parent: But, that isn't dinner!
b. Parent 1: Let's go to the Thalia. Best snacks. Parent 2: But, R-rated movie.

There is no reckoning (69) without a good parenting handbook. Any expert system understanding and guiding parents would need to axiomatize it, but this is too boring a chore for linguists. It suffices to recognize in (69) that there is such a handbook that the parents know and know they know, and that the knowledge it codifies is indexed when its language is the language in use in conversation or thought. The pragmatics of (69) is doubly boring—too boring to strain a complacent, Gricean view of what passes between semantics and pragmatics (*contra* Fox & Hackl 2006) and boring in its details about parenting. So it is that language explicit about margins of error indexes a handbook of measurement (including (55)-(63), (68)) consulted in a boring imitation of Krifka-Fox-Hackl implicature. But, if (43)-(54) veil unpronounced language as explicit about margins of error, then imitation is nature and Krifka-Fox-Hackl implicature is truly boring. Back now to Spock talking about margins of error and to its effect on scalar implicature—

[*Time-traveling aboard the USS Enterprise (NCC-1701) to Los Angeles*, *9 September 1965*,
Kirk: Spock, so how much has Koufax pitched?
Spock: Captain, it appears that…
(70) a. Sandy Koufax pitched pitches measuring 113 to the nearest pitch.
b. Sandy Koufax pitched pitches measuring 113 with a margin of error less than 1.
(71) (70) ⊩ Sandy Koufax did not pitch 114 pitches.

(72) a. Sandy Koufax pitched pitches measuring 113, give or take a couple.
b. Sandy Koufax pitched pitches measuring 113 with a margin of error [±2].
(73) (72) ⊮ Sandy Koufax did not pitch 114 pitches.

(74) (70) ⊩ Sandy Koufax did not pitch 118 pitches.
(75) (72) ⊩ Sandy Koufax did not pitch 118 pitches.

It matters for the scalar implicature (71) that a 114th pitch does not fall within the margin of error; doing so undoes it, as in (73)— (71) *vs.* (73). A 118th pitch is however outside the margin of error for both (70) and (72), and so both implicate that there were not 118, (74)-(75). The reasoning is that *Original Error* (58) and its corollary *Original μ-Error* (63) *entail* that for any quantity within the margin of error, it is *not* known whether or not what is measured is that much. That is, the *known-whether-or-not* and *measured-whether-or-not* implicatures are explicitly denied and pre-empted for values falling within the margin of error, thereby undoing for (71) the step from ignorance implicature to scalar implicature.[15] Reasoning about dense measurement and its margins of error is the same *mutatis mutandis*:

(76) a. Sandy Koufax pitched time on the mound measuring 103 min. to the minute.
b. Sandy Koufax pitched time on the mound measuring 103 min. with a margin of error under 1 minute.

(77) (76) ⊩ Sandy Koufax did not pitch 104 minutes.

(78) a. Sandy Koufax pitched time on the mound measuring 103 minutes, give or take a couple.
b. Sandy Koufax pitched time on the mound measuring 103 minutes with a margin of error [±2 min.].

(79) (78) ⊮ Sandy Koufax did not pitch 104 minutes.

A central, unifying observation is that ignorance implicatures about values within the margins of error do not graduate to scalar implicatures; but, ignorance implicatures about values beyond the margins of error do.[16] And, so, manipulating the margins of margins of error manipulates the distribution of scalar implicatures. Knowing (56) that all measurements come with margins of error, there is no limit to Spock's creativity describing their margins, as bounded ((80), (82)) or unbounded ((84), (86)), all of which shield the values within from scalar implicature:

(80) a. Sandy Koufax pitched pitches counted at around 113.
b. Sandy Koufax pitched pitches counted to within two pitches of 113.
c. Sandy Koufax pitched pitches measuring (to) between 110 and 115.

(81) (80) ⊮ Sandy Koufax did not pitch 114 pitches.

(82) a. Sandy Koufax pitched time on the mound clocked somewhere between 100 and 105 minutes.
b. Sandy Koufax pitched tonight for a couple minutes more or less than last night's 103 minutes.

(83) (82) ⊮ Sandy Koufax did not pitch 104 minutes.

(84) a. Sandy Koufax pitched pitches counted (to) (somewhere) past 112.
b. Sandy Koufax pitched pitches measured (somewhere) (at) more than 112.

(85) (84) ⊮ Sandy Koufax did not pitch 114 pitches.

(86) a. Sandy Koufax pitched time on the mound clocked in (somewhere) at above his record 102 minutes.
b. Sandy Koufax pitched time on the mound measured (somewhere) (at) more than 102 minutes.

(87) (86) ⊮ Sandy Koufax did not pitch 104 minutes.

It has been convenient to illustrate in the surrogative, numeric language of pitch counts and minutes on the mound explicit talk about margins of error. The effect of such talk on implicature and the reasoning from it are the same when the bounded and unbounded margins of error are the outcome of measurements measuring directly the empirical relations, as should be given the reach of Krifka-Fox-Hackl implicature:[17]

(88) A pain the pain of a moderate headache give or take some sinus pressure persisted.
$\Vdash$ No opioid is indicated.
(89) A pain between the pain of a mild headache and that of a cracked rib persisted.
$\nVdash$ No opioid is indicated.
(90) A pain beyond the pain of a mild headache persisted.
$\nVdash$ No opioid is indicated.
(91) A pain more than the pain of a mild headache persisted.
$\nVdash$ No opioid is indicated.

(92) a. Undercover agents as American as Boris Badenov with or without a Ford pick-up are on it.
b. As much of an American as Boris Badenov with or without a Ford pick-up truck is on it.
$\Vdash$ The mission is doomed.
(93) Undercover agents as American as Boris Badenov up to Clint Eastwood are on it.
$\nVdash$ The mission is doomed.
(94) a. Undercover agents more American than Boris Badenov are on it.
b. More of an American than Boris Badenov is on it.
$\nVdash$ The mission is doomed.

That is, Krifka-Fox-Hackl implicature as distilled in (43)-(50) is imitated in language expanded as in (95)-(103), for which the pragmatics is boring and yet universal to measurement discrete and dense, numeric and non-numeric, wherever is found a contrast between "*x* is *that*-much/many" and "*x* is more than *that*":

(Krifka-Fox-Hackl) For some *q*, some *r*, $r \succ$ *this* $\succ q \succ$ *that* and *q* is not at *that*:
(95) Things/stuff measured [to a margin of error with values] (at) *that* are/is thus.
(96) (95) $\Vdash$ Not any such measured [to a margin of error with values] more than *q* are/is thus.
(97) (95) $\Vdash$ Not any such measured [to a margin of error with values] more than *r* are/is thus.

(98) Things/stuff measured [to a margin of error with values] between *that* & *this* are/is thus.
(99) (98) $\nVdash$ Not any such measured [to a margin of error with values] more than *q* are/is thus.
(100) (98) $\Vdash$ Not any such measured [to a margin of error with values] more than *r* are/is thus.

(101) Things/stuff measured [to a margin of error with values] more than *that* are/is thus.
(102) (101) $\nVdash$ Not any such measured [to a margin of error with values] more than *q* are/is thus.
(103) (101) $\nVdash$ Not any such measured [to a margin of error with values] more than *r* are/is thus.

With a boring pragmatics to comfort, let imitation be nature. Sentences (43)-(50) are said aloud and heard but (95)-(103) are the real thoughts. Logical form for (43)-(50), dressier than (51)-(54), is a formal translation of (95)-(103).

3. *Measurement talk*

Anyone can answer a naïve question about how much or many with a cautious how so measured and be understood to have answered the question. Talking to himself or other Vulcans, for Spock to ask how much or many is itself just to ask how much or many it measures. As this is always understood among telepathic Vulcans, without humans in the conversation, constant reminders of measurement's error and the speakers' fallibility are elided in speech without compromise to the thought or to the logical form of what is transmitted and received. Then, if to utter (1) and (5) is to think thoughts with logical forms resembling "Sandy Koufax pitched pitches measured somewhere at 113" and "Sandy Koufax pitched pitches measured somewhere more than 112" (*v.* (95), (101)), their pragmatics also resembles what their syntax and semantics resembles. Implicature will emerge from (1) and (5) as it does from their verbose paraphrases. How then to read into *113 pitches* and *more than 112 pitches* logical forms paraphrased "(some) pitches measured somewhere at 113" and "(some) pitches measured somewhere more than 112"?

The key to the language of measurement in its entirety is in the syntax, semantics and pragmatics of its prepositional numerals, such as *at n*, *around n*, *near n*, *above n*, *under n*, *up to n*, *between m and n*, *etc.*, derived from locative prepositions, which Corver & Zwarts (2006) say "have the same global semantic structure as spatial PPs....The main difference is that while spatial PPs are interpreted with respect to three-dimensional physical space, numerical PPs are interpreted with respect to a one-dimensional space of natural[*sic*] numbers,"[18] as the preposition *between* illustrates in the prepositional numerals in (104) (*v.* (82)), in the unmetrized measurement of a pain (105), in the temporal location of a lull between assaults, and in a spatial location between the sites of those assaults (107) (also, (108), (119)-(121)):

(104) between 110 and 115 pitches
(some) pitches (measured) (somewhere) between 110 and 115 (in number)

(105) There was (a) pain between a punch in the nose and a kick to the groin.
(106) There was a lull between a punch in the nose and a kick to the groin.
(107) There was an unbroken rib between a punch to the nose and a kick in the groin.
(108) There is a plain between a punchbowl and foothills.

The preposition *between* is the same in (109) whether it orders the 1965 Dodgers starters according to their seating on the bench, their position in the starting rotation, their height, their time on the mound, or some other sabermetric:

(109) Sandy Koufax is between Don Drysdale and Claude Osteen.

If it be fact that the very same preposition is tokened in descriptions of metric and spatiotemporal spaces, then as much as the locative prepositions are known to denote relations to events or states, 'at(e,x)' 'around(e,x)', 'near(e,x)' 'above(e,x)', 'under(e,x)', 'up-to(e,x)', 'between(e,x)',[19] in the best, Davidsonian analysis of adverbial modification (Parsons 1990), they denote the same in prepositional numerals. If it is the very same preposition and yet the Davidsonian analysis dare be rejected, there can be no alternative analysis of prepositional numerals that does not then dare an alternative analysis of locative adverbial modification in natural language. Good luck with that. Embedded in DPs, prepositional numerals, being prepositional phrases, modify. Describing a location in metric space, they modify measurement, *i.e.*, measurement events — "pitches measured between 110 and 115", '[∃X: pitches(X)… [∃e: Past(e)](measure(e) &…& between(e, *110 and 115*))]', "pitches measured at 113", '[∃X: pitches(X)…[∃e: Past(e)](measure(e) &…& at(e, 113))]'. A verb of measurement need not be pronounced:

(110) Muhammad Ali, the weighing in at 225 lb. greatest heavyweight champion ever, faced Joe Frazier weighing in at 216 lb. at the Thrilla in Manila.
(111) Muhammad Ali, the at 225 lb. greatest heavyweight champion ever, faced Joe Frazier at 216 lb. at the Thrilla in Manila.

It is understood in (111) that the prepositional phrases "at 225 lb." and "at 216 1b." locate events (or states) in metric space. For reasoning from (111), its logical form need not itself token silent verbs of measurement, since it is common knowledge that to be an event in metric space is to be a measurement event:

(112) $\vdash$ (quantity(q) & at(e, q)) $\rightarrow$ $\text{measure}_{\mathfrak{c}}(e)$
(113) $\vdash$ (quantity(q) & $\text{P}_{\text{Locative}}(e, q)$) $\rightarrow$ $\text{measure}_{\mathfrak{c}}(e)$

For (114) merely to host the prepositional numeral is for its DP to have at least the logical form (115), which in conjunction with (113) renders (114) equivalent to (116) (=(80)a.):

(114) Sandy Koufax pitched around 113 pitches.
(115) … [∃X: ∃e(Patient(e, X) & around(e,113)) & pitches(X)]…

(116) Sandy Koufax pitched pitches measured at around 113.
(117) … [∃X: pitches(X) & [∃e: Past(e)]($\text{measure}_{\mathfrak{c}}(e)$ & Patient(e, X) & around(e,113))]…

That is, *around 113 pitches* practically reads itself into "some pitches measured (somewhere) around 113" just to accommodate the prepositional numeral *qua* locative prepositional phrase—a simple point of syntax. So do all the prepositional numerals read themselves into equivalences with their paraphrases in (80)-(86). Under that equivalence in their syntax and semantics, prepositional numerals have the same pragmatics, in particular the same distribution of scalar implicatures as in (80)-(86), which distinguishes prepositions describing bounded space, *at*, *near, around, between, within* and those describing unbounded space, *above, over, past, beyond*, etc.

The same preposition *between* with a semantics uniform and austere enough to hold of all the spaces it describes omits their eccentricities. It is left to the modified lexical item, the verb, *ricochet*, *find* or *lose,* in (118)-(121) to say what hanky-panky must go on in that space between the ping pong tables for it to be ricochet, find or loss:

(118) …Verb(e) & [the X: ping pong tables(X)] between(e, X)…
(119) The ricocheted ping pong ball ricocheted between the ping pong tables.
(120) The ricocheted ping pong ball was found between the ping pong tables.
(121) The ricocheted ping pong ball was lost between the ping pong tables.

In (121), *lose* entails that it was *not known* where between the tables the ball was. It would not be lost there otherwise:

(122) $\vdash$ (lose(e) &Theme(e,y) & between(e, X)) $\rightarrow$
(overlaps(e,r) $\rightarrow \neg$Known$_e$ $\vdash$ [$\exists e$': overlaps(e,e')](Theme(e',y) & at(e',r)))

To have instead found the ball, it must have been known where between the tables it was, *find* thus imposing a contrary epistemic condition and *ricochet*, no epistemic condition at all. Similarly, it is left to the semantics of the language of measurement to say that prepositional numerals describe for it *margins of error* (*v.* (55)-(63), (68)):[20]

(123) $\vdash$ (measure$_{\mathfrak{c}}$(e) & between(e, X)) $\rightarrow$
$\forall m$(margin-of-error$_e$(m) $\rightarrow$ [$\exists e$': at(e', m)] between(e', X))
(124) $\vdash$ (measure$_{\mathfrak{c}}$(e) & P$_{\text{Locative}}$(e, X)) $\rightarrow$
$\forall m$(margin-of-error$_e$(m) $\rightarrow$ [$\exists e$': at(e', m)] P$_{\text{Locative}}$(e', X))

Saying so, given both (113) and (123), now reads *between 110 and 115 pitches* in (125) into an equivalence with "(some) pitches measured to a margin of error with values between 110 and 115" (*v.* (98)):

(125) Sandy Koufax pitched between 110 and 115 pitches.
Sandy Koufax pitched between those-many pitches.
(126) … [$\exists Y$: $\exists e$(Patient(e, Y) & between(e, X)) & pitches(Y)]…

(127) Sandy Koufax pitched pitches measured to a margin of error with values between 110 and 115.
Sandy Koufax pitched pitches measured to a margin of error with values between those values.

Thus, *between*, a trifle with all the other prepositions *qua* prepositional numerals, is the camel's nose under the tent, slipping inside what speaker and hearer know and take others to know about measurement, measurement set-up, error, and margins of error, so that the reasoning from prepositional numerals as they occur in the likes of (125) unfolds as it does for its paraphrase in (127).

There is more to trifle with here. If the reasoning unfolds from the meaning of the locative preposition *pro se*, similar reasoning might be expected in settings in which it occurs outside prepositional numerals *per se*. For an orienteering race from Washington DC to Québec PQ, the orienteers are equipped with transponders to track and measure their progress through backcountry. In this setting, to check in *via* transponder is to measure current position in the race, under spotty coverage:

(128) $\vdash$ check-in(e) $\rightarrow$ measure$_{\mathfrak{C}}$(e)

In (129)-(134), the prepositional phrases describe ordinary physical rather than metric space in reports of current position. But, given (128) and (123)-(124), they now also describe margins of error in the measurement of Tove's position in the race:

(129) Tove has checked in at the New York-New Jersey border.
(130) (129) $\Vdash$ Tove has not checked in in the Catskill Mountains.

(131) a. Tove has checked in between the NY-NJ and NY-Vermont borders.
b. Tove has checked in in New York State.
(132) (131) $\nVdash$ Tove has not checked in in the Catskill Mountains.

(133) a. Tove has checked in past the New York-New Jersey border.
b. Tove has checked in north of the New York-New Jersey border.
(134) (133) $\nVdash$ Tove has not checked in in the Catskill Mountains.

Tove has already checked in at various points before New York. If (129) is the current, most informative report of her position, it is implicated that Tove is not further in the race, not in the Catskill Mountains far from the border. If, instead, (131) is current, Tove is currently lost within the state and perhaps already in the Catskills. Similarly, for (133), provided that *past* and *north* are not understood as "just past" or "just north" to exclude the Catskill Mountains from a just past or just north margin of error, within which her position is unknown. This is yet another example of non-numeric, unmetrized measurement, that Tove's position is as much as—as far as—the New York-New Jersey border ('t ≈ ny/nj'), in an ordering of positions in the race from start to finish. Distances *en route* between check-ins might be unknown and only their relative order charted without regard to scale or to their relative, uneven spacing. The reasoning that denies *past the New York-New Jersey border* the scalar implicature affirmed for *at the New York-New Jersey border* neither assumes that a mathematical structure mirrors the empirical one nor engages any surrogative reasoning about numbers. The prepositional phrases aren't even about a metric space—not directly— since the New York-New Jersey border is not a quantity but a place for the weary to rest.

Recall that Krifka-Fox-Hackl implicature as distilled in (43)-(50) is imitated in language expanded as in (95)-(103) to talk direct about margins of error. That language forges as a matter of its lexical semantics and the knowledge of measurement it indexes essential connections among measurement, ignorance, accuracy, error and margins of error (*v.* (55)-(63), (68), (123)-(124)). The pragmatics as a result is boring, yet universal to measurement discrete and dense, numeric and non-numeric, wherever is found a contrast between "*x* is *that*-much/many" and "*x* is more than *that*". Ignorance implicatures about values within the margins of error do not graduate to scalar implicatures; ignorance implicatures about values beyond the margins of error do. Manipulating the margins of margins of error manipulates the distribution of scalar implicatures. The remarks on *between* (and on *at* and *past*) now observe that sentences with prepositional numerals, equivalent to their paraphrases, are destined for the same boring pragmatics. To utter (125) is as if to have all along thought and been understood to have thought (127), and to reason from it accordingly. That is, the sentences (43)-(50) just are, in effect, their expansion in (95)-(103). The heavy lifting is done by and for the syntax, semantics and pragmatics of prepositional numerals *qua* locative prepositional phrases. The final step is trivial. Tell me a theory of the syntax, semantics and pragmatics of prepositional numerals, and I will tell you it is the same for *113 glorious pitches* and *more than 112 glorious pitches* and the rest of (1)-(8), as these too all contain prepositional numerals in light disguise. Tell me less and miss that the problem of Krifka-Fox-Hackl implicature subsumes reasoning in Sassoon's (2010) unmetrized language augmented with Corver & Zwarts' (2010) prepositional numerals.

What is told is that *113 glorious pitches*, and all bare measure terms contain an unspoken locative preposition 'AT', *AT 113 glorious pitches* (Jackendoff 1979: 173).[21] Whether embedded in such a DP or not, as in *The Apostles are twelve, Paul is twelve,* or *Ali is 225 lb.*, they all work like *Mama Bear is home* (*v.* Larson 1985)—*Mama Bear is AT home*, *The Apostles are AT twelve, Paul is AT twelve, Ali is AT 225 lb*— containing 'AT(e, x)' in logical form:

(135) Sandy Koufax pitched 113 pitches. (*v*. (114)-(115))
(136) … [$\exists X$: $\exists e$(Patient(e, X) & AT(e,113)) & pitches(X)]…

AT, like *at, around* and *near*, describes a bounded neighborhood. With a certain syntax for *more-than q*, the comparative rather describes an unbounded region, like *above*, *beyond*, and *past*, rendering *more than 112 pitches* equivalent to *above 112 pitches*:

(137) Sandy Koufax pitched above 112 pitches.
(138) … [$\exists X$: $\exists e$(Patient(e, X) & above(e,112)) & pitches(X)]…

(139) Sandy Koufax pitched more than 112 pitches.
(140) … [$\exists X$: $\exists e$(Patient(e, X) & more-than(e,112)) & pitches(X)]…

(141) $\vdash$ (quantity(q) $\rightarrow$ (more-than(e, q) $\leftrightarrow$ above(e, q))

The problem of Krifka-Fox-Hackl implicature encountered in (1)-(8) is resolved when its equative language, *e.g.*, *113 glorious pitches* and its comparative language, *e.g.*, *more than 112 glorious pitches*, are recognized to properly contain prepositional numerals— 'AT 113' for '113' and the likes of 'AT more than 112' for 'more than 112'—insinuating reference to margins of error along with what a handbook on measurement says about them. It would be incoherent to confine such a revision in the elementary vocabulary of measurement to the nominal expressions occurring in citation examples of Krifka-Fox-Hackl implicature. To measure is to err, so that to predicate measurement as in (142)-(143) (*v.* also (129)-(134)) is for a prepositional numeral (144)-(145) to again insinuate reference to a margin of error (*v.* n. 16, II-III):

(142) a. The Apostles are twelve.
b. Tonight's attendance at Dodger Stadium was (counted) 29,139.
(143) a. The Apostles are more than eleven.
b. Tonight's attendance at Dodger Stadium was (counted) more than 29,138.

(144) a. The Apostles are AT twelve.
b. Tonight's attendance at Dodger Stadium was (counted) AT 29,139.
(145) a. The Apostles are AT more than eleven.
b. Tonight's attendance at Dodger Stadium was (counted) AT more than 29,138.

So it goes, as §4 argues, for the most elementary sentences in the language of measurement:

(146) a. The mètre des Archives is 1 meter.
b. The mètre des Archives measures 1 meter.
(147) a. The mètre des Archives is AT 1 meter.
b. The mètre des Archives measures AT 1 meter.

Sentences (142) and (146) exemplify the simplest of measurement statements (*cf.* '$q_0 \approx q_1$', '$q_0 = q_1$'), but there is also *more-than* in (145) and the rest of Corver & Zwarts' (2006) prepositional numerals. A semantics for *more-than* isolated in degree semantics is reducible to a primitive relation between points ('$q_0 \succ q_1$', '$q_0 > q_1$'), even in (148):

(148) a. The Apostles are more than the other NFL teams.
b. Tonight's attendance at Dodger Stadium was more than the last seven games.

But, that cannot be said for *above* and the other prepositional numerals *qua* locative prepositions, which express relations between extended regions as well as points, for which a semantics is sketched in §5 prior to its application to metric spaces to derive (141), so that *more-than* proves to be just another prepositional numeral.

Understanding a measurement report is to understand which measurements are within the margin of error and which not. With a language for measurement and error comes knowledge thereof, a handbook of measurement, error, measurement set-up, instrumentation and protocol, from which *Original Error* (58), *Original μ-Error* (63), and *Accuracy* (68) have already been excerpted to explain some implicatures, to which §6 will add *Resolution* (213), *Standard Precision* (214), *Approximation* (215), *Standard Instrumentation* (216), *Calibration* (217), *Instrument Detection Range* (218), and *Silent Detection* (220) to explain yet others. The point here is both due diligence and intimidation. After all, a few examples like (69) quickly summon a practical reasoning that draws upon guidelines on nutrition and entertainment enumerated in a good parenting handbook, which are better *not* enumerated by us. The further excerpt from the handbook of measurement and its issue in devious implicature point to practical reasoning so eccentric and dependent on specific measurement practices that it overpowers the instinct of formal pragmatics to see itself everywhere and forces a confession that reasoning to scalar implicatures does indeed engage the particulars of measurement second nature to its good parents—every tinker, tailor, baker, candlestick maker and anyone else counting and measuring.

What is said in §§2-7 about reasoning in a language for measurement and error is then applied in §8 to reasoning with negated measurement reports, such as *Sandy Koufax did* not *pitch 114 pitches* and *Tove has* not *checked in past the New York State border.*

Fox & Hackl (2006) appeal to density, to the difference between open and closed intervals, to deny comparative dense measures scalar implicatures. In §9, lengths, areas, volumes, weights, and velocities and their continuous real-number measurement are, in fact, blind to any difference between open and closed intervals. Without good use for density, a boring pragmatics is just as well. Section §10 returns to more general questions about its architecture. Fox & Hackl (2006) join a formal pragmatics (Krifka 1999, Rooth 1985) that obtains the scalar implicatures for (1) from its entailment relations with sentences that substitute alternative numerals α, as illustrated above in the reasoning that pitching 113 pitches implicates not pitching 114:

(149) Sandy Koufax pitched α pitches.

Yet, a contrast between bare and comparative measures survives both when there are no substitute α in mind, (150)-(151), and when there are no entailment relations between the sentence and its substitution instances, (152)-(155):

(150) Sandy Koufax pitched the record-breaking number of pitches for a perfect game.
(151) Sandy Koufax pitched more than the record-breaking number of pitches for a perfect game.

(152) 613 telephones connect to all and only each other.
(153) More than 613 telephones connect to all and only each other.

(154) Eighteen nations are in a perfect balance of their power.
(155) More than eighteen nations are in a perfect balance of their power.

The pragmatics for bare and comparative measures proposed here looks instead only to the language of measurement and error, the meaning of its primitive relations and their subsentential tokens within measure phrases, indifferent to whether their relata are named by a numeral or described as a forgotten milestone, and indifferent to the host sentence's entailment relations to certain substitution instances.

4. *Measurement vocabulary: AT*

Number meets not the natural world except in measurement and error, so that (156) is understood as (157), where the second measurement inherits the first's error, re-using the tape measure or applying the circumference formula to obtain a measurement to the nearest mm:

(156) The platinum cylinder's diameter is 1 m. Its circumference is 3.141 m.
(157) The platinum cylinder's diameter *measures* 1 m. Its circumference *measures* 3.141 m.

Absent expected error, since numbers are not themselves measured, there is no tolerance for it in (158) or (159) (*cf.* (160)), in contrast to (156):

(158) # π is 3.141.
(159) # π measures 3.141.
(160) F π equals 3.141.

A bar forged in Paris in 1799 is the platinum standard for measurement by the meter:

(161) a. The mètre des Archives is 1 meter.
b. The mètre des Archives measures 1 meter.

Preliminary to a language enlarged with *more-than* and prepositional numerals, even this most elementary sentence (161) engages reference to margins of error. Sentences (161) and (162) measure the same length, and '1' and '1.00000' refer to the same number, and so (161) and (162) ought mean the same:

(162) a. The mètre des Archives is 1.00000 meter.
b. The mètre des Archives measures 1.00000 meter.

(163) a. The mètre des Archives is more than 0.99999 meter.
b. The mètre des Archives measures more than 0.99999 meter.

But, equipped in her science class with the International System of Units and significant digits, no middle schooler confuses (161) and (162). Saying out loud all the zeros in (162), she recognizes that it entails (163), (162) ⊢ (163); yet, she squirms at the thought that (161) does, too, * (161) ⊢ (163). For her, either (161) and (162) don't mean the same or she has some other excuse to treat them different. Given that '1' and '1.00000' refer to the same number, where in meaning or thought do (161) and (162) diverge?

On her field trip to the Archives, the middle schooler Eudora confirmed (161) using a Sears® laser ruler that displayed three decimal places, '1.000 m', accurate to within ±1 mm, according to the technical specifications that she carefully read in advance of her measurement result that:

(164) a. The mètre des Archives is between 0.999 m and 1.001 m.
b. The mètre des Archives measures between 0.999 m and 1.001 m.

Given this margin of error for the instrument in hand and given her result (164), she accepts (165) but rejects (163):

(165) a. The mètre des Archives is no less than 0.999 meter.
b. The mètre des Archives measures no less than 0.999 meter.

From the practical reasoning or pragmatics of measurement, it ought to follow that in any context in which measurement is accurate to only the nearest 1mm, (161) does not impart (163). Despite its apparent singular reference to a single point, a surrounding margin of error inhibits (163). Eudora knows as much, and knows that (162) is unfaithful, unlike (161), to her results with a Sears® laser ruler, unfit for her lab notebook. Her knowledge of measurement error is at the root of her discrimination between (161) and (162). Ought this amount to a difference in their meaning paraphrased as in (166) and (167)?

(166) The mètre des Archives measures *somewhere (undetermined) within the measurement set-up's margin of error for* 1 meter.
(167) The mètre des Archives measures *somewhere (undetermined) within* ±.00001m *of* 1.00000 meter.

With their direct mention of measurement and margins of error, the virtue in these paraphrases is that their logic is plainly different while the reference of '1 meter' and '1.00000 meter' plainly the same. Diverging in their syntax and semantics, the paraphrases then also diverge in their pragmatics. If, instead, one holds that (161) and (162) are identical and their syntax impoverished, Eudora's variable behavior and others' even more erratic behavior end in despair that a truth is ever expressed. Allan (1983), in his review of Katz (1981), reading the target sentence as 'miles|NYC-Boston| = 200' *simpliciter*, remarks:

> "To illustrate a difficulty with semantic representations and truth values, Chomsky 1977a invokes a problem (first noted by J. L. Austin) posed by such sentences as *New York is two hundred miles from Boston*. K (129) insists that such a sentence is not 'true enough for some occasions', as Austin supposed,[22] but is true only if New York is exactly 200 miles from Boston. K's position leads to absurdity: suppose that the actual distance were in fact 200 miles and 1/10,000,000 inch; then, on K's plan, the statement would be false— but meaninglessly false. Or suppose that the distance were found to be exact on two separate days— but, because of effects on the measuring device, or movement of the earth's crust, inexact on an intervening day; then, on K's plan the statement would be 'true, false, true' on a daily basis. Surely this makes a mockery of truth, and especially of linguistic truth."

Despite the mockery of linguistic truth, Lasersohn (1999) foresees and cautions against repair by semantic reference to a margin of error, as in (166) and (167). Badly done, (168) might validate (169) badly rendered as (170):

(168) The ball in free fall reached 50 m/sec in 5 seconds at 12:00:05.
(169) a. F The ball in free fall reached 50 m/sec in 5 seconds at noon and after noon.
b. F The ball in free fall reached 50 m/sec in 5 seconds at noon and after.
(170) T The ball in free fall reached 50 m/sec in 5 seconds at *some point within seconds of* noon and after *some (other) point within seconds of* noon.

Rather, (169) should be done up as (171):

(171) F The ball in free fall reached 50 m/sec in 5 seconds in *the margin of error for* noon and after *the margin of error for* noon.

Context decides the margin of error, as it does for (161), while grammar insists that two tokens of the same tacit, definite description, (169)a., or definite description and null anaphor, (169)b., refer the same. If so, that (172) is known suffices for (171) and hence for (169) to be judged false just like (173):

(172) x is at/in $y \rightarrow \neg\, x$ is after y
(173) F Lenny died at/in his retirement party and after it.

There is then no objection to a semantic proposal that the morpheme *noon* naming a point occurs within an unspoken phrase referring to a margin of error around it. If that is the project, (161), (162), (168) and (169) and any other elementary measure sentences are promised logical forms transparent to truth and logic and to their variable pragmatics, in a language that is uncompromising in its realism about measurement, precision, and error.[23]

Now revisit (161) to compare it to (174):

(174) a. The mètre des Archives is 1 meter within a margin of error [±1 mm].
b. The mètre des Archives measures 1 meter to within [±1 mm].

Neither (161) nor (174) entails (163), *i.e.*, (161) $\nvdash$ (163) and (174) $\nvdash$ (163). Sentences (174) overtly mention their margins of error expanding on (175) given the Davidsonian logical form of locative prepositional phrases:

(175) a. *The mètre des Archives is 1 meter*…& within(*e*, *a margin of error* [±*1 mm*]).
b. *The mètre des Archives measures 1 meter* …& to within(*e*, [±*1 mm*]).

But, then, the logical form of first saying that the mètre des Archives is or measures 1 meter long cannot be that its length in meters when measured at *e* equals 1:

(176) a. * $\exists e$(Patient(e, mdA) & length_{m}(e, mdA) =1 & within(e, *a margin of error* [±*1 mm*]))
b. * $\exists e$(Patient(e, mdA) & measure(e) & length_{m}(e, mdA) =1 & to within(e, [±*1 mm*]))

No matter the fancy things the prepositional phrase says about error in (176), it's just an aside about measurement conditions, leaving (176) to entail (163), (176) $\vdash$ (163). It must rather be that no identity has been asserted. It is first said only that the measurement is *near* 1 meter with a follow-up mention of a margin of error to say how near:

(177) a. $\exists e$(Patient(e, mdA) & AT(e, *1 meter*) & within(e, *a margin of error* [±*1 mm*]))
b. $\exists e$(Patient(e, mdA) & measure(e) & AT(e, *1 meter*) & to within(e, [±*1 mm*]))

Absent a revealed identity and given the mentioned margin of error, (177) plainly does not entail (163), as desired, (177) $\nvdash$ (163). The logical form for (161) is the same as that for (174) with omission of the optional locative phrase, namely:

(178) a. $\exists e$(Patient(e, mdA) & AT(e, *1 meter*))
b. $\exists e$(Patient(e, mdA) & measure(e) & AT(e, *1 meter*))

Similarly, when the measure is discrete, (179) is understood without squirming to entail (180) just in case its implicit margin of error is understood as in (181)-(182) (*v.* n.16,III):

(179) a. Tonight's attendance at Dodger Stadium was 29,139.
b. Tonight's attendance at Dodger Stadium tallied 29,139.

(180) Tonight's attendance at Dodger Stadium was more than 29,138.

(181) a. Tonight's attendance at Dodger Stadium was 29,139 to the nearest fan.
b. Tonight's attendance at Dodger Stadium tallied 29,139 to the nearest fan.

(182) a. $\exists e$(Patient(e, ta) & AT(e, 29,139) & to(e, *the nearest fan*))
b. $\exists e$(Patient(e, ta) & tally(e) & AT(e, 29,139) & to(e, *the nearest fan*))

To be 1 meter or 29,139 is to be AT 1 meter or AT 29,139, with margins of error dead center in even the most elementary measurement sentence.[24]

5. *Measurement vocabulary: aspect & spatial prepositions*
Subject to a margin of error, measurement results are always *approximative* (van Fraassen (2008: 163)), never 'point-like sharp', despite point-like language neglecting to pronounce 'AT'. For Eudora, the point-like language voices the read-out from the Sears® laser ruler for a measurement that is still approximative at [$\pm\varepsilon$] for ε=1mm, yet optimal, sharp for her set-up. For dull performance—a sneeze unsteadying hand or eye, a skittish laser, or noisy trial—the language of measurement expands to accommodate dulled measurement behavior, from the sharp *103 minutes* and *113 pitches* to the dull *between 101 and 105 minutes*, *more than 112 pitches*, *etc.* *Approximations* so-called lose the measured quantity within a margin of error that expands to include a margin [$\pm k\varepsilon$], $k > 1$, strictly greater than the relatively sharp [$\pm\varepsilon$]. Expanding the margin of error, expands ignorance's breadth so that, again (*Original Error* (58), *Original* μ*-Error* (63)), if q is the quantity e measures and margin-of-error$_e$(m), it is not known as of e that q is not m, over an even broader range for m. Prepositional numerals, 'AT q' and 'more than q' and the others, 'near q', 'around q', 'above q', 'over q', 'under q', 'up to q', 'between p and q', *etc.*, differ in how they shape their margins of errors, from which have come their different implicatures (§§2-3).

All numerals in measure phrases[25] are prepositional numerals, and the language of measurement makes explicit that prepositional numerals describe margins of error. That is, if a measurement e is located AT 113, more than 112, above 113, or between 109 and 116, its margin of error is so located as in (124) repeated in (183) (*v.* n. 20):

(183) $\vdash$ (measure$_{\mathfrak{C}}$(e) & P$_{\text{Locative}}$(e, X)) $\rightarrow$
$\forall m$(margin-of-error$_e$(m) $\rightarrow$ [$\exists e'$: at(e', m)] P$_{\text{Locative}}$(e', X))

The bare and comparative measures (1)-(8) cite differ semantically, differing in the spatial prepositions they contain. Describing different regions of metric space, they describe different margins of error for their measurements and therefore differ in what they leave unknown and in what ignorance their use implicates, as in §§2-3.

Remarks such as (183) come from the pages of a handbook that measurement vocabulary indexes, exploited in reasoning to the implicatures of utterances in which bare and comparative measures occur. It suffices for pragmatics and practical reasoning that (183) along with (55)-(63) from §2 codify knowledge that has become second nature. There need be nothing definitional, analytic, constitutive of meaning, or essential to the concepts mentioned in (183) or (55)-(63) that bitter, universal experience has not taught. Alongside these, among what is known about a measurement is that its set-up includes an instrument and protocol μ_e, on which the measurement outcome and its error are contingent:

(184) $\text{measure}_{\mathfrak{C}}(e) \rightarrow \exists\mu_e(\text{instrument}_{\mathfrak{C}}(e, \mu_e)\ \&\ \forall x(\text{instrument}_{\mathfrak{C}}(e, x) \rightarrow x = \mu_e))$

Some measurement "collapses into" a result *n* such as the digits in the read-out from Eudora's Sears laser pointer, which is within the measurement's margin of error and reflects error compounded from imprecision in the set-up, from flaws or limitations of design or manufacture of the instrument or protocol, from sloppy execution of the protocol and from error from random ambient conditions (Tal 2017):

(185) $\text{measure}_{\mathfrak{C}}(e) \rightarrow (\exists n\ \text{collapse-into}(e, n) \rightarrow \exists n \forall x(\text{collapse-into}(e, x) \rightarrow x = n)))$
(186) $\text{measure}_{\mathfrak{C}}(e) \rightarrow \forall x(\text{collapse-into}(e, x) \rightarrow \text{margin-of-error}_e(x))$, *i.e.*,
(187) $\text{collapse-into}(e, x) \rightarrow \text{AT}(e, x)$

The language of measurement has vocabulary dedicated to it, *i.e.*, relations to events that relate only to measurement events:

(188) a. $\text{instrument}(e, x) \rightarrow \exists\mathfrak{C}\,\text{measure}_{\mathfrak{C}}(e)$
b. $\text{quantity}(e, x) \rightarrow \exists\mathfrak{C}\ \text{measure}_{\mathfrak{C}}(e)$
c. $\text{collapse-into}(e, x) \rightarrow \exists\mathfrak{C}\ \text{measure}_{\mathfrak{C}}(e)$
d. $\text{margin-of-error}_e(x) \rightarrow \exists\mathfrak{C}\ \text{measure}_{\mathfrak{C}}(e)$

Similarly, location in metric space is dedicated to measurement so that (189) (=(112)) confers on (191) a brevity that renders it equivalent to (192) where both sentences are understood to imply that the measurement events are contemporary with the match, as per boxing regulations:[26]

(189) $\vdash (\text{quantity}(q)\ \&\ \text{at}(e, q)) \rightarrow \text{measure}_{\mathfrak{C}}(e)$
(190) $\vdash (\text{quantity}(q)\ \&\ \text{P}_{\text{Locative}}(e, q)) \rightarrow \text{measure}_{\mathfrak{C}}(e)$

(191) Muhammad Ali at 225 lb. faced Joe Frazier at 216 lb. at the Thrilla in Manila.
(192) Muhammad Ali weighing in at 225 lb. faced Joe Frazier weighing in at 216 lb. at the Thrilla in Manila.

Accordingly, *103 minutes* and *113 pitches* come to mean "103 then-measured minutes" and "113 so-counted pitches" without translation as verbose, as remarked in §3.

A measurement is a telic event, and so to make a stative out of it as (161) does is to say that the mètre des Archives repeatedly—consistently— measures 1 meter. The aspect of (161) has a modal flavor with respect to a protocol for measurement, intending report of a property that is persistent, constant or replicable across conditions that conform to protocol. Let's see this as plural reference to measurement events,[27] without regard for the modal flavor, so that (161) translates as (195) and Alice's measurements vary in *Alice in Wonderland* under the influence of Drink-Me potions and Eat-Me cakes as in (196):

(193) $\text{measure}_{\mathfrak{C}}[E] \leftrightarrow_{def} \exists eEe \ \&\ \forall e(Ee \rightarrow \text{measure}_{\mathfrak{C}}(e))$[28]
(194) $\theta[E, X] \leftrightarrow_{def} \forall x(Xx \leftrightarrow \exists e(Ee \ \&\ \theta(e, x)))$ (Pietroski 2003: 282)
$\theta[E, x] \leftrightarrow_{def} \exists X(\forall y(Xy \leftrightarrow y = x) \ \&\ \theta[E, X])$

(195) $[\iota E{:}then[E]]([\text{the } x{:}\ \text{mdA}(x)]\ \text{Patient}[E, x] \ \&\ \text{measure}_{\mathfrak{C}}[E] \ \&\ \text{AT}[E, 1])$

(196) a. Alice measures between 1 and 5 feet.
b. $[\iota E{:}\ \textit{Alice in Wonderland}[E]]$
$([\iota x{:}\ \text{Alice}(x)]\ \text{Patient}[E, x] \ \&\ \text{measure}_{\mathfrak{C}}[E] \ \&\ \text{between}[E, \textit{1 and 5}])$[29]

Any token in logical form of a verb denoting events, V(e), or of a relation to events, $\theta(e, x)$, invites temporal and aspectual quantification over the events denoted. It cannot be a surprise that "weighing 225 lb." or "at 225 lb." comes with as much, as just described.

Of those relations that occur within the scope of aspectual quantification, Corver & Zwarts' (2006) essential insight is that prepositional numerals "have the same global semantic structure as spatial PPs," containing the familiar spatial prepositions, *at, around*, *near*, *above*, *over*, *under*, *up to*, *between*, *etc.*, for which there is a native context-dependent, topological relation '*x is within y's neighborhood'*, where the choice of neighborhood is context-dependent. The same geometric and topological concepts apply and the between-ness is the same whether (197) orders the 1965 Dodgers starters according to their seating on the bench, their position in the starting rotation, their height, their time on the mound, or some other sabermetric:

(197) Sandy Koufax is between Don Drysdale and Claude Osteen.

The neighborhood relation underlying *at the checkpoint*, *near the Catskill Mountains, in the woods, among the trees, past the New Jersey-New York state line, past New Jersey* is reflexive, *x is within x's neighborhood* and well-defined for all the above-mentioned relata: there is a way for point, interval, area or volume to be within the neighborhood of point, interval, area or volume. The plural relation is reflexive and distributively reflexive in its first argument, *X is within X's neighborhood*[sg] and [∀*x*: *Xx*]*x is within X's neighborhood*, while it remains that a neighborhood for any one of the *X* might be too small a neighborhood for them all, *i.e.*, it is not true that [∀*x*: *Xx*]*x is within X's neighborhood* → ((*Xx* & *Xy*) → *x is within y's neighborhood*). More ought to be said about the neighborhood of some scattered things, perhaps, that it "shrink wraps" these things in a continuous region that contains them.[30]

The scattered things to which a neighborhood is fitted are its *landmarks*, and neighborhoods are finite regions in *n*-dimensional space (where time is the *n*+1 dimension).[31] To talk about neighborhoods inhabited *for a while,* say instead that *x is* *within the (pop-up) neighborhood e of its landmarks Υ,* a spatiotemporal region:

(198) within(e, x) & neighborhood$_{\mathfrak{C}}$(e) & landmark[e, Υ][32]

For many landmarks, singular, plural or mass, to be within their neighborhood is to be amid or among them: *amid the forest*, *amid the trees, amid the foliage, amid the chaos, amid the conflict, amid the conflicts.* But, the paraphrase falters when rather than a region the only landmark named is a point, which nothing is within, amid, or among. Instead, location by reference to any landmark or landmarks is understood in terms of a neighborhood they landmark: *at the meeting point* is "within the neighborhood of the meeting point" and *at the meeting area* is "within the neighborhood of the meeting area". The contextual scaling of the neighborhood is left at 'neighborhood$_{\mathfrak{C}}$', leaving unpacked all that is tacitly known to tie it to limits on representation from the physical constraints on perception and instrumentation. Let it be that to landmark is to landmark a neighborhood:

(199) landmark[e, Υ] → ∃$\mathfrak{C}$ neighborhood$_{\mathfrak{C}}$(e)

For any spatial preposition *Loc*, such as *amid* or *past*, parse and abbreviate the prepositional phrase, *amid the forest* or *past the forest*, as in (200)-(201):

(200) '*Loc*(e) & landmark[e, Υ]' abbreviated '*Loc*[e, Υ]'
(201) 'amid(e) & landmark[e, *the forest*]' abbreviated 'amid[e, *the forest*]'
'past(e) & landmark[e, *the forest*]' abbreviated 'past[e, *the forest*]'

Locative prepositional phrases '*Loc*[e,Υ]' describe spatiotemporal locations in which locanda (or Themes) are located:

(202) Locandum[e,X] & *Loc*[e, Υ] (*v.* (194), n. 28.)
(203) Locandum[e, rh] & amid[e, sf]. "Robin Hood is amid Sherwood Forest."

In expanding the language of measurement to include (1)-(8)'s bare and comparative measures, both are assimilated to prepositional numerals (Corver & Zwarts 2006), derived from locative prepositions with their ordinary meaning. Bare measures contain an unspoken locative preposition 'AT'. The bare cardinal predicate spoken '113' in *113 pitches* is translated as a predicate of measurement events meaning "(counted) at 113" or "(measuring) at 113":[33]

(204) Sandy Koufax pitched 113 pitches.
(205) …pitch(e) & [∃X: ∃E(Patient[E, X] & AT[E,113] & pitches(X))]Theme(e, X)

(206) Sandy Koufax pitched above 112 pitches.
(207) …pitch(e) & [∃X: ∃E(Patient[E, X] & above[E,112] & pitches(X))]Theme(e, X)

For comparative measures, there is little daylight between *above 112* in (206) and *more than 112* in (208), which means "(counted) above 112":

(208) Sandy Koufax pitched more than 112 pitches.
(209) …pitch(e) & [$\exists X$: $\exists E$(Patient[E, X] & -er$_{Loc}$[E,112] & pitches(X))]Theme(e, X)
(210) -er$_{Loc}$[e, n] $\rightarrow \exists \mathfrak{C} \exists q$(measure$_{\mathfrak{C}}$($e$) & Locandum($e$,$q$) & above($e$, n))
(Locandum(e,q) & above(e, n)) $\rightarrow q > n$

The logical syntax for the comparative construction in quantifiers (*v.*, *e.g.,* Hackl (2001, 2002, 2009)) quantifies into a primitive relation between degrees so-called that the comparative morpheme introduces, '-er(d_1, d_0)'. Let there instead be a locative preposition for numeric space, with the meaning in (210), which passes through landmarks and neighborhoods to get to what it means for numeric space, *viz.*, "$q > n$". The logical form (209) parses '-er' like any other preposition, *above* or *past*, in prepositional numerals. If however the comparative construction demands from '-er' a relation between degrees, one is provided in (211):

(211) '-er$_{Loc}$[e , d_1, d_0]' for 'Locandum[e, d_1] & -er$_{Loc}$[e, d_0]'
(212) …pitch(e) & [$\exists X$: $\exists E$(Patient[E, X] & $\exists d_1$ -er$_{Loc}$[E, d_1,112] & pitches(X))]Theme(e, X)

Translating (208) as (212) still shortchanges the comparative construction *more…than* (*v.*, *e.g.,* Hackl (2001, 2002, 2009)) but in a harmless way assuming that any syntax embedding '-er(d_1, d_0)' can do just as well with '-er$_{Loc}$[e, d_1, d_0]' *mutatis mutandis* in an event semantics.

Assimilating all bare and comparative measures to prepositional numerals unifies the semantics for the spatial and the numeric and ushers in a uniform pragmatics for them as in §§2-3 and §7.

6. *Measurement handbook*

Original Error (58), *Original μ-Error* (63), and *Accuracy* (68) have already been excerpted from the handbook, which contains several ancillary principles of measurement, *Resolution* (213), *Standard Precision* (214), *Approximation* (215), *Standard Instrumentation* (216), *Calibration* (217), *Instrument Detection Range* (218), and *Silent Detection* (220).

The heights of NBA players are measured in the locker room, feet and head to the measuring stick. Formalismo semantics has Thales measure the Great Pyramid of Giza the same way. There is rather an embarrassment of measurement set-ups, instruments and protocols, which aggravates guessing at the intended measurements when, as usual, none is explicitly mentioned. "[A] measurement outcome does not display what the measured entity is like but what it 'looks like' in the measurement set-up" (van Fraassen (2008: 2)). Absent technical specifications, a default to *standard precision* assumes a good fit among what is to be measured, what it looks like, and the measurement set-up. In compliance with standard precision, sighting a difference between the Great Pyramid and the Pyramid of Khafre, Thales must also have seen it in their shadows or found better shadows, closer in or under better light, before planting his measuring stick in the sand to measure the Great Pyramid at 280 cubits, which reflects whatever margin of error can be expected from that measurement set-up and optical resolution at that perspective on what is to be measured. Measurement is "for the use of the industrial designer, the navigator, and …the traveler, to plan and direct action, bringing with it an inevitable perspective and indexicality" (van Fraassen (2008: 76)), and with that, an inevitable perspective and indexicality to its margin of error, too. Given what might be known about the qualitative conditions and perspective that are context for report that the Great Pyramid is 280 cubits, and the good fit that standard precision imposes on measurement, it can be inferred that the margin of error does not erase the visible differences among the pyramids in Giza but it swallows the birds nesting on them. As a default, standard precision also assumes that the set-up in place is good for a good fit— with nothing broken anywhere in the range of what is to be measured. Contexts in which the set-up is known to be broken in places will vary their ignorance and scalar implicatures (§7).

An important aspect of the measurement set-up $\mathfrak{C}$ guiding selection of instrument and protocol μ_e is location and perspective on the quantities to be measured and the resolution it affords of those quantities. As a matter of optical resolution, the distance between two binary stars cannot be measured if under current magnification and haze the two appear unresolved as one. Two quantities are resolved in the measurement set-up $\mathfrak{C}$ only if it is perceived there that they are different:

(213) (*Resolution*) $\text{Known}_{\mathfrak{C}} \vdash (\exists e_0\ \text{quantity}(e_0, q_0)\ \&\ \exists e_1\ \text{quantity}(e_1, q_1)\ \&\ q_0 \neq q_1)$[34]

Resolution limits the precision of any measurement set-up that measures proximal quantities to measure distal quantities (when, for example, the image distance in mm is not the distance in astronomical units).[35] For many instruments, optical or acoustic, there is a familiar inverse relation between resolution and range—increasing resolution diminishes field of view, the range. Perceptual resolution at the scene may surpass the instrument to the extent, for example, that there are lengths between .999 m and 1.000 m discernible to the naked eye all of which the Sears® laser ruler "collapses" to either .999 m or 1.000 m. On such occasions, one is uncomfortable that the measurement set-up has been less than ideal. Without a particular instrument in hand, one assumes and is taken to assume that the instrumentation for the measurement reported is equal to perceptual resolution (*v. Standard Precision* (214)). But given the perceptual, computational and physical limits of agent and instrument, measurement at time t_0 is always imprecise with finite resolution that "collapses" the measurement of quantities it cannot resolve. Language calibrates its scalar entailments and implicatures to resolution, exempting anyone who utters or thinks (161) from any thoughts such as (163) about resolutions that are too high. That is, any quantities that the measurement set-up cannot resolve from the measured quantity fall within its margin of error, subject to *Original Error* (58) and *Original μ-Error* (63).

A measurement set-up $\mathfrak{C}$ conforms to *standard precision* just in case for any quantities (*e.g.*, the heights of two Giza pyramids) that are discerned at $\mathfrak{C}$ to be distinct, $\mathfrak{C}$ has the instrumentation to measure them as such:

(214) (*Standard Precision*) standard precision($\mathfrak{C}$) $\leftrightarrow_{\text{def}}$

$$(\text{Known}_{\mathfrak{C}} \vdash (\exists e_0\, \text{quantity}(e_0, q_0) \;\&\; \exists e_1\, \text{quantity}(e_1, q_1) \;\&\; q_0 \neq q_1)) \rightarrow$$
$$\exists n_0 \exists n_1 \exists e_0 \exists e_1 \exists \mu_{e0} \exists \mu_{e1} (n_0 \neq n_1 \;\&$$
$$\text{measure}_{\mathfrak{c}}(e_0) \;\&\; \text{instrument}_{\mathfrak{c}}(e_0, \mu_e) \;\&\; \text{quantity}(e_0, q_0) \;\&\; \text{collapse-into}(e_0, n_0) \;\&$$
$$\text{measure}_{\mathfrak{c}}(e_1) \;\&\; \text{instrument}_{\mathfrak{c}}(e_1, \mu_e) \;\&\; \text{quantity}(e_1, q_1) \;\&\; \text{collapse-into}(e_1, n_1))$$

Relative to any measurement set-up $\mathfrak{C}$, a particular measurement is a (disappointing) *approximation* if its margin-of-error spreads over what other measurements under that set-up could resolve, often prompting a do-over:[36]

(215) (*Approximation*) approximate$_{\mathfrak{c}}$(e) $\leftrightarrow_{\text{def}}$

$$\text{measure}_{\mathfrak{c}}(e) \;\&\; \exists n_0 \exists n_1 \exists e_0 \exists e_1 (n_0 \neq n_1 \;\&\; \text{margin-of-error}_e(n_0) \;\&\; \text{margin-of-error}_e(n_1) \;\&$$
$$\text{measure}_{\mathfrak{c}}(e_0) \;\&\; \text{collapse-into}(e_0, n_0) \;\&\; \text{measure}_{\mathfrak{c}}(e_1) \;\&\; \text{collapse-into}(e_1, n_1))$$

Utterances of (3) or (156)-(157) do describe measurement events entailing no relief from ignorance about values within their margins of error, as *Original Error* (58), *Original μ-Error* (63), and *Accuracy* (68) say they do. But, the utterances themselves merely *implicate* that the utterer does not have within reach even better, different measurements to further probe the margins of error of the measurements under report (*v.* n. 16). With any of these utterances, the speaker lets on that she has certain measurements in hand with their understood margins of error (as Spock did (64)-(66), (70)-(72)). She implicates, the *best-measurement* implicature (§2), that she has nothing better under the assumption (*Standard Instrumentation*) that she has selected the best instrument from among those the set-up provides and is thus unable to fine tune to anything m within the margin of error for the measurements e reported:

(216) (*Standard Instrumentation*)

$$(\text{Known}_{\mathfrak{c}+e} \vdash (\text{measure}_{\mathfrak{c}}(e) \;\&\; \text{quantity}_{\mathfrak{c}}(e, q) \;\&\; \text{margin-of-error}_e(m))) \rightarrow$$
$$\neg \exists e \exists \mu_e (\text{measure}_{\mathfrak{c}}(e) \;\&\; \text{instrument}_{\mathfrak{c}}(e, \mu_e) \;\&\; \text{quantity}_{\mathfrak{c}}(e, q) \;\&\; \text{collapse-into}(e, m))$$

If she could have done better, she would have, to have been more informative.

Measurement is "for the use of the industrial designer, the navigator, and …the traveler, to plan and direct action" (van Fraassen (2008: 76)). Such plans for action are thwarted unless repeated measurement of the same quantity by the same instrument and protocol μ_e measures it the same, repeatedly reading out an n that can be the same despite ambient, random noise or error:

(217) (*Calibration*) calibrated(μ_e, $\mathfrak{C}$) $\leftrightarrow_{\text{def}}$
(measure$_{\mathfrak{C}}$(e_0) & instrument(e_0, μ_e) & quantity(e_0, q) &
measure$_{\mathfrak{C}}$(e_1) & instrument(e_1, μ_e) & quantity(e_1, q))
$\rightarrow \exists n$(collapse-into(e_0, n) & collapse-into(e_1, n))

Measure twice—get the same answer—cut. If the Sears® laser ruler first measures the mètre des Archives to be 1.000m, collapse-into(e_0, 1.000), and later to be 1.001m, collapse-into(e_1, 1.001), it is a different instrument in need of re-calibration, or Eudora is unsteady and needs a tripod to create a different measurement set-up $\mathfrak{C}'$, or a different quantity is measured, perhaps after an uncontrolled temperature increase lengthens the platinum bar.

Set-ups more complex deploy multiple instruments with dedicated ranges, perhaps for cross-correction or redundancy. An instrument μ_e may be calibrated at t for a range that includes m, so that if there is a quantity q that is within range at t and measures m, μ_e detects q and measures it m:

(218) (*Instrument Detection Range*) "m is in μ_e's range at time t in set-up $\mathfrak{C}$".[37]
range$_{\mathfrak{C}}$ (μ_e, t, m) $\leftrightarrow_{def}$ calibrated(μ_e, $\mathfrak{C}$) & $\forall q \forall e' \forall \mathfrak{C}'$(
(time(e') = t & measure$_{\mathfrak{C}'}$(e') & quantity$_{\mathfrak{C}'}$(e', q) & collapse-into(e', m)) $\rightarrow$
$\exists e$(time(e)=t & measure$_{\mathfrak{C}}$(e) & instrument$_{\mathfrak{C}}$(e, μ_e) & quantity$_{\mathfrak{C}}$(e, q) & collapse-into(e, m)))

Despite the weak transponder transmission that puts Tove somewhere unknown within New York State (219), its margin of error, the race course may be seeded with checkpoints, other instruments, such that it may be inferred from their silence that Tove was not at the time of transmission at any of the checkpoints scattered throughout the state, and therefore still quite far from where the race crosses into Vermont:

(219) (=(131)) Tove has checked in in New York State.
(220) (*Silent Detection* at m by μ_e)
(time(e)=t & measure$_{\mathfrak{C}}$(e) & quantity$_{\mathfrak{C}}$(e, q) & margin-of-error$_e$(m))) $\rightarrow$
(($\exists \mu_e$ range$_{\mathfrak{C}}$(μ_e, t, m) & $\neg\exists e$(time(e)=t & measure$_{\mathfrak{C}}$(e) & collapse-into(e, m))) $\rightarrow$
$\neg\exists \mathfrak{C} \exists e$(measure$_{\mathfrak{C}}$($e$) & quantity$_{\mathfrak{C}}$($e$, q) & collapse-into(e, m))))

Thus, despite a measurement with a state-sized margin of error, if one knows of other instruments, a narrower margin of error can be inferred.

7. *Measurement afoot.*

In reporting the progress of a cross-country orienteering race in which the 18th checkpoint is at mile 248 at the New York State border and the 48th checkpoint is at mile 592 at the Vermont border, (221)-(226) all locate a quantity within or at a neighborhood, using prepositions that describe a bounded region (Corver & Zwarts 2006). Sentences (221)-(223) describe a small neighborhood *at the New York State border,* AT *248 miles, at 18* *checkpoints*, and (224)-(226) describe a large one, *in New York State, between 248 and 592 miles, from 18 to 48 checkpoints*:

(221) Tove has checked in at the New York State border.
(222) Tove has checked in 248 miles from DC.
(223) Tove has checked in at 18 checkpoints.

(224) Tove has checked in in New York State.
(225) Tove has checked in between 248 and 592 miles from DC.
(226) Tove has checked in at from 18 to 48 checkpoints.

Sentences (227)-(232) describe unbounded regions that extend past the neighborhoods of the named landmarks, regions reaching the finish line in Sherbrooke, *P.Q.* Sentences (227)-(229) describe the region past the small neighborhood at the New York State border, and (230)-(232) describe the region past the large neighborhood that New York State landmarks:

(227) Tove has checked in past the New York State border.
(228) Tove has checked in more than 248 miles from DC.
(229) Tove has checked in at more than 18 checkpoints.

(230) Tove has checked in past New York State.
(231) Tove has checked in more than between 248 and 592 miles from DC.
(232) Tove has checked in at more than 18 to 48 checkpoints.

Sentences vary in their implicatures about Tove's locations *en route*, where Nyack, NY is *not* at the New York State border but ten miles inside the state, Lake Champlain is the furthest point within New York State just prior to the Vermont border, and Stowe, Vermont is deep in the Green Mountains:

(233) NJ|Palisades, NY ➢ Nyack ➢...➢ Lake Champlain|VT ➢...➢ Stowe ➢ NNE

What tells apart bare measure *248 miles* and comparative *more than 248 miles*, Fox & Hackl's (2006)'s initial worry, is what tells apart *at the New York State border* and *past the New York State border*, or *at the border checkpoint* and *past the border checkpoint.* From the meaning of 'measure$_{\mathfrak{c}}$(*e*)' and knowledge of measurement and error, it is known that any measurement comes with a margin of error (*Original Error* (58), *Original μ-Error* (63)). In this orienteering race, the check-ins measure current position:

(234) $\vdash$ check-in(*e*) $\rightarrow$ measure$_{\mathfrak{c}}$(*e*)

Sentences (221)-(232) report events and current position in physical space rather than metric space; but, given (234) and (183), they now also describe margins of error in the measurement of Tove's position in the race. Pragmatics or practical reasoning next joins the syntax and semantics of the language of measurement (§§2-5) and the knowledge of measurement and error (§6) to derive (221)-(232)'s implicatures.

Out of the blue, (221)-(226) assert that Tove is checked in within the neighborhood. They implicate that Tove has not yet checked in in Vermont, outside both small and large neighborhood:

(235) (221)-(226) ⊩ Tove has not checked in in Vermont.

But, neighborhood size matters. In contrast to (224)-(226) which do not implicate it, (221)-(223) implicate that Tove is neither in Nyack nor at Lake Champlain, outside the small neighborhood but still within the larger neighborhood at opposite ends:[38]

(236) (221)-(224) ⊩ Tove has not checked in Nyack, NY.
(237) (221)-(224) ⊩ Tove has not checked in at Lake Champlain.

(238) (224)-(226) ⊮ Tove has not checked in Nyack, NY.
(239) (224)-(226) ⊮ Tove has not checked in at Lake Champlain.

As far as (224)-(226) go, it is enough that all New York State is within the margin of error to disarm an implicature that Tove is not checked in at its far end, at Lake Champlain on the Vermont border. New York State's location within the margin of error follows directly from the semantics for (224)-(226), (234) and *Original Error*. It remains however that (224)-(226) implicate (235) that Tove has not yet checked in beyond wherein she is said to be, not yet in Vermont.

In contrast to (221)-(226), (227)-(232) all locate a quantity past a neighborhood, using prepositional phrases describing an unbounded region (Corver & Zwarts 2006). The unbounded regions, past the New York State border and past New York State reach to the finish line in Sherbrooke, PQ. But, (227)-(229) describing it as past the small neighborhood of the New York State border sustain an implicature that Tove is not as far from the border as Lake Champlain or Vermont, despite their location within the margin of error, a region now unbounded:

(240) (227)-(229) ⊩ Tove has not checked in at Lake Champlain.
(241) (227)-(229) ⊩ Tove has not checked in in Vermont.

Lake Champlain and Vermont are within the margin of error, and (227)-(229) report measurement events whose ignorance of Tove's location anywhere within their margin of error is absolute. For any location between the New York State order and Sherbrooke, PQ, how could uttering (227)-(229) implicate that Tove has not checked in there? The default *Standard Precision* (214) slaves the distribution of measurement instruments across the range from DC to Sherbrooke, PQ to the perspective and resolution of the given measurement set-up. Suppose any of (227)-(229) is uttered in report of a measurement for which a transient glitch in transponder or tracking equipment has amplified the margin of error. Conditions prior to this measurement, such as Tove's last known location, that are presumed to be the background and warrant for measurement with the New York State border in its range, sustain an inference, given *Standard Instrumentation* (216), *Silent Detection* (220), and *Standard Precision* (214), that although Tove is now lost with whereabouts unknown past the New York State border, she is still not at Lake Champlain or in Vermont, an order of magnitude away for this measurement. Thus, both (221)-(224) and (227)-(229), the latter by a more crooked path, arrive at implicating that Tove was neither at Lake Champlain ((237),(240)) nor in Vermont ((235),(241)). The reasoning is however fully general. It characterizes for any location *m*, properly within a margin of error of arbitrary size or outside it, how it is implicated that the measured quantity is *not* at that location *m* despite measurements and reports that declare the executed measurements' ignorance of the very point implicated. Given *Standard Instrumentation* (216) and *Silent Detection* (220), if the quantity were at *m*, then *other* measurements would have indicated as much, and if it is assumed that no such other measurements have been obtained, it is implicated that the quantity is indeed not at *m.* For the sake of concreteness, three kinds of silent detection or its absence might be distinguished. First, there are the smallest margins of error, where because of error intrinsic to the measurement set-up, its instrumentation and protocol, it is known *a priori* that there is no detection of quantities within the margin of error. Second, there are complex measurement set-ups and their scattered instrumentation for scattered detection, from which it can be inferred from their silence that the target quantity is not within any of the regions so monitored. This measurement is ignorant of Tove's position within New York State, but since she is not yet checked-in at the next checkpoint, it is still implicated that she is not yet there. The third kind, let's call silent detection by order of magnitude. The measurement set-up for the orienteering race does not include flight altimeters for sudden ascents to 36,000 feet nor instrumentation that includes within its range locations on the St. Lawrence River, and yet it is implicated that Tove has not checked in at 36,000 feet above the St. Lawrence River.

What tells apart bare measure *248 miles* (*v.* (221)-(223)) and comparative *more than 248 miles* (*v.* (227)-(229)), Fox & Hackl's (2006)'s worry, are the implicatures denied in (243):

(242) (221)-(223) $\Vdash$ Tove has not checked in in Nyack, NY, *i.e.*,
(221)-(223) $\Vdash$ Tove has not checked in 258 miles from DC.

(243) (227)-(229) $\nVdash$ Tove has not checked in in Nyack, NY, *i.e.*,
(227)-(229) $\nVdash$ Tove has not checked in 258 miles from DC.

In the same measurement set-up and context fixed for (221)-(223) and (227)-(229), the neighborhood of the New York State border, at 248 miles from DC, does not include locations ten miles away. Even so, in contrast to (221)-(223), (227)-(229) report measurements with margins of error that include Nyack, NY, which *prima facie* denies implicature that Tove is not there, which denial is robust given the assumption *Standard*

Instrumentation (216) that the reported measurement is the best available, ruling out more precise reports such as bare measures (221)-(223). Fox & Hackl (2006) mistakenly assume that comparative measures are bereft of all scalar implicatures, but Cummins *et al.* (2012) discover, in effect, that it is still implicated that Tove is not yet across the entire state, for which she would have had to skip detection elsewhere to find herself an order of magnitude beyond the possible distance for an orienteer in the time since her last reported position:

(244) (227)-(229) ⊩ Tove has not checked in at Lake Champlain, *i.e.*,
(227)-(229) ⊩ Tove has not checked in 592 miles from DC.[39]

The order of states or of counties or of any other checkpoints from DC to Québec is neither dense nor a scale, given the uneven distances between state or county line or between arbitrary checkpoints. Yet, (245) and (246)-(247) (=(224)-(226)) describing the same New York segment of the race have implicated that she is not yet as far as Vermont or Québec. Nothing is said about her position within New York State, and, in fact, so blurred has our surveillance been that she might have just crossed into New York or be about to cross into Québec. Similarly, at a finer resolution, (248)-(250) implicate that she is not yet as far as Orange County, NY, without a fix on her position within Rockland County:

(245) Tove has checked in in New York State.
(246) Tove has checked in 248 and 569 miles from DC.
(247) Tove has checked in at between 18 and 48 checkpoints.

(248) Tove has checked in in Rockland County.
(249) Tove has checked in between 248 and 276 miles from DC.
(250) Tove has checked in at between 18 and 22 checkpoints.

Sentences (230)-(232) and (245)-(250) refer to extended regions that intersect extended stretches of any route through them within which the orienteer's position on her route is uncertain. As a measure of her progress, the regions referred to are, in effect, their own margins of error. In (221), in contrast, the location named, the New York State border, intersects the route at a point, and thus, (221) resembles sentences from the elementary language of measurement (161) and *Tove is 248 miles into the race* in referring to a point which a margin of error envelopes. Of the convenience stores and gas stations within a stone's throw of the state line, (221) hardly means that Tove was straddling the border and not transponding her check-in from one of those, although the sentence itself does not identify the neighborhood, the intended margin of error, for which the speaker and hearer must rely on custom. An order of magnitude between locandum—a body— and location—a county or state, as in (245)-(250), the sentences with comparative measures and prepositional numerals, signals an *approximation* (215) that disappoints *the standard precision* (214) expected at the given resolution for tracking and broadcasting a race, even from drone or helicopter.

In reportage covering an orienteering race, *at the New York State border* is a bare measure like *248 miles*, and *past the New York State border*, a comparative measure like *more than 248 miles.* Outside such contexts, mere spatial location is not measurement, and here *at the New York State border* and *past the New York State border* are unmetrized measurements of position in an empirical relation, 'x_1 is farther in the race than x_0'. A race provides the structure, a weak ordering, in which spatial location becomes measurement. It makes sense to ask of Tove whether she is farther along now *at the New York state border* than she was yesterday *at Independence Hall, Philadelphia*. No numbers, so what? Eudora presented with a desk-top replica of the Parthenon is set to scale it up tenfold in styrofoam. Being the very model of a modern middle-schooler, she knows all about √2 and π and realizes that she would need to do better for the pieces to fit than the 1.41 and 3.14 she remembers, but she is deprived of calculator, formula, or tables to calculate it. Instead she has a drafting compass and unmarked straight edge, scissors, and dental floss. Eudora is now Eudoxus of Cnidus (390-337 BCE) (*v.* §1). Measuring the desk-top Parthenon and then measuring styrofoam, twice to cut, are a series of measurements with outcomes that do not represent quantity—lengths, areas, volumes— in the language of real numbers but in a *mixed* language of primitive, geometric magnitudes—lengths, areas, volumes— a relation of proportion between them (ratio), and natural numbers. In this geometrical language of measurement, Eudora logs measurement results, *e.g.,* what circumferences are greater than what lengths, which areas are equal, whether or not a styrofoam column hits its mark at tenfold the volume of a column in the desk-top Parthenon. Tove, the orienteer, can join Eudora in the historical moment. Orienteering is a map & magnetic compass race, in which the orienteers are provided a topographical map of the course. For Tove's race, remove all indications of scale from the topographical map and make it a mountain bike race without bike computer so that Tove cannot count her paces. She gets a fix on her position by visual triangulation between sighted, charted landmarks. Her thought that she is after a full hour only halfway between two checkpoints measures large real-world distances, for which the representation in her estimation of those distances are just small geometric lengths on the topographical map. A meaner race hands her a map projection that distorts distance. For a homespun example, to gift a Christmas tree, I need to measure its girth to measure the wrapping paper. I hug it and then translating circumference to linear length, I lie down on the wrapping paper with arms outstretched. What numbers here? –unless it is just '1' in the event I need to wrap tree clones and need multiple lengths of wrapping paper in Christmas-tree-girth units. A *proper* subset of measurement corresponds to measure functions into the real numbers, but measuring the earth, geo-metry, has once upon a time been geometrical and remains so in some live measurement set-ups. If Hippocrates, Eudoxus, Tove, Eudora and I think of all our methods as measurement and our language reflects as much, its syntax, semantics and pragmatics ought to be equally comprehensive (*v.* §1; nn. 4-6).

8. *Negated measurement*
The mètre des Archives is not 1.002 m, and the Sears® laser ruler did not and does not measure it so:

(251) The mètre des Archives did not measure 1.002 m.
(252) The mètre des Archives does not measure 1.002 m.

Despite the instrument's known precision ±1 mm, neither (251) nor (252) implicates a measurement of 1.001 m. For a given quantity, measurements of it that "collapse into" or read out one number do not entail a measurement of it that would "collapse into" or read out any other number; nor does failure to measure and read out one number entail failure at any other. Thus, it would be neither more nor less informative to say rather than (251)-(252) that the mètre des Archives measures not 1.001 m or not 1.003 m. Negated measurement (251)-(252) bears no scalar implicatures.

Negated measurement reports that describe margins of error that are bounded, given locative prepositions describing bounded regions (AT *x*, *at x*, *amid x, in x, etc.*) (Corver & Zwarts (2006)) similarly undo scalar implicature. Absent orienteering race in progress, denials of spatial location within a bounded neighborhood engage no implicatures with other bounded neighborhoods:

(253) Tove has not checked in in New York.
(254) Tove has not checked in in New Jersey.
(255) Tove has checked in in New Jersey.

Having not checked in in New York neither implicates nor is implicated by having not checked in in New Jersey. On the other hand, for an orienteering race in progress, subject to incoming, updated, cumulative progress reports (with Standard Precision), (254) implicates under such measurement$_{\mathfrak{c}}$ conditions that Tove has not yet checked in in New York, but (253) does not implicate that she has not checked in in New Jersey. That is, if (254) were known, it would be more informative than (253)'s assertion. It is therefore implicated that (254) is not known, an ignorance implicature. To be ignorant of (254), it has to have been either already reported or not yet reported that Tove has checked in in New Jersey, which does not implicate (255) that she in fact has, a scalar implicature undone. When point or line rather than region landmarks the margin of error, the same reasoning applies *mutatis mutandis* to excuse the negated measurement reports in (256)-(259) from a scalar implicature that Tove has checked-in in the immediately preceding neighborhood$_{\mathfrak{c}}$ or at the immediately preceding checkpoint:

(256) Tove has not checked in at the New York State border.
(257) Tove has not checked in 248 miles from DC.
(258) Tove has not checked in at 18 checkpoints.
(259) (256)-(258) $\nVdash$ Tove has not checked in at 17 checkpoints.

Negated measurement reports that describe unbounded margins of error, with locative prepositions or prepositional numerals (Corver & Zwarts (2006)) describing unbounded regions (*past x*, *above x, more..than, etc.*), (260) are also excused from scalar implicatures (261), by a different reasoning:

(260) Tove has not checked in past/north of New York.
(261) (260) $\nVdash$ Tove has checked in past/north of New Jersey.
(262) Tove has not checked in past/north of New Jersey.

Since for fixed direction, to be not past/north of New Jersey *is* to be not past/north of New York and the reverse not true, (262) is strictly more informative than (260). No matter the protocol for measurement$\mathfrak{c}$, any assertion of (260) therefore implicates that (262) is not known. So, either it is not known whether Tove has checked in past/north of New Jersey, or it is implicated that she has. But, if it is known that she has checked in past/north of New Jersey and that she has not checked in past/north of New York (as per (260)), it is known that she has checked in *in* New York. Note that to report position *in* New York would have been strictly more informative than (260)'s report that she is *not past/north of* New York. If her being in New York were known, it should and would have been reported as such, and so (260) also implicates that it is not known that she was in New York and therefore *not* known that she was past/north of New Jersey, vacating a scalar implicature from (260) to (261), as desired. Again, when point or line rather than region landmarks the margin of error, the same reasoning applies *mutatis mutandis* to excuse the negated measurement reports in (263)-(265) from a scalar implicature that she was past the George Washington Bridge, a checkpoint south, before the New York-New Jersey border:

(263) Tove has not checked in past/north of the New York State border.
(264) Tove has not checked in more than 248 miles from DC.
(265) Tove has not checked in at more than 18 checkpoints.
(266) (263)-(265) $\nVdash$ Tove has checked in past/north of the George Washington Bridge.
(267) Tove has not checked in past/north of the George Washington Bridge.

Assertion of (263) implicates that (267) is not known. So, either it is not known whether Tove has checked in past/north of the George Washington Bridge, or it is implicated that she he. But, if it is known that she has checked in past/north of the George Washington Bridge and that she has not checked in past/north of the New York State border (as per (263)), it is known that she has checked in *between* the George Washington Bridge and the New York State border, which, if it were known, should have and would have been reported as such, implicating that it is not known that she was between bridge and border and therefore *not* known that she was past/north of the George Washington Bridge, again vacating as desired a scalar implicature from (263) to (266).

The reasoning above passes between sentences with locative prepositions and prepositional numerals describing unbounded regions and sentences with those describing bounded regions. In measurement reports, margins of error all, unbounded or bounded, are thus described. One proffers the best measurement one can at the moment, which implicates that one does not know of a better or more informative one. That is, one knows of no measurement that narrows better the margin of error, whatever the words to describe it. *In New York* and *between the George Washington Bridge and the New York State border* describe narrower margins of error than *not past/north of New York* and *not past/north of the New York State border*. Thus when (260) measures progress, it is indeed implicated that Tove is not known to be *in* New York. When it is (263), it is implicated that she is not known to be *between* the George Washington Bridge and the New York State border. If a formal calculus for (260)'s and (263)'s scalar implicatures confines itself to substitutions for '$\boldsymbol{\alpha}$' in (268), placing (269) and (270) *hors de combat*, so much the worse for the formal calculus:

(268) Tove has not checked in past/north of $\boldsymbol{\alpha}$.
(269) Tove has checked in in New York.
(270) Tove has checked in between the George Washington Bridge and the New York State border.[40]

9. *Measureability open and closed*

Recall (§0) that Fox & Hackl (2006) appeal to the density of time to burden the comparative measure in (273) with an open interval with no first moment, in order to excuse it from the unwanted implicature, in contrast to the bare measure in (271):

(271) Sandy Koufax pitched 103 minutes.
(272) (271) ⊩ Sandy Koufax did not pitch 104 minutes.

(273) Sandy Koufax pitched more than 102 minutes.
(274) (273) ⊮ Sandy Koufax did not pitch more than103 minutes.

But, lengths, areas, volumes, weights, and velocities and their continuous real-number measurement are, in fact, blind to any difference between the open and the closed. The time that Sandy Koufax pitched has the same duration with or without first or last moments. Lengths, areas, volumes, weights, and velocities are measured, compared, and multiplied in language fluent in the measurement of the open and closed alike, as the next section illustrates. And, it turns out that a missing first moment does not explain the contrast between (271) and (273). A new (whimsical) counterexample to Fox & Hackl (2006) is constructed in the subsequent section, in which among the dense quantities measured, heights in this case, there *is* a largest *h* such that the heights presented are more than, which denies Fox & Hackl (2006) their explanation for the comparative measure continuing to differ from bare measure in scalar implicature. If density has no hand even in the contrast between bare and comparative dense measure, there is no reason to extend it *via* the Universal Density of Measurement to the bare and comparative discrete measures in (275)-(277):

(275) Sandy Koufax pitched 113 pitches.

(276) (275) $\Vdash$ Sandy Koufax did not pitch 114 pitches.

(277) Sandy Koufax pitched more than 112 pitches.
(278) (277) $\nVdash$ Sandy Koufax did not pitch more than 113 pitches.

Open & closed intervals. Before renovation, the Coney Island Cyclone, a roller coaster, restricted its 24 riders (in three cars of eight) to those above 50 lbs. and below 150 lbs., with a maximal load to remain below 2400 lbs. Renovation doubled the range so that it was restricted to those above 50 lbs. and below 250 lbs., with a maximal load to remain below 4800 lbs. Or, was it that the old Cyclone had a minimum of 50 lbs. and a maximum of 150 lbs. per rider, with a maximal load of 2400 lbs. and the new Cyclone, a different maximum per rider of 250 lbs. and maximal load of 4800 lbs.? Either way, although the fate of 50-pounders and 250-pounders sits in the balance, renovation doubled the Cyclone's range and maximal load. Next, in a high security zone with a posted speed limit of 60 mph, one vehicle accelerating from 0 mph to120 mph in 4 minutes trips a security alarm in 2 minutes, and another accelerating to 120 mph in 2 minutes trips it in 1 minute. Or, did the signpost warn that vehicles at 60 mph and above would trip the alarm? Either way, the first vehicle trips it in 2 minutes and the second, in 1 minute, half the time. Also, a point light source is visible for 1 mile to the naked eye, and another that quadruples its light output doubles its visibility to 2 miles. The space the first light source illuminates is a sphere of $^4/_3\,\pi$ cubic miles, and the second light source illuminates a sphere of $10\ ^2/_3\,\pi$ cubic miles. It matters for none of these measurements whether the radial lengths are maxima for visibility or minima for invisibility. *Because,* if the closed unit interval [0,1] is of length 1 and its endpoints of length zero (279), by the additivity of length (280), the open interval (0,1) is also of length 1 (281), and similarly for other dimensions:

(279) $\mu([0,1]) = 1.\ \mu(0) = \mu(1) = 0.$
(280) $\mu([0,1]) = \mu(0) + \mu((0,1)) + \mu(1)$
(281) $\mu((0,1)) = 1.$

Ranges, loads and visibility are doubled, trip times halved, and volumes made eight-fold, and none of the measurements is lost measuring open intervals, areas or volumes. In measure theory, Lebesgue measure generalizes measure to arbitrary subsets of n-dimensional Euclidean space so as to coincide with the standard measures, as exemplified in the measure of the open and closed unit interval. Lebesgue measure may however be too conservative for perception and cognition and for the natural language of measurement if, for example, six points (a finite subset, Lebesgue measure 0) in coordinated motion are perceived as a 1 cubic-inch cube (Johansson 1971, 1975). Whether or not there is this urge to project as a volume just about any scattered things in coordinated motion, a merely open texture or porous surface does not do much to derail ordinary talk about measurement at Coney Island or elsewhere.

Alongside measurement of what there is, ordinary talk of measurement comes to include measurement of what there isn't, when what there is faces limits—maximal loads, maximal speeds, finite fuel supplies, etc. The new Coney Island Cyclone may have a maximal capacity of 24 riders and 4800 lbs; but, it operates with a safety margin, so that:

(282) The Coney Island Cyclone carries 4500 lbs.
(283) The Coney Island Cyclone carries 20 riders.

Then, in answer to how much of its capacity it doesn't use:

(284) The Coney Island Cyclone doesn't carry 300 lbs, in the interest of safety.
(285) The Coney Island Cyclone doesn't carry 4 riders, in the interest of safety.

If an onboard battery is fully charged at 4800 kilowatt hours for a run at its maximal load of 4800 lbs., operating at safe limits:

(286) The Coney Island Cyclone consumes 4500 kwH.
(287) The Coney Island Cyclone doesn't consume—300 kwH.

Revisiting the universal density of measurement (*Fox & Hackl 2006*). The basic contrast is that bare measures license scalar implicatures that comparative measures do not (Krifka 1999):

(288) Dolph Schayes is 201cm.
(289) (288) $\Vdash$ Dolph Schayes is not 202 cm.

(290) Dolph Schayes is more than 200 cm.
(291) a. (290) $\nVdash$ Dolph Schayes is not more than 201 cm.
b. (290) $\nVdash$ Dolph Schayes is not 202 cm.

Fox & Hackl's (2006) favored account for the missing implicatures (291) relies on the failure of the definite description "the largest height *h* that Dolph Schayes is more than" to refer. If Schayes is 201 cm, there is no such largest *h*, since there is always a larger *h* on asymptotic approach from 200 to 201. Their alternative account, reviewed below, relies on density in essentially the same way. But, Krifka (1999) objects that the reasoning about density that exempts the comparative measure from scalar implicatures mistakenly commits the superlative *at least 201cm* to implicating *exactly 201cm*:

(292) Dolph Schayes is at least 201 cm.
(293) a. (292) $\nVdash$ Dolph Schayes is not more than 201 cm.
b. (292) $\nVdash$ Dolph Schayes is exactly 201 cm.

Despite the now closed interval of heights of at least 201cm, the superlative measure *at least 201cm* is still denied scalar implicatures, just like the comparative measure *more than*

200 cm. It should have been the end of the Universal Density of Measurement; but, Fox & Hackl (2006) elude the objection from superlative measures,[41] so that the present exercise is to reformulate it for the comparative measure itself. It takes an angelic basketball league, a heavenly host as dense as the real numbers, where every positive, real-value increment ε has a Cherub exactly $200+\varepsilon$ cm tall. *The Cherubim are more than 200cm* still does not implicate that they are not more than 201cm—comparative measures never do. But, now there is a largest height, *viz.*, 200cm, that the Cherubim are more than. As detailed below, Fox & Hackl's (2006) accounts fail to explain then why this example is relieved of scalar implicatures just the same. Accounts based on density, and open *vs*. closed intervals, fail even for dense comparative measures such as *more than 200cm*.

The Cherubim, for whom Dolph Schayes plays, have a minimum height that players must strictly exceed:

(294) The Cherubim are (on average) 201 cm.
(295) (294) ⊩ The Cherubim are not (on average) 202 cm.
(296) The Cherubim are (each) more than 200 cm (by league rules).
(297) (296) ⊮ The Cherubim are not (each) more than 202 cm (by league rules).

(290)'s *ignorance* implicatures that for every real number $\varepsilon > 0$, it is not known that Dolph Schayes is more than $200+\varepsilon$ cm cannot become the *scalar* implicatures, among which is (291)a., that for every real number $\varepsilon > 0$, Dolph Schayes is not more than $200+\varepsilon$ cm. As Fox & Hackl (2006: 542) remark, the class of these scalar implicatures is inconsistent with (290)'s assertion that for some $\varepsilon > 0$, Dolph Schayes is indeed $200+\varepsilon$ cm. Such reasoning to explain the failed scalar implicatures does not however extend to the Cherubim. Lost in the hierarchy of angels, suppose the height minimum for the Cherubim is forgotten. Yet aware that Dolph Schayes plays for them, (296) is correctly inferred from an estimate of his height, accompanied with ignorance implicatures that for every real number $\varepsilon > 0$, it is not known that the Cherubim are (each) more than $200+\varepsilon$ cm. But, here the corresponding scalar implicatures are not inconsistent with (296)'s assertion. If, indeed, the league minimum is 200 cm, then for every real number $\varepsilon > 0$, there will a Cherub with height h cm, $200 < h < 200+\varepsilon$, who validates the scalar implicature that the Cherubim are not (each) more than $200+\varepsilon$ cm. The comparative measure is no more congenial in (296) to scalar implicature than it is in (290), without a forthcoming explanation.

Fox & Hackl (2006: 559) favor and develop an alternative that derives all scalar implicatures from an original implicature that refers by singular definite description to a degree or number. That is,

(298) (288) ⊩ The largest h such that Dolph Schayes is (at least) h cm is 201.
(299) (298) ⊩ (289)

But,

(300) (290) ⊮ The largest *h* such that Dolph Schayes is more than *h* cm is 200.
(301) (300) ⊮ (291)

Since, there isn't a largest height that Dolph Schayes' height is more than, the failure of singular reference in (300) is said to deprive the comparative measure in (290) of all scalar implicatures. This alternative does no better denying scalar implicatures to the comparative measure in (296):

(302) (296) ⊩ The largest *h* such that the Cherubim are (each) more than *h* cm is 200.
(303) (302) ⊩ (297)

The implicature (302) is simply true, given that the minimum height for Cherubim, which they strictly exceed, is 200 cm, and the Cherubim densely throng the heights between 200 cm and that of any one of their number. Even if the ordinary objects instantiating quantities measured in the real numbers— lengths, areas, volumes, weights, durations, *etc*.— are all individuals with closed topologies, plural reference makes it easy enough, since these quantities are as dense as the real numbers, to aggregate them in open topologies, in which each individual length, say, is itself a closed line segment, an angel's height, with length *h* for some real number. For any open interval (m, n), "the largest *h* such that $\forall x(m < x < n \rightarrow h < x)$" names *m*, and "the least *h* such that $\forall x(m < x < n \rightarrow x < h)$" names *n*. For closed intervals $[m, n]$, "the largest *h* such that $\forall x(m \leq x \leq n \rightarrow h < x)$" and "the least *h* such that $\forall x(m \leq x \leq n \rightarrow x < h)$" both fail to refer. The Commissioner might convene the Cherubim under a different rule to recruit 200-cm angels to the league, and then there would not be a largest height that the Cherubim are more than. But, surely the failure of the comparative measure in (296) to license scalar implicature hardly depends on whether the heights of Cherubim are an open or closed interval.

The Cherubim thus defeat both accounts of scalar implicature, the one first mentioned that would derive scalar implicatures directly from ignorance implicatures and this favored last that derives them *via* singular reference to a number or degree. They are defeated when plural reference to Cherubim measurements refers to *open*, dense intervals of them, a neglected case. The Universal Density of Measurement (Fox & Hackl 2006: 542) declares the discrete dense so as to extend these accounts to the discrete where the contrast in scalar implicature persists:

(304) The Universal Density of Measurement (UDM): measurement scales needed for natural language semantics are always dense.

(305) Dolph Schayes started 613 games in the Cherubim.
(306) (305) ⊩ Dolph Schayes did not start 614 games in the Cherubim.
(307) Dolph Schayes started more than 612 games in the Cherubim.
(308) (307) ⊮ Dolph Schayes did not start more than 613 games in the Cherubim.

The Universal Density of Measurement is however to be rejected in rejecting the defective accounts of scalar implicature it is based on,[42] inviting an explanation of the contrast between bare and comparative measures that does without it (*v.* §§2-7).

10. *Conclusion*

To measure is to err. Its natural language is a language of measurement that refers to margins of error. That language embraces numeric measurements but also non-numeric, measurements for orderings dense and discrete. To realize that generality, the language of measurement and error is spatiotemporal language for location in numeric and non-numeric spaces (Corver & Zwarts 2006), calibrating its margins of error to the perspective and resolution of the measurement set-up (§§2-6). Delicate, it is. From the semantics for the language of measurement and error, it is known that any measurement comes with a margin of error. Understanding a measurement report is to understand which measurements are within the margin of error and which not. Such reports and reasoning from them invoke what is known and what is known to be known about measurement subject to error and about the use of spatiotemporal location in measurement, among which are *Original Error* (58), *Original μ-Error* (63), and *Accuracy* (68) and defaults to *Standard Precision* (214), *Standard Instrumentation* (216), and *Silent Detection* (220), and *Standard Precision* (214).

In their semantics and in their license of implicature, there is a contrast between spatial vocabulary (*at the New York State border*, *amid the forest*) describing bounded regions and vocabulary (*past the New York State Border, above the trees*) describing unbounded regions (Corver & Zwarts 2006). The contrast between bare measures (*248 miles*, *18 checkpoints*) and comparative measures (*more than 248 miles*, *more than 18 checkpoints*) follows from their assimilation to spatial vocabulary as prepositional numerals (Corver & Zwarts 2006), in which bare measures describe their margins of error as bounded regions (AT *248 miles,* AT *18 checkpoints*) with help from an unspoken preposition 'AT', and comparative measures with a certain syntax for *-er* describe them as unbounded regions ("above 248 miles", "over18 checkpoints") (§§4,5). A central, unifying observation is that ignorance implicatures about values within the margins of error do not graduate to scalar implicatures; but, ignorance implicatures about values beyond the margins of error do.

With a language for measurement and error comes knowledge thereof (§6), a native theory of measurement, error, measurement set-up, instrumentation and protocol, excerpts from which have been called upon to elucidate the language's lexical semantics. From the language's semantics and the apprentice's mastery of measurement, it is left to pragmatics or practical reasoning to draw out the implicatures of utterances expressed in that language. In poker, what one says, what one is taken to have said, and what one says so as to be taken to have said it is a decision that rests on several competences, several special theories— the rules of the game, a theory of best strategy given the current state of the game, a theory of human gambling behavior, knowledge of prior history with the seated opponents, *etc.* a list to include a theory of measurement practice if a sentence in its language is to be uttered during the game. One among the theories knowledge of which crowd the decision crafting a message is a theory of conversation, maxims for effective and cooperative communication (Grice 1975). With as much in mind, a measurement report with a small margin of error is more informative than comparable report with a larger one that properly contains it. Anyone who says that Tove the orienteer is past the New York State border (*en route* to Canada) knowing that she is still in New York is less informative than she should be.[43] The implicature that the cooperative speaker must not know that Tove is still in New York rests on the knowledge that *in New York* describes the smaller margin of error than *past the New York State border*. As Jim Higginbotham might have said, skeptical of formal pragmatics but never of syntax, semantics or phonology, there is semantic competence and competence in many subjects; the decision what to say at what moment is poker. Yet, despite a poker game always in progress among gamblers using all their guile, it may be, except for some concluding remarks, that there is also a modular formal pragmatics, a pachinko machine in the room to be consulted for its calculation of scalar implicatures. It is equally benign that such a formal pragmatics derive its scalar implicatures, as in Fox & Hackl (2006), all from sentences that token covert singular definite descriptions (their exhaustivity operator, *exh*). But, whether poker, pachinko, or formal pragmatics (*v.* Sauerland (2012) for discussion), the decision— deductive, inductive, abductive, probabilistic or not, open or not to introspection— to bet on one utterance rather than another is a game played on thoughts whole and complete. Under the Universal Density of Measurement (304), the semantic processing of *613 games* pauses blinded to the games discrete and counted and sees instead gaming dense enough to demand measurement by real numbers. Scalar implicatures are registered shielded from an eventual meaning true to the games as they are. A language adapted to the universal delicacy of measurement is spared the universal density of measurement, and it restores semantics and pragmatics to a living arrangement less entwined, and mercifully boring.

The formal pragmatics for scalar implicature humming in Fox & Hackl (2006) "follow[s] Krifka (1999)," as Geurts & Nouwen (2007: 539) put it, "in assuming that [its] scales are built from focus-induced alternatives," where "it is usually assumed that focus on an expression α serves to induce a set of alternatives to α's denotation $[\![\alpha]\!]$ [as in Rooth 1985]." In understanding (309), onions, potatoes, blueberries, and sweetcorn are a set of alternatives in the greengrocer's inventory, which does not include poodles until meat is a vegetable and poodle, a meat:

(309) In high summer, the greengrocer doesn't showcase *onions* and *potatoes*.
$\Vdash$ In high summer, the greengrocer showcases *blueberries* and *sweetcorn*.
$\nVdash$ In high summer, the greengrocer showcases *poodles*.

What is or is not understood to have been implicated owes something to knowledge of the marketplace, without assuming that the greengrocer's inventory is built from focus-induced alternatives. Yet, formal pragmatics gazes at the sextants, astrolabes, compasses and time pieces in the British Museum and beholds scales built from focus-based alternatives, the tail wagging the dog when it comes to scalar implicature. Whether it is poker, produce, good parenting, or measurement, it is not the view here that a formal pragmatics privileges any of it—implicatures about poker are about hands and stakes, those about produce are not about poodles, and those about measurement are about quantities, scales when metrized, and error, it being the special nature of measurement to be about all that.

Pace such reservations about Krifka (1999), let me turn to question whether implicature as it concerns Fox & Hackl (2006) is indeed a game played on sentences that substitute alternatives for the focused expression (Rooth 1985), questioning both whether it is substitutional and whether it is sentential.

The contrast in the implicatures of bare and comparative measures that citation examples such as (310)-(311) illustrate survives just the same when the numeral under focus is replaced with a description in (312)-(313) and similarly in (314)-(315):

(310) Sandy Koufax pitched 113 pitches.
(311) Sandy Koufax pitched more than 113 pitches.

(312) Sandy Koufax pitched the record-breaking number of pitches for a perfect game.
(313) Sandy Koufax pitched more than the record-breaking number of pitches for a perfect game.

(314) From the charges to the cabal's credit card, a whole number multiple of the (number of) pizzas ordered for the last full moon were ordered for this one.
(315) From the charges to the cabal's credit card, more than a whole number multiple of the (number of) pizzas ordered for the last full moon were ordered for this one.

The bare measures implicate that not one more pitch was pitched and not one more pizza ordered than the number described, and the comparative measures do not implicate that it was not one more than one more, despite a forgotten baseball record and secret pizza deliveries. None of this is unexpected when §§2-7 derive the contrast without consideration for numbers other than the number referred to or for how they might be named or described. But, in the formal pragmatics, what are the alternatives that focus-induced scales provide to substitute for the number descriptions in (312)-(315) to generate the sentences that the bare measures but not the comparative measures implicate?

In §§2-7, a lexical semantics and a sub-sentential logical syntax for it, rendering *613 telephones* as "telephones counted AT 613" join what is known about measurement and error. The reasoning that then distinguishes the implicatures of bare and comparative measures proceeds just the same in (316)-(323) despite their collective, non-monotonic predicates:

(316) 613 telephones connect to all and only each other.
(317) More than 613 telephones connect to all and only each other.

(318) Some telephones connect to all and only each other. 613 are in the network.
(319) Some telephones connect to all and only each other. More than 613 are in the network.

(320) Eighteen nations are in a perfect balance of power.
(321) More than eighteen nations are in a perfect balance of power.

(322) Some nations are in a perfect balance of power. Eighteen are in the group.
(323) Some nations are in a perfect balance of power. More than eighteen are in the group.

The telephones counted 613 are not 614, and the nations counted eighteen are not 19. But, the telephones counted more than 613 are not not more than 614, and the nations counted more than eighteen are not not more than 19. It matters little to §§2-7 that what is said is said in one sentence as in (316)-(317) and (320)-(321) or across two as in (318)-(319) and (322)-(323), since semantically the one sentence is in effect a reduced version of the two, and one reasons from a one-sentence paragraph or an equivalent two-sentence paragraph just the same.

The sentential alternatives that formal pragmatics provides to the one-sentence (316)-(317) and (320)-(321) substitute other numerals for α in:

(324) α telephones connect to all and only each other.
(325) More than α telephones connect to all and only each other.

(326) α nations are in a perfect balance of power.
(327) More than α nations are in a perfect balance of power.

There are no entailments among the sentential alternatives for any of (324)-(327), no asymmetries in their informativity and thus nothing forthcoming about the implicatures that distinguish bare and comparative measures in one-sentence and two-sentence variants alike, except perhaps for a Universal Distributivity of Measurement, that measurement domains needed for natural language semantics are always distributive, where one deviation from realism can keep company with another.

Appendix: *Pragmatic slack and rounding*
The essential problem of Krifka-Fox-Hackl implicature is that (§1) there are quantities *q* discrete and dense about which equative but not comparative language implicates not more than *q*. It subsumes reasoning directly in Sassoon's (2010) unmetrized language about a relation between empirical magnitudes ('x_0 is more than x_1', '$x_0 \succ x_1$') rather than surrogatively in a numeric language ('$\mu(x_0) > \mu(x_1)$') about numbers. If error and margins

of error have anything to do with it, the relevant concepts of measurement, error, and margins of error had better not be confined to numeric measurement or numerals. If the contrast in implicature between equative and comparative measures survives Spock's exactitude in his use of language (§§2-3), it better not have much to do with an intention to slacken off and speak more loosely. This appendix is an aggressive defense against a violent objection and the confusion that measurement, error, margins of error, and Krifka-Fox-Hackl implicature ought to be about pragmatic slack, rounding, and what there is in them adjacent to error and margins of error. Against objection and confusion, the appendix considers how the syntax and semantics for the language of measurement proposed here would fit in to a treatment of pragmatic slack and rounding when those phenomena, quite separate from Krifka-Fox-Hackl implicature, are eventually taken up.

Any utterance of (i) is said to be (ii) in the explicit language of measurement, with parameters for measurement set-ups and margins of error:

(i) The mètre des Archives is 1 meter.
(ii) The mètre des Archives *measures somewhere within the measurement set-up's margin of error* $\pm k$m *for* 1 meter.

When neither measurement set-up nor margin of error is mentioned or demonstrated, custom and habit intervene with practices that fit measurement set-ups and error tolerances to task and purpose (*v.* §6)—to pace out the area of a room to measure its room capacity for fire and safety codes, or to purchase for it flooring for a biohazard containment lab with pathogen-tight seams—and then if (i) is uttered without qualification it is understood to report a customary measurement for the purpose at hand. What is the distance between New York and Boston if not odometer distance, for any conversation outside the pilot lounge? It does not read 200 miles. "Pragmatic slack", as Lasersohn (1999) aptly calls it, lets the false be true enough, with notice of its deviation from the more accurate measurement that customary practice affords. Lasersohn (1999) happens to agree with Katz (1981) that (iii) is always false (*v.* p. 26 above), because (iii) is taken to assert with absolute precision an absolute falsehood (iv) of an absolute distance, rather than the banal truth (v):

(iii) New York is 200 miles from Boston.
(iv) distance(New York-Boston) = 200 miles.
(v) New York *measures somewhere within the measurement set-up's margin of error* $\pm$ 20 miles 200 miles from Boston.
(vi) New York *measures somewhere within measurement set-up* $\mathfrak{C}$*'s margin of error* $\pm k$ *miles* 200 miles from Boston.

Here, instead of certain falsehood, the logical form of (iii) scandalizes conversation with an embarrassment of truths and falsehoods aligned with different values for the parameters in (vi), which context and conversation may hedge with further description and deferral:

(vii) New York *measures somewhere within the margin of error for* 200 miles *under the measurement set-up customary for present purposes* from Boston.

But, if (iii) is uttered without qualification and (vii) understood, then as fast as one can say "odometer", (iii) conveys a falsehood, unless alert to "pragmatic slack", one is alert to something like (v), report from a different set-up for an occasion when a map and the back of an envelope for lack of better do no harm and may even hasten action.

"Pragmatic slack" (Lasersohn (1999)) and the rounding practices flagging the amount of it (*to the nearest ten miles vs. to the nearest hundred miles, etc.*) have dedicated languages distinct from the language of measurement enlisted in the semantics of bare and comparative measures. Its margins of error are not just incidental to pragmatic slack or to rounding conventions. Pragmatic slack and rounding name behaviors distinct from measurement precision and measurement error. There is however a lawful relation between pragmatic slack and measurement precision sketched below, alongside the logical syntax for these languages, which holds them formally apart. Discussion of rounding and of its language follows, again, to dislodge it from the language of measurement and measurement error. It thus builds a wall around it to show that the language of measurement and measurement error is not reducible to accounts of pragmatic slack or of rounding; and, for good measure, it will be shown that their accounts in the literature are not good for their intended purpose either.

For Lasersohn (1999), an utterance stoops to true enough if it deviates within a *pragmatic halo* of near misses, near enough in ignoring just those details that an elaboration of Grice's (1975) theory of conversation says are safe to ignore on the occasion, even at the cost of not telling the truth (Lasersohn 1999: 525). Since "pragmatic slack" messes with assertion itself, it's good that pragmatic halos are aglow not only above measurement sentences but elsewhere too, when, for example, (ix) is true enough despite a few insomniacs and (x), despite the immaterial existence of perfect spheres:

(viii) Mary arrived at 3 o'clock.
(ix) The townspeople are asleep.
(x) The ball is spherical.

If my rival is truly a frenemy, pragmatic slack allows it to be close enough to the truth for some purposes to sometimes say that he is a friend and other times, that he is an enemy. As Lasersohn says (1999: 533), "pragmatic slack is not the same thing as scalarity or truth-conditional vagueness, but a separate phenomenon over and on top of it," distinguishing "between authentic semantic vagueness, in which the extension of a predicate really does not have well-defined borders [e.g., *bald*, *heap*], and mere pragmatic looseness of speech, in which we allow speakers a certain leeway even in the use of predicates whose extensions are quite sharply defined [e.g., There are 600 beans in the jar] (535)." Lasersohn (1999:535) endorses that standards of precision enter the semantics of predicates like *bald* and *heap*, which give rise to the *sorites* paradox, in order to avert in (xi) the contradiction in (xii), which no amount of pragmatic slack alleviates:

(xi) John is bald with a few hairs behind the ears.
(xii) # John has no hair with some behind the ears.

Lasersohn also recognizes that standards of precision and the extremes of scientific precision are features of measurement *per se* (525). Of course, pragmatic halos are not indifferent to the margins of error that come with measurement. Aglow above a measurement sentence, a pragmatic halo contains the measurement's margin of error and properly contains it when slacking off. A Gricean theory with access to a native theory of measurement will say that it is safe to ignore precision that the measurement in hand is too imprecise to supply anyway. What is known about error and precision and represented in sentences about measurement and what is known about vagueness and represented in sentences with *sorites*-inducing predicates are independent of each other, and also independent of the syntax and semantics of the plural definite description in (x). And, all these questions are both independent and prior to whether or not there is a unified theory of pragmatic slack and pragmatic halos. Suppose there are 613 beans in the jar and 600 townspeople asleep in a town of 613. How much the same is the reasoning that renders it true enough that there are 600 beans in the jar and true enough that the townspeople are asleep? Error, precision and margins of error are intrinsic to measurement, and well-established as such. They enter the language of measurement without detour through semantic or pragmatic commitment to the *true enough*.

In support of Lasersohn's thought that pragmatic slack is indeed a separate phenomenon *over and on top of* the haloed sentence, suppose it time to place a bet on the outcome of a supercomputer's month's work on a prime factorization problem, for which purpose (xiii) has become *true enough*, which (xiv) states explicitly with an adverb that modifies a graded truth predicate, as in (xv) and (xvi), since prime number-hood is even better than death or pregnancy at being ungraded:

(xiii) Your 10^{18}-digit number is prime.
(xiv) Probably, your 10^{18}-digit number is prime.
(xv) It is more likely now than last month that your 10^{18}-digit number is prime.
(xvi) It is truer now than last month that your 10^{18}-digit number is prime, enough to bet your retirement savings on it.

Lasersohn (1999) offers a theory of pragmatic slack regulation and its vocabulary of adverbs— *Yes, New York is 200 miles and*—more precisely, *212 miles from Boston*—which ought to extend to a large class of adverbial constructions, *e.g., to be more precise, speaking loosely, back of the envelope, as best as can be estimated, (if) rounding/rounded to the nearest posted mileage,…etc.*, which could all be construed to shift the pragmatic halo for what counts as true enough, relaxing or tightening the pragmatic slack. The syntax, semantics and pragmatics of these adverbs and adverbial constructions, their relation to evidential and epistemic adverbs, and whether they shift a contextual parameter or occur as adverbs of quantification quantifying into their prejacent clauses are all interesting questions, one aspect of which is relevant.

As there is nothing gradient about being a prime number and no objective probability to it (*cf.* 10^{18} coin tosses all heads), the adverb in (xiv) is plainly epistemic or evidential, applying to some gradient conception of truth, subjective probability, credence or confidence. Unsurprisingly degree of confidence and the margin of error allowed are related by a *law of direct proportionality*, (xviii) *vs.*(xix):

(xvii) a. Probably, your 10^{18}-digit number is prime.
b. There is .85 confidence that your 10^{18}-digit number is prime.

(xviii) a. F Probably, there are 600 beans in the jar to the nearest bean.
b. F There is .85 confidence that there are 600 beans [±1 bean].

(xix) a. T Probably, there are 600 beans in the jar to the nearest hundred beans.
b. T There is .85 confidence that there are 600 beans [±100 bean].

A law is all it is. It cannot be that the syntax, semantics, or pragmatics of the adverb in (xviii)-(xix) differs from its occurrence in (xvii), where it quantifies degrees of confidence—*without* quantifying a margin of error or anything else within the prejacent clause. Likewise, it is known from what is known about measurement and measurement reports that a measurement off by 1/600th is not so loose, and a margin of error as much as 1/6th the reported value is none too precise, from which the infelicity of (xx)-(xxi) is inferred without *loosely* or *precisely* loosening or precisifying a parameter which *[±n beans]* or *to the nearest n beans* has already spoken for:

(xx) # Speaking very loosely, there are 600 beans in the jar [±1 bean]/to the nearest bean.
Loosely, there are 600 beans in the jar [±1 bean]/to the nearest bean.

(xxi) # Reporting with utmost precision, there are 600 beans in the jar [±100 beans]/ to the nearest hundred beans.
Precisely, there are 600 beans in the jar [±100 beans]/ to the nearest hundred beans.

Exaggerating the dissociation, the infelicity of (xxii) is merely inferred from the contradiction that the illocutionary adverbial clause describes a measurement activity that cannot result in the measurement reported, since 1% accuracy is already a margin of error of [±6 beans]:

(xxii) #Measuring more precisely this time than the first method with 10% accuracy and less precisely than the second method with 1% accuracy and double- and triple-checking, there are 600 beans in the jar [±1 bean]/to the nearest bean.

It is a familiar Davidsonian conceit that adverbs *loosely* and *precisely*, and the rest of the adverbs that Lasersohn (1999) deputizes as pragmatic slack operators, are event predicates, 'loose(*e*)' and 'precise(*e*)', that here denote measurement or speech events, and in *fit loosely* and *cut precisely*, events that engage a tailor. There is little except descriptive content to tell apart *loosely* and *precisely* from the more verbose adverbial clauses in (xx)-(xxi) and thus little to the pretense that *loosely* and *precisely* are operators with a syntax, semantics and pragmatics *sui generis* that assimilates neither the more verbose adverbial clauses serving the same function nor the meanings of these morphemes as they occur elsewhere in the language. Such skepticism about pragmatic slack operators (and, their variant semantic operators in subsequent literature) denies the letter of Lasersohn's (1999) proposal but fortifies his wall around the language of measurement and the conclusion that pragmatic slack and its language are a separate problem concerned with epistemic, evidential, and illocutionary adverbial phrases, which relate to a gradient and measured subjective probability, reliability, confidence, or credence in that which the prejacent clause expresses autonomously whether it is about prime numbers or anything else. There is no locus where the language of pragmatic slack and the language of measurement and measurement precision intersect, and no occasion for one to do double duty for the other.

So separated from the language of measurement, the language of pragmatic slack is irrelevant to the questions at issue, as is the existence of pragmatic slack itself, the Austinian behavior that finds convenient what is true enough. The contrast in implicature between bare and comparative measures is immutable under any precision:

The mètre des Archives measures 1.00000… meter.
The mètre des Archives measures more than 1.00000… meter.

If density has anything to do with it as Fox & Hackl (2006) allege, it isn't manifest in Austinian contexts. If pragmatic slack licenses as true enough report and reasoning to the closest minute, then *103 minutes* and *more than 102 minutes* ought equally implicate not 104 minutes:

(3) Sandy Koufax pitched 103 minutes.
(3) ⊩ Sandy Koufax did not pitch 104 minutes.
(7) Sandy Koufax pitched more than 102 minutes.
(7) ⊮ Sandy Koufax did not pitch 104 minutes.

That is, there is a universal *discreteness* to slack measurement that blunts inquiry into the exact meaning of dense measurement and how bare and comparative dense measures differ in their implicatures in all contexts, including the unforgivingly precise.[44] Acknowledging as much, Cummins *et al.* (2012: 161f.) rejoin Fox & Hackl (2006) in their explanation *via* density for when there is no pragmatic slack (*v.* n. 2).

In subsequent literature (Krifka (2009), Sauerland & Stateva (2007, 2011), Cummins *et al.* (2012), Solt (2014, 2015)), pragmatic slack becomes *imprecision* so called, with a focus on numerical rounding careful to distinguish the imprecision that results from it from the imprecision in metrology, as the term is used here. Rounding is notable in two respects:

(xxiii) Rounding to the nearest hundred, there are 600 beans in the jar.
(xxiv) Rounding to the nearest ten, there are 600 beans in the jar.
(xxv) To the bean, there are 600 beans in the jar.

(xxvi) #Rounding to the nearest hundred, there are 613 beans in the jar.
(xxvii) #Rounding to the nearest ten, there are 613 beans in the jar.
(xxviii) To the bean, there are 613 beans in the jar.

(xxix) #Rounding to the nearest three, there are 612 beans in the jar.

Rounding to the nearest hundred, ten, or unit fits decimal numerals, and rounding to the nearest three does not, (xxix). Rounding to the nearest sixteen presumably fits hexadecimal numerals. The first respect in which rounding is notable is that the numerals to which numbers are rounded are not arbitrary, '613', for example, not being among round decimals, (xxvi)-(xxviii) (*v.* Krifka (2009)). The numerical system determines its class of rounding functions, where rounding functions can be either the rounding conventions learned in middle school or the rounding functions that populate software languages such as 'ROUND' in Excel. The second respect in which rounding is notable is the disambiguation problem that occurs when the round numeral exists in the range of multiple rounding functions and (xxx) doesn't say which of (xxiii)-(xxv) is intended:

(xxx) There are 600 beans in the jar.

Similarly, the citation Austinian example (xxxi) is ambiguous among (xxxii)-(xxxiv):

(xxxi) NYC is 200 miles from Boston.
(xxxii) Rounding to the nearest hundred, NYC is 200 miles from Boston.
(xxxiii) Rounding to the nearest ten, NYC is 200 miles from Boston.
(xxxiv) To the mile, NYC is 200 miles from Boston.

Disambiguation engages speaker and hearer in some fancy game theory (Krifka 2009). Granted then is that something matches rounding functions to numerical system and something guides guessing the one intended when none is mentioned, exactly as in Krifka (2009), if you like. Let's turn to the language of rounding in the adverbial modifiers of (xxxii)-(xxxiv):

(xxxv) π rounds to 3.14, to the nearest hundredth.
π is rounded to 3.14, to the nearest hundredth.
ENIAC rounded π to 3.14, to the nearest hundredth.

(xxxvi) Having rounded to the nearest hundredth, 3.14 unrounds to real numbers *n*, $3.135 \leq n < 3.145$.

If (xxxv)-(xxxvi) contain 'round(*e*) & to(*e*,*n*)', it is uncontroversial that the verb denotes processes or computations that are the rounding functions mentioned above, and that the spatial preposition '*to*(*e*,*x*)', denoting paths *e* to destination *x*, denotes here processes or computations *e* ending at *n* in the range of a rounding function. That is, the rounding functions of middle school and code school are at home in the semantics of natural language. Note however that the round numbers in (xxiii)-(xxv) and (xxx)-(xxxiv), although they are the numbers rounded to, are not introduced '*to n*'. Rather, the round number referred to is the outcome of measurement that a rounding-to has antecedently tampered with, as in (xxxvii)-(xxxviii), no different except in its legality from the tampering in (xxxix)-(xl):

(xxxvii) a.i. π is 3.14, (rounded) to the nearest hundredth.
a.ii. π is 3.14, ((after/as a result of) rounding) to the nearest hundredth.
b.i. (Rounded) to the nearest hundredth, π is 3.14.
b.ii. ((After/As a result of) rounding) to the nearest hundredth, π is 3.14.

(xxxviii) a.i. π comes out as 3.14, (rounded) to the nearest hundredth.
a.ii. π comes out as 3.14, ((after/as a result of) rounding) to the nearest hundredth.
b.i. (Rounded) to the nearest hundredth, π comes out as 3.14.
b.ii. ((After/As a result of) rounding) to the nearest hundredth, π comes out as 3.14.

(xxxix) Putting a finger on the scale for foes and under it for friends, this customer's produce weighed 1.8 kg.

(xl) Cooking the books to conceal from the IRS 20% of revenue, this week's receipts total $6,130.

These remarks sketch a take on pragmatic slack and rounding congenial to the views taken up in the text. The first move is to decline anything about *approximately* and *precisely* that will not apply just as well to *now approximating, after quick approximation, more precisely now than before, to a precision never before seen*—just to spin on two examples. It will apply to these just in case a common adverbial syntax and semantics emerges and the morphemes *approximat-* and *precis-* receive a uniform meaning here and throughout the language, which commits in my view to 'approximate(*e*)' and 'precise(*e*)'. The view is that *approximately* and *precisely* are reduced adverbial clauses. When there is pragmatic slack, when rounding is true enough, speakers and hearers saying and hearing (xxxi) are thinking something like (xxxii) or (xxxiii). Formal semanticists would rather drink poison than accept the abstract syntax that assimilates *approximately* or *precisely* to (xxxix)-(xl) let alone a silent adverbial clause as a prefix to (xxxi). I don't have other than a poison chalice to offer if *precisely* and *more precisely now than before* are to belong to the same language, but one need not swallow a silent adverbial clause in (xxxi), too. At issue, after all, is (xxxi)'s pragmatics. It suffices that such sentences are accepted as true enough just in case the common ground contains the understanding that measurements for present purposes are *rounded to the nearest ten,* or *to the nearest hundred.* It suffices that the content of the silent adverbial clause be off-loaded onto a background assumption. Either way, the language of rounding in (xxxv) belongs to the natural language. It appears in adverbial clauses silent or overt and in the language in which background assumptions are stated. The language of rounding in (xxxv) has a semantics that refers to the ordinary rounding functions of middle school or code school, which join error, precision and margins of error, equally familiar, when the language of rounding joins the language of measurement.

The literature on rounding subsequent to Lasersohn (1999) (Krifka (2009), Sauerland & Stateva (2007, 2011), Cummins *et al.* (2012), Solt (2014, 2015)) introduces *granularity functions* into semantic interpretation. As such, the domain of a granularity function is a set of round numerals to be interpreted, *0, 100, 200,*…, if rounding to the nearest hundred, and its range is a set of real-number intervals—$\text{gran}_{100}(\mathit{0}) = [0,50)$, $\text{gran}_{100}(\mathit{100}) = [50, 150)$, $\text{gran}_{100}(\mathit{200}) = [150, 250)$… Granularity functions are inverse rounding functions. Under no regime that I can think of do they replace rounding functions, as these are independently necessary, as noted above, for the language of rounding in (xxxv). But, they are introduced in service of a language that treats *approximately, precisely, etc.* as operators *sui generis* and applies them to sentences containing round numerals. As sketched above, the larger natural language with a semantics in which rounding functions are necessary manages with just these to take on pragmatic slack and rounding, idling granularity functions. There is also a faint resemblance between granularity functions and the margins of error that torment all measurement. At granularity zero, gran_0, there is no rounding and the interpretation of a numeral is a solitary number. Absent rounding, gran_0 doesn't distinguish a class of round numerals to which to restrict its domain. If numerals were as numerous as real numbers, the domain of gran_0 would be numerals as dense as the real numbers so that $\text{gran}_0(\mathit{n}) = \text{n}$, for any real number n and its numeral *n.* As I understand the letter of these proposals, there is no granularity function other than this one interpreting all numerals, since coarser granularity functions restrict their domains to a designated class of round numerals. Yet, one could imagine a family of granularity-zero functions that interpret all the numerals for all the real numbers, assigning a numeral *n* a real-number interval, corresponding to a margin of error for measurements that read *n.* Even so, it is not the same. With rounding in mind, granularity functions are constrained to assign equal intervals to numerals. As remarked in the text, it is not in the nature of margins of error to display such consistency across the range of a measurement set-up. If so, granularity functions are an alternative to neither rounding functions nor margins of error. Measurement error, measurement precision, margins of error and rounding functions furnish ordinary measurement practice and approximation. Except to be excused from discussion of scientific measurement altogether, granularity functions are imposed without comparison to related precedent and common coin.

Lasersohn's (1999) pragmatic slack operators and the granularity functions of Krifka (2009), Sauerland & Stateva (2007, 2011), Cummins *et al.* (2012), Solt (2014, 2015)) find no place in natural language, except to disguise the adverbial phrases that modify a gradient predicate of truth, subjective probability, credence or confidence or those that introduce the rounding that resulted in the reported result. These adverbial phrases do not cross paths with the language of measurement and measurement error that surrounds bare and comparative measures, *113 pitches* and *more than 113 pitches.*

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Notes

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[1] In predicative position, cardinal expressions are unambiguous attributions of cardinality, in that (i) is false if we are114, without a meaning equivalent to (ii):

(i) We are 113. (ii) We are at least 113.

The same expression with the same meaning recurs in, *every 113 pitches, any 113 pitches, all 113 pitches, the 113 pitches, some 113 pitches, a certain 113 pitches, many a 113 pitches, no more than 113 pitches, fewer than 113 pitches,* and finally *exactly 113 pitches* and a focus of interest, *113 pitches.* Fox & Hackl (2006) and I have been persuaded that it is an unambiguous indefinite description "some 113 pitches" in illustration of classic Gricean implicature. But, a null article is anybody's plaything. If a half dozen readings for *Four hundred Dutch companies bought five hundred American computers* merit semantic disambiguation (Scha 1981), a suite of operators in van der Does (1992, 1993) is ready to derive homonymous DPs, among which a meaning for *113 glorious pitches* means "exactly 113 glorious pitches", which entails rather than implicates that there were not 114. Sentences (1)b. and (3)b. take away the plaything to make explicit an indefinite description *a glorious 113 pitches* or *a glorious 103* minutes, examples of Ionin & Matushansky's (2006) modified cardinal construction. Sentences (1)b. and (3)b. plainly do not entail but implicate (2) and (4) respectively. Sentences (1)a. and (1)b. are alike toward (2), and (3)a. and (3)b., toward (4). Any account of the semantics and pragmatics of (1)b. and (3)b. flips to account for (1)a. and (1)b. under the assumption that *113 glorious pitches* and *103 glorious minutes* are also indefinite descriptions, "*some* 113 glorious pitches" and "*some* 103 *glorious minutes*". On the other hand, the frisson that their null article might be the occasion for "exactly 113 glorious pitches" and "exactly 103 minutes" to entail (2) and (4) is not a pleasure embracing *<u>a</u> glorious 113 pitches* and *<u>a</u> glorious 103 minutes.* Sentences (1)a. and (1)b. flag those to follow that sink their measures deep within indefinite descriptions:

(iii) a. Into the Valley of Death rode a brigade 600 strong.
b. Cavalry 600 strong rode into the Valley of Death.

(iv) a. Into the Valley of Death rode a brigade more than 600 strong.
b. Cavalry more than 600 strong rode into the Valley of Death.

(iii)a. is Lord Tennyson's report of a charge against implicature, and yet the bare equative measure merely *implicates* that there were not brigades with 600 and more strong between them, not 601 cavalry in the Valley of Death, which the comparative measure in (iv)a. does not implicate. The equative and comparative measures in (iii)b. and (iv)b. similarly contrast in implicature. The question for Fox & Hackl (2006) and for us all is how to award equative measures and yet deny comparative measures scalar implicatures that plainly neither entails in the relevant contexts, whether or not *113 glorious pitches* and *103 glorious minutes* join *a glorious 113 pitches* and *a glorious 103 minutes* among them.

As much as the language already has *exactly 113 pitches* and *some 113 pitches*, and I am promiscuous with null vocabulary, it could just as well contain a null article to mean the one, the other, or both, or a couple null articles. Yet, as might be imagined for a semantics from 1981 disambiguating *113 pitches*, it has been under review whether ambiguity between "exactly 113 pitches" and "some 113 pitches" is a good thing.

For my part, ambiguity seems semantically defective. No meaning of *eight cannoli* in (v) limits how many overstuff a box for a half dozen, in contrast to *exactly eight cannoli* in (vi). No meaning for it validates the inference in (vii) that more cannoli will not also fit into boxes that are as many as 9, again, in contrast to *exactly 8 cannoli* in (viii):

(v) Eight cannoli will overstuff a box for a half dozen.
(vi) F Exactly 8/ Eight and no more than 8 cannoli will overstuff a box for a half dozen.
(vii) Eight cannoli fit into as many boxes with room for more.
$\nvdash$ Nine cannoli do not fit into as many boxes with room for more.
(viii) Exactly 8/Eight and no more than 8 cannoli fit into as many boxes with room for more.
$\vdash$ Nine cannoli do not fit into as many boxes with room for more.

If *eight cannoli* is ambiguous, why can't it mean "exactly eight" in (v) or (vii)? The problem is dire for late proposals that *eight cannoli* means *only* "exactly eight" (*v.* Spector 2013). Admittedly, it is uncollegial of me, I am told, to think that a semantics for *eight cannoli* ought still work applied to the collective predicates in (v)-(viii) when prior discussion is agreed to confine itself to distributive predicates, such as those in (1)-(8).

Since 1981, it has been observed (Liu 1990, Szabolcsi 1997, 2010) that *eight cannoli* and *exactly eight cannoli* contrast in the semantic conditions they impose on descriptive anaphora, and subsequent accounts of descriptive anaphora (Schein 1993: 215-237, Schein 2017: 793-810) have thus been shy to confound the syntax and semantics of antecedents *eight cannoli* and *exactly eight cannoli.*

Directly relevant to the pragmatics of cannoli counting— if *any* numeric quantifier in *any* language in *any* construction in that language finds itself within the scope of event quantification, and the scalar implicature survives, all the fuss about the meaning of the quantifier *per se* is in vain, given the plenitude of events:

(ix) Hesh ate eight cannoli.

(x) Hesh ate more than eight cannoli.

(xi) $\exists e$ [exactly ten x: cannoli (x)] eat(e, h, x)

Sentence (ix) implicates that Hesh did not eat ten cannoli, in contrast to (x). Yet, on his way to eating ten cannoli is first an event of eating exactly the first eight, (xii). Note however that (xi) remains more informative than (xii):

(xii) $\exists e$ [exactly eight x: cannoli (x)] eat(e, h, x)

An event of eating ten cannoli entails the existence of an event of eating eight, and not vice versa. So, in declining to assert (xi) rather than (ix) (=(xii)), it is implicated that not ten were eaten, which returns the question to Fox & Hackl (2006) to deny (x) a similar scalar implicature, having gained nothing from the meaning alleged for *eight cannoli*.

Since Parsons (1990), event quantification has been essential for verbal polyadicity, adverbial modification, nominalization, causativization, and tense and aspect. Since Kroch (1979) and especially since Liu (1990), Szabolcsi (1997, 2010) and references cited therein, it has been known that increasing and non-increasing quantifiers and bare and modified quantifiers vary in the syntactic positions in which they originate and in those to which they move to vary their scope (*v.* Szabolcsi 2010: Table 8.1, p. 125), which correlates with the distribution of scalar implicatures (*v.* Szabolcsi 2010: 150f.). Enquiring minds enquire whether from their variable positions within the clause there is also variation in their scope relations to the clause's event quantifiers. As Szabolcsi (2010: 151) pointedly remarks, there are in Hungarian syntactic positions where *két problémát* "two problems" cannot entail but continues to implicate "not three problems" (which *több, mint két problémát* "more than two problems" in that position does not do (p.c.)). English traps within the scope of event quantification the direct objects of what Quine (1960) calls *notional predicates* and Kratzer (1995), predicates with *ill-behaved objects*, which do not coherently export, in that to *have some friends* or *make some enemies* is not for there to be some friends that are had or for there to be some enemies that are made, among which are poster children for implicature and its cancellation:

(xiii) Once upon a time, an errant king had three illegitimate offspring, and therein were made three enemies sworn to all and only each other to claim the throne. Perhaps, more illegitimate offspring plotted, too.

(xiv) #F Once upon a time, an errant king had exactly three/three and no more than three illegitimate offspring and therein were made three enemies sworn to all and only each other to claim the throne. Perhaps, more illegitimate offspring plotted, too.

(xv) Once upon a time, an errant king had at least three/more than two illegitimate offspring and therein were made at least three/more than two enemies sworn to all and only each other to claim the throne.

More illegitimate offspring might covet the throne, too. The first sentence of (xii) only implicates that the king stopped at three, in contrast to *exactly three* or *three and no more than three* in (xiv). And, then, with implicature cancelled, (xiv) rests not at meaning "at least three", since (xiii) in contrast to (xv) is not true if four be so sworn, with any three among them also sworn to a fourth. The quantifiers in these and the following examples are ill-behaved and even mean enough not to quantify in restrictions. Plainly, one understands that the king has three illegitimate offspring or makes three enemies without having in mind there be certain people or a certain kind that he has exactly three of as illegitimate offspring or has made exactly three enemies of (*pace* Breheny 2008). The scalar implicatures survive the unforgiving syntax in (xiii)-(xv) and again in (xvi)-(xix), where again having some scratches or migraines is not for there to some scratches or migraines that one has:

(xvi) After the kids' work with sharp pencils, the table had eight scratches all connected to each other.

(xvii) After the kids' work with sharp pencils, the table had more than eight scratches all connected to each other.

(xviii) After it, I had eight migraines that rapidly succeeded each other without any others after.

(xix) After it, I had more than eight migraines that rapidly succeeded each other without any others after.

Whatever derives the contrast derives it from a logical syntax that traps both the bare and comparative measures within the scope of event quantification and *eight* has no occasion to effectively mean "exactly eight":

(xx) $\ldots\exists e(\ldots[\exists X\text{: } 8(X)\ \&\ \mathrm{NP}(X)]\Phi(e,X)\ldots)\ldots$

(xxi) $\ldots\exists e(\ldots[\exists X\text{: more than } 8(X)\ \&\ \mathrm{NP}(X)]\Phi(e,X)\ldots)\ldots$

As the Gricean expects, scalar implicature is an inference from what is known, unknown, and known unknown. It is a game of espionage that varies with the deep background:

(xxii) There are a hidden six survivors of an espionage ring in secret retirement in the former Soviet Union.

(xxiii) There are a hidden six or more survivors of an espionage ring in secret retirement in the former Soviet Union.

There is an ignorance implicature that the informant does not know of a seventh survivor. It's then spy craft that implicates there isn't one and folly to think that the debriefers mistake the informant to assert that six and no more than six survivors are hidden. Yet, it remains that the comparative measure in (xxiii) is immune from the scalar implicature that apt epistemic conditions prompt for (xxii).

To return the question about *113 pitches* to Fox & Hackl (2006) as one of scalar implicature as such, it suffices to find the contrast between bare and comparative measures in constructions within the scope of an existential event quantifier as above. This is just another iteration of the observation that sentences (1)b. and (3)b. flag. The occurrence of *113 pitches* within the scope of existential quantification either in its restriction, as in *a glorious 113 pitches* in (1)b., or within its matrix scope as just surveyed undoes the hope for "exactly 113 pitches" to pluck an entailment from the jaws of scalar implicature.

Breheny (2008), Kennedy (2015), and Spector (2013) resurrect for *113 pitches* a meaning of "exactly 113 pitches" without regard for the decades of argument against it. Spector (2013) chronicles that the field of cardinal semantics is divided in three parts: one of which the lower-bounders inhabit ("some 113 pitches"), another the upper-bounders ("exactly 113 pitches"), and a third the mixed lower- and upper-bounders ("some 113 pitches", "exactly 113 pitches"). It is advanced in §2 that if *113 pitches* is an unambiguous indefinite description, as it is for the lower-bounders, it means a tedious "some pitches counted to a margin of error with values near 113". Thrilled to do it all for less, for a glib "exactly 113 pitches", the upper- and mixed- bounders might gloat, unaware that "exactly 113 pitches" is play money. But, in truth, as also noted above, *113 pitches* isn't the hill to die on. It doesn't matter whether it is ambiguous or even means only "exactly 113 pitches". The problem of scalar implicature as such and not as entailment in disguise, to which Fox & Hackl (2006) address themselves, is richly attested elsewhere. For brevity and from conviction, although unnecessary, I will continue to treat *113 pitches* as an unambiguous indefinite description, without constant recourse to *a glorious 113 pitches* and the like.

[2] But still smaller than the garden of bare, comparative, and superlative modifiers (*v.*, *e.g.*, Geurts & Nouwen (2007), Geurts *et al.* (2010), Nouwen (2008b, 2010, 2015)), from which the Universal Density of Measurement still needs uprooting. Nouwen (2008b: 280ff.), for example, while pursuing *at least 112 pitches* and *no more than 112 pitches*, defers to the Universal Density of Measurement to look after *more than 112 pitches* as above.

Cummins *et al.* (2012) discover counterexamples to the alleged empirical generalization (Krifka (1999), Fox & Hackl (2006)) that comparative measures are without any scalar implicatures at all:

(i) There are more than 600 beans in the jar.

(ii) (i) ⊮ There are not more than 613 beans in the jar.

(iii) (i) ⊩ There are not more than 700 beans in the jar.

(iv) (i) ⊩ There are not more than 6000 beans in the jar.

Like above, *more than 600 beans* does not implicate not more than 613 beans; but, rounding to the nearest hundred, *more than 600 beans* does implicate not more than 700 beans, and *a fortiori* implicates that one is not off by an order of magnitude. Even absent any rounding or innocence of the facts, one cannot escape implicating that one is not off by an order of magnitude:

(v) I ate more than one cannoli.

(vi) (v) ⊮ I did not eat six cannoli.

(vii) (v) ⊩ I did not eat six dozen cannoli.

Confessing to more than one cannoli leaves room for six but not for six dozen. These surviving scalar implicatures are derived in §7 with a comparison in n. 39 to Cummins *et al.* (2012). But, the question remains *how to exclude the excluded (ii) and (vi)*? Here, they, too, concur with Fox & Hackl (2006)'s density account of comparative dense measures, *more than 100g of cannoli*, *etc.* and are ambivalent about comparative discrete measures, *more than 600 beans, more than one cannoli*. They (2012: 164) allow Fox & Hackl (2006) strong evidence for the Universal Density of Measurement from contexts that embed comparative discrete measures under modals (*v.* n. 40 below). But, in unembedded contexts, the unwanted scalar implicatures are said to be vacated, as follows, without help from the Universal Density of Measurement. If I do know how many cannoli I ate, then to say (v) *would* implicate that I ate exactly two, but it would have been communicatively more efficient to have just said so, "two cannoli" rather than "more than one cannoli". So, I don't say so, unless I have truly lost count of my cannoli (or, wish not to confess to gluttony). Either way, fault of ignorance or evasion, the scalar implicature is vacated. The slender thread here is the communicative efficiency that "two cannoli" enjoys over "more than one cannoli". In collecting signatures for a ballot initiative, every signature counts and needs verification. (Despite the big numbers, no rounding is allowed):

(viii) a. Less than ten thousand signatures were collected.
b. Nine thousand nine hundred ninety-nine signatures were collected.

(ix) a. Less than one times ten to the four [$1*10^4$] signatures were collected.
b. Nine point nine nine nine times ten to the three [$9.999*10^3$] signatures were collected.

Now that the bare numeral is the mouthful, the scalar implicature ought to be restored to the comparative measure and embraced for its more efficient communication of a signature count of 9,999, which it isn't, and the comparative measures exclude the scalar implicatures just the same.

Better not trust word length for the crucial contrast in implicature between bare and comparative measures. Better not rely on numeric nomenclature and rounding conventions when none are in sight as in the examples below. Implicature is to be derived uniformly for dense and discrete measures, without appeal either to the density of the measure or the measured or to the UDM in any context.

3 I am indifferent whether '$x_0 \succ x_1$' orders concrete objects with respect to their intrinsic magnitudes or orders abstract empirical magnitudes intrinsic to concrete objects, *v.* Mundy 1987, Swoyer 1987, Tal 2017, Wolff 2020.

4 As van Fraassen (2008) explains, even pictorial perspective is a variety of measurement in Dürer's (1525) *Treatise on Measuring*.

5 After Stevens (1945), a baseball team's numbered jerseys measure it because it talks numbers and only numbers. Eudoxus of Cnidus and Hippocrates of Chios measure nothing, since, lacking √2 and π, they resort to non-numerical magnitudes in the mathematical structure chosen to mirror aspects of the natural world. Not a good start for measurement in language or cognition. The mature Representational Theory of Measurement is not so stupid. Gallistel et al. (2005) argue for a primitive system of mental magnitudes and analog computation that is prior in evolution and separate from discrete arithmetic in those organisms that both measure magnitudes and count. Gallistel (2018) then turns to how computation on and about continuous magnitude is executed in a sub-cellular, molecular binary code.

6 It is constructible to draw lengths *OA* and *OB*, *O*----*B*--------*A*, incommensurable so that their proportion *OA/OB* correspond to the irrational numbers π, √2, or golden ratio φ. Then, a pantograph (Hero of Alexandria (c. 10CE-70CE), Christoph Scheiner (1603)) with fixed point at *O* and pivots at *A* and *B* scales blueprints up or down by that irrational number, all without numeric calculation. Well-aware of the real numbers, some applications prior to the digital age preferred graphical, geometric methods of measurement to tedious numeric calculation.

7 It is not to be expected from the Representational Theory of Measurement, which is not itself a metrology (Tal 2017)), to be concerned with the laboratory conditions for measurement.

8 If (55) looks too much like a brutal assertion that a measurement event is itself an error, then: i) rewrite 'error(e)' as 'errant(e)'; or, ii) for a spatiotemporal or causal relation R, let $\text{measure}_{\mathfrak{C}}(e) \rightarrow \exists e'(\mathrm{R}(e, e')$ & error(e')); or, iii) let the values for 'e' in (55) be the spatiotemporal regions that distinct events and states may occupy. It is known that to measure is to err. In other words, where there is measurement, therein is error, too, *i.e.,* (55). I do not see that the metaphysics of events are relevant enough to refine it further.

9 See the discussion of metrology in Tal (2017).

10 A critic has demanded a *definition*[*sic*] of "margin of error". I defer to the disgraced former Governor of New York, Andrew Cuomo, who, at his daily briefings about the COVID-19 pandemic, projected next to every statistic "Margin of Error: [±n]", and to the chapter in my lost 7th-grade science textbook on scientific measurement. I mean what they mean. If the following discussion of margins of error doesn't scream "Boring!", it isn't doing its job.

11 $q_i \succcurlyeq q_j \leftrightarrow_{\text{def}} q_i \succ q_j \vee q_i \approx q_j$.

12 My impression is that there is a musical chairs among the terms *amount, magnitude, quantity,* and *numerosity*, which I stop arbitrarily to seat three of them.

13 A critic objects to the absurdity, "not easily fixable," that a margin of error declares what remains unknown:

(i) Today 100 people got married in the chapel with a margin of error [±10].

Since both 100.5 and 101 are within the margin of error, the critic charges that the speaker has been committed to not knowing that neither 100.5 nor 101 is the number married. But, it is known prior to any count at the chapel that only even numbers of persons marry, and thus, according to *Original Error* (58) and *Original μ-Error* (63), (i)'s speaker remains ignorant of just which even number within the margin of error is the number married. What isn't broken is indeed not easily fixable.

14 A premise that is cancellable—
Kirk: "Spock, so how much has Koufax pitched?"
Spock: "Captain, sensors measure 113 pitches."

Kirk: [(68) ⊢] "Ah hah, so, *not* 114 pitches—I win my bet against Bones."
Spock: [¬(68) ⊢] "But, Captain, the space-time wormhole may have disrupted the sensors."

[15] Note that (64), with margin of error unspecified, nevertheless implicates not 114, patterning with (70) with it specified to the nearest pitch rather than with (72) with margin of error [±2]. In uttering (64), Spock knows that all measurements come with margins of error only some of which support the scalar implicature. If unspecified, there must be a convention that guides (64) to a default margin of error, such as the conventions for rounding numerals, *v.* Krifka (2009), Sauerland & Stateva (2007, 2011), Cummins *et al.* (2012), Solt (2014, 2015); Appendix. Nutrition, like measurement, has its conventions special to the subject and its jargon. All eating is eating of something. Yet, a bare "The child has eaten" implicates that dinner was not candy corn, which "The child has eaten candy corn" does not. There is little of form or grammar in this nutritional implicature, nor need there be for (64) to find its way to its default margin of error.

[16] I. Under scrutiny is indeed scalar implicature, as the contrast between (ii) and (iv), confirms with further embedding under collective nouns.

Kirk: "Spock, so how much did Koufax pitch?" Spock: "Captain,..

(i) a. Koufax pitched a passel of pitches registering 113 to the nearest pitch."
b. Koufax fired off a battery of pitches counted 113 with a margin of error less than one."

(ii) (i) ⊩ Sandy Koufax did not pitch 114 pitches.

(iii) a. Koufax pitched a passel of pitches registering 113 give or take a couple."
b. Koufax fired off a battery of pitches counted 113 with a margin of error [±2]."

(iv) (iii) ⊮ Sandy Koufax did not pitch 114 pitches.

Kirk's question is answered and the scalar implicature in (ii) is sound only supplemented with the premises that all the pitches pitched were pitched in passels or fired off in batteries and Spock knows not of other passels or batteries but would have known and said so if there had been any. Yet, it remains that the scalar implicature is sound only if a 114th pitch does not fall within the margin of error as now described deeply embedded within an indefinite description of *a* passel or of *a* battery of pitches. Examples like these can never be lit up enough to torch the zombie idea that *113 pitches* sometimes mean "113 and no more than 113" to entail rather than implicate that there were not 114 (*v.* n. 1).

II. A critic protests that (i)'s speaker never doubts and never suspects an error in their count:

(i) I have more than two toes. In fact, I have ten (just like a human).

But then, to "How many toes have you, Spock?", Spock, too, answers variously:

(ii) I have toes measured to more than two. In fact, I have them measured at ten (just like a human).

(iii) I have a measurement of more than two. In fact, I also have a measurement of ten (just like a human).

To recap then the parallel, Gricean reasoning, ending the reply after the first sentence implicates equally weirdly in (i)-(iii) that the speaker does not know the number of their own toes. If a speaker responds with only that first sentence, the speaker implicates the *ignorance* implicature that they do not know whether they have more than three. The ignorance implicature that the speaker knows no better derives in (ii)-(iii) from the speaker's implicating (the *best-measurement* implicature) that an approximate measurement is their best. Next, the second sentence goes on to cancel the ignorance implicature of the first, by cancelling in the case of (ii)-(iii) the best-measurement implicature and revealing the better information of another, better measurement (*v.* n. 14). The conversational structure of (i)-(iii) is unremarkable except to raise the same question: what invites leading with an ignorance implicature only to cancel it? Spock's exactitude in his choice of language (ii)-(iii) implies no more self-doubt nor global ignorance of bodily condition than (i) does. The critic must not have completed the first season of *Star Trek*.

III. If speaking like himself in (64), Spock says (iv), a margin of error is understood even if, as in (64), none is specified (*v.* n. 15):

(iv) I have toes measured at ten.

Yet, for the same critic, it embarrasses an understood margin of error that Spock is absolutely certain of his toes. The margin of error the values of which are near enough to be "at ten" for (iv) or "at 113" for (64) is left to context if unspecified (*cf.* (70) or (72)). Nothing holds Spock and his audience back from reckless confidence for this toe count in a [$\pm\varepsilon$]-margin-of-error for $\varepsilon = 0$ (*v.* (62)). Elsewhere, stadium gatekeepers with hand-held counters aggregate attendance so that at the end of the perfect game in Los Angeles, 9 September 1965, the game announcer announced that today's attendance was 29,139[*sic*], about which Spock commented:

(v) I have toes measured at ten; and,

(vi) Today's Dodgers fans have them measured at 291,390.

The only imperative is that the syntax and semantics for (v) and (vi) be the same, and for (vii) and (viii), too:

(vii) I have ten toes.

(viii) Today's Dodger fans have 291,390 toes.

If (vi) warrants tacit reference to a margin-of-error parameter, then it occurs in (v), too, and similarly for (viii) and (vii). (It could be further speculated that the humor in zeugma derives from the expectation that the values of unspoken parameters are presumed the same from one conjunct to the next.) Yes, it could be that for (iv), (v) and (vii), its value is [±0], as is self-evident to the critic. But, it could just as well be that there are no true zero margins of error, that in (iv), (v) and (vii), it is nonzero but insignificant, say [±.0001], because when the very same set-up counting ten toes is set up to count Dodger fan toes an error of one toe per 10,000 creeps in (not to mention the occasional missing toe or rare extra). The inquiry is to discover the invariance in the structure of natural language that hosts the variation in its use in context. That ordinary people ordinarily use it counting their toes or the donuts on their plate without attention to or explicit mention of the limits to which they have become habituated in their daily measurements is an irrelevant observation and incoherent objection to an analysis of the language fit to both ordinary and extraordinary conditions.

[17] Similar examples can be constructed for (33)-(36) and (37)-(38).

[18]Nouwen (2008a) observes, with different examples, the same global semantic structure in physical and numeric space. The scatter conforming to *up*'s topology is indifferently pinpricks ((i) *vs.* (ii)) or measured quantities ((iii) *vs.* (iv), (v) *vs.* (vi)):

(i) # The pinprick is up to the collar.
(ii) The pinpricks are up to the collar.
(iii) # Hamlet's Act III Scene 1 soliloquy is up to 33 lines.
(iv) Hamlet's soliloquies are up to 34 lines.
(v) #Last night, Hamlet's Act III Scene 1 soliloquy was up to 4 minutes.
(vi) Last night, Hamlet's soliloquies were up to 4 minutes.

[19] The internal structure of 'up-to' is short-changed. In "between *x* and *y*", *between*, I assume, occurs as it does in "between the goal posts" and "between those values" denoting a relation between event or state and a plurality, which a deferred analysis of "*x* and *y*" supplies. But, for present purposes, 'between(*e,x,y*)' would do as well provided it is the same preposition in both "between 110 and 115" and "between the left goal post and the right goal post".

[20] When the preposition describes bounded space, *at*, *near, around, between, within*, the margin of error is understood to be within the bounded space so described, as say (123)-(124). It could be strengthened to place all the space described, bounded and unbounded *above, over, past, beyond*, etc. within the margin of error: $\vdash$ (measure$\mathfrak{e}$(e) & $\mathrm{P}_{\mathrm{Locative}}(e, X)$) $\rightarrow \forall m$(margin-of-error$_e$(m) $\leftrightarrow$ [$\exists e$': at(e', m)] $\mathrm{P}_{\mathrm{Locative}}(e', X)$)
Then, if Koufax pitches above 112 or more than 112 pitches, 29,139 is strictly speaking in the margin of error for the measurements reported as such. But, all this amounts to, according to *Original Error* (58), (63), is that if one didn't already know that Koufax could not have pitched 29,139 pitches in a single game, nothing in the measurement reported tells otherwise. It is also no comment on the existence of other measurements with greater accuracy (*v.* n. 13). That the margin of error attaches only to the measurement event to which it is attached is why a *best-measurement* implicature was needed to supplement *Accuracy* (68) to derive a global ignorance implicature for the speaker reporting nothing more informative than more than 112 pitches. I do not see then that a margin of error coincident with the space the locative phrase describes risks the attribution of too much ignorance to the speaker. Note that the text's weaker formulation (124) that the margin of error is merely *within* the space described itself needs a further turn of the pragmatic screw, *viz.*, that in saying of a margin of error merely that it is *within* the region above 112, the speaker does not know of a bound within that region for this margin of error for said measurement. It cannot then be assumed of anything above 112, such as 114, that it is not within the margin of error, denying then this measurement report a scalar implicature that the pitches were not 114. From the point of view of its license of scalar implicature, it seems to matter little whether the semantics says that the margin of error is within or coincident with the space described.

[21] As Joost Zwarts points out to me, Jackendoff (1979) has already discovered that the fallacy in Montague's (1973) (i)-(iii) is to have mistaken (i) for an identity, rather than as synonymous with (iv):

(i) The temperature is ninety. (ii) The temperature is rising. (iii) Therefore, ninety is rising.
(iv) The temperature is *at* ninety.

[22] That is, (Chomsky 1977ab), if New York is 212 miles from Boston, it may be true enough that it is 200 miles from Boston for ball-parking travel time and the number of rest stops on the way but not true enough for calculating fuel reserves to destination.

[23] The "pragmatic slack" (Lasersohn (1999)) that lets the false be true enough and the rounding practices flagging the amount of slack (*to the nearest ten miles vs. to the nearest hundred miles, etc.*) have a dedicated literature discussed in Appendix, with a vocabulary of its own for what is related but distinct from measurement precision and margins of error.

[24] To measure is to err within a margin of error whether or not overtly described. The convention of significant digits that Eudora learned in middle school encodes in the numeral used the margin of error

intended abbreviating the need for a prepositional phrase. In (161) and (162), '1' and '1.00000' name the same number but the latter encodes a precision that supports the inference to (163):

(i) a. $\exists e$(Patient(e, mdA) & AT(e, *1 meter*) & within(e, *a margin of error* [±0.00001 *m*]))

b. $\exists e$(Patient(e, mdA) & measure(e) & AT(e, *1 meter*) & to within(e, [±0.00001 *m*]))

In understanding (161), which neglects to describe a margin of error, either that of the instrument in hand, Eudora's Sears® laser ruler, is assumed or it defaults to a precision significant to the nearest meter, neither of which is an understanding that entails (163) (*v.* Appendix).

[25] The restriction to measure phrases exempts the numerals in arithmetic sentences (158) and (160), which do not contain measure phrases.

[26] See *adverbialization* in Schein (2012: 289f.; 2017: §§1.6.2, 8.3, chapters 11-12) for a semantics that links the weighings in at *n.* lb. to the facing off. With the cardinal predicates in (i)-(iv) denoting different measurement events with different protocols, the very same things may be counted 3000 and 1000 (Gupta 1980: 23f.), and restriction on the temporal location of such events explains why (i) cannot be continued with (iv) into the present (Moore 1994, Doetjes & Honcoop 1997):

(i) Three million passengers crowded National Airlines' routes in 1980.

(ii) One million frequent flyers crowded National Airlines' routes in 1980.

(iii) The three million passengers who crowded National Airlines' routes in 1980 were the one million frequent flyers loyal to it.

(iv) The three million passengers are flying frequently now.

[27] See Alexeyenko 2018 for recent argument that habitual aspect is plural reference to events, and antecedents for this view in Carlson 1977, van Geenhoven 2004, Rimell 2004, Ferreira 2005, Kratzer 2008, Boneh & Doron 2013.

[28] In (193)-(194) and *passim*, a notation is borrowed from elementary logic in which square brackets enclose variables free in the formula to which they are suffixed. It is used here to flag an expression that abbreviates a complex formula. In (194), the pluralization of relation θ is a formula with free variables 'E' and 'X' defined in terms of the primitive first-order relation '$\theta(e, x)$'. The pluralized relation may itself then relate to a plurality of one, for which it is convenient to introduce the notation shown.

[29] The translations in (195) and (196) neglect an important aspect. Imagine a balloon that remains a cylinder under inflation and deflation varying only its length (like a spring) between 2ft. and 5ft:

(i) The balloon measures between 2ft. and 5ft.

The balloon is between 2ft. and 5ft. long.

If, instead, the balloon oscillates between a 2ft. long cylinder and a 5ft. diameter sphere, (i) is infelicitous. It is awkward to attribute length to the balloon for a period which includes a sphere, for which length is not defined. Here is a quick and dirty shortcut. Suppose there is a finite, extensible list of dimensions, *Dim*, among which are Length for lengths, Diameter for diameters, Weight for weights, Work Hours for work hours, etc.:

(ii) a. $\Pi_\mu(E) \rightarrow (\text{measure}_{\mathfrak{c}}[E] \rightarrow [\exists D\text{: } Dim(D)]\ [\exists Q\text{: } D[Q]]\ \text{quantity}[E, Q])$

b. $\text{Length}[Q] \leftrightarrow_{def} \exists q Qq\ \&\ \forall q(Qq \rightarrow \text{Length}(q))$, *etc.*

The conditions for a session Π_μ of stable, coherent measurement demand that the measurements E probe the same dimension. Sentences (i) are infelicitous to the extent that their nomic, dispositional aspect invites the assumption that the measurements referred to conform to Π_μ. Coherent stable measurement might also demand more, for example, the same instrument and therefore the same limits on range and resolution throughout:

(iii) $\Pi_\mu(E) \rightarrow (\text{measure}_{\mathfrak{c}}[E] \rightarrow \exists \mu_e \forall e(Ee \rightarrow \text{instrument}_{\mathfrak{c}}(e, \mu_e)))$

Instead of quick and dirty, more elaborate translations of (161) and (i) could directly manage the relevant parameters. See Schein (2017: Chapter 12 §1.1) for related discussion.

[30] Herskovits (1986) Outlines rather than "shrink-wraps" these things to get to their "footprint", as Galton & Duckham (2006) call it, or their concave hull as in Moreira & Santos (2007) and Kalinina et al. (2018). Insisting on a definition for "shrink-wrap" when none is needed, a critic has clamored for the convex hull, which Herskovits (1986) has already rejected in proposing her Outline function citing counterexamples such as the following, which recur in the references just cited. Imagine an ivy vine trained in the form of a tree with foliage covering an expansive canopy and trunk:

(i) Alongside the ivy vine, the hummingbirds hummed.

(ii) Alongside the ivy leaves, the hummingbirds hummed.

The hummingbirds are in close to canopy or trunk, but the convex hull of vine or leaves tents the whole plant and keeps the hummingbirds outside, which is not a defect of the concave hull of vine or leaves, their Outline, "footprint", "shrink wrap", or silhouette. All these authors presuppose a prior concept in perception or cognition that a scientific theory or computer algorithm attempts to formalize or model. The existence of that prior concept in perception or cognition and an evocative word for it, "shrink-wrap", is all that matters here. For overviews on the semantics of spatial prepositions, *v.* Casati & Varzi (1999), Crangle

& Suppes (1989), Herskovits (1986), Landau & Jackendoff (1993), Pustejovsky & Moszkowicz (2011), Zwarts (2017), *inter alia*.

[31]That is, 1-dimensional neighborhoods are spatial intervals of finite length, 2-dimensional neighborhoods are of finite area, 3-dmensional neighborhoods are of finite volume. *etc*.

[32] Landmark[e, Y] $\leftrightarrow \forall y(Yy \leftrightarrow$ Landmark(e, y)), *i.e.* The Y are each landmark and the only landmarks determining the neighborhood *e*. (*v*. (194), n. 28.)

[33] Recall that plural reference to measurement events E stands in for the consistent measurement*s* of stable measurement.

[34] Perception is the focal source of knowledge, but I hesitate to introduce yet more vocabulary, "perceived that…'. Note that the variables q_i occur *de re* anyway in an attribution of knowledge that somehow makes direct reference to these quantities.

[35] It is worth noting that facts about resolution are physical and psychophysical and prior to metrization. It is not, after all, a numerical observation that distinct stars fail to correspond to distinct images or that two shades of yellow or two pitches are not resolved except under bright light or loud volume. For that matter, one can go as far as (i) and still leave room to dispute the metrization of yellowness (Fechner 1860/1966) and launch the Representational Theory of Measurement to defend it (Luce & Suppes 2004).

(i) Butter is yellower than milk, a daffodil is yellower than butter, and a daffodil is more yellower than butter than butter is yellower than milk.

[36] Other measurements include the abstract, non-actual ones for the scale and resolution assumed that could in principle be more precise than this approximation has been.

[37] Note that identifying true measurement with the universal quantification over measurement set-ups trusts in their instruments' accuracy and thus that the content of a measurement event e, measure$_{\mathfrak{C}}$(e), is veridical.

[38] N.B. (221)-(223) merely implicate and do not entail not being in Nyack or at Lake Champlain. Tove has already checked in at the New York State border when she has later checked in at Lake Champlain, where having checked in at the border cannot entail not having checked in at the lake.

[39] Cummins *et al.* (2012) confine themselves to numeric measurement by ideal rulers that vary in their granularity, *i.e.*, in the density of their marks, but all evenly spaced, by cm or by mm, etc. The cm-ruler is the special case where the instrumentation provides for (visual) detection of a quantity at every cm. Visual inspection sees that a quantity overshoots the 5cm-mark while unable to see that it is not more than 5.1 cm, and it is reported that the quantity is more than 5 cm. Of course, it is not implicated that it is not more than 5.1cm, yet it is still implicated that it is not 6cm, given this instrumentation, the cm-ruler. If the quantity had been 6cm or more, it would have been seen at the 6cm-mark as such. Even-spacing is not a general characteristic of measurement set-ups and their resolution over their range.

[40] Fox & Hackl (2006) discuss at length that comparative quantifiers under a necessity modal as in (ii) acquire scalar implicatures that are denied them under an existential modal as in (iv). As (i) and (iii) show, the same can be said for the locative prepositions and prepositional numerals to which they are assimilated:

(i) Tove was required to be past/north of the NYS border.
$\Vdash$ Tove was not required to be past/north of Nyack, NY.

(ii) Tove was required to be at more than 18 checkpoints (from DC).
$\Vdash$ Tove was not required to be at more than 19 checkpoints (from DC).

(iii) Tove was allowed to be past/north of the NYS border.
$\nVdash$ Tove was not allowed to be past/north of Nyack, NY.

(iv) Tove was allowed to be at more than 18 checkpoints (from DC).
$\nVdash$ Tove was not allowed to be at more than 19 checkpoints (from DC).

Given the assimilation of numeric and non-numeric measurement, whatever accounts for the contrast between (i) and (iii) ought to do it for (ii) and (iv), whereas it seems that Fox & Hackl's (2006) account of the latter does not extend to the former.

[41] Fox & Hackl (2006: 540 n.4) cite Geurts & Nouwen's (2007) conclusion that despite the superficial equivalence of *more than six* and *at least seven*, the latter conceals an epistemic modal *at least* with variable scope, exposed in examples from Kay (1992):

(i) In that big train wreck, at least several people were saved.
(ii) In that big train wreck, at least several people were killed.
(iii) At least in that big train wreck, several people were saved.
(iv) At least in that big train wreck, several people were killed.

One might go further and deny *more than six people* and *at least seven people* a common parse, with *at least* always adjoining to a DP: [$_{DP}$more than six people] *vs.*[at least [$_{DP}$(more than)(the) seven people]]. In support of Fox & Hackl's pleading, dismiss discussions that suppress (iii)-(iv) to pretend that *at least several people* is a generalized quantifier despite the sentential operator within (dismissing also discussions of *six or seven people* that ignore *six or possibly even as much as seven people*).

[42] See n. 2 for some to be disappointed, that *contra* Fox & Hackl (2006), density plays no role in denying scalar implicatures to comparative measures, to neither the dense *more than 200cm* nor the discrete *more than 613 games*.

[43] Unless you, the reader, have already compensated for this and accommodated a perspective that zooms in your live map or focal attention to the interior of New York State, with the border at one edge and *past the New York State border* understood as "just past it."

[44] Moreover, comparative measures are themselves not subject to pragmatic slack. As Solt (2014: 516) observes, "the potential looseness around the interpretation of *fifty meters* in the comparative [*more than fifty meters*] seems not to involve such a coarse interpretation, but rather relate to the maximum possible degree of precision compatible with a given context, which may be limited by factors such as measurement error." "In short, the flexibility to interpret round numbers at different levels of precision disappears when they are embedded in comparative statements." If so, the implicatures of comparative measures are fruitfully contrasted with bare comparatives only when the latter are interpreted without slack, too.