Tying Free Choice in Questions to Distributivity

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Abstract. The idea that *wh*-phrases can quantify over generalized quantifiers emerged following two main observations: (i) disjunctive answers to modalized questions lead to free choice inferences if the *wh*-phrases's restrictor is plural and (ii) questions with collective predicates do not lead to uniqueness presuppositions. Such proposals, however, fail to derive the connection between (i-ii) and plurality. We propose a novel analysis in which (i-ii) are derived via the presence of an existential distributivity operator. By tying these phenomena to distributivity, our analysis is able to establish the desired connection to plurality.

Keywords: question semantics, free choice, distributivity, exhaustivity

1 Introduction

The semantics literature has seen growing discussion on the topic of free choice in *wh*-interrogatives [1, 5, 14–16]. It has been noted that questions like (1-a) can receive disjunctive answers such as (1-b) which carry free choice inferences.

(1) a. Q: Which books are we required to read?
b. A: The French books or the Russian books.
$\rightsquigarrow$ $\Diamond$we read the French books $\wedge$ $\Diamond$we read the Russian books

A popular analysis of these facts involves assuming that *wh*-items have the option of quantifying over generalized quantifiers. In the case above, this allows *which books* to quantify over an existential quantifier ranging over books which scopes below the universal modal. This in turn allows us to apply familiar theories of universal free choice to the disjunctive answer.

It has been noted that the free choice effects discussed above do not arise when the restrictor of the *wh*-phrase is singular. The disjunctive answer in (2-b) to the question in (2-a) leads to ignorance rather than free choice.

(2) a. Q: Which book are we required to read?
b. A: The French book or the Russian book.
$\not\rightsquigarrow$ $\Diamond$ we read the French book $\wedge$ $\Diamond$ we read the Russian book

The interaction between free choice and number has been left largely unexplored in the literature. We propose to establish this link by discussing a new account

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of free choice in interrogatives which involves the presence of a covert existential distributivity operator [2]. We show that this move allows us to derive free choice for plural *wh*-interrogatives, whilst deriving no such inference for the singular case. Furthermore, we show that this does not result in bad predictions for other data that have served as motivation for higher-order quantification, namely the lack of a uniqueness presupposition in (3).

(3) Which students formed a group.
does not presuppose: $\exists! x[\mathsf{students}(x) \wedge \mathsf{formed\text{-}a\text{-}group}(x)]$

We show that under the assumption that questions are felicitous when the exhaustified set of their answers forms a partition on the context [7], and assuming that the exhaustification of alternatives involves a step which allows us to assert some of those alternatives [3], we can derive the lack of uniqueness of (3).

The use of a distributivity operator to derive free choice establishes a natural connection between the presence of this inference in plural *wh*-interrogatives and its absence in their singular counterparts. Furthermore, as we will show, restrictions on the kind of generalized quantifiers *wh*-items quantify over are naturally derived from our proposal. We therefore argue that our proposal offers a number of advantages over the view that *wh*-items quantify over generalized quantifiers, both on conceptual and empirical grounds.

2 Background

Following [8, 9], we assume questions to denote the set of their answers, as shown in (4)[1]. The question denotation in (4-c) is compositionally derived from the LF in (4-b), where the interrogative complementizer **?** denotes [9]'s proto-question operator and the *wh*-phrase an existential quantifier, as in (5).

(4) a. Who arrived?
b. λ_p who λ_x [**?** p] [x arrive]
c. $[\![\text{(4-b)}]\!] = \{\mathsf{arrive}(x) \mid \mathsf{human}(x)\}$

(5) a. $[\![\text{who}]\!] = \lambda f_{et}.\ \exists x \in \mathsf{human} : f(x)$
b. $[\![\mathbf{?}]\!] = \lambda p_{st}.\lambda q_{st}.\ p = q$

The question in (4-a) presupposes that someone arrived. To derive this, we follow [4] in assuming interrogatives to fall within the scope of a covert answerhood operator ANS. The LF schema for (4-a) should, therefore, be represented as (6). We adopt [7]'s formulation of ANS, shown in (7), in which ANS takes a question denotation as its argument and ouputs the set of its exhaustified answers only if this set partitions the context set C (in (7), we take C to be a parameter of $[\![\cdot]\!]$, but we supress it from the definitions to follow).

[1] For ease of exposition, we use set-theoretic notation for question denotations. We could have equivalently written (4-c) as $[\lambda p_{st}.\ \exists x \in \mathsf{human}_w : p = \lambda w'.\ \mathsf{arrive}_{w'}(x)]$. We furthermore suppress intensional details.

(6) ANS [λ_p who λ_x [**?** p] [x arrive]]

(7) $[\![\text{ANS}]\!]^C = \lambda Q_{stt} : \mathsf{Partition}(Q, C).\ \{\mathsf{Exh}_Q(q) \mid q \in Q\}$
where $\mathsf{Partition}(Q, C)$ iff $\{\mathsf{Exh}_Q(q) \cap C \mid q \in Q\}$ partitions C

Exhaustification is assumed to proceed as proposed by [3], where, in order to exhaustify a proposition p with respect to a set of alternatives Q, one must first determine two sets. The set of *innocently excludable* alternatives of p given Q is the the maximal set of elements of Q that can be consistenly negated if p is true, while the set of *innocently includable* alternatives of p given Q is the maximal set of elements of Q that can be consistently asserted if p is true and its innocently excludable alternatives are false. Exhaustification of p given Q thus consists in negating all of its innocently excludable alternatives and asserting all of its innocently includable alternatives. This procedure is formalized in (8).

(8) $\mathsf{Exh}_Q(p) := \forall q \in Q[q \in \mathsf{IE}_Q(p) \rightarrow \neg q] \wedge \forall q \in Q[q \in \mathsf{II}_Q(p) \rightarrow q]$

a. $\mathsf{IE}_Q(p) = \bigcap\{C' \subseteq C : C'$ is a maximal subset of C s.t. $\{\neg p : p \in C'\} \cup \{p\}$ is consistent$\}$

b. $\mathsf{II}_Q(p) = \bigcap\{C'' \subseteq C : C''$ is a maximal subset of C s.t. $\{r : r \in C''\} \cup \{p\} \cup \{\neg q : q \in \mathsf{IE}_Q(p)\}$ is consistent$\}$

We derive (4-a)'s presupposition that someone left in the following way. Due to ANS, (6) will only be defined if the application of pointwise exhaustification to the question denotation in (4-c) induces a partition on the context set. In (9), this set is defined under the assumption that $\mathsf{human} = \{\mathsf{a}, \mathsf{b}\}$[2]. In (10), we illustrate the exhaustification of a singular alternative of (4-c). The exhaustification of $\mathsf{a}\oplus\mathsf{b}$ is vacuous, given that it is the strongest alternative in the set.

(9) $$\{\mathsf{Exh}_{(4\text{-c})}(q) \mid q \in (4\text{-c})\} = \{\mathsf{Exh}_{(1\text{-c})}(\mathsf{a}), \mathsf{Exh}_{(1\text{-c})}(\mathsf{b}), \mathsf{Exh}_{(1\text{-c})}(\mathsf{a}\oplus\mathsf{b})\}$$
$$= \{\mathsf{a} \wedge \neg\mathsf{b}, \mathsf{b} \wedge \neg\mathsf{a}, \mathsf{a}\oplus\mathsf{b} (= \mathsf{a} \wedge \mathsf{b})\}$$

(10) a. $\mathsf{IE}_{(1\text{-c})}(\mathsf{a}) = \{\mathsf{b},\ \mathsf{a}\oplus\mathsf{b}\}$[3]

b. $\mathsf{II}_{(1\text{-c})}(\mathsf{a}) = \{\mathsf{a}\}$

c. $\mathsf{Exh}_{(1\text{-c})}(\mathsf{a}) = \mathsf{a} \wedge \neg\mathsf{b}$

The set in (9) will impose a partition on a context set C only if (i) for each proposition p in (9), there is at least one world in C in which p is true, (ii) for each world in C, there is a proposition p in (9) such that p is true in that world. Therefore, if there is a single world in the context set in which no one

[2] Note that, given that *arrive* is distributive, $\mathsf{a}\oplus\mathsf{b}$ is equivalent to $\mathsf{a} \wedge \mathsf{b}$

[3] The inclusion of the alternative $a\oplus b$ in $\mathsf{IE}_{(1\text{-c})}(a)$ depends on whether we take $\neg(a\oplus b)$ to mean $\neg(a \wedge b)$ or $\neg(a \vee b)$. Although logically it should be the former, the natural language sentence *A and B didn't arrive* seems to be interpreted as the latter (a phenomenon known as homogeneity, which will be discussed below). Nothing in the above analysis depends on which of these is the right answer: if $\neg(a \oplus b) = \neg(a \wedge b)$, it will be vacuously included to $\mathsf{IE}_{(1\text{-c})}(\mathsf{a})$; if $\neg(\mathsf{a} \oplus \mathsf{b}) = \neg(\mathsf{a} \vee b)$, it won't be either in $\mathsf{IE}_{(1\text{-c})}(a)$ or $\mathsf{II}_{(1\text{-c})}(a)$, and therefore won't affect the final result.

has arrived, the presupposition of ANS won't be satisfied; no proposition in (9) is true in that world.

3 Higher-Order Quantification in Questions

3.1 Free Choice

[14, 15] is to our knowledge the first to discuss the presence of free choice effects in complex *wh*-interrogatives. In the presence of a universal modal, disjunctive answers to plural complex *wh*-interrogatives lead to a free choice inference.

(11) a. Q: Which books are we required to read?
b. A: The French books or the Russian books.
$\rightsquigarrow$ $\Diamond$we read the French books $\wedge$ $\Diamond$we read the Russian books

If we follow standard assumptions and assume *which books* quantifies over regular individuals, it becomes difficult to derive free choice from the utterance of the disjunctive answer in (11-b). To see this, consider the exhaustified set of answers to (11-a) in (12-b), where we assume *books* denotes the set in (12-a).

(12) a. $[\![\text{books}]\!] = \{\mathsf{f}, \mathsf{r}, \mathsf{f} \oplus \mathsf{r}\}$
b. $\left\{\begin{array}{c} \Box \text{ we read f} \wedge \Diamond \neg \text{ we read r}, \\ \Box \text{ we read r} \wedge \Diamond \neg \text{ we read f}, \\ \Box \text{ we read f} \wedge \Box \text{ we read r} \end{array}\right\}$

While this question is well-formed insofar as it partitions the context, it is not clear how any answer should lead to free choice here. While each of the propositions in (12-b) is compatible with free choice, only "$\Box$ we read f $\wedge$ $\Box$ we read r" entails it. It is not clear that how this last proposition can be equated to a disjunctive answer. In fact, the disjunctive answer in (11-b) does not seem to correspond straightforwardly to any of the answers in (12-b), and might as such be treated as a partial answer denoting the disjunction of two propositions in the set. It is unclear how disjoining two such propositions should lead to an enriched meaning incorporating free choice.

The free choice effect of disjunctive answers such as (11-b) is derivable if we assume the disjunction to take narrow scope below the modal. This can be achieved if we provide a meaning for *which books* such that it can quantify over generalized quantifiers (GQs). More specifically, we can assume the *wh*-phrase can quantify over generalized disjunctions of individuals rather than quantifying over individuals proper.

(13) $[\![\text{which books}]\!] = \lambda P_{(ett)t}.\exists Q \in \{\lambda f_{et}.\ \exists x \in X : f(x) \mid X \subseteq \mathsf{books}\} : P(Q)$

The answers in (12-b), prior to the application of ANS, can be interpreted using the LF schema in (14-a), where *which books* binds a variable of type *ett*, which itself binds a type e variable. This in turn denotes the set of propositions described in (14-b)[4].

(14) a. λ_p which books λ_Q [? p] [$\Box$ Q_{ett} λ_x [we read x_e]]

b. $$\left\{\begin{array}{c} \Box\ \mathsf{we\text{-}read(f)}, \\ \Box\ \mathsf{we\text{-}read(r)}, \\ \Box\ \mathsf{we\text{-}read(f \oplus r)}, \\ ①\Box\ \mathsf{we\text{-}read(f)} \vee \mathsf{we\text{-}read(r)} \end{array}\right\}$$

The answer in (11-b) can be taken to correspond to the proposition in ①. Once the ANS operator is applied to the set in (14-b), each of its members is exhaustified relative to the others. The answer in ① is entailed by every member of (14-b). Furthermore, each member of (14-b) is innocently excludable relative to ①, and will as a result be negated. The exhaustified meaning of ① will be the one in (15), which entails the free choice inference.

(15) $$\begin{array}{l} \mathsf{Exh}_{(14\text{-b})}(\Box\ \mathsf{we\text{-}read(f)} \vee \mathsf{we\text{-}read(r)}) = \\ (\Box\ \mathsf{we\text{-}read(f)} \vee \mathsf{we\text{-}read(r)}) \wedge \neg\ \Box\ \mathsf{we\text{-}read(f)} \wedge \neg\ \Box\ \mathsf{we\text{-}read(r)} \Rightarrow \\ \hfill \Diamond\ \mathsf{we\text{-}read(f)} \wedge \Diamond\ \mathsf{we\text{-}read(r)} \end{array}$$

3.2 Collective Predicates

Questions involving collective predication, such as (16-a), allow for answers where multiple groups were formed.

(16) a. Q: Which students formed a group?
b. A: Al and Bob, and Bob and Carl.

[16] notes that if we assume *which students* to range over individuals, we predict answers such as (17-b) to be unavailable. Let (17-a) be the contextually relevant set of students. We predict (16-a) to denote (17-b) before the application of ANS.

(17) a. $[\![\text{students}]\!] = \{\mathsf{a}, \mathsf{b}, \mathsf{c}, \mathsf{a}\oplus\mathsf{b}, \mathsf{a}\oplus\mathsf{c}, \mathsf{b}\oplus\mathsf{c}, \mathsf{a}\oplus\mathsf{b}\oplus\mathsf{c}\}$

b. $$\left\{\begin{array}{c} \mathsf{formed\text{-}group(a\oplus b)}, \\ \mathsf{formed\text{-}group(a\oplus c)}, \\ \mathsf{formed\text{-}group(b\oplus c)}, \\ \mathsf{formed\text{-}group(a\oplus b\oplus c)} \end{array}\right\}$$

Given the collective nature of the predicate *formed a group*, the alternatives in (17-b) are logically independent from one another. As a result, ANS will presuppose that a unique group was formed. Indeed, the partition on the context created through the application of Exh on each member of (17-b) will only contain propositions in which exactly one group was formed. This is because given the logical independence of the alternatives in (17-b), all those distinct from the prejacent are negated.

[4] The following are equivalent:
$\Box\ \exists x \in \{\mathsf{f}, \mathsf{r}\} : \mathsf{we\text{-}read}(x) = \Box\ \mathsf{we\text{-}read(f)} \vee \mathsf{we\text{-}read(r)}$

This problem can be avoided if we once again assume that complex *wh*-interrogatives have the option to range over GQs. More specifically, we can avoid uniqueness inferences if we assume that *which students* can range over generalized conjunctions of students.

(18) $[\![\text{which students}]\!] = \lambda P.\, \exists Q \in \{\lambda f.\forall x \in X : f(x) \mid X \subseteq \mathsf{students}\} \;:\; P(Q)$

Assuming for (16-a) a structure similar to (14-a), we obtain the following set of alternatives.

(19) $$\left\{\begin{array}{c} \mathsf{formed\text{-}group}(\mathsf{a}{\oplus}\mathsf{b}), \\ \mathsf{formed\text{-}group}(\mathsf{a}{\oplus}\mathsf{c}), \\ \vdots \\ \mathsf{formed\text{-}group}(\mathsf{a}{\oplus}\mathsf{b}) \wedge \mathsf{formed\text{-}group}(\mathsf{a}{\oplus}\mathsf{c}), \\ ②\mathsf{formed\text{-}group}(\mathsf{a}{\oplus}\mathsf{b}) \wedge \mathsf{formed\text{-}group}(\mathsf{b}{\oplus}\mathsf{c}), \\ \vdots \end{array}\right\}$$

The application of ANS to (19) will yield a partition of the context. The proposition in ② can be taken to correspond to the answer in (16-b). Once exhaustified, it will negate all innocently excludable alternatives in (19), generating the meaning in (20), which states that $\mathsf{a} \oplus \mathsf{b}$ and $\mathsf{b} \oplus \mathsf{c}$ each formed a group, and that nobody else did.

(20) $$\begin{array}{l} \mathsf{Exh}_{(19)}(\mathsf{formed\text{-}group}(\mathsf{a}{\oplus}\mathsf{b}) \wedge \mathsf{formed\text{-}group}(\mathsf{b}{\oplus}\mathsf{c})) = \\ \mathsf{formed\text{-}group}(\mathsf{a}{\oplus}\mathsf{b}) \wedge \mathsf{formed\text{-}group}(\mathsf{b}{\oplus}\mathsf{c}) \wedge \\ \qquad\qquad \neg\exists x \in \{\mathsf{a}{\oplus}\mathsf{c}, \mathsf{a}{\oplus}\mathsf{b}{\oplus}\mathsf{c}\} : \mathsf{formed\text{-}group}(x) \end{array}$$

3.3 Problems

Assuming that *wh*-items can quantify over GQs runs into a number of problems. On the one hand, it has been pointed out that modalized complex *wh*-interrogatives with singular restrictors do not lead to free choice inferences when answered with a disjunction of atomic books.

(21) a. Q: Which book are we required to read?
b. A: The French book or the Russian book.
$\not\rightsquigarrow$ $\Diamond$ we read the French book $\wedge$ $\Diamond$ we read the Russian book

A free choice effect is predicted to arise if we allow for *which book* to quantify over generalized disjunctions of books. To avoid this, one must stipulate that this sort of higher-order quantification is reserved for *wh*-items with plural restrictors.

A further problem with the account so far presented is in the choice of quantifiers over which complex *wh*-interrogatives can range. We have proposed that these items can range over at least generalized disjunctions and generalized conjunctions. However, the theory so far presented suggests that they cannot always range over both. Consider the case of collective predicates, repeated below.

(22) Which students formed a group?

Let us assume for *which students* the lexical entry in (23), where the phrase ranges over both generalized conjunctions and disjunctions of students.

(23) a. $[\![\text{which students}]\!] = \lambda P.\exists Q \in \text{(23-b)} : P(Q)$
b. $\{\lambda f.\ \exists x \in X : f(x) \mid X \subseteq \mathsf{student}\} \cup$
$\{\lambda f.\ \forall x \in X : f(x) \mid X \subseteq \mathsf{student}\}$

The set of answers denoted by (22) will contain propositions formed through all the disjunctions and conjunctions of students. However, such an answer set will fail to partition the context, resulting in undefinedness given our semantics for ANS[5]. To see this, consider the exhaustified meanings of the sentences in (24).

(24) a. $\mathsf{Exh}_{[\![(22)]\!]}(\mathsf{formed\text{-}group}(\mathsf{a}\oplus\mathsf{b}) \vee \mathsf{formed\text{-}group}(\mathsf{b}\oplus\mathsf{c})) =$
$(\mathsf{formed\text{-}group}(\mathsf{a}\oplus\mathsf{b}) \vee \mathsf{formed\text{-}group}(\mathsf{b}\oplus\mathsf{c})) \wedge$
$\neg\ \mathsf{formed\text{-}group}(\mathsf{a}\oplus\mathsf{c}) \wedge \neg\ \mathsf{formed\text{-}group}(\mathsf{a}\oplus\mathsf{b}\oplus\mathsf{c}) \wedge$
$\neg(\mathsf{formed\text{-}group}(\mathsf{a}\oplus\mathsf{b}) \wedge \mathsf{formed\text{-}group}(\mathsf{b}\oplus\mathsf{c}))$
b. $\mathsf{Exh}_{[\![(22)]\!]}(\mathsf{formed\text{-}group}(\mathsf{a}\oplus\mathsf{b})) =$
$\mathsf{formed\text{-}group}(\mathsf{a}\oplus\mathsf{b}) \wedge \neg\ \mathsf{formed\text{-}group}(\mathsf{a}\oplus\mathsf{c}) \wedge$
$\neg\ \mathsf{formed\text{-}group}(\mathsf{b}\oplus\mathsf{c}) \wedge \neg\ \mathsf{formed\text{-}group}(\mathsf{a}\oplus\mathsf{b}\oplus\mathsf{c})$

The intersection of the propositions in (24-a) and (24-b) is non-empty, hence no partition of the context can be made from a set which contains both. We must therefore assume an ambiguity for complex *wh*-interrogatives insofar as they can be taken to denote generalized disjunctions or generalized conjunctions of individuals. This leads into a further problem with the proposal, namely what restrictions exist on the type of quantifiers over which complex *wh*-phrases can in principle range. As has been noted, the set of GQs over which complex *wh*-phrases can range must be restricted to at least upward-monotone quantifiers. This is due to the fact that allowing for downward-monotone quantifiers would make it possible for (25-a) to be answered by prohibitions.

(25) a. Q: Which books are we required to read?
b. A: #None of the French books.
$\not\rightsquigarrow \Box\neg\mathsf{we\text{-}read}(\mathsf{f})$

Summarizing, assuming that *wh*-phrases can quantify over GQs forces us to make three stipulations. On the one hand, we must assume this type of quantification to be available only when the restrictor of the *wh*-item is singular. We must further stipulate that *wh*-items are ambiguous with respect to whether they quantify over generalized disjunctions or conjunctions. Finally, we must assume that these items cannot quantify over downward-monotone quantifiers.

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[5] This problem only arises if the meaning assumed for ANS is that of [7]. It will not arise if we follow [4] and assume ANS to presuppose only that there is within the question denotation a maximally informative true answer.
$[\![\text{ANS}]\!] = \lambda Q_{(st)t} : \exists! p \in Q[p(w) \wedge \forall q \in Q[q(w) \rightarrow p \subseteq q]].\ Q$

4 A Novel Approach

Higher-order readings of questions seem related to plurality: the first phenomenon, free choice, is only observed with plural *wh*-interrogatives, whereas the second is due to collective predication. In this section, we offer a novel account of these readings that derives their connection with plurality, rather than stipulate it. The resulting theory furthermore allows us to stick with the standard assumption that *wh*-items are quantifiers that range over individuals.

We argue that the source of higher-order readings of questions can be explicated by (i) the presence of an existential distributivity operator [2], and (ii) the possibility of binding the cover that serves as the restrictor of this operator by an existential quantifier. These two assumptions are independently motivated.

The first ingredient in our proposal, an existential covert distributivity operator, was proposed by [2] to account for homogeneity effects. These are illustrated in (26): distributivity is interpreted as a universal quantifier in positive sentences, but as an existential in negative ones.[6]

(26) a. Rico and Sonny are Italian.
$\rightsquigarrow$ *Both* Rico and Sonny are Italian.
b. Rico and Sonny aren't Italian.
$\rightsquigarrow$ *Neither* Rico nor Sonny are Italian.

[2] proposes an implicature account of the effects in (26). The proposal is that the covert distributivity operator is lexically weak: it denotes an existential quantifier, as shown in (27). However, in upward entailing environments, it can be exhaustified into a universal quantifier.

(27) $[\![\exists\text{-}\textsc{Dist}_C]\!] = \lambda x_e.\lambda f_{et} : \mathsf{Cov}(C, x).\ \exists x' \in C : x' \leq x \wedge f(x')$
where $\mathsf{Cov}(C, x)$ iff C covers x

The strengthening of $\exists\text{-}\textsc{Dist}_C$ is possible due to the exhaustification procedure presented in section 2 coupled with certain assumptions about the alternatives of sentences involving distributivity. We present a rough rendition of [2]'s analysis of (26-a). This sentence is associated with the LF in (28), where a covert exhaustification operator, which has Exh as its denotation, takes scope over the sentence (see [6]). In (28-a) we have the denotation of the prejacent and in (28-b) the set of its alternatives. A crucial property of the set of alternatives in (29-b) is that it is not closed under conjunction, which results in the set of innocently excludable alternatives being empty. Given that all of the prejacent's alternatives are innocently includable, they are all asserted, giving rise to universal quantification.

[6] Although it is possible that in (26-b) negation takes scope below the distributive quantifier, [10] shows that, in at least some cases, this is not a possible line of argumentation: *No boy read his books* is interpreted as implying that there isn't a single boy who read *any* of his books. Given that the definite description *his books* is bound by the negative generalized quantifier *no boy*, it must be interpreted in its scope, and, therefore, under the scope of negation.

(28) [$\text{EXH}_{\text{ALT}(\phi)}$ [$_{\phi}$ [Rico and Sonny $\exists$-DIST_C] italian]]

(29) a. $\exists x \leq \mathsf{r} \oplus \mathsf{s} : \mathsf{italian}(x) = \mathsf{italian}(\mathsf{r}) \vee \mathsf{italian}(\mathsf{s})$
b. $\text{ALT}(\phi) = \{\mathsf{italian}(\mathsf{r}) \vee \mathsf{italian}(\mathsf{s}), \mathsf{italian}(\mathsf{r}), \mathsf{italian}(\mathsf{s})\}$

(30) a. $\mathsf{IE}_{\text{ALT}(\phi)}(\mathsf{italian}(\mathsf{r}) \vee \mathsf{italian}(\mathsf{s})) = \emptyset$
b. $\mathsf{II}_{\text{ALT}(\phi)}(\mathsf{italian}(\mathsf{r}) \vee \mathsf{italian}(\mathsf{s})) = \text{ALT}(\phi)$
c. $\mathsf{Exh}_{\text{ALT}(\phi)}(\mathsf{italian}(\mathsf{r}) \vee \mathsf{italian}(\mathsf{s})) = \mathsf{italian}(\mathsf{r}) \wedge \mathsf{italian}(\mathsf{s})$

Note that the same won't happen in negative sentences: since the prejacent (*none of Rico and Sonny are Italian*) is the strongest alternative, exhaustification is vacuous. The contrast in (26) is thus naturally captured in this framework.

The second ingredient in our analysis is the assumption that the cover that serves as the restrictor of the covert distributivity operator can be existentially quantified over. [13], who defended an analysis of distributivity that crucially relied on pragmatically given covers, argues against such a possibility.

Nonetheless, [13]'s proposal seems too strong. First, it requires speech participants to have full knowledge about the organization of the objects in the world, and most of the time that is not the case (see [12] for similar point). Furthermore, there are cases in which covers do seem to be existentially quantified over. Among such cases we note the command in the first half of (31), where it is irrelevant how the subject is distributed over the VP (i.e., which cover is fed to the distributivity operator) so long as there is one such cover.[7]

(31) You three need to fix these bikes, and I don't care who fixes which.

We now have the tools to present our analysis of higher-order readings in questions. We first discuss free choice readings, then collective readings.

4.1 Free Choice

We propose the LF schema in (32-b) as representing the question in (32-a), where (i) $\exists$-DIST is stranded under the scope of the modal and (ii) existential closure over the covers scopes above **?**.

(32) a. Which books are we required to read?
b. $\text{ANS}[\lambda_p \exists \lambda_C [\text{which books} \lambda_x [\boldsymbol{?} p] [\text{require}[\exists\text{-DIST}_C\, x] \lambda_y [\text{you read } y]]]$

The meaning assigned to the prejacent of ANS is shown (33). Note that the proposition denoted by *We are required to read the French books or the Russian books* is in the set in (33): as shown in (34), this is made possible when the value of x in (33) is the sum of the French and the Russian books and the value of C is a set containing the French books and the Russian books.

[7] A reviewer points out that (31) is actually an instance of cumulativity, and should thus be handled by [11]'s $*$-operator. However, the $*$-operator can be seen as nothing more than universally quantifying over elements of an existentially quantified cover. Thus, we believe that, rather than having a lexical opposition between $*$ and $\exists$-DIST, we could simply have the latter, with the option of sometimes existentially quantifying over it.

(33) a. $\{\Box\, \exists x' \in C : x' \leq x \wedge \textsf{we-read}(x') \mid \textsf{books}(x) \wedge \textsf{Cov}(C, x)\}$

b. $\left\{\begin{array}{l} \Box\ \textsf{we-read}(\textsf{f}), \\ \Box\ \textsf{we-read}(\textsf{r}), \\ \Box\ \textsf{we-read}(\textsf{f}{\oplus}\textsf{r}), \\ \Box\ \textsf{we-read}(\textsf{f}) \vee \textsf{we-read}(\textsf{b}), \end{array}\right\}$

(34) $\Box\, \exists x' \in \{\textsf{f}, \textsf{r}\} : x' \leq \textsf{f}{\oplus}\textsf{r} \wedge \textsf{we-read}(x') = \Box\ \textsf{we-read}(\textsf{f}) \vee \textsf{we-read}(\textsf{r})$

(34) is in fact equivalent to the question set one obtains under the GQ analysis. However, a number of stipulations made under that analysis aer derivered here. ∃-Dist, being an existential quantifier, will restrict the set of answers to upward entailing quantifiers, and, furthermore, account for the plural-singular asymmetry. Indeed, given that complex *wh*-phrases restricted by singular nouns only range over singularities, ∃-Dist will apply vacuously. We thus predict the lack of free choice inferences with singular restrictors.

4.2 Collective Predicates

We assign for a question like (35-a) the set of of alternatives in (35-b). This set differs from the one predicted in a GQ theory insofar as it does not contain conjunctive answers such as "$\textsf{formed-group}(\textsf{a}{\oplus}\textsf{b}) \wedge \textsf{formed-group}(\textsf{a}{\oplus}\textsf{c})$", where multiple groups were formed.

(35) a. Which students formed a pair?

b. $\{\exists x' \in C : x' \leq x \wedge \textsf{form-group}(x') \mid \textsf{students}(x) \wedge \textsf{Cov}(C, x)\}$

c. $\left\{\begin{array}{l} \textsf{form-group}(\textsf{a}{\oplus}\textsf{b}), \\ \textsf{form-group}(\textsf{a}{\oplus}\textsf{b}), \\ \textsf{form-group}(\textsf{b}{\oplus}\textsf{c}), \\ \textsf{form-group}(\textsf{a}{\oplus}\textsf{b}{\oplus}\textsf{c}), \\ \textsf{form-group}(\textsf{a}{\oplus}\textsf{b}) \vee \textsf{form-group}(\textsf{a}{\oplus}\textsf{c}), \\ \textsf{form-group}(\textsf{a}{\oplus}\textsf{b}) \vee \textsf{form-group}(\textsf{a}{\oplus}\textsf{c}), \\ \textsf{form-group}(\textsf{a}{\oplus}\textsf{c}) \vee \textsf{form-group}(\textsf{b}{\oplus}\textsf{c}), \\ \vdots \end{array}\right\}$

where $[\![\text{students}]\!] = \{\textsf{a}, \textsf{b}, \textsf{c}, \textsf{a}{\oplus}\textsf{b}, \textsf{a}{\oplus}\textsf{c}, \textsf{b}{\oplus}\textsf{c}, \textsf{a}{\oplus}\textsf{b}{\oplus}\textsf{c}\}$

While it may appear as though this account fails to predict the lack of a uniqueness presupposition for (35-a), this is not so. Given that exhaustification is built into the semantics of the ANS operator, and given that conjunctive answers are absent from the set in (35-b), we predict disjunctive answers to become conjunctions after the application of ANS. As shown in the discussion of [2]'s account of distributivity, exhaustifying disjunctive alternative with respect to a set not closed under conjunction will give rise to conjunctive propositions. For example, an answer such as $\textsf{a}{\oplus}\textsf{b} \vee \textsf{b}{\oplus}\textsf{c}$ will take on a conjunctive meaning following the application of ANS. We therefore do not predict a uniqueness presupposition to arise in questions involving collective predication.[8]

(36) a. $\mathsf{IE}_{\text{(15-b)}}(\mathsf{a}{\oplus}\mathsf{b} \vee \mathsf{a}{\oplus}\mathsf{c}) = \{q \in \text{(15-b)} \mid \mathsf{a}{\oplus}\mathsf{b} \not\subseteq q \wedge \mathsf{a}{\oplus}\mathsf{c} \not\subseteq q\}$
b. $\mathsf{II}_{\text{(15-b)}}(\mathsf{a}{\oplus}\mathsf{b} \vee \mathsf{a}{\oplus}\mathsf{c}) = \text{(35-b)} - \mathsf{IE}_{\text{(13-b)}}(\mathsf{a}{\oplus}\mathsf{b} \vee \mathsf{a}{\oplus}\mathsf{c})$
c. $\mathsf{Exh}_{\text{(13-b)}}(\mathsf{a}{\oplus}\mathsf{b} \vee \mathsf{a}{\oplus}\mathsf{c}) = \mathsf{a}{\oplus}\mathsf{b} \wedge \mathsf{a}{\oplus}\mathsf{c} \wedge \neg\mathsf{b}{\oplus}c \wedge \neg\mathsf{a}{\oplus}\mathsf{b}{\oplus}\mathsf{c}$

4.3 A Possible Issue

Our analysis – as in the case of GQ theories – seems to make bad predictions for a sentence like (37-a). If this sentence had the LF in (37-b), its question denotation would be the set in (37-c), which is equivalent to (37-d) if $\mathsf{books} = \{\mathsf{a}, \mathsf{b}, \mathsf{a}{\oplus}\mathsf{b}\}$. Pointwise exhaustification of (37-d) yields the set in (38), which is unable to induce partition of a context set. The presupposition of ANS is thus not satisfied.

(37) a. Which books arrived?
b. ANS [λ_p $\exists$ λ_C which books λ_x [**?** p] [[$\exists$-DIST$_c$ x] λ_y [y was sold]]]
c. $\{\exists x' \in C : x' \leq x \wedge \mathsf{arrive}(x) \mid \mathsf{books} \wedge \mathsf{Cov}(C, x)\}$
d. $\{\mathsf{arrive}(\mathsf{a}), \mathsf{arrive}(\mathsf{b}), \mathsf{arrive}(\mathsf{a}) \vee \mathsf{arrive}(\mathsf{b}), \mathsf{arrive}(\mathsf{a}{\oplus}\mathsf{b})\}$

(38) $\{\mathsf{arrive}(\mathsf{a}) \wedge \neg\mathsf{arrive}(\mathsf{b}), \mathsf{arrive}(\mathsf{b}) \wedge \neg\mathsf{arrive}(\mathsf{a}), \mathsf{arrive}(\mathsf{b}) \wedge \mathsf{arrive}(\mathsf{a}),$
$(\mathsf{arrive}(\mathsf{a}) \vee \mathsf{arrive}(\mathsf{b})) \wedge \neg(\mathsf{arrive}(\mathsf{a}) \wedge \mathsf{arrive}(\mathsf{b}))\}$

The problem with the question denotation in (37-d) is the presence of disjunctive alternatives: once these are ignored, pointwise exhaustification of the question set of (37-a) is once again able to partition a context. The source of these alternatives is the presence of an existential quantifier in the question nucleus. Removing the existential quantifier from the question nucleus would also remove these alternatives from the question set. We see at least three different ways of doing this.

We could in principle say that the presence of ∃-DIST is optional. This solution is however problematic: if ∃-DIST was optional, we would expect homogeneity effects be so as well, which is not the case. Another solution would be to allow ∃-DIST to take scope outside the question nucleus. This solution would be equivalent to simply not having ∃-DIST in the sentence. Yet another approach would be to have EXH in the question nucleus – it would exhaustify ∃-DIST into a universal quantifier, and thus also eliminate disjunctive alternatives from the question set.

[8] Note that the conclusion from this section are not confined to our analysis of higher order readings of questions. It in fact is arguing against the necessity of having generalized conjunction in the question denotation of interrogatives given that one can access conjunctive readings via a more sophisticated procedure of exhaustification.

5 Conclusion

We have shown that assuming the presence of a covert existential distributivity operator allows us to derive the free choice effects of questions with universal modals in them, as well as the lack of uniqueness in questions involving collective predication.

Our proposal allows us to derive why singular *wh*-interrogatives differ from their plural counterparts insofar as that they do not generate free choice effects. We believe that to account for this difference in terms of the interaction between the number of the *wh*-item's restrictor and a distributivity operator is a natural path to follow.

Our analysis finally derives why, under the GQ view, it was necessary to restrict the domain of higher-order quantifiers to upward monotone GQs. This follows from the fact that the answers to questions are obtained via the presence of an existential quantifier in the question nucleus.

We take this work to provide new insight into the semantics of *wh*-interrogatives by incorporating new developments in the semantics of plurality. We do not believe the relationship between these two fields of study to be accidental. The semantics of interrogatives and plurality constitute areas of research which have seen fruitful development come from analyses involving exhaustification [2, 7]. The main insight of the present work is to establish a firm connection between plurality and questions.

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