Roads to necessitarianism

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Abstract

We show that each of three natural sets of assumptions about the conditional entails necessitarianism: that anything possible is necessary.

1 Introduction

We show that, given three different sets of assumptions about the logic of conditionals—each of which is very natural and each of which has been defended or assumed by many in the literature—we can prove that necessitarianism is true: that is, that anything which is possible is necessary—there are no merely contingent truths. For those disinclined to this conclusion, this provides motivation to reject at least one assumption from each set.

We work with a standard modal propositional language (with '$\supset$' the material conditional, '$\equiv$' the material biconditional, and '$\wedge$' and '$\neg$' conjunction and negation, respectively, and '$\Box$' the necessity operator as usual), augmented with the two-place connective '$>$' meant to correspond to 'If…then…' (indicative or subjunctive). We assume that the connectives other than '$>$' receive their standard semantical interpretation, with the connectives given their usual classical Boolean values and our modal operators receiving a normal modal semantics characterized by the *K* axioms (in the proofs that follow we distinguish those inferences that follow by the *Propositional Calculus* (*PC*) alone from those that follow from *K*, though of course *K* encompasses the propositional calculus). We assume further that our modal operators satisfy the *5* axiom, which is required to prove the second and third claims (though is not really essential for establishing the basic tensions we point to here).

We use '$\Gamma \models p$' to mean that in every possible world in every intended model where all the elements of Γ are true, p is true (when Γ is a singleton $\{q\}$ we write '$q \models p$', and when Γ is empty we write '$\models p$'). Note that a biconditional *Deduction Theorem* will obtain for our logic: $\Gamma \models (q \supset r)$ iff $\Gamma \cup \{q\} \models r$.[1]

Our three results all start with the following core assumptions, with Γ any set of sentences and p, q, r any sentences:

- *Restricted Aristotle's Thesis*: $\Gamma \models (\neg p > p) \supset \Box p$
- *Conditional Quodlibet*: $\Gamma \models \Box(\neg p) \supset (p > q)$
- *Import-Export*: $\Gamma \models (p > (q > r)) \equiv ((p \wedge q) > r)$

The first two principles are standard assumptions in the theory of conditionals, both validated, for instance, by the theories of Stalnaker (1968), Lewis (1973), Kratzer (1981). The first follows as one half of the standard definition in the logic of conditionals of $\Box p$ as $(\neg p > p)$; the idea is that $(\neg p > p)$ could only be true if p were in some sense guaranteed to be true no matter what. (The principle is a restricted version of a principle known as *Aristotle's Principle*, which says that $(\neg p > p)$ is never true. Modern theories of the conditional hold that *Aristotle's Principle* is valid only in the restricted sense specified here.) The second principle is closely related in intuitive motivation: it says that if p is necessarily false, then $(p > q)$ is true for any q. In other words, once p is definitively ruled out, the supposition that p is true is contradictory, and anything whatsoever follows from that supposition.

1 The *Deduction Theorem* famously does not hold on certain ways of thinking about entailment in modal logic, but it *will* hold for systems in which entailment is defined as preservation of truth in all worlds in all intended models, as we have defined it here (rather than preservation of validity). See Hakli & Negri 2011 for helpful discussion; see in particular §3 of that paper for a full exposition of the kind of system we have in mind here, and a proof that the *Deduction Theorem* is valid in that system. Note that we omit corner quotes throughout the paper for readability, and omit parentheses when there is no risk of ambiguity.

The third principle says that conditionals of the form $(p > (q > r))$ and of the form $((p \wedge q) > r)$ are always equivalent. This principle is more controversial than the first two. It has been difficult to find intuitive counterexamples to *Import-Export* (though see Kaufmann 2005, Etlin 2008 for attempts), and *Import-Export* is validated by many theories of the conditional (see e.g. Kratzer 1981, McGee 1985, Gillies 2009, Khoo & Mandelkern 2018, Mandelkern 2018). But Gibbard (1981) showed that, together with *Modus Ponens* (which says that $\{p, p > q\} \models q$) and *Conditional Deduction* (which says that, if $p \models q$, then $\models (p > q)$), *Import-Export* leads to the collapse of the natural language conditional '$>$' to the material conditional '$\supset$'. As we discuss in the conclusion, our conclusions here are closely related to this collapse (a collapse which is widely taken to be unacceptable), and, like Gibbard's, show a tension between *Import-Export* and other apparently natural principles. The tensions we point to, however, are different from those Gibbard identifies, because unlike Gibbard, we do not assume *Modus Ponens* anywhere, and we assume *Conditional Deduction* only in the second proof.

2 *Right Monotonicity*

We now explore three different ways of augmenting these three core principles with further principles which allow us to obtain our absurd result. The first, and simplest, approach takes on board the following principle:

- *Right Monotonicity*: If $\models p > q$ and $s \models q \supset r$ then $\Box s \models p > r$

Right Monotonicity encodes the very natural and widely validated assumption that, if $(p > q)$ is always true; and if we are at a world where all accessible worlds are s-worlds; and if every s world which is a q-world is also an r-world, then $(p > r)$ is also true at our world. In other words, if '$\Box$' delimits the worlds that are accessible in evaluating our conditional at some given world; if, at any world, all the p-worlds relevant to evaluating the conditional are q-worlds; and if all the accessible q-worlds from our given world must also be r worlds, then $(p > r)$ must also be true at our given world. (We don't mean to commit to any particular analysis of the conditional in this paper, but this way of talking is neutral across many different analyses, and can be helpful in seeing the motivation for principles like this one.)

Right Monotonicity, together with the three assumptions at the outset, entails $\Diamond p \models \Box p$. The proof is in an Appendix.[2]

3 *Conditional Deduction* and *Triviality*

The second approach takes on board two further principles:

- *Conditional Deduction:* If $p \models q$ then $\models p > q$
- *Triviality*: If $\models p > q$ and $\models p > \neg q$ then $\models \neg p$

2 A slightly different result can be obtained by an even weaker form of monotonicity, plus a second principle:

- *Weak Right Monotonicity*: If $\models p > q$ and $\models q \supset r$ then $\models p > r$
- *Just Add r*: If $\models p > q$ then $\models (p \wedge r) > (q \wedge r)$

Weak Right Monotonicity is weaker than *Right Monotonicity* in that nothing ever appears to the left of '$\models$'. *Just Add r* says that if $(p > q)$ is invariably true, then adding the same sentence to the antecedent and consequent should not affect the truth value. We could motivate it as follows: $(r > r)$ is always true: all the relevant r-worlds are always r-worlds. If $(p > q)$ is also always true, then all the relevant p-worlds are always q-worlds. But then all the relevant worlds which are both p-worlds and r-worlds should also be both q-worlds and r-worlds. *Weak Right Monotonicity* and *Just Add r*, together with our three core assumptions, let us argue to the conclusion that $\models (\Diamond p > \Box p)$ (this conclusion slightly differs from the main one we focus on here, but is in the same spirit). The proof is left to the reader: the key steps comprise showing that $\models (\neg p > (p > \neg p))$ in the usual way, from which it follows by *Just Add r* that $\models (\neg p \wedge \Diamond p) > ((p > \neg p) \wedge \Diamond p)$. Then we show from *Restricted Aristotle's Thesis* that $\models ((p > \neg p) \wedge \Diamond p) \supset p$. *Weak Right Monotonicity* gives us $\models (\neg p \wedge \Diamond p) > p)$; *Import-Export*, *Weak Right Monotonicity*, and *Restricted Aristotle's Thesis* get us to the conclusion that $\models (\Diamond p > \Box p)$.

Conditional Deduction says that, if p entails q, then $(p > q)$ is a theorem; this is the very natural idea that the natural language conditional is bounded from above by entailment. *Triviality* is essentially a corollary of *Restricted Aristotle's Thesis*, under mild further assumptions, and can be motivated in the same way.

These two principles, in the presence of our three core principles, again let us prove that $\Diamond p \models \Box p$; the proof, again, is in the Appendix.

4 *Nothing Added*

The final approach takes on board just one further principle:

- *Nothing Added*: If $\models p > q$ then $\Gamma \models (p > (q > r)) \equiv (p > r)$

Nothing Added is intuitive for roughly the same reason that *Conditional Deduction* is. If $(p > q)$ is a theorem, then q has no intercessionary power, as it were, in a conditional of the form $(p > (q > r))$: q contributes nothing more beyond what p already contributed (see Mandelkern 2018 for further discussion). Once more, *Nothing Added* in the presence of our three core principles lets us prove that $\Diamond p \models \Box p$; the proof, again, is in the Appendix.

5 Conclusion

Thus our three core assumptions—*Restricted Aristotle's Thesis, Conditional Quodlibet*, and *Import-Export*—paired either with *Right Monotonicity*, or with *Conditional Deduction* and *Triviality*, or with *Nothing Added*, let us prove that anything possible is necessary. From a metaphysical perspective, this would mean that there are no merely contingent truths. From a model-theoretic perspective, this would mean that any canonical model for our language must be solipsistic (i.e., every world can access at most itself under the accessibility relation for possibility and necessity).

From the perspective of most modern theories of the conditional, this would mean that the natural language conditional '$>$' is the material conditional '$\supset$', since most theories of the conditional coupled with a solipsistic accessibility relation collapse to the material conditional. From any perspective, most (apart from already committed necessitarians) would agree that the conclusion is insupportable: in particular, there is near consensus that '$>$' is not '$\supset$' (see Khoo & Mandelkern 2018 for recent discussion). Thus some of these principles must be rejected.

A Appendix

A.1 First Proof: *Right Monotonicity*

i. $\models \Box(\neg(\neg p \wedge p)) \supset ((\neg p \wedge p) > \neg p)$ — *Conditional Quodlibet*

ii. $\Box(\neg(\neg p \wedge p)) \models ((\neg p \wedge p) > \neg p)$ — *Deduction Theorem*, (i)

iii. $\models ((\neg p \wedge p) > \neg p)$ — *K*, (ii)

iv. $\models (\neg p > (p > \neg p))$ — *Import-Export*, (iii)

v. $\models \Diamond p \supset \neg(p > \neg p)$ — *Restricted Aristotle's Thesis, PC*

vi. $\models \Diamond p \supset ((p > \neg p) \supset p)$ — *PC*, (v)

vii. $\Diamond p \models ((p > \neg p) \supset p)$ — *Deduction Theorem*, (vi)

viii. $\Box(\Diamond p) \models \neg p > p$ — *Right Monotonicity*, (iv), (vii)

ix. $\Box(\Diamond p) \models \Box p$ — *Restricted Aristotle's Thesis*, *PC*, (viii)

x. $\Diamond p \models \Box(\Diamond p)$ — *5*

xi. $\Diamond p \models \Box p$ — *PC*, (ix), (x)

A.2 Second Proof: *Conditional Deduction* and *Triviality*

We note first that these the following is entailed by our definition of entailment and our classical semantics for '$\wedge$':

- *Monotonicity:* If $p \models q$ then $(p \wedge r) \models q$

Then:

i. $\models \Box(\neg((\Diamond p \wedge \neg p) \wedge p)) \supset (((\Diamond p \wedge \neg p) \wedge p) > \neg p)$

Conditional Quodlibet

ii. $\Box(\neg((\Diamond p \wedge \neg p) \wedge p)) \models ((\Diamond p \wedge \neg p) \wedge p) > \neg p$

Deduction Theorem, (i)

iii. $\models ((\Diamond p \wedge \neg p) \wedge p) > \neg p$ — *K*, (ii)

iv. $\models (\Diamond p \wedge \neg p) > (p > \neg p)$ — *Import-Export*, (iii)

v. $\models \Diamond p \supset \neg(p > \neg p)$ — *Restricted Aristotle's Thesis, PC*

vi. $\Diamond p \models \neg(p > \neg p)$ — *Deduction Theorem*, (v)

vii. $(\Diamond p \wedge \neg p) \models \neg(p > \neg p)$ — *Monotonicity*, (vi)

viii. $\models (\Diamond p \wedge \neg p) > \neg(p > \neg p)$ — *Conditional Deduction*, (vii)

ix. $\models \neg(\Diamond p \wedge \neg p)$ — *Triviality*, (iv), (viii)

x. $\models \Diamond p \supset p$ — *PC*, (ix)

xi. $\models \Box\Diamond p \supset \Box p$ — *K*, (x)

xii. $\models \Diamond p \supset \Box\Diamond p$ — *5*

xiii. $\models \Diamond p \supset \Box p$ — *PC*, (xi), (xii)

xiv. $\Diamond p \models \Box p$ — *Deduction Theorem*, (xiii)

A.3 Third Proof: *Nothing Added*

i.	$\models \Box(\neg(\neg p \wedge p)) \supset (\neg p \wedge p) > \neg p)$	*Conditional Quodlibet*
ii.	$\Box(\neg(\neg p \wedge p)) \models (\neg p \wedge p) > \neg p)$	*Deduction Theorem*, (i)
iii.	$\models (\neg p \wedge p) > \neg p)$	*K*, (ii)
iv.	$\models (\neg p > (p > \neg p))$	*Import-Export*, (iii)
v.	$\models \Diamond p \supset \neg(p > \neg p)$	*Restricted Aristotle's Thesis, PC*
vi.	$\models \Box\Diamond p \supset \Box(\neg(p > \neg p))$	*K*, (v)
vii.	$\models \Diamond p \supset \Box\Diamond p$	*5*
viii.	$\models \Diamond p \supset \Box(\neg(p > \neg p))$	*PC*, (vi), (vii)
ix.	$\Diamond p \models \Box(\neg(p > \neg p))$	*Deduction Theorem*, (viii)
x.	$\Diamond p \models \Box(\neg(p > \neg p)) \supset ((p > \neg p) > (\neg p > p))$	*Conditional Quodlibet*
xi.	$\Diamond p \models (p > \neg p) > (\neg p > p)$	*PC*, (ix), (x)
xii.	$\Diamond p \models ((p > \neg p) \wedge \neg p) > p$	(xi), *Import-Export*
xiii.	$\Diamond p \models (((p > \neg p) \wedge \neg p) > p) \equiv ((\neg p \wedge (p > \neg p)) > p)$	*PC*
xiv.	$\Diamond p \models (\neg p \wedge (p > \neg p)) > p$	(xii), (xiii), *PC*
xv.	$\Diamond p \models \neg p > ((p > \neg p) > p)$	(xiv), *Import-Export*
xvi.	$\Diamond p \models (\neg p > ((p > \neg p) > p)) \equiv (\neg p > p)$	(iv), *Nothing Added*
xvii.	$\Diamond p \models \neg p > p$	(xv), (xvi), *PC*
xviii.	$\Diamond p \models \Box p$	*Restricted Aristotle's Thesis*, (xvii), *PC*

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