Semantic Expressivism for Epistemic Modals*

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Abstract

Expressivists about epistemic modals deny that 'Jane might be late' canonically serves to express the speaker's acceptance of a certain propositional content. Instead, they hold that it expresses a lack of acceptance (that Jane isn't late). Prominent expressivists embrace pragmatic expressivism: the doxastic property expressed by a declarative is not helpfully identified with (any part of) that sentence's compositional semantic value. Against this, we defend semantic expressivism about epistemic modals: the semantic value of a declarative from this domain is (partly) the property of doxastic attitudes it canonically serves to express. In support, we synthesize data from the critical literature on expressivism—largely reflecting interactions between modals and disjunctions—and present a semantic expressivism that readily predicts the data. This contrasts with salient competitors, including: pragmatic expressivism based on domain semantics or dynamic semantics; semantic expressivism à la Moss [2015]; and the bounded relational semantics of Mandelkern [2019].

Keywords: epistemic modals; expressivism; domain semantics; update semantics; state-based semantics; assertibility logic.

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1 Introduction

Expressivism about a domain of discourse is, broadly, the thesis that a canonical utterance from that domain does not state a fact but, rather, expresses a property of the speaker's attitudes, so as to engender coordination on that property among her interlocutors. For instance, an expressivist about the normative domain may hold that such sentences canonically express properties of an agent's planning state [Gibbard, 2003] or preferences [Silk, 2015, Starr, 2016a]. Expressivism is typically glossed as a theory of meaning. As [Gibbard, 2003, pp.5-6] frames it: "to explain the meaning of a term, explain what states of mind the term can be used to express". Stated barely, this exhibits various ambiguities: what is it to express a state of mind? What aspect of meaning is relevant? What form of explanation suffices? For our part, it will be useful to construe expressivism about a domain, more precisely, as the thesis that the *object of assertion* ('communicative value') of an assertion from that domain is a property of attitudes that is (i) determined, in context, by that sentence's *compositional semantic value* and (ii) is not a belief in a descriptive content.[1]

Typically, expressivism has been associated with a stronger thesis: the state of mind that a sentence in the domain expresses serves as (part of) that sentence's compositional semantic value. Call this *semantic* expressivism. Compare Rosen [1998], discussing Blackburn [1993]:

> The centerpiece of any quasi-realist [expressivist] 'account' is what I shall call a psychologistic semantics for the region: a [recursively defined] mapping from statements in the area to the mental states they 'express' when uttered sincerely.[Rosen, 1998, p.387].

Certainly, prominent *critics* of metaethical expressivism have targeted semantic expressivism. The classic problem in the area is the so-called *Frege-Geach problem*, loosely described as the worry that a credible 'psychologistic semantics' is not possible [Geach, 1965, Schroeder, 2008a].

However, recent work on expressivism (especially, but not solely limited to, that on epistemic modals) has tended to reject semantic expressivism. As we see it, Yalcin [2011, 2012], Charlow [2015], Starr [2016a] and Willer [2017] fall in this camp. These are *(merely) pragmatic* expressivists, holding that the property of attitudes expressed by a sentence is *non-trivially* determined by - and so distinct from - the components of the sentence's compositional semantic value, along with context.[2]

An (alleged) advantage of merely pragmatic expressivism is that it neutralizes Frege-Geach worries [Yalcin, 2018a]. This is intended to flow from its avoidance of a sharp bifurcation of the semantics, and better sensitivity to the distinctive theoretical roles of compositional semantic values and objects of assertion. Although the discourse role of sentences from the target domain (e.g. moral language expressing preferences) differs from the role of standard, factual sentences (e.g. straightforwardly descriptive language expressing beliefs), this difference need not, for the pragmatic expressivist, trickle backwards to semantics. She can assign an entirely

[1] On the distinction between compositional semantic value and object of assertion, see Dummett [1973], Lewis [1980], Stanley [1997], Ninan [2010], Rabern [2012], Yalcin [2014]. The term 'expressivism' has a long history. We do not pretend that our construal accommodates every forerunner, or explicit usage to which it has been put. It fits uneasily with, for instance, emotivism, or the tradition Yalcin [2018a] labels *compositional force expressivism*. A metaethical expressivist in the latter tradition holds that an utterance of 'Stealing is wrong' performs a non-assertoric speech act and so is misleadingly classed as an assertion. Compare: Incurvati and Schlöder [2019] argue that 'might' is best explicated in terms of the speech act of *weak assertion*, rather than assertion *per se*.

[2] We leave the qualifier 'merely' implicit in the rest of the text.

uniform type of entity (e.g. a function from a tuple of parameters to an extension, or a context change potential) as the semantic value of all expressions in their language, and then explain the distinctly expressivist features of the domain in question in terms of pragmatics/postsemantics.[3] This invites the standard techniques of formal semantics, which are well-equipped for explaining how sentences from the target domain interact with (e.g. compose with) and stand in logical relationships (entailment, contradiction, etc.) with sentences not in the domain. The rejection of semantic orthodoxy is avoided, along with its special burdens.

This paper, however, defends semantic expressivism for the domain of epistemic modals. Our primary argument will be empirical: we argue that our version of semantic expressivism provides a better account of the interaction of epistemic modals and disjunction than extant pragmatic expressivists. As a passing remark, we also find it plausible that our system shares the aforementioned advantages of pragmatic expressivism: it treats compositional semantic values as a sufficiently uniform type and is significantly continuous with standard formal semantics. To foreshadow: in our proposed system—*modal propositional assertibility semantics*—sentences will be evaluated only with respect to an information state (representing a doxastic state). Roughly, p will hold at state $\mathbf{s}$ if p is assured to be true by $\mathbf{s}$ and $\neg p$ will hold if p is ruled out by $\mathbf{s}$. The modal claim $\Diamond p$—that p might be the case—holds if $\mathbf{s}$ is compatible with p. A disjunction will hold if $\mathbf{s}$ can be split into an exhaustive pair of substates, each supporting a disjunct. Altogether, our system encapsulates plausible *assertibility conditions*. Indeed, our system is fruitfully interpreted as treating assertibility conditions as primary in semantics, rather than, say, truth conditions or context change potentials. This yields expressivism directly, in accord with the 'assertability expressivism' advocated by Schroeder [2008b]. Abstractly speaking, the form of a basic 'semantic descriptivism' will be mimicked, but, crucially, possible worlds are replaced with doxastic states.

Our system is an entry in a rich and growing tradition of state-based semantics for epistemic modals. Our most proximate forerunners are Hawke and Steinert-Threlkeld [2015, 2018], Aloni [2016b], Lin [2016] and Steinert-Threlkeld [2017]. Less proximately, such systems may be seen as variations on the data semantics of Veltman [1985] that incorporate the intensional account of disjunction mentioned above.[4] This disjunction has itself a storied history, having been deployed for possibility semantics [Cresswell, 2004], dependence logic [Väänänen, 2008] and inquisitive logic [Punčochář, 2016].[5]

The state-based systems in question yield various important linguistic applications: in particular, natural accounts of *free choice effects* and *epistemic contradictions* [Hawke and Steinert-Threlkeld, 2015, Aloni, 2016b, Lin, 2016, Hawke and Steinert-Threlkeld, 2018]. Against this backdrop, we argue that our system has notable empirical advantages over nearby variations in the literature and a host of other important rivals. In particular, we leverage these advantages to bolster semantic expressivism over pragmatic expressivism. We focus, therefore, on an in-house conflict: assuming sympathy for expressivism in general, what is the best way to *be* an expressivist? We will not engage with anti-expressivist positions such as contextualism or relativism.

§2 presents preliminaries on expressivism for epistemic modals and a logical setting for comparing various pragmatic and semantic expressivist theories. §3 collects and analyzes myriad data concerning epistemic modals and disjunctions, boiling down to four logical constraints. §4 shows that two basic and prominent pragmatic expressivist theories (domain semantics and

[3] See MacFarlane [2014] for the term 'postsemantics' and Yalcin [2011, 2018a] for its use in pragmatic expressivism.

[4] See Cariani [2017] for a critical overview of the broad family of 'intensional accounts' of disjunction.

[5] It is also in the spirit of a semantics for disjunction proposed by Beth: see van Dalen [1986, pp. 246-252].

dynamic semantics) do not meet the constraints. §5 presents our own semantic expressivist theory. We show that it meets all the constraints from §3 and is otherwise logically conservative. We also note advantages over similar systems. §6 presents and critiques more radical and sophisticated rivals: in particular, recent proposals due to Moss [2015] and Mandelkern [2019]. Finally, §7 considers some alterations to our system that yield additional logical constraints that have initial plausibility. We reject both the alterations and the proposed constraints.

2 Preliminaries

2.1 *Information, Expression and Assertibility*

Throughout, we use a standard formal propositional language to capture the (apparent) logical form of ordinary claims of interest. We think of our atoms as atomic indicative claims (i.e. simple predications). Unless explicitly stated otherwise, we use p and q as variables for *literals* in our language: ranging over atoms and negated atoms. $\Diamond\varphi$ is to be read as 'it might be that φ' and $\neg\Diamond\varphi$ as 'it can't be that φ'. We use $\vee$, $\wedge$ and $\neg$ to formalize natural language disjunction, conjunction and negation.

We need a precise mathematical model of an agent's doxastic state and of a property of such a state. Following tradition, we model doxastic states as sets of possible worlds: those compatible with what the agent knows/believes.[6] We model a property of such states as a set of such states: those that have the property in question.

Definition 1 (Information Model)**.** An *information model* $\mathscr{M}$ is a pair: a set of worlds W and a valuation V for the propositional atoms.

Definition 2 (Information State, Information Type)**.** Given information model $\mathscr{M}$, an *information state* $\mathbf{s}$ is a subset of W. An *information type* is a set of information states. We also refer to information types as *properties of an information state*.

To model doxastic states as information states is, intuitively, to model them in terms of their *total descriptive content*. This yields welcome flexibility: nothing in our *formal* theory commits one to taking descriptive claims to express belief rather than, more broadly, acceptance, or an alternative attitude like knowledge. Emphasis on belief is just a convenience.[7]

Given $\mathscr{M}$, we write $\mathbf{s} \Vdash \varphi$ to mean that φ is *assertible* relative to information state $\mathbf{s}$. We also say that $\mathbf{s}$ *supports* φ. An *account of* $\Vdash$ is a formal framework that aims to capture the properties of ordinary language assertibility. Throughout 'assertibility' is understood in a purely *epistemic* sense: p is assertible, in context, exactly when the speaker accepts a body of information that *establishes* p. Likewise, p is deniable exactly when the speaker's information *refutes* p. Assertibility, therefore, is prior to Gricean considerations of rational cooperation. For salience, we follow the literature on indicative conditionals in using 'assert*i*bility' rather than 'assert*a*bility', with the latter incorporating Gricean considerations [Bennett, 2003, Burgess, 2009].

Definition 3 (Assertoric Consequence)**.** Given an account of $\Vdash$, we say that ψ is an *assertoric consequence* of $\varphi_1, \ldots, \varphi_n$, and write $\varphi_1, \ldots, \varphi_n \Vdash \psi$, just in case, for every information model $\mathscr{M}$, if $\mathbf{s}$ supports all of $\varphi_1, \ldots, \varphi_n$ then $\mathbf{s} \Vdash \psi$.

[6]See Yalcin [2018b] for attractive refinements that only distract from the issues in the present paper. Also see §6.2 for a probabilistic refinement.

[7]It also means that the system introduced in this paper can be understood as a novel *informational* account of logic, as per Bledin [2014]. A semantic expressivist must depart from a purely 'informational account' when she turns to language that requires *multiple contents* or a *mix of diverse attitudes* for its evaluation.

Read $\varphi \Vdash \psi$ as: an agent's information is such that φ is properly assertible only if their information renders ψ is assertible.

We make *expression* precise via an intuitive connection between assertibility and expression: the type of state expressed by φ is the property of rendering φ assertible.

Definition 4 (Expression)**.** Given $\mathscr{M}$ and $\Vdash$, the *property/type expressed by* φ is

$$\{\mathbf{s} \subseteq W : \mathbf{s} \Vdash \varphi\}$$

As we see it, this is a precise development of the proposal by Schroeder [2008b] that *expression* is best explained by appeal to assertibility conditions.

2.2 *Expressivism: Semantic vs Pragmatic*

A primary motivation for expressivism about epistemic modals—expressions such as 'might', 'must', 'likely', 'probably'—comes from considering ordinary conversations.[8]

(1) A: I can't find my keys.

B: They might be in the drawer.

A: No, I already checked there.

Consider *descriptivism*. Suppose that B's utterance were factual, i.e. it states the fact that the keys being in the drawer is compatible with some contextually-determined body of information and attempts to deposit that fact in the common ground.[9] For B to be warranted in making the utterance, she must have good epistemic status with respect to that fact.[10] A natural candidate for the contextually-determined body of information is B's own belief state. But if what B's utterance does is state the fact that the keys being in the table is compatible with her belief state, then A is unjustified in rejecting the assertion as she does: B knows her own doxastic state better than anyone. If the contextually-determined body of information is expanded to one where A's rejection would be justified (e.g. the information compatible with both A's and B's beliefs), then B's utterance would be unwarranted. So it is difficult to fit this run-of-the-mill conversation into a standard, fact-stating key.

Expressivism offers an elegant alternative analysis. B's utterance expresses that the keys being in the drawer is compatible with her beliefs. A's final utterance expresses that she believes that the keys are not in the drawer. Assuming that A and B's beliefs respectively match the states that their utterances canonically express, and that these particular beliefs are transparent via introspection, the expressivist finds no mystery in why A and B *rightly* assert as they do. Likewise, there is no mystery as to why their stand in disagreement: a single agent cannot hold beliefs that render both utterances appropriate. While more arguments for expressivism about epistemic modals are available, our purpose here is not to defend it from contextualist [von Fintel and Gillies, 2011, Dowell, 2011] and relativist [MacFarlane, 2011] rivals. Rather, we take expressivism for granted and adjudicate in favor of specifically semantic expressivism.

We now render *semantic* expressivism about *epistemic modals* with precision. For the purposes of this paper, every expressivist about epistemic modals characteristically holds:

[8] See von Fintel and Gillies [2011], MacFarlane [2011], Yalcin [2011] for similar analyses.

[9] As in the canonical picture of assertion from Stalnaker [1978].

[10] For present purposes, we do not care whether that status is knowledge (Williamson [1996]), reasonable belief (Lackey [2007]), or something else besides.

(2) **EXPRESSIVISM ABOUT $\Diamond$:** Given $\mathscr{M}$ and $\Vdash$, the *property/type expressed by* $\Diamond p$ is

$$\{\mathbf{s} \subseteq W : \mathbf{s} \nVdash \neg p\}$$

This characterization is sensible if one grants an idealization: that both $\Diamond p$ is assertible and $\neg p$ is not assertible exactly when one's information is compatible with p. This echoes the influential expressivist proposal of Yalcin [2011]. However, Yalcin [2011] argues that more nuance is ultimately desirable: Jane's current information might not rule out that it is raining in Topeka but, since she's never even heard about Topeka, she is not aptly described as believing, or positioned to assert, that it might be raining there. However, every system we consider is compatible with Yalcin's proposed refinement (i.e. doxastic states should be modeled as topic-sensitive) and, anyway, the issue seems orthogonal to our core discussion. We put it aside.

A formal semantic theory, standardly understood, recursively defines an interpretation function $[\![\cdot]\!]$ that assigns an extension to every linguistic expression, relative to certain parameters. The compositional semantic value of φ is usefully understood as $[\![\varphi]\!]$, a function from a tuple of parameters to an extension. Then, we characterize a *psychologistic semantics* by:

(3) **PSYCHOLOGISTIC SEMANTICS:** $[\![\varphi]\!]$ is a (potentially partial) function from information states to truth-values– 0 or 1.

More broadly, a psychologistic semantics represents a sentence's semantic value as an assignment of truth values to mental states. This usage of 'truth-values' is technical. 'Assertibility-values' is a perhaps more natural label: if $[\![\varphi]\!]^{\mathbf{s}} = 1$ indicates assertibility of φ given $\mathbf{s}$, then $[\![\varphi]\!]^{\mathbf{s}} = 0$ is naturally taken to indicate deniability. Partiality allows for $[\![\varphi]\!]$ and $\mathbf{s}$ where the former assigns *no* truth-value to the latter, accommodating effective trivalence. Then:

(4) **SEMANTIC EXPRESSIVISM:** A *semantic expressivist* characteristically accepts: (i) expressivism about $\Diamond$, (ii) a psychologistic semantics and (iii) a *direct account* of $\Vdash$ i.e. $\mathbf{s} \Vdash \varphi$ just in case $[\![\varphi]\!]^{\mathbf{s}} = 1$.

We take *pragmatic* expressivism, precisely, as expressivism about $\Diamond$ that rejects (4).[11]

There is a core sense in which holding (4) earns one the label of *'semantic'* expressivist. Assuming that Definition 4 fruitfully explicates *expression*, the mental type expressed by φ is characterized in terms of assertibility relation $\Vdash$. Now, (4) says that assertibility conditions are primary in semantics: the definition of $\Vdash$ is an essential part of that of $[\![\cdot]\!]$. In this case, the expressed mental type is given *directly* by the semantics. On the other hand, a 'pragmatic' expressivist, precisely understood, allows for a *gap* between the compositional semantic theory and $\Vdash$. For instance, she can posit that truth conditions (or context change potentials) are semantically primary, with $\Vdash$ defined post-semantically. Theorists that agree on the semantics might differ on this definition.

Recall two traditional glosses of 'semantic expressivism': the property of attitudes expressed by declarative φ plays the role of the compositional semantic value of φ; the semantic framework recursively assigns a property of attitudes to each declarative [Rosen, 1998]. Does our precise account honor these thoughts? We say 'yes', though it is important to note where

[11] As an anonymous reviewer notes, (4) allows for psychologistic semantics without expressivism. For instance, a descriptivist about $\Diamond$ can embrace a psychologistic semantics, with the semantic value of $\Diamond p$ assigning 1 to every doxastic state that accepts the fact described by $\Diamond p$.

we grant flexibility. A standard model of a property is its *characteristic function*: the function that assigns 1 to every object that has the property in question. By these lights, (4) understands $[\![\varphi]\!]$ as capturing the property of rendering φ assertible (identical to the property of attitudes expressed by φ). Indeed, if 0 indicates deniability, then $[\![\varphi]\!]$ intuitively captures *binary, opposed properties*. Compare the function that represents being short versus being tall by assigning 1 to short people, 0 to tall ones.

An equivalent way to model a property is as a set of objects: intuitively, those with the property. Now, a semantics that observes (4) is equivalent to a semantics that assigns two sets of information states to each φ, via mutual recursion: an *assertibility set* $\{\mathbf{s} \subseteq W : [\![\varphi]\!]^{\mathbf{s}} = 1\}$ and *deniability set* $\{\mathbf{s} \subseteq W : [\![\varphi]\!]^{\mathbf{s}} = 0\}$. The compositional semantic value of φ is thus equivalent to a *pair* of properties. The first component serves as the object of assertion for φ. Compare a sensible development of *semantic descriptivism*: the view that the compositional semantic value of a declarative is always a descriptive content. Suppose we model descriptive contents as information states. Then a semantics is, precisely, a version of *semantic descriptivism* just in case it recursively assigns, to each φ, a function $[\![\varphi]\!]$ from possible worlds to truth values. If trivalence is accepted, $[\![\varphi]\!]$ is partial, and so may assign an irreducible pair of information states to φ: a truth set $\{w \in W : [\![\varphi]\!]^{w} = 1\}$ and falsity set $\{w \in W : [\![\varphi]\!]^{w} = 0\}$. The first component is the object of assertion of φ. Semantic expressivism follows this template, replacing worlds with information states.

We present our own version of semantic expressivism in §5. First, we develop some criteria for assessing such a theory and thereby cast doubt on pragmatic expressivism.

3 Relevant Data

We now gather linguistic data that have been influential in recent literature on epistemic modals. As we proceed, we generalize away apparently arbitrary features, yielding some plausible constraints on the logic of assertibility: wide-scope free choice; inheritance; and systematic failures of disjunctive syllogism. The explanatory power of these generalizations (regarding the data) is taken as *prima facie* support for their truth. Likewise, a theory of assertibility that satisfies them is thereby bestowed abductive support. In contrast, while violating these constraints is not a death knell for a theory, it pressures the theorist to provide a similarly attractive explanation of the data. Indeed, we will argue that expressivists face peculiar difficulty in providing an alternative: standard Gricean maneuvers seem especially unconvincing through an expressivist lens.

3.1 *(Weak) Wide-Scope Free Choice*

Consider the following, asserted in March 2016:

(5) Bernie Sanders might or might not win the Democratic nomination.

As has been widely noted, if (5) is assertible, then intuitively so are:[12]

[12]Free choice issues for deontic modals were highlighted by Kamp [1973]. Zimmermann [2000] extends the discussion to epistemic modals and defends a semantic account of free choice phenomena. The debate between semantic and pragmatic approaches remains unresolved: see, for instance, Kratzer and Shimoyama [2002], Simons [2005], Geurts [2005], Aloni [2007], Chemla [2009], Barker [2010], Franke [2011], Aloni [2016b], Fusco [2014]. We do not endorse a definitive resolution. We claim only that there is pressure on *expressivists* to offer a semantic treatment of wide-scope free choice.

(6) Bernie Sanders might win the Democratic nomination.

(7) Bernie Sanders might not win the Democratic nomination.

Call this the *free choice effect*. It invites a natural expressivist construal. What property of their doxastic state did the asserter express with (5)? Intuitively, uncertainty: the speaker neither believes that Bernie will win nor that he will not. This is bolstered by noting that an interlocutor can disagree in two ways:

(8) a) No, he has no chance of winning.

b) No, he will definitely win.

Without offering a full theory of disagreement,[13] here's an appealing assessment: (5) expresses a lack of acceptance that Bernie will lose. If sincere, the speaker truly stands in that state. Pragmatically, her assertion invites her interlocutors to share in that lack. Someone who sincerely disagrees via (8a) firmly believes that Bernie has no chance. So, her doxastic state is incompatible with the speaker's. Further, she is well understood, pragmatically, as refusing the invitation to alter it. *Mutatis mutandis* for (8b).

Consider an explanatory hypothesis:

(9) $\Diamond p \vee \Diamond \neg p \Vdash \Diamond p \wedge \Diamond \neg p$

Call this *weak wide-scope free choice*. A bolder hypothesis is similarly compelling:

(10) $\Diamond p \vee \Diamond q \Vdash \Diamond p \wedge \Diamond q$

Call this *wide-scope free choice*.[14] Our theory in §5 delivers the stronger principle. It is generally convenient to emphasize weak wide-scope free choice, however.

To endorse (9) is to explain the free choice effect as exemplifying a logical principle governing assertibility. (Even pragmatic expressivists, who offer indirect accounts of $\Vdash$, take this logic to operate at a level prior to Gricean pragmatic reasoning.) Is a rival Gricean explanation easily at hand? Consider a plausible principle:[15]

(11) **DISJUNCTION IMPLICATURE:** if a speaker asserts $\varphi \vee \psi$ (with φ and ψ relevant to the discourse), this pragmatically implicates that the speaker believes neither $\neg\varphi$ nor $\neg\psi$.

Assuming that φ is compatible with an agent's beliefs if and only if the agent does not believe $\neg\varphi$, expressivists can render the consequent of (11) as saying that $\Diamond\varphi$ and $\Diamond\psi$ are both assertible. So, by expressivist-cum-Gricean lights, an assertion of $p \vee q$ implicates $(p \vee q) \wedge \Diamond p \wedge \Diamond q$: a cooperative speaker avoids misleading a hearer by saying $p \vee q$ if she believes $p \wedge \neg q$ or $\neg p \wedge q$. Now, apply (11) to $\Diamond p \vee \Diamond \neg p$: in this case $(\Diamond p \vee \Diamond \neg p) \wedge \Diamond\Diamond p \wedge \Diamond\Diamond\neg p$ is implicated. This is plausibly equivalent to $\Diamond p \wedge \Diamond \neg p$, as desired.

On closer inspection, this reasoning is flawed. Indeed, we claim that (11) is valid only if φ and ψ are $\Diamond$-free. To see this, we exhibit the Gricean reasoning in detail for $p \vee \neg p$ and $\Diamond p \vee \Diamond \neg p$. In what follows, we assume the expressivist thesis that $\Diamond p$ expresses that the agent does not believe $\neg p$ and that classical boolean logic holds for ordinary language disjunction (on the presumption the attraction of the Gricean strategy is to preserve this).

A cooperative, rational speaker X asserts $p \vee \neg p$. Hearer Y reasons that X's doxastic state is one of three exhaustive types:

[13] See Khoo [2015] for a friendly account.

[14] We consider *narrow-scope free choice* in §7.2.

[15] See(41) on p. 50 and pp. 59-61 of Gazdar [1979] and [Aloni, 2016a, Sect.4].

(a) X believes p

(b) X believes $\neg p$

(c) X neither believes p nor believes $\neg p$

However, if X were in (a) or (b), X would not have asserted $p \vee \neg p$. For, in each case, she could have asserted something more informative and briefer (respectively, p or $\neg p$). So, her state must be of type (c). So Y can infer that X is prepared to assert both $\Diamond p$ and $\Diamond \neg p$.

Compare: X asserts $\Diamond p \vee \Diamond \neg p$. Y considers. As before, X has to be in (a), (b) or (c). But now expressivism plus boolean disjunction produces a curious outcome: for *each* of (a), (b) and (c), X is in a position to say something more informative than $\Diamond p \vee \Diamond \neg p$. Respectively: $\Diamond p$; $\Diamond \neg p$; and $\Diamond p \wedge \Diamond \neg p$, each of which asymmetrically entails $\Diamond p \vee \Diamond \neg p$ by current lights. Thus Gricean reasoning gets Y nowhere: she fails to pinpoint a type of state for X that makes sense of her choice of assertion. Y can only conclude that X is not cooperative or rational after all. Thus, (11) does not straightforwardly generalize to assertions that contain epistemic vocabulary, and the Gricean must turn to more sophisticated strategies to account for free choice effects.

A bolder conclusion: expressivism plus boolean disjunction plus standard Gricean mechanisms has an unacceptable empirical consequence: $\Diamond p \vee \Diamond \neg p$ can never be rationally asserted. Far from undermining its support, standard Gricean mechanisms motivate the expressivist to accept weak wide-scope free choice: the troublesome Gricean reasoning of the previous paragraph derails if we deny that $\Diamond p \vee \Diamond \neg p$ is strictly weaker than $\Diamond p \wedge \Diamond \neg p$.

A final argument for a non-Gricean explanation: if the free choice effect merely reflected cooperative discourse, it would be cancellable and reinforceable. But, as Zimmermann [2000] observes, cancellation seems blocked. Consider some attempts:

(12) Bernie might or might not win. I don't know which.

(13) Bernie might or might not win. I'm uncertain whether he might win.

(14) # Bernie might or might not win. However, he definitely won't.

Far from canceling the implication of $\Diamond p \wedge \Diamond \neg p$, the second sentences in (12) and (13) intuitively *reinforce* this implication, by communicating uncertainty about p. Further, (14) sounds contradictory (unless one takes the second sentence to signal belief revision).[16]

What's more, reinforcement feels as redundant as reinforcing an entailment:

(15) Bernie might or might not win. In fact, he both might win and might not win.

[16] An anonymous reviewer suggests that wide-scope free choice *does* seem cancellable. Imagine this exchange in a game of Mastermind:

(15) A: It might be red or it might be blue.

(16) B: Yes, that's true, but only because it might be red.

B's response isn't obviously infelicitous. However, it strikes us as somewhat unusual, and tricky to paraphrase. Indeed, it becomes much worse if rephrased as an *explicit* rejection of 'it might be blue':

(1) # B: Yes, it might be red or it might be blue, even though it can't be blue.

Thus, it's unclear that 'only because' is interpreted by default as explicitly canceling one of the disjuncts, as needed. That said, such judgments are subtle and require investigation. Steinert-Threlkeld [2017, Appendix B] reports a preliminary experiment on similar exchanges, finding that B's assertion is judged worse for a wide-scope disjunction than for a narrow-scope one. We emphasize: our endorsement of wide-scope free choice (*qua* logical principle) stems not only from sampled judgments, but theoretical challenges a pragmatic treatment faces for expressivists in particular.

3.2 *Inheritance*

Dorr and Hawthorne [2013] observe that an utterance of (16) is typically heard as equivalent to (17).[17]

(16) Jennifer is at home and might be watching a movie.

(17) Jennifer is at home and might be watching a movie at home.

So, (16) seems unassertable by a speaker that believes that Jennifer only ever watches movies at the cinema. Call this phenomenon *inheritance*.

The order of the conjuncts does not seem to affect inheritance:

(18) Jennifer might be watching a movie and she is at home.

Indeed, the following sounds odd:

(19) # It can't be that Jennifer is watching a movie at home, but she might be watching a movie and she is at home.

We claim *only* that (16) and (18) uniformly exhibit inheritance. We need *not* claim that, in the final analysis, these utterances are equivalent in meaning or conversational effect. For instance, we agree that (18), while ultimately interpretable, sounds awkward compared to (16). Further, we do not deny that conjunction gives rise to systematic order effects. Consider:

(20) Maria got sick and was admitted to hospital.

(21) Maria was admitted to hospital and got sick.

(22) Maria might have gotten sick and was admitted to hospital.

(23) Maria was admitted to hospital and might have gotten sick.

(20) has a different meaning in conversation to (21), and (22) to (23). This difference—whether semantic or pragmatic—reflects time-sensitive or cause-sensitive features of ordinary language 'and'. We put this aside as a distraction (our system can be tweaked to incorporate such effects, but discussion is best left for elsewhere). Note that inheritance is evident in (22): the speaker presumably would deny that it might be that Maria got sick but was *not* admitted to hospital. Inheritance is evident in (23): the speaker presumably would deny that it might be that Maria was *not* admitted to hospital but got sick.

Dorr and Hawthorne [2013] also observe that inheritance survives embeddings. For instance, embed (16) and (18) under a disjunction:[18]

(24) Jennifer is (either) at home and might be watching a movie, or she is working late at the office.

(25) Jennifer is (either) working late at the office, or she might be watching a movie and she is at home.

Intuitively, the first can be paraphrased as: 'Jennifer is either at home and might *in this case* be watching a movie, or she is working late at the office'. The second as: 'Jennifer is either working late at the office, or she might be watching a movie at home'.

Finally, inheritance does not seem amenable to cancellation:

[17] Such observations are familiar from the literature on modal subordination: see Roberts [1989].

[18] Compare example (56) on page 153 of Klinedinst and Rothschild [2012].

(26) # Jennifer is at home and might be watching a movie, but isn't watching a movie at home.

(Also see (19).) All this bears against a *pragmatic* explanation of inheritance. First, the apparent symmetry is in tension with the pragmatic account of inheritance of Dorr and Hawthorne [2013]. On their account, as Cariani [2017] and Mandelkern [2019] observe, inheritance is an effect of processing order and should favor sentences where the constraining factual claim comes before the modal.[19] Second, robustness under embeddings and resistance to cancellation are *prima facie* evidence of a semantic phenomenon. In particular, doubt is cast on a broadly Moorean explanation of the data.[20]

Generalization yields plausible, symmetric assertibility principles:

(27) $p \wedge \Diamond q \Vdash \Diamond(p \wedge q)$

(28) $\Diamond q \wedge p \Vdash \Diamond(p \wedge q)$

(29) $(p \wedge \Diamond q) \vee \varphi \Vdash (p \wedge \Diamond(p \wedge q)) \vee \varphi$

(30) $\varphi \vee (p \wedge \Diamond q) \Vdash \varphi \vee (p \wedge \Diamond(p \wedge q))$

(31) $(\Diamond q \wedge p) \vee \varphi \Vdash (\Diamond(p \wedge q) \wedge p) \vee \varphi$

(32) $\varphi \vee (\Diamond q \wedge p) \Vdash \varphi \vee (\Diamond(p \wedge q) \wedge p)$

Call (29)-(32) the *disjunctive inheritance principles*.

3.3 *(Disjoined) Epistemic Contradictions*

Inheritance bears on a key issue. Following Yalcin [2007], recent discussion of expressivism has focused on *epistemic contradictions*—claims of the form $p \wedge \Diamond\neg p$ and $\neg p \wedge \Diamond p$—and *reverse epistemic contradictions*—claims of the form $\Diamond\neg p \wedge p$ and $\Diamond p \wedge \neg p$. For example:

(33) # Jo isn't tall but she might be.

(34) # Jo might be tall but she isn't.

These have an air of contradiction, which the expressivist explains as the expression of an impossible state. Note that reversing the order of the conjuncts makes little difference. Further, Yalcin [2007] observes that the infelicity survives embedding under an attitude verb or in the antecedent of a conditional. Mandelkern [2019] further observes that embedding two epistemic contradictions under a disjunction sounds as bad as a bare epistemic contradiction.

[19] Silk [2017] offers further criticisms of Dorr and Hawthorne [2013].

[20] Consider the simple descriptivist: she can account for (16) as follows. First, she claims that $p \wedge \Diamond q$ is assertible just in case the speaker knows $p \wedge \Diamond q$. Second, the speaker knows $p \wedge \Diamond q$ just in case every world compatible with her knowledge is a world where $p \wedge \Diamond q$ is true. It follows that every world compatible with her knowledge is a p and $\Diamond q$ world: so she knows p and knows $\Diamond q$. Third, since the statement of (16) is in a typical context, the speaker knows $\Diamond q$ just in case a world at which q is true is compatible with her knowledge. But this world must then also be a p world: so $p \wedge \Diamond(p \wedge q)$ is assertible. Or so the descripitivist's story goes. However, this explanatory strategy fails for (24). The speaker asserts $(p \wedge \Diamond q) \vee r$. The simple descriptivist concludes: every world compatible with the speaker's knowledge is either a $p \wedge \Diamond q$ world or an r world. Suppose that both types of world are compatible with her knowledge. It follows from the (second conjunct of the) former that her knowledge is compatible with a world at which q holds. But there is no reason to posit that any such world is a p world: they may all be $\neg p \wedge r$ worlds, in which case the speaker still rightly accepts $(p \wedge \Diamond q) \vee r$, by the simple descriptivist's lights. Descriptivists that embrace a more sophisticated truth-conditional theory - e.g. the weak bounded relational semantics of Mandelkern [2019] - have more room to maneuver. Our assessment of the bounded framework in §6.3 is therefore pertinent.

(35) # Jo isn't tall but she might be, or Jim isn't tall but he might be.

This is evidence that the infelicity has semantic roots. For our part, we take the infelicity of (35) to show that sentences of the forms $(\neg p \wedge \Diamond p) \vee (\neg q \wedge \Diamond q)$ and $(p \wedge \Diamond \neg p) \vee (q \wedge \Diamond \neg q)$ are never assertible. However, there is a caveat. Moss [2015] and [Silk, 2017, Sect. 3.4] suggest the data is nuanced: in certain contexts, a reverse epistemic contradiction can seem felicitous:

(36) If a tiger might be in the bushes but there isn't one, you should still run away. One can never be too careful. [Silk, 2017, p.1789]

Should this be granted as data? To our ears, (36) sounds odd. Things smooth out significantly if an information shift is made explicit: 'If, according to your current information, there might be a tiger, then you should run away—even if there isn't one'. We grant, therefore, that (36) is felicitous if it is sufficiently clear that the evaluation of the modal involves an implicit shift from the information available to the *speaker*. An expressivist can embrace this, for she can posit an implicit information-shifting operator, at the level of logical form [Yalcin, 2012]. However, all hands agree that 'out-of-the-blue' epistemic contradictions *generally* sound infelicitous, and that there is an intuitive explanation: epistemic modals communicate features of a salient information state and, without additional cues, the 'default' information is that which is available to the speaker. In that case, an epistemic contradiction communicates the impossible: this information both supports p and is compatible with $\neg p$. Thus, even if one grants that epistemic contradictions can be felicitous, there is a familiar, default use of 'might' that renders them infelicitous. A key expressivist claim, as we see it, is that this 'default' infelicity requires an expressivist explanation, in order to predict that epistemic contradictions *embed* like contradictions [Yalcin, 2007, 2011]. In this spirit, we proceed in treating epistemic contadictions as inassertible, on the understanding that we have the default use of 'might' in mind.

Now for the main moral of this section. We assume that assertions of the forms $\Diamond(p \wedge \neg p)$ and $\Diamond(\neg p \wedge p)$ are never assertible and embed like contradictions. In conjunction with inheritance principles, this predicts that epistemic contradictions (with a 'default' reading) are never assertible, and embed like contradictions. Consider these instances of (28) and (the combination of) (29) and (30):

(37) $\Diamond \neg p \wedge p \Vdash \Diamond(\neg p \wedge p)$

(38) $(p \wedge \Diamond \neg p) \vee (q \wedge \Diamond \neg q) \Vdash (p \wedge \Diamond(p \wedge \neg p)) \vee (q \wedge \Diamond(q \wedge \neg q))$

Since the conclusion is never assertible, the premise is never assertible.

The data that supports the oddness of epistemic contradictions is no more or less compelling than the data for inheritance. As Dorr and Hawthorne [2013] suggest, it would be *ad hoc* to use different mechanisms to account for inheritance and the infelicity of (reverse) epistemic contradictions. Going forward, we emphasize inheritance: the infelicity of (reverse) epistemic contradictions is a basic consequence.

3.4 *Disjunctive Syllogism and Schroeder's Constraints*

We assume, as is standard, that 'might' and 'must' are duals. We assume little else about 'must'. For instance, we do *not* assume that 'Jane must be late' is semantically or pragmatically equivalent to 'It is certain that Jane is late'.

Now, the literature has noted apparent failures of disjunctive syllogism.[21]

[21] This is reported, with different examples, in Schroeder [2015, Sect.5.2], in work that has been circulated since at least 2011, and in Klinedinst and Rothschild [2012, Sect. 3.2], crediting Seth Yalcin. For an overview and discussion, see Cariani [2017]. Also see Moss [2015], Swanson [2016], Yu [2016].

Context: Meg flips a coin. We consider the coin fair. As it lands, she covers it, then puts it back in her pocket. None of us saw whether it landed heads or tails. Sam then walks into the room and asks us what happened.

Given our information, cautious responses are appropriate:

(39) It might have landed heads and it might have landed tails.

(40) Either it landed heads or it must have landed tails.

(41) It might be that it didn't land heads.

(42) It might be that it didn't land tails.

On the other hand, the following responses seem inappropriate:

(43) # It landed heads.

(44) # It must have landed heads.

(45) # It landed tails.

(46) # It must have landed tails.

Assume that (42) is assertorically equivalent to 'It isn't the case that the coin must have landed tails' and that (41) disagrees with 'It landed heads'. In that case, note a seeming counter-example to the validity of disjunctive syllogism: we have located p and q (namely, 'It landed heads' and 'It landed tails') such that $p \vee \Box q$ and $\neg\Box q$ are assertible in context, but p is not.[22]

Compare: suppose we are told that the coin did not land heads. Then, on the basis of (40), we can rightly assert both (45) and (46). Indeed, we find no reason to doubt that, in general, $\Box q$ is assertible whenever both $p \vee \Box q$ and $\neg p$ are. Disjunctive syllogism is respected. We generalize as follows:

(47) $\{\neg p, p \vee \Box q\} \Vdash \Box q$

(48) $\{\Diamond\neg q, p \vee \Box q\} \nVdash p$

As its form shows, it isn't *crucial* for accepting (48) that one reads this as a failure of disjunctive syllogism: if one rejects the duality of 'might' and 'must' - in particular, the equivalence of $\Diamond\neg q$ and $\neg\Box q$ - then one can accept (48) without impugning disjunctive syllogism. For simplicity, our own system (and surveyed rivals) will respect the orthodoxy of duality.

(48) is closely related to a simple observation: typically, a disjunction may be assertible though neither disjunct is assertible. In particular, (40) may be assertible though neither disjunct is (i.e. neither (43) nor (46)). How to represent this as a logical principle? In line with the 'epistemic contradictions' discussed in §3.3, we take it that if $\Diamond\neg\varphi$ is assertible, then φ is not. So $\Diamond\neg\varphi$ is a handy expression of the unassertibility of φ. Hence:

[22] Another counter-example to disjunctive syllogism might be afoot. To our ears, the following is also an appropriate claim in the context of our coin scenario:

(*) It must have landed heads or it must have landed tails.

In this case, it seems we have a case where $\Box p \vee \Box q$ and $\neg\Box q$ are assertible in context, but $\Box p$ is not. The judgment that (*) is appropriate seems widespread. Variants are endorsed by Geurts [2005], Klinedinst and Rothschild [2012], Rothschild [2012], Moss [2015], Schroeder [2015] and Dorr and Hawthorne [2013]. Further, Cariani [2017, pg. 173] retrieves a selection of felicitous uses of such expressions 'from the wild'. However, in our experience, it is not universally accepted that judgments of the form of (*) are appropriate in response to cases akin to our coin example. For that reason, the main text emphasizes (40) as our central data.

(49) $\{\Diamond\neg\Box q, p \vee \Box q\} \not\Vdash p$

(50) $\{\Diamond\neg p, p \vee \Box q\} \not\Vdash \Box q$

Following Cariani [2017, Sect. 4.1], call these *Schroeder's constraints*, since Schroeder [2015] observes that such principles create trouble (as we shall see) for the domain semantics of Yalcin [2007] supplemented with boolean disjunction.[23]

3.5 *Summary: Some Constraints on Assertibility Logic*

We summarize with a short list of constraints on the logic of assertibility. We have argued that expressivists should accept these constraints. First, the following constraints are plausible examples of valid reasoning:

WFC $\Diamond p \vee \Diamond\neg p \Vdash \Diamond p \wedge \Diamond\neg p$

DIN $(\Diamond p \wedge q) \vee r \Vdash (\Diamond(p \wedge q) \wedge q) \vee r$

Second, the following capture plausible failures of validity:

DSF $\{\Diamond\neg q, p \vee \Box q\} \not\Vdash p$

SCH $\{\Diamond\neg p, p \vee \Box q\} \not\Vdash \Box q$

4 Classic Versions of Merely Pragmatic Expressivism

We now show that the constraints of §3.5 are missed by two influential frameworks: domain semantics [Yalcin, 2007, 2011], supplemented with boolean disjunction, and update semantics [Heim, 1982, 1983, Veltman, 1996]. Both are forms of pragmatic expressivism. Domain semantics is notably *conservative*: it lightly departs from orthodox modal propositional logic (the Kripke-Hintikka-Kratzer tradition) but significantly accommodates the badness of bare epistemic contradictions. Further, it descends from the pioneering expressivist framework of Gibbard [2003]. Update semantics also has independent motivation [Beaver, 2001] and a compelling account of the badness of epistemic contradictions [Veltman, 1996]. There is growing support for an expressivism founded on a 'dynamic semantics' [Willer, 2013, Yalcin, 2015, Charlow, 2015, Willer, 2017].

4.1 *Domain Semantics with Boolean Disjunction*

Yalcin [2007], following an early version of MacFarlane [2011], interprets sentences relative to an information state—to provide the domain of quantification for modals—*and* a possible world—to provide truth-values for factual sentences. So compositional semantic values for him are functions from a world-state pair to a truth-value.

Definition 5 (Domain Semantics, Boolean Disjunction)**.** Given information model $\mathscr{M} = \langle W, V\rangle$, information state $\mathbf{s} \subseteq W$ and world w, define $[\![\cdot]\!]^{w,\mathbf{s}}$ as follows:

- if p is an atom: $[\![p]\!]^{w,\mathbf{s}} = 1$ iff $w \in Vp)$

[23] Cariani [2017, Sect. 4.1] generalizes Schroeder's conclusion. See Charlow [2015] for an alternative take on Schroeder's constraints. Due to space constraints, we delay assessing its strengths and weaknesses for elsewhere.

- $[\![\neg\varphi]\!]^{w,\mathbf{s}} = 1$ iff $[\![\varphi]\!]^{w,\mathbf{s}} = 0$
- $[\![\varphi \wedge \psi]\!]^{w,\mathbf{s}} = 1$ iff $[\![\varphi]\!]^{w,\mathbf{s}} = 1$ and $[\![\psi]\!]^{w,\mathbf{s}} = 1$
- $[\![\varphi \vee \psi]\!]^{w,\mathbf{s}} = 1$ iff $[\![\varphi]\!]^{w,\mathbf{s}} = 1$ or $[\![\psi]\!]^{w,\mathbf{s}} = 1$
- $[\![\Diamond\varphi]\!]^{w,\mathbf{s}} = 1$ iff there exists $u \in \mathbf{s}$ such that $[\![\varphi]\!]^{u,\mathbf{s}} = 1$

Yalcin [2007] does not explicitly define a disjunction, but says that "logical connectives will be defined as usual" (p. 464) and defines negation and conjunction as we do here. Also following logical orthodoxy, we define $\Box\varphi := \neg\Diamond\neg\varphi$.

Because $[\![\varphi]\!]$ is not a function from information states (i.e. doxastic states) to truth-values, the above is not a psychologistic semantics. Thus, it is not a form of semantic expressivism. However, a natural account of expression is available: say that state $\mathbf{s}$ *accepts* φ just in case φ is satisfied by every pair $\langle \mathbf{s}, w\rangle$, where $w \in \mathbf{s}$. This can be interpreted as an account of the assertibility relation $\Vdash$: $\mathbf{s} \Vdash \varphi$ just in case $\mathbf{s}$ accepts φ. Expressivism about $\Diamond$ follows i.e. $\{\mathbf{s} \subseteq W : \mathbf{s} \Vdash \Diamond p\} = \{\mathbf{s} \subseteq W : \mathbf{s} \nVdash \neg p\}$. At least, this is essentially so, ignoring an edge case: the empty information state. Note the definition of assertibility is not direct, as semantic expressivism requires (i.e. the compositional semantic value of an expression is not its acceptance conditions). Yalcin [2007] notes a pleasing consequence of his definition: no information state renders $p \wedge \Diamond\neg p$ assertible, and so bare epistemic contradictions express a degenerate property (the empty set). However, this success does not extend to the constraints of §3.5. The following result crystallizes much of the critical literature on Yalcin's framework:[24]

Theorem 1. *The account of $\Vdash$ from domain semantics fails to validate **WFC** and **DIN**. It violates **SCH** and **DSF** by rendering the corresponding patterns valid.*[25]

4.2 *Update Semantics*

Turn to update semantics: the basic semantic framework recursively assigns, to each sentence φ, a function $[\![\varphi]\!]$ that maps information state $\mathbf{s}$ to information state $\mathbf{s}[\![\varphi]\!]$: the *update* of $\mathbf{s}$ given φ. $[\![\varphi]\!]$ is often called the *context-change potential* of φ.[26] Thus, we do not have semantic expressivism: the compositional semantic value of φ is not a function from information states to truth-values. Here are the semantic clauses from Veltman [1996].[27]

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[24] In particular, Schroeder [2015], Dorr and Hawthorne [2013], Mandelkern [2019].

[25] *Proof.*

WFC Let $\mathbf{s}$ contain a single p world. Then it accepts $\Diamond p \vee \Diamond\neg p$, but not the corresponding conjunction.

DIN Let $\mathbf{s}$ consist of one $\neg r \wedge \neg p \wedge q$ world and one $r \wedge p \wedge \neg q$ world. Because every world is either r or q, and there is a p-world amongst $\mathbf{s}$, $\mathbf{s}$ accepts $(\Diamond p \wedge q) \vee r$. But $\mathbf{s}$ does not accept $(\Diamond(p \wedge q) \wedge q) \vee r$, because it does not accept r and because there are no worlds satisfying the left disjunct.

DSF Let $\mathbf{s}$ accept $p \vee \Box q$ as well as $\Diamond\neg q$. So for every world therein, $[\![p \vee \Box q]\!]^{w,\mathbf{s}} = 1$ and $[\![\Diamond\neg q]\!]^{w,\mathbf{s}} = 1$. Thus, there exists $u \in \mathbf{s}$ such that $[\![q]\!]^{u,\mathbf{s}} \neq 1$. Since $[\![\Box q]\!]^{w,\mathbf{s}} = 1$ for some $w \in \mathbf{s}$ only if $[\![q]\!]^{w,\mathbf{s}} = 1$ for every $w \in \mathbf{s}$, it follows that $[\![\Box q]\!]^{w,\mathbf{s}} = 1$ for no $w \in \mathbf{s}$. So $[\![p]\!]^{w,\mathbf{s}} = 1$ for every w, i.e. $\mathbf{s}$ accepts p.

SCH Let $\mathbf{s}$ accept $p \vee \Box q$ as well as $\Diamond\neg p$. So for every world therein, $[\![p \vee \Box q]\!]^{w,\mathbf{s}} = 1$ and $[\![\Diamond\neg p]\!]^{w,\mathbf{s}} = 1$. Thus, there exists $u \in \mathbf{s}$ such that $[\![p]\!]^{u,\mathbf{s}} \neq 1$. It follows that $[\![\Box q]\!]^{u,\mathbf{s}} = 1$. But then $[\![\Box q]\!]^{w,\mathbf{s}} = 1$ for every $w \in \mathbf{s}$, i.e. $\mathbf{s}$ accepts $\Box q$. $\Box$

[26] Standard notation uses $[\cdot]$ instead of $[\![\cdot]\!]$.

[27] Veltman uses intersection for conjunction. We've followed the norm in using sequential update, introduced nearly contemporaneously in Groenendijk et al. [1996].

Definition 6 (Update Semantics)**.** Relative to information model $\mathscr{M}$, $[\![\varphi]\!]$ maps information state $\mathbf{s}$ to information state $\mathbf{s}[\![\varphi]\!]$, as follows:

- if p is an atom: $\mathbf{s}[\![p]\!] = \{w \in \mathbf{s} : w \in V(p)\}$
- $\mathbf{s}[\![\neg\varphi]\!] = \mathbf{s} - \mathbf{s}[\![\varphi]\!]$
- $\mathbf{s}[\![\varphi \wedge \psi]\!] = \mathbf{s}[\![\varphi]\!][\![\psi]\!]$
- $\mathbf{s}[\![\varphi \vee \psi]\!] = \mathbf{s}[\![\varphi]\!] \cup \mathbf{s}[\![\psi]\!]$
- $\mathbf{s}[\![\Diamond\varphi]\!] = \{w \in \mathbf{s} : \mathbf{s}[\![\varphi]\!] \neq \varnothing\}$
- $\mathbf{s}[\![\Box\varphi]\!] = \{w \in \mathbf{s} : \mathbf{s}[\![\varphi]\!] = \mathbf{s}\}$

$[\![\Diamond\varphi]\!]$ and $[\![\Box\varphi]\!]$ are *tests*: they either return $\mathbf{s}$ (if, respectively, $\mathbf{s}$ is compatible with φ or $\mathbf{s}$ assures φ) or, otherwise, the empty set.

Pragmatic expressivism may now be generated. Say that $\mathbf{s}$ *accepts* φ just in case $\mathbf{s}[\![\varphi]\!] = \mathbf{s}$. This offers a natural account of assertibility: $\mathbf{s} \Vdash \varphi$ just in case $\mathbf{s}$ accepts φ. Then, as usual: φ *expresses* the property $\{\mathbf{s} \subseteq W : \mathbf{s} \Vdash \varphi\}$ relative to information model $\mathscr{M}$. Expressivism about $\Diamond$ follows i.e. $\{\mathbf{s} \subseteq W : \mathbf{s} \Vdash \Diamond p\} = \{\mathbf{s} \subseteq W : \mathbf{s} \nVdash \neg p\}$. At least, this is essentially so, ignoring an edge case: the empty information state. Again, this account has the nice outcome that epistemic contradictions like $p \wedge \Diamond\neg p$ can never be asserted, given a non-degenerate information state.

However:

Theorem 2. *The account of $\Vdash$ from update semantics fails to validate **WFC** and **DIN**. It violates **DSF** by rendering the corresponding pattern valid.*[28]

5 Positive Proposal

5.1 *Modal Propositional Assertibility Semantics*

We now develop a novel semantic expressivism that *does* satisfy the constraints of §3.5. As a form of semantic expressivism, our compositional interpretation function $[\![\cdot]\!]$ will be a partial function from information states to truth-values.[29] Intuitively, a state $\mathbf{s}$ can *accept* a sentence (the information establishes that the sentence holds), *reject* a sentence (the information establishes that the sentence does not hold), or be neutral. 1 and 0 mark the first two; lacking a truth value marks the third.[30] We will also say that 'φ is assertible relative to $\mathbf{s}$' to mean that $[\![\varphi]\!]^{\mathbf{s}} = 1$ and *mutatis mutandis* for 'deniable' and value 0. After presenting the semantics, we prove that it meets the constraints from §3 in a logically conservative manner: non-classical behavior only arises from the interaction of connectives with modals.

[28] *Proof.*

WFC Counter-model: a single world w, with V assigning p to w.

DIN Let $\mathbf{s}$ consist of $\neg r \wedge \neg p \wedge q$ world w_1 and $r \wedge p \wedge \neg q$ world w_2. Because every world is either r or q, and there is a p-world amongst $\mathbf{s}$, $\mathbf{s}$ accepts $(\Diamond p \wedge q) \vee r$: $\mathbf{s}\ [\![\Diamond p]\!][\![q]\!] = \{w_1\}$ and $\mathbf{s}\ [\![r]\!] = \{w_2\}$. But $\mathbf{s}$ does not accept $(\Diamond(p \wedge q) \wedge q) \vee r$, since $\mathbf{s}\ [\![\Diamond(p \wedge q)]\!][\![q]\!] = \varnothing$.

DSF Let $\mathbf{s}$ accept $p \vee \Box q$ as well as $\Diamond\neg q$. So: $\mathbf{s} = \mathbf{s}[\![p]\!] \cup \mathbf{s}[\![\Box q]\!]$. Since $\mathbf{s}[\![\Diamond\neg q]\!] = \mathbf{s}$, it follows that $\mathbf{s}[\![\Box q]\!] = \varnothing$. Hence, $\mathbf{s} = \mathbf{s}[\![p]\!]$, i.e. $\mathbf{s}$ accepts p. □

[29] Compare Aloni [2016b], about which more later.

[30] This is equivalent to a bilateral system, simultaneously defining acceptance and rejection clauses. Compare Veltman [1985], Groenendijk and Roelofsen [2010], Fine [2014], Incurvati and Schlöder [2019].

Definition 7. Given an information model, the interpretation function $[\![\cdot]\!]$ is defined as follows (where $\mathbf{s}$ is an information state and φ is a sentence, read '$\mathbf{s}$ refutes φ' as short-hand for 'for all non-empty $\mathbf{t} \subseteq \mathbf{s}$: $[\![\varphi]\!]^{\mathbf{t}} \neq 1$'):

- if p is an atom: $[\![p]\!]^{\mathbf{s}} = 1$ iff $\mathbf{s} \subseteq V(p)$
  if p is an atom: $[\![p]\!]^{\mathbf{s}} = 0$ iff $\mathbf{s}$ refutes p
- $[\![\neg\varphi]\!]^{\mathbf{s}} = 1$ iff $[\![\varphi]\!]^{\mathbf{s}} = 0$
  $[\![\neg\varphi]\!]^{\mathbf{s}} = 0$ iff $[\![\varphi]\!]^{\mathbf{s}} = 1$
- $[\![\varphi \wedge \psi]\!]^{\mathbf{s}} = 1$ iff $[\![\varphi]\!]^{\mathbf{s}} = 1$ and $[\![\psi]\!]^{\mathbf{s}} = 1$
  $[\![\varphi \wedge \psi]\!]^{\mathbf{s}} = 0$ iff $\mathbf{s}$ refutes $\varphi \wedge \psi$
- $[\![\varphi \vee \psi]\!]^{\mathbf{s}} = 1$ iff there exists $\mathbf{s}_1, \mathbf{s}_2$ such that: $\mathbf{s} = \mathbf{s}_1 \cup \mathbf{s}_2$, $[\![\varphi]\!]^{\mathbf{s}_1} = 1$ and $[\![\psi]\!]^{\mathbf{s}_2} = 1$
  $[\![\varphi \vee \psi]\!]^{\mathbf{s}} = 0$ iff $\mathbf{s}$ refutes $\varphi \vee \psi$
- $[\![\Diamond\varphi]\!]^{\mathbf{s}} = 1$ iff $[\![\varphi]\!]^{\mathbf{s}} \neq 0$
  $[\![\Diamond\varphi]\!]^{\mathbf{s}} = 0$ iff $\mathbf{s}$ refutes $\Diamond\varphi$

We define 'must': $\Box\varphi := \neg\Diamond\neg\varphi$. This yields pleasing results: $\Box\varphi$ is assertible exactly when no amount of further information can undermine the assertibility of φ. Further, $\Box\varphi$ is deniable (i.e. $\neg\Box\varphi$ is assertible) exactly when $\Diamond\neg\varphi$ is assertible.

Reflection on inquiry motivates our clauses. An information state represents the worlds that have not been ruled out as candidates for the actual world. Intuitively, a sentence without modal components is assertible exactly when the information state has already established that it holds, whatever further information is forthcoming. Such a sentence is deniable exactly when no further possible information could establish it. This notion of deniability is directly reflected in the clauses for atoms, conjunction, and disjunction.[31] Thus, atomic p is assertible relative to an information state if the current information state only contain worlds at which p is true. A disjunction $\varphi \vee \psi$ will be assertible when the current information state is *covered* by a φ zone and a ψ zone.[32] That is: the current information guarantees that one of φ or ψ holds, though neither may yet be established. This is an *intensional* approach to disjunction.[33] Neither $\mathbf{s}_1$ nor $\mathbf{s}_2$ are insisted to be *non-empty* in the clause for disjunction. This turns out to be significant for preserving the classicality of disjunction in non-modal contexts. Negation switches between expressing acceptance and expressing rejection. The possibility operator expresses a *lack* of rejection.[34]

In line with the tenets of semantic expressivism codified in (4), we define: $\mathbf{s} \Vdash \varphi$ iff $[\![\varphi]\!]^{\mathbf{s}} = 1$. The corresponding entailment relation meets all of our desiderata. Proof is left to an appendix.

Theorem 3. *The entailment relation $\Vdash$ validates* ***WFC*** *and* ***DIN****. It satisfies* ***SCH*** *and* ***DSF*** *by rendering the corresponding patterns invalid.*

[31] It is also the sense of 'negation' that is used in inquisitive semantics [Ciardelli et al., 2018].
[32] This disjunction is used in team semantics for dependence logic [Väänänen, 2008, Yang and Väänänen, 2017].
[33] See Cariani [2017] for an overview of this approach.
[34] This idea appears in Veltman [1996], Punčochář [2015] and Hawke and Steinert-Threlkeld [2018].

5.2 *Nearby Variations*

We now pinpoint features of our system that set it apart from close variants in the literature.

In our system, the assignment of 0 is not handled uniformly: $\neg\varphi$ receives special treatment. In pursuit of elegance, one might be tempted toward a more uniform treatment, for instance:

(51) For every φ: $[\![\neg\varphi]\!]^{\mathbf{s}} = 0$ iff for all non-empty $\mathbf{t} \subseteq \mathbf{s}$: $[\![\varphi]\!]^{\mathbf{t}} \neq 1$

In this case, rejection clauses need not be explicitly stated: they can be recovered from the acceptance clause for negation: $[\![\varphi]\!]^{\mathbf{s}} = 0$ iff $[\![\neg\varphi]\!]^{\mathbf{s}} = 1$. This is essentially the approach of Ciardelli and Roelofsen [2015]. Acceptance for $\Diamond\varphi$ then becomes:

(52) $[\![\Diamond\varphi]\!]^{\mathbf{s}} = 1$ iff $[\![\neg\varphi]\!]^{\mathbf{s}} \neq 1$

Alternatively, one could replace (51) with a weaker (but still uniform) account:

(53) For every φ: $[\![\neg\varphi]\!]^{\mathbf{s}} = 1$ iff $[\![\varphi]\!]^{\{w\}} \neq 1$ for all $w \in \mathbf{s}$

This yields the system from Hawke and Steinert-Threlkeld [2018].

However, these approaches to rejection and negation share drawbacks. First, $\neg\neg\varphi$ is no longer generally equivalent to φ. In particular, in the current settings, $[\![\neg\neg\Diamond\neg p]\!]^{\mathbf{s}} = 1$ just in case $[\![\neg p]\!] = 1$ (proof is left to the interested reader). This has the further consequence that $\Box p$, as usually defined, no longer has the intuitively correct relationship with $\Diamond$: for now $[\![\neg\Box p]\!]^{\mathbf{s}} = 1$ just in case $[\![\neg p]\!]^{\mathbf{s}} = 1$, as opposed to $[\![\Diamond\neg p]\!]^{\mathbf{s}} = 1$. Whatever the theoretical value of elegance, it is offset by this failure in empirical coverage.

Turn to the nearby system of Lin [2016]. Here, the key divergences from our system, as we see it, are encapsulated by:

(54) if p is an atom: $[\![p]\!]^{\mathbf{s}} = 0$ iff $\mathbf{s} \cap V(p) = \varnothing$ and $\mathbf{s} \neq \varnothing$

(55) $[\![\Diamond\varphi]\!]^{\mathbf{s}} = 1$ iff there exists non-empty $\mathbf{t} \subseteq \mathbf{s}$ s.t. $[\![\varphi]\!]^{\mathbf{t}} = 1$[35]

(56) $[\![\Diamond\varphi]\!]^{\mathbf{s}} = 0$ iff there is no non-empty $\mathbf{t} \subseteq \mathbf{s}$ s.t. $[\![\varphi]\!]^{\mathbf{t}} = 1$

(54) leads to trouble. Presumably, a correct semantics renders the following principle valid: $p \Vdash p \vee \neg p$. However, let $\mathbf{s}$ be an information state composed only of p worlds. In this case, $[\![p]\!]^{\mathbf{s}} = 1$, by Lin's lights. However, since Lin embraces our acceptance clauses for $\neg$ and $\vee$, his system yields $[\![p \vee \neg p]\!]^{\mathbf{s}} \neq 1$.[36] Thus, by his lights, p is not predicted to entail $p \vee \neg p$.

We are not aware of strong arguments for favoring our clauses for $\Diamond\varphi$ over (55) and (56). At any rate, the choice has logical ramifications. For instance, by Lin's lights: $[\![\Diamond(\neg\Diamond p)]\!]^{\mathbf{s}} = 1$ just in case $[\![\Diamond\neg p]\!]^{\mathbf{s}} = 1$. By our lights: $[\![\Diamond(\neg\Diamond p)]\!]^{\mathbf{s}} = 1$ just in case $[\![\neg p]\!]^{\mathbf{s}} = 1$.[37]

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[35] Compare [Humberstone, 1981].

[36] *Proof.* No non-empty subset of $\mathbf{s}$ accepts $\neg p$, since each is a set of p worlds. However, $\varnothing$ doesn't accept $\neg p$ either, by the stipulation in (54). Thus, $\mathbf{s}$ cannot be divided into two subsets such that one subset accepts $\neg p$.

[37] *Proof.* For Lin: $[\![\Diamond(\neg\Diamond p)]\!]^{\mathbf{s}} = 1$ iff there exists non-empty $\mathbf{t} \subseteq \mathbf{s}$ s.t. $\mathbf{t}$ has no non-empty subsets that accept p. Hence, $\mathbf{t}$ must itself reject p. For us: $[\![\Diamond(\neg\Diamond p)]\!]^{\mathbf{s}} = 1$ iff $[\![\neg\Diamond p]\!]^{\mathbf{s}} \neq 0$ iff $[\![\Diamond p]\!]^{\mathbf{s}} \neq 1$ iff $[\![p]\!]^{\mathbf{s}} = 0$ iff $[\![\neg p]\!]^{\mathbf{s}} = 1$.

5.3 *Logical Conservativeness*

Further features of our semantics enhances its appeal. First, we can recover a notion of propositional content by considering assertibility at singletons, via the following slogan: what is true is what would be assertible in the absence of uncertainty, i.e. at the end of inquiry. Define the *truth set of* φ as follows:

Definition 8. $[\varphi] = \{w : \{w\} \Vdash \varphi\}$

One can prove that, under the semantics in Definition 7, truth sets behave classically for the fragment without epistemic modals. First, a lemma.

Lemma 1. $[\![\varphi]\!]^{\{w\}} = 0$ *iff* $[\![\varphi]\!]^{\{w\}} \neq 1$.[38]

Proposition 1. *(Classicality) If* φ *and* ψ *do not contain* $\Diamond$*, then:*[39]

$$[p] = V(p)$$

$$[\neg\varphi] = W \backslash [\varphi]$$

$$[\varphi \wedge \psi] = [\varphi] \cap [\psi]$$

$$[\varphi \vee \psi] = [\varphi] \cup [\psi]$$

While it is often intuitive that φ is assertible at **s** iff it is true throughout **s**, the clause for epistemic 'might' undercuts this thought. The modal reflects a global property of information states, one that does not distribute to properties of the individual worlds therein. More precisely, sentences $\Diamond p$ are not *persistent* (i.e. preserved under sub-states of information states): there are **s** and $\mathbf{t} \subsetneq \mathbf{s}$ such that $\mathbf{s} \Vdash \Diamond p$ but $\mathbf{t} \nVdash \Diamond p$.[40] It turns out that 'might' is the sole source of such failures:

Proposition 2. *(Distributivity) Let* φ *be a* $\Diamond$*-free sentence. Then:*[41]
$[\![\varphi]\!]^{s} = 1$ *iff* $[\![\varphi]\!]^{\{w\}} = 1$ *for every* $w \in s$
$[\![\varphi]\!]^{s} = 0$ *iff* $[\![\varphi]\!]^{\{w\}} = 0$ *for every* $w \in s$

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[38]*Proof.* Notice that $\{w\}$ refutes φ just in case $\{w\} \nVdash \varphi$, since $\{w\}$ is the only non-empty subset of $\{w\}$. Hence, it suffices to show that the property in question is closed under applications of negation. Assume that $[\![\varphi]\!]^{\{w\}} = 0$ exactly when $[\![\varphi]\!]^{\{w\}} \neq 1$. We then have that $[\![\neg\varphi]\!]^{\{w\}} = 0$ iff $[\![\varphi]\!]^{\{w\}} = 1$ iff, by our assumption, $[\![\varphi]\!]^{\{w\}} \neq 0$, which holds iff $[\![\neg\varphi]\!]^{\{w\}} \neq 1$. □

[39]*Proof.* A largely straightforward induction. For the negation case, one uses the previous lemma. For the disjunction case, first note: $\{w\} \Vdash \varphi \vee \psi$ iff there exists $\mathbf{s}_1, \mathbf{s}_2$ such that: $\{w\} = \mathbf{s}_1 \cup \mathbf{s}_2$, $\mathbf{s}_1 \Vdash \varphi$, $\mathbf{s}_2 \Vdash \psi$. Note that at least one of $\mathbf{s}_1$ or $\mathbf{s}_2$ has to be $\{w\}$, and so either $\{w\} \Vdash \varphi$ or $\{w\} \Vdash \psi$. For the converse, note that if, e.g., $\{w\} \Vdash \varphi$, then $\{w\} \Vdash \varphi \vee \psi$, by letting $\mathbf{s}_2 = \varnothing$. □

[40]For an example, let **s** contain one p world and one $\neg p$ world, with **t** being the singleton containing the $\neg p$ world. Persistence plausibly demarcates fact-stating from non-factual domains of discourse. Both of the systems of Väänänen [2008] and Ciardelli and Roelofsen [2015] evaluate sentences at information states and use a clause similar to ours for disjunction. But in both systems, truth at an information state is persistent. This shows that our 'might' cannot be defined in their systems.

[41]Proof. By induction. The atomic and negation cases are straightforward. Consider conjunction. $\mathbf{s} \Vdash \varphi \wedge \psi$ iff $\mathbf{s} \Vdash \varphi$ and $\mathbf{s} \Vdash \psi$ iff (by the inductive hypothesis) $\{w\} \Vdash \varphi$ and $\{w\} \Vdash \psi$ iff $\{w\} \Vdash \varphi \wedge \psi$ for every $w \in \mathbf{s}$. For the second case, suppose there is a $w \in \mathbf{s}$ such that $[\![\varphi \wedge \psi]\!]^{\{w\}} \neq 0$. By Lemma 1, $[\![\varphi \wedge \psi]\!]^{\{w\}} = 1$. But then **s** does not refute the conjunction, so we have that $[\![\varphi \wedge \psi]\!]^{\mathbf{s}} \neq 0$, as desired. For the other direction, assume that $[\![\varphi \wedge \psi]\!]^{\mathbf{s}} \neq 0$. Then there must exist non-empty $\mathbf{t} \subseteq \mathbf{s}$ such that $[\![\varphi \wedge \psi]\!]^{\mathbf{t}} = 1$. By the induction hypothesis, it follows that $[\![\varphi \wedge \psi]\!]^{\{w\}} = 1$ for every $w \in \mathbf{t}$. By Lemma 1, it follows that $[\![\varphi \wedge \psi]\!]^{\{w\}} \neq 0$ for some $w \in \mathbf{s}$. The other cases are similar. □

The following is now virtually immediate.

Theorem 4. *If φ and ψ are $\Diamond$-free, then $\varphi \Vdash \psi$ holds just in case φ entails ψ according to classical propositional logic.*

Proof. Using Proposition 2, $\varphi \Vdash \psi$ holds just in case for every information model and world w: $\{w\} \Vdash \varphi$ holds only if $\{w\} \Vdash \psi$ holds. But we also know that assertibility at a singleton behaves classically, by Proposition 1. $\square$

6 Sophisticated Rivals

We now evaluate more sophisticated rivals. The constraints of §3.5 all involve the interplay between modals and disjunction. Assertibility semantics meets the constraints largely because of its treatment of disjunction. Thus, we consider a version of domain semantics with a non-boolean disjunction. We next consider a rival semantic expressivism, based on Moss [2015]. Finally, we consider the *weak bounded relational semantics* of Mandelkern [2019], which shares key features with domain semantics but has better empirical coverage. Our conclusion: the surveyed rivals do not meet all our constraints, and exhibit other costs.

6.1 *Domain Semantics with Intensional Disjunction*

Recall domain semantics, as in §4.1. Recall the associated notion of *acceptance*: $\mathbf{s}$ accepts φ (written $\mathbf{s} \Vdash \varphi$) just in case φ holds at every pair $\langle w, \mathbf{s} \rangle$, where $w \in \mathbf{s}$. Now, replace the clause for boolean disjunction with the following intensional variation:

Definition 9 (Domain Semantics, Intensional Disjunction)**.** $[\![\varphi \vee \psi]\!]^{w,\mathbf{s}} = 1$ iff there exists $\mathbf{s}_1$ and $\mathbf{s}_2$ such that: $\mathbf{s} = \mathbf{s}_1 \cup \mathbf{s}_2$, $\mathbf{s}_1$ accepts φ, and $\mathbf{s}_2$ accepts ψ.

This is in the spirit of the clause for disjunction in assertibility semantics. The resulting system outperforms boolean domain semantics: the interested reader can check that constraints **DSF** and **SCH** are met.

However, the system does not meet **WFC** and **DIN**. We prove the first. A counter-model: W consists of a single world, at which p holds. Now, here is a key feature of the assertibility logic generated by domain semantics: the empty information state $\varnothing$ vacuously accepts every φ. Hence, for our counter-model, W can be divided into $\mathbf{s}_1$ and $\mathbf{s}_2$ that respectively accept $\Diamond p$ and $\Diamond \neg p$: $\mathbf{s}_1$ is W and $\mathbf{s}_2$ is $\varnothing$. Hence, W accepts $\Diamond p \vee \Diamond \neg p$, but clearly doesn't accept $\Diamond p \wedge \Diamond \neg p$.

The present system exhibits further disturbing traits, rooted in a core feature: if $[\![p \vee q]\!]^{w,\mathbf{s}} = 1$ for some $w \in \mathbf{s}$, then $[\![p \vee q]\!]^{u,\mathbf{s}} = 1$ for every $u \in \mathbf{s}$. This, for instance, renders $p \vee q$ and $\Diamond(p \vee q)$ as co-assertible, since $\mathbf{s}$ accepts the former just in case it accepts the latter. *Proof.* Suppose that non-empty $\mathbf{s}$[42] accepts $\Diamond(p \vee q)$. Thus, there exists $w \in \mathbf{s}$ such that $[\![p \vee q]\!]^{w,\mathbf{s}} = 1$. It follows that $[\![p \vee q]\!]^{u,\mathbf{s}} = 1$ for every $u \in \mathbf{s}$. Hence, $p \vee q$ is accepted and, so, is assertible.

To illustrate, assume that 'Jones has a sibling' is equivalent to 'Jones has a brother or Jones has a sister'. The present system predicts that if an agent rightly asserts 'Jones might have a sibling' then she is positioned to assert 'Jones has a sibling'.

[42] Since everything is accepted by the empty set, the non-emptiness requirement is not substantive.

6.2 *Semantic Expressivism via Measures and Partitions*

Moss [2015, 2018] offers a probabilistic framework that she interprets as a 'hybrid' semantic expressivism. It has two primary innovations: (i) while non-epistemic language can receive its standard boolean semantics, epistemic vocabulary operates on sets of probability measures, and so such a set will be the semantic value of sentences; (ii) both epistemic and logical vocabulary can be sensitive to contextually-salient *partitions* of the space of possible worlds.

This requires positing a bifurcation in sentence meaning and a thorough-going ambiguity for logical vocabulary. Absent any epistemic vocabulary, a sentence is assigned a set of possible worlds and an item of logical vocabulary denotes a standard operation on sets of worlds. But an epistemic modal denotes an operation on a set of probability measures. To reconcile this, Moss posits a type-shifter $\mathscr{C}$ (a 'certainty' operator) which takes a set of possible worlds and returns a set of measures: those assigning the set of worlds probability 1. Then, logical vocabulary is taken to be systematically ambiguous, denoting operations on sets of possible worlds in certain linguistic contexts, and on sets of probability measures in others.[43] Further, to implement (ii), modals and logical operators are indexed, with context assigning a partition to each index. With this background in place, we can present the semantics.

A *partitioned information model* is an information model supplemented with a set $\{\pi_i : i \in \mathbb{N}\}$ of partitions on W. Intuitively, think of the pairing of a particular partition with index i (i.e. π_i) as fixed by context.[44] We understand *partition* in the standard way: as a set of non-empty and pairwise disjoint sets (*cells*) that together cover W. Let $\mathbf{p}, \mathbf{q}$ range over 'propositions' (i.e. information states: $\mathbf{p} \subseteq W$, $\mathbf{q} \subseteq W$). Let m range over probability measures. Let $m|_{\mathbf{p}}$ be the result of conditioning m on the set of worlds $\mathbf{p}$ (i.e. $m|_{\mathbf{p}}(\mathbf{q}) = m(\mathbf{q}|\mathbf{p})$ for every $\mathbf{q}$).

Definition 10 (Semantic Expressivism via Partitions)**.** Given a partitioned information model (the partitions supplied by context), we define the interpretation of our language as follows:

- If φ has no epistemic vocabulary: $[\![\mathscr{C}\varphi]\!]^m = 1$ iff $m([\![\varphi]\!]) = 1$ (where $[\![\varphi]\!]$ is the set of worlds where φ is true according to the standard truth-conditions)
- $[\![\varphi \vee_i \psi]\!]^m = 1$ iff $\forall \mathbf{p} \in \pi_i$ s.t. $m(\mathbf{p}) > 0 : [\![\varphi]\!]^{m|_{\mathbf{p}}} = 1$ or $[\![\psi]\!]^{m|_{\mathbf{p}}} = 1$
- $[\![\varphi \wedge_i \psi]\!]^m = 1$ iff $\forall \mathbf{p} \in \pi_i$ s.t. $m(\mathbf{p}) > 0 : [\![\varphi]\!]^{m|_{\mathbf{p}}} = 1$ and $[\![\psi]\!]^{m|_{\mathbf{p}}} = 1$
- $[\![\neg_i \varphi]\!]^m = 1$ iff $\forall \mathbf{p} \in \pi_i$ s.t. $m(\mathbf{p}) > 0 : [\![\varphi]\!]^{m|_{\mathbf{p}}} \neq 1$
- $[\![\Diamond_i \varphi]\!]^m = 1$ iff $\exists \mathbf{p} \in \pi_i$ s.t. $m(\mathbf{p}) > 0$ and $[\![\varphi]\!]^{m|_{\mathbf{p}}} = 1$
- $[\![\Box_i \varphi]\!]^m = 1$ iff $\forall \mathbf{p} \in \pi_i$ s.t. $m(\mathbf{p}) > 0 : [\![\varphi]\!]^{m|_{\mathbf{p}}} = 1$

(Compare [Moss, 2015, pg. 70].) Assertibility then gets a direct definition: $m \Vdash \varphi$ just in case $[\![\varphi]\!]^m = 1$. And logical consequence is preservation thereof: $\Gamma \Vdash \varphi$ just in case $m \Vdash \varphi$ if $m \Vdash \gamma$ for every $\gamma \in \Gamma$, relative to any partitioned information model.

Note that when quantifying over cells in a partition, we impose the restriction: $m(\mathbf{p}) > 0$. This assures that evaluation of $[\![\varphi]\!]^m$ proceeds without evoking undefined conditional measures. We consider this a charitable extrapolation of Moss [2015]. It isn't clear to us how she prefers to handle undefined conditional measures. But the other salient option is to drop the restriction $m(\mathbf{p}) > 0$ and let $[\![\varphi]\!]^m$ be undefined if its purported evaluation evokes an undefined measure.

[43] See Mandelkern [2019] for more worries that arise from this reliance on type-shifting.

[44] This lightly departs from Moss [2015], who *formalizes* the interaction with context, as a function $g_c(i)$ accepting a context c and index i and returning a partition. This difference has no bearing on our observations.

This has disastrous consequences. For instance, if φ, ψ, χ are epistemic (i.e. denote sets of measures; for example, are of the form $\mathscr{C}p$):

$$[\![\varphi \wedge_1 \Box_1 \psi]\!]^m = [\![\varphi \wedge_1 (\psi \wedge_1 \chi)]\!]^m = [\![\neg_1 \Diamond_1 \psi]\!]^m \neq 1$$

for every measure m in every context where π_1 has more than one cell.[45,46]

We count the above system as a form of semantic expressivism. In §2.2, we characterized psychologistic semantics as taking semantic values to be functions from information states to truth-values. This reflects the tentative modeling assumption that a doxastic state is best characterized as a mere information state. Moss [2015] models a doxastic state in a more refined way: as a probability distribution (a 'credence function') on a set of worlds. Given this, her semantics takes semantic values to be functions from doxastic states to truth-values, in the spirit of semantic expressivism.[47] As she writes: "The [semantic value] of a declarative sentence is a property that credences can have. Formally, [semantic values] are sets of probability measures. In a paradigmatic case of assertion, when you assert a sentence with a certain [semantic value], I come to have a credence distribution that is contained in that [semantic value]."[48] Semantic values are properties of mental states, and assertion functions to coordinate on the property expressed by the sentence asserted. Finally, the above system embraces a refinement of expressivism about $\Diamond$: relative to a model, $\{m : m \Vdash \Diamond_i \mathscr{C}p\} = \{m : m \nVdash \neg_i \mathscr{C}p\}$.

The system makes headway with our constraints. **DSF** and **SCH** are met, by rendering the corresponding inference patterns invalid.[49] Further, the logic meets **DIN**.[50]

However, the logic misses **WFC**. *Proof.* Consider this counter-model: $W = \{w\}$, with w a p world; $\pi_1 = \{\{w\}\}$; and $m(\{w\}) = 1$. Then $[\![\Diamond_1 \mathscr{C}p \vee_1 \Diamond_1 \mathscr{C}\neg p]\!]^m = 1$ but $[\![\Diamond_1 \mathscr{C}p \wedge_1 \Diamond_1 \mathscr{C}\neg p]\!]^m \neq 1$.

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[45] *Proof.* By the lights of the proposal: $[\![\varphi \wedge_1 \Box_1 \psi]\!]^m = 1$ iff $\forall \mathbf{p} \in \pi_1 : [\![\varphi]\!]^{m|\mathbf{p}} = 1$ and $\forall \mathbf{q} \in \pi_1 : [\![\psi]\!]^{m|\mathbf{p}|\mathbf{q}} = 1$. Further, $[\![\varphi \wedge_1 (\psi \wedge_1 \chi)]\!]^m = 1$ iff $\forall \mathbf{p} \in \pi_1 : [\![\varphi]\!]^{m|\mathbf{p}} = 1$ and $\forall \mathbf{q} \in \pi_1 : [\![\psi]\!]^{m|\mathbf{p}|\mathbf{q}} = 1$ and $[\![\chi]\!]^{m|\mathbf{p}|\mathbf{q}} = 1$. Further, $[\![\neg_1 \Diamond_1 \psi]\!]^m = 1$ iff $\forall \mathbf{p} \in \pi_1 : \forall \mathbf{q} \in \pi_1 : [\![\psi]\!]^{m|\mathbf{p}|\mathbf{q}} \neq 1$. But, in every case, $m|_{\mathbf{p}}|_{\mathbf{q}}$ is undefined when $\mathbf{p} \neq \mathbf{q}$.

[46] We note that the solution to undefined measures we present here is subtly different than Moss' appeal to *quasi-validity* (pp. 56-57 of Moss [2015]), which is validity restricted to *well-behaved* context/measure/argument triples. Her definition: c is well-behaved with respect to m and A just in case m assigns non-zero mass to every cell of every partition associated by context with every index in the argument. Our point here, however, is that measure-zero cells can be systematically introduced by the compositional semantics even for well-behaved triples, and so something more must be done. Whence our restricted quantification in the semantics.

[47] At least for sentences containing any epistemic vocabulary.

[48] See Moss [2015], p. 23. We have replaced 'content' by 'semantic value', as is intended in her writing, and to avoid confusions discussed in the introduction.

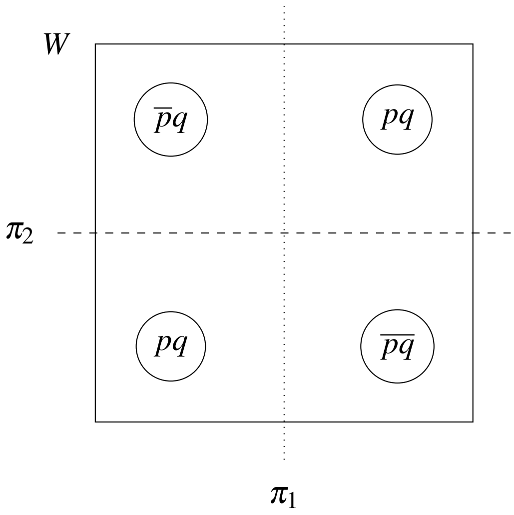
[49] Consider **DSF**. *Proof.* Here is a counter-model. W has three worlds: w_1, a p and q world; w_2, a $\neg p$ and q world; and w_3, a p and $\neg q$ world. Let $\pi_1 = \{\{w_1, w_2\}, \{w_3\}\}$ (i.e. it distinguishes the q from the $\neg q$ worlds), and let m be an arbitrary measure that places non-zero mass on all three worlds. We have that $[\![\Diamond_1 \mathscr{C}\neg q]\!]^m = 1$, since $m|_{\{w_3\}}$ is defined and assigns probability 1 to $[\![\neg q]\!]$. We also have that $[\![\mathscr{C}p \vee_1 \Box_1 \mathscr{C}q]\!]^m = 1$, since $[\![\mathscr{C}p]\!]^{m|\{w_3\}} = 1$ and $[\![\Box_1 \mathscr{C}q]\!]^{m|\{w_1, w_2\}} = 1$. (Note the latter relies on the fact that $m|_{\{w_1, w_2\}}(\{w_3\}) = 0$.) But $[\![\mathscr{C}p]\!]^m \neq 1$, since m assigns non-zero mass to w_2, a $\neg p$ world.

For **SCH**, the reader can verify that letting π_1 be the finest partition (containing the singleton of every world) provides a counter-model.

[50] *Proof.* In what follows, we implicitly restrict quantification over partition cells to those with non-zero measure, relative to whatever measure is salient. Assume that $[\![((\Diamond_1 \mathscr{C}p) \wedge_2 \mathscr{C}q) \vee_3 \mathscr{C}r]\!]^m = 1$, i.e. for every $\mathbf{p}_3 \in \pi_3$, either $[\![(\Diamond_1 \mathscr{C}p) \wedge_2 \mathscr{C}q]\!]^{m|\mathbf{p}_3} = 1$ or $[\![\mathscr{C}r]\!]^{m|\mathbf{p}_3} = 1$. If the latter holds for every cell in π_3, then $[\![(\Diamond_1 \mathscr{C}(p \wedge q) \wedge_2 \mathscr{C}q) \vee_3 \mathscr{C}r]\!]^m = 1$, as desired.

Otherwise, let $\mathbf{p}_3$ be an arbitrary cell such that $[\![(\Diamond_1 \mathscr{C}p) \wedge_2 \mathscr{C}q]\!]^{m|\mathbf{p}_3} = 1$. This entails that for every $\mathbf{p}_2 \in \pi_2$, $[\![\mathscr{C}q]\!]^{m|\mathbf{p}_3|\mathbf{p}_2} = 1$ (i.e. $m([\![q]\!]|\mathbf{p}_3, \mathbf{p}_2) = 1$) and $[\![\Diamond_1 \mathscr{C}p]\!]^{m|\mathbf{p}_3|\mathbf{p}_2} = 1$, i.e. for some $\mathbf{p}_1 \in \pi_1$ we have that $m([\![p]\!]|\mathbf{p}_3, \mathbf{p}_2, \mathbf{p}_1) = 1$. But since, by assumption, all the mass throughout $\mathbf{p}_3 \cap \mathbf{p}_2$ is on $[\![q]\!]$, we also have that $m([\![q]\!]|\mathbf{p}_3, \mathbf{p}_2, \mathbf{p}_1) = 1$. Together, these give us that $[\![\mathscr{C}(p \wedge q)]\!]^{m|\mathbf{p}_3|\mathbf{p}_2|\mathbf{p}_1} = 1$. Moving backwards through the unpacked definitions, it follows that $[\![((\Diamond_1 \mathscr{C}(p \wedge q)) \wedge_2 \mathscr{C}q) \vee_3 \mathscr{C}r]\!]^m = 1$.

The problems go deeper: the logic breaks conjunction elimination. To see that $\Diamond_1 p \wedge_2 \Diamond_1 q \not\Vdash \Diamond_1 p$, consider the below counter-model, with the mass from m distributed equally between worlds (i.e. singletons):



The intuitive diagnosis: modals can be interpreted differently when embedded than when unembedded, since operators like conjunction quantify over cells of a partition and conditions the measure at which the modals are evaluated on those cells. While one may try to pragmatically restrict admissible indexing to rule out such counter-examples, the features that cause this discrepancy are essential to a key motivation for Moss' system: its account of embedded modals. This, in general, requires different indices on modals and connectives to get the right readings.

Before moving on, note a similarity between Moss' system and domain semantics:[51] both build on a standard truth-conditional base semantics, by enriching the possible semantic values in order to handle epistemic vocabulary. In both, the fragment without modals can be evaluated for truth entirely at a single possible world. Such meanings are then 'lifted' to interact with modals, either by adding an index of evaluation (Yalcin) or a type-shifter (Moss). Both maneuvers lead to trouble, as outlined above. By contrast, our thorough-going semantic expressivist system has no such bifurcation: non-epistemic sentences are evaluated at information states in the same way that sentences with epistemic vocabulary are. We preserve a 'boolean base'—the logic of the fragment without epistemic modals is classical propositional logic—but without starting from a traditional worldly semantics and lifting it. Comparison with 'hybrid' systems suggests that preserving classical logic in this way is especially promising.

6.3 *Bounded Modality*

Finally, we consider the *weak bounded relational semantics* proposed in [Mandelkern, 2019, Sect. 7.1].[52] It enhances standard relational modal semantics in two key ways. First, drawing on Schlenker [2009], constituents of sentences are evaluated relative to both a world and a *local context* (i.e. an information state, in our parlance), with the latter sensitive to the syntactic environment of the constituent. Second, well-defined modal claims are subject to the presupposition that local context worlds can only access local context worlds.

[51] Thanks to an anonymous reviewer for discussion.

[52] Mandelkern [2019] also studies a system he calls the *strong bounded theory*. Mandelkern [2019, Sect. 7.1] presents worries about this variation that we find convincing. Hence, we pass over it.

The bounded framework robustly accounts for the wayward behavior of embedded (reverse) epistemic contradictions and, more broadly, inheritance.[53] Further, it echoes domain semantics: evaluation requires both a world and an information state. But, unlike domain semantics, the prospects for building expressivism on top of it are unexplored.[54] Hence, our current preoccupations demand a thorough discussion of the framework. For reasons of space, we confine ourselves to sketching a few immediately problematic features.

We evaluate formulas relative to a *relational model* $\mathscr{M}$: a set of worlds W, an accessibility relation R between worlds and a valuation V for atoms. Let $R(w)$ be the set of worlds accessible from world w i.e. $\{u \in W : wRu\}$. For our purposes, $R(w)$ may be interpreted as the agent's information at w. The interpretation function will be a function from worlds and information states to truth-values.[55] In what follows, set $[\varphi]_{\mathbf{s}} = \{w \in \mathbf{s} : [\![\varphi]\!]^{w,\mathbf{s}} = 1\}$.

Definition 11 (Weak Bounded Relational Semantics)**.** Given $\mathscr{M} = \langle W, R, V \rangle$, information state $\mathbf{s} \subseteq W$ and world w, interpretation $[\![\cdot]\!]$ is recursively defined in accordance with:

- $[\![p]\!]^{w,\mathbf{s}} = 1$ iff $w \in V(p)$
- $[\![p]\!]^{w,\mathbf{s}} = 0$ iff $w \notin V(p)$
- $[\![\neg\varphi]\!]^{w,\mathbf{s}} = 1$ iff $[\![\varphi]\!]^{w,\mathbf{s}} = 0$
- $[\![\varphi \wedge \psi]\!]^{w,\mathbf{s}} = 1$ iff $[\![\varphi]\!]^{w,\mathbf{s} \cap [\psi]_{\mathbf{s}}} = 1$ and $[\![\psi]\!]^{w,\mathbf{s} \cap [\varphi]_{\mathbf{s}}} = 1$
- $[\![\varphi \vee \psi]\!]^{w,\mathbf{s}} = 1$ iff $[\![\varphi]\!]^{w,\mathbf{s} \setminus [\psi]_{\mathbf{s}}} = 1$ or $[\![\psi]\!]^{w,\mathbf{s} \setminus [\varphi]_{\mathbf{s}}} = 1$
- $[\![\Diamond\varphi]\!]^{w,\mathbf{s}} = 1$ iff $\forall u \in \mathbf{s}$, $R(u) \subseteq \mathbf{s}$ and there exists $u \in R(w)$ s.t. $[\![\varphi]\!]^{u,\mathbf{s}} = 1$
- $[\![\Diamond\varphi]\!]^{w,\mathbf{s}} = 0$ iff $\forall u \in \mathbf{s}$, $R(u) \subseteq \mathbf{s}$ and for all $u \in R(w)$, $[\![\varphi]\!]^{u,\mathbf{s}} = 0$

The condition that $\forall u \in \mathbf{s}$, $R(u) \subseteq \mathbf{s}$ serves as a semantic presupposition on $\Diamond\varphi$: if it fails, $\Diamond\varphi$ receives no truth value. The above constraints do not uniquely determine $[\![\cdot]\!]$: the clauses for $\wedge$ and $\vee$ receiving value 0 are outstanding. This choice will not bear on the remarks below.

The above yields an eccentric logic. To start, conjunction introduction fails: $\Diamond p$ and $\Diamond q$ is predicted not to entail $\Diamond p \wedge \Diamond q$;[56] further, p and $\Diamond q$ is predicted not to entail $p \wedge \Diamond q$.[57] Indeed, Mandelkern [2019, Sect. 5.1] utilizes the failure of conjunction introduction to explain the behavior of embedded (reverse) epistemic contradictions. This explanatory benefit is not obviously outweighed by the costs of invalidating an uncontroversial logical principle. Whatever a framework's successes, it is counter-intuitive to predict contexts where, say, 'Jane might eat ice cream' and 'Jane might eat cake' are both true yet 'Jane might eat ice cream and might eat cake' is not. Further, there are frameworks (e.g. §5) that render (reverse) epistemic contradictions genuine contradictions while preserving conjunction introduction.

[53] See [Mandelkern, 2019, Sect. 5].

[54] As Mandelkern [2019, fn. 15] notes.

[55] Departing from Mandelkern [2019], we forgo variable assignments and indexed modals, for simplicity.

[56] *Proof.* Consider a model where $\mathbf{s} = \{w_1, w_2\}$; neither p nor q holds at w_1; both p and q hold at w_2; $R(w_1) = \{w_1\}$ and $R(w_2) = \mathbf{s}$. Thus, $[\![\Diamond p]\!]^{w_2,\mathbf{s}} = 1$ and $[\![\Diamond q]\!]^{w_2,\mathbf{s}} = 1$. But $[\![\Diamond p \wedge \Diamond q]\!]^{w_2,\mathbf{s}} \neq 1$: since there are no p or q worlds in $R(w_1)$, we have $[\![\Diamond p]\!]^{w_1,\mathbf{s}} \neq 1$ and $[\![\Diamond q]\!]^{w_1,\mathbf{s}} \neq 1$. So $\mathbf{s} \cap [\Diamond q]_{\mathbf{s}} = \{w_2\} = \mathbf{s} \cap [\Diamond p]_{\mathbf{s}}$. Hence $[\![\Diamond p]\!]^{w_2,\mathbf{s} \cap [\Diamond q]_{\mathbf{s}}} \neq 1$ and $[\![\Diamond q]\!]^{w_2,\mathbf{s} \cap [\Diamond p]_{\mathbf{s}}} \neq 1$, since $R(w_2) \nsubseteq \mathbf{s} \cap [\Diamond q]_{\mathbf{s}}$ and $R(w_2) \nsubseteq \mathbf{s} \cap [\Diamond p]_{\mathbf{s}}$.

[57] *Proof.* Let R be an equivalence relation. Suppose $R(w) = \mathbf{s}$ and that $\mathbf{s}$ exhaustively divides into $p \wedge \neg q$ and $\neg p \wedge q$ worlds. Finally, suppose $w \in V(p)$. Thus $[\![\Diamond p]\!]^{w,\mathbf{s}} = 1$ and $[\![q]\!]^{w,\mathbf{s}} = 1$. But $[\![\Diamond p \wedge q]\!]^{w,\mathbf{s}} \neq 1$: $\mathbf{s} \cap [q]_{\mathbf{s}}$ is the set of q worlds in $\mathbf{s}$, a proper subset of $\mathbf{s}$. But $R(w) = \mathbf{s}$. Hence $[\![\Diamond p]\!]^{w,\mathbf{s} \cap [q]_{\mathbf{s}}} \neq 1$.

Conjunction elimination also fails: the bounded framework predicts that $\Diamond p \wedge \Diamond q$ does not entail $\Diamond p$.[58] Notably, our counter-model hinges on evaluating at an initial context $\mathbf{s}$ that is a proper subset of the worlds accessible from the worlds in $\mathbf{s}$. This feature can be dropped, however, in counter-models to closely related principles: for instance, the bounded framework predicts that $r \vee (\Diamond p \wedge \Diamond q)$ does not entail $r \vee \Diamond p$.[59]

The framework also predicts that $\Diamond p \wedge \Diamond q$ does not entail $\Diamond p \vee \Diamond q$.[60] Similarly, it predicts that $\Diamond p \vee \Diamond q$ does not entail $\Diamond p \wedge \Diamond q$.[61] This is a tricky position: the framework can neither explain wide scope free choice as an entailment, nor by using standard Grice-like pragmatic mechanisms, as $\Diamond p \wedge \Diamond q$ is not predicted to be a straightforward strengthening of $\Diamond p \vee \Diamond q$.

Can these worries be explained away? Mandelkern [2019, fn. 26] notes that every classical logical validity will be *Strawson* valid in the bounded framework: valid if all the premises and conclusions are defined.[62] But it is not clear that this helps. On its face, the framework predicts that contexts arise where, for instance, conjunction introduction fails because certain modal claims 'lack sense' in that context. However, we cannot identify pre-theoretic examples that corroborate this prediction. In their absence, the bounded framework requires an independently motivated story as to why ordinary contexts systematically assign accessibility relations that steer away from undefined modal claims.[63]

7 Objections, Variations, Replies

We consider some assertibility principles that hold initial intuitive appeal but are not validated by our framework. In most cases, we suggest a variation of our system that yields these principles while pointing out drawbacks to the alteration. Along the way, we cast doubt on the validity of the principles in question.

7.1 *Addition*

Start with the *principle of addition* i.e. disjunction introduction.

(57) $\varphi \Vdash \varphi \vee \psi$

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[58]*Proof.* Suppose p and q both hold at w_2, and neither holds at w_1. Let $R(w_1) = R(w_2) = \{w_1, w_2\}$. Let $\mathbf{s} = \{w_1\}$. Then $[\![\Diamond p]\!]^{w_1,\mathbf{s}}$ and $[\![\Diamond p]\!]^{w_2,\mathbf{s}}$ are both undefined, since $R(w_1) \nsubseteq \mathbf{s}$. Similarly, $[\![\Diamond q]\!]^{w_1,\mathbf{s}}$ and $[\![\Diamond q]\!]^{w_2,\mathbf{s}}$ are both undefined. Thus, $[\Diamond p]_{\mathbf{s}} = [\Diamond q]_{\mathbf{s}} = \varnothing$. It follows that $[\![\Diamond p]\!]^{w_1,\mathbf{s}\cap[\Diamond q]_{\mathbf{s}}} = [\![\Diamond p]\!]^{w_1,\varnothing} = 1$, since it is vacuously true that $\forall w \in \varnothing, R(w) \subseteq \varnothing$. Similarly, $[\![\Diamond q]\!]^{w_1,\mathbf{s}\cap[\Diamond p]_{\mathbf{s}}} = [\![\Diamond q]\!]^{w_1,\varnothing} = 1$. Thus $[\![\Diamond p \wedge \Diamond q]\!]^{w_1,\mathbf{s}} = 1$.

[59]*Proof.* Suppose p, q, r all hold at w_2 and none hold at w_1. Let $R(w_1) = R(w_2) = \mathbf{s} = \{w_1, w_2\}$. Thus: $[r]_{\mathbf{s}} = \{w_2\}$. Further, $[\![\Diamond p]\!]^{w_1,\mathbf{s}\setminus[r]_{\mathbf{s}}}$ and $[\![\Diamond p]\!]^{w_2,\mathbf{s}\setminus[r]_{\mathbf{s}}}$ are both undefined, since $R(w_1) \nsubseteq \{w_1\}$. Similarly, $[\![\Diamond q]\!]^{w_1,\mathbf{s}\setminus[r]_{\mathbf{s}}}$ and $[\![\Diamond q]\!]^{w_2,\mathbf{s}\setminus[r]_{\mathbf{s}}}$ are both undefined. Hence: $[\Diamond p]_{\mathbf{s}\setminus[r]_{\mathbf{s}}} = [\Diamond q]_{\mathbf{s}\setminus[r]_{\mathbf{s}}} = \varnothing$. Since, in addition, $[\![r]\!]^{w_1,\mathbf{s}\setminus[\Diamond p]_{\mathbf{s}}} = 0$, it follows that $[\![r \vee \Diamond p]\!]^{w_1,\mathbf{s}} \neq 1$. On the other hand, $[\![\Diamond p]\!]^{w_1,(\mathbf{s}\setminus[r]_{\mathbf{s}})\cap[\Diamond q]_{\mathbf{s}\setminus[r]_{\mathbf{s}}}} = [\![\Diamond p]\!]^{w_1,\varnothing} = [\![\Diamond q]\!]^{w_1,(\mathbf{s}\setminus[r]_{\mathbf{s}})\cap[\Diamond p]_{\mathbf{s}\setminus[r]_{\mathbf{s}}}} = [\![\Diamond q]\!]^{w_1,\varnothing} = 1$. Hence, $[\![\Diamond p \wedge \Diamond q]\!]^{w_1,\mathbf{s}\setminus[r]_{\mathbf{s}}} = 1$. Hence, $[\![r \vee (\Diamond p \wedge \Diamond q)]\!]^{w_1,\mathbf{s}} = 1$.

[60]*Proof.* Let $\mathbf{s} = \{w_1, w_2, w_3\}$, with w_1 and w_2 both being $\neg p \wedge \neg q$ worlds and w_3 being a $p \wedge q$ world. In addition to reflexive loops at all three worlds, we have that $w_1 R w_2$ and $w_2 R w_3$. Then we have that $[\![\Diamond p \wedge \Diamond q]\!]^{w_2,\mathbf{s}} = 1$ but $[\![\Diamond p \vee \Diamond q]\!]^{w_2,\mathbf{s}}$ will be undefined, since each disjunct will be evaluated at $w_2, \{w_1\}$, but $R(w_1) \nsubseteq \{w_1\}$.

[61]*Proof.* Let $R(w) = \mathbf{s} = \{w\}$, with w a $p \wedge \neg q$ worlds. Then $[\![\Diamond p \vee \Diamond q]\!]^{w,\mathbf{s}} = 1$ since $[\![\Diamond p]\!]^{w,\mathbf{s}\setminus[\Diamond q]_{\mathbf{s}}} = [\![\Diamond p]\!]^{w,\mathbf{s}} = 1$. But $[\![\Diamond p \wedge \Diamond q]\!]^{w,\mathbf{s}} \neq 1$: the second conjunct will be evaluated at $w, \varnothing$ and no q world is accessible from w.

[62]As per von Fintel [1999].

[63]A simple proposal for epistemic modals: in ordinary contexts, the accessible worlds at a point of evaluation are, typically, all and only the worlds in the local context of evaluation. As we see it, this formally yields a hybrid of domain semantics and bounded semantics. However, Mandelkern [2019, Sect. 6.2] correctly notes that this theory itself makes problematic predictions.

Our system's logic of assertibility exhibits an interesting pattern. On one hand, (57) is validated for $\Diamond$-free sentences. On the other hand, it yields:

(58) $\Diamond p \not\Vdash \Diamond p \vee \Diamond q$

This is not surprising: Kamp [1973] observes that the conjunction of a free choice principle with standard boolean logic results in paradox.

(59) a) $\Diamond p$.

b) Conclude $\Diamond p \vee \Diamond q$, by unrestricted addition.

c) Conclude: $\Diamond p \wedge \Diamond q$, by wide-scope free choice.

d) Conclude: $\Diamond q$, by simplification.

If valid, this licenses the conclusion 'Jones might be a murderer' from the premise 'Jones might be late for work'. Fortunately, despite observing **WFC** and simplification, our system does not endorse this reasoning.

The paradox suggests that acceptance of weak wide-scope free choice and rejection of unrestricted addition are a package deal. So if the arguments of 3.1 are compelling, our system is well-positioned.

The paradox does *not* license the rejection of addition for purely descriptive, $\Diamond$-free claims. Indeed, validating this explains the ordinary utility of modes of reasoning like disjunctive syllogism and De Morgan's laws. Consider:

(60) Does Jones have (either) a brother or a sister?

(61) Does Jones have a sibling?

(62) Jones has a brother.

(63) Does Jones have a sister?

(64) Jones has neither a brother nor a sister.

(65) Jones does not have a sibling.

(62) is an overly informative answer to (60) and the equivalent question (61): compare the information so communicated to that communicated by simply answering 'Yes'. (64), or the equivalent (65), is an overly informative answer to (63). A simple explanation: descriptive p entails $p \vee q$. In short, it seems desirable to *cordon off* the unusual logical behavior exhibited by sentences with epistemic vocabulary, as our system does.

7.2 *Narrow-scope Free Choice*

Turn to *narrow-scope free choice*:

(66) $\Diamond(p \vee q) \Vdash \Diamond p \wedge \Diamond q$

Our system does not yield this. One easily establishes with our clauses that $\mathbf{s} \Vdash \Diamond(p \vee q)$ holds just in case there exists non-empty $\mathbf{t} \subseteq \mathbf{s}$ such that either $\mathbf{t} \Vdash p$ or $\mathbf{t} \Vdash q$. Thus, if $\mathbf{s}$ is non-empty, $\mathbf{s} \Vdash p$ entails $\mathbf{s} \Vdash \Diamond(p \vee q)$. But $\mathbf{s} \Vdash p$ is compatible with $\mathbf{s} \nVdash \Diamond q$.

Some semanticists have found narrow-scope free choice attractive; a free choice effect is plausibly exhibited by the following.[64]

(67) Bernie might win or not win the nomination.

(68) Mr. X might be in Victoria or in Brixton [Zimmermann, 2000].

(69) Jones might have a brother or a sister.

As with wide-scope free choice, Gricean explanations of this data seem questionable. For instance, one observes similarly odd (expressivist) outcomes if one attempts a straightforward Gricean explanation, assuming 'or' is boolean, $\Diamond$ is closed under consequence and φ entails $\Diamond\varphi$. Suppose X asserts $\Diamond(p \vee \neg p)$. Hearer Y considers. X either believes p, believes $\neg p$ or suspends belief on the question of p. But $\Diamond(p \vee \neg p)$ is not maximally informative in any of these cases. A rational speaker should rather have asserted, respectively, p, $\neg p$ or $\Diamond p \wedge \Diamond\neg p$. By assumption, these strictly entail $\Diamond(p \vee \neg p)$.

Therefore, a semantic explanation of the data holds initial attraction. A clear instance of this strategy is to endorse narrow-scope free choice. The following alteration of our clause for disjunction delivers this:

(70) $[\![\varphi \vee \psi]\!]^{\mathbf{s}} = 1$ iff for some $\underline{\text{non-empty}}$ $\mathbf{s}_1, \mathbf{s}_2$ such that $\mathbf{s} = \mathbf{s}_1 \cup \mathbf{s}_2$: $[\![\varphi]\!]^{\mathbf{s}_1} = 1$ and $[\![\psi]\!]^{\mathbf{s}_2} = 1$

This alteration brings our system closer to 'System B' in Aloni [2016b].[65] It has drawbacks. First, the validity of addition is undone even for $\Diamond$-free sentences. For instance, if $\mathbf{s} \Vdash p \wedge \neg q$ for non-empty $\mathbf{s}$, then q is false at every world in $\mathbf{s}$, and so there can be no non-empty $\mathbf{t} \subseteq \mathbf{s}$ such that $\mathbf{t} \Vdash q$. Hence, $\mathbf{s} \nVdash p \vee q$. Thus, according to the current alteration, if one accepts that Fred has only a brother, then one is not positioned to assert that that Fred has a sibling.

Second, De Morgan's laws are undone for this system, even for $\Diamond$-free sentences. To see this, note that $\mathbf{s} \Vdash \neg(p \wedge \neg p)$ holds for every $\mathbf{s}$ in the altered system (as it should be). However, $\mathbf{s} \Vdash \neg p \vee \neg\neg p$ need not hold in this system: for instance, this does not hold if $\mathbf{s} \Vdash p$ holds.

Finally, $p \vee q$ itself entails $\Diamond p \wedge \Diamond q$. While this seems like a valid defeasable principle based on Gricean reasoning, it's less clear that it should be a consequence of assertibility logic. Certainly, 'Jane has a sibling' is assertible in some contexts where 'Jane might have a brother' is rightly denied.

In total, our attempt to validate (66) dramatically abandons logical conservativeness.

Is (66) desirable on reflection? The theoretical cost of its endorsement is higher than for wide-scope free choice. Another paradox (Zimmermann [2000], following Kamp [1973]):

(71) a) $\Diamond p$.

b) Conclude $\Diamond(p \vee q)$, by addition (for descriptive claims) and the principle that $\Diamond$ is closed under logical entailment (for descriptive claims).

c) Conclude: $\Diamond p \wedge \Diamond q$, by narrow-scope free choice.

[64] See Simons [2005], Fusco [2015], Aloni [2016a,b], Starr [2016b], Willer [2018] for semantic approaches and Kratzer and Shimoyama [2002], Fox [2007], Franke [2011], Santorio and Romoli [2017] for pragmatic approaches.

[65] In a modification, Aloni [2019] recovers this type of disjunction via a mechanism of pragmatic enrichment, rather than through the semantic clause for disjunction.

d) Conclude: $\Diamond q$, by simplification.

If one assumes narrow-scope free choice, the only tempting options for defiance are to reject addition for descriptive claims, or the closure of $\Diamond$ under entailment for descriptive claims. Both are costly. For instance, they rob us of a simple explanation of the contradictory sound of:

(72) # Jones might have a brother, but it can't be that she has a sibling.

Explanations with more promise are available. Consider two tentative suggestions. First, our pessimistic assessment of a pragmatic approach was perhaps too hasty: criticisms aside,[66] pragmatic mechanisms have shown success in delivering narrow-scope free choice, in contrast to wide-scope free choice.[67] Appendix B shows that the standard recursive pragmatic explanation – following Kratzer and Shimoyama [2002], Fox [2007], Franke [2011], among others – can be used in our system.

Second: it is worth exploring the hypothesis that assertions like (67), (68) and (69) are not, typically, best represented by the logical form $\Diamond(\varphi \vee \psi)$. Notice that re-phrasing (67) and (68) to *explicitly* exhibit this form does not obviously preserve their typical meaning in conversation and the free choice effect.

(73) It might be that either Bernie wins or he does not.

(74) It might be that Jones has a sibling (that is, a brother or a sister).

Unlike (67), (73) is a strangely weak claim: something that obviously *has to* be the case—either Bernie wins or he does not—is claimed to *might* be the case. (74), meanwhile, seems perfectly assertible if, say, the question under discussion is 'Does Jones have a sibling?' and one knows that Jones does not have a sister, but is unsure whether Jones has a brother. (Context: the registrar at Jones' school has announced that siblings of existing students receive a fees discount, and one is wondering whether Jones' family is eligible.)

An intriguing, and under-explored, elaboration of this line claims that the free choice effect of (67), (68) and (69) reflects that such assertions are typically *elliptical*.[68]

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[66] For instance, see Fusco [2014].

[67] For instance, see Fox [2007], especially Sect. 11.2.

[68] See Schwarz [1999] for an analysis of elided disjunctions that sits well with this. An elliptical analysis won't be plain-sailing, however. Compare:

(A) The keys might be upstairs and downstairs.

(B) The keys might be upstairs or downstairs.

Consider candidate paraphrases:

(A1) It might be that both the keys are upstairs and the keys are downstairs.

(A2) The keys might be upstairs and the keys might be downstairs.

(B1) It might be that either the keys are upstairs or the keys are downstairs.

(B2) The keys might be upstairs or the keys might be downstairs.

The elliptical analysis of (B)—accounting for the free-choice effect—posits that (B2) is typically the most prominent reading. As an anonymous reviewer points out, the most prominent reading of (A) seems to be (A1), even if (A2) suggests itself in certain contexts. Why the asymmetry? A semantic expressivist can (at least) explain why (B2) is more prominent than (B1). Suppose p, q and $p \vee q$ are all relevant to the conversation. Then an assertion of $\Diamond(p \vee q)$ is especially weak: if the speaker is unsure about only p, she should assert the stronger $q \wedge \Diamond p$; if unsure about only q, she should assert the stronger $p \wedge \Diamond q$; if unsure about both, she should assert the

(75) Bernie might win or [Bernie might] not win the nomination.

(76) Mr. X might be in Victoria or [Mr. X might be] in Brixton.

(77) Jones might have a brother or [Jones might have] a sister.

Hence, wide-scope free choice accounts for the observed free choice effect.[69]

7.3 *De Morgan in Full Generality*

Our assertibility semantics does not validate De Morgan's laws in full generality.

(78) $\neg\Diamond p \vee \neg\Diamond\neg p \nVdash \neg(\Diamond p \wedge \Diamond\neg p)$

Proof. The key is that $\neg\Diamond p \vee \neg\Diamond\neg p$ expresses a logical truth in our system: it holds at *every* information state. Indeed, once the details are worked out, it turns out to be equivalent to $p \vee \neg p$. On the other hand, $\neg(\Diamond p \wedge \Diamond\neg p)$ does not express a logical truth. Rather, it holds exactly when one of either p or $\neg p$ is accepted.[70]

One way to preserve De Morgan's laws is to alter the system to have properly symmetrical clauses for $\vee$ and $\wedge$, as in Steinert-Threlkeld [2017, Sect. 3.5.1]:

(79) $[\![\varphi \wedge \psi]\!]^{\mathbf{s}} = 1$ iff $[\![\varphi]\!]^{\mathbf{s}} = 1$ and $[\![\psi]\!]^{\mathbf{s}} = 1$
$[\![\varphi \wedge \psi]\!]^{\mathbf{s}} = 0$ iff there exists $\mathbf{s}_1, \mathbf{s}_2$ such that: $\mathbf{s} = \mathbf{s}_1 \cup \mathbf{s}_2$, $[\![\varphi]\!]^{\mathbf{s}_1} = 0$ and $[\![\psi]\!]^{\mathbf{s}_2} = 0$

(80) $[\![\varphi \vee \psi]\!]^{\mathbf{s}} = 1$ iff there exists $\mathbf{s}_1, \mathbf{s}_2$ such that: $\mathbf{s} = \mathbf{s}_1 \cup \mathbf{s}_2$, $[\![\varphi]\!]^{\mathbf{s}_1} = 1$ and $[\![\psi]\!]^{\mathbf{s}_2} = 1$
$[\![\varphi \vee \psi]\!]^{\mathbf{s}} = 0$ iff $[\![\varphi]\!]^{\mathbf{s}} = 0$ and $[\![\psi]\!]^{\mathbf{s}} = 0$

However, the altered system preserves (78) in an unacceptable way: it has the consequence that $\neg(\Diamond p \wedge \Diamond\neg p)$ (like $\neg\Diamond p \vee \neg\Diamond\neg p$) is a logical truth.[71] This is odd. First, it means that there exist information states $\mathbf{s}$ such that $\mathbf{s} \Vdash \neg(\Diamond p \wedge \Diamond\neg p) \wedge (\Diamond p \wedge \Diamond\neg p)$. In other words: the altered system egregiously allows for the assertibility of contradictions. Second, since $\Diamond p \wedge \Diamond\neg p$ intuitively expresses uncertainty concerning p, it is natural to think that $\neg(\Diamond p \wedge \Diamond\neg p)$ expresses *certainty* on the question of p (i.e. it is assertible by an agent that is either committed to p or to $\neg p$). But having such certainty is a contingent state, with the consequence that we would *not* expect $\neg(\Diamond p \wedge \Diamond\neg p)$ to always be assertible.

Our unaltered system offers a more satisfying account: $\neg(\Diamond p \wedge \Diamond\neg p)$ is indeed expressible exactly when the information settles the question of p. We find this so natural that it bolsters the invalidity of (78). It does not offend intuition to read $\neg\Diamond p \vee \neg\Diamond\neg p$ as equivalent to $\neg p \vee \Box p$, which in turn is plausibly equivalent to the logical truth $p \vee \neg p$. But if $\neg\Diamond p \vee \neg\Diamond\neg p$ does not express anything substantial, and $\neg(\Diamond p \wedge \Diamond\neg p)$ expresses a substantial state of mind (certainty on the question of p), we shouldn't expect the assertibility of the first to entail that of the second.

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stronger $\Diamond p \wedge \Diamond q$. So, since $\Diamond(p \vee q)$ tends not to be assertable, (B) is naturally interpreted as communicating (B2). A similar explanation is not available for favoring (A2) over (A1). Indeed, if (B) conventionally communicates (B2), then this helps to explain why (A1) is prominent over (A2). A speaker says (A). If she had meant to unambiguously communicate (A2), she would have used (B). So she must have intended to communicate (A1).

[69] Simons [2005] takes the opposite approach: she uses Across-the-Board Movement to reduce wide-scope disjunctions to narrow-scope ones and explains free choice for the latter. For criticism, see Alonso-Ovalle [2006].

[70] *Proof.* $\mathbf{s} \Vdash \neg(\Diamond p \wedge \Diamond\neg p)$ holds iff for every non-empty $\mathbf{t} \subseteq \mathbf{s}$: either $\mathbf{t} \nVdash \Diamond p$ or $\mathbf{t} \nVdash \Diamond\neg p$. This holds iff for every non-empty $\mathbf{t} \subseteq \mathbf{s}$: either $[\![p]\!]^{\mathbf{t}} = 0$ or $[\![\neg p]\!]^{\mathbf{t}} = 0$. This holds iff for every non-empty $\mathbf{t} \subseteq \mathbf{s}$: either $\mathbf{t} \Vdash \neg p$ or $\mathbf{t} \Vdash p$. But this is equivalent to: either $\mathbf{s} \Vdash \neg p$ or $\mathbf{s} \Vdash p$.

[71] *Proof.* $\mathbf{s} \Vdash \neg(\Diamond p \wedge \Diamond\neg p)$ iff for some $\mathbf{s}_1, \mathbf{s}_2$ such that $\mathbf{s} = \mathbf{s}_1 \cup \mathbf{s}_2$: $[\![\Diamond p]\!]^{\mathbf{s}_1} = 0$ and $[\![\Diamond\neg p]\!]^{\mathbf{s}_2} = 0$. This holds iff: for some $\mathbf{s}_1, \mathbf{s}_2$ such that $\mathbf{s} = \mathbf{s}_1 \cup \mathbf{s}_2$: $\mathbf{s}_1 \Vdash \neg p$ and $\mathbf{s}_2 \Vdash p$. But every $\mathbf{s}$ can be covered in this way.

8 Conclusion

This paper presented a novel semantic expressivism in the domain of epistemic modals. We motivated some plausible constraints on the logic of assertibility that the leading versions of purely pragmatic expressivism violate. By contrast, our version of semantic expressivism—modal propositional assertibility semantics—satisfies the constraints. We canvassed more sophisticated rivals and found them wanting, and concluded by discussing some constraints that our system does not satisfy, but attempted to cast doubt on their plausibility. Much work remains. Most pressing: how to extend the framework to handle probability operators, graded modals, questions, quantifiers and deontic modals, without offending the linguistic data?

A Proof of Theorem 3

We work with $\Vdash$, as defined in §5.1. First, three preliminaries.

Proposition 3. *If φ is $\Diamond$-free, then $\mathbf{s} \Vdash \Diamond\varphi$ holds iff there exists $w \in \mathbf{s}$ such that: $\{w\} \Vdash \varphi$.*

Proof. $\mathbf{s} \Vdash \Diamond\varphi$ holds iff $[\![\varphi]\!]^{\mathbf{s}} \neq 0$. We know from §5.3 that $[\![\varphi]\!]^{\mathbf{s}} = 0$ iff $[\![\varphi]\!]^{\{w\}} = 0$ for every $w \in \mathbf{s}$. Thus, $[\![\varphi]\!]^{\mathbf{s}} \neq 0$ iff $[\![\varphi]\!]^{\{w\}} \neq 0$ for some $w \in \mathbf{s}$, which in turn holds iff $\{w\} \Vdash \varphi$ for some $w \in \mathbf{s}$. □

Proposition 4. *If φ is $\Diamond$-free, then $\mathbf{s} \Vdash \Box\varphi$ holds iff $\mathbf{s} \Vdash \varphi$ holds.*

Proof. $\mathbf{s} \Vdash \Box\varphi$ holds iff $\mathbf{t} \Vdash \varphi$ for every non-empty $\mathbf{t} \subseteq \mathbf{s}$. Since φ is $\Diamond$-free, and therefore persistent, $\mathbf{t} \Vdash \varphi$ for every $\mathbf{t} \subseteq \mathbf{s}$ holds iff $\mathbf{s} \Vdash \varphi$. □

Proposition 5. *If φ is $\Diamond$-free, then $\mathbf{s} \Vdash \Box\varphi$ holds iff $\{w\} \Vdash \varphi$ for every $w \in \mathbf{s}$.*

Proof. A consequence of the previous proposition and the results in §5.3. □

Now our main result: the system from §5 meets the constraints distilled in §3. The validities:

WFC Assume $\mathbf{s} \Vdash \Diamond p \vee \Diamond\neg p$. Thus, there exists $\mathbf{s}_1, \mathbf{s}_2$ that cover $\mathbf{s}$ and $\mathbf{s}_1 \Vdash \Diamond p$ and $\mathbf{s}_2 \Vdash \Diamond\neg p$. Hence, by Proposition 3, we have: there exist $u, v \in \mathbf{s}$ such that $\{u\} \Vdash p$ and $\{v\} \Vdash \neg p$. Hence: $\mathbf{s} \Vdash \Diamond p$ and $\mathbf{s} \Vdash \Diamond\neg p$.

DIN Assume $\mathbf{s} \Vdash (\Diamond p \wedge q) \vee r$. Thus, there exists $\mathbf{s}_1, \mathbf{s}_2$ s.t. $\mathbf{s} = \mathbf{s}_1 \cup \mathbf{s}_2$ with $\mathbf{s}_1 \Vdash q$, $\mathbf{s}_1 \Vdash \Diamond p$, and $\mathbf{s}_2 \Vdash r$. By the first conjunct: for every $w \in \mathbf{s}_1$, $\{w\} \Vdash q$. By the second conjunct: there exists $u \in \mathbf{s}_1$ such that $\{u\} \Vdash p$. Hence, $\{u\} \Vdash p \wedge q$, and so $\mathbf{s}_1 \Vdash \Diamond(p \wedge q)$ by Proposition 3. We thus have that $\mathbf{s} \Vdash (\Diamond(p \wedge q) \wedge q) \vee r$, as desired.

To show that **DSF** and **SCH** are invalid consider this counter-model: take any $\mathbf{s}$ such that every world in $\mathbf{s}$ is either a $p \wedge \neg q$ world or a $\neg p \wedge q$ world, and there exists at least one $p \wedge \neg q$ world in $\mathbf{s}$ and at least one $\neg p \wedge q$ world in $\mathbf{s}$.

B Pragmatic Derivation of Narrow-scope Free Choice

Here, we demonstrate that a variant of the standard pragmatic derivations of narrow-scope free choice—due to Kratzer and Shimoyama [2002], Fox [2007], Franke [2011] and others—can be carried out in our system. First, we note that our system distinguishes between assertibility, deniability, and mere lack of assertibility. When considering how to treat alternative utterances in a neo-Gricean framework, this last attitude seems to be the appropriate one: we can infer that the speaker did not utter an alternative because it was not assertible, not necessarily because it was deniable. So our general scalar reasoning principle will be:

(81) If the speaker uttered φ and ψ asymmetrically entails φ, infer that ψ was not assertible.

Similarly, given our clause for epistemic 'might' in terms of lack of deniability and a negation which flips assertibility and deniability, we have:

(82) φ is not assertible at $\mathbf{s}$ if and only if $\Diamond\neg\varphi$ is assertible at $\mathbf{s}$.

We aim to show that $\Diamond(p \vee q)$ in some sense implicates $\Diamond p \wedge \Diamond q$ via broadly Gricean mechanisms. We begin by considering the alternatives to an utterance of narrow-scope disjunction.

(83) Alternatives to $\Diamond(p \vee q)$: $\Diamond p$; $\Diamond q$; $\Diamond(p \wedge q)$.

Note that $\Diamond(p \wedge q)$ asymmetrically entails each of the simpler alternatives. For the first step of the derivation, we enrich the alternatives, in line with the two principles above:

(84) Enriched alternatives: $\Diamond p \wedge \Diamond\neg\Diamond q$; $\Diamond q \wedge \Diamond\neg\Diamond p$; $\Diamond(p \wedge q)$.

Now, we enrich the meaning of $\Diamond(p \vee q)$ with the unassertibility of each already-enriched alternative. Consider the first one. We have:

$$\begin{aligned}
\mathbf{s} \Vdash \Diamond\neg(\Diamond p \wedge \Diamond\neg\Diamond q) &\text{ iff } \mathbf{s} \nVdash \Diamond p \wedge \Diamond\neg\Diamond q \\
&\text{ iff } \mathbf{s} \nVdash \Diamond p \text{ or } \mathbf{s} \nVdash \Diamond\neg\Diamond q \\
&\text{ iff } [\![p]\!]^{\mathbf{s}} = 0 \text{ or } \mathbf{s} \nVdash \Diamond\neg\Diamond q \\
&\text{ iff } [\![p]\!]^{\mathbf{s}} = 0 \text{ or } [\![\neg\Diamond q]\!]^{\mathbf{s}} = 0 \\
&\text{ iff } [\![p]\!]^{\mathbf{s}} = 0 \text{ or } [\![\Diamond q]\!]^{\mathbf{s}} = 1
\end{aligned}$$

Now consider the conjunction of $\Diamond(p \vee q)$ with $\Diamond\neg(\Diamond p \wedge \Diamond\neg\Diamond q)$ and $\Diamond\neg(\Diamond q \wedge \Diamond\neg p)$. For these to be assertible at an information state $\mathbf{s}$, we have: (i) there is either a p world or a q world; (ii) either there are no p worlds or there is a q world; (iii) either there are no q worlds or there is a p world. The reader can verify that these three together entail that there is a p and a q world in $\mathbf{s}$, and so $\mathbf{s} \Vdash \Diamond p \wedge \Diamond q$, as desired.

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