# Functional Quantification

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**Abstract**

This paper develops a unified analysis of "functional" anaphora and wide-scope indefinites. A new operator is added to Jacobson's variable-free semantics of functional readings, which leads to an analysis of theses readings using the general *Skolem function* interpretation of wide-scope indefinites. This accounts for the distributional, technical and intuitive similarities between the two phenomena. Moreover, after formally characterizing the class of generalized quantifiers that are treated by the proposed mechanism, it is argued that this class is a good approximation of the quantifiers that empirically support functional readings.

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## 1 Introduction

Traditional formal semantics assumes that quantification in natural language operates only on atomic entities. However, much recent work in natural language semantics has shown the advantages of more complex forms of quantification that involve functions over atomic domains. Two areas of functional quantification have received special attention. One area concerns the so-called *functional* and *pair-list* interpretations of questions and copular sentences. Another area deals with the *wide scope* interpretation of indefinite NPs. Quantification over *Skolem functions* is used to model both kinds of phenomena. Some theories restrict the usage of Skolem functions to the simple case of *choice functions*: Skolem functions that map any non-empty set to an entity in this set.

Despite the similarity between the mechanisms that are used for describing functional readings and wide scope indefinites, no attempt has so far been made to

bring them into one framework. This paper proposes such a unified mechanism. It argues that functional readings and the interpretation of wide scope indefinites are restricted by the same principles – those that govern quantification over Skolem functions. Under this view, quantification over functions in natural language is existential only, and is furthermore restricted to a small subclass of noun phrases. This accounts for the similar distribution of functional quantification and wide scope phenomena, as well as for some formal relations between functional quantification and Generalized Quantifier Theory. A novel hypothesis characterizes the class of quantifiers that license functional interpretations and explains their restricted distribution.

In a nutshell, the two general phenomena with which this article deals can be illustrated by the following two sentences.

(1) a. The (only) woman that every man loves is his mother.

b. Every man loves a (certain) woman.

Sentence (1a) illustrates a *functional reading*. In this reading, the pronoun *his* is "bound" by the noun phrase *every man* although it is not within its syntactic scope. Another familiar scope problem appears in sentence (1b). This sentence has a reading where the indefinite *a* (*certain*) *woman* is interpreted as taking *wide scope* over the subject, although the subject is not within its syntactic scope.

Popular analyses of both phenomena involve Skolem functions under some version or another. To illustrate these approaches, consider first the following intuitive analyses of sentences (1a) and (1b).

(2) a. The (only) function in the set $\{f : f$ maps every man to a woman he loves$\}$ is the function that maps every man to his mother.

b. There is a choice function $f$ such that every man loves $f(\{x : x$ is a woman$\})$.

We refer to nominal expressions such as *woman* and *woman that every man loves* as the *restricting predicate*, or *restriction*, of the relevant noun phrase. The functions in (2a) are in the denotation of the surface restriction *woman that every man loves*. Consequently, this analysis derives a "bound" reading of the pronoun *his* without assuming that *every man* takes the pronoun within its scope. Similarly, since the function $f$ in (2b) applies to the restriction *woman* in its surface position, there is no standard scope of the object over the subject in this analysis.

However, there is one important difference between the two analyses. In the analysis (2a) of the functional reading, it is the restricting predicate itself (i.e.

*woman that every man loves*) that is assumed to range over functions. In the functional analysis (2b) of the scope of indefinites, the function is used as a variable that only applies to the restricting predicate *woman*, but it does not belong to the restricting predicate itself. Accordingly, existential quantification over functions in (2b) is assumed as a (possibly contextual) default mechanism, and not as the denotation of the indefinite article *a* (*certain*) in (1b). By contrast, the quantifier *the* (*only*) over functions in (2a) is assumed to be the denotation of the definite article *the* (*only*) in (1a).

The first aim of this paper is to bridge this gap between the two kinds of analyses. It is argued that a unified analysis is not only justified by the technical and conceptual similarity between the two theories, but also because functional readings with copular sentences, questions and wide scope indefinites appear with the same class of NPs. The first step in unifying the two mechanisms, which is quite uncontroversial, is to generalize the choice function analysis of wide scope indefinites to Skolem functions of arbitrary arity. This is independently necessary for treating indefinites such as *a certain woman he knows*, where the pronoun *he* is bound from outside the indefinite. The second step is to modify Jacobson's (1994) analysis of functional readings in variable-free semantics. The revised analysis allows the Skolem function mechanism of wide scope indefinites to apply to the restricting predicate also with functional copular sentences, while keeping the quantification over functions existential only.

The main ingredient in unifying Jacobson's account with the general treatment of indefinites is a novel type-shifting operator from objects of type $(ee)t$ (sets of functions from entities to entities) to binary relations of type $e(et)$ (functions from entities to sets of entities). Over finite domains this mapping can be one-to-one only under certain restrictions on its domain of $(ee)t$ objects. It is shown that this is naturally guaranteed when the quantifier within the functional NP (e.g. *every man* in (1a)) is a so-called *bounded* quantifier – a quantifier that can be expressed as an intersection of a positive universal quantifier $every(A)$ with a negative universal quantifier $no(B)$, for some sets $A$ and $B$. This characterization leads to the linguistically plausible hypothesis that only bounded quantifiers can give rise to functional readings.

The structure of this paper is as follows. Section 2 briefly overviews the problems of functional readings and wide scope indefinites, gives necessary technical details about previous approaches and discusses the motivation for a unified analysis. Section 3 introduces the mapping that allows an extended theory to treat both phenomena, and exemplifies its applications. Section 4 motivates the restrictions on the proposed mapping, and proves the relations between this restriction and the

class of bounded quantifiers. Section 5 points out the strong relationships between the present work and the data and mechanisms that are studied in a recent work by Jacobson (2002).

# 2 Functional readings and wide scope indefinites

## 2.1 Functional readings

The so-called *functional* reading of questions can be illustrated by the following familiar question-answer pairs.

(3) a. Which woman does every man love? His mother.

b. Which woman does no man love? His mother-in-law.

The problem of interpreting questions that exhibit this kind of readings was discussed extensively in the literature.[1] The puzzle is often related (cf. Chierchia 1993) to the problem of *pair-list* readings of questions, as illustrated by the following short discourse.

(4) Which woman does every man love? John loves Mary, Bill loves Sue, etc.

In order to analyze both kinds of readings that questions exhibit, it has been proposed that quantification over functions plays a role in the interpretation. Similar mechanisms have been proposed for the functional readings of *copular sentences* like the following.

(5) a. The woman that every man loves is his mother.

b. The woman that no man loves is his mother-in-law.

Sharvit (1999) convincingly argues that such copular sentences have the same distribution and syntactic/semantic properties of functional questions as in (3). In order to illustrate the mechanisms that will be considered, we will therefore concentrate on such indicative copular sentences, without getting into the more intricate semantics of questions.

A fully worked-out account of functional readings in copular sentences as in (5) is given in Jacobson (1994) and Jacobson (1999). Jacobson claims that sentences like (5a) and (5b) cannot be treated by giving the noun phrase *every man* sentential scope, as informally illustrated below.

[1] See Engdahl (1986:ch.4) and Groenendijk and Stokhof (1984:ch.3) for two classical works on this topic.

(6) For every (no) man $x$, the woman that $x$ loves is $x$'s mother (mother-in-law).

One reason is syntactic: to obtain such an analysis of the sentences in (5), the noun phrases *every/no man* would have to cross a complex NP (*the woman that...loves*), which is generally acknowledged to be a scope island. For instance, sentence (7a) does not have the reading that is paraphrased in (7b).

(7) a. The woman that no man loves came to the party.

b. For no man $x$, the woman that $x$ loves came to the party.

Moreover, noun phrases normally do not bind pronouns that are not within their syntactic scope (their *c-command domain*) as witnessed by the fact that the informal analysis in (8b) is not an available reading of sentence (8a).[2]

(8) a. The woman that no man loves pinched him.

b. For no man $x$, the woman that $x$ loves pinched $x$.

Another argument against a "scopal" analysis, which Jacobson attributes to Dahl (1981), is semantic. Consider sentence (9a) below. This sentence is clearly not equivalent with (9b), which is obtained by giving the noun phrase *no man* sentential scope.

(9) a. The only woman that no man loves is his mother-in-law.

b. For no man $x$, the only woman that $x$ loves is his mother-in-law.

To see that, consider a situation in which John is a man who loves both his wife and his mother-in-law. In this situation (9a) is false. However, (9b) may still be true, as long as also other men do not love *only* their mother-in-law.

Jacobson's account of functional readings is based on her general theory of variable-free semantics. The theory itself is introduced in much detail in Jacobson (1999,2000), and I will not try to review all of its parts here. Jacobson's assumptions that are important for our present purposes are the following, which for convenience are given the names (J1)-(J4).

[2] Sharvit (1999) claims, following Doron (1982), that if *no* in (8a) is replaced by *every*, then binding may sometimes become possible, but argues that this option should be derived by the same mechanism that derives pair-list readings with *every* as in (4). A similar point may be related to the fact that some speakers find the statement in (7b) more acceptable as a reading of (7a) when *no* is replaced by *every* or *each*. Since this paper does not deal with pair-list readings, these points are not directly relevant to its main purposes.

(J1) An expression $E$ that contains a "free pronoun" $P$ denotes a function from entities of the standard type of $P$ to entities of the standard type of $E$.

Consider for instance the noun phrase *the woman who gave him birth* or, equivalently *his mother* (as in (5)). Assume that the standard type of NPs is $e$ and that this is also the standard type of the pronoun *him*. Jacobson therefore assumes that the whole NP denotes a function of type $ee$: a function from entities to entities. In the example, this is the function that maps every (adult male human) entity to its mother.

(J2) Transitive predicates like *love*, of the standard type $e(et)$, have a secondary meaning of type $(ee)(et)$ that ranges over $ee$ functions in the object argument. This reading enables the subject NP to "bind" a pronoun within the object. The operator that derives this additional meaning of transitive predicates is denoted 'Z' and is defined as follows.

(10) $\mathrm{Z}_{(e(et))((ee)(et))} \stackrel{def}{=} \lambda R_{e(et)}.\lambda f_{ee}.\lambda x_e.R(f(x))(x)$

In words: the Z function maps a binary relation $R$ to the relation $\mathrm{Z}(R)$ that holds exactly between those $ee$-type functions $f$ and entities $x$ that satisfy $R(f(x))(x)$. For instance, the following example (11a) is analyzed as in (11b).[3]

(11) a. Every man loves his mother.

b. $\mathbf{every}'_{(et)((et)t)}(\mathbf{man}'_{et})(\mathrm{Z}(\mathbf{love}'_{e(et)})(\mathbf{his_mother}'_{ee}))$
$\Leftrightarrow \mathbf{every}'(\mathbf{man}')(\lambda x_e.\mathbf{love}'(\mathbf{his_mother}'(x))(x))$
$\Leftrightarrow \forall x[\mathbf{man}'(x) \rightarrow \mathbf{love}'(\mathbf{his_mother}'(x))(x)]$

The determiner *every* here standardly denotes the subset relation between sets, or in lambda format: $\lambda A_{et}.\lambda B_{et}.\forall x_e[A(x) \rightarrow B(x)]$.

(J3) Items like the definite article *the*, the relative pronoun *that* and the copula *be* can range over $ee$ functions as well as "ordinary" $e$-type entities. In essence,

[3]Throughout this paper, we employ a typed lambda calculus with equality over the extensional types $e$, $t$ and their functional compounds. In this formalism, the propositional connectives are easily defined, as well as the logical quantifiers $\exists$ and $\forall$ over any type (see Van Benthem (1991:p.7)). Lambda terms are freely mixed with the set-theoretical correlates of their denotations, where the domain of entities $D_e$ is also denoted '$E$'. For instance: a binary relation is viewed either as a lambda term of type $e(et)$ or as a subset of $E^2$; a generalized quantifier is viewed either as a lambda term of type $(et)t$ or as a subset of $\wp(E)$; etc.

we can assume that such items denote the (polymorphic) *iota*, intersection and identity functions respectively.

(J4) Intransitive restricting predicates like *woman*, of the standard type $et$, have a second meaning of type $(ee)t$. This meaning ranges over $ee$ functions and allows the restriction to combine with functional relative clauses. I use 'N' to denote the operator that derives this additional meaning of transitive predicates. This operator is defined as follows.

(12) $\mathrm{N}_{(et)((ee)t)} \stackrel{def}{=} \lambda P_{et}.\lambda f_{ee}.\forall x_e[P(f(x))]$

For instance, the $(ee)t$ denotation of the noun *woman* that the N operator derives is the set of $ee$ functions that map each entity to a woman. Note that this set is empty when there are no women in the model.

For sake of exposition, I will use here a slightly modified version of the Z operator that Jacobson uses for binding. This revised operator, which is denoted '$\mathrm{Z}^0$', allows generalized quantifiers of type $(et)t$, rather than $e$ type entities, to combine directly with the binary relation that is modified by the operator. Its definition follows.

(13) $\mathrm{Z}^0{}_{(e(et))(((et)t)((ee)t))} \stackrel{def}{=} \lambda R_{e(et)}.\lambda Q_{(et)t}.\lambda f_{ee}.Q(\lambda x_e.R(f(x))(x))$

This operator has essentially the same consequences of Jacobson's Z operator, but its arguments are now a quantifier and an $ee$ function (in this order), instead of an $ee$ function and an entity as in Jacobson's analysis. This revised formulation of Z only comes to allow a generalized quantifier such as *every man* in the relative clause *that every man loves* to "saturate" the subject argument of the transitive predicate *loves*, without getting into complex questions concerning the derivation of this option within a general categorial theory.[4]

Now we can get back to the sentences in (5) and illustrate their analysis in Jacobson's approach. The meaning derivation of sentence (5a) is summarized in figure 1. The type variable $\tau$ stands for any monomorphic type. It is not hard to verify that the result of this derivation is tantamount to the following statement:

(14) There is only one $ee$ function $f$ s.t. $f$ maps every man to a woman he loves, and this function is the function that maps every man to his mother.

[4] Jacobson's mechanism achieves this using (a modified version of) the "Geach Rule" and Function Composition.

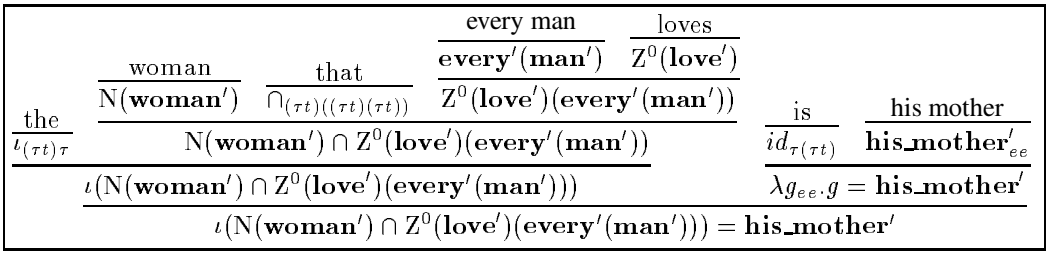


Figure 1: Jacobson's derivation of meaning for sentence (5a)

Jacobson argues that this paraphrase captures the intuitive meaning of sentence (5a).[5] Furthermore, this meaning is derived without any scope shifting of the noun phrase *every man*, which, as mentioned above, would have been problematic.

Alexopoulou and Heycock (2002) observe that copular sentences exhibit functional readings also in cases where their subject is not necessarily a singular definite as in (5). Some examples for this observation follow.

(15) a. One/a (certain) woman that every man loves is his mother.

b. One/a (certain) woman that no man loves is his mother-in-law.

(16) A woman that every/no man would be happy to see again is his childhood sweetheart.

(17) a. (The) two women that every Frenchman admires are his mother and Brigitte Bardot. (after Engdahl (1986:ch.4))

b. (The) two women that no Frenchman admires are his mother-in-law and Margaret Thatcher.

Jacobson does not mention such sentences, and their functional readings are not immediately captured by her mechanism. However, along the lines of her proposal, it is natural to assume that pre-nominal items such as *one, a* and (*the*) *two* should also be analyzed as polymorphic operators, similar to the polymorphic *iota* operator that Jacobson assigns to the definite article. For instance, the items *one* and *a* in (15) and (16) could be analyzed as polymorphic existential determiners,

[5] Strictly speaking, this is not true, since requiring uniqueness of an $ee$ function is much too strong a requirement. In fact, Jacobson argues that domain restriction of the definite article plausibly leads here only to a requirement about the uniqueness of a function from men to women with the properties required above and which is furthermore "natural" in the given discourse.

of type $(\tau t)((\tau t)t)$. When $\tau$ is the $ee$ type, of functions from entities to entities, this would allow the determiner to compose with a restricting predicate of type $(ee)t$ as in Jacobson's polymorphic analysis of the definite article. Consider the existential determiner of the relevant type.

(18) $\mathbf{a}'_{((ee)t)(((ee)t)t)} \stackrel{def}{=} \lambda F_{(ee)t}.\lambda G_{(ee)t}.\exists f_{ee}[F(f) \wedge G(f)]$

Substituting this existential determiner for $\iota$ in Jacobson analysis in figure 1, we get the following meaning.

(19) $\mathbf{a}'(\mathrm{N}(\mathbf{woman}') \cap \mathrm{Z}^0(\mathbf{love}')(\mathbf{every}'(\mathbf{man}')))(\lambda g_{ee}.g = \mathbf{his_mother}')$
$\Leftrightarrow \exists f[(\mathrm{N}(\mathbf{woman}') \cap \mathrm{Z}^0(\mathbf{love}')(\mathbf{every}'(\mathbf{man}')))(f) \wedge f = \mathbf{his_mother}']$
$\Leftrightarrow (\mathrm{N}(\mathbf{woman}') \cap \mathrm{Z}^0(\mathbf{love}')(\mathbf{every}'(\mathbf{man}')))(\mathbf{his_mother}')$

This analysis makes the requirement of the mother function that is intuitively expressed by sentence (15a), and similar analyses of the sentences in (15)-(17) can be derived in the same method. However, a general analysis of determiners as ranging over $ee$ functions would be unnecessarily strong. This is because other NPs do not lend themselves so easily to functional readings. Consider for instance the following unacceptable sentences.

(20) ??At most/at least one woman that every man loves is his mother.

(21) ??No woman that every man would be happy to see again is his childhood sweetheart.

(22) ??Between two and three women that every Frenchman admires are his mother, Brigitte Bardot and possibly Isabelle Adjani.

In (22), for instance, it could be expected that the sentence should entail that there are between two and three functions that send each Frenchman to a woman he admires, in a similar way to the entailment from (17a) to the existence of two such functions. However, sentence (22) is quite incoherent. Thus, it would not be too helpful to assign a polymorphic meaning to all the determiners in natural language, since we only need a proper subset of them (if any) to range over functions.[6] In this paper I propose that in fact, *no* determiner should range over

[6] An anonymous reviewer points out that quantified NPs like those in (20)-(22) are also infelicitous in copular sentences without functional interpretations:

(i) ??At most/at least one woman that every man loves is Sue.

functions. According to the proposed mechanism, functional quantification is only existential and it is derived by the same general functional process that is responsible for the interpretation of wide scope indefinites. The unified process that will emerge will also cast some doubts on the usefulness of the other polymorphic entries that are assumed in (J3) and of the $\mathrm{N}$ operator that is assumed in (J4).[7]

## 2.2 Wide scope indefinites

The main reason for introducing functions in the semantic analysis of indefinites is their ability to take scope beyond syntactic islands, which has received much attention in the literature since Fodor and Sag (1982). Consider for instance the following example.

(23) If some/a (certain) girl I know arrives to the party then John will be happy.

This sentence has the reading that is paraphrased in (24) below, which makes a statement about a "specific girl".

(24) There is a girl $x$ I know such that if $x$ arrives to the party then John will be happy.

We say that under this reading, the indefinite takes *wide scope* (WS) over the conditional. This reading is sometimes sloppily referred to as the *WS reading* of the indefinite. This behavior of indefinites is in sharp contrast with the behavior of other NPs. For instance, the following sentence in (25a) does not have the analysis that is paraphrased in (25b).

(25) a. If every girl arrives to the party then John will be happy.

b. For every girl $x$, if $x$ arrives to the party then John will be happy.

This claim can be attested by considering that sentence (25a) can be intuitively false in situations where the statement in (25b) is true. For instance, when John

(ii) ??No woman that every man would be happy to see again is Sue.

(iii) ??Between two and three women that every Frenchman admires are Catherine Deneuve, Brigitte Bardot and possibly Isabelle Adjani.

Thus, while I am not claiming here to have an account of unacceptabilities as in (20)-(22) or (i)-(iii), my claim is more modest than that: that a 'functional' meaning of determiners such as *at most one, at least one, no, between two and three* etc. is unnecessary for analyzing functional readings.

[7] See the derivation in (41) below.

would be happy if *any* girl comes to the party, while not wanting *all* the girls to come to the party together.

The special scopal behavior of indefinites calls for an explanation. In recent years, many works have followed Reinhart (1992,1997) and Kratzer (1998), and assumed that the problem should be solved by allowing indefinites to be interpreted using *choice functions* (CFs), whose definition is given below.

**Definition 1 (choice functions)** *Let $E$ be a non-empty set. A function $f$ from $\wp(E)$ to $E$ is a* choice function *over $E$ iff for every $A \subseteq E$: if $A$ is not empty then $f(A) \in A$.*

In extensional type logical frameworks like the one assumed throughout this paper, the set $CH^\tau$ of *choice functions over type $\tau$* is defined as follows:

(26) $CH^\tau \stackrel{def}{=} \lambda f_{(\tau t)\tau}.\forall P_{\tau t} \neq \emptyset[P(f(P))]$

*Convention*: we often write '$CH$' instead of '$CH^e$'.

The WS behavior of indefinites is treated using CFs by letting a free CF variable apply to the restriction of the indefinite. We assume that an existential quantifier applies to this variable at the matrix level, and derives the following interpretation of sentence (23).

(27) $\exists f[CH(f) \wedge [\mathbf{arrive}'(f(\mathbf{girl}')) \rightarrow \mathbf{glad}'(\mathbf{j}')]]$

The introduction of an existential quantifier over CFs – especially at levels lower than the matrix level – is controversial. Some authors, notably Kratzer (1998), favor a usage of CFs as "deictic" entities, without existential quantification over them. This controversy is not central for the purposes of this paper, and readers may easily modify the analyses below to the "deictic" version of CFs. Another problematic point which is not directly relevant to our purposes here is the treatment of indefinites with an empty restricting predicate. For example, consider sentence (23) when there happens to be no girl in the given situation. In Winter (1997) and Winter (2001:ch.3) this case is given a solution using CFs of a higher type. For the sake of presentation I will not employ here this more complex analysis.

We have seen that indefinites with *some* and *a certain* show WS behavior beyond the "adjunct island" of the conditional. The same holds for such indefinites in other syntactic environments that behave like scope islands.[8] However, many

[8] See Ruys (1992:ch.3) for a concise description of the facts.

other NPs (e.g. universal NPs as in (25a)) do not show this island-free behavior. It is therefore important to characterize those NPs to which the CF mechanism applies. This question is taken up in Winter (2001:ch.4), where it is argued that in addition to simple indefinites as in (23), the CF mechanism also applies to simple numeral indefinites (e.g. *three students*), *WH* phrases and singular and plural definite NPs. The WS potential of simple numerals can be easily illustrated by the interpretation (28b) of sentence (28a).

(28) a. If three girls I know arrive to the party then John will be happy.

b. There is a set $A$ of three girls that I know such that if the girls in $A$ arrive to the party then John will be happy.

The WS potential of *WH* elements is exemplified by Reinhart (1997) using question-answer pairs like the following:

(29) Who will be offended if we invite which philosopher? John will be offended if we invite Putnam.

To interpret the noun phrase *which philosopher*, early semantic theories of questions would have to assign it sentential scope over the conditional. Reinhart however shows that the CF mechanism can treat such effects of *WH in situ* in a similar fashion to the treatment of indicative indefinite that was reviewed above. As for definite NPs, because of their uniqueness requirement it is not easy to test their scopal behavior. I refer the reader to Winter (2001:ch.4) for other interpretative effects with definites that are accounted for by their CF interpretation.

Crucially, we have seen above that all these kinds of NPs – simple singular/plural (in)definites and WH elements – also lead to functional readings (cf. (3), (5) and (15)-(17)). Conversely, NPs as in (20)-(22) do not give rise to wide scope readings beyond islands, and therefore they do not require the CF analysis. Consider for instance the contrast between *at least one* in the following sentence and *some* in (23) above.

(30) If at least one girl (I know) arrives to the party then John will be happy.

This sentence does not have the reading that is paraphrased below.

(31) There is at least one girl I know $x$ such that if $x$ arrives to the party then John will be happy.

Similar cases where WS interpretations are unavailable can also be illustrated for the other NPs in (20)-(22). This distinction between NPs where the CF analysis is available and other NPs where it is unavailable, is addressed by the syntactic-semantic mechanism that is proposed in Winter (2000, 2001:ch.4). In these works I propose that the two kinds of nominals correspond to two different syntactic classes of DP structures. It is proposed that so-called *flexible DPs*, like simple (in)definites and WH phrases, are basically predicative, and therefore in argument position they are interpreted using CFs. Other DPs, which are not amenable to application of flexibility principles, are accordingly called *rigid*. These NPs include complex numerals (e.g. *at least one* or *exactly one*) and universal quantifiers, and involve a full DP structure. Consequently, these DPs are purely quantificational and have no CF interpretation.

Of course, the parallelism we observed between WS interpretation and the availability of functional readings is not accounted for in the absence of a unified theory of both phenomena. However, before moving on to the details of this theory, we need to discuss a crucial ingredient of the proposal: general Skolem functions of arbitrary arity.

## 2.3 General Skolem functions

It is interesting to note that, quite independently of the scope of indefinites that have occupied semantic theories in recent years, some earlier works had proposed to use general *Skolem functions* (SFs) for capturing other semantic effects with indefinites and interrogatives. Intuitively, we can think of an $n$-ary Skolem function as a choice function with $n$ parameters. While a choice function $f$ maps any non-empty set $X$ to a member of $X$, a general Skolem function $f_n$ of arity $n$ maps such a set $X$, together with tuple of $n$ parameters $\langle x_1, ..., x_n \rangle$, to a member of $X$. Since we want to treat parameters in such tuples as "free variables" in Jacobson's variable-free framework, we assume that the input to a Skolem function is an $n$-place *function* that has the power-set $\wp(E)$ as its range. A variable-free Skolem function modifies such a function to a function that has $E$ as its range. This is formally stated in the following definition.

**Definition 2 (Skolem functions of arity $\geq$ 1)** *Let $E$ and $A$ be non-empty sets. A function $f$ from $(\wp(E))^A$ to $E^A$ is a* Skolem function *iff for every function $g \in (\wp(E))^A$, for every $x \in A$ s.t. $g(x) \neq \emptyset$: $(f(g))(x) \in g(x)$. If $A$ is a Cartesian product of arity $n \geq 1$ then we say that $f$ is a Skolem function of arity $n$.*

| | | $g$ | $h_1$ | $h_2$ | $h_3$ | $h_4$ |
|---|---|---|---|---|---|---|
| $a$ | $\mapsto$ | $\{c\}$ | $c$ | $c$ | $c$ | $c$ |
| $b$ | $\mapsto$ | $\{a,b\}$ | $a$ | $a$ | $b$ | $b$ |
| $c$ | $\mapsto$ | $\{a,b\}$ | $a$ | $b$ | $a$ | $b$ |

Table 1: an $e(et)$ function and some $ee$ functions

In a purely functional type-theoretical format, the set $A$ cannot be a Cartesian product. However, standard 'Currying' allows us to simulate products using functions and generalize the notion of Skolem functions. SFs of arity $n \geq 0$ are defined as objects of the following type scheme:

$$(\tau_1(...(\tau_n(\tau t))...))(\tau_1(...(\tau_n\tau)...)).$$

In this type scheme, the argument of an SF is a function of type $\tau_1(...(\tau_n(\tau t))...)$, which is isomorphic to a function from the Cartesian product $D_{\tau_1} \times ... \times D_{\tau_n}$ to the sets of $\tau$-type elements. An SF maps such a function to a function of type $\tau_1(...(\tau_n\tau)...)$, which is isomorphic to a function from the Cartesian product $D_{\tau_1} \times ... \times D_{\tau_n}$ to $\tau$-type elements.

The set $SK^n$ of general Skolem functions of this type is defined as follows.

(32) $SK^n \stackrel{def}{=} \lambda f.\forall g_{\tau_1(...(\tau_n(\tau t))...)}\forall x_1...\forall x_n$
$[g(x_1)...(x_n) \neq \emptyset \rightarrow (g(x_1)...(x_n))((f(g))(x_1)...(x_n))]$

*Convention*: We usually assume that $\tau_1 = ... = \tau_n = \tau$, and say that a function in $SK^{\tau,n}$ is a *Skolem function* (of arity $n$) *over type* $\tau$. We often write '$SK^n$' instead of '$SK^{n,e}$'. With this notation it is clear that the SF of arity 0 over type $\tau$ are simply the CFs over $\tau$.

Consider the following example for a Skolem function $f$ of arity 1. Assume that $E = \{a, b, c\}$. Let $g$ be a function of type $e(et)$ that maps $a$ to the singleton set $\{c\}$ and maps both $b$ and $c$ to the set $\{a, b\}$. Then the function $f(g)$ must be one of the four $ee$ functions $h_1, ..., h_4$ that are described in table 1. Note that $h_2$ and $h_3$ illustrate the liberty of $f(g)$ to "choose" a different element of the set $\{a, b\}$ for each of the two "parameter values" $b$ and $c$. It is in this sense that SFs of arbitrary arity are more powerful than CFs.

The close relationships between WS indefinites and general SFs on the one hand, and Jacobson's treatment of functional readings on the other hand, are most easily demonstrated by the following examples due to Groenendijk and Stokhof (1984:ch.3) and Hintikka (1986), respectively.

(33) Every man loves a (certain) woman – his mother.

(34) According to Freud, every man unconsciously wants to marry a certain woman – his mother.

Both Groenendijk & Stokhof and Hintikka propose to treat such examples using an existential quantifier over SFs that takes matrix scope. In the current setting, this means that the restricting predicate that the noun *woman* denotes must be a two-place relation, of type $e(et)$. This is surprisingly reminiscent of Jacobson's assumption (J4), according to which restricting predicates are systematically of a higher type. Jacobson's $\mathrm{N}$ operator maps the set of women to the set of $ee$ functions that map any entity to some woman or other. Instead, now we need to map the set of women to a "parameterized set of women": the constant function of type $e(et)$ that maps any entity to the set of women. In view of this similarity, let us use the notation '$\mathrm{N}^0$' to denote the general operator that derives such functions from ordinary denotations of nouns. Formally, the $\mathrm{N}^0$ operator is defined as follows.

(35) $\mathrm{N}^0{}_{(et)(e(et))} \stackrel{def}{=} \lambda P_{et}.\lambda x_e.\lambda y_e.P(y)$

Using SFs, we can now model discourses such as (33) and (34) by existential quantification over an SF at the matrix level. The result is that each of these sentences is analyzed as a statement about the existence of a certain function, where the second part of each sentence specifies its identity. Even without getting into the technical derivation of this "discourse specification", it is clear that existential quantification over SFs accounts for the interpretation of such sentences.[9] To give an illustration of this treatment, consider the following analysis of (33).

[9]Manfred Krifka (p.c.) points out to me that the discourse processing of similar examples may be more complicated than a simple substitution of a Skolem function for a variable. He mentions examples like the following:

(i) Every man loves at most two women – his mother, and if he has one – his sister.

Here, no Skolem function is likely to be operational. However, Skolem functions are required if we like to account for contrasts like the following, where the indefinite NP is within a (complex NP) island.

(ii) Every child loves every man who admires a certain woman – his mother.

(iii) ?Every child loves every man who admires at most two women – his mother, and if he has one – his sister.

We see that the same discourse process that applies in (i) does not apply in (iii), although simple indefinites, as expected, lead to wide scope interpretations beyond islands, as in (ii), as expected by the Skolem function mechanism.

(36) $\exists f_{(e(et))(ee)}[SK^1(f) \wedge \mathbf{every'}(\mathbf{man'})(\mathrm{Z}(\mathbf{love'})(f(\mathrm{N}^0(\mathbf{woman'}))))]$

In words: there is an Skolem function $f$ of arity $1$, which maps the function $\mathrm{N}^0(\mathbf{woman'})$ – the constant function that maps every entity to the set of women – to an $ee$ function $h$. This $h$ function furthermore has the property that every man is in $\mathrm{Z}(\mathbf{love'})(h)$. More simply: every man loves the woman that $h$ assigns to him. Assuming that the set of women is not empty, this is equivalent to the standard analysis of sentences like (33), with narrow scope existential quantification over $e$-type entities:

(37) $\forall x \in \mathbf{man'} \exists y \in \mathbf{woman'}[\mathbf{love'}(y)(x)]$

We see that in these examples, general SFs are employed to account for *narrow scope* readings of indefinites that due to the anaphora have a wide scope "functional flavor". It is curious to note that Reinhart's usage of the simpler $0$-arity SFs (i.e. CFs) for deriving "ordinary" *wide scope* readings of indefinites was discovered rather late in the development of the theory.

In addition to the use of general Skolem functions in the treatment of indefinites as in (33) and (34), there are (at least) four other types of motivations that were given in the literature for their introduction:

1. Kratzer (1998) objects to the existential closure of CFs in Reinhart's analysis. Instead, Kratzer proposes to use general SFs as a means for deriving readings that under Reinhart's account require existential quantification over CFs with scope that is narrower than matrix scope.

2. Schlenker (1998) and Winter (1998,2001:ch.3) argue (independently) that certain sentences with indefinites show readings that are inexpressible using CFs alone and require SFs of arbitrary arity.[10]

3. In Winter (2001:ch.3) it is argued that SFs are required in order to eliminate problematic analyses that are derived using CFs for indefinites with free pronouns (e.g. *a woman he knows*).

4. Chierchia (2001) argues that certain effects with indefinites, including weak crossover and *de re* interpretations, require introduction of SFs of arbitrary arity.

[10]Following Hintikka, Schlenker also uses general SFs for deriving "branching readings" with indefinites. I will not discuss this phenomenon here.

I will not discuss these issues in this paper, but simply assume, as in most works on this subject, that general SFs are required for the treatment of indefinites. The exact restrictions on their introduction is a topic that is still under extensive investigation.[11] The next section shows how this assumption allows us to achieve a unified treatment of functional readings and WS indefinites.

# 3 A unified mechanism for functional readings and wide scope indefinites

In the previous section we have seen technical, intuitive and distributional reasons to assume that "functional readings" and "wide scope indefinites" are two phenomena that should be derived by the same mechanism. Technically, the SF treatment of the scope of indefinites involves functions from entities to entities, like the ones that were independently employed by Jacobson and others for treating sentences with functional readings. Intuitively, the "wide scope functional" interpretation of discourses as in (33) and (34) seems to be essentially the same kind of thing as the functional interpretation of questions such as (3) or copular sentences such as (5). From a distributional point of view, we saw that the same NPs that are treated using SFs and can show WS effects beyond islands also show "functional" effects. It is therefore natural to expect the two phenomena to be amenable to the same treatment.

However, as mentioned in the introduction, one link is missing between the two kinds of theories that were reviewed above. Jacobson assumes that the restricting predicate in NPs with functional readings *ranges* over functions. Consequently, in her account any NP should in principle allow quantification over functions. By contrast, the CF treatment assumes that choice functions *apply to* the restricting predicate, which denotes a set of "ordinary" entities of type $e$. Quantification over CFs, if needed at all, is only existential, and applies syncategorematically: independently of the syntax/semantics of the NP. The same holds of the way SFs of higher arity are used for treating scopal effects with indefinites.

The main argument of this section is that this discrepancy can be resolved by renouncing Jacobson's general polymorphic scheme of quantification over functions. In the proposed modification of her mechanism, functions are involved in the internal semantics of the NP as part of the variable-free mechanism in assump-

[11] In addition to Chierchia (2001), see also Schlenker (1998) and Schwartz (2001) for some recent work in progress that deal with this topic.

tions (J1) and (J2). However, the restricting predicate and the determiner range over ordinary entities and are standardly treated using the general SF mechanism. This allows us to renounce Jacobson's assumption (J4) about the N operator. Further polymorphism as in (J3) will be needed only with the copula construction and not with determiners or relative pronouns.

To achieve this unified analysis, the main new part of the proposal is an operator RG ('range' operator) that is defined below.

**Definition 3 (RG operator)** *Given two non-empty sets, $A$ and $B$, the* RG *operator is a function from $\wp(B^A)$ (the sets of functions from $A$ to $B$) to $(\wp(B))^A$ (the functions from $A$ to subsets of $B$) s.t. for all $F \subseteq B^A$, for all $x \in A$:* $(\mathrm{RG}(F))(x) \stackrel{def}{=} \{f(x) : f \in F\}$.

In other words, if $F$ is a set of functions from $A$ to $B$, then $\mathrm{RG}(F)$ maps each entity $x$ in $A$ to the subset of $B$ that consists of the images of $x$ by the functions in $F$. As an example, reconsider the functions in table 1 above, and observe that $\mathrm{RG}(\{h_1, h_2, h_3, h_4\}) = g$.

For our purposes, the most useful instance of RG is when $A$ and $B$ are both equal to the domain of entities. The RG operator of the respective type is given below in lambda format.

(38) $\mathrm{RG}_{((ee)t)(e(et))} = \lambda F_{(ee)t}.\lambda x_e.\lambda y_e.\exists f \in F[f(x) = y]$

To illustrate how this operator allows us to unify the two mechanisms, consider again Jacobson's analysis of sentence (5a) that was illustrated in figure 1 above. In the revised analysis, the denotation of the "gapped" constituent *every man loves* still denotes the same $(ee)t$ set of functions as in Jacobson's analysis. This is the set $F = \mathrm{Z}^0(\mathbf{love}')(\mathbf{every}'(\mathbf{man}'))$ – the set of functions that map every man to something he loves. The RG operator maps $F$ to a binary predicate – the function that maps every man to the *set of* things he loves, provided that every man loves something.[12] Let us henceforth denote this binary relation $\mathrm{RG}(F)$ by '$S$'. The binary relation $S$ should be intersected with the denotation of common noun *woman*. This can be achieved by lifting the set $\mathbf{woman}'$ using the $\mathrm{N}^0$ operator that was defined in (35) above, and was used in the treatment of sentence (33) in (36).[13] Consequently, the restricting predicate *woman that every man loves*

[12] If there is a man who does not love anything, then $F$ is empty and consequently $\mathrm{RG}(F)$ sends everything to the empty set.

[13] Later we will see that this special operator is actually not needed here, and its postulation reflects a more general property of the "binding" mechanism.

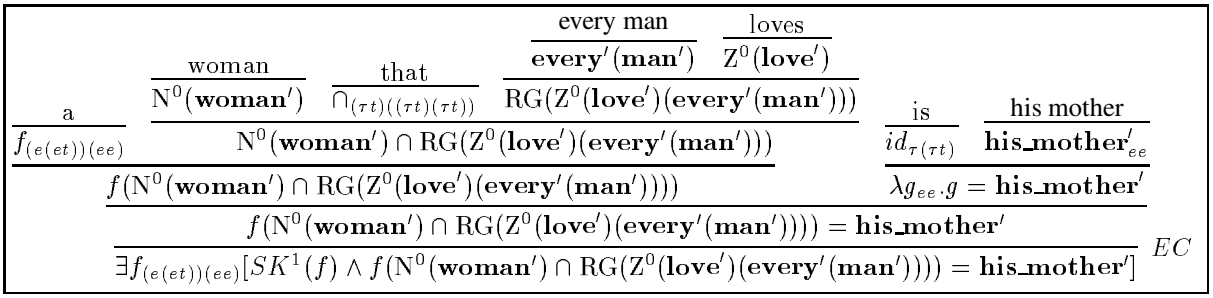


Figure 2: Derivation of meaning for sentence (39)

denotes the function that maps every man to the set of *women* he loves. Let us refer to this binary relation $(\mathrm{N}^0(\mathbf{woman}') \cap S)$ by the letter '$R$'. To the binary relation $R$ we can apply the general SF mechanism, as in (36). It is easier to see how this SF mechanism works with indefinites. Therefore, let us consider how the meaning of sentence (39) below is derived. This sentence (=(15a)) is a slight variation on Jacobson's example (5a).

(39) A (certain) woman that every man loves is his mother.

The analysis of this example using SFs is given in figure 2, where $EC$ stands for "existential closure" – here of an SF of arity $1$. In this analysis, the Skolem function $f$ applies to the denotation of the restricting predicate *woman that every man loves*: the binary relation $R$ that sends each man to the set of women he loves. Assuming that $R(x)$ is non-empty for every man $x$, any SF sends $R$ to an $ee$ function that maps every man to one of the women he loves. The statement that is derived in figure 2 claims that one of these $ee$ functions is the mother function. This is the desired interpretation of sentence (39), and it is equivalent to the Jacobsonian analysis of the same example in (19) above.

The proposed treatment of sentence (39) as described above is presented in a way that highlights the main differences from Jacobson's treatment. These differences (cf. figure 1 vs. figure 2) are the introduction of the $\mathrm{RG}$ operator and the application of SFs to the restricting predicate. However, a more general treatment is obtained if we adopt the mechanism of Winter (2001:ch.3) for handling "free variables". This mechanism uses a special functional type constructor "$\rightarrow$" in order to distinguish the type of denotations that contain "free variables" from the type of other functions.[14] An expression of type $\tau \rightarrow \sigma$ denotes a function

[14] Jacobson (1999) identifies expressions that contain "free variables" by their syntactic category

from $D_\tau$ to $D_\sigma$, but it behaves compositionally like an expression of type $\sigma$ that contains a "gap" (a free variable) of type $\tau$. For instance, a transitive predicate such as *love* gets the standard type $e(et)$. By contrast, a binary relation like the relation $R$ that is denoted by the expression *woman that every man loves* gets the type $e \rightarrow (et)$: it behaves compositionally like an ordinary noun of type $et$, but involves a "free variable" of type $e$. In order to capture this double nature of expressions with gapped meanings, we use the following categorial rule.

$$\frac{\Gamma, \sigma_1 \vdash \sigma_2}{\Gamma, \tau \rightarrow \sigma_1 \vdash \tau \rightarrow \sigma_2}\ \mathrm{ACOND} \qquad \frac{X, y_1(x_\tau) \Rightarrow y_2(x_\tau)}{X, y_1 \Rightarrow y_2}$$

This rule, presented here in sequent format with the appropriate semantics, is called ACOND (for "Argument Conditionalization"), and it is a restricted version of the general Conditionalization rule of the Lambek Calculus (see Van Benthem, 1991). An example for its application is the following derivation of meaning for the constituent *that every man loves* in (39). Unlike Jacobson, we assume now that the relative pronoun *that* standardly denotes the *monomorphic* intersection function of type $(et)((et)(et))$. The derivation of meaning for the constituent *every man loves* proceeds as in figure 2. Recall that we denote this meaning by '$S$'. Accordingly, we assume the following types and meanings:

(40) $[\![\text{that}]\!] = \cap_{(et)((et)(et))} \stackrel{def}{=} \lambda A_{et}.\lambda B_{et}.\lambda x_e.A(x) \wedge B(x)$

$[\![\text{every man loves}]\!] = S_{e \rightarrow (et)} \stackrel{def}{=} \mathrm{RG}(\mathrm{Z}^0(\mathbf{love}')(\mathbf{every}'(\mathbf{man}')))$

Composition of these two types and meanings is achieved by the following application of ACOND:

$$\frac{(et)((et)(et)),\ et \vdash (et)(et)}{(et)((et)(et)),\ e \rightarrow (et) \vdash e \rightarrow ((et)(et))}\ \mathrm{ACOND}$$

To see the use semantics of this rule, note the following valid derivation (using application):

$\cap,\ S(x) \Rightarrow \cap(S(x))$,

which is tantamount to:

$\cap,\ (\lambda y.S(y))(x) \Rightarrow (\lambda y.\cap(S(y)))(x)$

Using ACOND, we can use this derivation as follows:

$$\frac{\cap,\ (\lambda y.S(y))(x) \Rightarrow (\lambda y.\cap(S(y)))(x)}{\cap,\ S \Rightarrow \lambda y.\cap(S(y))}\ \mathrm{ACOND}$$

rather than by their semantic type, and changes the categorial syntax accordingly.

Intuitively, when composing $\cap$ and $S$, this rule amounts to: introduction of a "fresh free variable" $y$, application of $\cap$ to $S(y)$ and abstraction over $y$. Motivations for this particular formulation and for the "$\rightarrow$" constructor are given in Winter (2001:ch.3), and I will not repeat them here.

In a similar way, the ACOND rule allows us to derive the meaning of the constituent *woman that every man loves*, without applying the $\mathrm{N}^0$ operator to *woman* as in figure 2. This derivation is given below.

$$\dfrac{\dfrac{\dfrac{et,\ (et)(et) \vdash et}{et,\ e \rightarrow ((et)(et)) \vdash e \rightarrow (et)}\ \mathrm{ACOND}}{\mathbf{woman}',\ \cap(S(x)) \Rightarrow \mathbf{woman}' \cap (S(x))}}{\dfrac{\mathbf{woman}',\ (\lambda y.\cap(S(y)))(x) \Rightarrow (\lambda y.\mathbf{woman}' \cap (S(y)))(x)}{\mathbf{woman}',\ \lambda y.\cap(S(y)) \Rightarrow \lambda y.\mathbf{woman}' \cap (S(y))}\ \mathrm{ACOND}} \tag{41}$$

Note that the result is the same as the relation $R$, which was obtained above by intersecting $S$ with $\mathrm{N}^0(\mathbf{woman}')$ (cf. figure 2). Formally, let $\cap_1$ and $\cap_2$ denote the intersection operators of one-place and two-place predicates respectively. Then we have:

$$\begin{aligned} (42)\ [\![\text{woman that every man loves}]\!] &= R \\ &= (\mathrm{N}^0(\mathbf{woman}')) \cap_2 S \\ &= (\lambda y.\lambda z.\mathbf{woman}'(z)) \cap_2 S \\ &= \lambda y.((\lambda z.\mathbf{woman}'(z)) \cap_1 S(y)) \\ &= \lambda y.\mathbf{woman}' \cap_1 S(y) \end{aligned}$$

In words, for any binary relation $S$: the intersection of $S$ with the relation that sends each entity to the set of women, is the relation that sends each entity $y$ to the set of women in $S(y)$. This means that when the second conjunct in the construction *woman that...* ranges over functions as in Jacobson's account, the outcomes of the $\mathrm{N}^0$ operator are now derived rather than stipulated.[15]

The revised analysis of sentence (39) using the ACOND rule is given in figure 3, with the abbreviations $S$ and $R$ for the denotations of the expressions *every man loves* and *woman that every man loves* respectively, as derived above. The meaning that is derived in this way is equivalent to the meaning that is derived using the more *ad hoc* mechanism in figure 2. It is important to mention that the introduction of the free SF variable in this analysis (and the following one) is

[15]Note that for cases such as (33) or (34) above, where there is no functional conjunct, an operation like $\mathrm{N}^0$ is still required in order to introduce the "implicit variable" within the noun *woman*. I do not address here the linguistic status of such implicit variables.

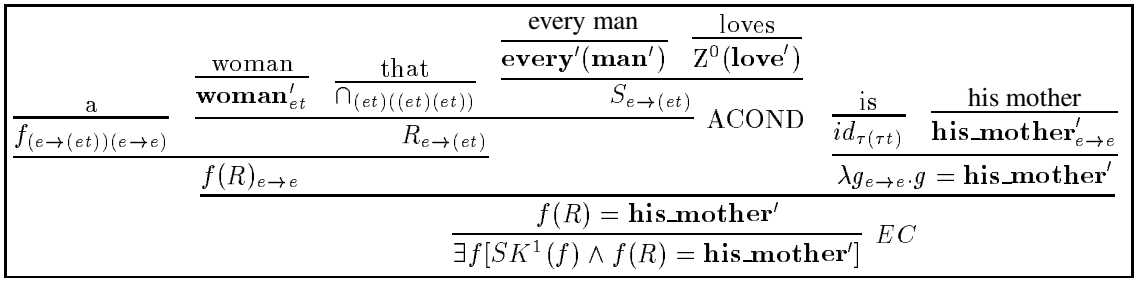


Figure 3: Revised derivation of meaning for sentence (39)

only for the sake of presentation. In fact, the ACOND rule as introduced above is designed to get rid of such free variables, as explained in Winter (2001:ch.3).

With the ACOND rule, it is also no longer necessary to adopt Jacobson's assumption (J3) that items such as relative pronouns (e.g. *that*) or the definite article (i.e. *the*) are polymorphic. We adopt the treatment in Winter (2001:ch.3) of the definite article as a predicate modifier that imposes singularity on the restricting predicate. The Strawsonian (presuppositional) version of this operator is defined as follows.

(43) $\mathbf{the}'_{(et)(et)} \stackrel{def}{=} \lambda A_{et}.\lambda x_e.x = \iota(A)$

Thus, the $\mathbf{the}'$ operator sends a set to itself if it contains a unique element, and is undefined otherwise.[16] Using these assumptions, the analysis of sentence (5a) in figure 4 becomes analogous to the analysis of sentence (39) in figure 3.[17]

A similar analysis becomes now possible for copular sentences with bare numerals as illustrated in (17). We adopt the common assumption that the numeral *two* is a predicate modifier, similarly to the definite article. Concretely, given a cardinality function *card* on "plural" $e$-type entities, we can assume the following denotation for the numeral *two*:

(44) $[\![\text{two}]\!] = \mathbf{two}'_{(et)(et)} \stackrel{def}{=} \lambda A_{et}.\lambda x_e.card(x) = 2 \wedge A(x)$

[16] A Russellian (truth-conditional) version of the definite operator of this type is: $\mathbf{the}'_{(et)(et)} \stackrel{def}{=} \lambda A_{et}.\lambda x_e.|A| = 1 \wedge A(x)$. I use here the Strawsonian operator for sake of compatability with Jacobson's analysis.

[17] The only material difference between the two analyses is that the SF variable corresponds to the indefinite article in figure 3, and to a null element in figure 4. In fact, in Winter (2001) it is argued that SF variables correspond to null elements in both cases, and the deviation from this treatment is here only for the sake of simplicity.

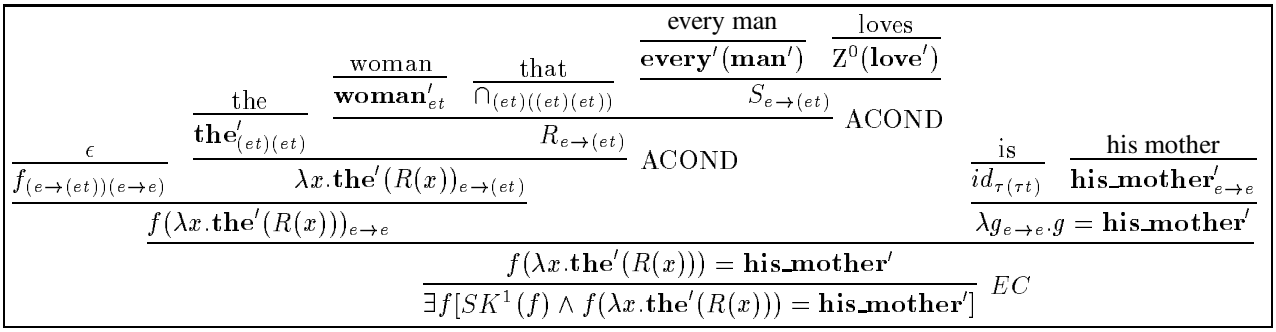


Figure 4: Derivation of meaning for sentence (5a)

This makes the analysis of (17) similar to the analysis of (5a) in figure 4.[18]

Note that in the proposed modification of Jacobson's system, the only polymorphic item in the analysis of functional readings is the copula *be*. This is not coincidental. The polymorphism of *be* is what allows predicates such as *be his mother* to range over functions of type $e \rightarrow e$. With a monomorphic transitive predicate such as *love* or *pinch* this is not possible. In the proposed system, there are two ways to analyze a verb phrase such as *love his mother*. The object *his mother* is of type $e \rightarrow e$. The transitive predicate *love* is standardly of type $e(et)$. One way to compose their denotations is using ACOND; another is using the Z (or $\mathrm{Z}^0$) function and direct application. These two options are illustrated below.

(45) a. $\mathbf{love}'_{e(et)}$, $\mathbf{his_mother}'_{e\rightarrow e} \overset{\text{ACOND}}{\Rightarrow} (\lambda x.\mathbf{love}'(\mathbf{his_mother}'(x)))_{e\rightarrow(et)}$

b. $\mathrm{Z}(\mathbf{love}')_{(e\rightarrow e)(et)}$, $\mathbf{his_mother}'_{e\rightarrow e} \Rightarrow (\lambda x.\mathbf{love}'(\mathbf{his_mother}'(x))(x))_{et}$

The first option accounts for the deictic interpretation of the pronoun (as in *Mary loves his mother*). The second option accounts for the bound interpretation of the pronoun (as in *every man loves his mother*). However, the VP denotation cannot combine with $e \rightarrow e$ functions that are denoted by NPs. This accounts for the impossibility to get a functional reading in non-copular sentences such as the following (cf. Sharvit (1999)).

[18] Post-copular nominals such as *his mother and Brigitte Bardot* are treated using the ACOND rule. Suppose (for the sake of presentation alone) that *and* denotes here the i-sum operator of Link (1983). This operator, of type $e(ee)$ composes using ACOND with the $e$-type denotation of *Brigitte Bardot* and the $(e \rightarrow e)$-type denotation of *his mother*, to derive another $(e \rightarrow e)$-type function that maps each man to the i-sum of his mother and Brigitte Bardot. A similar analysis can be derived within the more complex treatment of plurals and predication that is proposed in Winter (2001:ch.4).

(46) a. The woman that no man admires hates his mother.

b. The woman that no man loves pinched him.(=(8a))

Assuming that no man admires his mother-in-law, sentence (46a) cannot mean something like "a mother-in-law always hates her son-in-law's mother". Thus, the sentences in (46), as opposed to (5b), do not have a functional interpretation. This is accounted for by our assumption that ordinary transitive predicates, as opposed to *be*, do not have a polymorphic meaning.[19]

As pointed out by an anonymous reviewer, relying on polymorphism for the derivation of functional readings has some further implications. One implication is that the functional interpretation of questions as in (3) should also involve a polymorphic item. It seems natural to assume that *wh* elements can indeed range over functions, which is the source of this interpretation. Another phenomenon that Jacobson (1999) proposed to treat using her functional mechanism is the case of "unexpected" (or "sloppy") inferences. Consider for instance the following inference, of the type that Jacobson (1999) attributes to Reinhart (1990).

(47) Mary will buy what(ever) Bill buys; Bill buys his favorite car
$\Rightarrow$ Mary will buy her favorite car.

Jacobson shows that her binding mechanism using $ee$ functions accounts for this kind of inferences and shows some advantages of her analysis over previous one. Again, in order to achieve this in the present setting of Jacobson's mechanism we need a polymorphic item to blame. I think that polymorphism is quite natural to assume for the relative pronoun *what*(*ever*), but I will not try to substantiate this suggestion here, and leave for further study the details of the adaptation of the present account for the sake of analyzing functional questions and "unexpected" inferences.

Let us summarize the main proposals of this section:

- Sets of $ee$ functions are mapped to binary relations using the $\mathrm{RG}$ operator.
- This allows the general SF mechanism to derive functional readings.
- The analysis of functional readings does not require Jacobson's $\mathrm{N}$ operator or polymorphic meanings of items such as *that* or *the*. These are treated using a more general variable-free compositional mechanism.

[19] In fact, as argued in Winter (2001:ch.4), it may be advantageous to treat copulas as meaningless rather than polymorphic, but whether this alternative is adopted or not is irrelevant for our present purposes.

- Polymorphism of the copula *be* is required in order to account for its licensing of functional readings as opposed to other transitive predicates.

Note that SFs of arity greater than one are needed in order to analyze sentences with more than one pronoun in the post-copular NP. For example:

(48) The present that every man will send to every woman is the present that she asked him to send her.

For sake of exposition, I will not consider such complex examples in the sequel.

# 4 Generalized quantifiers and functional readings

Mapping sets of functions to binary relations using the $\mathrm{RG}$ operator may potentially result in loss of information. To see that, note that in a model with $n$ entities, the cardinality of the $(ee)t$ domain is $2^{(n^n)}$, whereas the cardinality of the $e(et)$ domain is only $(2^n)^n = 2^{(n^2)}$. Consequently, the $\mathrm{RG}$ operator is not one-to-one. This means that the present proposal is, at least in principle, less expressible than Jacobson's original quantification over functions. In this section it is argued that this loss of expressibility is not only innocuous, but actually desired. In the mechanism that was introduced in the previous section, a restricting predicate such as *woman that Q loves* denotes a set of functions $F$ for any quantifier $Q$. This section introduces a notion of *closed sets of functions*, for which the $\mathrm{RG}$ operator is one-to-one, and shows that the set $F$ is closed if and only if $Q$ is a *bounded quantifier*: an intersection of a principal filter and a principal ideal. We will then hypothesize that it is exactly the bounded quantifiers that are licensed in functional interpretations of questions and copular sentences.

To illustrate the intuitive reasoning that underlies the formal discussion in this section, consider the following contrast.

(49) A woman that $\left\{ \begin{array}{l} \text{no} \\ \text{*at most one} \end{array} \right\}$ man loves is his mother-in-law.

As in Jacobson's analysis, the denotation of the constituent *woman that no man loves* is the set of functions that send no man to a woman he loves. Let us denote this set of functions by $F_{no}$. Assuming that $F_{no}$ is not empty, the relation $\mathrm{RG}(F_{no})$ sends every man to the set of women he does not love. Applying any Skolem function of arity $1$ to $\mathrm{RG}(F_{no})$ gives us again a function in $F_{no}$. This is not the case when we consider the determiner *at most one*, which does not support

a functional reading in (49). Let us denote the set of functions that send at most one man to a woman he loves by $F_{\leq 1}$. Assume that the men are John and Bill and that the women are Mary and Sue. Assume further that John loves Mary and hates Sue, and that Bill loves Sue and hates Mary. Thus, the set $F_{\leq 1}$ includes the following functions $f_1, f_2$ and $f_3$:

$f_1$ : $\text{John} \mapsto \text{Mary}, \quad \text{Bill} \mapsto \text{Mary}$
$f_2$ : $\text{John} \mapsto \text{Sue}, \quad \text{Bill} \mapsto \text{Sue}$
$f_3$ : $\text{John} \mapsto \text{Sue}, \quad \text{Bill} \mapsto \text{Mary}$

This means that $\text{RG}(F_{\leq 1})$ sends both John and Bill to the set $\{\text{Mary}, \text{Sue}\}$. Consequently, a Skolem function of arity 1 may map $\text{RG}(F_{\leq 1})$ to the function that sends each of the two men to the woman he loves. But this function is not in $F_{\leq 1}$. This formal distinction between the quantifiers that the noun phrases *no woman* and *at most one woman* denote will be given a general characterization in this section, in terms of bounded and unbounded quantifiers. It will be hypothesized that (un)boundedness of quantifiers is responsible for contrasts as in (49).

## 4.1 Closed sets of functions and bounded quantifiers

Consider first the following definition.

**Definition 4 (closed sets of functions)** *Let $F \subseteq B^A$ be a set of functions from $A$ to $B$. The* closure of $F$ *is the set of functions $\overline{F} \subseteq B^A$ that is defined by:*

$\overline{F} \stackrel{def}{=} \{f \in B^A : \textit{for every } x \in A \textit{ there is } g \in F \textit{ s.t. } f(x) = g(x)\}$.

*We call $F$ a* closed set of functions *if $F = \overline{F}$.*

**Example**: Consider the functions in table 1 above. The set $\{h_1, h_2\}$ is closed. By contrast, the set $\{h_1, h_4\}$ is not closed: its closure is $\{h_1, h_2, h_3, h_4\}$.
Note that $F \subseteq \overline{F}$ trivially holds for any set of functions $F$. It is also easy to observe the following fact.

**Fact 1** *If $F$ and $G$ are sets of functions in $B^A$ then $\text{RG}(F) = \text{RG}(G)$ iff $\overline{F} = \overline{G}$.*

As a result, the RG operator is one-to-one when it is restricted to apply to closed sets of functions:

**Corollary 2** *If $F$ and $G$ are closed sets of functions in $B^A$ then $\text{RG}(F) = \text{RG}(G)$ iff $F = G$.*

A related fact about closed sets of functions, which was informally mentioned and illustrated above, concerns their relations with Skolem functions and the RG operator.

**Fact 3** *A non-empty set $F$ of functions in $B^A$ is closed iff for every Skolem function $f$ from $(\wp(B))^A$ to $B^A$: $f(\mathrm{RG}(F)) \in F$.*

**Proof**
($\Rightarrow$): Assume that $F$ is a non-empty closed set of functions in $B^A$ and that $f$ is an SF of the given signature. For every $x \in A$: from $F \neq \emptyset$ it follows that $(\mathrm{RG}(F))(x) \neq \emptyset$, and by definition of SFs: $(f(\mathrm{RG}(F)))(x) \in (\mathrm{RG}(F))(x)$. By definition of RG, this means that for every $x \in A$ there is $g \in F$ such that $(f(\mathrm{RG}(F)))(x) = g(x)$. Hence $f(\mathrm{RG}(F)) \in \overline{F}$. But $F = \overline{F}$ by closure of $F$, hence $f(\mathrm{RG}(F)) \in F$.
($\Leftarrow$): Assume that for every Skolem function $f$ of the given signature: $f(\mathrm{RG}(F)) \in F$. By definition of SFs, this means that for every function $g \in B^A$: if $g$ sends every $x \in A$ to a member of $(\mathrm{RG}(F))(x)$ then $g \in F$. But this means that $\overline{F} \subseteq F$, hence $\overline{F} = F$, so $F$ is closed. □

Thus, we know that for a non-empty closed set of functions $F$, sequential application of RG and a Skolem function maps $F$ to one of its members. Recall that in the compositional mechanism of section 3, the set $F$ is the denotation of a restricting predicate such as (*woman that*) *every man loves*, which is derived by applying the $\mathrm{Z}^0$ operator to a quantifier $Q$ (e.g. the denotation of *every man*) and a binary relation $R$ (e.g. the denotation of *loves*). Let us use the following abbreviation:

$$(50)\ F_{QR} \stackrel{def}{=} \mathrm{Z}^0(Q_{(et)t})(R_{e(et)}) = \lambda f_{ee}.Q(\lambda x_e.R(f(x))(x))$$

Further modification of the set $F_{QR}$ within the relative clause is immaterial for our present purposes, hence ignored.[20] In set-theoretical format, for any quantifier $Q \subseteq \wp(E)$ and binary relation $R \subseteq E^2$, the set of functions $F_{QR}$ is $\{f \in E^E : \{x : R(f(x), x)\} \in Q\}$.

The main theorem of this section characterizes the class of quantifiers $Q$ that guarantee that $F_{QR}$ is a closed set of functions for every $R$. It will be shown

[20] As an illustration for why this is justified, consider a modified restricting predicate of the form *A that Q R*, where $A$ is the denotation of a noun (e.g. *woman*). Our analysis of this construction in section 3 is $\mathrm{N}^0(A) \cap \mathrm{RG}(F_{QR})$, which is equal to $\mathrm{RG}(\mathrm{N}(A) \cap F_{QR}) = \mathrm{RG}(F_{QR'})$, of the form of the analysis, where $R' = \lambda x.\lambda y.R(x)(y) \wedge A(x)$.

| Noun Phrase: | Quantifier bounded by: | |
|---|---|---|
| *every student* | $\mathbf{student}'$ | $E$ |
| *no student* | $\emptyset$ | $E \setminus \mathbf{student}'$ |
| *every student but no teacher* | $\mathbf{student}'$ | $E \setminus \mathbf{teacher}'$ |
| *every student but Mary* | $\mathbf{student}' \setminus \{\mathrm{m}'\}$ | $E \setminus \{\mathrm{m}'\}$ |
| *no student but Mary* | $\{\mathrm{m}'\}$ | $E \setminus (\mathbf{student}' \setminus \{\mathrm{m}'\})$ |

Table 2: Bounded quantifiers

that this class is precisely the class of *bounded* quantifiers: those quantifiers that are an intersection of a principal filter (e.g. the denotation of *every student*) and a principal ideal (e.g. the denotation of *no teacher*). This class of quantifiers is officially defined below.

**Definition 5 (bounded quantifiers)** *A quantifier $Q \subseteq \wp(E)$ is called* bounded *iff there are two sets $X$ and $Y \subseteq E$ s.t. $Q = \{A \subseteq E : X \subseteq A \subseteq Y\}$. In this case we say that $Q$ is bounded by $X$ and $Y$.*

Table 2 gives some NPs that denote bounded quantifiers with the sets that bound them.

**Theorem 4** *A quantifier $Q \subseteq \wp(E)$ is bounded iff the set of functions $F_{QR}$ is closed for any binary relation $R \subseteq E^2$.*

The proof of this theorem makes use of the *convexity*[21] property of quantifiers, which is defined below.

**Definition 6 (convex quantifiers)** *A quantifier $Q \subseteq \wp(E)$ is* convex *iff for all $A \subseteq B \subseteq C \subseteq E$: if $A \in Q$ and $C \in Q$ then $B \in Q$.*

For instance, the noun phrase *between three and five students* denotes a convex quantifier in any model, but this is not true of the disjunctive noun phrase *exactly three or exactly five students*.

The following simple fact will be useful in proving theorem 4.

**Fact 5** *A quantifier $Q \subseteq \wp(E)$ is bounded iff $Q$ is convex and closed under arbitrary intersections and unions. In this case $Q = \{A \subseteq E : \cap Q \subseteq A \subseteq \cup Q\}$.*

[21] Thijsse (1983), who introduced this notion into generalized quantifier theory, gave it the misnomer *continuity*, which was used in some subsequent works.

**Proof of theorem 4**

($\Rightarrow$): We assume that $Q \subseteq \wp(E)$ is bounded by $X, Y \subseteq E$ and show that $F_{QR}$ is a closed set of functions for any $R \subseteq E^2$. It is enough to show that any $f_0 \in \overline{F_{QR}}$ is also in $F_{QR}$. Thus, we need to show that for any $f_0 \in \overline{F_{QR}}$, for any $R \subseteq E^2$: the set $A_0 \stackrel{def}{=} \{x \in E : R(f_0(x), x)\}$ is in $Q$, which holds iff $X \subseteq A_0 \subseteq Y$.

To show that $A_0 \subseteq Y$, we assume $x_0 \in A_0$ and show $x_0 \in Y$.
From $f_0 \in \overline{F_{QR}}$ it follows that some $g_0 \in F_{QR}$ satisfies $g_0(x_0) = f_0(x_0)$.
Because $g_0 \in F_{QR}$, we conclude from the definitions of $F_{QR}$ and $Q$ that $\{x \in E : R(g_0(x), x)\} \subseteq Y$. (i)
From the assumption $x_0 \in A_0$, it follows that the relation $R(f_0(x_0), x_0)$ holds.
Because $g_0(x_0) = f_0(x_0)$, we conclude that $R(g_0(x_0), x_0)$ holds too. (ii)
Facts (i) and (ii) entail that $x_0$ is in $Y$.

The proof that $X \subseteq A_0$ is similar.

($\Leftarrow$): Assume that for a quantifier $Q \subseteq \wp(E)$, the set of functions $F_{QR}$ is closed for any binary relation $R \subseteq E^2$. By fact 5, we need to show that $Q$ is convex and closed under arbitrary intersections and unions. The proofs of the convexity and closure properties of $Q$ are all similar.

*Convexity of* $Q$: Suppose, for contradiction, that there are $A \subsetneq B \subsetneq C \subseteq E$ s.t. $A, C \in Q$ but $B \notin Q$. We conclude that $|E| \geq 2$ and denote $a, b \in E$ for two arbitrary elements $a \neq b$. Consider the relation $R \stackrel{def}{=} \{a\} \times E$ and the characteristic functions $\chi_A, \chi_B$ and $\chi_C$ in $\{a, b\}^E$ of the sets $A, B, C \subseteq E$. By saying that $\chi_Y \in \{a, b\}^E$ is the characteristic function of a set $Y \subseteq E$ we mean that $\chi_Y$ is the function that satisfies for any $x \in E$:

$$\chi_Y(x) = \begin{cases} a & x \in Y \\ b & x \notin Y \end{cases}$$

Observe now the following equalities:

$\{x \in E : R(\chi_A(x), x)\} = A$
$\{x \in E : R(\chi_B(x), x)\} = B$
$\{x \in E : R(\chi_C(x), x)\} = C$

By definition of $F_{QR}$ and the non-convexity assumption about $Q$ it therefore follows that $\chi_A, \chi_C \in F_{QR}$, but $\chi_B \notin F_{QR}$.
However, by the assumption $A \subsetneq B \subsetneq C$, we have for every $x \in E$:

$x \in B \quad \Rightarrow \quad \chi_B(x) = a = \chi_C(x)$
$x \notin B \quad \Rightarrow \quad \chi_B(x) = b = \chi_A(x)$

Thus, for every $x \in E$ there is $f \in \{\chi_A, \chi_C\}$ s.t. $\chi_B(x) = f(x)$.

We conclude that $\chi_B \in \overline{\{\chi_A, \chi_C\}} \subseteq \overline{F_{QR}}$, which means that $\overline{F_{QR}} \neq F_{QR}$.
By assuming non-convexity of $Q$, we contradicted the assumption that $F_{QR}$ is a closed set of functions for any relation $R$. Hence: $Q$ is convex.

*Closure of $Q$ under intersections*: Suppose, for contradiction, that there is a set $\mathcal{A} \subseteq Q$ s.t. $\cap\mathcal{A} \notin Q$ ($|\mathcal{A}| \geq 2$). Again, we conclude that $|E| \geq 2$, denote $a, b \in E$ for two arbitrary elements $a \neq b$, and consider the binary relation $R = \{a\} \times E$. Now consider the characteristic functions in $\{a, b\}^E$ of the sets in $\mathcal{A}$ and of their intersection $\cap\mathcal{A}$. Observe now the following equalities:

$\{x \in E : R(\chi_A(x), x)\} = A$, for every $A \in \mathcal{A}$
$\{x \in E : R(\chi_{\cap\mathcal{A}}(x), x)\} = \cap\mathcal{A}$

By definition of $F_{QR}$ and the assumption about $Q$ it therefore follows that $\chi_A \in F_{QR}$ for every $A \in \mathcal{A}$, but $\chi_{\cap\mathcal{A}} \notin F_{QR}$.

However, for every $x \in E$:

$x \in \cap\mathcal{A} \;\Rightarrow\; \chi_{\cap\mathcal{A}}(x) = a = \chi_A(x)$ for any $A \in \mathcal{A}$
$x \notin \cap\mathcal{A} \;\Rightarrow\; \chi_{\cap\mathcal{A}}(x) = b = \chi_B(x)$ for at least one $B \in \mathcal{A}$ .

Thus, for every $x \in E$ there is $f \in \{\chi_A \in \{a, b\}^E : A \in \mathcal{A}\}$ s.t. $\chi_{\cap\mathcal{A}} = f(x)$.
We conclude that $\chi_{\cap\mathcal{A}} \in \overline{\{\chi_A \in \{a, b\}^E : A \in \mathcal{A}\}} \subseteq \overline{F_{QR}}$, which means that $\overline{F_{QR}} \neq F_{QR}$.
This contradiction to our assumption about the closure of $F_{QR}$ entails that $Q$ is closed under arbitrary intersections.

The proof of the closure of $Q$ under unions is similar. □

## 4.2 The distribution of quantifiers in functional readings

Consider the following examples.

(51) The/A woman that { every/no man (but John) / *at most one man / ?exactly/at least one man } loves is his mother.

(52) Which woman does { every/no man (but John) / *at most one man / ?exactly/at least one man } love? His mother.

These examples show a contrast in acceptability of functional readings between bounded quantifiers such as *every, no* etc. and unbounded quantifiers such as

*exactly one, at most/least one* etc. The following hypothesis makes a natural connection between this linguistic contrast and the formal properties that were proven in the previous subsection.

(53) **Hypothesis**: *the* $\mathrm{RG}$ *operator applies only to closed sets of functions.*

Let us examine how this hypothesis makes the desired connection. As shown by theorem 4, it is exactly the bounded quantifiers that generate closed sets of functions. Thus, if hypothesis (53) is correct, then the functional mechanism is analyzed as sensitive to the (un)boundedness property of the quantifier. This is due to theorem 4. Such sensitivity is not at all trivial to explain in compositional frameworks, because the functional mechanism applies at a much higher syntactic level than the level where the quantifier is present. However, theorem 4 makes the necessary connection between the restrictions on the set of functions that is generated, and the generalized quantifier that participates in its generation.

But *why* should the $\mathrm{RG}$ operator be sensitive to whether the set of functions it applies to is closed or not? Fact 3 gives at least a partial answer to this question. According to this fact, these are exactly the closed sets $F$ of functions that guarantee that Skolem functions do not return functions outside $F$. For instance, an SF that applies to the denotation $\mathrm{RG}([\![$*woman that every man loves*$]\!])$, does not return functions that were not already in the basic denotation of *woman that every man loves*. Thus, if the hypothesis in (53) is correct, then in a sense it makes the grammar of functional quantification semantically "optimal": functional readings derive "natural functions" exactly in the cases when they are grammatically licensed.[22]

Despite this attractiveness of hypothesis (53), there are (at least) two potential empirical problems that it has to face. One problem concerns the felicitous functional interpretation of sentences such as the following, which is based on examples due to Alexopoulou and Heycock (2002).

(54) The woman that almost every/no man loves is his mother.

The natural interpretation of the noun phrase *almost every man* is the following generalized quantifier.

(55) $\{A \subseteq E : 1 \leq |\overline{A} \cap \mathbf{man}'| \leq n\}$,
where $n$ is a (small) number that is determined by the context.

[22] I believe that a similar reasoning underlies Barwise and Cooper's (1981) semantic account of the grammaticality of *there* sentences with various NPs, using a non-triviality assumption.

This quantifier in not bounded. However, sentence (54) can also be interpreted as in the rough paraphrase below.

(56) There is a (small) set of men $B$ s.t. the woman that every/no man except for the men in $B$ loves is his mother.

In other words: the "exception set" $B$ that *almost* quantifiers require can take here sentential scope, which allows the noun phrase *almost every/no man* to be interpreted as a bounded quantifier.[23]

Another problem for the hypothesis in (53) may come from certain facts that are pointed out by Sharvit (1999:(18)-(21)). Sharvit considers plural sentences in Hebrew that are parallel to the following English sentence.[24]

(57) The woman that most of the students invited was their mother.

The determiner *most* is unbounded, hence functional anaphora is *a priori* not expected here by hypothesis (53).

However, with noun phrases such as *most of the students*, also discourse anaphora as in the following example may appear and complicate the picture.

(58) Most of the students admire their mother. They invited her.

By contrast, discourse anaphora is unacceptable with the other quantifiers that illustrated (un)availability of functional readings in the above examples. For instance:

(59) No/at most one student hates his mother. *He invited her.

This means that the contrast we observed above between *no* and *at most one* cannot originate from different discourse anaphora potentials, and therefore it supports hypothesis (53). The functional reading in Sharvit's example (57) may result from discourse anaphora and therefore it does not seriously challenge this hypothesis. For recent works that deal with discourse anaphora using a mechanism of functional quantification see Steedman (1999) and Peregrin and Von Heusinger (2001).

[23] This is not always the case with *almost* quantifiers. For instance, a sentence such as *every Frenchman admires almost every actress*, is compatible with a situation where one Frenchman admires every actress but Brigitte Bardot, while another Frenchman admires every actress but Isabelle Adjani.

[24] Sharvit also judges as felicitous sentences like (57) with *more than two* and *at most two* instead of *most of the*. However, my Hebrew informants disagree, and consider these cases significantly worse than the Hebrew parallel of (57).

# 5 Further motivation: a note on Jacobson (2002)

In her recent article, Jacobson shows motivation for an operator that is very similar to the $\mathrm{RG}$ operator, although her motivation comes from different phenomena than the ones that were addressed throughout this paper. The examples that motivate Jacobson's proposal are all similar to the following ones.

(60) The woman *he loves* that every/no man invited (to his wedding) is his mother.

(61) A (certain) woman *he loves* that every/no man invited (to his wedding) is his mother.

Jacobson (1994) lifts the denotation of the noun *woman* to a set of $ee$ functions. However, in her variable-free treatment of anaphora, the denotation of the relative *he loves*, which should be intersected with the denotation of *woman*, is a binary relation. Hence, Jacobson (2002) proposes to shift this binary relation into a set of functions using an operator that she calls '$\mathbf{m}$'. The effect of this operator is to map any binary relation $R \subseteq A \times B$ to the set of functions $\{f \in B^A$ : for all $x \in A$ : $R(x, f(x))\}$. In type-theoretical format:

(62) $\mathbf{m}_{(e(et))((ee)t)} \stackrel{def}{=} \lambda R_{e(et)}.\lambda f_{ee}.\forall x_e.R(f(x))(x)$

In Jacobson's categorial treatment of extraction, the relative *he loves* denotes the following relation:

(63) $\mathbf{he_loves}' = \lambda x.\lambda y.\mathbf{love}'(y)(x)$

Intersection with the functional denotation of the noun *woman* leads to the following analysis:

(64) $\mathrm{N}(\mathbf{woman}') \cap \mathbf{m}(\mathbf{he_loves}') =$
$(\lambda f_{ee}.\forall z[\mathbf{woman}'(f(z))]) \cap (\lambda f_{ee}.\forall u[\mathbf{love}'(f(u))(u)]) =$
$\lambda f_{ee}.\forall z[\mathbf{woman}'(f(z))] \wedge \forall u[\mathbf{love}'(f(u))(u)]$

In words, this is the set of $ee$ functions that send every $x$ to a woman that $x$ loves. Once this set of functions is derived, the rest of the analysis of sentences (60) and (61) is analogous to the analysis of sentences (5a) and (39) in figure 1 (see also (19)). For instance, the analysis of sentence (61) using Jacobson's $\mathbf{m}$ operator is given below.

(65) $\mathbf{a}'(\mathrm{N}(\mathbf{woman}')\cap\mathbf{m}(\mathbf{he_loves}')\cap\mathrm{Z}^0(\mathbf{invite}')(\mathbf{every}'(\mathbf{man}')))(\lambda g_{ee}.g = \mathbf{his_mother}')$
$\Leftrightarrow \exists f[(\mathrm{N}(\mathbf{woman}')\cap\mathbf{m}(\mathbf{he_loves}')\cap\mathrm{Z}^0(\mathbf{invite}')(\mathbf{every}'(\mathbf{man}')))(f)\wedge f = \mathbf{his_mother}']$
$\Leftrightarrow (\mathrm{N}(\mathbf{woman}') \cap \mathbf{m}(\mathbf{he_loves}') \cap \mathrm{Z}^0(\mathbf{invite}')(\mathbf{every}'(\mathbf{man}')))(\mathbf{his_mother}')$

In the present proposal, since the noun *woman* is anyway lifted to a binary relation using the $\mathrm{N}^0$ operator, there is no need to use further shiftings, and the analysis of the relative nominal *woman he loves* is as follows.

(66) $\mathrm{N}^0(\mathbf{woman}') \cap (\mathbf{he_loves}') =$
$(\lambda x.\lambda y.\mathbf{woman}'(y)) \cap (\lambda x.\lambda y.\mathbf{love}'(y)(x)) =$
$\lambda x.\lambda y.\mathbf{woman}'(y) \wedge \mathbf{love}'(y)(x)$

From this point the analysis is as in figure 2:

(67) $\exists f_{(e(et))(ee)}[SK^1(f) \wedge$
$f(\mathrm{N}^0(\mathbf{woman}')\cap(\mathbf{he_loves}')\cap\mathrm{RG}(\mathrm{Z}^0(\mathbf{invite}')(\mathbf{every}'(\mathbf{man}')))) = \mathbf{his_mother}']$

This statement is equivalent to the one that is derived above in the Jacobsonian analysis (65).

We conclude, in agreement with Jacobson, that her strategy is in a sense the inverse of the present proposal. Jacobson shifts relatives from binary relations into sets of functions, while in this paper we used the opposite direction – from sets of functions to binary relations – using the $\mathrm{RG}$ operator. It is notable that the $\mathrm{RG}$ mechanism, which was developed for entirely different reasons than Jacobson's $\mathbf{m}$ operator, also handles the problem that motivated this latter operator. I also believe that it is likely that Jacobson's mechanism can be adapted to handle at least some of the problems that were treated in the present paper.

# 6 Conclusions

In this paper we have seen intuitive, distributional and technical support for the claim that "functional" readings of questions and copular sentences and "wide scope" readings of indefinites are two names for the same phenomenon. One simple operator that maps sets of functions to binary relations was used as the key for unifying separate mechanisms that had been previously introduced for treating these phenomena using Skolem functions. This unified mechanism establishes some new relations between functional quantification and generalized quantifier theory. In particular, it was shown that there are both empirical and mathematical

reasons to expect functional quantification to be restricted to the newly introduced class of bounded quantifiers. Of course, many hard linguistic-logical questions about functional quantification in natural language are still open. However, I believe that the results in this paper exemplify the benefits that can be gained by the on-going study into this challenging area.

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