Weak Reciprocity without the Cumulative Operator

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Abstract. Reciprocal sentences can have the so-called Weak Reciprocity (WR) interpretation. [5] gives an elegant analysis to reciprocal sentences, but it cannot derive WR. [10] and [2] propose that the cumulative operator helps to derive WR. We show that WR can be derived following [5] without the cumulative operator. We provide new empirical evidence showing the need for Skolemized covers. This innovation improves the explanatory power of covers and helps derive WR without cumulativity.

Keywords: Reciprocity · Cumulativity · Non-maximality · Cover

1 Introduction

In this paper we focus on reciprocal sentences with the so-called Weak Reciprocity (WR) interpretation, as shown in (1).

(1) The children kicked each other.

(1) is true in a scenario where every child kicked some other child, and every child was kicked by some other child. The interpretation is referred to as WR. It can be formalized as below.

(2) \[ \forall x [x \leq A \rightarrow \exists y [y \leq A \land x Ry \land x \neq y]] \land \forall y [y \leq A \rightarrow \exists x [x \leq A \land x Ry \land x \neq y]] \]

When a sentence has a WR interpretation, every individual in the antecedent \( A \) must be the subject and the object of the relation. When an individual is the subject of the relation, the object must be some other individual in \( A \), and conversely when an individual is the object of the relation, the subject must be some other individual in \( A \).

The antecedents of reciprocal pronouns are always plural. [7] compares the interpretations of reciprocal sentences and plural sentences, and proposes that the former should be derived from the logical properties of the latter. In this paper, we investigate the logical property of plural sentences that derives WR.

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[10] and [2] propose that the cumulative operator is involved in deriving WR. The cumulative operator helps derive the cumulative interpretation in plural sentences, as shown in (3).

(3) Three Spanish students borrowed five French textbooks.

(3) has the so-called cumulative interpretation. The sentence is true in a scenario where three Spanish students each borrowed at least one French textbook and five French textbooks were each borrowed by at least one Spanish student. To formalize the cumulative interpretation, [6] introduces the cumulative operator as defined below.

\[
\left[ \ast \ast \right] = \lambda P_{e,t} . \lambda x . \lambda y . \forall x'[x' \leq x \rightarrow \exists y'[y' \leq y \land P(x')(y')]] \land \forall y'[y' \leq y \rightarrow \exists x'[x' \leq x \land P(x')(y')]] 
\]

The cumulative operator takes a two-place predicate \( P \) and two individuals, \( x \) and \( y \), as its arguments. The operator requires every part of the argument \( x \) must be the subject of the relation, every part of the argument \( y \) must be the object of the relation. Moreover, when an individual in \( x \) is the subject of the relation, the object must be some individual in \( y \). Conversely, when an individual in \( y \) is the object of the relation, the subject must be some individual in \( x \).

We will show how Beck uses the cumulative operator to derive WR. Beck proposes that each other is a definite noun. Following [5], the semantics and the logical form of each other are as below.

\[
\left[ \text{each other} \right]^g = ix[x \leq g(2) \land x \neq g(1)]
\]

\[\text{LF: } [\text{the} [\text{other}_1 \text{ of Pro}_2]]\]

Each other contains two variables. One variable is bound by the antecedent of each other, the other variable is a pronoun which is co-referent to the antecedent. \( ix[\phi x] \) means the unique \( x \) such that \( \phi x \). Each other denotes the group containing all members of the antecedent, minus the individuals which are distributed over.

With the semantics of each other and the cumulative operator, [2] derives the WR interpretation as below.

\[
\left[ \text{the children} \right]_{[\text{Pro}_2]} [\left[ \text{the children} \right]_{1} [\ast \ast \left[ 1 \left[ 2 \left[ t_1 \text{ kicked } [\text{the} [\text{other}_1 \text{ of Pro}_2]]] \right] \right]]] \\
[\left[ S \right] = \forall z'[z' \leq \text{the children} \rightarrow \exists y'[y' \leq \text{the children} \land \text{KICK}(z')(ix[x \leq y' \land x \neq z'])]] \land \forall y'[y' \leq \text{the children} \rightarrow \exists z'[z' \leq \text{the children} \land \text{KICK}(z')(ix[x \leq y' \land x \neq z'])]]
\]

The sentence thus means for every child, there is some other child that he or she kicked. Besides, for every child, he or she is kicked by some other child.

In this paper, we challenge the proposal that the cumulative operator is needed to give rise to the WR interpretation and propose a simplification of Beck’s account. In Section 2, We provide two pieces of evidence challenging the view that the cumulative operator derives WR. The first is on scope islands. The second is to do with the compatibility with quantifiers like all. In Section
3, we outline a new proposal with no cumulative operators involved. WR does not involve quantificational weakness, but the non-maximality effect. We extend Brisson’s account of non-maximality to some new cases and introduce the Skolemized covers. We derive the WR interpretation with the Skolemized covers. In Section 4, we conclude and discuss some future directions.

2 Problems with the cumulative operator approach

To derive the WR interpretation, Beck uses two weakening mechanisms, i.e. weakening by introducing the quantificationally weak cumulative operator as introduced in Section 1, and weakening by pragmatics using ill-fitting covers as we will introduce in Section 3. Beck uses the former as the main device. In this section, we challenge this choice. We give two pieces of evidence against the presence of the cumulative operator.

2.1 QR from the specifier position

The cumulative operator takes a two-place predicate and gives the predicate a cumulative interpretation. Quantifier raising (QR) is involved in sentences with the cumulative operator, as shown in [3]. [3] shows that the cumulative interpretation is restricted by the same island effects as QR. We take (3) as an example and show its LF below.

(7) Three Spanish students borrowed five French textbooks.

LF: \[
[\text{five French textbooks}_2 \[\text{three Spanish}_1 \[\ast \ast \[\text{[t}_1 \text{borrowed t}_2]\]\]\]\]\]\]

In (7), the subject \textit{three Spanish students} and the object \textit{five French textbooks} QR to sentence-initial positions, thus creating a two-place predicate lower in the structure, as shown in the LF. Similar to the cumulative sentence, [2] also requires QR for WR sentences. We repeat the LF of (6) below.

(8) The children kicked each other.

LF: \[
[\text{Pro}_2 \[\text{the children}_1 \[\ast \ast \[\text{[t}_1 \text{kicked the}_1 \text{other}_1 \text{of t}_2]\]\]\]\]\]

In (8), the antecedent of the reciprocal and a pronoun which is co-indexed with the antecedent inside \textit{each other} QR, creating a two-place predicate, which is taken by the cumulative operator.

If we suppose that WR involves the cumulative operator, as shown in (8), and that the cumulative operator requires QR, as shown in [3], we predict that WR interpretation is possible only when QR of the antecedent and the pronoun are possible. We show a case where the prediction is contradicted below.

First, we show that quantifiers cannot take scope out of the possessor of a complex DP. (9) below interprets as there is no president \(x\) such that \(x\)'s biography discusses \(x\)'s death. The quantifier \textit{no president} can QR from the specifier position of the complex DP to scope over the bound variable \textit{his}.

\[
(9) \text{There is no president such that his biography discusses his death.}
\]
(9) No president’s biography discusses his death.
   \( \neg \exists x[\text{president}(x) \land x’s \text{ biography discusses } x’s \text{ death}] \)

When the quantifier is inside the possessor, however, the quantifier can no longer take scope out of the possessor.

(10) Some biography about no president’s final chapter discusses his death.
    Not available: \( \neg \exists x[\text{president}(x) \land \exists y[\text{biography}(y) \land \text{about}(y)(x) \land y’s \text{ final chapter discusses } x’s \text{ death}]] \)

The intended reading of (10) is that there is no president \( x \) such that the final chapter of some biography about \( x \) discusses \( x \)’s death. The interpretation is not available, showing that the quantifier no president cannot QR from the position. In (10), no president is inside the specifier of the DP. It is in the complement of the complex DP in the specifier.

The contrast above shows that the specifier position of a complex DP is an island for QR. Quantifiers can QR from the specifier of a DP, but quantifiers cannot QR from the inside of the specifier of a DP.

As mentioned above, the cumulative operator requires the cumulated arguments to QR. We show with (10) that the specifier of a DP is an island for QR. Beck assumes that in WR sentences, the antecedent and a pronoun which is co-indexed with the antecedent QR. We thus predict them to show up in places where QR can take place. It is not possible for the pronoun which is co-indexed with the antecedent to be inside the specifier of a DP. The prediction is not borne out, as shown by the example below.

(11) The students graded each other’s papers.

(11) is true in a scenario where every student graded the paper of some other student, and every student’s paper was graded by some other student. It has a WR interpretation. Following [2], the LF and syntactic structure of the sentence will be as below.

(12) The students graded each other’s papers.
    LF: [[Pro2 [the students]1 [** [1 [2 [t1 graded [the [other x1 (of) t2]’s papers]]]]]]]

In (12), as shown in the LF, each other is the specifier of a DP. The pronoun which is co-indexed with the antecedent in each other is inside the specifier of the DP. As mentioned above, the specifier of a DP is an island for QR. Thus, the pronoun cannot QR from the position it occupies in the sentence. If the required QR cannot take place, we predict that the cumulative interpretation and WR should be unavailable. This is not correct, as (11) still has a WR interpretation.

A natural question to ask is whether it is possible for the cumulative operator to take a two-place predicate without quantifier raising the arguments. It is possible, but the semantics cannot compose, as we will show below.

(13) LF: [[#the students2] [** [graded]] [[the [other x1 (of) Pro2]’s papers]]]
In (13), we have a two-place predicate $graded$. If we let the cumulative operator to take the predicate as its argument, and give it a cumulative inference, there will be an unbounded variable in each $other$, i.e., $x_1$. To solve the problem, one may QR the students to get the free variable bound, as shown in the LF below.

\[(14) \text{LF: } [[\text{the students}]_{2} [1 [[t_{1} [\ast \ast ] [\text{graded}]]] \text{ [[the [other $x_1$ (of) Pro$_2$]'s papers]]}]\]

When no other pluralization operators are involved in (14), the meaning of each $other$ is not as intended. The semantics of (14) is as below.

\[(15) \llbracket (14) \rrbracket = \forall x [x \leq \text{i}z{'}s \text{papers} \land z \neq \text{the students}] \rightarrow \exists y [y \leq \text{the students} \land y \text{ graded } x] \land \forall y [y \leq \text{the students} \rightarrow \exists x \leq \
\]

\[\text{i}z{'}s \text{papers} \land z \neq \text{the students} \land y \text{ graded } x]\]

The semantics in (15) involves a unique $z$ which is a part of the students and not identical to the students, but there is no such a thing. Thus, the LF in (14) is also invalid.

As shown in (13) and (14), when no QR is involved, one gets false predictions. This emphasizes the necessity of QR for Beck’s proposal. The unavailability of QR and the availability of WR in the same context challenge the view that WR involves the cumulative operator.

2.2 WR, all and cumulativity

In some sentences involving quantifiers like all, most and every, the cumulative readings become unavailable. Here we take all as an example. [12] notices the following contrast.

\[(16) \begin{align*}
\text{a.} & \quad \text{The students read ten books.} \\
\text{b.} & \quad \text{All the students read ten books.}
\end{align*}\]

The cumulative reading is available in (16-a). The sentence is true in a scenario where every student read some book, and every book was read by some student. The reading is unavailable in (16-b). The only available reading of (16-b) is a distributive one. The sentence is true in a scenario in which each of the students read ten (possibly different) books.

The contrast does not show up when all is in the object, as shown below.

\[(17) \begin{align*}
\text{a.} & \quad \text{The students read the ten books.} \\
\text{b.} & \quad \text{The students read all the ten books.}
\end{align*}\]

The cumulative reading is available in (17-a). The reading is still available in (17-b). The sentence is true in a scenario in which every student read some of the ten books and every book was read by some student.

If reciprocal sentences are interpreted as WR because of cumulativity as observed in typical cumulative sentences as in (16-a), we predict that when all shows up in the subject, the WR reading is not available for reciprocal sentences. The prediction is not borne out, as shown by the following pairs of examples.
(18) a. The pirates stared at each other.
b. All the pirates stared at each other.

A WR interpretation is available in (18-a). The sentence is true in a scenario where each pirate stared at some other pirates, and each pirate was stared at by some other pirate. If WR comes from cumulativity, we predict that when a cumulative interpretation is blocked, WR should not be available. In (18-b), we have all in the subject of the sentence. It will block the cumulative interpretation, as we showed above, but (18-b) still has a WR interpretation. It is true in the same scenario where ((18-a) is true. This challenges the view that WR involves the cumulative operator.

3 Proposal: WR with Skolemized covers

We showed how WR can be derived with the cumulative operator and gave two pieces of evidence against the proposal that the cumulative operator is involved in deriving WR. Besides the cumulative operator, [2] uses ill-fitting covers to derive certain exceptional cases. In this section we will introduce ill-fitting covers and show that ill-fitting covers alone can derive WR without the cumulative operator. We provide new empirical evidence showing the need for skolemized covers. This innovation improves the explanatory power of covers and helps derive the WR interpretation. The new proposal will be theoretically simpler, and it avoids the above-mentioned problems.

3.1 Non-maximality with ill-fitting covers

Before delving into the new proposal, a detour into covers is needed. We will introduce what covers and ill-fitting covers are following [9] and [4].

[9] proposes that the pluralization operators distribute over a certain universe of discourse. The universe of discourse has internal structures, where each item is in a cell with possibly other items. Cover, a concept in topology, is a good model of the above-mentioned universe of discourse, thus it is introduced to natural language semantics in [9]. The definition of cover is given below.

(19) C is a cover of a set A if and only if:
a. C is a set of subsets of A.
b. Every member of A belongs to some set in C.
c. \( \emptyset \) is not in C.

[9] gives the following updated definition to distributive operators.

(20) \[ [D_i]^g = \lambda P(c,i).\lambda x.\forall y[y \leq x \land y \in \text{cov}_i \to P(y)] \]

The distributive operator takes a predicate and an individual as its arguments. It is co-indexed with a contextually assigned cover, with the cover providing the domain of the distributive operator. The predicate applies to every part of the argument which is also a member of the contextually given cover.
[1] gives the following updated definition of the cumulative operator by restricting the domain of the cumulative operator with covers, following [9].

\[
[**] g = \lambda P(\langle e, \langle e, t \rangle \rangle) \cdot \lambda x \cdot \lambda y \cdot \forall x'[x' \leq x \land x' \in cov_i \rightarrow \exists y'[y' \leq y \land y' \in cov_i \land P(x')(y')]] \land \forall y'[y' \leq y \land y' \in cov_i \rightarrow \exists x'[x' \leq x \land x' \in cov_i \land P(x')(y')]]
\]

The cumulative operator takes a two-place predicate and two individuals, \(x\) and \(y\), as its arguments. For the sentence to be true, every part of the argument \(x\) which is also in the cover must be the subject of the relation, every part of the argument \(y\) which is also in the cover must be the object of the relation. Moreover, when an individual in \(x\) is the subject of the relation, the object must be some individual in \(y\). Conversely, when an individual in \(y\) is the object of the relation, the subject must be some individual in \(x\).

We will show how [2] uses the above concepts to derive some exceptional cases of WR.

(22) Scenario: There are six pirates. Five pirates stared at some other pirate and was stared at by some other pirate. One pirate was neither staring at other pirate nor being stared at by other pirates.
The pirates stared at each other.

The scenario above verifies the sentence in (22) according to [2]. The scenario is weaker than WR. The WR interpretation of the sentence is that every pirate stared at some other pirate, and every pirate was stared at by some other pirate. In the scenario described above, however, one pirate neither stared at some other pirate nor was stared at by some other pirates. This reading is entailed by WR. According to Beck, (22) is an example of WR. The scenario which verifies the sentence is weaker than WR because of non-maximality or ill-fitting covers. [2] gives the LF and semantics of (22) as below.

(23) The pirates stared at each other.

\[
\text{LF: } [\text{Pro}_2 [\text{the pirates}]_1[**]_i [1 \text{ [Cov}_i [2 \text{ [Cov}_i [t_1 \text{ [stared at ] the [other } x]_1

\text{(of)} t_2] \ldots ]]])]]] \land \forall z'[z' \leq \text{the pirates } \land z' \in \text{cov}_i \rightarrow \exists y'[y' \leq \text{the pirates } \land y' \in \text{cov}_i \land \text{STARE}\! - \! \text{AT}(z')(x[x \neq z' \land x \leq y'])] \land \forall y'[y' \leq \text{the pirates } \land y' \in \text{cov}_i \land \text{STARE}\! - \! \text{AT}(z')(x[x \neq z' \land x \leq y'])]
\]

The ill-fitting cover chosen by the context is as below.

(24) \(\text{Cov}_i = \{ \{p_1\}, \{p_2\}, \{p_3\}, \{p_4\}, \{p_5\}, \{\{p_6, q\ldots\}\} \ldots \} \)

In the cover, pirates \(p_1, p_2, p_3, p_4\) and \(p_5\) occupy individual cells in the cover. \(p_6\) is in a cell with some non-pirates, thus being silently ignorable in the scenario. (24) is true in a scenario where five of the six pirates stared at some other pirate, and were stared at by some other pirate, while one of the pirates neither stared at some other pirate nor was stared at by some other pirate.
The current definition of covers, however, has the problem of under-generation. For instance, it cannot capture the interpretation of the following example.

(25) Scenario: There are six pirates. Five pirates stared at some other pirate and was stared at by some other pirate. One pirate stared at some other pirate, but he was not stared at by any other pirates.

The pirates stared at each other.

In (25), the sentence is natural under the scenario described. The reading is slightly different from that of (22). In (22), a same pirate is ignored both as a subject and as an object of the relation. The non-maximal interpretation is symmetric. In (25), a pirate is ignored only as an object of the relation. The non-maximal interpretation is non-symmetric. [2] will analyse (25) with the same LF as that of (22). There is no cover, however, which can capture the interpretation. The cumulative operator is co-indexed with a cover which restricts the domains of both the subject and the object. Thus, what can be ignored as a subject must also be ignorable as an object. In the scenario described in (25), one pirate is ignorable as an object, but not as a subject. This reading is stronger than that of (22), but weaker than WR. Although the reading entails the semantics given in (23) and the cover given in (24), this cannot work as a general solution to the problem. Imagine an extreme scenario where for every pirate, there is a pirate who didn’t stare at him. In that case, one needs a cover in which all the pirates are ignored. This does not fit with people’s intuition and understanding of the scenario.

The LF in (23) is not the only possible LF of reciprocal sentences in [2]. [2], following [5], assumes that reciprocal sentences can be interpreted without the cumulative operator. Reciprocal sentences can be interpreted with the distributive operator, and the analysis is necessary for deriving the so-called Strong Reciprocity interpretation. With the distributive operator, the sentence in (23) will have a LF and semantics as below.

(26) The pirates stared at each other.

\[
\text{LF: } [[\text{The pirates}]]_1 [D_1 [1 [\text{[the } other \ x_1] \ (of) \ Pro_3]]_2 [D_2 [2 [t_1 \ starred \ at \ t_2]...]]
\]

\[
[S]^{\Pi} = \forall x'[x'] \leq the \ pirates \land x' \in cov_1 \rightarrow \forall y'[y'] \leq \Omega z[z \neq x' \land z \leq the \ pirates] \land y' \in cov_1 \rightarrow x' \ stared \ at \ y']
\]

According to the LF, (26) means for every subgroup of the pirates who are in \(cov_i\), they stared at every other subgroup of the pirates who are in \(cov_j\).

This analysis is better than the cumulative analysis of the same example, in that now the non-maximal interpretations of the subject and the object are governed by two separate covers, thus we are no longer restricted by symmetric non-maximality. But it cannot derive the intended interpretation, as the covers still cannot co-vary with any higher quantifiers.

In this section, we show how [2] derives some exceptional cases of WR with ill-fitting covers, and we point out that the current cover cannot derive all the available readings. In the next section, we will show that the current definition of
covers has the problem of under-generation independent of reciprocal sentences. We will discuss these problems and provide an updated definition of covers. The updated definition improves the explanatory power of covers, and will be helpful in deriving WR.

3.2 The Skolemized covers

An updated definition of cover is independently needed given examples as below.

(27) Scenario: The syntax seminar has three graduate students and two undergraduate students. There are a total of four notes. The graduate students are expected to read almost all of the notes. There is no such requirements on undergraduate students. As long as they read some notes, that’s considered enough. When the professor was asked how the students performed in the class, given all the students satisfied the course requirements, the professor answered:

The students (each) read the notes.

LF: \[
\forall x[\text{x \leq \text{the students}} \land x \in cov_i \rightarrow \forall y[y \leq \text{the notes} \land y \in cov_j \rightarrow x \text{ read } y]]
\]

Suppose that the graduate students are \(G_1, G_2\) and \(G_3\). The undergraduate students are \(U_1\) and \(U_2\). The notes are \(N_1, N_2, N_3\) and \(N_4\). \(G_1\) and \(G_2\) read all the notes. \(G_3\) did not read \(N_1\) but he read all the other notes. \(U_1\) only read \(N_1\). \(U_2\) only read \(N_3\) and \(N_4\). The scenario verifies (27). We will show that under the current proposal, no cover can derive the intended interpretation.

(28) \(cov_i = \{\{G_1\}, \{G_2\}, \{G_3\}, \{U_1\}, \{U_2\}, \ldots\}\)

\(cov_j = \{\{N_3\}, \{N_4\}, \{N_1, N_2, T\}, \ldots\}\)

For (27), we suppose \(cov_i\) and \(cov_j\) have the values as listed in (28). The students each occupies an individual cell in \(cov_i\), thus the predicate applies to each of the students. In \(cov_j\), all the notes except for \(N_1\) and \(N_2\) are in independent cells. \(N_1, N_2\) and a non-notes \(T\) are in a cell, thus being silently ignorable in the scenario. With the covers, (27) means all the students read every notes except for \(N_1\) and \(N_2\). This is not the intended interpretation, where only \(U_2\) read every notes except for \(N_1\) and \(N_2\).

Other choices of covers will run into similar problems. Under the current proposal, non-maximality comes from ill-fitting covers. Ill-fitting covers are selected by pluralization operators. With each pluralization operator, one cover is contextually-assigned. The cover cannot vary. In (27), the non-maximal interpretation of the plural definite the notes co-varies with the students. There’s no way for the current cover to derive the co-variation. We thus give the following updates to covers.

We propose that covers can have a more complicated structure than the current definition. There are two types of covers. The first type of covers \(cov^1\) are as proposed in [9] and later used in [4] and [2]. The second type of covers \(cov^2\) are
Skolem functions which take individual variables. The values of the Skolem functions and the individual variables are determined by the assignment function. The Skolem function maps the individuals to the covers. The new semantics of covers is as below.

\[(29) \quad \text{cov}_1^g = g(i) \iff g(i) \text{ is a cover of the universe of discourse.} \]
\[(29) \quad \text{cov}_2^g = g(j) \iff g(j) \text{ is a function from entities to covers of the universe of discourse.} \]

We will show the explanatory power of the new semantics of covers with (27). The intended reading can be derived with the covers in (30).

\[(30) \quad \text{cov}_1^g = \{\{G_1\}, \{G_2\}, \{G_3\}, \{U_1\}, \{U_2\}, \ldots\} \]
\[
\text{cov}_2^{x_1^1 \rightarrow G_1} = g(j)(G_1) = \{\{N_1\}, \{N_2\}, \{N_3\}, \{N_4\}, \ldots\} \\
\text{cov}_2^{x_1^2 \rightarrow G_2} = g(j)(G_2) = \{\{N_1\}, \{N_2\}, \{N_3\}, \{N_4\}, \ldots\} \\
\text{cov}_2^{x_1^3 \rightarrow G_3} = g(j)(G_3) = \{\{N_2\}, \{N_3\}, \{N_4\}, \{N_1, T\}, \ldots\} \\
\text{cov}_2^{x_1^4 \rightarrow U_1} = g(j)(U_1) = \{\{N_1\}, \{N_2, N_3, N_4, T\}, \ldots\} \\
\text{cov}_2^{x_1^5 \rightarrow U_2} = g(j)(U_2) = \{\{N_3\}, \{N_4\}, \{N_1, N_2, T\}, \ldots\} \]

Cov_1 is a Type 1 cover. It is contextually assigned a value. Each of the students is an element in the cover, thus the predicate distributes to each of the students. Cov_j is a Type 2 cover. It is a Skolem function which takes an individual variable and maps the individual to a cover. In (30), it takes a variable which is bound by the students. The value of the cover thus co-varies with each student. When the variable equals G_1 or G_2, the function maps them to a cover in which each note occupies an independent cell. Thus, G_1 and G_2 each read all the notes. When the variable equals G_3, the function maps the individual to a cover in which N_1 is in a cell with non-notes, being silently ignorable in the context. Thus, G_3 read all the notes except for N_1. When the variable equals U_1, the function maps the individual to a cover in which only N_1 is not in a cell with non-notes. Thus, U_1 only read N_1. When the variable equals U_2, the function maps the individual to a cover in which N_1 and N_2 are in a cell with non-notes. Thus, U_1 read N_3 and N_4. This gives us the intended interpretation, in which for each student, the non-maximal interpretation of the notes is different.

### 3.3 Deriving WR

In Section 2, we provide two pieces of evidence against the presence of the cumulative operator in WR. Earlier in this section, we introduced how covers help derive certain reciprocal interpretations, and we gave an updated definition to covers. In this section, we will show how to derive WR without the cumulative operator, using only the analysis in [5] and the updated covers.

\[(31) \quad \text{The men followed each other.} \]
\[\text{LF: } [\text{[The men]} | D_1 | [\text{[the other } x_1 \text{ (of) } Pro_3] | [D \text{ [cov}_2^{x_1 \rightarrow Pro_1]} ]] | [2 \text{ [t}_1 \text{ followed } t_2]]] \ldots\]
\[
[5]^{g} = \forall x [x \leq \text{the men} \land x \in g(i) \rightarrow \forall y [y \leq i \exists z \leq g(3) \land z \neq g(1)] \land y \in g(j)(\text{pro1}) \rightarrow x \text{ followed } y]
\]

(31) is true in a scenario where there were four men, namely \(m_1, m_2, m_3\) and \(m_4\). \(n\) is not a man. \(m_1\) followed \(m_2\), \(m_2\) followed \(m_3\), \(m_3\) followed \(m_4\). The covers in (32) derive the intended interpretation.

\[
\begin{align*}
[cov^1_{\text{pro}}]^{g} &= g(i) = \{\{m_1\}, \{m_2\}, \{m_3\}, \{m_4, n\}, \ldots\}, \\
[cov^2_{\text{pro1}}]^{g[1\rightarrow m_1]} &= g(j)(m_1) = \{\{m_2\}, \{m_1, m_3, m_4\}, \ldots\}, \\
[cov^2_{\text{pro1}}]^ {g[1\rightarrow m_2]} &= g(j)(m_2) = \{\{m_3\}, \{m_1, m_2, m_4\}, \ldots\}, \\
[cov^2_{\text{pro1}}]^{g[1 \rightarrow m_3]} &= g(j)(m_3) = \{\{m_4\}, \{m_1, m_2, m_3\}, \ldots\},
\end{align*}
\]

\(\text{Cov}\_1\) has a value as in (32). In the cover, \(m_1\), \(m_2\) and \(m_3\) occupy individual cells in the cover. \(m_4\) is in a cell with non-men. Thus, the predicate applies to \(m_1\), \(m_2\) and \(m_3\). \(m_4\) is silently ignorable. \(\text{Cov}\_2\) takes a variable bound by \text{the men}. When assigned different values, the function maps the individual to different covers. When the variable equals \(m_1\), \(m_1\), \(m_3\) and \(m_4\) are in a subset with non-men, thus being silently ignorable in the context. This gives us \(m_1\) followed \(m_2\). When the variable equals \(m_2\), then \(m_1\), \(m_2\) and \(m_4\) are in a subset with non-men, thus being silently ignorable in the context. This gives us \(m_2\) followed \(m_3\). When the variable equals \(m_3\), then \(m_1\), \(m_2\) and \(m_4\) are in a subset with non-men, thus being silently ignorable in the context. This gives us \(m_3\) followed \(m_4\). The covers above give us the intended interpretation.

We derive WR with ill-fitting covers or non-maximality. Thus, we predict that the availability of WR is determined by the availability of non-maximality. The prediction is borne out. Non-maximality is crucial to examples like (32). Given a finite line of students, there must be a first student, who did not follow anyone, and a last student, who was not followed by anyone. Maximal readings of the plurals will violate our world knowledge. When the antecedent of the reciprocal pronoun has a small cardinality, a non-maximal interpretation is not preferred. Besides, \textit{all} removes the non-maximality effect of the associated nouns. We predict that when small cardinality or \textit{all} is used in reciprocal sentences like (32), the sentence should become unnatural. The prediction is borne out.

(33)
\[
\begin{align*}
\text{a.} & \quad \text{The students followed each other.} \\
\text{b.} & \quad \# \text{All the students followed each other.} \\
\text{c.} & \quad * \text{All the three students followed each other.}
\end{align*}
\]

When we add factors which force a maximal interpretation, the reciprocal sentences become unnatural, as shown in (33).

4 Conclusions and future works

In this section we will outline some potential future directions.

A first open issue is that although reciprocal sentences behave differently from classical cumulative examples, they are similar to co-distributive sentences as defined in [11] and exemplified in (34).
Scenario: At a shooting range, each soldier was assigned a different set of targets and had to shoot at them. At the end of the shooting we discovered that:

The soldiers hit the targets.

Both the co-distributive interpretation and the WR interpretation are compatible with all, as shown below.

(35) a. All the soldiers hit the targets.
   b. All the soldiers hit each other.

It is worth investigating further what (35) shows about reciprocal sentences. A possibility is that instead of cumulativity, co-distributivity or collectivity is involved in reciprocal sentences.

Furthermore, our proposal has the potential problem of over-generalisation. A theory on the criteria underlying the selection of covers is needed, as empirical evidence indicates that not all covers are equally favored across different reciprocal sentences. Further studies are required here, but we find proposals like the Maximal Typicality Hypothesis [8] promising.

References