MULTIPLICITY AND NON-MONOTONIC ENVIRONMENTS*

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Abstract

In this squib, building on the insights of Spector (2007) and Zweig (2008, 2009), I propose a compositional system for deriving the "multiplicity" component of the meaning of bare plurals in non-monotonic environments in terms of recursive exhaustification.

1 Introduction

It is a well-known fact that while bare plurals give rise to the multiplicity ("more than one") meaning in sentences like (1a), this meaning disappears in downward-entailing (DE) environments ((1b), see Krifka 2004, Sauerland et al. 2005, Spector 2007):

(1) a. Dogs are barking. (= '**More than one** dog is barking.')
b. It's not true that dogs are barking. (= 'It's not the case that **one or more** dogs are barking.')
(≠ 'It's not the case that more than one dog is barking.')

The fact that the multiplicity ("more than one") component disappears in DE environments presents a challenge to the view that multiplicity is a part of the denotation of bare plurals and gives credibility to the idea that bare plurals are number neutral (i.e. they mean "one or more", cf. Sauerland et al. 2005, Spector 2007) and multiplicity arises through some sort of

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strengthening. A reasonable hypothesis is that multiplicity comes about as a scalar implicature (then its disappearance in DE contexts is not surprising). The most obvious step to make would be to say that the plural form has its singular counterpart as a scalar alternative. However, as noted in Sauerland et al. (2005), Spector (2007), Zweig (2008), if no extra assumptions are made, the two alternatives would often be equivalent ((3) obviously would entail (2), and (2) would entail (3) due to distributivity), so no implicature could be generated.

(2) Dogs are barking.

(3) A dog is barking.

Another interesting property of bare plurals was originally noted in Spector (2007). It is the fact that in non-monotonic environments the multiplicity component behaves in an unusual way: it survives in the "positive part" of the meaning, but not in the "negative" part. For example, the most natural interpretation of (4) is the one in (5).

(4) Exactly one of my students has solved difficult problems.

(5) '[$_{(POS)}$ One student of mine has solved more than one difficult problem]
and [$_{(NEG)}$ no other student of mine solved any problem at all].'

In what follows, I will review two existing implicature-based theories of bare plurals (Spector 2007, Zweig 2008, 2009) with particular attention as to how they handle these properties of multiplicity: its presence in UE contexts, its disappearance in DE environments and its "partiality" in non-monotonic environments. Then I will try to improve on these approaches and provide a new unified account of multiplicity.

2 Previous implicature-based approaches

2.1 Spector (2007)

The problem of the equivalence of the alternatives with distrubutive predicates is addressed in Spector (2007). In order to solve it, Spector assumes that not only does the singular have the plural as its alternative, it also forms a scale with numerals: ⟨ a(SG), two, three ⟩.

Spector offers a mechanism of generating higher-order implicatures along the following lines: in cases where there is a symmetric entailment between the singular and the plural, as in 1, the singular first get strengthened to the "exactly one" meaning (by negating the stronger "two"-alternative) and next multiplicity arises as a result of a higher-order implicature (by comparing the plural with the enriched version of the singular). See a schematic derivation below:

(6) a. A dog is barking. ⟶ 'A dog is barking and it's not true that two dogs are barking.' = 'Exactly one dog is barking.' (*enriched version of the singular*)
b. Dogs are barking. ⟶ 'One or more dogs are barking and it's not true that exactly one dog is barking.' = 'More than one dog is barking.' (*enriched version of the plural*)

The absence of the multiplicity in DE environments is not surprising: the singular alternative does not get strengthened to the "exactly one" in this case, as the "two"-alternative is now weaker than the singular one, hence no higher-order implicatures can be generated.

The mechanism proposed by Spector (2007) also handles the non-monotonic case quite efficiently. Consider (7a) and its singular alternative (7b):

(7) a. Exactly one of my students solved difficult problems.
    b. Exactly one of my students solved a difficult problem.

Again, this is a case in which two alternatives are equivalent, i.e., they entail each other. Using Spector's recipe, what we can do is to first strengthen the singular alternative in (7b) by negating the *two*-alternative, as shown below:

(8) 'Exactly one of my students solved a difficult problem and it's not true that exactly one of my students solved at least two difficult problems.' = 'Exactly one of my students solved exactly one difficult problem and no other student solved any problem at all.'

And now we can compute the strengthened meaning of (7a), by negating the alternative in (8):

(9) 'Exactly one of my students solved one or more difficult problems and it's not true that [exactly one of my students solved exactly one difficult problem and no other student solved any problem at all]'.

Spector proves that (9) is equivalent to (10), and this is the desired meaning:

(10) 'Exactly one of my students solved at least two difficult problems and no other student solved any problem at all.'

2.2 Zweig (2008, 2009)

Zweig (2008, 2009) gives a different solution to the problem of deriving the multiplicity meaning as an implicature. He assumes that only the singular is the plural's scalemate, but additionally he assumes that implicature calculation can happen at the predicate level, and this is a level where two alternatives are not equivalent.

Specifically, he argues that the implicature calculation happens at the event predicate level[1]. For example, for (1), the plural alternative at the event predicate level can be represented as follows:

(11) $\lambda e.\exists X[\text{*Dog}(X) \wedge \text{*Bark}(e)(X)]$

The singular alternative is shown below:

(12) $\lambda e.\exists X[\text{*Dog}(X) \wedge X \text{ is atomic} \wedge \text{*Bark}(e)(X)]$

Note that both (11) and (12) denote sets of events and it can be shown that the set in (12) is a subset of (11). If an event belongs to the set in (12), it obviously belongs to the set in (11) ((12) contains events with atomic agents, whereas (11) contains events with both atomic and non-atomic agents). However, the opposite is not true. Consider an event in which two dogs A and B are barking. This event, which has a plural agent, belongs to the set in (11), but it does not belong to the set in (12), as (12) only contains events with atomic agents.

Since the alternative in (12) is stronger than the alternative in (11), we can negate it (let's treat negation at the predicate level as set subtraction):

[1]The same results can be derived in a system without events. The necessary assumption would be to allow implicatures to be calculated at the predicate level. In our particular case, it would have to take place at the level of the predicate *dog*: $\lambda X.\text{*Dog}(X)$.

(13) $\lambda e.\exists X[{*}\text{Dog}(X) \wedge {*}\text{Bark}(e)(X)] \wedge \neg\exists X[{*}\text{Dog}(X) \wedge X \text{ is atomic} \wedge \text{Bark}(e)(X)] =$
$= \lambda e.\exists X[{*}\text{Dog}(X) \wedge {*}\text{Bark}(e)(X) \wedge \mid X \mid > 1]$

After the event closure is applied, the following meaning is derived:

(14) $\exists e.\exists X[{*}\text{Dog}(X) \wedge {*}\text{Bark}(e)(X) \wedge \mid X \mid > 1]$

The disappearance of multiplicity in DE contexts can be taken care of by using some form of a principle that allows implicatures to be calculated only if it strengthens the overall meaning.

In general, Zweig's approach seems to be more attractive on the conceptual grounds, as it stipulates fewer alternatives. However, it has an empirical problem: while, it can capture the cases like (1a)–(1b)[2], it has no way of deriving (4). Note that *exactly one* has to scope above the event closure, leading to the following LF:

(15) $[\![\text{exactly one student}]\!]\ (\lambda x.\exists e.\exists Y[{*}\text{solve}(e)(x)(Y) \wedge {*}\text{DifficultProblem}(Y)])$

In Zweig's system we derive the implicature at the level of the event predicate. At that level the two alternatives in (16) are not equivalent, and the negation of the singular alternative (16b) leads to the meaning in (17):

(16) a. $\lambda e.\exists Y[{*}\text{Solve}(e)(x)(Y) \wedge {*}\text{DifficultProblem}(Y)]$
b. $\lambda e.\exists Y[{*}\text{Solve}(e)(x)(Y) \wedge {*}\text{DifficultProblem}(Y) \wedge Y \text{ is atomic}]$

(17) $\lambda e.\exists Y[{*}\text{Solve}(e)(x)(Y) \wedge {*}\text{DifficultProblem}(Y) \wedge \mid Y \mid > 1]$

Applying existential closure and the quantifier *exactly one of my students* will lead to the meaning informally represented in (18):

(18) 'Exactly one of my students is such that that student has solved more that one difficult problem.'

The meaning in (18) is too weak. It makes the sentence (4) true, for example, in a situation in which there is one student who solved more than one difficult problem and there is another student

[2]Another advantage of Zweig's approach is that it can derive dependent plural readings for sentences like "*Three friends of mine attend good schools*". The dependent plural reading can be paraphrased as 'Each of the three friends attends at least one good school and more than one good school is attended overall.'

Zweig argues that dependent plural readings are a subcase of cumulative readings and the overall multiplicity requirement should be derived as a scalar implicature, just the same way it is derived for cases like (1). Spector's account has a problem with deriving dependent plural readings that can be briefly summarized as follows. Under the assumption that dependent plural readings are a subcase of cumulative ones, the plural and the singular alternatives can be represented as '*There is a plurality of schools (one or more) that is attended by the three friends*' and '*There is an atomic school that is attended by the three friends*', correspondingly. In this case the alternatives are not equivalent (in a situation in which the three friends each attend a different school, the plural alternative is true, but the singular one is false), hence Spector's mechanism will negate the stronger singular alternative, leading to the following meaning '*There is a plurality of schools (one or more) that is attended by the three friends but there is no atomic school that is attended by them*'. However, this reading seems to be too strong, as it requires there not to be any atomic school that is attended by the three friends. It seems that dependent plural reading has weaker requirements, but we are leaving a more detailed examination of this issue for further research, as the phenomenon of dependent plurality is not the focus of the paper.

who solved exactly one difficult problem. But our desired reading (5) is incompatible with such a scenario.

Note that deriving the implicature after event closure is applied wouldn't help, because there the two alternatives, shown below, become equivalent:

(19) a. $\exists e.\exists Y[\text{*Solve}(e)(x)(Y) \wedge \text{*DifficultProblem}(Y)]$
b. $\exists e.\exists Y[\text{*Solve}(e)(x)(Y) \wedge \text{*DiffucultProblem}(Y) \wedge Y \text{ is atomic}]$

(19b) entails (19a), but (19a) also entails (19b) due to the distributivity of the predicate *solve* (the presense of an event in which one or more than one problems were solved entails the presence of an event in which an atomic problem was solved). So, computing an implicature anywhere higher than the event predicate level will be vacuous and wouldn't derive the desired reading either.

3 Proposal

I would like to offer an account that is able to explain all the facts above without stipulating alternatives the way Spector (2007) does (namely, appealing to two different scales). Following Zweig (2008, 2009), I will assume that the singular is an alternative to the plural and that the implicature calculation can happen at an event predicate level.

I would also like to pursue the intuition that in order to account for the meaning of (4), we somehow need to make sure that we are comparing the sentence to an alternative that is equivalent in meaning to 'exactly one student of mine has solved exactly one difficult problem'. If we had this alternative available to us, we would derive the desired meaning:

(20) 'Exactly one student of mine has solved one or more problems and it's not true that exactly one student solved exactly one difficult problem.'
= 'Exactly one student solved more than one difficult problem and no one else solved any problem at all.'

The task is to understand how we can get to such an alternative in the first place. First, let me present an informal idea due to Irene Heim and Danny Fox (p.c.) that I will later try to formalize. Even if the formalization is not very successful in the end, I think the idea can be of some value.

What I intended to show in the previous section is that it is quite conceivable that the plural gets its multiplicity ("more than one") meaning by competing with its singular alternative, which is stronger at a certain level of semantic derivation (namely, at the predicate level). What I would like to add now is that just like the plural gets multiplicity by competing with its singular alternative, the singular, in its turn, gets its "exactly one" meaning by competing with the enriched ("more than one") version of the plural, at a later stage.

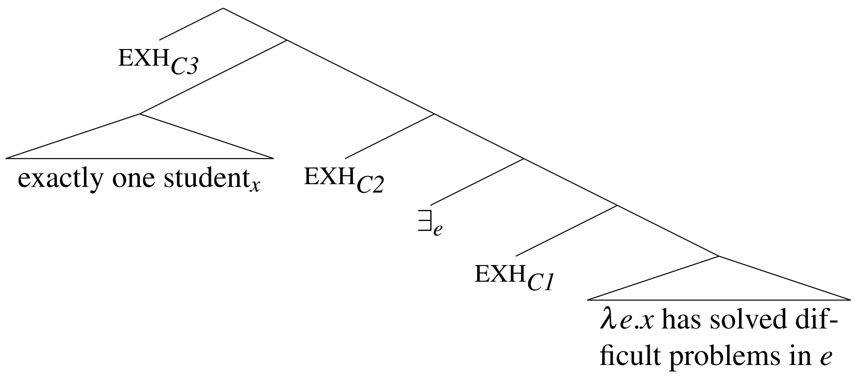
To derive the multiplicity meaning for the plural alternative, we negate the stronger singular alternative at the event predicate level (as was shown above). However, notice that after we apply event closure, the scalar relationship between the alternatives reverses: the enriched plural alternative, e.g., [$\exists e$[x has solved more than one difficult problem in e]] in (4) is stronger than the singular alternative, [$\exists e$[x has solved a difficult problem in e]]. And since the plural alternative is now stronger than the singular one, we can go through the exhaustification process (by negating the enriched plural alternative) and derive the "exactly one" meaning for the singular alternative.

Before trying to give a formalization of this idea, and since I already mentioned "exhaustification", let me first lay out the necessary assumptions concerning the implicature calculation process.

I assume that implicatures are brought about by a covert exhaustivity operator EXH akin to *only* (whose purpose is to negate the alternatives not entailed by the prejacent, cf. Chierchia et al. 2012), which can apply at both sentence and predicate levels. I will also follow Magri (2009) in assuming that EXH is obligatory at every level, so in our case it would be equal to saying that we have EXH in both places – at the event predicate level and after the event closure applied.

With these ideas in mind, I argue that the LF presented schematically below gives the meaning of (4) we are after:

(21)



The three exhaustificators operate on three different sets of alternatives (C1, C2, C3). Importantly, I assume that *embedded* EXH-operators do not affect their prejacents[3], but only exhaustify the alternatives in their given sets.

Let's show how the alternative set is generated at each of the levels.

At the level C1, we are dealing with the set consisting of our plural alternative and another alternative that is derived by replacing the bare plural with a corresponding singular NP:

(22) C1 = $\{[\lambda e.x$ solved (one or more) problems in $e], [\lambda e.x$ solved a problem in $e]\}$

The set of alternatives at a higher level, C2, will be of the form $\{\exists e[\text{EXH}_{\text{C1}}$[A]: A is a member of C1$]\}$, i.e. they will contain members of C1 exhaustified with respect to the set in C1. The first member of C1 exhastified with respect to the alternatives in C1 will give rise to the multiplicity alternative and the second member of C1 will remain untouched, as it is the strongest alternative in the set:

(23) C2 = $\{[\exists e[x$ solved more than one problem in $e]], [\exists e[x$ solved a problem in $e]]\}$

The alternative set at the level C3, in turn, will be of the form $\{[⟦$exactly one student$⟧$ $(\text{EXH}_{\text{C2}}$[A]: A is a member of C2$)]\}$. This will give us the following set (the exhaustification of the first member of C2 is vacuous, as it is the strongest alternative in the set; the exhaustification of the second member will derive the "exactly one" meaning, $\exists e[x$ solved a problem in $e]$ and $\nexists e[x$ solved more than one problem in $e]$):

[3] I leave the explanation of this restriction for future research. We can think of EXH being vacuous at certain levels as a result of the *pruning* of alternatives. At this stage, we can assume the following architecture: first, sets of alternatives are generated, and this process is blind to pruning; second, the pruning mechanism comes in, allowing us to prune all the alternatives from C1 and C2, thus leading to EXH being vacuous at those two levels.

(24) C3 = {[Exactly one student solved more than one problem], [exactly one student solved exactly one problem]}

Note that the set in C3 contains the needed alternative "exactly one student solved exactly one problem", but on the other hand, it also contains a second alternative, which is a harmful one: if we were to negate both of these alternatives, we wouldn't derive the right meaning. In order to rule out this situation, I stipulate a constraint on sets of alternatives[4]:

(25) The set of alternatives of ϕ cannot contain an exhaustified version of ϕ
Definition: β is an exhaustified version of α, if when all EXHs are dropped in β, we get the same LF in α and β.

This constraint will allow us to get rid of the first alternative in C3, as it *is* an exhaustified version of the prejacent of EXH_{C3}. Thus, the topmost EXH_{C3} will negate the only remaining alternative in C3, giving rise to the reading 'exactly one student has solved one or more difficult problems and it's not the case that exactly one student solved exactly one difficult problem', i.e. the reading we were after: 'exactly one student solved more than one difficult problem and no other student solved any difficult problems.'

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[4]The constraint of this sort is needed independently for more basic cases with other scalar items like *or* in *Every boy talked to Mary or Sue*. If the proposed architecture is right, we would need to somehow rule out the alternative "Every boy talked to Mary or Sue but not both". If the alternative were there, the sentence would be predicted to entail that some boy talked to both, which is not correct. The constraint in (25) takes care of it. Thanks to Danny Fox (p.c.) for his help in formalizing this constraint.