THE COMPARATIVE AS SUBSETHOOD: HOW DEGREE-REFERRING RESTRICTORS FIT IN*

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Abstract

Heim's (2006) definition of the comparative morpheme is tested against constructions containing bare degree-referring expressions in *than*-clauses. The combination is shown to require nontrivial assumptions about the structure of *than*-clauses, but assumptions that nonetheless find independent support in Hackl (2000).

1 Introduction

Comparative forms in English are found for a variety of expressions. In some cases the expression is uncontroversially classifiable as an adjective, like for example *tall(-er)* and *big(-ger)*. In others, e.g. *few(-er)* and *more*, the classification is arguably less straightforward.[1] More relevantly for this squib, we also find comparative forms for expressions of opposite polarity, as in the antonym pairs *taller/shorter*, *bigger/smaller*, and *more/fewer*. In light of this distribution, we ask whether it is possible to assign a semantics to *-er*, the comparative morpheme, that can produce the correct truth conditions for each of these cases. Here, our interest will be the case of antonyms: can we define ⟦*-er*⟧ so that it combines correctly with both members of a given antonym pair, and importantly, will the composition generate the desired results? It is in response to this question that Heim (2006) proposes her subset-based semantics of the comparative morpheme.

The goal of this squib is to describe an apparent difficulty for the subset-based definition of *-er*, and to sketch a solution that finds independent motivation elsewhere. The difficulty is presented by so-called 'measure-phrase comparatives', comparatives that contain bare numerals or degree-referring expressions in their *than*-clauses. The solution is an adaptation of Hackl's (2000) intensionalization of degree sets.

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[1]Following Bresnan (1973) and much of the literature since, I take *more* to be a suppletive of *many-er*.

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2 Background

I will assume that *-er* composes first with the semantic content of its *than*-clause, and that a process of extraposition moves the *than*-clause to the position where it is pronounced (Bhatt and Pancheva 2004). I assume also that *-er* denotes a relation between sets of degrees, type $\langle dt, \langle dt, t \rangle \rangle$: its first argument is the set of degrees denoted by the *than*-clause, and its second is the set of degrees denoted by the matrix clause. Together with the *than*-clause, *-er* denotes a generalized quantifier over degrees (see e.g. Heim 2000, and see Beck 2011 for a recent survey).

To keep things simple, I will use examples with the adjectives *tall* and *short*. I take these to denote relations between degrees and individuals, as shown in (1) and (2).[2]

(1) $[\![\textit{tall}]\!] = [\lambda d \,.\, \lambda x \,.\, \text{HEIGHT}(x) \geq d]$

(2) $[\![\textit{short}]\!] = [\lambda d \,.\, \lambda x \,.\, \text{HEIGHT}(x) \leq d]$

Before we see how these entries interact with the comparative morpheme, let us quickly see (1) at work with a measure phrase, e.g. in (3).[3]

(3) John is 6 feet tall.
John is [6ft-tall]
$[\![(3)]\!] = [\![\textit{tall}]\!]([\![\text{6ft}]\!])([\![\text{John}]\!])$
$= 1 \text{ iff HEIGHT}([\![\text{John}]\!]) \geq 6 \text{ ft}$

tall appears in the comparative in (4).

(4) John is taller than Bill (is).

(4) compares John's height to Bill's. It compares, that is, the maximal degree that John's height reaches, to the maximal degree that Bill's height reaches. The sentence is true iff the former exceeds the latter. Given our decision to treat *-er* (together with its *than*-clause) as denoting a quantifier over degrees, and given the resulting need to interpret [*-er* than ...] at the clausal level, we arrive at the LF in (5). Note, crucially, the additional unpronounced material appearing in the *than*-clause: I assume, following Bresnan (1973) and much of the literature thereafter, that *than*-clauses contain material that parallels the content of the matrix clause. The material undergoes ellipsis under an identity condition, a process known (since Bresnan) as Comparative Deletion.[4]

(5) [*-er* than ~~λd~~ Bill (is) ~~d-tall~~] $\lambda d'$ [John is d'-tall]]

We may now write the denotation of *-er* as in (6).

[2] $[\![\textit{short}]\!]$ and $[\![\textit{tall}]\!]$ are defined independently here. The considerations of this squib are equally applicable to the so-called 'decompositional' analyses of antonymy, where the negative of a pair of antonyms is split into an "antonymizing" negative component, together with the positive counterpart (examples of these include von Stechow 2006, Heim 2006, and Büring 2007. For a detailed assessment, see Heim 2008).

[3] *short*, like many of the negatives in antonym pairs, are not compatible with measure phrases. See e.g. Rett (2008) and Breakstone (2012) for discussion.

[4] For more on ellipsis in the context of comparatives, see e.g. Lechner (2004).

(6) $[\![\text{-}er]\!] = \lambda D.\lambda D'.\max(D') > \max(D)$ (to be revised)

The entry for *-er* in (6) takes the maxima of its arguments, and returns True iff the first falls below the second. Suppose that John's (maximal) height is $6'$, while Bill's is $5'10''$. Then (4)—with the LF in (5)—would be predicted, correctly, to be true, since Bill's height falls below John's. If we change the scenario and assume that Bill and John have the same height, or that Bill's exceeds John, the matrix set's maximum will not exceed that of the *than*-set, and *-er* will correctly return false. The composition of (5), using the entry for the comparative in (6), is shown below.

(7) $[\![(4)]\!] = [\![(5)]\!] = [\![\text{-}er]\!](\lambda d.[\![\text{Bill is } d\text{-tall}]\!])(\lambda d'.[\![\text{John is } d'\text{-tall}]\!])$
$= 1 \text{ iff } \max(\lambda d'.[\![\text{John is } d'\text{-tall}]\!]) > \max(\lambda d.[\![\text{Bill is } d\text{-tall}]\!])$
$= 1 \text{ iff } \max(\lambda d'.\text{HEIGHT}(j) \geq d') > \max(\lambda d.\text{HEIGHT}(b) \geq d)$

Let us now turn to *short*, shown in the comparative in (8).

(8) Bill is shorter than John (is)

We will now see that the definition in (6) derives incorrect truth conditions for (8). First recall our assumption that *than*-clauses contain deleted material, and that deletion is licensed under identity. In the case of (8), this means that *short* must appear in both the *than*-clause and the matrix clause, as shown in (9).

(9) [[*-er* than ~~λd~~ John (is) ~~d-short~~] $\lambda d'$ [Bill is d'-short]]

The *than*-clause in (9) denotes the set of degrees that are above or equal to John's height. The matrix clause denotes the set of degrees that are above or equal to Bill's height. These denotations follow from the definition of *short* in (2): if John is $6'$-tall, what are the degrees d that verify 'John is d-short'? They are the degrees d that verify the proposition that $\text{HEIGHT}(j) \leq d$. These are $6'0''$, $6'1''$, $6'2''$ and so on. And if Bill is $5'10''$, then the degrees d' that verify 'Bill is d'-short' begin at $5'10''$, and also continue upwards to infinity.

(10) If $\text{HEIGHT}(j) = 6'$, and $\text{HEIGHT}(b) = 5'10''$, then
a. $[\lambda d\,.\,[\![\text{John (is) } d\text{-short}]\!] = [6', \cdots)$
b. $[\lambda d'\,.\,[\![\text{Bill (is) } d'\text{-short}]\!] = [5'10'', \cdots)$

Neither of these sets has a maximal element, so the current entry for the comparative will not do. What we want instead is to compare the *minimal* elements of (10a) and (10b): in the current scenario it is true that Bill (at $5'10''$) is shorter than John (at $6'$). The comparative should therefore require that the *minimal* element of (10a), which is 6 feet, *exceed* the minimal element of (10b), $5'10''$.

(11) $[\![\text{-}er]\!] = \lambda D \lambda D'.\min(D') < \min(D)$

If we now compare (6) to (11), we find two differences: where the former refers to max, the latter refers to min, and where the former requires $>$, the latter requires $<$. Heim (2006) reconciles these differences by redefining $[\![\text{-}er]\!]$ as in (12).

(12) $[\![\text{-}er]\!] = \lambda D \lambda D'. D \subset D'$ (Heim 2006)

(12) requires proper inclusion between its two arguments. If (12) holds of two sets of degrees D and D', then there must be a degree in D' that is not in D. In the case of *taller*, *-er* takes two sets that share a lower bound, e.g. [λd. Bill is d-tall] and [λd. John is d-tall], so the only way for the first to be a proper subset of the second is for the former to have a lower maximal element, i.e. a lower upper bound. This is equivalent to the requirement that the latter set of degrees have a greater maximum than the former. In the case of *shorter*, *-er* compares two sets D, D' that extend infinitely upward. D can only be a proper subset of D' if D has a higher minimal element than D', i.e. if it has a greater lower bound, which is the same as the requirement in (11).[5]

3 Measure phrase comparatives

Consider the comparative in (13).

(13) John is taller than 6 feet.

If (13) is to be interpreted using the semantic entry in (12), there needs to be two sets of degrees, one denoted by the matrix clause [λd.John is d-tall], and the other by the *than*-clause. But what set of degrees does ‘than 6 feet’ refer to?

Here is a first stab: suppose we create a singleton set consisting of the degree named by expression ‘6 feet’, and designate this as the denotation of the *than*-clause in (13). [-*er*] will now require that this singleton be contained in $\{d$: John is d-tall$\}$. If John is $5'10''$, the sentence is predicted to be false—as desired—since $\{6'\}$ is not contained in $\{0, \cdots, 5'10''\}$. If John is $6'2''$, the sentence is predicted to be true—again as desired—since $\{6'\}$ is contained in $\{0, \cdots, 6'2''\}$. But the problem comes in the scenario where John is exactly $6'$: the singleton $\{6'\}$ *is* properly contained in $\{0, \cdots, 6'\}$, so the sentence is predicted be true, but it isn’t; if John is 6 feet tall, then it is false that he is taller than $6'$.

The source of this problem is easy to see: the subset condition behaves well when we compare intervals that have the same scalarity. In example (4), repeated,

(4) John is taller than Bill (is)

we compared John’s degrees of height to those of Bill. In this case both sets are downward-scalar: if John/Bill is d-tall, then for every $d' \leq d$, John/Bill is d'-tall. This means that the two sets will share their lower bound, and subsethood will hold depending on how far up the two sets extend. If John and Bill are of equal heights, the sets are identical, and in consequence the sentence is (correctly) predicted to be false.[6] Now, in the case of (13) we want to derive the same result, but the singleton approach chops away the bottom segment of one of [-*er*]’s arguments, and as a result it allows subsethood to hold for the wrong reasons. If we want to maintain Heim’s proposal, we will want to abandon the singleton idea, and somehow retrieve the set $\{0, \cdots, 6'\}$ from $\{6'\}$; if we do, [-*er*] will produce correct results for (13), just like it does for (4).

What, then, makes it possible to retrieve the desired set of degrees from the *than*-clause and the degree name contained in it? The answer I propose is based primarily on an idea from

[5] In von Stechow (2006) there is a similarly advantageous unification in the semantics of the positive morpheme.

[6] This can be restated for *short*, which makes the two sets upward-scalar.

Hackl (2000), who diagnoses a problem with comparative constructions and argues for a solution involving intensionalized sets of degrees. In the next section I present the problem and describe Hackl's solution. I must note that my description (and implementation) of Hackl's solution is slightly different from the original, but I believe both are equally applicable.

3.1 Hackl's problem, and application

The problem noted by Hackl is shown in (14).

(14) a. *More than one person gathered.
b. *John introduced more than one person to each other.

Intuitively, (14a,b) are true iff the number of (gatherers)/(people introduced by John) is greater than 1. Our lexical knowledge tells us that the verb *gather*, and the predicate *be introduced to each other*, cannot combine with atomic/singular individuals. But the understood subject in (14) is not atomic, because the comparative requires that at least two people gather/be introduced to each other. So there is no obvious reason why the sentences should sound as odd as they do. One could try to explain (14) by appealing to the relative weakness of the predicted truth conditions: if there was any gathering at all, then it will follow that (14a) is true. But if this kind of explanation is on the right track, we would expect to see the same oddness in the acceptable (15), since the truth conditions are weak in the same way.

(15) a. (At least) two people gathered
b. John introduced (at least) two people to each other.

To Hackl, (14) suggests an analysis of *-er* that uses an intensionalized maximality operator. I will not incorporate the details of that particular proposal here. Instead, I will show the intuition behind it, and apply that intuition to the cases that concern us.

The reason behind (14)'s oddness, according to Hackl, becomes clearer once we consider the plausible paraphrases in (16).

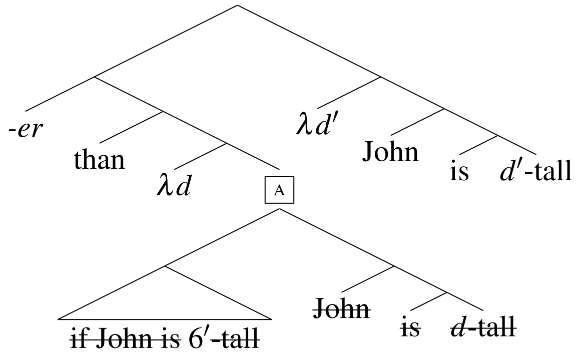
(16) a. More people gathered than (would have been the case if) one person gathered.
b. John introduced more people to each other than (would have been the case if) John introduced one person to each other.

As I will now show, this intuition will facilitate the move from the traditional account of the comparative, where two degrees are compared, to Heim's subset condition, where the comparative operates on two sets of degrees. Consider (13) again.

(13) John is taller than 6 feet.

For simplicity I will take the paraphrases in (16) literally, noting that different implementations of the core idea are conceivable. Let us assign the following LF to (13).

(17)



The *than*-clause in (17) contains a conditional. Except for the overt expression '6 feet', the antecedent of the conditional is identical to material in the matrix clause. This licenses the deletion of the antecedent, leaving behind the degree-referring '6 feet', and 'if'. There is no matching material that might license the deletion of 'if', so let us interpret (17) as containing a modal restriction (paraphrasable as a conditional), rather than an explicit conditional, in its *than*-clause.

We can now construct the meaning of the *than*-clause in (17): it is a property that holds of a degree d iff it satisfies the denotation of A, i.e. the proposition in (18).

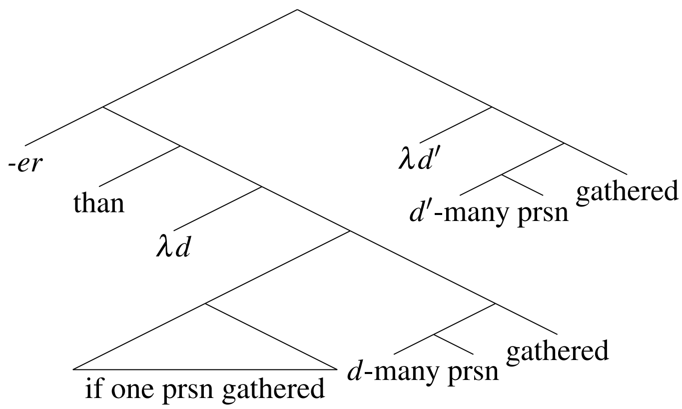
(18) If John is (at least) 6 feet tall, John is d-tall

If John is (at least) 6 feet tall, then the consequent will hold of every degree from $6'0''$ downwards, because in that case John's height will be at least $5'11''$, at least $5'10''$, at least $5'9''$, and so on. The consequent will not hold of $6'1''$, however, because not all worlds where John is at least 6 feet tall are worlds where he is at least $6'1''$. With $\{0, \cdots, 6'0''\}$ as the denotation of the *than*-clause, the entire comparative will be true only if John's height is greater than 6 feet, because that is the only way for the matrix clause to denote a proper superset of the denotation of the *than*-clause.

Before I turn to *short*, let me quickly comment on Hackl's original problematic example. On the current proposal the sentence in (14a) will have the LF in (19).

(14a) #More than one person gathered

(19)

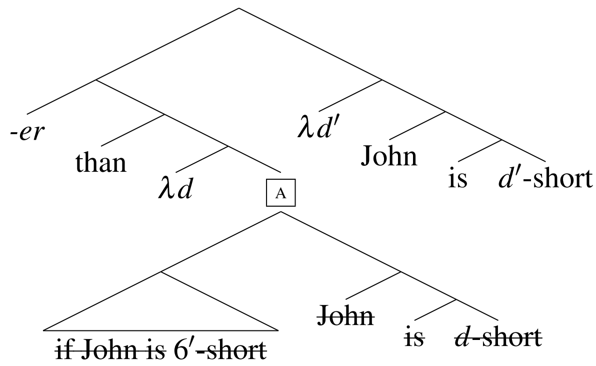


The conditional in (19) contains an antecedent where the (singular) subject *one person* co-occurs with the collective predicate *gather*. Since this combination results in a semantic conflict—we remain agnostic about its precise details for reasons of brevity— the conditional, and consequently the construction containing it, is illicit.

We now turn to *short*. Here things might be trickier. In order for the story to work, the relevant LF will need to feature *short* in the same places where *tall* appears in (17). The structure is shown in (21).

(20) John shorter than 6 feet.

(21)



(21) generates the correct truth conditions in the same way as (17): the antecedent of $\boxed{\text{A}}$ denotes the proposition that John's height is 6 feet or less. This, recall, is what it means for John to be 6 feet *short*. $\boxed{\text{A}}$ will therefore hold of a degree d provided that, if John's height is 6 feet or less, then John's height is d or less. The degrees that satisfy this conditional begin at $6'0''$ and range infinitely upward: if John's height is 6 feet or less, then it follows that his height is $6'1''$ or less, $6'2''$ or less, etc. The entire comparative will therefore be true iff the set $\{6'0'', 6'1'', \cdots\}$ is properly included in the degrees d' to which John is d'-short, and this in turn holds only if John's maximal height is strictly less than 6 feet, as desired.

A strange aspect of (21) is that *short* appears with a measure phrase in the unpronounced conditional. We know that this combination is ungrammatical in English, and if the reason turns out to be semantic, then the same conflict that was appealed to in ruling out Hackl's (14a/19) might also rule out (21). I leave this issue unresolved here, but point the reader to Alxatib (2013), where I discuss the possibility of replacing the (potentially) problematic *short* with an exhaustified parse of *tall*.[7]

[7] See specifically Chapter 6. There are a few issues involved in making this move. First it must be noted that, with $6'$-*tall* in place of $6'$-*short* in (21), it becomes trickier to justify the silence of the antecedent, since there are no other occurrences of *tall* in the LF that might license ellipsis. However, if we assume that antonyms like *short* are composites of an antonymizer (a degree negation operator) together with the positive form, e.g. *tall*, the deletion of the antecedent material becomes easier to explain (see Footnote 2 for references). Second, as the reader may verify, replacing $6'$-*short* with $6'$-*tall*, without exhaustification, will render the *than*-clause denotation empty. This is discussed in detail in Section 6.1 of Alxatib (2013), where I also show how exhaustification generates the correct results for both "taller than" and "shorter than". Recently, Nussbaum (2014) has argued that similar effects are derivable if the conditional is assumed to quantify over minimal situations, in the style of e.g. Kratzer (2014).

4 Summary

We discussed the motivation for Heim's (2006) definition of *-er*, as requiring subsethood, and discussed a possible difficulty in applying it to measure phrase comparatives. The difficulty is posed by the measure phrase in the *than*-clause, which, as was argued, cannot be taken to denote a singleton consisting of the named degree. I argued that Hackl's (2000) idea of intensionalizing the semantics of *-er* provides a solution to the problem, and implemented the solution by adding a silent (conditional-like) modal restriction in the *than*-clause.

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