Opposition Inferences and Generalized Quantifiers[1]

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Abstract

This paper develops a theory on opposition inferences – quantifier inferences involving the contradictory, contrary and subcontrary relations. After the basic notions associated with opposition inferences, including opposition properties (OPs), o-sensitivities, etc., are defined as generalizations of the notions associated with monotonicity inferences, a number of theorems for determining the o-sensitivities of various types of monadic generalized quantifiers (GQs), including determiners, type <1> GQs and structured GQs, are proposed and proved, resulting in a classification of the most commonly used monadic GQs according to their OPs. For iterated polyadic GQs, the notion of OP-chain is defined. A principle that enables one to determine the o-sensitivities of an iterated GQ according to the o-sensitivities of its constituent monadic GQs is then proposed. The o-sensitivities of GQs viewed as sets and arguments of other GQs and logical operators, particularly the negation operator, are also discussed. Finally, opposition inferences are compared and contrasted with monotonicity inferences. It is finally concluded that o-sensitivities are independent of monotonicities, and opposition inferences are not subsumable under monotonicity inferences.

1. Introduction

Opposition inferences constitute an important type of immediate inferences studied in Classical Logic. These are inferences involving four types of relations defined on the classical square of opposition: subalternation, contradictoriness, contrariety and subcontrariety. Based on the definitions of these relations (which will be given below), one can immediately obtain the following instances of opposition inferences:

(1) (Given that there is some student.)
Every student sang. ⇒ Some student sang.

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[1] This is the author-final version of a paper in Béziau, J.-Y. and Georgiorgakis, S. (eds.), *New Dimensions of the Square of Opposition*, München: Philosophia Verlag GmbH, pp. 155–199, 2017. The final publication is available at www.lehmanns.de. When I wrote this paper about three years ago, I used the notation CC→CC, CC→SC, SC→SC and SC→CC to denote the 4 opposition properties studied in this paper. While these notations fit definition (24) in this paper well, they sound strange to the ear because they are not like English words. I now propose to call these 4 properties by the names "homo-exclusive", "anti-exclusive", "homo-exhaustive" and "anti-exhaustive", respectively.

(2) (Given that there is some student.)

No student sang. $\Rightarrow$ It is not the case that every student sang.

Apart from these mundane examples, opposition inferences can also help us solve some logical puzzles that are not so straight-forward, such as the following:

(3) Three persons A, B and C each made a remark. Suppose there is some student, John is a student and there is only one true statement among the three remarks. Which is the only true statement?

A: Some student sang.

B: Not every student sang.

C: John sang.

To solve this puzzle, we first note that A's and B's remarks satisfy the subcontrary relation, i.e. they cannot be both false and so one of them must be true. Since there is only one true statement among the three, C's remark must be false, i.e. John did not sing. This means that B's remark must be true, because otherwise it contradicts the fact that John did not sing. Thus, we conclude that B's remark is the only true statement.

However, the applicability of classical opposition inferences is limited because Classical Logic only studied quantified statements headed by the four classical quantifiers: "every", "no", "some" and "not every". The advent of modern Generalized Quantifier Theory (GQT) has opened up possible ways to extend the classical opposition inferences. Not only can we now consider opposition inferences of quantified statements headed by non-classical quantifiers such as "most", "all … except John", but we can also consider inferences that have very different structures than (1) – (3) above such as the following:

(4) (Given that there is some member.)

Every member is elderly. $\Rightarrow$ It is not the case that every member is a teenager.

Note that although (2) and (4) both make use of the contrary relation, the contrariety in (2) is between the quantifiers "no" and "every", whereas the contrariety in (4) is between the predicates "be elderly" and "be a teenager".

Apart from applications to logical reasoning, opposition inferences also have linguistic applications. One such application is to determine the incompatibility between two predicates. For instance, from (4) above, we know that "clubs all members of which are teenagers" and "clubs all members of which are elderly" are incompatible, whereas "clubs of which all teenagers are members" and "clubs of which all elderly are members" are not (because it is logically possible to have a club that includes all teenagers and elderly as members). As incompatibility is an essential element of antonyms that feature in certain linguistic structures, such as those identified by Jones (2002), the determination of incompatibility can help us determine the well-formedness of certain linguistic structures.

For example, "X rather than Y" is a structure where X and Y should be antonyms. Thus, based on the above discussion, we know that the following sentence is well-formed:

(5) I would rather work for a club all members of which are teenagers than a club all members of which are elderly.

Nevertheless, this does not mean that (5) will necessarily become not well-formed if it becomes

(6) I would rather work for a club of which all teenagers are members than a club of which all elderly are members.

because when appearing in an antonymy context like "X rather than Y", the meanings of X and Y will often be construed contrastively so as to become mutually incompatible. This is the pragmatic process called "narrowing" in Geurts (2010).

According to Geurts (2010), narrowing is a common phenomenon in antonymy contexts. The purpose of this strategy is to narrow down the extensions of one or all of the lexical items in contrast by enriching their intensions, thereby sharpening their meaning and avoiding semantic oddity. The following is an example from Geurts (2010):

(7) Around here, we don't like coffee, we love it.

In the above example, "like" and "love" are not antonyms according to their original meanings. But here narrowing has occurred and the meaning of "like" has indeed been narrowed down to "like but not love", which then becomes contrary to "love". That is why "like" and "love" can appear in the above antonymy context. Similarly, in (6) the meanings of "club of which all teenagers / elderly are members" may be narrowed down to say "club that includes all and only teenagers / elderly as members", so as to make the two types of clubs contrary to each other. Thus, the results of opposition inferences can help us determine in what occasion narrowing has occurred in sentences with complex quantifier structures in antonymy contexts.

In this paper we will develop a new theory on opposition inferences using some notions and results of modern GQT. It will turn out that the classical inferences involving the contradictory, contrary and subcontrary relations are just special cases of the opposition inferences studied under this new theory. The organization of the rest of this paper is as follows. Section 2 provides an account of the basic notions used in this paper. Sections 3 and 4 discuss the opposition properties of monadic generalized quantifiers (GQs). Section 5 discusses the opposition properties of iterated GQs. Section 6 discusses the o-sensitivities of GQs that are viewed as sets and arguments of other GQs / logical operators. Section 7 compares and contrasts opposition inferences and monotonicity inferences. Section 8 concludes the paper.

2. Basic Notions

Since many notions of the opposition inferences studied in this paper are generalizations of the corresponding notions of monotonicity inferences studied under GQT, we will first give an introduction and review of GQT and monotonicity inferences.

A GQ can be seen as a second-order predicate with first-order predicates as arguments. Different GQs may differ in terms of the number and arities of their arguments, where "arities" refer to the number of arguments of the first-order predicates. Lindström (1966) devised a special notation to denote the type of a GQ. The notation takes the form of a sequence of natural numbers $<n_1, \ldots n_k>$ where k is the number of arguments of the GQ and $n_1, \ldots n_k$ are the arities of each argument. If all the numbers in the sequence are 1, the GQ is "monadic". Otherwise, it is "polyadic". Thus, type <1> GQs are GQs with one unary argument. Determiners (or type <1,1> GQs) are GQs with two unary arguments. Structured GQs are GQs with three or more unary arguments.

In this paper, we will basically adopt the notation of Keenan (2002) with some modifications for representing GQs. Under this notation, a sentence with determiner is represented in the following format[2]:

(8) Q(A, B)

where Q is a determiner, A is the nominal or left argument of Q (representing the sentential subject) and B is the predicative or right argument of Q (representing the sentential predicate). For example, the sentence "Every boy sang" will be represented as

(9) EVERY(BOY, SING)

The semantics of a GQ is delineated by its truth condition which is expressed by a

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[2] Note that Keenan (2002)'s original notation is Q(A)(B), where the arguments A and B are put in two brackets. In this paper we put all arguments in one bracket because this "flat" structure is more convenient for defining properties for GQs which are applicable to all sorts of argument structure (such as monotonicities).

set-theoretic proposition[3]. For example, the truth condition of EVERY is as follows:

(10) EVERY(A, B) $\Leftrightarrow$ A $\subseteq$ B

The representation of other types of GQs is similar to that of determiners with some modifications. Since a type <1> GQ, e.g. NOBODY, only requires a predicative argument, the representation of these GQs consists of only one argument. For example, the sentence "Nobody sang" may be represented as:

(11) NOBODY(SING)

Structured GQs are monadic GQs with more than two arguments. According to Beghelli (1994), there are several types of structured GQs. In this paper, we will consider two main types. The first type is the quantity comparative structured GQs, which may occur in different argument structures. In this paper, we will only consider the one with two nominal arguments and one predicative argument (denoted $<1^2,1>$). An example of a sentence with this kind of GQs is "More boys than girls sang", which may be represented as:

(12) (MORE … THAN …)(BOY, GIRL, SING)

The second type is the identity comparative structured GQs, which may only occur in an argument structure with one nominal argument and two predicative arguments (denoted $<1,1^2>$). An example of a sentence with this kind of GQs is "Different girls sang than danced", which may be represented as:

(13) (DIFFERENT … THAN …)(GIRL, SING, DANCE)

If a sentence contains n-ary predicates with n > 1, then it has to be represented by polyadic GQs. According to Keenan (1996) and Keenan and Westerståhl (2011), there are various types of polyadic GQs. In this paper we will only consider iterated polyadic GQs. These are GQs built up from several constituent monadic GQs by an operation called iteration. The precise definition of iteration can be found in any standard work on GQT such as Peters and Westerståhl (2006). For illustration, the following is the representation of the sentence "Every boy sang some song":

(14) EVERY(BOY, {x: SOME(SONG)({y: SING(x, y)})})

The iterated GQ shown above is composed of two constituent monadic GQs, namely EVERY and SOME. Intuitively, the above formula reads "every boy x is such that for some song y, x sang y".

GQT also studies various operations and properties of GQs. Among these operations and properties, outer negation, inner negation, dual, converse, symmetry,

[3] See Appendix 1 for the truth conditions of some commonly used monadic GQs.

contrapositivity and monotonicity will be useful in this paper and their definitions are given below. Let Q be a monadic GQ with n arguments, the outer negation (denoted $\neg Q$), inner negation in the i^{th} argument ($1 \leq i \leq n$) (denoted $Q\neg_i$) and dual in the i^{th} argument ($1 \leq i \leq n$) (denoted Q^{di}) are defined as follows: for all $X_1, \ldots X_n$,

(15) $(\neg Q)(X_1, \ldots X_n) \Leftrightarrow \neg(Q(X_1, \ldots X_n))$

(16) $(Q\neg_i)(X_1, \ldots X_i, \ldots X_n) \Leftrightarrow Q(X_1, \ldots \neg X_i, \ldots X_n)$

(17) $(Q^{di})(X_1, \ldots X_i, \ldots X_n) \Leftrightarrow \neg(Q(X_1, \ldots \neg X_i, \ldots X_n))$

The following three notions are only defined on determiners:

(18) For a determiner Q, its converse (denoted Q^{-1}) is a determiner such that for all A, B, $Q(A, B) \Leftrightarrow Q^{-1}(B, A)$.

(19) A determiner Q is symmetric iff for all A, B, $Q(A, B) \Leftrightarrow Q(B, A)$.

(20) A determiner Q is contrapositive iff for all A, B, $Q(A, B) \Leftrightarrow Q(\neg B, \neg A)$.

We next turn to monotonicity. Note that monotonicity is definable on both GQs and logical operators.

(21) Let Q be a GQ / logical operator with n arguments. Q is increasing in the i^{th} argument ($1 \leq i \leq n$) iff for all $X_1, \ldots X_i, X_i', \ldots X_n$, $X_i \leq X_i' \Rightarrow Q(X_1, \ldots X_i, \ldots X_n) \leq Q(X_1, \ldots X_i', \ldots X_n)$.

(22) Let Q be as above. Q is decreasing in the i^{th} argument ($1 \leq i \leq n$) iff for all $X_1, \ldots X_i, X_i', \ldots X_n$, $X_i \geq X_i' \Rightarrow Q(X_1, \ldots X_i, \ldots X_n) \leq Q(X_1, \ldots X_i', \ldots X_n)$.

Q is called monotonic in the i^{th} argument iff it is either increasing or decreasing in that argument. Otherwise, it is called non-monotonic in the i^{th} argument. In the above definitions, "≤" is a general partial order relation. When used between two sets, it represents the subset relation; when used between two propositions, it represents the entailment relation[4]. Here is an instance of monotonicity inferences:

(23) Every child is jogging. ⇒ Every boy is doing exercises.

Since CHILD ≥ BOY and JOG ≤ DO-EXERCISES, this example illustrates a general fact established in GQT, i.e. EVERY is decreasing in the left argument and increasing in the right argument.

In the definitions above, "≤" and "≥" are just two of the possible binary relations between sets / propositions. If we replace "≤" and "≥" by general binary relations (denoted by R_1 and R_2), and write them in prefix form (i.e. "$R_1[X, Y]$" instead of "X

[4] According to the Boolean Semantics developed by Keenan and Faltz (1985), propositions and various word classes (modeled as sets) in natural language form Boolean algebras. Under this approach, the entailment relation between propositions and the subset relation between sets are indeed the same relation, namely the domination relation (represented by "≤") of a Boolean algebra. Note that for convenience in this paper, sometimes we will use the general symbol "≤", and sometimes we will use the particular symbols "⊆" for sets and "⇒" for propositions.

R_1 Y"), then we obtain the following more general definition:

(24) Let Q be a GQ / logical operator with n arguments. Q is $R_1 \rightarrow R_2$ in the i^{th} argument ($1 \leq i \leq n$) iff for all $X_1, \ldots X_i, X_i', \ldots X_n$, $R_1[X_i, X_i'] \Rightarrow R_2[Q(X_1, \ldots X_i, \ldots X_n), Q(X_1, \ldots X_i', \ldots X_n)]$.

Under this definition, the increasing and decreasing monotonicities may be represented by "≤→≤" (or equivalently "≥→≥") and "≥→≤" (or equivalently "≤→≥"), respectively.

As mentioned above, "≤" and "≥" are two binary relations between sets / propositions. In fact, these two relations can be seen as combinations of even more basic binary relations between sets / propositions. There are seven basic binary relations: equivalence, subalternation, superalternation, contradictoriness, contrariety, subcontrariety and loose relationship. The names of these seven relations are adapted from Brown (1984). They are defined as follows. Let X and X' be sets / propositions (in what follows, "=" represents the equality relation between sets or equivalence relation between propositions; "<" represents the proper subset relation between sets or unilateral entailment relation between propositions; "¬" represents the complement of sets or negation of propositions).

(25) (a) X is equivalent with X' iff $X = X'$;

(b) X is subalternate to X' iff $X < X'$;

(c) X is superalternate to X' iff $X > X'$;

(d) X is contradictory with X' iff $X = \neg X'$;

(e) X is contrary to X' iff $X < \neg X'$;

(f) X is subcontrary to X' iff $\neg X < X'$;

(g) X is loosely related to X' iff X and X' do not satisfy (a) – (f) above.

Now "≤" and "≥" are just two possible disjunctions of these seven binary relations, i.e. ≤ = subalternate or equivalent; ≥ = superalternate or equivalent. In this paper we will study two other possible disjunctions of these relations. They are "contrary or contradictory" (denoted by "CC" for short) and "subcontrary or contradictory" (denoted by "SC" for short), which can be defined using the definitions in (25):

(26) $CC[X, X'] \Leftrightarrow X \leq \neg X'$; $SC[X, X'] \Leftrightarrow \neg X \leq X'$

From the above definitions and the contrapositive law, it is easily seen that

(27) $CC[X, X'] \Leftrightarrow CC[X', X]$; $SC[X, X'] \Leftrightarrow SC[X', X]$

(28) $CC[X, X'] \Leftrightarrow SC[\neg X, \neg X']$

When X and X' are propositions, we can also interpret the CC and SC relations alternatively as follows: two propositions satisfy the CC relation iff they cannot be both true, and they satisfy the SC relation iff they cannot be both false. For example, we have CC[TEENAGER, ELDERLY] and SC[AGED-OVER-50, AGED-BELOW-51] because an individual cannot be a teenager and elderly at the same time, whereas an individual must be either aged over 50 or aged below 51.

By instantiating R_1 and R_2 in definition (24) as CC and SC, we then have four possible properties of Q: "CC→CC", "CC→SC", "SC→CC" and "SC→SC". These four properties will henceforth be called "opposition properties" (OPs). We say that Q is "o(pposition)-sensitive" in a certain argument iff it possesses any of the aforesaid four OPs in that argument. Otherwise, it is o-insensitive in that argument[5]. Moreover, we will denote the sets of GQs possessing or not possessing a certain OP in a certain argument by placing a "+" or "–" sign on the left-hand side (representing the left or nominal argument(s)) and right-hand side (representing the right or predicative argument(s)) of the name of the OP. For example, –CC→CC+ denotes the set of those GQs that are CC→CC in the right but not left argument.

For example, it will be shown below that EVERY ∈ –CC→CC+. An illustration of the fact that EVERY is CC→CC in the right argument can be found in (4) above. In that example, the two right arguments "is elderly" and "is a teenager" satisfy the CC relation and the two propositions "Every member is elderly" and "Every member is a teenager" also satisfy the CC relation. To illustrate that EVERY is **NOT** CC→CC in the left argument, we note that although "elderly" and "teenager" satisfy the CC relation, the two propositions "Every elderly is member of this club" and "Every teenager is member of this club" do not satisfy the CC relation (i.e. they can be both true), because we can imagine a club that includes every elderly and teenager (in a suitable universe) as its member.

Note that sometimes it is useful to view the entailment and equivalence relations between quantified statements as set-theoretic relations between GQs. To this end, we first reinterpret GQs as sets. Using determiners as an example (note that the following definitions can be generalized to other types of GQs), we can interpret any determiner as a second-order set of ordered pairs of sets. For example, we have

(29) EVERY = $\{\langle A, B\rangle: A \subseteq B\}$

Based on the above reinterpretation, we can then define the following set-theoretic relations between determiners. Let Q, Q' be determiners.

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[5] Note that we may talk about "o-sensitivity" on either the GQ level or the argument level. Using EVERY(A, B) as an example, on the GQ level, we say that the o-sensitivity of EVERY is, subject to certain conditions, SC→CC in the left argument and CC→CC in the right argument; on the argument level, we say that the o-sensitivities of the arguments A and B under EVERY are SC→CC and CC→CC, respectively.

(30) $Q \subseteq Q'$ iff with respect to every model and every A, B, $Q(A, B) \Rightarrow Q'(A, B)$.

(31) $Q = Q'$ iff with respect to every model and every A, B, $Q(A, B) \Leftrightarrow Q'(A, B)$.

For illustration, using the truth conditions of GQs, one can easily derive

(32) (EXACTLY r OF) $\subseteq$ (AT LEAST r OF)

(33) MOST = (MORE THAN 1/2 OF)

In Classical Logic, we have the subalternate relation "Every A is B $\Rightarrow$ Some A is B". But under the modern interpretation of EVERY, this relation is valid only under the condition that A is non-empty. Now, under the above reinterpretation, we can express this conditionally valid relation as

(34) Within the domain $\{<A, B>: A \neq \varnothing\}$, EVERY $\subseteq$ SOME.

3. O-Sensitivities of Monadic GQs (Single OP)

Having defined the necessary notions, our next task is to derive rules for determining the o-sensitivities of monadic GQs. We first state and prove the following general theorems:

Theorem 1 Let Q be a GQ with n arguments. Then with respect to the i^{th} argument, Q possesses a certain OP iff each of $\neg Q$, $Q\neg_i$ and Q^{di} possesses a different OP according to the following table:

Q	**¬Q**	**$Q\neg_i$**	**Q^{di}**
CC→CC	CC→SC	SC→CC	SC→SC
CC→SC	CC→CC	SC→SC	SC→CC
SC→CC	SC→SC	CC→CC	CC→SC
SC→SC	SC→CC	CC→SC	CC→CC

Proof: Here we only prove the first row of the table. The remaining rows can be proved similarly. By definitions (24) and (26), Q is CC→CC in the i^{th} argument iff

(35) $CC[X_i, X_i'] \Rightarrow Q(X_1, \ldots X_i, \ldots X_n) \leq \neg Q(X_1, \ldots X_i', \ldots X_n)$

Now (35) is equivalent to

(36) $CC[X_i, X_i'] \Rightarrow \neg(\neg Q)(X_1, \ldots X_i, \ldots X_n) \leq (\neg Q)(X_1, \ldots X_i', \ldots X_n)$

Substituting the arbitrary X_i and X_i' by their negations and using (28) and the definitions of inner negation and dual, (35) and (36) can be rewritten as

(37) $SC[X_i, X_i'] \Rightarrow (Q\neg_i)(X_1, \ldots X_i, \ldots X_n) \leq \neg(Q\neg_i)(X_1, \ldots X_i', \ldots X_n)$

(38) $SC[X_i, X_i'] \Rightarrow \neg(Q^{di})(X_1, \ldots X_i, \ldots X_n) \leq (Q^{di})(X_1, \ldots X_i', \ldots X_n)$

From (36) – (38), we may conclude that ¬Q is CC→SC, $Q\neg_i$ is SC→CC and Q^{di} is SC→SC in the i^{th} argument. □

Theorem 2 Let Q_1 and Q_2 be GQs of the same type with $Q_1 \subseteq Q_2$.

(a) If Q_2 is CC→CC (SC→CC) in the i^{th} argument, so is Q_1.

(b) If Q_1 is CC→SC (SC→SC) in the i^{th} argument, so is Q_2.

Proof:

(a) Suppose CC[X_i, X_i'] and $\|Q_1(X_1, \dots X_i, \dots X_n)\| = 1$ [6], then since $Q_1 \subseteq Q_2$, we have $\|Q_2(X_1, \dots X_i, \dots X_n)\| = 1$. But since Q_2 is CC→CC in the i^{th} argument, we have $\|Q_2(X_1, \dots X_i', \dots X_n)\| = 0$. By $Q_1 \subseteq Q_2$ again, we have $\|Q_1(X_1, \dots X_i', \dots X_n)\| = 0$. We have thus proved that CC[$Q_1(X_1, \dots X_i, \dots X_n)$, $Q_1(X_1, \dots X_i', \dots X_n)$] i.e. Q_1 is CC→CC in the i^{th} argument. The proof for the case SC→CC is exactly the same.

(b) Suppose CC[X_i, X_i'] and $\|Q_2(X_1, \dots X_i, \dots X_n)\| = 0$, then since $Q_1 \subseteq Q_2$, we have $\|Q_1(X_1, \dots X_i, \dots X_n)\| = 0$. But since Q_1 is CC→SC in the i^{th} argument, we have $\|Q_1(X_1, \dots X_i', \dots X_n)\| = 1$. By $Q_1 \subseteq Q_2$ again, we have $\|Q_2(X_1, \dots X_i', \dots X_n)\| = 1$. We have thus proved that SC[$Q_2(X_1, \dots X_i, \dots X_n)$, $Q_2(X_1, \dots X_i', \dots X_n)$], i.e. Q_2 is CC→SC in the i^{th} argument. The proof for the case SC→SC is exactly the same. □

Theorem 3 Let Q be a determiner and Π be one of the four OPs.

(a) Q is Π in one argument iff Q^{-1} is Π in the other argument.

(b) If Q is symmetric, then Q is Π in both or neither of its arguments.

Proof:

(a) Here we only prove the case when Π = CC→CC. The proofs of the other cases are similar. Suppose CC[X_1, X_1'] and Q is CC→CC in the left argument. Then we have $Q(X_1, X_2) \Rightarrow \neg Q(X_1', X_2)$, which may be rewritten as $(Q^{-1})(X_2, X_1) \Rightarrow \neg(Q^{-1})(X_2, X_1')$. This shows that Q^{-1} is CC→CC in the right argument. Similarly, we can prove that Q is CC→CC in the right argument iff Q^{-1} is CC→CC in the left argument.

(b) Let Q be symmetric, then by (18) and (19), Q is self-converse, i.e. $Q = Q^{-1}$. So by (a), Q is Π in one argument iff it is Π in the other argument, i.e. Q is Π in both or neither of its arguments. □

Theorem 4 Let Q be a contrapositive determiner. Then Q is CC→CC in an argument iff it is SC→CC in the other argument. Q is CC→SC in an argument iff it is SC→SC in the other argument.

Proof: Suppose Q is CC→CC in the right argument and SC[A, A'], which by (28) is equivalent to CC[¬A, ¬A']. Let $\|Q(A, B)\| = 1$. By contrapositivity of Q, this is equivalent to $\|Q(\neg B, \neg A)\| = 1$. But then we must have $\|Q(\neg B, \neg A')\| = 0$. By

[6] In this paper, we use $\|p\|$ to denote the truth value of a proposition p.

contrapositivity of Q again, this is in turn equivalent to ‖Q(A', B)‖ = 0. We have thus proved that SC[A, A'] ⇒ CC[Q(A, B), Q(A', B)], i.e. Q is SC→CC in the left argument. Similarly, we can prove that if Q is SC→CC in the left argument, then Q is CC→CC in the right argument. The proofs for the cases that Q is CC→CC in the left argument and CC→SC in either argument follow the same line. □

The above are general principles. We also need the following particular result:

Theorem 5 (AT LEAST r OF) (1/2 < r < 1) is CC→CC in the right argument. (MORE THAN r OF) (1/2 ≤ r < 1) is CC→CC in the right argument. (BETWEEN q AND r OF) (0 < q ≤ r < 1) is not CC→CC in the left argument.

Proof: We first prove (AT LEAST r OF) (1/2 < r < 1) is CC→CC in the right argument. Let ‖(AT LEAST r OF)(A, B)‖ = 1 and CC[B, B']. Then by (26), B ⊆ ¬B'. Since (AT LEAST r OF) is increasing in the right argument by a standard result in GQT, we have ‖(AT LEAST r OF)(A, ¬B')‖ = 1, which is equivalent to ‖(AT MOST 1 – r OF)(A, B')‖ = 1. Since 1/2 < r < 1, this entails ‖(LESS THAN r OF)(A, B')‖ = 1, which is equivalent to ‖¬(AT LEAST r OF)(A, B')‖ = 1. We have thus shown that CC[(AT LEAST r OF)(A, B), (AT LEAST r OF)(A, B')]. Thus, (AT LEAST r OF) is CC→CC in the right argument. The fact that (MORE THAN r OF) (1/2 ≤ r < 1) is CC→CC in the right argument can be proved similarly.

Next we show that (BETWEEN q AND r OF) (0 < q ≤ r < 1) is not CC→CC in the left argument by devising a method for constructing counterexamples for any 0 < q ≤ r < 1. Choose any rational number x/y such that q ≤ x/y ≤ r. Construct two finite sets A and A' such that |A| = |A'| = y and A ∩ A' = ∅. Choose a subset X of A and a subset X' of A' such that |X| = |X'| = x. Then set B = X ∪ X'. It is easy to check that with these predicates, we have CC[A, A'] and ‖(BETWEEN q AND r OF)(A, B)‖ = ‖(BETWEEN q AND r OF)(A', B)‖ = 1. In other words, we do not have CC[(BETWEEN q AND r OF)(A, B), (BETWEEN q AND r OF)(A', B)], thus completing the proof. □

Based on this particular result and the general theorems above, we can then determine the o-sensitivities of the proportional determiners. For example, let 1/2 < r < 1, then since (EXACTLY r OF) ⊆ (AT LEAST r OF) and (EXACTLY r OF) = (BETWEEN r AND r OF), from Theorems 5 and 2, we have (EXACTLY r OF), (AT LEAST r OF) ∈ –CC→CC+ for 1/2 < r < 1. Next let 1/2 ≤ r < 1. By Theorem 5, we already know that (MORE THAN r OF) is CC→CC in the right argument. Moreover, since (EXACTLY r + ε OF) ⊆ (MORE THAN r OF), where ε represents any small positive number such that $1/2 < r + \varepsilon < 1$, by Theorem 2, we know that (MORE THAN r OF) is not CC→CC in the left argument. Thus, we have (MORE THAN r OF) ∈ –CC→CC+ for $1/2 \le r < 1$.

We next consider the classical determiner SOME. First we observe that there is the relation (AT LEAST r OF) ($0 < r \le 1/2$) ⊆ SOME, on condition that A ≠ ∅[7]. Now it can be shown that (AT LEAST r OF) is SC→SC in the right argument for $0 < r \le 1/2$[8]. So by Theorem 2(b), we know that SOME is SC→SC in the right argument on condition that A ≠ ∅. Note that this condition is essential because when A = ∅, ‖SOME(∅, B)‖ = 0 for any B, and so we can never have SC[B, B'] ⇒ SC[SOME(∅, B), SOME(∅, B')]. As for the left argument of SOME, by symmetry of SOME and Theorem 3(b), we know that SOME is SC→SC in the left argument subject to certain condition. One can easily verify that this condition is B ≠ ∅. The above fact will be represented succinctly by SOME ∈ +SC→SC+ (B ≠ ∅; A ≠ ∅)[9].

Since EVERY is the right dual of SOME, by Theorem 1, we may conclude that EVERY is CC→CC in the right argument subject to certain condition. One can easily verify that this condition is A ≠ ∅. Since EVERY is contrapositive according to Zuber (2007), by Theorem 4, EVERY is SC→CC in the left argument subject to B ≠ U where U represents the universe. Again this condition is essential because when B = U, ‖EVERY(A, U)‖ = 1 for any A, and so we can never have SC[A, A'] ⇒ CC[EVERY(A, U), EVERY(A', U)]. The above fact will be represented succinctly by EVERY ∈ +SC→CC– ∩ –CC→CC+ (B ≠ U; A ≠ ∅)[10]. The o-sensitivities of some other determiners can be determined in a similar way.

Regarding the absolute determiners and quantity comparative structured GQs, we have the following negative results:

Theorem 6 Every absolute numerical determiner listed in Appendix 1, i.e. (AT LEAST n) ($n > 1$), (AT MOST n) ($n > 0$), (MORE THAN n) ($n > 0$), (FEWER THAN n) ($n > 1$), (EXACTLY n) ($n > 0$), (BETWEEN m AND n) ($0 < m < n$), (ALL EXCEPT n) ($n > 0$), is o-insensitive in all

7 By Appendix 1, when A = ∅, then for any B, (AT LEAST r OF)(A, B) is trivially true while SOME(A, B) is trivially false, and the relation (AT LEAST r OF) ⊆ SOME cannot hold.

8 By Theorem 5, (MORE THAN r OF) ($1/2 \le r < 1$) is CC→CC in the right argument. Since the right dual of (MORE THAN r OF) is (AT LEAST 1 – r OF), by Theorem 1, (AT LEAST 1 – r OF) ($0 < 1 - r \le 1/2$) is SC→SC in the right argument. Replacing the arbitrary 1 – r by r, we obtain the result: (AT LEAST r OF) ($0 < r \le 1/2$) is SC→SC in the right argument.

9 The conditions B ≠ ∅; A ≠ ∅ are ordered such that the first (second) condition corresponds to the left (right) argument of the determiner.

10 The fact that EVERY is neither SC→CC in the right argument nor CC→CC in the left argument can be established by constructing counterexamples.

arguments.

Proof: According to Table 1 (see below), for a certain OP and a certain argument, there will be a classical determiner or a proportional determiner (with a certain range of r) that does not possess that OP in that argument. Now, an absolute numerical determiner can be made equivalent to a classical or proportional determiner by setting an appropriate cardinality of its left or right argument. Thus, given an absolute numerical determiner Q, a certain OP and a certain argument, we can construct a model in which Q is equivalent to a suitable classical or proportional determiner that does not possess that OP in that argument. This model will also be a counterexample showing that the absolute numerical determiner does not possess that OP in that argument. Thus, every absolute numerical determiner is o-insensitive in all arguments.

For example, to show that (AT LEAST n) ($n > 1$) is not SC→SC in the right argument, we first observe that (AT LEAST n)(A, B) is equivalent to EVERY(A, B) in a model where $|A| = n$. Since EVERY is not SC→SC in the right argument, we then construct a model in which $|A| = n$ and EVERY is not SC→SC in the right argument, such as the following: $U = \{x_1, \dots x_{n+1}\}$, $A = \{x_1, \dots x_n\}$, $B = \{x_1, \dots x_{n-1}, x_{n+1}\}$, $B' = \{x_2, \dots x_n, x_{n+1}\}$. Then we have SC[B, B'], and $\|$EVERY(A, B)$\|$ = $\|$EVERY(A, B')$\|$ = 0, i.e. SC[EVERY(A, B), EVERY(A, B')] is not true in this model. Note that this model is also a counterexample showing that (AT LEAST n) ($n > 1$) is not SC→SC in the right argument. ☐

Theorem 7 Every quantity comparative structured GQ listed in Appendix 1, e.g. (MORE … THAN …), (PROPORTIONALLY MORE … THAN …), etc, is o-insensitive in all arguments.

Proof: To prove this, we may construct counterexamples to show that a certain GQ does not possess a certain OP in a certain argument. Some of these counterexamples are given in Appendix 2. Note that apart from the counterexamples shown there, we can derive many others based on the properties of these GQs. For example, since (EXACTLY AS MANY … AS …) is symmetric with respect to its first and second arguments, i.e. (EXACTLY AS MANY … AS …)(A_1, A_2, B) ⇔ (EXACTLY AS MANY … AS …)(A_2, A_1, B), from a counterexample for the first argument of this GQ, e.g. (e) in Appendix 2, one can immediately derive a counterexample for the second argument by exchanging the roles of A_1 and A_2 in (e). Likewise, since (MORE … THAN …) and (FEWER … THAN …) are converses with respect to the first and second arguments, i.e. (MORE … THAN …)(A_1, A_2, B) ⇔ (FEWER … THAN …)(A_2, A_1, B), from a counterexample for the third argument of (MORE … THAN …), e.g. (c) in Appendix 2, one can immediately derive a counterexample for the third argument of (FEWER … THAN …) by exchanging the roles of A_1 and A_2 in (c). Moreover, we can also make use of the entailment relations between these GQs. For example, (a) in Appendix 2 can be used as a counterexample to show that (MORE … THAN …) is not CC→CC in the first argument, i.e. CC[A_1, A_1'] and ‖(MORE … THAN …)(A_1, A_2, B)‖ = ‖(MORE … THAN …)(A_1', A_2, B)‖ = 1. But since ‖(MORE … THAN …)(A_1, A_2, B)‖ = 1 ⇒ ‖(FEWER … THAN …)(A_1, A_2, B)‖ = 0, one can also use (a) to show that ‖(FEWER … THAN …)(A_1, A_2, B)‖ = ‖(FEWER … THAN …)(A_1', A_2, B)‖ = 0. Thus, (a) also serves as a counterexample showing that (FEWER … THAN …) is not CC→SC in the first argument. In this way, one can construct all the required counterexamples based on those given in Appendix 2. □

Based on the above results, we can derive valid inferences. For example, the following are instances exemplifying the facts that (AT LEAST 3/4 OF) is CC→CC in the right argument and SOME is SC→SC in the right argument on condition that its left argument is non-empty:

(39) CC[“At least 3/4 of the members are teenagers”, “At least 3/4 of the members are elderly”]

(40) (Given that there is some member.)
SC[“Some member is aged over 50”, “Some member is aged below 51”]

4. O-Sensitivities of Monadic GQs (Multiple OPs)

In the previous section, we have only considered the case in which a GQ possesses a single OP in an argument. In this section, we will consider the possibility that a GQ may possess more than one OP in the same argument. To do this, we need to introduce some new notions[11]:

(41) Let Q be a GQ with n arguments. Q is perfectly consistent in the i^{th} argument iff Q(X_1, … X_i, … X_n) ⇒ ¬Q(X_1, … Y, … X_n) where Y is any subset or superset of ¬X_i.

(42) Let Q be a GQ with n arguments. Q is perfectly complete in the i^{th} argument iff ¬Q(X_1, … X_i , … X_n) ⇒ Q(X_1, … Y, … X_n) where Y is any subset or superset of ¬X_i.

We have the following theorem:

Theorem 8 Let Q be a GQ with n arguments. With respect to the i^{th} argument,

[11] The notions of “perfect consistency” and “perfect completeness” are generalizations of the notions of “consistency” and “completeness” in Zwarts (1996).

(a) It is impossible for Q to be CC→CC and CC→SC.

(b) It is impossible for Q to be SC→CC and SC→SC.

(c) Q is CC→CC and SC→SC iff Q is self-dual and increasing.

(d) Q is SC→CC and CC→SC iff Q is self-dual and decreasing.

(e) Q is CC→CC and SC→CC iff Q is perfectly consistent.

(f) Q is CC→SC and SC→SC iff Q is perfectly complete.

Proof:

(a) Suppose Q is CC→CC and CC→SC. Take an arbitrary X_i. For any particular set of $X_1, \ldots X_{i-1}, X_{i+1}, \ldots X_n$, $\|Q(X_1, \ldots X_i, \ldots X_n)\| = 1$ or 0. Let $\|Q(X_1, \ldots X_i, \ldots X_n)\| = 1$. Since $CC[X_i, \neg X_i]$, we have $\|Q(X_1, \ldots \neg X_i, \ldots X_n)\| = 0$. Since $CC[\neg X_i, \varnothing]$, we then have $\|Q(X_1, \ldots \varnothing, \ldots X_n)\| = 1$. But since $CC[\varnothing, X_i]$, we then have $\|Q(X_1, \ldots X_i, \ldots X_n)\| = 0$. Thus, starting from $\|Q(X_1, \ldots X_i, \ldots X_n)\| = 1$, we can derive $\|Q(X_1, \ldots X_i, \ldots X_n)\| = 0$. Similarly, starting from $\|Q(X_1, \ldots X_i, \ldots X_n)\| = 0$, we can derive $\|Q(X_1, \ldots X_i, \ldots X_n)\| = 1$. This contradiction shows that it is impossible for Q to be CC→CC and CC→SC. The proof of (b) follows a similar line of reasoning.

(c) First let Q be CC→CC and SC→SC. Then since $CC[X_i, \neg X_i]$ and $SC[\neg X_i, X_i]$, we have $Q(X_1, \ldots X_i, \ldots X_n) \Rightarrow \neg Q(X_1, \ldots \neg X_i, \ldots X_n)$ and $\neg Q(X_1, \ldots \neg X_i, \ldots X_n) \Rightarrow Q(X_1, \ldots X_i, \ldots X_n)$, respectively. Combining the above, we have $Q(X_1, \ldots X_i, \ldots X_n) \Leftrightarrow \neg Q(X_1, \ldots \neg X_i, \ldots X_n)$. By (17), $Q = Q^{di}$, i.e. Q is self-dual. Next, let $\|Q(X_1, \ldots X_i, \ldots X_n)\| = 1$ and $X_i \subseteq X_i'$. Then since $CC[X_i, \neg X_i']$, we have $\|Q(X_1, \ldots \neg X_i', \ldots X_n)\| = 0$. But since $SC[\neg X_i', X_i']$, we have $\|Q(X_1, \ldots X_i', \ldots X_n)\| = 1$. Thus, Q is increasing.

Next let Q be self-dual and increasing. Suppose $\|Q(X_1, \ldots X_i, \ldots X_n)\| = 1$ and $CC[X_i, X_i']$. Since Q is self-dual, i.e. $Q = Q^{di}$, by (17), we have $\|\neg Q(X_1, \ldots \neg X_i, \ldots X_n)\| = 1$. From $CC[X_i, X_i']$ we have $X_i' \subseteq \neg X_i$. Since Q is increasing, by a standard result in GQT, ¬Q is decreasing and so we have $\|\neg Q(X_1, \ldots X_i', \ldots X_n)\| = 1$, i.e. $\|Q(X_1, \ldots X_i', \ldots X_n)\| = 0$. So Q is CC→CC. Similarly, one can prove that Q is also SC→SC, thus completing the proof of (c). The proof of (d) follows a similar line of reasoning.

(e) First let Q be CC→CC and SC→CC. When Y is a subset of $\neg X_i$, we have $CC[X_i, Y]$. From this we have $Q(X_1, \ldots X_i, \ldots X_n) \Rightarrow \neg Q(X_1, \ldots Y, \ldots X_n)$. When Y is a superset of $\neg X_i$, we have $SC[X_i, Y]$. From this we also have $Q(X_1, \ldots X_i, \ldots X_n) \Rightarrow \neg Q(X_1, \ldots Y, \ldots X_n)$. So by definition (41), Q is perfectly consistent.

Next let Q be perfectly consistent and CC[X_i, X_i']. By (26), $X_i' \subseteq \neg X_i$. So by (41) we must have $Q(X_1, \ldots X_i, \ldots X_n) \Rightarrow \neg Q(X_1, \ldots X_i', \ldots X_n)$. Thus, Q is CC→CC. Similarly, one can prove that Q is also SC→CC, thus completing the proof of (e). The proof of (f) follows a similar line of reasoning. □

From Theorem 8(a) and (b), we can deduce that it is impossible for any GQ to possess three or four of the OPs in the same argument. Therefore we need not consider these cases.

According to Theorem 8(c) and (d), we can find GQs that are both CC→CC and SC→SC, or both SC→CC and CC→SC from among the self-duals identified in Keenan (2003, 2008). For example, since singular proper names are increasing self-duals, they are both CC→CC and SC→SC[12]. Moreover, according to Keenan (2003, 2008), (LESS THAN 1/2 OF) (where the left argument is odd) is a decreasing right self-dual, we thus know that this determiner is both SC→CC and CC→SC in the right argument.

According to Theorem 8(e), we can find GQs that are both CC→CC and SC→CC from among perfectly consistent GQs. But what GQs are these? Among the GQs listed in Appendix 1, the absolute numerical and proportional GQs are in general not perfectly consistent, because their truth conditions are dependent on the cardinalities or proportionalities rather than the member composition of their arguments. For illustration, consider (EXACTLY 3/4 OF). Let me show that this determiner is not perfectly consistent in the right argument by constructing a counterexample. Define finite sets A and B such that $\|(\text{EXACTLY 3/4 OF})(A, B)\| = 1$, i.e. $|A \cap B| / |A| = 0.75$. That means $|A \cap \neg B| / |A| = 0.25$. Also define a subset X of $A \cap B$ such that $|X| / |A| = 0.5$. Since ¬B and X are disjoint, we must have $|A \cap (\neg B \cup X)| / |A| = 0.25 + 0.5 = 0.75$, and so we have $\|(\text{EXACTLY 3/4 OF})(A, \neg B \cup X)\| = 1$. This model shows that (EXACTLY 3/4 OF) is not perfectly consistent in the right argument.

Thus, perfectly consistent GQs can only be found from among GQs that are not essentially numerical or proportional. It turns out that exceptive determiners, such as (ALL … EXCEPT X_1, … AND X_n), whose truth conditions are in the form of a set-theoretic equation are such GQs. Since the truth of a set-theoretic equation depends on the membership composition of the sets involved, changing a set X to a subset or superset of ¬X will in general make a true equation become false. Consider

[12] As for plural proper names, since a plural proper name can be represented by EVERY, for example, "John and Mary" can be represented by EVERY({j, m}), the OP in its unique argument is the same as that of EVERY in the right argument, i.e. a plural proper name is CC→CC in its unique argument.

(ALL … EXCEPT X_1, … AND X_n) as an example. Suppose ‖(ALL … EXCEPT X_1, … AND X_n)(A, B)‖ = 1, then we have $A – B = \{x_1, … x_n\}$. That means $\{x_1, … x_n\}$ is disjoint from B. So provided that $A – \{x_1, … x_n\} \neq \varnothing$, we must have $A – Y \neq \{x_1, … x_n\}$, where Y is any subset or superset of ¬B. Thus we conclude that (ALL … EXCEPT X_1, … AND X_n) is perfectly consistent and is thus both CC→CC and SC→CC in the right argument on condition that $A – \{x_1, … x_n\} \neq \varnothing$. In a similar fashion, one can conclude that (ALL … EXCEPT X_1, … AND X_n) is both CC→CC and SC→CC in the left argument on condition that $B \cup \{x_1, … x_n\} \neq U$.

Apart from the exceptive determiners, the identity comparative structured quantifier (THE SAME … AS …) is also perfectly consistent in its second and third arguments subject to different conditions. In what follows, we will prove that this GQ is perfectly consistent in the second argument on condition that $A – (A \cap B_2) \neq \varnothing$. Let ‖(THE SAME … AS …)(A, B_1, B_2)‖ = 1, i.e.

(43) $A \cap B_1 = A \cap B_2$

This entails

(44) $A \cap B_2 \subseteq B_1$

First suppose $Y \subseteq \neg B_1$. Then we have $A \cap Y \subseteq \neg B_1$. Comparing this with (44), one can see that $A \cap Y \neq A \cap B_2$, i.e. ‖(THE SAME … AS …)(A, Y, B_2)‖ = 0. Next suppose $Y \supseteq \neg B_1$. On the one hand, by (44) above, we have

(45) $(A \cap B_2) \cap \neg B_1 = \varnothing$

On the other hand, we have $(A \cap Y) \cap \neg B_1 = A \cap \neg B_1$ (because $\neg B_1 \subseteq Y$). Now $A \cap \neg B_1 \neq \varnothing$ because otherwise we would have $A \subseteq B_1$, which is equivalent to $A \cap B_1 = A$. But then by (43) we would have $A \cap B_2 = A$, which contradicts the condition that $A – (A \cap B_2) \neq \varnothing$. Based on the above, we have

(46) $(A \cap Y) \cap \neg B_1 \neq \varnothing$

By (45) and (46), we conclude that $A \cap Y \neq A \cap B_2$, i.e. ‖(THE SAME … AS …)(A, Y, B_2)‖ = 0. We have thus shown that (THE SAME … AS …)(A, B_1, B_2) ⇒ ¬(THE SAME … AS …)(A, Y, B_2) where Y is any subset or superset of $\neg B_1$. Hence, (THE SAME … AS …) is perfectly consistent in the second argument.

In a similar fashion, one can prove that (THE SAME … AS …) is perfectly consistent in the third argument on condition that $A – (A \cap B_1) \neq \varnothing$. By Theorem 8(e), (THE SAME … AS …) is both CC→CC and SC→CC in the second and third arguments subject to different conditions. Moreover, one can prove that (THE SAME … AS …) is SC→CC (on condition that $B_1 \neq B_2$) and does not possess other OPs in the first argument (see Appendix 3 for the proof). The above fact can be succinctly represented as (THE SAME … AS …) ∈ –CC→CC++ ∩ +SC→CC++ ($B_1 \neq B_2$; $A – (A \cap B_2) \neq$

$\varnothing$; $A - (A \cap B_1) \neq \varnothing$)[13].

Finally, regarding GQs that are both CC→SC and SC→SC, i.e. perfectly complete GQs, by Theorem 8(e), (f) and Theorem 1 we know that this kind of GQs can be found from the outer negations of perfectly consistent GQs. It turns out that there is only one such GQ listed in Appendix 1. This is the identity comparative structured quantifier (DIFFERENT … THAN …)[14], which is the outer negation of (THE SAME … AS …). Thus, we have (DIFFERENT … THAN …) ∈ –CC→SC++ ∩ +SC→SC++ ($B_1 \neq B_2$; $A - (A \cap B_2) \neq \varnothing$; $A - (A \cap B_1) \neq \varnothing$).

The following exemplifies the fact that (ALL … EXCEPT X_1, … AND X_n) is both CC→CC and SC→CC in the right argument on condition that $A - \{x_1, \ldots x_n\} \neq \varnothing$:

(47) (Given that John and Mary are not the only members.)
CC["All members except John and Mary are teenagers", "All members except John and Mary are elderly"] and CC["All members except John and Mary are aged over 50", "All members except John and Mary are aged below 51"]

The following table summarizes the OPs of the GQs listed in Appendix 1[15]:

Table 1: OPs of some GQs

OP Type	GQ
CC→CC+	plural proper names
CC→CC+ ∩ SC→SC+	singular proper names
–CC→CC+	MOST, (MORE THAN r OF) (1/2 ≤ r < 1), (AT LEAST r OF) (1/2 < r < 1), (EXACTLY r OF) (1/2 < r < 1), (BETWEEN q AND r OF) (1/2 < q < r < 1), (ALL EXCEPT r OF) (0 < r < 1/2)
–CC→SC+	(LESS THAN r OF) (1/2 < r < 1), (AT MOST r OF) (1/2 ≤ r < 1)
+SC→CC+	NO ($B \neq \varnothing$; $A \neq \varnothing$)
–SC→CC+	(LESS THAN r OF) (0 < r ≤ 1/2), (AT MOST r OF) (0 < r < 1/2), (EXACTLY r OF) (0 < r < 1/2), (BETWEEN q AND r OF) (0 <

[13] Since (THE SAME … AS …) has one nominal and two predicative arguments, there is one "+/–" sign on the left and two "+/–" signs on the right of the names of the OPs.
[14] According to Beghelli (1994), there is a "weak" version and a "strong" version of (DIFFERENT … THAN …). This paper only considers the "weak" version.
[15] Only those OP types with at least one o-sensitive argument are listed here. Thus, GQs listed in Appendix 1 that are not listed below are understood to be o-insensitive in all arguments. For example, (EXACTLY n) ∈ –CC→CC– ∩ –CC→SC– ∩ –SC→CC– ∩ –SC→SC–.

	$q < r < 1/2$), (ALL EXCEPT r OF) ($1/2 < r < 1$)
+SC→SC+	SOME ($B \neq \varnothing$; $A \neq \varnothing$)
–SC→SC+	(MORE THAN r OF) ($0 < r < 1/2$), (AT LEAST r OF) ($0 < r \leq 1/2$)
–CC→CC+ ∩ –SC→SC+	(MORE THAN 1/2 OF) (|A| is odd)
–SC→CC+ ∩ –CC→SC+	(LESS THAN 1/2 OF) (|A| is odd)
+CC→CC+ ∩ +SC→CC+	(ALL … EXCEPT X_1, … AND X_n) ($B \cup \{x_1, \ldots x_n\} \neq U$; $A - \{x_1, \ldots x_n\} \neq \varnothing$), (NO … EXCEPT X_1, … AND X_n) ($B - \{x_1, \ldots x_n\} \neq \varnothing$; $A - \{x_1, \ldots x_n\} \neq \varnothing$)
+SC→CC– ∩ –CC→CC+	EVERY ($B \neq U$; $A \neq \varnothing$)
+SC→SC– ∩ –CC→SC+	(NOT EVERY) ($B \neq U$; $A \neq \varnothing$)
–CC→CC++ ∩ +SC→CC++	(THE SAME … AS …) ($B_1 \neq B_2$; $A - (A \cap B_2) \neq \varnothing$; $A - (A \cap B_1) \neq \varnothing$)
–CC→SC++ ∩ +SC→SC++	(DIFFERENT … THAN …) ($B_1 \neq B_2$; $A - (A \cap B_2) \neq \varnothing$; $A - (A \cap B_1) \neq \varnothing$)

5. O-Sensitivities of Iterated GQs

In the study of monotonicity inferences, one can determine the monotonicities of an iterated GQ based on the monotonicities of its constituent monadic GQs. This has been studied by van Benthem (1986), Sanchez Valencia (1991), van Eijck (2007), etc. For example, consider the following iterated GQ:

(48) (AT MOST 1/2 OF)(A_1, $\{x_1$: NO(A_2, $\{x_2$: $B(x_1, x_2)\})\}$)

Since A_2 is within the left argument of NO and the right argument of (AT MOST 1/2 OF), and both NO and (AT MOST 1/2 OF) are decreasing in both of their arguments, by a standard result in GQT, we know that A_2 is increasing.

In parallel to monotonicity inferences, we also hope to formulate a principle that determines the o-sensitivity of an iterated GQ based on those of its constituent monadic GQs. Before stating the principle, we first need a definition:

(49) Let X be a predicate under an iterated GQ. Suppose X is within the i_k^{th} argument of Q_k ($1 \leq k \leq n$), i_{k-1}^{th} argument of Q_{k-1}, … i_1^{th} argument of Q_1, where Q_k, Q_{k-1}, … Q_1 are constituent monadic GQs of the iterated GQ ordered from the innermost to the outermost layers. Then X has an OP-chain

<R_k, R_{k-1}, … R_0>, where each of R_k, R_{k-1}, … R_0 is one of {CC, SC}, iff Q_k is $R_k \rightarrow R_{k-1}$ in the i_k^{th} argument, Q_{k-1} is $R_{k-1} \rightarrow R_{k-2}$ in the i_{k-1}^{th} argument, … Q_1 is $R_1 \rightarrow R_0$ in the i_1^{th} argument.

For instance, in the argument structure of the iterated GQ given in (48), A_2 is within the left argument of NO and the right argument of (AT MOST 1/2 OF). Since, according to Table 1, NO is SC→CC in the left argument on condition that its right argument is non-empty and (AT MOST 1/2 OF) is CC→SC in the right argument, A_2 has an OP-chain <SC, CC, SC> on condition that $\{x_2: B(x_1, x_2)\} \neq \varnothing$. One can also easily check that B has an OP-chain <SC, CC, SC> on condition that $A_2 \neq \varnothing$ while A_1 has no OP-chain.

We now consider the case in which a predicate is not within the scope of any GQ / logical operator. Let X and X' be predicates. A predicate not within the scope of any GQ / logical operator can be thought to be within the scope of the identity operator ι defined by ι(X) = X for any predicate X. Now it is obvious that if CC[X, X'], then CC[ι(X), ι(X')]. The same is true for the case of SC[X, X']. Thus, ι is CC→CC and SC→SC in its argument. We conclude that a predicate not within the scope of any GQ / logical operator is CC→CC and SC→SC.

We next consider the case in which a predicate is within the argument of some GQ / logical operator. We need the following theorems:

Theorem 9 Let P be a predicate. Then $\{x: \neg P(x)\} = \neg\{x: P(x)\}$.

Proof: For any member x of U, $x \in \{x: \neg P(x)\} \Leftrightarrow \|\neg P(x)\| = 1 \Leftrightarrow \|P(x)\| = 0 \Leftrightarrow x \notin \{x: P(x)\} \Leftrightarrow x \in \neg\{x: P(x)\}$. Thus $\{x: \neg P(x)\} = \neg\{x: P(x)\}$. □

Theorem 10 Let P and P' be n-ary predicates and R be one of {CC, SC}, then $R[P_1, P_2] \Rightarrow R[\{x_i: P(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)\}, \{x_i: P'(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)\}]$ for any $1 \leq i \leq n$ and any particular set of $x_1, \ldots x_{i-1}, x_{i+1}, \ldots x_n$.

Proof: Here we only prove the case in which R = CC. The case in which R = SC is similar. Suppose CC[P, P']. By (26), this is equivalent to $P \subseteq \neg P'$. Then for any set of $x_1, \ldots x_{i-1}, x_{i+1}, \ldots x_n$, and any particular x_i, we have $P(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n) \Rightarrow \neg P'(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)$, and so we have $\{x_i: P(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)\} \subseteq \{x_i: \neg P'(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)\}$. Now by Theorem 9, $\{x_i: \neg P'(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)\} = \neg\{x_i: P'(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)\}$. Thus, by (26) again, we have $CC[\{x_i: P(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)\}, \{x_i: P'(x_1, \ldots x_{i-1}, x_i, x_{i+1}, \ldots x_n)\}]$. □

With the above theorems, we can then conclude that a predicate is $R_n \rightarrow R_0$ if it has an OP-chain $<R_n, \dots R_0>$. In what follows, we will provide a proof sketch for this important result. Suppose we have an iterated GQ in the following form:

(50) $Q_1(A_1, \{x_1: \dots Q_n(A_n, \{x_n: B(x_1, \dots x_n)\}) \dots \})$

We focus on the o-sensitivity of B (the o-sensitivities of other predicates can be similarly treated). Let B have an OP-chain $<R_n, R_{n-1}, \dots R_0>$ and $R_n[B, B']$. By Theorem 10, we have $R_n[\{x_n: B(x_1, \dots x_n)\}, \{x_n: B'(x_1, \dots x_n)\}]$ for any $x_1, \dots x_{n-1}$. Moreover, by definition (49), Q_n is $R_n \rightarrow R_{n-1}$ in $\{x_n: B(x_1, \dots x_n)\}$, and so we have $R_{n-1}[Q_n(A_n, \{x_n: B(x_1, \dots x_n)\}), Q_n(A_n, \{x_n: B'(x_1, \dots x_n)\})]$. The above reasoning can be seen as a kind of "upward derivation": from the R_n relation at the B-level, we derive the R_{n-1} relation at the Q_n-level. Now the process of determining the o-sensitivities of B is essentially a repetition of this upward derivation, i.e. from the B-level to the Q_n-level, and then to the Q_{n-1} level, and then … After n rounds of derivation, we will finally derive the R_0 relation at the Q_1 level. The net effect is thus $R_n[B, B'] \Rightarrow R_0[Q_1(A_1, \{x_1: \dots Q_n(A_n, \{x_n: B(x_1, \dots x_n)\}) \dots \}), Q_1(A_1, \{x_1: \dots Q_n(A_n, \{x_n: B'(x_1, \dots x_n)\}) \dots \})]$, showing that B is $R_n \rightarrow R_0$.

The above derivation relies on the condition that B has an OP-chain. This condition does not hold either when at least one of $Q_1, \dots Q_n$ is o-insensitive, or when the OPs possessed by $Q_1, \dots Q_n$ do not form a chain. In either case, the absence of the OP-chain blocks the upward derivation. With the above discussion and results, we can now formulate the following principle:

(51) A predicate not within the scope of any GQ / logical operator is CC→CC and SC→SC. A predicate is $R_k \rightarrow R_0$ iff it has an OP-chain $<R_k, \dots R_0>$.

We next use the above principle to determine the o-sensitivities of predicates in a quantified statement with an iterated GQ. Consider (48) renumbered as (52) below:

(52) (AT MOST 1/2 OF)$(A_1, \{x_1: NO(A_2, \{x_2: B(x_1, x_2)\})\})$

In the above, it has been found that A_1 has no OP-chain whereas A_2 and B both have the OP-chain <SC, CC, SC> subject to different conditions. Thus, according to (51), we know that A_1 is o-insensitive, A_2 is SC→SC on condition that $\{x_2: B(x_1, x_2)\} \neq \emptyset$ and B is SC→SC on condition that $A_2 \neq \emptyset$. From the above result, we can derive the following valid inference (by letting A_1 = CLUB, A_2 = AGED-OVER-50, A_2' = AGED-BELOW-51, B = ADMIT-AS-MEMBERS):

(53) (Given that every club admits somebody as member.)
SC["At most 1/2 of the clubs admit nobody aged over 50 as member", "At most 1/2 of the clubs admit nobody aged below 51 as member"]

The derivation process of the principle in (51) is not exclusively valid for (50). In fact, (51) is also applicable to iterated GQs in a form different than (50). For example, consider the following:

(54) NO(A ∩ {x: SOME(B, {y: C(x, y)})}, D)

The above iterated GQ represents a quantified statement whose subject contains a relative clause which is another quantified statement. The intersection operator "∩" shows that relative clauses function like intersecting adjectives. Let's determine the OP of the predicate B. Since B is within the left arguments of SOME and NO, which are SC→SC and SC→CC, respectively, both in its left argument on condition that its right argument is non-empty, B has an OP-chain <SC, SC, CC>. By (51), B is SC→CC subject to the condition that {y: C(x, y)} ≠ ∅ ∧ D ≠ ∅. From the above result, we can derive the following valid inference (by letting A = COMPANY, B = AGED-OVER-50, B' = AGED-BELOW-51, C = EMPLOY, D = GO-BANKRUPT):

(55) (Given that every company employed somebody and some company went bankrupt.)
CC["No company employing somebody aged over 50 went bankrupt", "No company employing somebody aged below 51 went bankrupt"]

Note that monotonicity inferences of iterated GQs are governed by the same principle as opposition inferences. If we represent increasing monotonicity as ≤→≤ or ≥→≥ and decreasing monotonicity as ≥→≤ or ≤→≥, then we can define an analogous notion of "MON-chain" by replacing {CC, SC} with {≤, ≥} in (49) and modify the principle in (51) as:

(56) A predicate not within the scope of any GQ / logical operator is increasing. A predicate is $R_k \to R_0$ iff it has a MON-chain <R_k, … R_0>.

This principle can then be used to determine the monotonicities of predicates under an iterated GQ.

For illustration, consider (48) renumbered as (57) below:

(57) (AT MOST 1/2 OF)(A_1, {x_1: NO(A_2, {x_2: B(x_1, x_2)})})

Let's determine the monotonicity of A_2. Now A_2 is within the left argument of NO and the right argument of (AT MOST 1/2 OF), and both NO and (AT MOST 1/2 OF) are decreasing in both of their arguments. Thus, A_2 has a MON-chain <≤, ≥, ≤> (or equivalently, <≥, ≤, ≥>)[16]. According to (56), we know that A_2 is ≤→≤ (or equivalently ≥→≥), i.e. increasing. This result is in accord with that obtained at the

[16] Note that since both increasing and decreasing monotonicities have two possible representations, the determination of MON-chains is more complicated than that of OP-chains. We may need to consider all possible representations of the monotonicities involved in order to determine whether a predicate has a MON-chain.

beginning of this section.

6. GQs as Sets and Arguments

As pointed out in Section 2, GQs can be seen as second-order sets and so they may enter into the CC and / or SC relations with other GQs. For example, it is easy to see that the following hold:

(58) Within the domain $\{<A, B>: A \neq \varnothing\}$, CC[EVERY, NO] $\wedge$ SC[SOME, (NOT EVERY)]

(59) CC[SOME, NO] $\wedge$ SC[SOME, NO]

Viewed as sets, a GQ not within the scope of any GQ / logical operator is both CC→CC and SC→SC according to (51). Based on (58) and (59), we can then derive the following contrary, subcontrary and contradictory relations in Classical Logic:

(60) Given that $A \neq \varnothing$, EVERY(A, B) $\Rightarrow$ ¬NO(A, B)

(61) Given that $A \neq \varnothing$, ¬SOME(A, B) $\Rightarrow$ (NOT EVERY)(A, B)

(62) SOME(A, B) $\Leftrightarrow$ ¬NO(A, B)

Thus, the classical contrary, subcontrary and contradictory relations can be seen as special examples of the opposition inferences studied in this paper.

Moreover, GQs viewed as sets may also act as arguments of other GQs / logical operators. For instance, consider (48) renumbered as (63) below:

(63) (AT MOST 1/2 OF)(A_1, $\{x_1$: NO(A_2, $\{x_2$: $B(x_1, x_2)\}$)$\}$)

Since NO is within the right argument of (AT MOST 1/2 OF), which is CC→SC in the right argument, we know that NO is CC→SC in (63). Now by (58), we have CC[NO, EVERY] on condition that the left arguments of NO and EVERY are both non-empty. We can thus derive the following valid inference (by letting A_1 = CLUB, A_2 = LOGICIAN, B = ADMIT-AS-MEMBERS):

(64) (Given that there is some logician.)
SC["At most 1/2 of the clubs admit no logician as members", "At most 1/2 of the clubs admit every logician as members"]

Particularly, since the negation operator "¬" is a logical operator, we may also discuss the o-sensitivity of this operator. We have the following theorem:

Theorem 11 "¬" is CC→SC and SC→CC and does not possess other OPs.

Proof: Suppose CC[X, X']. Then by (28), we have SC[¬X, ¬X'], thus showing that "¬" is CC→SC. We next show that "¬" is not CC→CC by constructing a counterexample. Let X and 0 be a non-trivial member and the zero member of a

Boolean algebra, respectively. Then we have CC[X, 0] (because $X \leq \neg 0$ for any X) but not CC[¬X, ¬0] (because $\neg X > 0$ for any non-trivial X). So "¬" cannot be CC→CC. The proofs that "¬" is SC→CC but not SC→SC are similar. □

With this theorem, we can determine the o-sensitivities of predicates within the scope of "¬". Consider the argument A_2 in the following iterated GQ:

(65) (LESS THAN 1/2 OF)(A_1, {x_1: SOME(¬A_2, {x_2: B(x_1, x_2)})})

Since A_2 is within the scope of "¬", the left argument of SOME and the right argument of (LESS THAN 1/2 OF), it has an OP-chain <CC, SC, SC, CC>. Therefore, A_2 is CC→CC on condition that {x_2: B(x_1, x_2)} ≠ ∅. Based on this result, we can derive the following valid inference (by letting A_1 = CLUB, A_2 = TEENAGER, A_2' = ELDERLY, B = ADMIT-AS-MEMBERS):

(66) (Given that every club admits somebody as member.)
CC["Less than 1/2 of the clubs admit some non-teenager as member", "Less than 1/2 of the clubs admit some non-elderly as member"]

7. Comparison with Monotonicity Inferences

From the discussion above, one can see that there is a parallel relation between opposition inferences and monotonicity inferences in terms of the basic notions and principles governing the inferential patterns of these two types of inferences. More importantly, the definitions of the CC / SC relations in (26) are expressed in the form of subset relations, a characteristic relation in the definitions of monotonicities. In view of this, one may doubt whether opposition inferences can be treated as a subtype of monotonicity inferences. Yet the GQs have non-parallel patterns of monotonicities and o-sensitivities. Consider the proportional determiner (AT LEAST r OF) as an example. While this determiner has a uniform monotonicity throughout the whole range of $0 < r < 1$ (i.e. it is non-monotonic in the left argument and increasing in the right argument within that range) according to a standard result in GQT, it has two different o-sensitivities within that range (i.e. it is –CC→CC+ for $1/2 < r < 1$ but –SC→SC+ for $0 < r \leq 1/2$) according to Table 1.

In fact, despite the similarity between the definitions of the CC / SC relations and those of monotonicities, one cannot derive results for the o-sensitivities of a GQ by simply referring to its monotonicities. Reviewing the proof of the o-sensitivity in the right argument of (AT LEAST r OF) (i.e. Theorem 5), one can find that it contains steps using the properties of right inner negation and outer negation, as well as a step that makes use of a property of proportional determiners (i.e. deriving ‖(LESS THAN r OF)(A, B')$\|$ = 1 from $\|$(AT MOST 1 – r OF)(A, B')$\|$ = 1 for 1/2 < r < 1). Note that these steps are not derivable from the right monotonicity of these determiners. Since the o-sensitivities of many other GQs depend on that of (AT LEAST r OF), we may thus conclude that o-sensitivities are independent of monotonicities, and opposition inferences are not subsumable under monotonicity inferences.

The inferential relations derived from the OPs of GQs are often weaker than those derived from their monotonicities. For instance, by (26) the inferential relation in (39) can be rewritten as the following entailment:

(67) At least 3/4 of the members are teenagers. ⇒ Less than 3/4 of the members are elderly.

Although valid, the conclusion above seems too weak because if we make use of the relation TEENAGER $\subseteq$ ¬ELDERLY, the right increasing monotonicity of (AT LEAST 3/4 OF) and the fact that the right inner negation of (AT LEAST 3/4 OF) is (AT MOST 1/4 OF), we can obtain the following sharper inference:

(68) At least 3/4 of the members are teenagers. ⇒ At most 1/4 of the members are elderly.

Thus, opposition inferences seem to generate weaker conclusions than monotonicity inferences.

However, entailment is not the only type of inferential relations that is of interest in logical studies. In some situations, we do need to establish some other types of inferential relations (such as the CC / SC relation) between sets / propositions. Consider the following puzzle which is analogous to (3) above:

(69) Three persons A, B and C each made a remark about the membership of a club. Suppose the club has some member, John is a member of the club and there is only one true statement among the three remarks. Which is the only true statement?

A: Not all members of the club are teenagers.

B: Not all members of the club are elderly.

C: John is a teenager.

Based on the fact that (NOT EVERY) is CC→SC in the right argument and CC[TEENAGER, ELDERLY], we may conclude that A's and B's remarks satisfy the SC relation, i.e. one of them must be true. Since there is only one true statement among the three, C's remark must be false, i.e. John is not a teenager. This means that A's remark must be true, because otherwise it contradicts the fact that John is not a teenager. Thus, we conclude that A's remark is the only true statement.

8. Conclusion

In this paper, we have developed a theory on a brand new type of quantifier inferences – opposition inferences. We have defined the basic notions associated with this type of inferences. We have also proposed and proved a number of theorems and the principle in (51) for determining the OPs of various types of GQs.

The establishment of the theory on opposition inferences is an important generalization of the monotonicitiy inferences studied under modern GQT. While monotonicity inferences can be seen as involving three of the seven basic binary relations between sets / propositions defined in (25) above, namely equivalence, subalternation and superalternation, opposition inferences involve another three of those seven relations, namely contradictoriness, contrariety and subcontrariety[17]. An important point to note here is that subalternation, contradictoriness, contrariety and subcontrariety make up the four relations defined on the classical square of opposition[18]. Thus, the opposition inferences studied in this paper is also a generalization of the inferences related to the square of opposition.

With the emergence of modern mathematical logic, some notions of Classical Logic such as the four relations defined on the square of opposition seem to have lost their importance in logical studies. But if we turn our attention from mathematical reasoning to natural language reasoning, we will find that these notions, when coupled with the powerful tools of GQT, will open up whole new areas of study to be explored by scholars. Opposition inferences constitute one such area and this paper has laid the foundation for the studies on this kind of inferences. Our next task is to extend the study to other GQs that have not been discussed in this paper. One group of such GQs includes the more complicated structured GQs that are studied in Beghelli (1994) but not listed in Appendix 1, such as (AT LEAST q MORE … THAN …), etc. Another group includes the non-iterated polyadic GQs that are studied in Keenan (1996) and Keenan and Westerståhl (2011). Moreover, in recent years, Zuber (2011, 2013) has begun to study some "generalized determiners". The OPs of these polyadic GQs / generalized determiners may also be the next target of studies.

Appendix 1: Truth Conditions of some GQs

[17] Since the seventh relation, i.e. loose relationship, is rather uninteresting, it will not play an important role in any studies on quantifier inferences.
[18] Also note that superalternation is just the converse of subalternation and equivalence is just the conjunction of subalternation and superalternation

In what follows, m, n are natural numbers with $0 < m < n$; q, r are rational numbers with $0 < q < r < 1$; $X_1, \ldots X_n$ are proper names representing individuals $x_1, \ldots x_n$ in the universe.

Argument Structure	**Truth Condition**
$(X_1, \ldots$ AND $X_n)(B)$	$\{x_1, \ldots x_n\} \subseteq B$
EVERY(A, B)	$A \subseteq B$
(NOT EVERY)(A, B)	$A - B \neq \varnothing$
SOME(A, B)	$A \cap B \neq \varnothing$
NO(A, B)	$A \cap B = \varnothing$
(ALL … EXCEPT $X_1, \ldots$ AND X_n)(A, B)	$A - B = \{x_1, \ldots x_n\}$
(NO … EXCEPT $X_1, \ldots$ AND X_n)(A, B)	$A \cap B = \{x_1, \ldots x_n\}$
(MORE THAN n)(A, B)	$\lvert A \cap B\rvert > n$
(FEWER THAN n)(A, B)	$\lvert A \cap B\rvert < n$
(AT LEAST n)(A, B)	$\lvert A \cap B\rvert \geq n$
(AT MOST n)(A, B)	$\lvert A \cap B\rvert \leq n$
(EXACTLY n)(A, B)	$\lvert A \cap B\rvert = n$
(BETWEEN m AND n)(A, B)	$m \leq \lvert A \cap B\rvert \leq n$
(ALL EXCEPT n)(A, B)	$\lvert A - B\rvert = n$
MOST(A, B)	$\lvert A \cap B\rvert > 0.5\lvert A\rvert$
(MORE THAN r OF)(A, B)	$\lvert A \cap B\rvert > r\lvert A\rvert$
(LESS THAN r OF)(A, B)	$\lvert A \cap B\rvert < r\lvert A\rvert$
(AT LEAST r OF)(A, B)	$\lvert A \cap B\rvert \geq r\lvert A\rvert$
(AT MOST r OF)(A, B)	$\lvert A \cap B\rvert \leq r\lvert A\rvert$
(EXACTLY r OF)(A, B)	$\lvert A \cap B\rvert = r\lvert A\rvert$
(BETWEEN q AND r OF)(A, B)	$q\lvert A\rvert \leq \lvert A \cap B\rvert \leq r\lvert A\rvert$
(ALL EXCEPT r OF)(A, B)	$\lvert A - B\rvert = r\lvert A\rvert$
(MORE … THAN …)(A_1, A_2, B)	$\lvert A_1 \cap B\rvert > \lvert A_2 \cap B\rvert$
(FEWER … THAN …)(A_1, A_2, B)	$\lvert A_1 \cap B\rvert < \lvert A_2 \cap B\rvert$
(EXACTLY AS MANY … AS …)(A_1, A_2, B)	$\lvert A_1 \cap B\rvert = \lvert A_2 \cap B\rvert$
(PROPORTIONALLY MORE … THAN …)(A_1, A_2, B)	$\lvert A_1 \cap B\rvert\, \lvert A_2\rvert > \lvert A_2 \cap B\rvert\, \lvert A_1\rvert$
(PROPORTIONALLY LESS … THAN …)(A_1, A_2, B)	$\lvert A_1 \cap B\rvert\, \lvert A_2\rvert < \lvert A_2 \cap B\rvert\, \lvert A_1\rvert$
(EXACTLY THE SAME PROPORTION OF … AS …)(A_1, A_2, B)	$\lvert A_1 \cap B\rvert\, \lvert A_2\rvert = \lvert A_2 \cap B\rvert\, \lvert A_1\rvert$
(THE SAME … AS …)(A, B_1, B_2)	$A_1 \cap B_1 = A \cap B_2$

(DIFFERENT … THAN …)(A, B_1, B_2)	$A_1 \cap B_1 \neq A \cap B_2$

Appendix 2: Some Counterexamples

(a) Counterexample showing that (MORE … THAN …) and (PROPORTIONALLY MORE … THAN …) are neither CC→CC nor SC→CC in the first argument: U = {a, b, c, d, e, f}, A_1 = {a, b, c}, A_1' = {d, e, f}, A_2 = {c, d}, B = {a, b, e, f}

(b) Counterexample showing that (MORE … THAN …) and (PROPORTIONALLY MORE … THAN …) are neither CC→CC nor SC→CC in the second argument: U = {a, b, c, d, e, f}, A_1 = {b, e}, A_2 = {a, b, c}, A_2' = {d, e, f}, B = {b, e}

(c) Counterexample showing that (MORE … THAN …) and (PROPORTIONALLY MORE … THAN …) are not CC→CC in the third argument: U = {a, b, c, d, e, f}, A_1 = {a, d}, A_2 = {e}, B = {a, c}, B' = {d, f}

(d) Counterexample showing that (MORE … THAN …) and (PROPORTIONALLY MORE … THAN …) are not SC→CC in the third argument: U = {a, b, c, d, e, f}, A_1 = {b, c, d, e}, A_2 = {a, f}, B = {a, b, c, d ,e}, B' = {b, c, d, e, f}

(e) Counterexample showing that (EXACTLY AS MANY … AS …) and (EXACTLY THE SAME PROPORTION OF … AS …) are neither CC→CC nor SC→CC in the first argument: U = {a, b, c, d}, A_1 = {a, b}, A_1' = {c, d}, A_2 = {b, c}, B = {a, c}

(f) Counterexample showing that (EXACTLY AS MANY … AS …) and (EXACTLY THE SAME PROPORTION OF … AS …) are neither CC→CC nor SC→CC in the third argument: U = {a, b, c, d}, A_1 = {a, c}, A_2 = {b, d}, B = {a, b}, B' = {c, d}

Appendix 3: Proofs of two Propositions

Proposition 1 (THE SAME … AS …) is decreasing in the first argument.
Proof: Let $A' \subseteq A$ and $\|$(THE SAME … AS …)(A, B_1, B_2)$\|$ = 1, i.e.

(70) $A \cap B_1 = A \cap B_2$

We will show that (THE SAME … AS …) is decreasing in the first argument by proving that

(71) $A' \cap B_1 = A' \cap B_2$

Let $x \in A' \cap B_1$. Since $A' \subseteq A$, we must have $x \in A \cap B_1$. From (70), we have $x \in A \cap B_2$. We have thus shown that $x \in A' \wedge x \in B_2$, i.e. $x \in A' \cap B_2$. Since x is arbitrary, we have $A' \cap B_1 \subseteq A' \cap B_2$. Similarly, we can also show that $A' \cap B_2 \subseteq A' \cap B_1$. Combining the above, we have (71) and hence the proposition is proved. □

Proposition 2 (THE SAME … AS …) is SC→CC (on condition that $B_1 \neq B_2$) and does not possess other OPs in the first argument.

Proof: We will prove that (THE SAME … AS …) is SC→CC (on condition that $B_1 \neq B_2$) by contradiction. So let SC[A, A'] and $\|$(THE SAME … AS …)(A, B_1, B_2)$\|$ = $\|$(THE SAME … AS …)(A', B_1, B_2)$\|$ = 1, i.e.

(72) $A \cap B_1 = A \cap B_2$

(73) $A' \cap B_1 = A' \cap B_2$

From SC[A, A'], we have $\neg A \subseteq A'$. By Proposition 1, if the set A' in (73) is replaced by any of its subset, the equality still holds. Thus, from (73) we have

(74) $\neg A \cap B_1 = \neg A \cap B_2$

Now B_1 and B_2 can be rewritten as

(75) $B_1 = (A \cap B_1) \cup (\neg A \cap B_1)$

(76) $B_2 = (A \cap B_2) \cup (\neg A \cap B_2)$

From (72), (74), (75), (76), we would then have $B_1 = B_2$, which contradicts the condition that $B_1 \neq B_2$. We have thus shown that CC[(THE SAME … AS …)(A, B_1, B_2), (THE SAME … AS …)(A', B_1, B_2)]. Hence, (THE SAME … AS …) is SC→CC in the first argument.

We next prove that (THE SAME … AS …) does not possess other OPs. First, by Theorem 8(b), we can immediately conclude that this GQ is not SC→SC in the first argument. To prove that (THE SAME … AS …) is neither CC→SC nor CC→CC in the first argument, we may construct counterexamples. First, consider the model: U = {a, b, c, d, e, f}, A = {a, b}, A' = {c, d}, B_1 = {a, c, e}, B_2 = {b, d, f}. Then we have CC[A, A'] and $\|$(THE SAME … AS …)(A, B_1, B_2)$\|$ = $\|$(THE SAME … AS …)(A', B_1, B_2)$\|$ = 0. This model thus shows that (THE SAME … AS …) is not CC→SC in the first argument. Finally, consider the model: U = {a, b, c, d, e, f}, A = {a, b}, A' = {c, d}, B_1 = {a, c, e}, B_2 = {a, c, f}. Then we have CC[A, A'] and $\|$(THE SAME … AS …)(A, B_1, B_2)$\|$ = $\|$(THE SAME … AS …)(A', B_1, B_2)$\|$ = 1. This model shows that (THE SAME … AS …) is not CC→CC in the first argument. □

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