On the Dynamics of Conversation*

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Abstract

On one familiar and traditional picture, linguistic communication is a matter of the expression and transmission of a proposition across a common ground, with the proposition determined as a function of its semantic value. What general properties of a system of linguistic communication indicate whether or not it can, even in principle, be modeled along these traditional lines? This is a fundamental question in natural language semantics and pragmatics, and one relevant to a full understanding and assessment of non-traditional models of communication—notably, those found in the dynamic semantics tradition. The question is, in part, about what properties make a semantics and pragmatics for a language fragment "robustly dynamic" as opposed to "static". We formalize one natural version of this question and answer it, in the process extending earlier results of van Benthem and Veltman. According to our result, the characterizing feature of the traditional picture—of 'staticness', on one precisification—is this: the updates to the common ground induced by every sentence of the relevant language fragment exhibit *idempotence* and *commutativity*. The result naturally raises the question whether natural languages exhibit failures of idempotence or commutativity. We examine the issue, bringing out some ways in which putative failures of idempotence and commutativity can, and cannot, be explained by appeal to context-sensitivity.

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1 Introduction

On one familiar and traditional picture, linguistic communication is a matter of the expression and transmission of a proposition across a common ground, with the proposition determined as a function of its semantic value. What general properties of a system of linguistic communication indicate whether or not it can, even in principle, be modeled along these traditional lines? This is a fundamental question in natural language semantics and pragmatics, and one relevant to a full understanding and assessment of models going beyond the traditional picture of communication—notably those found in the dynamic semantics tradition. The question is, in part, about what properties make a semantics and pragmatics for a language fragment "robustly dynamic" as opposed to "static".

In this paper, we formalize one natural version of this question and answer it, in the process extending earlier results of van Benthem and Veltman. According to our result, the characterizing feature of the traditional picture—of "staticness", on one attractive formalization of this notion—is this: the updates to the common ground induced by every sentence of the relevant language fragment exhibit *idempotence* and *commutativity*. (We make these notions precise below.) This result naturally encourages the question whether natural languages exhibit failures of idempotence or commutativity. We discuss the issue, bringing out some ways in which putative failures of idempotence and commutativity can, and cannot, be explained by appeal to context-sensitivity.

The plan of the paper is as follows. In §2, we say more about what we have in mind by the traditional static picture of communication, and about the abstract question we wish to raise concerning it. In §3 we offer a precise formalization of the relevant structural features of the traditional picture as we mean to investigate it, and we restate our target question accordingly. In §4 and §5, we review important previous results bearing upon our question, due to van Benthem [1986] and Veltman [1996]. §6 supplies the main result of the paper. Extending van Benthem and Veltman's work, we supply a representation theorem for the class of conversational systems to which the traditional static picture belongs, showing that these systems are characterized by the properties of idempotence and commutativity. The theorem illuminates character of the traditional picture, and supplies a route into the question to what extent fragments of natural language can be treated along traditional, static lines. In §7, we address the issue of whether there are failures of idempotence and commutativity in natural language. Here our chief aim is to clarify the basic contours of the debate, and to situate our formal result in the appropriate empirical context. As we discuss, it can often be tempting to explain away apparent failures of idempotence and commutativity by appeal to context-sensitivity. We provide a preliminary sense of the scope and limits of this strategy. As we explain, certain kinds of appeal to context-sensitivity, far from supplying a means of preserving idempotence and commutativity, actually entail that these properties are not jointly satisfied. In §8, we give a precise characterization of one important kind of context-sensitivity—what we call information-sensitivity—and explain the general way in which some kinds of information-sensitivity are compatible with the failure of idempotence and of commutativity. In the final section we draw some morals, discussing in particular the bearing of our investigations on the general form of a semantic theory for natural language.

2 The question

Here are four natural and widely assumed ideas in contemporary semantics and pragmatics. First, conversation takes place against a *common ground*, a body of information (set of propositions) mutually taken for granted, or presupposed, by the discourse participants in context. Second, the characteristic communicative effect of successful assertion is the addition of a proposition to the common ground. Third, to assert a proposition, one must utter a declarative sentence whose compositional semantic value is mutually understood to be identical to, or to determine, that proposition. Fourth, every declarative sentence has a compositional semantic value which is identical to, or determines, a proposition. Call these the *traditional assumptions*.

Natural as they are, the traditional assumptions are nontrivial modeling ideas at remove from anything like direct observation, and there is nothing inevitable about them. One can easily imagine systems of linguistic communication where things work differently. In particular, one can easily imagine languages for which it fails to be the case that every successful update induced by the tokening of a declarative sentence consists simply in the addition of a proposition to the common ground. Indeed, many theorists have argued that natural languages contain sentences of just this variety. Notably, as we review below, a range of *dynamic* approaches to natural language semantics have supplied analyses designed to yield precisely the result that for at least some declarative sentences, the conversational update they characteristically induce cannot be reduced to the addition of a proposition to the common ground. (The traditional assumptions are generally associated with so-called *static* approaches to semantics, such as truth-conditional semantics.) The question whether to theorize about natural language from the perspective of the traditional assumptions is thus connected to the question whether the best compositional semantics for natural languages will take a dynamic form.

These questions are contested. Our objective is to clarify what is required to answer them. To settle whether the traditional assumptions are appropriate to make in theorizing about any given language fragment, an obvious prior question we should like to address is the following:

> What general properties of a system of linguistic communication indicate whether or not it can, even *in principle*, be modeled in accord with something like the traditional assumptions?

Our focus below will be on answering this question. We are seeking an *independent characterization* of the traditional assumptions, one that will shed light on their content and facilitate investigation into the question whether any given linguistic system can be made to fit them. We will deliver such a characterization. The characterization will serve demarcate a class of linguistic systems for which the traditional assumptions do not (jointly) apply. Once this class is demarcated, we will raise the question whether natural languages, or certain fragments thereof, fall into that class.

3 The question formalized

To clarify our target question, we must formalize the relevant notion of a "linguistic system" or "system of linguistic communication". What we are interested in is a certain high-level description of a language, one which just gives a formal specification of the communicative effect of tokening sentences of the language in conversation. The level of abstraction we are after is supplied by what we will call the *conversation system* determined by the semantics and pragmatics of the language. To define the formal notion of a conversation system, we require two other formal notions: the idea of an *informational context*, and the idea of the *context-change potential* of a sentence.

The notion of an *informational context* (or, until further notice, just *context*) may be thought of as a generalization of the notion of a conversational common ground (as that notion occurs, for example, in Stalnaker [1974, 2002] and Lewis [1979]). We assume that to every conversation, there corresponds an informational context (appropriate to the relevant language), and we assume, in the same general spirit of the traditional assumptions, that the essential communicative import of a speech act is reflected in its characteristic effect on the context of a conversation. (Compare especially Stalnaker [1978] on assertion.) But whereas the traditional assumptions incorporate a specific conception of what an informational context is (viz., that a context is a set of propositions), the abstract notion of a conversation system we will define imposes no such requirement in advance.

From our modeling point of view, contexts will be what sentences serve to change. The *context-change potential* of a sentence is the characteristic kind of change to the context the sentence induces when tokened.[1] We model it formally as an operation on contexts. The context change potential of a sentence s is denoted $[s]$, and the result of updating a context c with s is denoted $c[s]$ (using postfix notation).

Now given a language (some set of sentences), a *conversation system* equips the language with some set of contexts, and associates each sentence of the language with a context-change potential defined on those contexts. Formally:

Def 1. A CONVERSATION SYSTEM is a triple $\langle L, C, [\cdot] \rangle$, where L is a set of sentences, C is a set of informational contexts, and $[\cdot]$ is an *update function* from L to a set of context-change potentials (unary operations) on C.[2]

Intuitively, a conversation system for a language specifies the characteristic communicative effect, between competent speakers, of tokening the sentences of the language. As noted, the definition imposes no formal requirements on the structure of contexts; and neither does it impose constraints on the structure of the language or on the structure of the update function. We take it that the semantics and pragmatics of a language will generally fix a conversation system, but not so the reverse. The general notion of a conversation system is more abstract. A single conversation system for a language might be compatible with many different compositional semantic theories for that language, and with many different ways of drawing the

[1] The term comes from Heim [1983b]. See also Heim [1982], Stalnaker [1975].

[2] The notion of a conversation system is equivalent to the computational notion of a deterministic labelled state transition system, and equivalent to what Veltman [1996] calls an *update system*.

boundary between compositional semantics and pragmatics. The high level of abstraction afforded by this notion will enable us to avoid begging questions on these matters as we investigate the traditional assumptions.

Now we may view the traditional assumptions as corresponding to a specific subclass of conversation systems, as follows.

First, making these assumptions in connection with some language L, it follows that every context can be identified with a set of propositions, and that every declarative sentence of L has a compositional semantic value which determines a proposition. From the latter, it follows that the semantics of the language supplies us with what we can call a *proposition map*:

Def 2. A PROPOSITION MAP is a triple $\langle L, P, [\![\cdot]\!] \rangle$, where L is a set of sentences, P is a set of propositions, and $[\![\cdot]\!]$ is a mapping with $[\![\cdot]\!] : L \to P$.

The traditional assumptions further entail that the context change potential of any sentence s in L is fixed by the proposition associated with s by the proposition map supplied by the semantics of L, as follows: s updates any context c by adding the proposition $[\![s]\!]$ to c. The idea of "adding a proposition to the context" might be understood and modeled in various ways. One interpretation would be this: the posterior context is simply the result of unioning the singleton of the proposition expressed with the prior context: $c \cup \{[\![s]\!]\}$.

If one can see the conversation system for a language as being induced by a proposition map in this way, we call the conversation system *incremental*. A preliminary definition of this notion would be the following: a conversation system $\langle L, C, [\cdot] \rangle$ is INCREMENTAL if and only if there exists a proposition map $\langle L, P, [\![\cdot]\!] \rangle$ such that for all $c \in C$ and $s \in L$, $c \cup \{[\![s]\!]\} = c[s]$. We should like a more general definition than this, however. After all, what about conversation systems that can be mapped one-to-one into an incremental conversation system, update for update? Such systems are "incrementally representable", and hence should count as incremental, too. Thus we settle on the following broader definition:

Def 3. A conversation system $\langle L, C, [\cdot] \rangle$ is INCREMENTAL if and only if there exists a proposition map $\langle L, P, [\![\cdot]\!] \rangle$ and a one-to-one function f from C to $\mathcal{P}(P)$ such that for all $c \in C$ and $s \in L$, $f(c) \cup \{[\![s]\!]\} = f(c[s])$.

Observe that like the traditional assumptions, the notion of an incremental conversation system is neutral about the nature of propositions. One might think that propositions are best modeled as structured n-tuples of objects and properties, or as structured n-tuples of modes of presentation, or as sets of possible worlds, or as sets of sentences, or something else. For all our abstract definition strictly requires, propositions could be natural numbers. As long as any conversational update is representable as the addition of a single proposition to the common ground, the system will be incremental—whatever propositions are.

Although the concept of an incremental conversation system comes close to formally distilling the spirit of the traditional assumptions, there is another notion that comes still closer. Consider the picture of conversational update familiar from Stalnaker [1974, 1975, 1978]. On this picture, propositions are modeled as sets of points, construed as possible worlds (*possible worlds propositions*). Contexts, too, are modeled as sets of possible worlds (*context sets*), the intended interpretation being that the propositions which are common ground are those true with respect to every world within the context set for the conversation.[3] Every declarative sentence s is semantically associated with a possible worlds proposition p, and the context change potential of s is given by the operation which intersects the semantically associated p with the prior context set. Thus the characteristic effect of successful assertion is to eliminate possibilities from the context set, those not compatible with the proposition semantically expressed.

For reasons that will become clear, we call this kind of conversation system *static*. A preliminary definition would be the following: a conversation system $\langle L, C, [\cdot] \rangle$ is STATIC if and only if for all $c \in C$ and $s \in L$, $c \cap [\![s]\!] = c[s]$. Again, however, we should like a more general definition. After all, what about conversation systems that can be mapped one-to-one into an static conversation system, update for update? Such systems are "statically representable", and hence should count as static, too—even if the contexts of such a conversation system are not strictly speaking sets. Thus we settle on the following broader definition:

Def 4. A conversation system $\langle L, C, [\cdot] \rangle$ is STATIC if and only if there exists a set of sets P, a proposition map $\langle L, P, [\![\cdot]\!] \rangle$, and a one-to-one function f from C to P such that for all $c \in C$ and $s \in L$, $f(c) \cap [\![s]\!] = f(c[s])$.

This kind of conversation system is entirely within the spirit of the traditional assumptions as we understand them. Every context corresponds to a set of propositions; every sentence corresponds to a proposition; assertions add propositions to

[3]Thus although context sets are not strictly sets of propositions, every context set determines a unique set propositions (which is closed under entailment).

the common ground; and the work done by update function can be factored into the two parts: first, an association of sentences with propositions, and second, a single general rule mapping a proposition and a prior context into a posterior context.

With respect to our intended modeling application, the intuitive difference between an incremental system and a static system is that updates within a static system *may* serve to add a *plurality* of propositions to the context, whereas updates within an incremental system can occur only one proposition at a time. Thus incremental systems are, formally, a certain limiting case of static systems. At the high level of abstraction at which we have defined these notions, every incremental system is static, but not every static system is incremental.[4]

The more general notion of a static conversation system is what we are after. It distills, for our purposes, the crucial structural features of the traditional assumptions. In what follows, it will serve as our formal surrogate for the picture of communication delivered by these assumptions. Accordingly, we reframe our target question as a question about what makes a conversation system static:

> What general properties of a conversation system indicate whether or not it is a static conversation system?

What we are seeking is an independent characterization of the static conversation systems, ideally one adverting to intuitive properties. If we can find some general properties necessary and sufficient for staticness in the sense defined, then we will have a deeper understanding of what the traditional assumptions require; and we can raise the question whether natural language supplies counterexamples to these properties. Moreover, we would be able to quickly settle, given any conversation system for a language, whether or not it admits of static reformulation.

Below, after reviewing relevant prior results, we will provide an answer to our target question. But before continuing, we flag two potential points of confusion about our use of the word 'static'.

First, it is important not to mistake the idea of a static *conversation system* with the idea of a static *compositional semantics*. These notions are at different levels. Unlike a static conversation system, a static compositional semantics says nothing about the context change potentials of the sentences of the language; and unlike a static

[4]See the appendix for proofs.

compositional semantics,[5] a conversation system (static or otherwise) says nothing *directly* about the compositional semantic structure of the language, as it abstracts from syntactic and subsentential structure altogether. (In fact, the abstract concept of a conversation system does not strictly require that the relevant language even admit of a compositional semantics.) The connection between the two notions is this: if a conversation system is static in the sense we have defined, then *if* the language has a compositional semantics at all, that semantics can be modeled as static, and in particular, as determining the relevant proposition map.

Second: in the current literature, the jargon of 'dynamic' versus 'static' is rarely made formally precise. Rather, it usually points to some quasi-technical intuitive distinction among ways of modeling the dynamics of discourse. Now consider the following thesis:

Thesis. The truly *dynamic* conversation systems are the conversation systems which fail to be static (in the technical sense of 'static' defined above).

This is a bit like the Church-Turing thesis: on the lefthand side we have a quasi-technical intuitive notion, and on the right a perfectly clear technical one. Not unlike the Church-Turing thesis, moreover, this claim can make for idle argument. Our terminological decision to use 'static' to refer to a certain specific class of conversation systems might give the impression that we embrace this thesis. But we do not—at least, not if this is taken to mean that what we have decided to call 'static' reflects the only interesting joint in linguistic nature in this vicinity. On the contrary, as we eventually argue below, we think that there is more than one worthwhile distinction among conversation systems to be drawn. Our view is simply that the notion we have fixed upon tracks *one* very important and widely employed concept of staticness (dynamicness).

[5] We assume a *static compositional semantics* for a language L supplies at least an interpretation function from expressions in L to semantic values such that (i) the semantic value of a complex expression is a function of the semantic values of of its constituents plus its syntactic structure; and (ii) the semantic value of any sentence determines a proposition, with that proposition, together with nontrivial, independently specified pragmatic rules, determining the context change potential of the sentence. For example, the conceptions of semantics we find in Stalnaker [1974] and in Lewis [1980] are static in the relevant sense.

4 van Benthem staticness

Return now to our target question. In the literature, there are two results that approach answering it. The first is due to van Benthem [1986], and is perhaps the most commonly cited observation on this issue.[6] To explain it, it will be convenient to define the notion of a *van Benthem static* conversation system:

Def 5. A conversation system $\langle L, B, [\cdot]\rangle$ is VAN BENTHEM STATIC iff there exists a Boolean algebra[7] B_A, $B_A = \langle B, \wedge, \vee, \neg, \top, \bot\rangle$, such that for all $c \in B$ and $s \in L$,

Eliminativity. $c[s] \vee c = c$

Finite distributivity. $(c \vee c')[s] = c[s] \vee c'[s]$

Call any such triple $\langle L, B_A, [\cdot]\rangle$ a van Benthem static conversation system WITH BOOLEAN STRUCTURE.

Then we can state the observation as follows:

Fact 1 (van Benthem 1986)**.** If $\langle L, B_A, [\cdot]\rangle$ is a van Benthem static conversation system with Boolean structure, where $B_A = \langle B, \wedge, \vee, \neg, \top, \bot\rangle$, then for all $c \in B$ and $s \in L$: $c[s] = c \wedge \top[s]$.[8]

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[6] See, e.g., Groenendijk and Stokhof [1991b], van Benthem [1996], von Fintel and Gillies [2007], van Eijck and Visser [2010], Muskens et al. [2011].

[7] A BOOLEAN ALGEBRA is a tuple $\langle B, \wedge, \vee, \neg, \top, \bot\rangle$, where B is a set, $\wedge, \vee$ are binary operations on B, $\neg$ is a unary operation on B, and $\top, \bot \in B$, such that: for any $x, y \in B$: (1) $x \vee (x \wedge y) = x$; (2) $x \wedge (x \vee y) = x$; (3) $x \vee \neg x = \top$; (4) $x \wedge \neg x = \bot$.

[8] See van Benthem [1986, p.86], where the point is made for set algebras in the context of a discussion of intersective adjectives.

We note that some authors, such as van Eijck and Visser [2010], cite van Benthem [1989] for Fact 1. That paper, however, presents a distinct claim:

> If $\langle C, \wedge, \vee, \neg, \top, \bot\rangle$ is a Boolean Algebra which is idempotent and distributive then "the whole information structure can be represented by a set structure of the 'eliminative' kind described earlier" [van Benthem, 1989, p. 38].

It is not entirely clear to us what the statement in quotes means, but from the context it seems that the weakest possible interpretation is as follows: there is a set S and a injective mapping f from C to $\mathcal{P}(S)$ such that for all $c \in C$ and $s \in L$, $f(cs) \subseteq f(c)$. If there is such an f we will say there is an ELIMINATIVE REPRESENTATION of $\langle C, \wedge, \vee, \neg, \top, \bot\rangle$.

The claim is not true. For a counterexample, let $C = \{\emptyset, \{a\}\}$, the Boolean operations have their usual interpretation for the powerset algebra of $\{a\}$, and $L = \{s_1, s_2\}$ such that for all $c \in C$, $c[s_1] = c\backslash\{a\}$ and $c[s_2] = c \cup \{a\}$. s_1 and s_2 are trivially distributive and idempotent, but there is no set-theoretic interpretation of this semantics. It is easy to see that the following three inconsistence properties would be needed for there to be an eliminative representation of

Proof.
$$\begin{aligned} c \wedge \top[s] &= c \wedge (c \vee \neg c)[s] \\ &= c \wedge (c[s] \vee (\neg c)[s]) && \text{(Finite distributivity)} \\ &= (c \wedge c[s]) \vee (c \wedge (\neg c)[s]) \\ &= c[s] \vee \emptyset && \text{(Eliminativity)} \\ &= c[s] \end{aligned}$$
□

If our conversation system is van Benthem static, then the update impact of any sentence s on c can be factored into two steps: first, let the sentence perform its update on the context corresponding to Boolean $\top$; second, take the resulting context and output the Boolean meet of it with c. Now the staticness of this kind of conversation system should be clear: in a system like this each sentence can be associated with some element in the space of contexts, and the update of every sentence on any c is equivalent to the Boolean conjunction of that element with c. And indeed, we can establish that any van Benthem static conversation system is static.

Fact 2. If a conversation system is van Benthem static, it is static.

(We will prove this claim in §6 below, as a corollary of a later result.) This supplies us with an illuminating, highly general sufficient condition for staticness.

The question now arises about the converse of Fact 2. If a conversation system is static, does it follow that it is van Benthem static? The answer is negative. This follows trivially from the fact that the set of contexts in a static conversation system needn't form a Boolean algebra. To see this, one need only consider (for example) any static conversation system with finitely many contexts n, such that n does not equal a power of 2.[9]

A great many interesting conversation systems admit of a natural Boolean structure, and in many such cases it will be clear that when evaluated with respect to that structure, the system will be van Benthem static. Still, if a conversation system fails to be van Benthem static with respect to a particular way of equipping it with Boolean structure, nothing yet follows. To conclude a system is not van Benthem static, we must check every possible way of equipping the system with Boolean structure. And even if we do find that a system is not van Benthem static, it does

this semantics: $f(\{a\}) \neq f(\emptyset)$ (since f is injective), $f(\emptyset) \subseteq f(\{a\})$ (since $\{a\}[s_1] = \emptyset$), and $f(\{a\}) \subseteq f(\emptyset)$ (since $\emptyset[s_2] = \{a\}$). (Note that van Benthem [1989] also mentions what he calls monotonicity in the context in which this proof arises: for all $s \in L, c, c' \in C$: $c \leq c'$ only if $c[s] \leq c[s']$. Adding this property to the claim does not help however, as the counterexample here also satisfies monotonicity.)

[9]See Figure 1 below for an example of such a conversation system.

not follow that it is not static. For again, there will be conversation systems which simply don't have Boolean structure, and in such cases van Benthem staticness does not usefully apply. These observations lead us to ask whether greater generality can be achieved.

5 Veltman staticness

Veltman [1996] answers this question affirmatively. This bring us to our second result. Rather that assuming Boolean structure for the space of contexts, Veltman assumes only that it forms what he calls an *information lattice*:

Def 6. A quadruple $\langle V, \top, \wedge, \leq \rangle$ is an INFORMATION LATTICE iff V is a set, $\top \in V$, $\wedge$ is a binary operation on V, and $\leq$ is a partial order on V such that for all $c, c' \in V$:

$\top \wedge c = c$

$c \wedge c = c$

$c \wedge c' = c' \wedge c$

$(c \wedge c') \wedge c'' = c \wedge (c' \wedge c'')$

$c \leq c'$ iff there is some c'' such that $c \wedge c'' = c'$.[10]

Any Boolean algebra determines some information lattice, but not so the converse. Notably, an information lattice need not include a Boolean $\bot$, and the cardinality of the space of contexts needn't be a power of 2. Figure 1 illustrates a simple Veltman static, but not van Benthem static, conversation system.

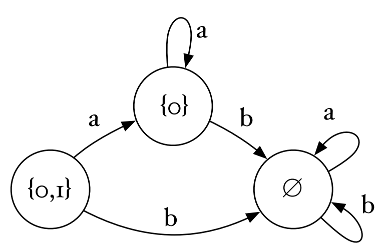

Figure 1: A Veltman static conversation system that is not van Benthem static. The information lattice is $\langle V = \{\emptyset, \{0\}, \{0,1\}\}, \top = \{0,1\}, \cap, \supseteq \rangle$. The conversation system is $\langle \{\text{a,b}\}, V, [\cdot] \rangle$, where for all $c \in V$, $c[\text{a}] = c \cap \{1\}$ and $c[\text{b}] = \emptyset$.

[10]The specification of $\leq$ adds no structure as it is induced by $\wedge$, but we will find the explicit specification convenient below. An intuitive gloss on $c \leq c'$ would be "c' is at least as informationally strong as c".

Using these weaker structural assumptions, we define Veltman's notion of staticness as follows:

Def 7. a conversation system $\langle L, V, [\cdot]\rangle$ is VELTMAN STATIC iff there exists an information lattice, V_I, $V_I = \langle V, \top, \wedge, \leq\rangle$, such that for all $c, c' \in V$ and $s \in L$,

Idempotence. $c[s][s] = c[s]$
Persistence. If $c[s] = c$ and $c \leq c'$ then $c'[s] = c'$
Strengthening. $c \leq c[s]$
Monotony. If $c \leq c'$ then $c[s] \leq c'[s]$

Call any such triple $\langle L, V_I, [\cdot]\rangle$ a Veltman static conversation system WITH INFORMATION STRUCTURE.

Now we observe a result analogous to van Benthem's.

Fact 3 (Veltman 1996)**.** If $\langle L, V_I, [\cdot]\rangle$ is Veltman static conversation system with information structure, where $V = \langle V, \top, \wedge, \leq\rangle$, then for all $c \in V$ and $s \in L$: $c[s] = c \wedge \top[s]$.

Proof. $c \leq c \wedge \top[s]$
$c[s] \leq (c \wedge \top[s])[s]$ (Monotony)
$c[s] \leq c \wedge \top[s]$ (Idempotence, Persistence)
For the other direction:
$\top \leq c[s]$
$\top[s] \leq c[s]$ (Idempotence, Monotony)
$c \wedge \top[s] \leq c[s] \wedge c$
$c \wedge \top[s] \leq c[s]$ (Strengthening)
$c \wedge \top[s] = c[s]$ □

As with van Benthem staticness, if a conversation system is Veltman static, then the update impact of any sentence s on c can be factored into two steps: first, let the sentence perform its update on the context corresponding to $\top$; second, take the resulting context and output the meet of it with c. And indeed, we can establish the staticness of any Veltman static conversation system.

Fact 4. If a conversation system is Veltman static, it is static.

(We will prove this claim in §6 below, as a corollary of a later result.) This supplies us with another illuminating sufficient condition for staticness, and one more general than van Benthem staticness.[11]

Again, the natural next question concerns the converse of Fact 4. If a conversation system is static, is it Veltman static? The answer is again negative: staticness and Veltman staticness do not coincide. This follows straightforwardly from the fact that the set of contexts in a static conversation system needn't form an information lattice. To see this, observe for instance that a static conversation system need not contain an element playing the $\top$-role.

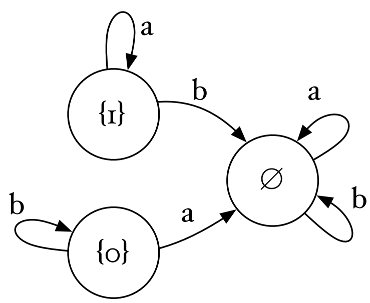

Figure 2: A static conversation system that is not Veltman static.

Example. Consider the static conversation system $\langle C, L, [\cdot]\rangle$ where $C = \{\{1\}, \{0\}, \emptyset\}$, $L = \{\text{a}, \text{b}\}$, with $c[\text{a}] = c \cap \{1\}$ and $c[\text{b}] = c \cap \{0\}$. (See figure 2.) Obviously, for every $c \in C$ either $c[\text{a}] = c$ or $c[\text{b}] = c$. Now suppose for contradiction that this system is Veltman static. Then there exists $\top \in C$; hence either $\top[\text{a}] = \top$ or $\top[\text{b}] = \top$. Suppose $\top[\text{a}] = \top$. Then, by Fact 3, for all $c \in C$, $c[\text{a}] = c \wedge \top[\text{a}] = c \wedge \top = c$. But not so, since $\{0\}[\text{a}] \neq \{0\}$. So $\top[\text{a}] \neq \top$. By symmetry $\top[\text{b}] \neq \top$. Hence $\top[\text{a}] \neq \top$ and $\top[\text{b}] \neq \top$. Contradiction.

□

The general thrust of our comments on van Benthem staticness apply *mutatis mutandis* to Veltman staticness. Many interesting conversation systems admit of a natural information lattice structure, and in many such cases it will be clear that

[11]Given Fact 1, it is easy to verify that any van Benthem static system determines an information lattice wherein idempotence, persistence, strengthening, and monotony hold, and hence van Benthem staticness implies Veltman staticness.

when evaluated with respect to that structure, the system will be Veltman static. However, if a conversation system fails to be Veltman static with respect to a particular way of equipping it with information lattice structure, nothing yet follows. To conclude a system is not Veltman static, we must check every possible way of equipping the system with information lattice structure. And even if we do find that a system is not Veltman static, it does not follow that it is not static. For again, there are static conversation systems which simply don't have information lattice structure.

The ideal, if we could have it, would be to be able state conditions which are both sufficient *and necessary* for staticness in the sense defined. This would supply a deeper perspective on the character of the traditional assumptions. Fortunately, this ideal can be achieved.

6 Staticness characterized

This takes us to the main result of the paper: a representation theorem for staticness. We show that a conversation system is static if and only if it has the properties of *idempotence* and *commutativity*.

Fact 5 (Static representation theorem)**.** A conversation system $\langle L, C, [\cdot]\rangle$ is static iff for all $s, s' \in L$ and $c \in C$,

Idempotence. $c[s][s] = c[s]$

Commutativity. $c[s][s'] = c[s'][s]$

We begin with the right-to-left direction.

Fact 5.1 If a conversation system is idempotent and commutative, then it is static.

Proof. Let $\langle L, C, [\cdot]\rangle$ be an idempotent and commutative conversation system. To show that the system is static, it will suffice to show that there exists a proposition map $\langle L, \mathcal{P}(C), [\![\cdot]\!]\rangle$ and an injective function $f : C \to \mathcal{P}(C)$ such that $f(c[s]) = f(c) \cap [\![s]\!]$, for all $s \in L$ and $c \in C$.

In order to define f and $[\![\cdot]\!]$, we first define a relation $\leq_U$ between contexts in an arbitrary conversation system U, as follows:

Def 8. For any conversation system U, and $c, c' \in C_U$, $c \leq_U c'$ iff there exist $s_1 \ldots s_n \in L_U$ such that $c[s_1]....[s_n] = c'$, or $c = c'$. (We will just write $\leq$ if the conversation system being discussed is clear from context.)

We will find the following abbreviation useful: since $[\cdot]$ is commutative, we can speak of the update of a set of sentences on a context irrespective of their sequential order:

Def 9. If S is a finite set of sentences $s_1....s_n$ from L, $c[S] =_{\text{DEF}} c[s_1]....[s_n]$.

We pause to observe that relative to any commutative idemopotent conversation system, $\leq$ is transitive, reflexive and anti-symmetric. Reflexivity is trivial. Transitivity: suppose $c_1 \leq c_2$ and $c_2 \leq c_3$. Then for some S, S', $c_1[S] = c_2$ and $c_1[S'] = c_3$; hence $c_1[S][S'] = c_3$, so $c_1 \leq c_3$. Anti-symmetry: suppose $c_1 \leq c_2$ and $c_2 \leq c_1$. Then for some S, S', $c_1[S] = c_2$ and $c_2[S'] = c_1$, and hence $c_1[S][S'] = c_1$. By commutativity it follows that $c_1[S'][S] = c_1$, and hence $c_1[S'][S][S] = c_1[S]$. By idempotence $c_1[S'][S][S] = c_1[S'][S]$, so substituting, $c_1[S'][S] = c_1[S]$; substituting again, $c_1 = c_2$.

Define $f : C \rightarrow \mathcal{P}(C)$ as follows: $f(c) =_{\text{DEF}} \{c' \in C : c \leq c'\}$. We observe f is an injection (i.e., if $f(c_1) = f(c_2)$ then $c_1 = c_2$, for all $c_1, c_2 \in C$.) Suppose $f(c_1) = f(c_2))$. Now $f(c_1) = \{c' \in C : c_1 \leq c'\}$, hence by reflexivity $c_1 \in f(c_1)$. Hence $c_1 \in f(c_2)$; hence $c_1 \in \{c' \in C : c_2 \leq c'\}$ and therefore $c_2 \leq c_1$. By parity, $c_2 \in f(c_1)$, and $c_1 \leq c_2$. By anti-symmetry, $c_1 = c_2$.

Now define $[\![\cdot]\!] : L \rightarrow \mathcal{P}(C)$ to be the minimum function such that $[\![s]\!] = \{c \in C : c[s] = c\}$. (Thus $[\![\cdot]\!]$ takes s to its fixed points on the update function $[\cdot]$.)

The preceding defines (i) a proposition map $\langle L, \mathcal{P}(C), [\![\cdot]\!]\rangle$ given an arbitrary commutative idempotent conversation system $\langle L, C, [\cdot]\rangle$, and (ii) a injective function f from $C \rightarrow \mathcal{P}(C)$. It remains to show that for all $c \in C$ and $s \in L$, $f(c[s]) = f(c) \cap [\![s]\!]$.

First we show that if $c_1 \in f(c[s])$, then $c_1 \in f(c) \cap [\![s]\!]$. Suppose $c_1 \in f(c[s])$. (i) Then $c_1 \in \{c' \in C : c[s] \leq c'\}$. So $c[s] \leq c_1$. By definition $c \leq c[s]$. So $c \leq c[s] \leq c_1$. Hence by transitivity $c \leq c_1$, hence $c_1 \in f(c)$. (ii) Now since $c[s] \leq c_1$, there exists some S such that $c[s][S] = c_1$. So $c[s][S][s] = c_1[s]$. By commutativity, $c[s][S][s] = c[S][s][s]$, which by idempotence equals $c[S][s]$, which by commutivity equals $c[s][S]$. So $c[s][S][s] = c[s][S]$. Here we substitute c_1 for $c[s][S]$, and we have $c_1[s] = c_1$. From this it follows that $c_1 \in [\![s]\!]$, since the latter just is $\{c \in C : c[s] = c\}$. So from (i) and (ii) we have $c_1 \in f(c) \cap [\![s]\!]$, the desired result.

Now let us show that if $c_1 \in f(c) \cap [\![s]\!]$, then $c_1 \in f(c[s])$. This is equivalent to showing that if $c_1[s] = c_1$ and $c \leq c_1$, then $c[s] \leq c_1$. Suppose $c \leq c_1$. Then there is some S such that $c[S] = c_1$. Suppose also $c_1[s] = c_1$. Then we have $c[S] = c_1[s] = c_1$. Therefore $c[S][s] = c_1$. By commutativity $c[s][S] = c_1$. And that means $c[s] \leq c_1$; and therefore $c_1 \in f(c[s])$.

□

The left-right direction completes the proof:

Fact 5.2 If a conversation system is static, then it is commutative and idempotent.

Proof. Any static system is idempotent and commutative, since intersection is idempotent and commutative. □

This theorem effectively supplies proofs of Fact 2 (If a conversation system is van Benthem static, it is static) and Fact 4 (If a conversation system is Veltman static, it is static). We need only observe that any system which is van Benthem or Veltman static is also idempotent and commutative. Now since any van Benthem static system is also Veltman static, it suffices to show that any Veltman static system is idempotent and commutative; and since any Veltman static system is idempotent by definition, it suffices to observe that if a conversation system is Veltman static, it is commutative. This point is easy to see: given Fact 3, for any Veltman static system $\langle L, V, [\cdot]\rangle$, there is an information lattice V_I, $V_I = \langle V, \top, \wedge, \leq\rangle$ such that for all $c \in V$ and $s \in L$. $c[s] = c \wedge \top[s]$. Since $\wedge$ is commutative, for any $s, s' \in L$ and $c \in V$, $c[s][s'] = c[s'][s]$.

We set out to answer the question what general properties are characteristic of a static conversation system. We have our answer: the static conversation systems are the commutative, idempotent conversation systems. This supplies a firm independent grip on a central and highly general concept of staticness.

We can employ the result as a test for staticness. The test is easily applied to artificially specified conversation systems. Thus, for example, consider the various conversation systems induced by file change semantics (Heim [1982, 1983a]), by dynamic predicate logic (Groenendijk and Stokhof [1991a]), and by update semantics (Veltman [1996]). We can say that these systems are all non-static for the following reason: they each allow for failures of commutativity.[12] Appropriately, our result

[12] For example, in Heim [1982, 1983a], sentence pairs of the form $Fx, \neg Gx$ are not commutative, in Groenendijk and Stokhof [1991a], $\exists x Fx, Gx$ are not commutative, and in Veltman [1996], $\neg\phi, \Diamond\phi$ are not commutative.

accords with the widely-held view that one of the distinctive features of dynamic semantics is the non-commutativity of conversational update, and of conjunction.[13]

Now the question: "To what extent do natural languages (or relevant fragments thereof) behave in accord with the traditional assumptions?" can be understood as this question:

> Do the conversation systems appropriate for modeling natural language respect idempotence and commutativity?

The balance of the paper approaches this subtle issue. Let us separate two questions:

(I) What empirical data motivates, or could motivate, the decision to model a natural language via a non-static conversation system, and what is the character of rival explanations of the data?

(II) Supposing that the conversation system appropriate to some natural language is non-static, what if anything follows concerning the character of the compositional semantics of the language?

We take these questions in order. Our main preoccupation will be with clarifying the terrain surrounding (I); we will come to (II) only at the close of the paper.

7 Dynamicness versus context-sensitivity

Natural language abounds in *prima facie* counterexamples to both idempotence and commutativity. The question is whether the putative counterexamples are bonafide, or whether they are instead better explained away along static lines. Our aim is not to settle the choice, but rather to deliver a preliminary sense of the issues by describing a small fraction of the data and bringing into focus some options for static treatment of that data. As will emerge, the natural static replies to apparent failures of idempotence and of commutativity generally involve appeals to context-sensitivity. We will distinguish three varieties context-sensitivity—what we will call

[13] To be clear, the fact that a conversation system for a language is static entails nothing by itself about the semantics of conjunction in the language. (The language needn't even *contain* a conjunction operator.) However, if we assume, following Stalnaker [1974] and Heim [1983b], that an unembedded conjunction is equivalent to consecutive assertions of the two conjuncts, then staticness will require unembedded conjunctions to be commutative.

setting-sensitivity, *information-sensitivity*, and *pragmatic context-sensitivity*—and describe some ways they can and cannot be invoked to preserve staticness.

7.1 Idempotence

Are the conversation systems appropriate to natural languages idempotent? We are not aware of any notable arguments to the contrary. Idempotence is certainly a natural assumption, as it helps to explain the banality that repeating yourself when you have already been heard typically serves no communicative purpose. Even within the dynamic semantics tradition, idempotence is a standard assumption. In fact, in many non-static dynamic semantic systems, idempotence is *required* by the way in which information-possession and entailment are conceptualized and modeled. This is because in many dynamic systems, for any s, the context change potential $[s]$ always takes a context into some fixed point of $[s]$. The fixed points of $[s]$ are conceived of as the contexts already incorporating the informational update associated with s, and entailment within such systems is usually defined in terms of the fixed points of the relevant sentences. If it is assumed that context change potentials always map sentences into fixed point contexts, idempotence is guaranteed.

We glance nevertheless at some superficial failures of idempotence, as these will enable us to introduce a range of distinctions useful for addressing the more central question of commutativity.

Consider the following discourses:

(1) a. *Speaker A:* I am sleepy.
    b. *Speaker B:* I am sleepy.

(2) a. This [*pointing to the lamp*] is old.
    b. This [*pointing to the table*] is old.

It is clear in each of these cases that the relevant (b.)-sentence serves to add new information to the common ground. It is likewise clear what the idempotence-friendly rejoinder to these examples will be, at least in general terms. It will be that although every sentence characteristically serves to add a proposition to the common ground along familiar static lines, the propositions expressed by the respective (b.)-sentences are different from the propositions expressed by the respective (a.)-sentences. In general, what proposition is expressed is a function of context—where by ‘context’ we now mean, not the body of information mutually taken for granted by the interlocutors, but rather the concrete discourse setting (or at least, the relevant features thereof). We refer to this kind of context-sensitivity as *setting-sensitivity*. We may think of a setting as corresponding to a centered world: a triple of a world, time, and speaker in conversation in the world at that time. Virtually everyone who conceives of assertion as in part a matter expressing propositions believes that in at least some cases, the proposition determined is a function of the discourse setting. Certainly, this has been the standard view about sentences containing indexicals and demonstratives since at least Kaplan [1977/1989].

What is setting-sensitivity from a conversation systems perspective? On one understanding, it is the idea that the ‘language’ of the conversation system is given, not by a set of sentences, but rather by a set of sentence-discourse setting pairs. Equivalently, if we picture the conversation system as a state transition system, it is the idea that the labels for the transitions of the state transition system are given by sentence-setting pairs. See figure 3 for a simple illustration of the idea in connection with example (1).

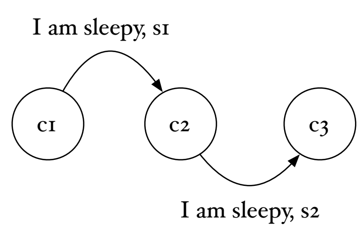

Figure 3: An idempotence-compatible fragment of a conversation system for example (1).

Plainly idempotence does not require $c2$ to equal $c3$, because the arrow connecting $c2$ and $c3$ does not share a common label with the arrow connecting $c1$ and $c2$. The basic move here is as simple and as it is powerful: any *prima facie* counterexample to idempotence might in principle be blamed in this way on tacit setting-sensitivity instead. The present point illustrates the way in which the notion of a ‘language’ appropriate to the conversation systems perspective may be forced to depart from the more usual one.

The same idea for preserving idempotence can be applied to example (2). But here we may add that there is a second reason idempotence is not impugned: it is the simple fact that the informational context which results from the update of (2-a) is not strictly identical to the context which (2-b) updates.[14] Figure 4 displays these two ways example (2) can be seen as compatible with idempotence:

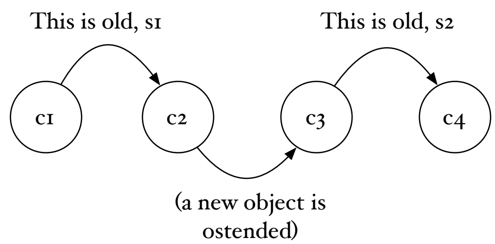

Figure 4: An idempotence-compatible fragment of a conversation system for example (2).

First, we do not have the same 'sentence' twice over, as idempotence requires; second, we do not have it that the update of the second sentence is applying to the immediate output of the first sentence, the kind of situation idempotence concerns.

In theorizing with conversation systems, it is frequently convenient and harmless to set aside changes to the informational context which are not strictly the result of the action of the context change potentials of sentences tokened in the situation; useful theorizing can occur in abstraction from the vicissitudes of non-semantic/pragmatic context change. But other times, such as here, it is entirely question-begging to ignore such changes. We return to this important point shortly, in connection with putative counterexamples to commutativity.[15]

It is interesting to compare (1) and (2) to the following superficial counterexample to idempotence:

[14] This remark in fact holds *mutatis mutandis* for (1) as well, a point that will become clearer below.

[15] One might think idempotence is sensible when dealing with declarative or information-transmitting sentences, but that it loses plausibility when we broaden our scope to include commands and other directives. For example, suppose a sergeant is inspecting a cadet's uniform. He shouts, "Turn around!" The cadet obliges. The sergeant, satisfied that the cadet's shirt is properly tucked, again shouts "Turn around!" The cadet then returns to his original position. Here it is clear that the sergeant's second command is not redundant, and nor is it merely adding emphasis to the first command. Nevertheless, this is no failure of idempotence, for the same reason noted here. The second command does not update the context resulting from the update due to the first command; rather it updates the context which results from the (presumed) satisfaction of the first command. If the cadet had not obliged by turning around after the first command, the sergeant's full discourse ("Turn around! Turn around!") would amount to one (emphasized) directive to turn 180 degrees, not a directive to turn 360 degrees.

(3) *Context: you are thinking up prime numbers with a friend. You've just said "I thought of a prime". You then say:*

a. I thought of another prime.
A moment later, you continue:

b. I thought of another prime.

Arguably (3) is a more subtle case. It will again be natural to blame the appearance that idempotence fails on the context-sensitivity of some expression in the sentences, here 'another'; but the kind of sensitivity present in this case looks distinctive. 'Another' is reminiscent of presupposition triggers such as 'too' which, as Kripke [2009] persuasively argued, are plausibly anaphoric to some aspect of the informational context of the conversation. Running with this idea from a static point of view, it will be tempting to hold that the propositions expressed by, e.g., (3-a) is partly a function of a specific feature of the setting of utterance, namely the informational context of the conversation (at the time immediately after the sentence is uttered, but before the context-change potential of the sentence is applied).

If the proposition expressed by a sentence tokened in a discourse setting is held to vary as a function of the informational context of the setting, we will say the sentence is *information-sensitive*. Information-sensitivity is not an alternative to setting-sensitivity, but rather a special instance of it. It is very much worth distinguishing from a conversation systems perspective, however, because in order to get a formal grip on information-sensitivity *per se*, we are not strictly required to view the language of the relevant conversation system as consisting of sentence-setting pairs. Conversation systems *already* encode the facts about what information is mutually presupposed, so such additional structure is not required. Below (§8) we show how demarcate the class of information-sensitive conversation systems, and we clarify the connection between information-sensitivity and the question of staticness.

7.2 Commutativity

Are the conversation systems appropriate to natural languages commutative? Many in the dynamic semantics tradition have thought not. As noted above, the best known dynamic semantic systems exhibit failures of commutativity; this, crucially, is what makes them fundamentally non-static in our sense. In the section we review a small handful of the data intended to motivate this modeling decision, and give an initial sense of certain static alternatives for analysis.

Here are three cases where commuting the sentences appears to influence felicity. The first concerns presupposition; the second, anaphora; the third, epistemic modality.

(4) a. Harry is married. Harry's spouse is a great cook.
b. ?Harry's spouse is a great cook. Harry is married.

(5) a. [A man]$_1$ walked in. He$_1$ was tall.
b. ?He$_1$ was tall. [A man]$_1$ walked in.

(6) a. Billy might be at the door.... Billy is not at the door.
b. ?Billy is not at the door... Billy might be at the door.

Problems about presupposition projection and anaphora were the driving motivations behind the original dynamic accounts of Kamp [1981] and Heim [1982, 1983b], with examples like (4) and (5) among the target explananda. Dynamic treatments of epistemic possibility modals emerged in Veltman [1996] (see also Stalnaker [1970], Veltman [1985]), with examples like (6) among the target explananda. We consider each case in turn, beginning with presupposition.

In (4-b) as contrasted (4-a), we have a textbook case of presupposition failure.[16] It is a basic observation in the presupposition literature that the order in which two sentences are uttered can determine whether or not they (as a pair) lead to a presupposition failure. Taking this order-sensitivity for granted, the question is to what extent such data requires the assumption of a non-commutative conversation system for the language. Perhaps the best known static-friendly treatment of such examples goes back to Stalnaker [1974], who sought to explain the facts pragmatically, and in a style compatible with the traditional assumptions. In the case of (4), the story begins like this: the sentence 'Harry's spouse is a great cook' expresses a proposition which is generally *inappropriate to assert* except in a context where the proposition that Harry is married is already presupposed. In the discourse (4-a), the sentence is tokened relative to such a context (at least assuming that the proposition expressed by first sentence is taken for granted after it is tokened, as it would be the normal course of events); but not so in (4-b). Thus the asymmetry in acceptability.[17]

[16]Alternatively, what we have is a case of accommodation (of the presupposition of the first sentence) together with redundancy (after accommodation) owing to the second sentence.

[17]Stalnaker would undoubtedly add that (4-b) is defective also because its second sentence is entailed by the first sentence, making the discourse redundant.

The story relies on the following idea:

> It is generally inappropriate to say 'Harry's spouse is a great cook' except in a context in which it is part of the mutually presumed background information that Harry is married.

Modulo cases covered by accommodation, this idea can hardly be denied. Appeal to it does take us a step—a very small step[18]—towards explaining the relevant asymmetry between (4-a) and (4-b). The present question, however, is whether the idea enables one to explain the presupposition failure data *in a manner compatible with the assumption of a static conversation system for the language.* To address this issue, we must ask: in virtue of what do the relevant appropriateness facts hold? Perhaps the best answer, whatever it is, is compatible with the traditional assumptions. But perhaps not. Perhaps the best account of the relevant appropriateness facts itself appeals to the idea that the conversation system for the language is non-static. If so, then appeal to such facts would preclude staticness, not help us to preserve it. (The idea that the relevant facts of appropriateness might be explained in part by appeal to non-staticness for the language is precisely the idea of Heim [1982, 1983b]. In that work, the inappropriateness observed in cases of presupposition failure was traced to undefinedness of context change potential: the context change potential of 'Harry's spouse is a great cook', for instance, was postulated to be defined only relative to contexts incorporating the information that Harry is married. This would generate the target asymmetry in felicity.)

The prospects for a Stalnakerian, static-friendly vindication of such basic presupposition data as (4) thus turn partly on whether the relevant notion of appropriateness can be plausibly cashed out without appeal to non-staticness. If the relevant appropriateness facts could be so cashed out, we would again have a case where a superficial failure of staticness really traces to context-sensitivity. This would not be context-sensitivity concerning what propositions are expressed—on Stalnaker's view, the propositions expressed by the sentences in (4-a) and the sentences in (4-b) are exactly the same, and in a certain strict sense the two discourses have the very same context change potentials. Rather, it would be context-sensitivity about the pragmatic appropriateness of what is expressed. For appropriateness would be a

[18]On the point that this is at best a small step, we think Stalnaker would agree: Stalnaker himself describes appropriateness facts such as these as the *explananda* of a theory of presupposition, not the explanans (see Stalnaker [1974, p.53]).

matter relative in part to the informational context. We could call this *pragmatic* context-sensitivity.[19]

Turn now to apparent commutativity failures owing to anaphora, such as in (5). Heim [1982] offers an explanation of such data effectively assimilating them to presupposition failure. The infelicity of a sequence like (5-b) was explained by the idea that the second sentence has a context change potential which is undefined relative to any context incorporating update triggered by the first sentence.[20] The total discourse (5-a), by contrast, is predicted to have a context change potential generally well-defined for all contexts. One of Heim's ambitions in supplying a compositional dynamic semantics yielding these results was to vindicate the intuition, voiced by Geach [1962], that truth-conditions of the discourse (5-a) are best supplied by:

(7) $\exists x$: x walked in $\wedge$ x was tall

Such an analysis would seem to require the possibility of quantification beyond the sentence boundary—an idea hard to understand from the point of view of the traditional assumptions, but achievable within a non-static, dynamic compositional semantic setting.[21]

The best known static-friendly approaches to anaphora do not supply (5-a) with truth-conditions along the lines of (7). Rather, the reference of the pronoun is understood to be mediated by appeal to a contextually salient description, one sensitive in some way to the preceding discourse. The literature contains a range of subtly different developments of this idea (see among others Evans [1977a,b], Cooper [1979], Kaplan [1977/1989], Heim [1990], Neale [1990], Heim and Kratzer [1998], Stalnaker [1998], Elbourne [2005]). But abstracting away from many details of implementation, we can say that on most of these approaches, putative counterexamples to commutativity such as (5) are explained away in a fashion analogous to the way in

[19]There is the further question whether the broadly Stalnakerian strategy can be scaled up to deal with the full range of presupposition projection facts, and in a manner not requiring appeal to non-staticness. This is an active area of contemporary investigation. For further discussion, see among others Schlenker [2006, 2008, 2009], Rothschild [2008, 2011], Abbott [2008], Stalnaker [2008, 2010]. Schlenker [2008, 2009], Chemla and Schlenker [2012], Rothschild [2008, 2011] are particularly notable in present context, for they argue that in many instances, presupposition projection is commutative, a fact hard to capture on standard dynamic accounts.

[20]There is a second reason the sequence is marked on Heim's account: the context change potential first sentence of (5-b) generally goes undefined when it is appears discourse-initial.

[21]We note in passing that the semantics of Heim [1982] was indeed perfectly compositional, despite puzzling suggestions in the literature to the contrary (for example, in Groenendijk and Stokhof [1991a]).

which the putative counterexamples to idempotence (1) and (2) were explained away above. Specifically, (5) fails to be a counterexample to commutativity in two ways: first, the the sentence 'He was tall' does not generally express the same proposition across (5-a) and (5-b), owing to the setting-sensitivity of the pronoun; and second, the informational context shifts between the sentences in (5-a) and between the sentences in (5-b), in a manner which precludes the possibility that these sequences could strictly be commutations of each other. We elaborate each of these points.

First, to what aspect of the setting is the pronoun in (5-a) supposed to be sensitive? On most of these views, part of what fixes the semantic contribution of the pronoun is the very fact that the first sentence ('A man walked in') was uttered. That is, the semantic contribution of the pronoun is sensitive to a fact about the history of the discourse itself. (*How* exactly it is sensitive depends on which account one considers.) That history is manifestly different across (5-a) and (5-b). These accounts lean on the simple fact that in making a successful assertion, one not only updates the informational context; one also changes the discourse setting, merely by talking. Thereby one fixes a fact about the history of the discourse. On this version of a static view, we would again need to resort to a richer 'language' to fully describe what is going on from a conversation systems perspective. The context change potentials of sentences containing pronouns will generally be fixed only relative to a setting. This setting-sensitivity will generally preclude (5-b) from being the commutation of (5-a).

Second, as in our discussion of idempotence, there is another way in which we have room to deny any problem for commutativity. Normally, a change in the history of the discourse—a fact about the setting—goes hand in hand with a change to the informational context. After all, in any normal conversation, the interlocutors are presupposing that the discourse has the history it actually has. Thus any successful assertion normally induces two kinds of change to the informational context. One kind of change is the one directly modeled from the conversation systems perspective: this is the update induced by the context change potential of the sentence. The second kind of change is the change resulting from the simple fact, manifest to all the interlocutors, that a certain speech act was made. Even if I don't accept your assertion, we will go on to presuppose that you uttered a certain sentence at a certain time, in an effort to add something to the common ground.[22]

The fact that assertions change the context in these two ways makes it much less

[22] The point is famously pressed by Stalnaker in many places; see in particular Stalnaker [1998].

straightforward than it may seem to probe commutativity in natural language. Commutativity is a generalization concerning the order-sensitivity—or lack thereof—of context change potentials. It abstracts from the secondary kind of change to the context that assertions make, the change induced by the fact of the utterance itself. But there is no way to escape this secondary kind of change in ordinary communication. As a consequence, merely reversing the order in which sentences are uttered does not generally result in commutation in the strictest sense. Figure 5 illustrates the basic difficulty.

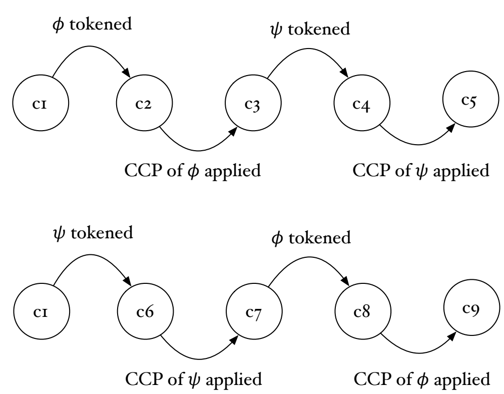

Figure 5: Merely reversing the order of sentences in natural language conversation does not result in commutation.

One might think that if a conversation system containing ϕ and ψ is commutative, it would follow that tokening ϕ, and then ψ, against an informational context c_1 would have to result in the same context as would result from tokening ψ, and then ϕ against c_1. But not so. For, first, the utterance of the sentences themselves will induce their own context changes, so that the context change potential of ϕ does not apply to a strictly identical context as the context change potential of ψ when the order is reversed; and second, there is no way to apply two context change potentials in immediate succession, for updates owing to the mutually presupposed facts of utterance will always interfere. This highlights the extent to which commutativity, while much closer to the linguistic surface than the abstract terms in which the notion of a static conversation system is defined, is still at a remove from direct empirical investigation. Whether any given case supports or undermines commutativity is in part a judgment call about the secondary effects on context induced by speech acts.

Earlier, in discussing 'another', we noted the idea of treating this expression as anaphoric and information-sensitive. One might apply this idea to anaphora generally. On such an approach, the aspect of the discourse setting the pronoun in (5-a) is sensitive to is (not so much the actual history of the discourse, but) the informational context: specifically, what is presupposed about the history of the discourse.[23] It is again worth distinguishing this possibility, since if anaphora exhibit only information-sensitivity, we needn't relativize sentences to discourse settings in order to model the language from a conversation systems perspective.

We come finally to the case of epistemic modals (repeated):

(6) a. Billy might be at the door.... Billy is not at the door.
b. ?Billy is not at the door... Billy might be at the door.

Pairs like this were first discussed by Veltman, and are cited approvingly by Groenendijk et al. [1995], Veltman [1996], Beaver [2001], and von Fintel and Gillies [2007], among others. We take it the point is supposed to be that pairs such as these manifest an asymmetry in markedness *in a certain kind of situation*: namely, one in which some kind of *monotonic* information growth happens in context mid-discourse. (For example, the door is opened.) The thought is that if we restrict to such cases we will see that (6-a) can sound fine, but not (6-b). One can predict this supposed asymmetry straightforwardly if one assumes a dynamic semantics for epistemic modals in the style of Veltman [1996] or (building on Veltman) Beaver [2001], wherein commutativity fails.

We think that insofar as there is something to be predicted here,[24] it can be accommodated easily by a wide class of theories. For consider this pair:

(8) a. I think it's possible that Billy is at the door... Billy isn't at the door.
b. Billy isn't at the door... I think it's possible Billy is at the door.

Few would argue that anything in (8) requires a dynamic treatment. If the shift in order leads to a change in how coherent the discourse strikes us, it would seem a real failure of theoretical imagination to suppose that a static picture of communication could not explain that. (8-b) indicates the sudden reopening, for the relevant agent,

[23]Compare Stalnaker [1998]: "What the context must determine, for the interpretation of a pronoun, is a function from worlds in the context set to individuals" (109).

[24]There is some question how robust the judgments are, and we doubt it is a straightforward matter intuiting whether any given case of information change is monotonic in the technical sense.

of a question recently closed; (8-a) does not. We would be sensitive to this difference even if the relevant assertions serve merely to add propositions to the common ground. The same point applies to (6). Indeed, on the classic account of epistemic modals due to Kratzer [1977, 1981, 1991], the truth-conditions of the sentences in (6) closely approximate those of (8) in certain contexts. On this account, modals are highly context-sensitive expressions: when unembedded, they will generally be sensitive to some salient body of information in the discourse setting—for instance, salient epistemic states, or other bodies of evidence within epistemic reach of agents in the discourse setting. As the epistemic states of agents in the discourse change, so too will what is expressed by such modals. As a result the epistemic modal sentence in (6-a) will generally not express the same proposition as the epistemic modal sentence in (6-b). Superficial non-commutativity can be explained away by appeal to context-sensitivity once again.

—Or can it? If epistemically modalized sentences express propositions but are context-sensitive, just what kind of context-sensitivity do they have? The answer is controversial,[25] but one live and relevant option would be treat them as information-sensitive: as expressing propositions determined in part as a function of the informational context.[26] In fact, suitably developed, such a move would enable one to associate epistemic modal sentences with exactly the context change potentials posited by dynamic approaches to these modals.[27]

This last fact is important, and not merely for the analysis of epistemic modals. Naively, one might have thought that appeal to context-sensitivity in the form of information-sensitivity would be just another means of preserving staticness. But the fact that some ways of leveraging information-sensitivity can yield conversation systems equivalent to non-static conversation systems reveals that this is not the case. Far from supplying a means of preserving staticness, some kinds of information-sensitivity may actually entail that the conversation system for the

[25] For an overview of recent debates, see the introduction to Egan and Weatherson [2011].

[26] Compare Klinedinst and Rothschild [2012], building on Yalcin [2007].

[27] To elaborate: the textbook dynamic treatment descending from Veltman [1996] assumes the following clause in a recursive specification of a dynamic semantic value function: $c[\Diamond\phi] = \{w \in c : c[\phi] \neq \emptyset\}$. (Contexts are construed as sets of possible worlds.) The idea can be captured in an intensional semantics incorporating information-sensitivity. Let our static semantic values be relativized to worlds and informational contexts, and assume the following semantic clause: $[\![\Diamond\phi]\!]^{c,w} = 1$ iff $\exists w' \in c : [\![\phi]\!]^{c,w'} = 1$. Then if the proposition added to a given context k by an utterance of $\Diamond\phi$ is just the proposition it expresses relative to k (that is, $\{w : [\![\Diamond\phi]\!]^{k,w} = 1\}$) the update impact of $\Diamond\phi$ will be the same as on Veltman's semantics. (Relevant discussion of this kind of static semantics occurs in Yalcin [2007, 2011] and Klinedinst and Rothschild [2012].)

language is *not* static.

8 Information-sensitivity characterized

To clarify this point, and the general relationship between information-sensitivity and staticness, we should like to characterize precisely the class of conversation systems where (i) each sentence serves to add a proposition to the common ground, as on the traditional assumptions, but (ii) at least some times, the proposition expressed is a function of the informational context. Call such conversation systems *information-sensitive*. Just as we defined the static conversation systems as those inducible by some proposition map for the relevant language, we may likewise define the class of information-sensitive conversation systems as those inducible by a certain kind of proposition map. This time, what we want is an *information-relative* proposition map, a mapping from sentence-informational context pairs to propositions:

Def 10. An INFORMATION-RELATIVE PROPOSITION MAP is a quadruple $\langle L, C, P, [\![\cdot]\!]\rangle$, where L is a set of sentences, C is a set of contexts, P is a set of propositions, and $[\![\cdot]\!]$ is a mapping with $[\![\cdot]\!]$: $L \times C \rightarrow P$.

Given such a mapping, we write the proposition expressed by $s \in L$ relative to $c \in C$ as $[\![s]\!]^c$. Then, the class of information-sensitive systems is defined as follows:

Def 11. A conversation system $\langle L, C, [\cdot]\rangle$ is INFORMATION-SENSITIVE if and only if there exists a set of sets P, an information-sensitive proposition map $\langle L, C, P, [\![\cdot]\!]\rangle$, and a one-to-one function f from C to P such that for all $c \in C$ and $s \in L$, $f(c) \cap [\![s]\!]^c = f(c[s])$.

It should be clear that any proposition map can be trivially construed as an information-relative proposition map, and that any static conversation system is trivially an information-sensitive conversation system. Information-sensitivity is a generalization of the notion of staticness. For an illustration of a non-static information-sensitive update system, see Figure 6.

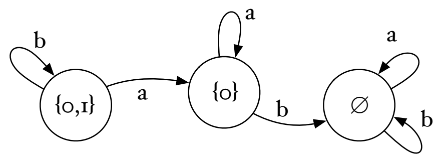

Figure 6: A information-sensitive conversation system that is not static. Observe that this system does not obey commutativity. Observe also that it is monotonic in the sense defined below.

We can now ask what formal properties characterize the information-sensitive conversation systems, just as we asked what formal properties characterize the static conversation systems. There is, it turns out, a rather simple answer to this question. A conversation system is information-sensitive just in case conversational update proceeds monotonically, in the sense that "there is no going back": once the conversation moves beyond a given context, it cannot return to it again. More precisely:

Def 12. A conversation system $\langle L, C, [\cdot] \rangle$ is MONOTONIC just in case for all $s_i \in L$ and $c \in C$, if $c[s_i] \neq c$, then for all ordered sequences $s_1 ... s_n$ of elements of L, $c[s_i][s_1]...[s_n] \neq c$.[28]

Then the observation is this:

Fact 6. A conversation system is information-sensitive just in case it is monotonic.

Proof. Recall the definition of $\leq_U$ in the proof of Fact 5 above:

Def 8. For any conversation system U, and $c, c' \in C_U$, $c \leq_U c'$ iff there exist $s_1 \ldots s_n \in L_U$ such that $c[s_1]....[s_n] = c'$, or $c = c'$.

Observe that a conversation system $U = \langle L, C, [\cdot] \rangle$ is monotonic just in case $\leq_U$ is a partial order. Thus it suffices to show that U is information-sensitive iff $\leq_U$ is a partial order. We first show that if U is information-sensitive, the corresponding $\leq$ is a partial order. Reflexivity and transitivity are immediate consequences of the definition of $\leq$. For anti-symmetry simply note that for c, d in C' $c \leq d$ only if $c \supseteq d$. It follows that if $c \leq d$ and $d \leq c$, $c = d$.

[28] Don't confuse this notion with the notion of monotony used to define the Veltman static systems in §5 above. The latter applies only in the context of information lattices.

We now show that if $\leq$ is a partial order, then U is information-sensitive. Let $f : C \to \mathcal{P}(C)$ be such that $f(c) : \{c' : c \leq c'\}$. Note that f is injective, as we showed in our proof of Fact 5 using only the fact that $\leq$ is a partial order. Now consider the information-relative proposition map $\langle L, C, \mathcal{P}(C), [\![\cdot]\!]\rangle$ where $[\![s]\!]$ is the minimal mapping such that for all $c \in C$ and $s \in L$, $[\![s]\!]^c = f(c[s])$. It remains to establish that for all $c \in C$ and $s \in L$, $f(c) \cap [\![s]\!]^c = f(c[S])$. First, we show that $[\![s]\!]^c \subseteq f(c)$. Note that $[\![s]\!]^c = f(c[s])$ and $c \leq c[s]$. Now, suppose $c' \in f(c[s])$, then, by definition, $c[s] \leq c'$. So, by transitivity of $\leq$, $c \leq c'$,and thus $c' \in f(c)$. So $[\![s]\!]^c \subseteq f(c)$. It follows immediately that: $f(c) \cap [\![s]\!]^c = [\![s]\!]^c = f(c[s])$.

□

The class of conversation systems that are information-sensitive is extremely broad. In fact, the class encompasses the conversation systems induced by all of the best known dynamic semantics systems (for example, Heim [1982], Groenendijk and Stokhof [1991a], Veltman [1996]), as these are all monotonic in the sense we have defined. Thus any of these systems can be represented as induced by an information-relative proposition map.[29] This helps to clarify the power of appeals to information-sensitivity.

Although information-sensitivity provides no guarantee of staticness in our technical sense, the picture of communication underlying the information-sensitive conversation systems is not far from the picture we get from traditional assumptions: we still have propositions determined in context, with updates still a matter of adding propositions to the common ground. We may therefore read the preceding result as confirming the idea that appeal to information-sensitivity is a viable means explaining failures of idempotence or commutativity within a framework that stays *close* to the traditional assumptions, if not within a framework that is strictly static.

9 Concluding remarks

We have noted a number of places where failures of idempotence and commutativity might in principle be explained by appeal to information-sensitivity concerning what proposition is expressed. We have also observed that information-sensitivity supplies

[29]Of course, whether it is best to approach the compositional semantics of any given fragment of language from the perspective of this point is a further question. We make some remarks on this question in the next section.

no guarantee of staticness. Thus insofar as such data can be fruitfully explained by appeal to non-static information-sensitivity—a matter still open to debate—dynamic approaches rejecting commutativity have gotten something importantly right.

On the other hand, it is important to note the indirectness of these issues on precise character of natural language semantics. The brings us, finally, to the second question raised at the end of section 5 above. Suppose that the conversation system appropriate to some natural language like English were non-static. What if anything would follow concerning the character of natural language semantics?

The answer is: not as much as one might have thought. Certainly, it would not follow that the compositional semantics of the language must be a dynamic semantics, assigning context change potentials as the compositional semantic values of the sentences. It could (for instance) take the form of an intensional semantics mapping sentences to functions from world-informational context pairs to truth values—that is, it could take the form of a static, but information-sensitive, semantics, one associating sentences with propositions relative to contexts. (Compare Yalcin [2007], Kolodny and MacFarlane [2010], Klinedinst and Rothschild [2012].) Whether one or the other is preferable cannot be settled merely by the fact of the non-staticness of conversation system for the language *per se*. Non-staticness of the conversation system for a language is one thing; dynamicness of its compositional semantics is quite another. It is an interesting question what considerations could settle the choice between these two possible formulations of a compositional semantics. We have not attempted to settle that choice here.

While the feature of dynamic semantics often emphasized is its identification of sentential semantic values with context-change potentials, dynamic semantic systems usually incorporate features which are independent of this identification but which lead to much of their power. The system of Heim [1982], for instance, includes unselective quantification (drawing on Lewis [1975]), the uniform treatment of pronouns and indefinites as variables, and the incorporation of discourse referents into the representation of context (drawing on Karttunen [1976]). These ideas do not fundamentally require the identification of sentential semantic values with context-change potentials in order to be implemented. We think it might prove worthwhile to explore static semantic systems incorporating these ideas, if only to gain further perspective on dynamic approaches.

This paper has brought into focus two classes of conversation systems: the static systems and the information-sensitive systems. We suspect that there are further interesting classes between the two. If such classes could be found and linked to linguistic phenomena, it could lead to a more refined hierarchy of dynamicness, allowing for a more precise characterization of how far current and future information-sensitive systems take us from the traditional picture of communication, and from traditional conceptions of semantics.

Appendix

We demonstrate that the class of incremental systems is a proper subset of the class of static systems.

Fact 7. If a conversation system is incremental, then it is static.

Proof. Suppose $\langle L, C, [\cdot]\rangle$ is an incremental conversation system. Then there exists a proposition map $\langle L, P, [\![\cdot]\!]\rangle$ and a one-to-one function f from C to $\mathcal{P}(P)$ such that for all $c \in C$ and $s \in L$, $f(c) \cup \{[\![s]\!]\} = f(c[s])$. Consider the proposition map $\langle L, \mathcal{P}(P), [\![\cdot]\!]'\rangle$ such that $[\![s]\!]' = P \backslash \{[\![s]\!]\}$ $(= \{[\![s]\!]\}^c)$, and consider the function $f' : C \to \mathcal{P}(P)$ such that $f'(c) = f(c)^c$. Then for all $c \in C$ and $s \in L$:

$$f(c) \cup \{[\![s]\!]\} = f(c[s])$$

$$(f(c) \cup \{[\![s]\!]\})^c = f(c[s])^c$$

$$f(c)^c \cap \{[\![s]\!]\}^c = f(c[s])^c$$

$$f'(c) \cap [\![s]\!]' = f'(c[s])$$

□

Fact 8. Not every intersective system is incremental.

Proof. Consider an intersective conversation system $\langle L, C, [\cdot]\rangle$ such that:

(i) $c_1[p \wedge q][p] = c_1[p \wedge q]$

(ii) $c_1[p \wedge q] \neq c_1[p] \neq c_1$.

Suppose the system is incremental. Then there exists some proposition map $\langle L, P, [\![\cdot]\!]\rangle$ and a one-to-one function f from C to $\mathcal{P}(P)$ such that for all $c \in C$ and $s \in L$, $f(c) \cup \{[\![s]\!]\} = f(c[s])$. Given such a map, we know

$$f(c_1[p \wedge q][p]) = f(c_1) \cup \{[\![p \wedge q]\!]\} \cup \{[\![p]\!]\}$$

Since by (i) we have $c_1[p \wedge q][p] = c_1[p \wedge q]$, it follows that

$$f(c_1) \cup \{[\![p \wedge q]\!]\} \cup \{[\![p]\!]\} = f(c_1) \cup \{[\![p \wedge q]\!]\}$$

This entails that either $[\![p]\!] \in f(c_1)$ or $[\![p]\!] \in \{[\![p \wedge q]\!]\}$ (n.b., $\{[\![p]\!]\}$ is a singleton). Suppose the former. Then $f(c_1) \cup \{[\![p]\!]\} = f(c_1) = f(c_1[p])$; but since f is one-one, this result is incompatible with (ii), which says $c_1[p] \neq c_1$. So suppose instead $[\![p]\!] \in \{[\![p \wedge q]\!]\}$. But this entails $[\![p]\!] = [\![p \wedge q]\!]$, meaning $f(c_1[p \wedge q]) = f(c_1[p])$. Since f is one-one, this result is incompatible with (ii), which says $c_1[p \wedge q] \neq c_1[p]$.

□

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