Free Choice in Deontic Inquisitive Semantics (DIS)*

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Abstract. We will propose a novel solution to the free choice puzzle. The approach is driven by empirical data from legal discourse and does not suffer from the same problems as implicature-based accounts. Following Anderson's violation-based deontic logic, we will demonstrate that a support-based radical inquisitive semantics will correctly model both the free choice effect and the standard disjunctive behaviour when disjunctive permission is embedded under negation. An inquisitive semantics also models the case when disjunctive permission is continued with "but I do not know which" which coerces an ignorance reading. We also demonstrate that a principled approach to negation provides a monotonic but restricted definition of entailment, which solves the problem of strengthening with a conjunct that is used as a counterargument against violation-based accounts.

1 Introduction

(1) A country may establish a research center or a laboratory.

When (1) is law, its salient reading gives permission to establish a research center and it gives permission to establish a laboratory. Although, it does not necessarily give permission to establish both. The problem lies in that a classical analysis of modality as quantification over worlds and disjunction as set union predicts that permission is given to do one or the other, which is a less salient reading. This so called free choice effect has become one of the better documented puzzles in semantics since it was investigated by Hans Kamp [13].

The puzzle is exacerbated by the observation made by Alonso-Ovalle [3], and Simons [19, p. 8] that embedding disjunctive permission under negation reverts disjunction to classical behaviour.

(2) A country may not establish a research center or a laboratory.

The salient reading of (2) says that permission is not granted to establish a research center nor is it granted to establish a laboratory. As this reading follows from standard accounts of modality and disjunction, a modification of disjunction to account for (1) will lead to problems with the salient reading of (2).

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A third puzzle arises from the fact that (1) can be coerced into a standard boolean reading of disjunction by adding "...but I do not know which."

(3) A country may establish a research center or a laboratory but I do not know which."

To utter (3) one must know that permission is granted to establish either a research center or laboratory, but it is not specified which of the two is permitted. This has become known as the ignorance reading.

This paper will propose a model that captures the truth conditions of all three of (1), (2) and (3), or in other words models the free choice effect, classical behaviour of disjunctive permission when embedded under negation and the ignorance reading. This sets it apart from prior accounts as is demonstrated in the next section.

2 Previous Accounts

Zimmermann [20] reignited interest in free choice by positing a pragmatic mechanism that reinterprets disjunction as a conjunctive list of epistemic possibilities: $\Diamond A \wedge \Diamond B$. Unfortunately a reinterpretation of disjunction as conjunction fails to provide the salient reading of (2) as the negation of $\Diamond A \wedge \Diamond B$ comes out as $\overline{\Diamond A} \vee \overline{\Diamond B}$

Many following accounts accepted that free choice is essentially a pragmatic effect and suggested implicature-based accounts. This approach is supported by (3), as there appears to be a way to cancel the free choice effect in disjunctive permission utterances. These accounts include Schulz [18], Eckardt [7], Fox [8] and the game theoretic implicature account by Franke [9]. As approaches to free choice have been extensively discussed in the literature, for example by Schulz [17] or more recently by Barker [5], we will concentrate on examining [7] to expatiate on general issues with implicature-based solutions.

Eckardt derives the free choice effect utilizing an implicature through the maxims of manner and quality. A simplified version of her account says that if an informed speaker uses disjunction $\Diamond(\varphi \vee \psi)$ then either disjunct would have been more economical. From this we infer that the governing permissions are best described by disjunction because either disjunct alone would be false. This provides the free choice effect: there must be some worlds where $\Diamond\varphi \wedge \overline{\Diamond\psi}$ and others where $\overline{\Diamond\varphi} \wedge \Diamond\psi$.

The weakness of this account lies in its conclusion as the intuition behind the deontic free choice effect in (1) is that the speaker believes that A and B are permitted. The reason why $\Diamond\varphi \wedge \overline{\Diamond\psi}$ and $\overline{\Diamond\varphi} \wedge \Diamond\psi$ worlds are the case is that there exist some worlds in the speaker's information state in which either φ or ψ is not permitted. But this is contrary to the intuition outlined above.

Simons [19, p. 14] argues generally against implicature based accounts on the grounds that there does not seem to be a distinction between what is said and what is implicated in examples such as (1). Compare this to a classic example of generalized implicature from Grice [10, p. 32].

(4) X is meeting a woman this evening.

Grice states that such a statement generally implicates that the woman being met is not X's wife, mother, sister, etc. Thus, there exists a clear distinction between that which is said (X will meet a woman) and that which is implicated (X will meet a potential romantic acquaintance). The lack of such distinctions in free choice sentences poses a challenge to any implicature based account.

Barker [5, p. 16] casts doubt on the existence of another marker of implicatures, namely cancellability. Observe the following example.

(5) You may eat an apple or a pear, although in fact you may not eat an apple.

When an implicature in cancelled, the utterance only has the meaning of what is said. If (4) were cancelled by "... but it's only her mother." then the utterance would lose the implicature that the woman is a romantic acquaintance. Yet, instead of reverting the phrase to that which is said, the added phrase in (5) appears to make the statement contradictory or offers a correction of the preceding information.

There appear to be other possible routes for cancellation, which is to utter either of the following continuations.

(6) You may eat an apple or a pear, although in fact you may not eat both.

The consequence of uttering (6) does not cancel the free choice effect. Permission is given to eat an apple and permission is given to eat a pear. Yet, the continuation provides the additional information that eating both an apple and a pear is prohibited. The additional information does not conflict with free choice readings.

But contrary to these facts, the ignorance reading in (3) does affect the free choice effect. Adding "...but I do not know which." intuitively suggests that the speaker does not know the governing permissions and thus such utterances do not give permission for both disjuncts. We will show that the ignorance reading can be accounted for as a scope effect, similar to one that is in effect in the following example.

(7) There isn't an apple or a pear on the table, but I do not know which is missing.

Assuming that it was expected that there would be an apple and a pear on the table, the utterance of (7) says that one of them is missing, but it is not necessary that both of them are missing as would be the case if the continuation "...but I do not know which." were omitted.

Barker [5] proposes a semantic approach similar to the one pursued here, by following Kanger [14] in positing a normative ideality δ such that if φ is obligatory, then if φ then δ. This view is a contrapositive view of Anderson's reduction [4] and, thus, similar to the proposal to follow, but in terms of details, a prior analysis of World Trade Organization (WTO) examples in [1] suggests

that legal reasoning does not concern idealities but rather violations. While this might be contingent on the deontic context, in terms of legal language, the violation-based solution remains preferable.

Also, while Barker's account of the free choice effect is entailment based, his semantics fail to predict the salient reading of negated disjunctive permission sentences such as (2) and he is forced to tell a pragmatic story to account for it. This observation also holds for the semantic account of Aloni [2]. The next section will show that the salient reading of disjunctive permission under negation can also be incorporated into the semantic account.

3 The Proposal

We observed that many previous accounts of free choice fail to capture the effect of negating disjunctive permission utterances. This led us to base the model on an independently motivated prior version of inquisitive semantics that focuses on the effects of negation - Radical Inquisitive Semantics. An earlier version of the language used here was developed and explored by Groenendijk and Roelofsen [12] and Sano [16]. Our proposal adds clauses for deontic permission and discusses entailment in the radical environment. Due to space constraints we must assume familiarity with standard inquisitive semantics and the above proposals.

We shall only consider a propositional language of a finite set of propositional variables and the operators: $\overline{\varphi}, \wedge, \vee, \rightarrow$. We also need to define worlds as binary valuations for atomic sentences and states as sets of worlds. σ and τ are variables that range over states, w is the variable that ranges over worlds and W is the set of all (classical) valuation functions.. Propositions expressed by sentences are defined through a support and reject relation. When a state supports φ then we write $\sigma \models^{+} \varphi$ and when a state rejects φ then we write $\sigma \models^{-} \varphi$.

Definition 1. *Radical inquisitive semantics (DIS).*

1. $\sigma \models^{+} p$ iff $\forall w \in \sigma : w(p) = 1$
   $\sigma \models^{-} p$ iff $\forall w \in \sigma : w(p) = 0$
2. $\sigma \models^{+} \overline{\varphi}$ iff $\sigma \models^{-} \varphi$
   $\sigma \models^{-} \overline{\varphi}$ iff $\sigma \models^{+} \varphi$
3. $\sigma \models^{+} \varphi \vee \psi$ iff $\sigma \models^{+} \varphi$ or $\sigma \models^{+} \psi$
   $\sigma \models^{-} \varphi \vee \psi$ iff $\sigma \models^{-} \varphi$ and $\sigma \models^{-} \psi$
4. $\sigma \models^{+} \varphi \wedge \psi$ iff $\sigma \models^{+} \varphi$ and $\sigma \models^{+} \psi$
   $\sigma \models^{-} \varphi \wedge \psi$ iff $\sigma \models^{-} \varphi$ or $\sigma \models^{-} \psi$
5. $\sigma \models^{+} \varphi \rightarrow \psi$ iff $\forall \tau \subseteq \sigma.(\ \tau \models^{+} \varphi$ implies $\tau \models^{+} \psi)$
   $\sigma \models^{-} \varphi \rightarrow \psi$ iff $\exists \tau.(\tau \models^{+} \varphi$ and $\forall \tau' \supseteq \tau.(\tau' \models^{+} \varphi$ implies $\sigma \cap \tau' \models^{-} \psi))$

Sano has also shown that in the propositional setting there are always maximal states under the $\subseteq$-relation that support or reject a sentence.

Proposition 1. *(Persistence).*
If $\sigma \models^{+} \varphi$ and $\tau \subseteq \sigma$ then $\tau \models^{+} \varphi$ and if $\sigma \models^{-} \varphi$ and $\tau \subseteq \sigma$ then $\tau \models^{-} \varphi$

We refer to the maximal states that support a sentence as the possibilities for that sentence, and denote it by $[\varphi]^+$. We refer to the maximal states that reject a sentence as the counter-possibilities for that sentence, and denote it by $[\varphi]^-$. Given persistence, this means that the meaning of a sentence φ is fully characterized by $\langle [\varphi]^+, [\varphi]^- \rangle$.

Definition 2. *Informativeness and inquisitiveness.*

1. *φ is informative iff $\bigcup [\varphi]^+ \neq W$.*
2. *φ is inquisitive iff $\bigcup [\varphi]^+ \notin [\varphi]^+$.*

It follows from these definitions that φ is informative if it eliminates worlds and φ is inquisitive if there are at least two possibilities for φ. The above definitions are standard in inquisitive semantics and have been used to characterize assertions, questions and hybrids in the way stated below (see for example [12]).

Definition 3. *Assertions, questions and hybrids.*

1. *φ is an assertion iff φ is not inquisitive.*
2. *φ is a question iff φ is not informative.*
3. *φ is a hybrid iff φ is inquisitive and informative.*

Unlike in standard inquisitive semantics, where negations are always assertions, in radical inquisitive semantics, negations can be inquisitive. Even when negation is applied to an assertion φ, the resulting sentence $\overline{\varphi}$ can be an inquisitive sentence. We can define a new characterization for sentences that are assertions both on the positive and negative side.

Definition 4. *Radical assertions.*
φ is a radical assertion iff both φ and $\overline{\varphi}$ are not inquisitive.

The following figures illustrate the clauses in the definitions. We draw possibilities on our figures such that any world outside of the connected line is eliminated and for counter-possibilities, any world outside of the dashed line is eliminated. For example, the possibility for the atom p is the set of worlds where p is the case and its counter-possibility is the set of worlds where p is not the case. The clause for negation flips between possibilities and counterpossibilities, so that the negation of p is the set of worlds that reject p. Atoms are radical assertions.

Disjunction is the source of inquisitiveness in standard inquisitive semantics. Figure 2 shows the possibilities and counter-possibilities for disjunction. As there can be more than one possibility for disjunction it can be inquisitive on the positive side but its negation is an assertion. Disjunction also eliminates the worlds where neither disjunct is the case, which makes it a hybrid.

In standard inquisitive semantics, conjunction is a radical assertion but in DIS the clause for the negation of conjunction allows inquisitiveness (see figure 3). This accounts for the fact that after a conjunction is rejected, one can ask: "Why?" and the other interlocutor can specify which conjunct was unacceptable.

So, while a conjunction is an assertion, the negation of conjunction is a hybrid. The current clauses make $p \wedge q$ symmetric with $\overline{p} \vee \overline{q}$. [1]

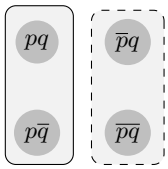

Fig. 1. p

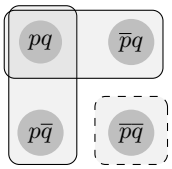

Fig. 2. $p \vee q$

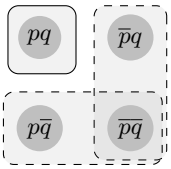

Fig. 3. $p \wedge q$

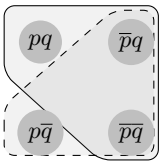

Fig. 4. $p \rightarrow q$

The clauses for conditionals are inspired by Ramsey Test literature and the original motivation is discussed at length elsewhere [12, pp. 18-23, 28-30]. The crucial idea is that the negation of conditionals should not make the antecedent the case. This means that unlike in classical models, a state that supports a conditional and a state that rejects a conditional do not have an empty intersection. This is demonstrated on figure 4. The negation of conditionals allows for inquisitiveness and thus conditionals are assertions.

The negation of conditionals also requires us to adapt the definition of entailment. Lewis and Langford [15] provided the intuition that if φ entails ψ then it should be impossible that $\varphi \wedge \overline{\psi}$. Yet, this does not hold under our clause for conditionals. According to standard inquisitive entailment, $\overline{p} \vee q \models p \rightarrow q$ but their intersection is $\overline{p}$ and thus not impossible.

Fortunately there is a principled way to characterize entailment in DIS. Standarly if $\varphi \models \psi$ then $\overline{\psi} \models \overline{\varphi}$. We want the implication to also hold in DIS as when you reject the weaker sentence, you should be able to reject the stronger one as well. As DIS looks both at the positive and negative side, then our definition of entailment must do the same and we must add $\neg\psi \models \neg\varphi$ to our definition of entailment.

Definition 5. *Radical inquisitive entailment.*

1. $\varphi \models \psi$ iff $\forall\sigma$ if $\sigma \models^{+} \varphi$ then $\sigma \models^{+} \psi$ and $\sigma \models^{-} \psi$ then $\sigma \models^{-} \varphi$.

This definition also allows us to characterize equivalence as mutual entailment. Radical inquisitive entailment restricts the number of available inferences as compared to standard inquisitive semantics and we will see that such a restriction invalidates some of the inferences that cause trouble for deontic semantics. But before we can discuss this, we need to introduce deontics into the model.

A violation-based deontic logic gravitates around the question whether an act violates a specific law. A permission sentence in a law text provides information on what is not a violation. Following Anderson [4] and the way in which WTO judges reason, we take permission statements to provide information about what

[1] In standard inquisitive semantics, the treatment of negation as a complement creates a situation where $p \wedge q \not\models \overline{p} \vee \overline{q}$.

is not a violation. This can be captured via introducing the atom v that provides the information that a specific violation has occurred.

Generally, v shall designate a specific law or regulation that is being violated. To account for different types of violations that can occur within a single legal framework, one can designate v_1, v_2, etc. for each specific violation. For example, v_1 may be taken as the proposition "Violation of law number 1 has occurred." As violation propositions are specific, violations can be reasoned about in the same manner as any other information. So the violation of one law does not lead to violations of other laws, nor does not violating one law save one from indictments due to other deeds. For simplicity, we shall assume that there is only one violation.[2] This will be defined in the semantics as follows.

Definition 6. *Permission.*

1. $\sigma \models^+ \Diamond\varphi$ iff $\forall\tau \subseteq \sigma.(\ \tau \models^+ \varphi$ implies $\tau \models^- v)$
   $\sigma \models^- \Diamond\varphi$ iff $\forall\tau \subseteq \sigma.(\tau \models^+ \varphi$ implies $\tau \models^+ v)$

As can be seen in figure 5, $\Diamond p$ coincides with the conditional: $p \to \overline{v}$. This similarity is not a general feature, though, as the negation of conditionals can be inquisitive but the negation of permission sentences cannot. So, permission is a radical assertion like atoms. This is intuitively correct as permission statements are generally made with authority, and thus the salient reading should be of an assertion.

Also, a permission sentence does not predetermine whether p is in fact the case. A state that supports $\Diamond p$ includes the world $< \overline{pv} >$ in which p is not the case. This accounts for the intuition that permission sentences do not require one to in fact perform the act that is permitted. Furthermore, the world $< \overline{p}v >$ is not eliminated. This world allows for a more fine-grained analysis of interaction between different permissions and prohibitions, as a permission for one thing, in this case p does not guarantee that a violation may not occur when another thing, for example q, is the case.

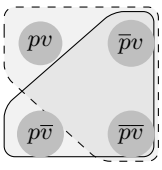

Fig. 5. $\Diamond p$

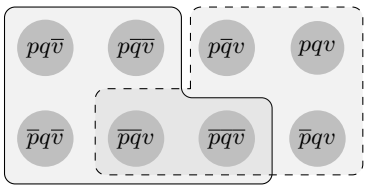

Fig. 6. $\Diamond(p \vee q)$

[2] This formulation should be tested on the Chisholm's paradox and the gentle murder paradox. Yet, as these fall out of the scope of describing the natural language semantics of permission, it will not be discussed in this article.

4 Puzzles Solved

For this account to provide a solution to the free choice puzzle, we must assume that permission takes scope over disjunction. And, indeed, this is supported by general observations regarding disjunction and scope. Following Eckardt [7, pp. 9-10] we assume that in case of ambiguities, one chooses the strongest of the alternatives. As $\Diamond(p \vee q) \models \Diamond p \vee \Diamond q$, permission scoping over disjunction provides the stronger reading. This is illustrated in figures 6 and 8.[3]

The semantics predicts that $\Diamond(p \vee q)$ eliminates three worlds: $< p\overline{q}v >$, $< \overline{p}qv >$ and $< pqv >$. The result is an assertion that includes the remaining worlds as shown in figure 6. A comparison with figure 7 shows that $[\Diamond(p \vee q)]^+$ is the same as $[\Diamond p \wedge \Diamond q]^+$, yet their negations differ and thus it follows that $\Diamond p \wedge \Diamond q \models \Diamond(p \vee q)$ but $\Diamond(p \vee q) \not\models \Diamond p \wedge \Diamond q$.

This appears to be in line with our intuitions regarding permission being granted for both disjuncts. Whether one enacts p or q, $\Diamond(p \vee q)$ guarantees that a violation does not occur. Note that while disjunctive free choice provides the information that doing either disjunct is permitted, inferring $\Diamond p$ from $\Diamond(p \vee q)$ is blocked by radical inquisitive entailment. This is due to the fact that disjunctive permission does not guarantee that doing both p and q simultaneously is not prohibited, which makes permission for either disjunct a contingent fact. By allowing $\Diamond(p \vee q) \models \Diamond p$ one gives each disjunct independence from the other, so that one can derive $\Diamond(p \wedge q)$ and $\Diamond p \wedge \Diamond q$ which we would not want.

The second puzzle concerns the fact that disjunctive permission embedded under negation as in (2) has the salient reading that neither disjunct is permitted. This result also straightforwardly follows from our clause for the negation of permission. As permission is a radical assertion, then $\overline{\Diamond(p \vee q)}$ is an assertion that eliminates all worlds in which doing either p or q would not result in a violation. This can can be seen by looking at the counter-possibility in figure 6.

The third puzzle concerned the fact that appending a disjunctive permission sentence with "... but I do not know which." such as is done in (3) gives it an ignorance reading. We do not take this effect to be cancellation of an implicature but rather one of blocking the modality from taking strongest scope - scoping over the disjunction. The result is a translation of (3) as a wide scope reading in which disjunction takes scope over permission such that "may" distributes into the disjuncts: $\Diamond p \vee \Diamond q$.

As disjunction is a hybrid, when disjunction scopes over permission it raises an issue for the speaker to solve, modelled as two possibilities as shown in figure 8 below.[4] It is no longer guaranteed that doing p or q will not incur a violation.

[3] This scope movement also accounts for examples of wide disjunction [20, p. 278].
[4] The counter-possibilities have been omitted for clarity.

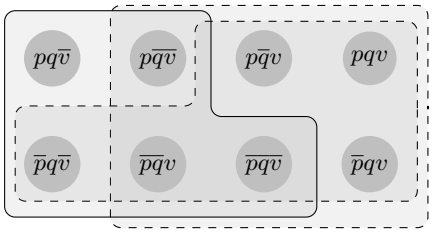

Fig. 7. $\Diamond p \wedge \Diamond q$

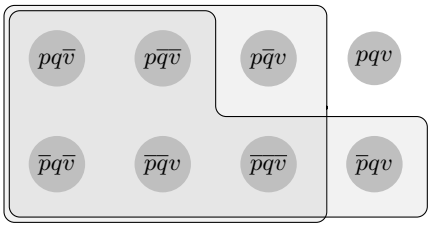

Fig. 8. $\Diamond p \vee \Diamond q$

5 Counterargument Countered

One might expect this account to suffer from the problem of strengthening with a conjunct. It should not follow from permission to eat an apple, that eating an apple and killing a postman does not incur a violation. As one can see in the following figures, this inference is blocked by radical inquisitive entailment.

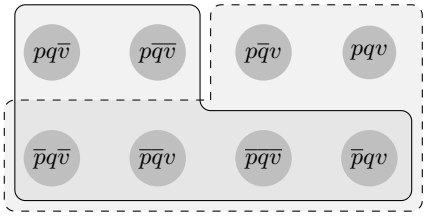

Fig. 9. $\Diamond p$

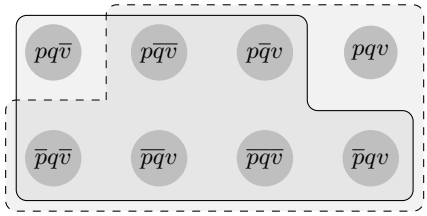

Fig. 10. $\Diamond(p \wedge q)$

As the comparison of figures 9 and 10 demonstrates, the counter-possibility for $\Diamond(p \wedge q)$ is not contained in the counter-possibility for $\Diamond p$, which means that according to radical inquisitive entailment $\Diamond(p \wedge q)$ is not entailed by $\Diamond p$.

6 Future work

There are a number of puzzles regarding inferences associated with deontic modality, among them Ross's paradox and the interaction of modality with conditionals (for discussion, see Cariani [6]). The treatment of modality and the introduction of radical inquisitive entailment in DIS provides a promising avenue for dealing with these issues.

For Ross's paradox, the solution will follow from the fact that $\Diamond p \not\models \Diamond(p \vee q)$, which can be observed by studying figures 9 and 6. For conditionals, the puzzle lies in the fact that in Kratzer semantics $p \rightarrow \Box p \models \Box p$, yet intuitively everything that is the case does not have to be the case. Assuming that obligation can be characterized as prohibition to not do p ($\overline{\Diamond \overline{p}}$), then in DIS $p \rightarrow \Box p \not\models \Box p$.

Yet, due to space constraints the full explication of these solutions must be left for future work.

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