

# A logic and semantics for imperatives

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Truth is undefined for imperative statements. However, if imperatives implicitly reference a *fact*, they can be rephrased as truth-valuable declaratives explicitly referencing that fact. But are there such facts? Kenny held that any imperative references a *set of wishes* held by its imperator. I extend his thesis by proposing that imperator wishes are *facts* implicitly referenced by imperatives and explicitly referencing them yields semantically isomorphic declaratives. I implement this thesis with modal operators for *wants* and *cause* with which declarative schemata are formed to automate translation of imperatives into semantically isomorphic declaratives called *proxy-imperatives*. The proxy-imperatives match imperative behavior and provide semantic validation of imperative arguments thereby integrating imperative reasoning into classic truth-valuable logic.

## 1. Introduction

### 1.1 The problem

Formal semantics defines criteria for evaluating the truth of declarative statements with respect to domains of discourse. However, imperative statements like “Shut the door!” are not obviously true or false in any domain and therefore fall outside the realm of truth-valuable statements. Without truth values imperative arguments cannot be shown to be truth preserving, or semantically valid, even if they are intuitively valid. As such, it is held that imperative arguments, which form a large body of everyday arguments, fall outside the scope of formal logical reasoning. This paper proposes to bring them in scope.

### 1.2 The path followed

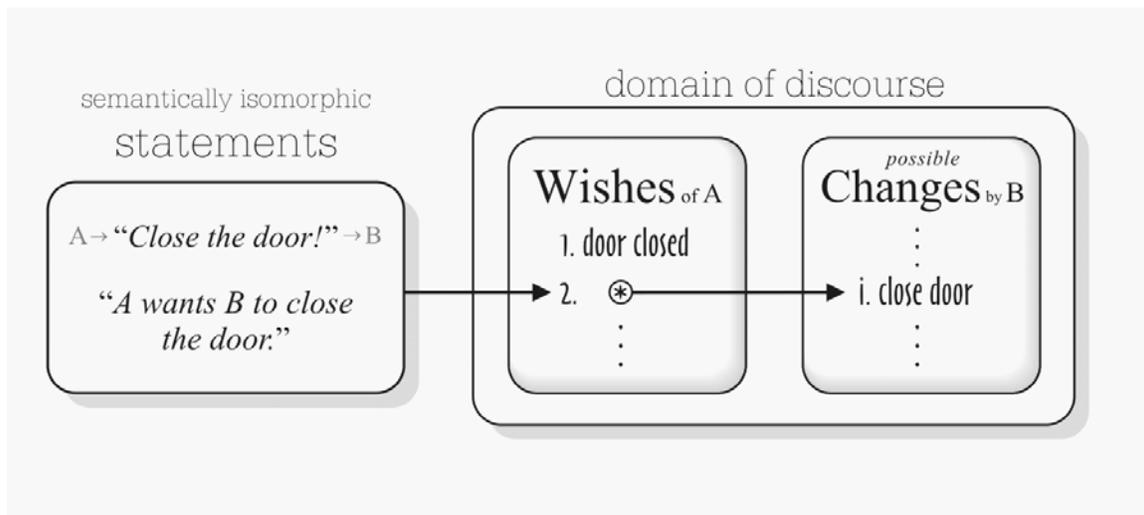
Given their lack of truth values, proposed imperative logics often define alternatives to truth. An important example is Anthony Kenny’s substitution of being *satisfactory* in place of being *true*. An imperative is satisfactory just in case obeying it will satisfy the *wishes* an imperator intends to express. So for example, if I want someone to close the door, the statement “Shut the door!” is satisfactory because its fulfillment by a listener will make my wish come true. Given this alternative to truth, Kenny proposed that just as the goal of classical declarative logic is to preserve truth, the goal of an imperative logic should be to preserve *wishes* from assumptions to conclusions in imperative argumentation.<sup>(1)</sup>

Kenny’s *wish-preserving* criterion is intuitive. For example, from “Shut the

door!” one should not expect it to be possible to infer any imperative that upon obeying keeps the door open. This gives us important insight into both what we want an imperative logic to *do* and what imperatives *are*. Even Kenny’s critics didn’t dispute his wish-preserving criterion but faulted his logic instead for its failure to actually preserve wishes.<sup>(2)(3)</sup> But for us here, what matters is Kenny’s insight that an imperative denotes a set of wishes -- an insight we’ll implement explicitly. We’ll avoid the problems raised against his logic by relying on classic truth values acquired by translating imperatives into truth-valuable declaratives that directly reference the contents of an imperator’s *set of wishes*.

## 2 The *wants* premise

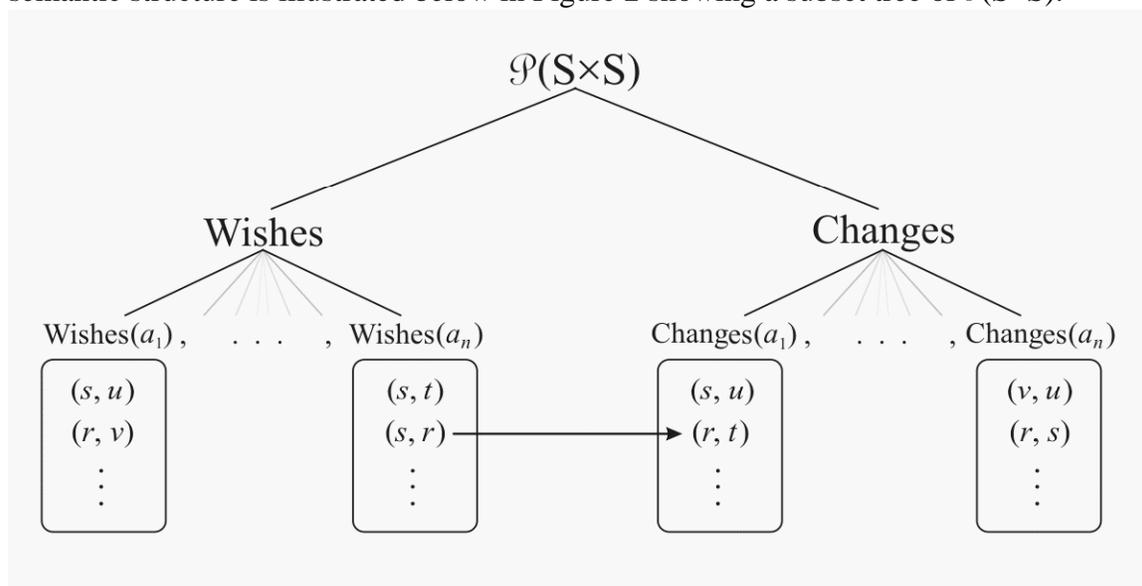
Kenny’s premise that an imperative expresses an imperator’s wish for a goal state is my premise too. By this premise, what an imperator *means* is what the imperator (or anyone compelling them to speak) *wants*. This is the natural interpretation of imperatives. If you are told “Do it!” you assume someone *wants* you to ‘do it’, otherwise the statement is not a true command. So we shall define declaratives that express the meaning of imperatives such that the declaratives are *true* just in case an imperator actually *wants* done what his or her imperative instructs listeners to do. Such imperatives are said to be *sincere*. Figure 1 below illustrates the concept underlying my thesis that shall be rigorously implement.



**Figure 1.** Thesis: for any imperative there’s a semantically isomorphic declarative that explicitly references an imperator’s wish for a listener-caused change (2) that makes a primary wish (1) true.

To flash forward briefly, the semantic structure in Figure 1 shall be explicitly implemented with modal operators that denote Wishes and possible Changes above. Given a set of conceivable states of affairs  $S = \{r, s, t, \dots\}$ , Wishes maps members of a set of agents  $A = \{a_1, \dots, a_n\}$  to members of the power set of the cross-product of  $S$ ,  $\mathcal{P}(S \times S)$ , whose members are sets of state pairs. So for example, suppose for simplicity that  $Wishes(a_1) = \{(s, u)\}$ , then the *set of wishes* of agent  $a_1$  contains one wish  $(s, u)$  that means: in state  $s$  agent  $a_1$  *wants* state  $u$ . Changes shares the same structure except that if  $Changes(a_1) = \{(s, u)\}$ , then  $(s, u)$  means: in state  $s$  agent  $a_1$  *can cause* state  $u$ . This

semantic structure is illustrated below in Figure 2 showing a subset tree of  $\mathcal{P}(\mathbf{S} \times \mathbf{S})$ .



**Figure 2.** A subset tree of  $\mathcal{P}(\mathbf{S} \times \mathbf{S})$  forms a modal frame used to formally implement the conceptual semantics seen in Figure 1. *Wishes* and *Changes* form subsets of  $\mathcal{P}(\mathbf{S} \times \mathbf{S})$  that contain sets of state pairs where each set represents an agent's wants or state-transitions they can cause. The arrow in Figure 2 replicates the arrow in Figure 1 (except agents *A* and *B* there are here  $a_n$  and  $a_1$  respectively). The arrow also represents the mapping of the proxy imperatives we'll define with *Wishes* and *Changes*.

Jumping back on track, let's define a language to implement the *wants* thesis.

## 2.1 A proxy-imperative language $\mathcal{L}$

The language  $\mathcal{L}$  defines agent-specific modal operators for *wants* and *cause*.  $\mathcal{L}$  is similar in construction to epistemic logics. For example, Fagin *et al.*<sup>(4)</sup> define knowledge operators indexed to intelligent agents such that for  $n$  agents there are  $K_1, \dots, K_n$  operators where each  $K_i$  means "Agent  $i$  knows" and so  $K_i\phi$  means "Agent  $i$  knows  $\phi$ " where  $\phi$  is a proposition variable. We'll also define modal *wants* and *cause* operators that are specific to agents. Let us then begin with a generative grammar for  $\mathcal{L}$ .

**DEFINITION 1** (*language  $\mathcal{L}$* ). Given a *vocabulary* =  $\langle \mathbf{P}, \mathbf{N}, \mathbf{U}, \mathbf{B}, \mathbf{M}, \mathbf{A} \rangle$  composed of six sets of *atomic propositions*  $\mathbf{P} = \{p, p', \dots\}$ , *names*  $\mathbf{N} = \{n_1, \dots, n_n\}$ , *unary connectives*  $\{\neg\}$ , *binary connectives*  $\mathbf{B} = \{\rightarrow, \wedge, \vee\}$ , *modalities*  $\mathbf{M} = \{[\omega], \langle \omega \rangle, [c], \langle c \rangle\}$ , and *auxiliary symbols*  $\{ (, ) \}$  the *formulae* of  $\mathcal{L}$  form the smallest set  $\mathbf{F}$  such that:

1. If  $p \in \mathbf{P}$ , then  $p \in \mathbf{F}$ .
2. If  $\phi \in \mathbf{F}$ , then  $\neg\phi \in \mathbf{F}$ .
3. If  $\bullet \in \mathbf{B}$ , and  $\phi, \psi \in \mathbf{F}$ , then  $(\phi \bullet \psi) \in \mathbf{F}$ .
4. If  $[\bullet], \langle \bullet \rangle \in \mathbf{M}$ ,  $n \in \mathbf{N}$ , and  $\phi \in \mathbf{F}$ , then  $[\bullet]n(\phi), \langle \bullet \rangle n(\phi) \in \mathbf{F}$ .

The syntactic structure for the proxy-imperatives we'll define appears in 1.4 above. The  $\omega$  modality means *wants* and the  $c$  modality means *cause*, which in Greek would be the *leipeic* and *aitic* modalities respectively. Each type  $\bullet$  has two modes, one mode  $[\bullet]$  expresses *necessity* and the other  $\langle\bullet\rangle$  *possibility*. Accordingly, these modes shall have these preferred English interpretations (where  $\varphi$  is an arbitrary formula in F):

1.  $[\omega]n(\varphi)$  reads: *n must have  $\varphi$ .*
2.  $\langle\omega\rangle n(\varphi)$  reads: *n accepts  $\varphi$ .*
3.  $[c]n(\varphi)$  reads: *n must cause  $\varphi$ .*
4.  $\langle c\rangle n(\varphi)$  reads: *n can cause  $\varphi$ .*

Other translations are possible. Instead of 'must have' in mode 1 above we could say 'requires'. As for mode 2 above, 'accepts' may in some cases be replaced with 'likes', and so 'loves' might replace 'must have' in mode 1 since *likes* and *loves* reflect weaker and stronger modes of wanting. A number of English terms point in similar directions and could be chosen to describe specific situations in various domains of discourse.

Both *wants* and *cause* modalities have classic modal negation transformations with intuitive translations (left as an exercise for curious readers).

$$\begin{array}{ll} [\bullet]n(\varphi) \Leftrightarrow \neg\langle\bullet\rangle n(\neg\varphi) & \langle\bullet\rangle n(\varphi) \Leftrightarrow \neg[\bullet]n(\neg\varphi) \\ [\bullet]n(\neg\varphi) \Leftrightarrow \neg\langle\bullet\rangle n(\varphi) & \langle\bullet\rangle n(\neg\varphi) \Leftrightarrow \neg[\bullet]n(\varphi) \end{array}$$

So for example,  $\langle\bullet\rangle n(\neg\varphi)$  is the negation-normal form of  $\neg[\bullet]n(\varphi)$ , and each can replace the other due to their equivalence. Now we introduce the proxy-imperative schemata.

## 2.2 Proxy-imperative schemata

From the modes of *wants* and *cause* the *proxy-imperative* schemata are formed:

1.  $[\omega]n[c]n'(\varphi)$  reads: *n must have it that n' must cause  $\varphi$ .*
2.  $[\omega]n\langle c\rangle n'(\varphi)$  reads: *n must have it that n' can cause  $\varphi$ .*
3.  $\langle\omega\rangle n[c]n'(\varphi)$  reads: *n accepts that n' must cause  $\varphi$ .*
4.  $\langle\omega\rangle n\langle c\rangle n'(\varphi)$  reads: *n accepts that n' can cause  $\varphi$ .*

These are the statement schemata we'll use for proxy-imperatives. It's essential to note that they do *not* denote imperative statements but rather *facts* that hold true about any imperator. The proxy-imperatives denote *preconditions* for the utterance of imperatives that also hold true concurrently with imperative utterance. Proxy-imperatives can be true even if no imperative is uttered. To denote in our translations that the utterance of an

imperative has occurred, ‘must have it’ in 1 above may be replaced with ‘demands’ or ‘commands’. And ‘asks’ or ‘requests’ may replace ‘accepts’ in 3 and 4. Not expressly denoting an utterance, ‘allows’ or ‘permits’ may also replace ‘accepts’. Many English terms point in similar directions.

Schema 1 denotes conditions underlying the strongest imperatives, *commands*, which denote the highest degree of wanting and of necessity of compliance. On the opposite end, schema 4 models *requests*, the least urgent and most polite imperatives like: “If possible, please pick up some milk after work,” or “Could you please pass the salt?” The four proxy-imperative schemata can cover a wide range of imperative statements.

**ℒ AXIOMS:** for all  $[\bullet], \langle \bullet \rangle \in M$ , all  $n, n' \in N$ , and any  $\varphi \in F$  we accept as true:

1.  $[\bullet]n(\varphi) \Rightarrow \langle \bullet \rangle n(\varphi)$
2.  $[\omega]n[c]n'(\varphi) \Rightarrow [\omega]n(\varphi)$

In the case of *wants* ( $\omega$ ), Axiom 1 says: *if n must have  $\varphi$ , then n accepts  $\varphi$* . Obviously, if I must win, I’ll accept winning. For *cause*, Axiom 1 says: *if n must cause  $\varphi$ , then n may cause  $\varphi$* . These are both not only intuitive but Axiom 1 also prevents vacuous truth for the necessary modes in our semantics, as we shall see shortly. Axiom 2 says: *if n must have it that n' must cause  $\varphi$ , then n must have  $\varphi$* . Obviously, if I must have it that you must close the door, then I must have the door closed. Axiom 2 says all imperatives are sincere.

### 2.3 Proxy-imperative behavior

Now we’ll compare proxy-imperatives with real imperatives. First, observe that because the minimal mode of wanting  $\langle \omega \rangle$  denotes what is *acceptable*, a negated proxy-imperative command is not a model for a contrary command but instead for contrary *permission*.

$$\neg[\omega]n[c]n'(\varphi) = \langle \omega \rangle n \langle c \rangle n'(\neg\varphi)$$

*It’s not the case that n demands n' must cause  $\varphi$  = n accepts that n' may cause not- $\varphi$*

That equivalence implies that the negation of “Shut the door!” is not the contrary command “Don’t shut the door!” but the contrary permit: “You may leave the door open.” So according to the proxy-imperatives of **ℒ**, a negated command *repeals* the command and *permits* contrary behavior. This in fact matches natural commands of which public laws are canonical. Take for example the military draft. What happens when we repeal a command by a *leader* that any man, let’s say *Jon*, must enlist? Let’s see (the proposition  $p$  that’s commanded to be made true is ‘*Jon is enlisted.*’)

(a)  $[\omega]leader[c]jon(\text{‘Jon is enlisted.’})$

Reads: *The leader commands that Jon must enlist.*

So the negation of command  $a$  above is by negation normalization b, c, and d:

(b)  $\neg[\omega]leader[c]jon('Jon\ is\ enlisted.')$

Reads: *It's not the case that the leader commands that Jon must enlist.*

(c)  $\langle\omega\rangle leader\neg[c]jon('Jon\ is\ enlisted.')$

Reads: *The leader accepts that Jon need not enlist.*

(d)  $\langle\omega\rangle leader\langle c\rangle jon('Jon\ is\ not\ enlisted.')$

Reads: *The leader accepts that Jon may not enlist.*

So according to both our proxy-imperatives and natural intuition, repealing a draft's command "Enlist!" does not mean "Don't enlist!" but rather: "You *may not* enlist." (This intuitive result suggests that there's an inherent modal structure in imperatives.) Obviously no person who understands the repeal of a draft would fear arrest for enlisting as they would not interpret its negation as a command against enlisting. The example above shows that natural language and intuition behave like the proxy-imperatives such that in both systems a negated command is not a contrary command but contrary *permission*.

## 2.4 A proxy-imperative semantics

And now let's explore the meaning, or semantics, of  $\mathcal{L}$  and its proxy-imperatives. We do that with a model for  $\mathcal{L}$  that defines a frame of objects and relations between them from which domains of discourse can be built and in which, by way of an interpretation, the statements of  $\mathcal{L}$  have their meaning. Here then is such a model for  $\mathcal{L}$ .

DEFINITION 2 (*model*). A model for language  $\mathcal{L}$  is  $\mathcal{M} = \langle S, A, \text{Wishes}, \text{Changes}, \alpha, V \rangle$  where  $\langle S, A, \text{Wishes}, \text{Changes} \rangle$  is a domain frame and  $\langle \alpha, V \rangle$  is an interpretation for  $\mathcal{L}$ :

1.  $S$  is a non-empty set of *conceivable states of affairs*:  $S = \{s, s', s'', \dots\}$ .
2.  $A$  is a non-empty set of intelligent *agents*:  $A = \{a_1, \dots, a_n\}$ .
3.  $\text{Wishes} : A \rightarrow \mathcal{P}(S \times S)$  assigns to each agent a *set of wishes* in  $\mathcal{P}(S \times S)$  containing state pairs such that if  $(s, s') \in \text{Wishes}(a)$ , then in state  $s$  agent  $a$  *wants* state  $s'$ .
4.  $\text{Changes} : A \rightarrow \mathcal{P}(S \times S)$  assigns to each agent a set of *causable* state transitions in  $\mathcal{P}(S \times S)$  such that if  $(s, s') \in \text{Changes}(a)$ , then in state  $s$  agent  $a$  *can cause*  $s'$ .
5.  $\alpha : \mathbb{N} \rightarrow A$  assigns *names* to *agents* such that  $\alpha(n)$  is the agent named  $n$ .
6.  $V : \mathcal{P} \rightarrow \mathcal{P}(S)$  is a *valuation function* that assigns to each  $\mathcal{L}$  *proposition* a set of states such that if  $V(p) = \{s, s'\}$ , then proposition  $p$  holds true in states  $s$  and  $s'$ .

The  $\mathcal{L}$  frame requires a wider set of states to draw from than an alethic frame because *wanting* casts a wider net over states than alethic *possibility* given that one can want the

*impossible*. For example, you could want to be as big as a mountain or to travel in time, but such conceivable states are not possible states. So in the  $\mathcal{L}$  frame the set of states  $S$  contains *conceivable states* that may be impossible but still wantable. On the other hand, Changes does assign access relations to agents. For all  $a \in A$  and all  $s, s' \in S$ , if  $(s, s') \in \text{Changes}(a)$ , then state  $s'$  is possible from state  $s$ , and perhaps because agent  $a$  can cause  $s'$ . Conceivable states are plausibly infinite and possible states are a proper subset of  $S$ .

2.5 defines a name-assignment function  $\alpha$  such that for any name  $n \in N$ ,  $\alpha(n)$  is the intelligent agent named  $n$  in the domain of discourse.<sup>+</sup> If for any agent  $a \in A$  we have it that  $a = \alpha(n)$ , then  $\text{Wishes}(a) = \text{Wishes}(\alpha(n))$ . So we may represent the arbitrary agent by either  $a$  or  $\alpha(n)$ , and we use  $\alpha(n)$  in Definition 3 below. Definitions 2.3 and 2.4 above are foundational to my imperative thesis and follow standard modal definitional structure with the exception that they serve to model modes of *wanting* and *causability* respectively rather than alethic *possibility*, deontic *obligation*, or epistemic *knowing*.

DEFINITION 3 (*semantics*). Given  $\mathcal{L}$  model  $\mathcal{M}$ , the *truth conditions* in any state  $s \in S$  are (where  $(s) \models \varphi$  is read: *in state  $s$ ,  $\varphi$  is true*):

1.  $(s) \models p$  iff  $s \in V(p)$ .
2.  $(s) \models \neg\varphi$  iff  $(s) \not\models \varphi$ .
3.  $(s) \models \varphi \rightarrow \psi$  iff  $(s) \not\models \varphi$  or  $(s) \models \psi$ .
4.  $(s) \models \varphi \wedge \psi$  iff  $(s) \models \varphi$  and  $(s) \models \psi$ .
5.  $(s) \models \varphi \vee \psi$  iff  $(s) \models \varphi$  or  $(s) \models \psi$ .
6.  $(s) \models [\omega]n(\varphi)$  iff for all  $s' \in S$ , if  $(s, s') \in \text{Wishes}(\alpha(n))$ , then  $(s') \models \varphi$ .
7.  $(s) \models \langle\omega\rangle n(\varphi)$  iff for a  $s' \in S$ ,  $(s, s') \in \text{Wishes}(\alpha(n))$  and  $(s') \models \varphi$ .
8.  $(s) \models [c]n(\varphi)$  iff for all  $s' \in S$ , if  $(s, s') \in \text{Changes}(\alpha(n))$ , then  $(s') \models \varphi$ .
9.  $(s) \models \langle c \rangle n(\varphi)$  iff for a  $s' \in S$ ,  $(s, s') \in \text{Changes}(\alpha(n))$  and  $(s') \models \varphi$ .

By Axiom 1, in any  $\mathcal{L}$ -model  $\mathcal{M}$ , if  $(s) \models [\omega]n(\varphi)$ , then  $(s) \models \langle\omega\rangle n(\varphi)$ . So by Definitions 3.6 and 3.7, every agent *wants* at least one conceivable state. Otherwise,  $[\omega]n(\varphi)$  can be vacuously true by Definition 3.6 when agent  $\alpha(n)$  wants *no* state. This *serial* condition also blocks vacuous truth in alethic modal logic and applies to the *cause* modality such that every agent *can cause* at least one state. Axiom 1 is intuitively valid as well.\* Definitions 3.6 through 3.9 are unique and implement my thesis.

Definitions for the proxy-imperatives follow directly from Definitions 3.6 through 3.9. However, it's worth presenting them explicitly. They are for brevity presented in meta-logic rather than the meta-language of English used for Definitions 3.6 - 3.9.

DEFINITION 3 (amendment - *proxy-imperative definitions*)

10.  $(s) \models [\omega]n[c]n'(\varphi)$  iff:

$$\forall s' \forall s'' [ ( (s, s') \in \text{Wishes}(\alpha(n)) \wedge (s', s'') \in \text{Changes}(\alpha(n')) ) \Rightarrow (s'') \models \varphi ]$$

11.  $(s) \models [\omega]n\langle c \rangle n'(\varphi)$  iff:

$$\forall s' [ (s, s') \in \text{Wishes}(\alpha(n)) \Rightarrow \exists s'' ( (s', s'') \in \text{Changes}(\alpha(n')) \wedge (s'') \models \varphi ) ]$$

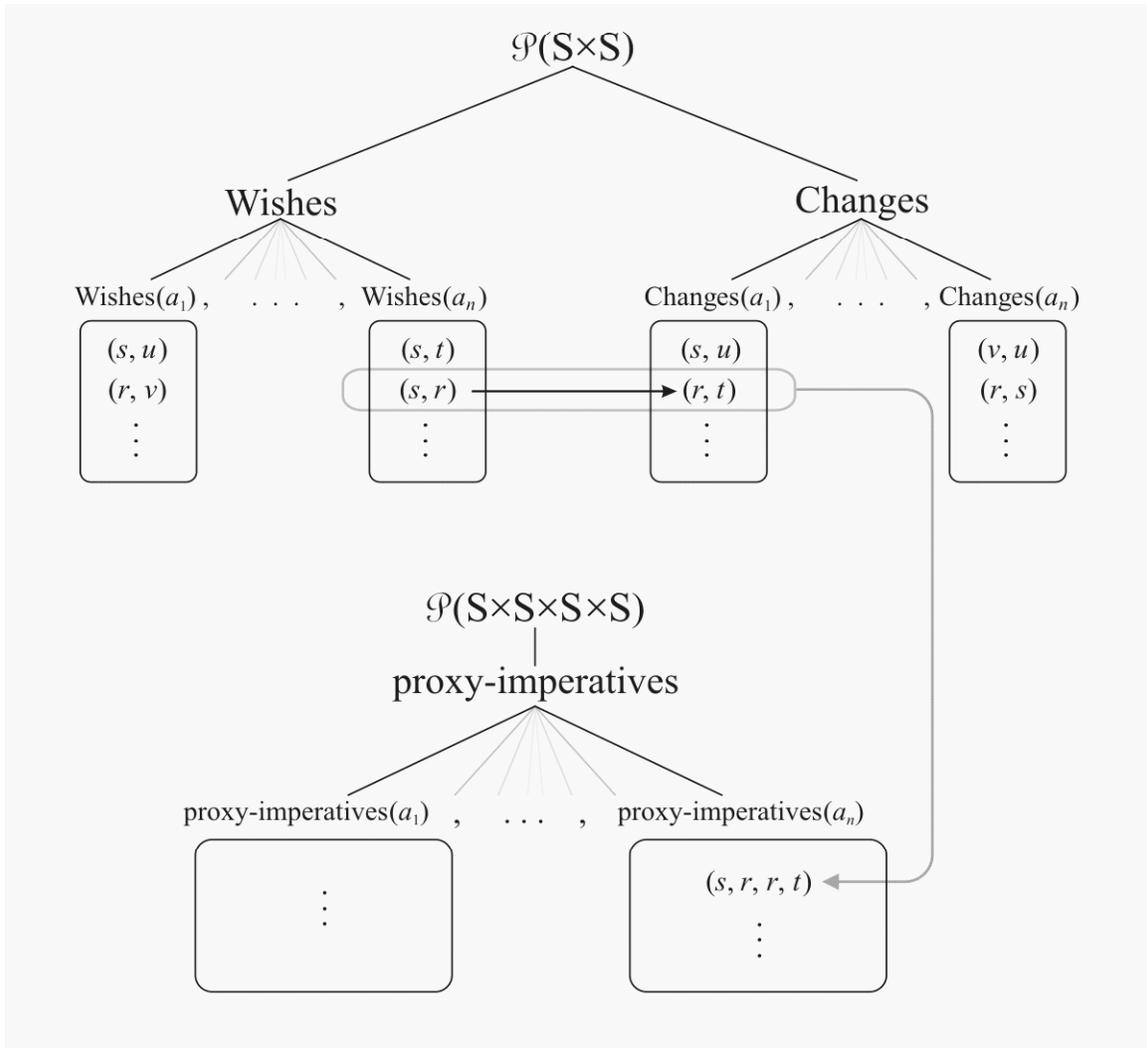
12.  $(s) \models \langle \omega \rangle n[c]n'(\varphi)$  iff:

$$\exists s' [ (s, s') \in \text{Wishes}(\alpha(n)) \wedge \forall s'' ( (s', s'') \in \text{Changes}(\alpha(n')) \Rightarrow (s'') \models \varphi ) ]$$

13.  $(s) \models \langle \omega \rangle n\langle c \rangle n'(\varphi)$  iff:

$$\exists s' \exists s'' [ (s, s') \in \text{Wishes}(\alpha(n)) \wedge (s', s'') \in \text{Changes}(\alpha(n')) \wedge (s'') \models \varphi ]$$

Figure 3 below extends Figure 2 by articulating the mapping on  $\mathcal{P}(S \times S)$  that builds the proxy-imperatives. Each proxy-imperative for some imperator agent  $\alpha(n)$  explicitly denotes the set  $\text{Wishes}(\alpha(n))$  which is that agent's *set of wishes*. This is an explicit implementation of Kenny's thesis that an imperative denotes its imperator's set of wishes, but we extend from his thesis by adding the *cause* modes that conjoin with the *wants* modes to form the proxy-imperatives as part of what is wanted is a change or null-change.



**Figure 3** the subset tree of  $S \times S$  branches into subsets *Wishes* and *Changes*, each divided into subsets, one for each agent  $\alpha(n)$ . Implementing the concept shown previously in Figure 1, here we have an example of agent  $\alpha(n_1)$  in state  $s$  wanting agent  $\alpha(n_2)$  in state  $r$  to cause state  $t$ . The resulting mapping to and from  $S \times S$  forms a subset of  $(S \times S) \times (S \times S)$  called *proxy-imperatives* which is divided above into subsets each of which contains state-quintuples that are preconditions for specific imperatives an agent may utter.

### 3 By proxy semantic validation of imperative argument

Now we put our proxy-imperatives to work to provide semantic proofs by proxy for imperative arguments. We assume that any meaningful imperative has an imperator and thus that there is at least one agent who *wants* it obeyed and whose name is  $i$ . Below, a natural-language imperative argument appears on the left (steps 1a, 2a, and 3a) and its translation into  $\mathcal{L}$  appears on the right (steps 1b, 2b, and 3b). Assume for this argument that proposition  $p = \text{'You see Jesse'}$  and  $q = \text{'The police are notified'}$ .

1a. If you see Jesse, call the police!

1b.  $p \rightarrow [\omega]i[c]n(q)$

- 2a. You see Jesse.  
3a. Call the police!

- 2b.  $p$   
3b.  $[\omega]i[c]n(q)$

PROOF: By 1b we assume that in model  $\mathcal{M}$ ,  $(s) \models p \rightarrow [\omega]i[c]n(q)$ . By Definitions 3.1 and 3.10 this means that we accept as true that if state  $s \in V(p)$ , then for all  $s', s'' \in S$ , if  $(s, s') \in \text{Wishes}(\alpha(i))$  and  $(s', s'') \in \text{Changes}(\alpha(n))$ , then  $(s'') \models q$ . Now, by 2b we have it as a fact that state  $s \in V(p)$ ; therefore, by assumption 1b and Definition 3.10 we also have it as a fact that for all conceivable states  $s'$  and  $s''$ , if agent  $\alpha(i)$  *wants* state  $s'$  and in state  $s'$  agent  $\alpha(n)$  *must cause* state  $s''$ , then in state  $s''$  proposition  $q$  is true, which is to say by Definition 3.10 again that we have it as a truth that:  $(s) \models [\omega]i[c]n(q)$ .  $\square$

Since perhaps most imperative arguments can be expressed in *modus-ponens* form as above, and because the proxy-imperatives are declaratives, it's trivial that we can provide semantic validation for any number of proxy-imperative translations of imperative arguments in the way shown above. So there's no need to further belabor the point that here we have a mechanism of providing *by proxy* semantic validation of imperative arguments.

But let's observe the semantic validation of an important proxy-imperative argument that acts as a proxy-imperative-reduction argument where 2a follows from 1a:

- 1a. Sue commands Jim to 'Close the door!'  
2a. Sue must have the door closed.

So if Sue *wants* Jim to *cause* the door to be closed, it follows that Sue *wants* the door closed. We can translate this into  $\mathcal{L}$  where a proposition  $p = \text{'The door is closed'}$  :

- 1b.  $[\omega]sue[c]jim(\text{'The door is closed.'})$   
2b.  $[\omega]sue(\text{'The door is closed.'})$

PROOF: By 1b we assume that in model  $\mathcal{M}$ ,  $(s) \models [\omega]sue[c]jim(\text{'The door is closed.'})$ . By Definition 3.10 this means we accept as true that in state  $s$ , for all states  $s'$  and  $s''$  if  $(s, s') \in \text{Wishes}(\alpha(sue))$  and  $(s', s'') \in \text{Changes}(\alpha(jim))$ , then  $(s'') \models \text{'The door is closed.'}$  Now, by Axiom 2, for all  $n, n' \in N$ , if  $[\omega]n[c]n'(\varphi)$  is true, then so is  $[\omega]n(\varphi)$ . In this case, by Definition 3.10 that means because 1b is true it's also true that  $(s, s'') \in \text{Wishes}(\alpha(sue))$ , which by Definition 3.10 again also means:  $(s) \models [\omega]sue(\text{'The door is closed.'})$ .  $\square$

## 4 Conclusion

The goal of this project has been to understand the semantic structure of natural imperatives and from such insight build a formal model of imperative semantics that can integrate imperatives into classical logic. So matching the behavior of natural imperatives has been both a goal and guide. Following the frequented path of defining alternatives to truth values in a new kind of logical system used only for imperatives was not an attractive option. My goal has been to facilitate semantic evaluation of imperatives within

*the same* semantic machinery used to evaluate declaratives. Such a model of imperatives would be the simplest model as it only requires preexisting modal-logic infrastructure. I believe and hope that the proxy-imperatives defined herein, which are declaratives that I posit as semantically isomorphic to imperatives, may be either a sufficient model of imperatives that brings them into the scope of classical logic or at least a useful start on that path.

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(1) Kenny, A.J. Practical Inference. *Analysis*, vol. 26 (1966), pp. 65-75.

(2) Geach, P.T. Dr. Kenny on Practical Inference. *Analysis*, vol. 26 (1966), pp 76-79.

(3) Gombay, A. What is imperative inference? *Analysis*, vol. 27 (1967), pp. 145-152.

(4) Fagin, R., Halpern, J.Y., Moses, Y., & Moshe Y. Vardi. *Reasoning about knowledge*. Cambridge, Massachusetts, MIT Press, 1995.