Asymmetrically distributive items and plural projection *

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Abstract

English *every* DPs, their German counterparts and distributive conjunctions in several languages give rise to cumulativity asymmetries: Such *asymmetrically distributive universals (ADUs)* allow for cumulative readings only if they occur in the scope of another semantically plural expression. We present a novel compositional analysis of these elements which has two advantages over existing accounts of cumulativity asymmetries with *every* DPs (Schein 1993, Kratzer 2003, Ferreira 2005, Zweig 2008, Champollion 2010): It is surface-compositional and it does not assume an inherent connection between cumulativity and events. More concretely, our analysis expands the 'plural projection' framework (Schmitt 2019), which derives cumulativity in a step-by-step process, following the syntactic structure. It does so by generalizing the notion of plurality to all semantic domains and by defining a new compositional rule that encodes cumulativity. Due to this rule, any constituent containing a semantically plural subexpression denotes a set of (possibly higher-type) pluralities, which reflect the part structure of embedded pluralities. We propose that ADUs operate on such sets of pluralities, which is why they have to be distributive w.r.t. their nuclear scope, and return another set of pluralities once they have combined with their scope argument. This correctly derives the cumulative readings of ADUs w.r.t. expressions outscoping them. Since the plural-projection mechanism preserves the part structure of material scopally dependent on ADUs, the approach extends to the more complex examples discussed by Schein (1993), where an ADU is 'sandwiched' between two plural expressions.

1 The puzzle

English DPs headed by *every* exhibit a well-known semantic asymmetry when co-occurring with plural expressions like *(the) two dogs* (Schein 1993, Kratzer 2003, Ferreira 2005, Zweig 2008, Champollion 2010). In some configurations, *every* DPs are restricted to a **distributive reading**: (1-b) is true in scenario (1-a-i) where the predicate *fed (the) two dogs* applies to each girl individually, but false in (1-a-ii). It thus lacks the **cumulative reading**, which would be true whenever each girl fed at least one dog, and each dog was fed by at least one girl.

(1) a. CONTEXT: Girls: Ada and Bea. Dogs: Carl and Dean.
  (i) SCENARIO: Ada fed Carl and Dean. Bea fed Carl and Dean.
  (ii) SCENARIO: Ada fed Carl. Bea fed Dean.
 b. ***Every girl** in this town fed (the) two dogs.* **true** in (1-a-i), **false** in (1-a-ii)

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But cumulative readings of *every* DPs are available if there is a semantically plural expression in a higher syntactic position: (2) is true in the 'cumulative' scenario (1-a-ii) and differs from (1-b) only in that the *every* DP occurs in object position, while *(the) two girls* is the subject.

(2) *(The) two girls fed* ***every dog*** *in this town.* **true** in (1-a-i), **true** in (1-a-ii)

This asymmetrical behavior of *every* DPs isn't shared by plural definites or indefinites: (3), where *(the) two girls* is the subject and thus in the same surface position as the *every* DP in (1-b), is true in scenario (1-a-ii) and hence has a cumulative reading.

(3) *The two girls in this town fed (the) two dogs.* **true** in (1-a-i), **true** in (1-a-ii)

We now sketch the compositionality problem raised by such asymmetries and our solution to it.

1.1 Basic asymmetries

The contrast between (1-b) and (2) illustrates the defining property of what we call **asymmetrically distributive universals (ADUs)**: They are limited to distributive readings relative to syntactically 'lower' plurals, but permit cumulative readings relative to syntactically 'higher' plurals.

The class of ADUs includes certain universal-quantifier DPs, like English *every* DPs or, for many speakers, their German correlates with *jed-* 'every'. It is a hitherto unnoticed fact that 'distributive conjunctions' (henceforth **D-conjunctions**) in German, Hungarian and Polish seem to exhibit the same asymmetry, suggesting that it reflects a cross-linguistic property of 'distributive' elements.

(4) exemplifies the asymmetry for the German D-conjunction pattern *sowohl A als auch B* 'A as well as B' (see § 4 for the other languages). As with *every* DPs, the position of the ADU relative to the other plural affects the availability of a cumulative reading: (4-b), where the ADU subject is syntactically higher than the definite plural, is false in scenario (4-a). (4-c), however, is true in the scenario and involves an ADU in object position, below the definite plural.

(4) German
  a. 'CUMULATIVE' SCENARIO: Two skiing races took place. Ada and Bea were the only German participants. Ada competed in the downhill and won. Bea competed in the slalom and won.
  b. *Heute haben* ***sowohl die Ada als auch die Bea*** *die zwei Rennen gewonnen!*
     today have PRT the Ada PRT also the Bea the two races won
     'Today, both Ada and Bea won the two races.' **false** in (4-a)
  c. *Heute haben die zwei Deutschen* ***sowohl die Abfahrt als auch den Slalom*** *gewonnen!*
     today have the two Germans PRT the downhill PRT also the slalom won
     'Today, the two Germans won both the downhill and the slalom.' **true** in (4-a)

The interpretative difference just observed could reflect a structural asymmetry, as implicitly assumed above, or a thematic asymmetry. In the latter case (see Kratzer 2003), the availability of cumulative readings would depend on the thematic roles of the ADU and the other plural expressions. Following Champollion (2010), we take the view that the asymmetry is structural and reflects scope, viewed as c-command at LF. (But see § 3.5 for more discussion.) This is supported by German sentences like (5-b), which permit the cumulative reading. In (5-b), the ADU is an agent, just like in (1-b). But unlike in (1-b), it occurs in the scope of the matrix subject, which is also a plural expression.

(5) a. SCENARIO: Detectives Ada and Bea were observing two suspects, Carl and Dean. Ada saw Carl sell cocaine. Bea saw Dean sell heroin.

b. *Gestern haben zwei Detektive jeden von diesen Kriminellen Drogen verkaufen gesehen.*
yesterday have two.NOM detectives.NOM every.ACC of these criminals drugs sell seen
'Yesterday, two detectives saw each of these criminals sell drugs.' **true** in (5-a)

1.2 Schein-sentences

Besides this simple structural asymmetry, ADUs pose another challenge for compositional semantics. A particular interaction between cumulativity and distributivity has been observed when ADUs are 'sandwiched' between two plural expressions, as in (6-b). We call these cases **Schein-sentences**, since they were first discussed in detail by Schein (1993).

(6) a. SCENARIO: Two dogs: Carl and Dean. Ada taught Carl tricks 1 and 2. Ada taught Dean trick 3. Bea taught Dean trick 2.

b. *Ada and Bea taught every dog two new tricks.* **true** in (6-a)
(adapted from Schein 1993)

(6-b) is true in scenario (6-a). On the relevant reading, *every dog* seems to cumulate with *Ada and Bea*, as it is not true that *each* of the girls taught every dog two tricks. But *every dog* is distributive w.r.t. *two tricks*, since each dog learns two (potentially different) tricks.

This reading cannot be straightforwardly described via a single cumulative relation between individuals. We cannot say that the relation [$\lambda x_e.\lambda y_e.y$ taught x two new tricks] holds cumulatively of the girls and the dogs: This would predict that for each girl x, some dog was taught two tricks by x, which is false in scenario (6-a) – Bea taught only one trick to one dog. Nor does the relation [$\lambda x_e.\lambda y_e.y$ taught x to every dog] hold cumulatively of the girls and some plurality of two tricks: This would entail that there are two tricks that each of the dogs was taught, but in scenario (6-a), the tricks are different for each dog. Finally, we cannot invoke a cumulative three-place relation [$\lambda x_e.\lambda y_e.\lambda z_e.$ z taught x to y]: This won't capture the distributivity requirement of *every dog* relative to *two new tricks*, predicting instead that the sentence can be true even if neither dog learned two tricks.

Accordingly, all three plural expressions seem to 'participate' in cumulativity, but this cannot reflect a single cumulative relation between individuals since *every dog* has scope over *two tricks*.

1.3 Sketch of our proposal

We saw that English and German have expressions – ADUs – that pattern as follows ((7-a) and (7-b), at least, can be replicated for Hungarian and Polish D-conjunctions):

(7) a. They permit cumulative readings w.r.t. syntactically higher plural expressions.
b. They prohibit cumulative readings w.r.t. syntactically lower plural expressions.
c. In Schein-sentences, they have a mixed cumulative/distributive reading that cannot reflect a single cumulative relation between individuals.

This paper presents a new account of ADUs which covers both singular universals and distributive conjunctions. It relies on a novel view of cumulativity proposed by Schmitt (2019), which derives cumulative readings 'step by step' via a special composition mechanism that is sensitive to syntactic structure.

The basic idea is that model-theoretic objects of higher types, like predicates or propositions, can also form pluralities that participate in cumulative relations. This permits the following paraphrase of the truth conditions of (6-b): We consider all predicates of the form $\lambda z.\mathbf{taught}(x)(y)(z)$, for a trick x and a dog y[1] and form a set containing all pluralities of such predicates that (i) 'cover' both of the dogs and (ii) relate each dog to two tricks. This set is sketched in (8), where + symbolizes a cross-categorial sum operation, defined in § 2.

(8) $\{\mathbf{taught(T_1)(C)} + \mathbf{taught(T_2)(C)} + \mathbf{taught(T_1)(D)} + \mathbf{taught(T_2)(D)},$
$\mathbf{taught(T_1)(C)} + \mathbf{taught(T_2)(C)} + \mathbf{taught(T_2)(D)} + \mathbf{taught(T_3)(D)},$
$\mathbf{taught(T_1)(C)} + \mathbf{taught(T_3)(C)} + \mathbf{taught(T_1)(D)} + \mathbf{taught(T_2)(D)}, \ldots\}$

If *taught every dog two new tricks* denotes this set, we can characterize the cumulative reading of (6-b) as follows: (6-b) is true iff Ada and Bea cumulatively satisfy a predicate sum from this set, i.e., some element P of (8) is such that each girl satisfies at least one predicate in P, and each predicate in P is satisfied by at least one girl.

To turn this into a convincing account of Schein-sentences, a principled way of deriving denotations like (8) is needed. Adapting Schmitt's 2019 proposal, we assume that whenever a constituent that would 'normally' be assigned semantic type a contains a plural, it actually denotes a set of pluralities of denotations of type a – a **plural set**. The property of denoting a plural set 'projects' from a constituent to its mother via special composition rules. Any node dominating a semantically plural expression will itself be plural unless an intervening operator blocks this process. Cumulativity falls out from the rules implementing this 'projection' mechanism and is thus derived in a surface-compositional way. We take this proposal to be superior to the existing accounts of ADUs (Schein 1993, Kratzer 2003, Ferreira 2005, Zweig 2008 but also Champollion 2010) as it generalizes to a larger set of cumulative sentences.

1.4 Structure of the paper

Section 2 introduces and motivates the compositional mechanism that underlies our analysis of *every* DPs and D-conjunctions in §§ 3 and 4, respectively. We discuss two competing accounts in § 5, and conclude in § 6.

2 Plural projection: Basic traits and motivation

We now outline the theory of plurals and cumulativity we will employ, the **Plural Projection analysis** ('**PPA**') from Schmitt (2019). It has two main features. First, **all semantic domains contain pluralities**: Besides pluralities of 'primitives' like individuals, we also have pluralities of properties, relations, propositions etc. For each type, these pluralities stand in a one-to-one correspondence with non-empty sets of atomic domain elements, but are not identified with such sets (Link 1983). Second, **pluralities 'project'** in the sense that, if a node α dominates an expression denoting a plurality, the denotation of α will reflect the part structure of that plurality. Our formal implementation of this idea resembles Alternative Semantics approaches to questions (Hamblin 1973), focus (Rooth 1985) and indefinites (Kratzer & Shimoyama 2002). Cumulativity will be **built into the composition rule** implementing this projection behavior.

[1]We represent denotations by boldfaced versions of (abbreviations of) the respective object-language expressions.

Here is a preview of the projection mechanism. Whenever a functor combines with an argument and at least one of the two constituents denotes a plurality, we obtain a plural value, as sketched in (9): When a non-plural functor combines with a plural argument, the result is the sum of all values obtained by applying the function to atomic parts of the argument. Likewise, if the functor is plural and the argument isn't, we get the sum of all values obtained by applying atomic parts of the function to the argument.

(9) $f(a) + f(b)$ [from f and $a + b$] $\quad$ $f(a) + g(a)$ [from $f + g$ and a]

However, whenever functor pluralities combine with argument pluralities, a single value plurality is insufficient: Cumulative truth conditions are compatible with several possible relations between function parts and argument parts. We therefore take plural expressions to denote sets of pluralities – **plural sets** – rather than single pluralities. Combining a set of function pluralities and a set of argument pluralities will yield a set of pluralities of the values obtained by applying atomic function parts to atomic argument parts. This set, schematized in (10), includes exactly those pluralities that 'cover' all the parts of some function plurality and all the parts of some argument plurality. Like (9), this denotation has accessible parts reflecting the structure of the original plurals.

(10) $\{f(a) + g(b), f(b) + g(a), f(a) + g(a) + g(b), f(b) + g(a) + g(b), f(a) + f(b) + g(a), f(a) + f(b) + g(b), f(a) + f(b) + g(a) + g(b)\}$ [from $\{f + g\}$ and $\{a + b\}$]

This operation is repeated at any node that dominates at least one plural: In cumulative sentences, it applies at each node intervening between the plural expressions participating in cumulativity. Sentences will thus denote plural sets of propositions, which count as true if at least one plurality in the set contains exclusively true propositions.

§§ 2.1 and 2.2 introduce this mechanism in greater detail. § 2.3 addresses its original motivation – which, as § 3 will show, is closely related to Schein-sentences.

2.1 Higher-order pluralities

The assumption that all semantic domains contain pluralities is motivated by the behavior of English *and*-conjunctions and German *und*-conjunctions (henceforth 'conjunctions'). Schmitt (2019) argues that conjunctions have cumulative readings regardless of the semantic type of their conjuncts, and should therefore be analyzed as denoting pluralities built from the conjunct denotations (pluralities of predicates, propositions etc.). As an illustration, consider the cumulative reading of a 'standard' sentence with two plural expressions like (3) above: It is true iff each of the girls fed at least one dog and each of the dogs was fed by at least one girl. Predicate conjunctions have an analogous reading, illustrated by scenario (11-a) for (11-b): For each of the girls it must hold that she made Gene do *P* or *Q* and for each of *P* and *Q*, there must be at least one girl who made Gene do it.[2]

(11) a. Scenario: Ada owns a nasty cat, Fay. Bea owns an aggressive dog, Dean. Both girls are going on a trip. Ada asked Gene to feed Fay. Bea asked Gene to brush Dean.

[2] Analyses of cumulative predicate conjunction like Link 1984, Krifka 1990, Heycock & Zamparelli 2005 won't work here: They require the predicate conjunction to directly take a plural argument, but the argument of the predicate conjunction in (11) is *him*.

b. *Poor Gene. The two girls made him [$_P$ feed Fay] and [$_Q$ brush Dean]!*
adapted from Schmitt (2019)

Propositional conjunctions pattern analogously: (12-b) is true in scenario (12-a), where each agency claimed at least one of p and q to be true, and for each of p and q, at least one agency claimed it to be true.

(12) a. Scenario: The Paris agency called and claimed that Macron was considering resignation. Later, the Berlin agency called and stated that Merkel had hired new bodyguards.
b. *The agencies claimed [$_p$ that Macron was considering his resignation] and [$_q$ (that) Merkel hired new bodyguards], but neither of them said anything about Brexit.*
adapted from Schmitt (2019)

Hence, predicate and propositional conjunctions pattern with type e plurals like *the (two) girls* or *Ada and Bea* concerning cumulativity. Schmitt (2019) therefore proposes that conjunctions of any type actually denote pluralities, whose atomic parts are the individual conjunct denotations. In analogy to Link's 1983 treatment of plural individuals, Schmitt adds pluralities to any semantic domain D_a. Pluralities are assumed to stand in a one-to-one correspondence with non-empty subsets of the domain: Just as D_e contains atomic individuals and pluralities thereof, $D_{\langle e,t\rangle}$ contains predicates and pluralities of predicates etc. So, *feed Fay and brush Dean* denotes the sum of the two atomic predicates $[\![$*feed Fay*$]\!]$ and $[\![$*brush Dean*$]\!]$, and the clausal conjunction in (12-b) denotes the sum of two propositions, p and q.

Admitting such pluralities gives us a new approach to cumulativity (Schmitt 2019). Consider first a 'standard' treatment of cumulative readings based on adding cumulation operators to the predicate (Link 1983, Krifka 1986, Sternefeld 1998, Beck & Sauerland 2000). This type of account, which we call the **predicate analysis**, derives the cumulative reading of sentence (3) by enriching the relation $[\![$*feed*$]\!]$ via the **-operator, which forms 'pointwise sums' of the pairs in the basic extension of *feed*, as shown in (13).

(13) a. $[\![feed]\!] = \{\langle a, c\rangle, \langle b, d\rangle\}$
b. $**[\![feed]\!] = \{\langle a, c\rangle, \langle b, d\rangle, \langle a + b, c + d\rangle\}$

On this approach, cumulativity always involves a cumulation operator (** for binary predicates, *** for ternary predicates, etc., see Sternefeld 1998) applying to a relation-denoting object language expression. This view is challenged by non-lexical cumulative relations, as in (14-a), which is true in the 'cumulative' scenario (14-b), but does not contain any surface constituent expressing the required relation, $[\lambda x_e.\lambda y_e.y$ wanted to feed $x]$. Beck & Sauerland (2000) suggest that in such cases, the two plurals move covertly to create a relation-denoting LF constituent that ** applies to (C in (14-c)).

(14) a. *The two girls wanted to feed the two dogs.* adapted from Beck & Sauerland (2000)
b. scenario: Ada wanted to feed Carl. Bea wanted to feed Dean.
c. [[the two girls] [[the two dogs] [** [$_C$ 2 [1 [t_1 wanted to feed t_2]]]]]]

Pluralities of functions, however, give us another way to derive cumulative readings. Consider first the simple example (15-a) in the cumulative scenario (15-b). In Schmitt's 2019 terms, this example involves an individual plurality ($[\![$*the two girls*$]\!]$) whose sister is a predicate plurality ($[\![$*fed F and brushed D*$]\!]$). This local configuration allows us to formulate a rule that encodes cumulativity without the ** operator. Informally speaking, for (15-a), this rule should yield all the propositional pluralities in (15-c), which 'cover' all atomic parts of the individual plurality and all atomic parts of the predicate plurality. Then (15-a) is true iff there is at least one such plurality that consists only of true atomic parts.

(15) a. *The two girls fed Fay and brushed Dean.*
b. Scenario: Ada fed Fay. Bea brushed Dean. (15-a) **true**
c. **fed(f)(a)+brushed(d)(b)**, **fed(f)(b)+brushed(d)(a)**, **fed(f)(a)+brushed(d)(b)+fed(f)(b)**, **brushed(d)(a)+brushed(d)(b)+fed(f)(b)**, **fed(f)(b)+brushed(d)(a)+fed(f)(a)**, **brushed(d)(b)+brushed(d)(a)+fed(f)(a)**, **fed(f)(b)+brushed(d)(b)+brushed(d)(a)+fed(f)(a)**

A type-independent formulation of this cumulation rule would give us a new approach to less local cases of cumulativity like (16) (= (3)), where the verb 'intervenes' between the two plurals: (16) reduces to two instances of the configuration in (15), a plurality of functions combining with a plurality of arguments. Our cumulation mechanism first combines **fed** with the object plurality ⟦*the two dogs*⟧, obtaining the predicate sum **fed(f)+fed(d)** as the denotation of the VP in (16). This sum combines with the subject plurality introduced by ⟦*the two girls*⟧ in the way sketched for (15). The derivation of (16) will therefore involve two applications of the cumulation rule, one at the VP node which yields a plural set of one-place predicates, and one for the IP which yields a plural set of propositions.

(16) *[$_{IP}$ The two girls [$_{VP}$ fed (the) two dogs]].*

We will now formalize this mechanism in detail. In effect, we will define a version of ** that generalizes to arbitrary function and argument types, and then apply this operation at any node dominating a plurality. The possibility of iterating the cumulation operation will remove the need for cumulative relations derived in covert syntax.

2.2 Plural projection: Details

Before discussing the new compositional rule, the ontological assumptions just sketched must be made explicit. (This section is based on Schmitt 2019, but introduces some new material.)

Ontology Motivated by cumulative readings of conjunctions, we assumed that conjunctions of any semantic category denote pluralities of the atomic domain elements. We therefore introduce a type-independent notion of sum: Sums of any type stand in a one-to-one correspondence to nonempty sets of atomic meanings of that type. Hence, the set $\{\mathbf{smoke}_{\langle e,t\rangle}, \mathbf{drink}_{\langle e,t\rangle}, \mathbf{dance}_{\langle e,t\rangle}\}$ corresponds to the sum $\mathbf{smoke}_{\langle e,t\rangle} + \mathbf{drink}_{\langle e,t\rangle} + \mathbf{dance}_{\langle e,t\rangle}$, which is a possible denotation of type $\langle e,t\rangle$ since it has three atomic parts of type $\langle e,t\rangle$.

As mentioned above, however, our approach to cumulativity requires a richer ontology in which semantically plural expressions denote sets of pluralities, so-called **plural sets**. Since ordinary unary predicates don't exhibit the compositional behavior of plurals, our type system will distinguish them from plural sets: For any type a, there is a matching type a^* for plural sets with elements of type a.

(17) The set T of **semantic types** is the smallest set such that $e \in T, t \in T$, for any $a, b \in T$, $\langle a,b\rangle \in T$, and for any $a \in T$, $a^* \in T$.

The domains of types a^* and $\langle a,t\rangle$ are disjoint, but will have the same algebraic structure. For instance, the plural set with elements $\mathbf{a}_e + \mathbf{b}_e$ and $\mathbf{a}_e + \mathbf{c}_e$ will be of type e^* and distinct from its counterpart in $D_{\langle e,t\rangle}$, the predicate $\lambda x_e.x = \mathbf{a}_e + \mathbf{b}_e \vee x = \mathbf{a}_e + \mathbf{c}_e$.

We formalize these ideas by introducing a distinction between the atomic domain A_a and the full domain D_a for each type a: A_a only contains the atomic domain elements, while D_a also contains their sums.

For instance, $\textbf{smoke}_{\langle e,t\rangle} + \textbf{drink}_{\langle e,t\rangle} + \textbf{dance}_{\langle e,t\rangle}$ is in $D_{\langle e,t\rangle}$, but not in $A_{\langle e,t\rangle}$. The atomic domains for basic types are stipulated in the usual way (18-a). For higher types – regular functional types and the starred types for plural sets – the atomic domain is defined recursively, based on the *full* domains of lower types (18-b,c). Thus, a function of type $\langle e, t\rangle$ may have plural individuals in its domain and return plural values. Finally, we stipulate that the domain of type a^* is disjoint from, but isomorphic to the set of unary predicates with a type a argument (18-c). This permits semantic rules sensitive to the distinction between plural sets (type a^*) and 'regular' predicates (type $\langle a, t\rangle$).

(18) a. $A_e = A$, the set of individuals; $A_t = \{0, 1\}^W$, where W is the set of possible worlds
b. For any types a, b: $A_{\langle a,b\rangle} = D_b^{D_a}$, the set of partial functions from D_a to D_b.
c. For any type a, A_{a^*} is a set that is disjoint from $A_{\langle a,t\rangle}$ and on which the operations $\cup$, $\cap$ and $\backslash$ are defined. Further, there is a function $pl^*_a : \mathcal{P}(D_a) \rightarrow A_{a^*}$ that is an isomorphism w.r.t. $\cup$, $\cap$ and $\backslash$.

(19-a) specifies the full domain D_a as follows: D_a contains all the pluralities made up of atomic domain elements from A_a. For simplicity, we assume a 'flat' plural semantics that is insensitive to subgroupings within a plurality (cf. Schwarzschild 1996 for discussion): The elements of D_a stand in a one-to-one correspondence to nonempty subsets of A_a. This correspondence is expressed by the isomorphism pl_a, which maps every nonempty subset of A_a to the corresponding plurality in D_a. (Unlike pl^*_a, pl_a is not a type-shifter, but an auxiliary notion needed to define D_a.) Since pl_a always combines with a set of *atomic* domain elements, we still need to define a sum operation on D_a that extends to non-atomic elements. This operation, called $\bigplus_a$, takes a nonempty subset of D_a as its argument and maps this set to its sum, a single element of D_a. (We could have started with a binary operation on elements of D_a, but our definition is more general since it permits sums of infinite cardinality.) Finally, clause (19-b) of the definition prevents us from directly identifying pluralities with sets of atomic meanings, which is necessary because the composition rules will treat them differently – e.g. $\mathbf{a}_e + \mathbf{b}_e$ should be an object of type e, not $\langle e, t\rangle$ or e^*.

(19) a. For each type a, there is an **atomic domain** A_a and a **full domain** D_a such that:
(i) $A_a \subseteq D_a$
(ii) There is a function $pl_a : \mathcal{P}(A_a)\backslash\{\emptyset\} \rightarrow D_a$ such that $pl_a(\{x\}) = x$ for each $x \in A_a$.
(iii) There is an operation $\bigplus_a : \mathcal{P}(D_a)\backslash\{\emptyset\} \rightarrow D_a$ such that pl_a is an isomorphism from $(\mathcal{P}(A_a)\backslash\{\emptyset\}, \bigcup)$ to $(D_a, \bigplus_a)$.
b. For any type $b \neq a$, D_a and D_b are disjoint.[3]

For readability, we introduce some notational conventions:

(20) a. We use 'starred' variables x^*, P^* etc. for types of the form a^*.
b. We sometimes omit type subscripts on cross-categorial operations like $\bigplus_a$ or pl_a.
c. For elements $x_1, \ldots, x_n$ of any type, we write $[x_1, \ldots, x_n]$ for the plural set $pl^*(\{x_1, \ldots, x_n\})$ with elements $x_1, \ldots, x_n$, and for any variable x, $[x \mid \phi]$ is the plural set $pl^*(\lambda x.\phi)$. Informally, square brackets replace the usual set brackets whenever we are characterizing plural sets.
d. For any type b and $x, y \in D_b$:
(i) $x +_b y =_{\text{def}} \bigplus_b(\{x, y\})$ (binary sum)
(ii) $x \leq y \Leftrightarrow_{\text{def}} x +_b y = y$ (parthood)
(iii) $x \leq_a y \Leftrightarrow_{\text{def}} x \leq y \wedge x \in A_b$ (atomic parthood)

[3]The empty partial function should be exempt from the disjointness conditions.

Here are some toy examples. (21) shows that D_e contains both atoms and pluralities of type e, and that the domain A_{e^*} of plural sets of individuals is isomorphic to $\mathcal{P}(D_e)$. (D_{e^*} then contains sums of such plural sets, like $\mathbf{[A] + [A + B]}$, plus the elements of A_{e^*}.) (22) illustrates that $D_{\langle e,t\rangle}$ contains sums of predicates in addition to the familiar 'atomic' predicates.

(21) a. $A_e = \{\mathbf{A}, \mathbf{B}\}$, $D_e = \{\mathbf{A}, \mathbf{B}, \mathbf{A} + \mathbf{B}\}$
b. $A_{e^*} = \{\mathbf{[\ ]}, \mathbf{[A]}, \mathbf{[B]}, \mathbf{[A + B]}, \mathbf{[A, B]}, \mathbf{[A, A + B]}, \mathbf{[B, A + B]}, \mathbf{[A, B, A + B]}\}$

(22) a. $A_{\langle e,t\rangle} = \{\mathbf{smoke}_{\langle e,t\rangle}, \mathbf{dance}_{\langle e,t\rangle}, (\lambda x.\mathbf{smoke}_{\langle e,t\rangle}(x) \vee \mathbf{dance}_{\langle e,t\rangle}(x)), \ldots\}$
b. $D_{\langle e,t\rangle} = \{\mathbf{smoke}_{\langle e,t\rangle}, \mathbf{dance}_{\langle e,t\rangle}, (\lambda x.\mathbf{smoke}_{\langle e,t\rangle}(x) \vee \mathbf{dance}_{\langle e,t\rangle}(x)),$
$\mathbf{smoke}_{\langle e,t\rangle} + \mathbf{dance}_{\langle e,t\rangle}, \mathbf{smoke}_{\langle e,t\rangle} + (\lambda x.\mathbf{smoke}_{\langle e,t\rangle}(x) \vee \mathbf{dance}_{\langle e,t\rangle}(x)),$
$\mathbf{dance}_{\langle e,t\rangle} + (\lambda x.\mathbf{smoke}_{\langle e,t\rangle}(x) \vee \mathbf{dance}_{\langle e,t\rangle}(x)), \ldots\}$

Semantics of plurals and conjunction This rich ontology allows us to give plural definites and indefinites a uniform type: Both denote plural sets of type e^*, as in (23). While *the girls* denotes a singleton containing the sum of all the girls, the indefinite *two pets* denotes the set of all sums of two pets.[4]

(23) a. $[\![$*the (two) girls*$]\!] = [\![$*the* [PL *girl*]$]\!] = \mathbf{[A + B]}$
b. $[\![$*two pets*$]\!] = [\![$*two* [PL *pet*]$]\!] = \mathbf{[C + D, C + E, D + E]}$

We assume that NPs like *girls* denote plural sets of predicates (type $\langle e, t\rangle^*$) to account for conjunctions or other plural expressions in the restrictor (e.g. *five cats and dogs*; cf. Heycock & Zamparelli 2005, Champollion 2016). Intuitively, the determiner combines with the set of those plural individuals whose atomic parts satisfy at least one of the predicates in the NP denotation – for *five cats and dogs*, *five* applies to the set of those pluralities that have only cats and dogs among their atomic parts.[5] We assume that this plural set is the output of a pluralization operator PL applying to the NP meaning (24-b).[6] Its definition relies on an auxiliary operator $\mathcal{A}$ (24-a) which extracts the atomic parts of all the pluralities in the NP denotation (24-a). (24-c) implements the familiar idea that the definite determiner selects the maximal plurality from the NP denotation. Numerals also combine with the output of PL, filtering out the sums of a certain cardinality (24-d).

(24) Plural definites and indefinites
a. $\mathcal{A}(P^*_{\langle e,t\rangle^*}) = \lambda x_e.(\exists P_{\langle e,t\rangle}.P \in pl^{*-1}(P^*) \wedge \exists P'_{\langle e,t\rangle}.P' \leq_a P \wedge P'(x))$
b. $[\![\textsc{pl}_{\langle\langle e,t\rangle^*,e^*\rangle}]\!] = \lambda P^*_{\langle e,t\rangle^*}.[x_e \mid \forall y_e(y \leq_a x \rightarrow \mathcal{A}(P^*)(y))]$
c. $[\![the_{\langle e^*,e^*\rangle}]\!] = \lambda x^*_{e^*} : \exists x \in pl^{*-1}(x^*)(\forall y \in pl^{*-1}(x^*)(y \leq x)).[\iota x \in pl^{*-1}(x^*).\forall y \in pl^{*-1}(x^*).y \leq x]$
d. $[\![two_{\langle e^*,e^*\rangle}]\!] = \lambda x^*_{e^*}.[x_e \mid pl^{*-1}(x^*)(x) \wedge |x| = 2]$, where $|x|$ is the number of atomic parts of x

Apart from the composition rule for cumulativity (discussed below), conjunction is the only binary operation in our system that directly combines two plural sets. The core of both operations is a cross-categorial operator $\oplus$ that combines multiple plural sets into one. This operator has a 'distributive' effect: It yields the set of all pluralities obtained by selecting one element from each argument set and summing up the selected elements. Thus, $[\![$*a girl and two pets*$]\!]$ in (25-a) denotes a plural set containing all sums of a girl and two pets. If the arguments are singleton sets as in (25-b), $\oplus$ produces a singleton, reflecting

[4]This is a generalization of Kratzer & Shimoyama's 2002 alternative semantics for indefinites.
[5]The set appears to be further restricted to pluralities with at least one cat part and at least one dog part (cf. Champollion 2016). If this restriction is a semantic phenomenon, it could be built into the PL operator (24-b).
[6]Determiners shouldn't directly take a type $\langle e, t\rangle^*$ argument: This would lead to a type mismatch in cases like *the two cats*.

the semantic parallelism between conjunctions (of any category) and plural definites.

(25) a. $[\![\textit{a girl and two pets}]\!]$
$= \mathbf{[A, B]} \oplus \mathbf{[C + D, C + E, D + E]}$
$= \mathbf{[A + C + D, A + C + E, A + D + E, B + C + D, B + C + E, B + D + E]}$
b. $[\![\textit{smoke and drink}]\!] = \mathbf{[smoke}_{\langle e,t\rangle}\mathbf{]} \oplus \mathbf{[drink}_{\langle e,t\rangle}\mathbf{]} = \mathbf{[smoke}_{\langle e,t\rangle} + \mathbf{drink}_{\langle e,t\rangle}\mathbf{]}$

(26) gives a general definition of this operation that permits arbitrarily many arguments. For arguments of a non-plural type, $\bigoplus$ coincides with the ordinary sum $+$. The 'distributive' effect emerges when the arguments are plural sets: In (26-b), all the different ways of choosing an element from each of the sets are considered. Each such choice corresponds to a possible value for the function variable f. The output is the set of all pluralities obtained by summing up the selected elements for some choice of f. (27) illustrates this for two functions that apply to the conjunct denotations in (25-a). If the selected elements are themselves plural sets, 'summing them up' means applying $\bigoplus$ recursively.[7]

(26) For any type a, the operation $\bigoplus_a : \mathcal{P}(D_a)\backslash\{\emptyset\} \to D_a$ is defined as follows:
For any nonempty $S \subseteq D_a$:
a. If a is a non-plural type (i.e. a is not of the form b^*): $\bigoplus_a S = +_a S$.
b. If $a = b^*$ for some type b: $\bigoplus_a S = \mathbf{[}\bigoplus_b(\{f(X^*) \mid X^* \in S\}) \mid f$ is a function from S to $D_b \wedge \forall X^* \in S : f(X^*) \in pl^{*-1}(X^*)\mathbf{]}$

(27) a. $f : \mathbf{[A, B]} \mapsto \mathbf{A}, \mathbf{[C + D, C + E, D + E]} \mapsto \mathbf{C + D}$
$\bigoplus_e(\{f(X^*) \mid X^* \in \{[\![\textit{a girl}]\!], [\![\textit{two pets}]\!]\}\}) = \mathbf{A + C + D}$
b. $f' : \mathbf{[A, B]} \mapsto \mathbf{B}, \mathbf{[C + D, C + E, D + E]} \mapsto \mathbf{D + E}$
$\bigoplus_e(\{f'(X^*) \mid X^* \in \{[\![\textit{a girl}]\!], [\![\textit{two pets}]\!]\}\}) = \mathbf{B + D + E}$

$\bigoplus$ provides an analysis of conjunction that integrates well with our DP meanings: According to (29), *and* requires two plural sets, so non-plural conjuncts must be shifted to singleton plural sets before combining with *and*. Given (29) and our semantics for definites and indefinites, the results in (25) follow.

(28) Notational convention: For any type a and any $x, y \in D_a$: $x \oplus_a y = \bigoplus_a(\{x, y\})$.

(29) $[\![\textit{and}_{\langle a^*,\langle a^*,a^*\rangle\rangle}]\!] = \lambda x_{a^*}.\lambda y_{a^*}.x \oplus_{a^*} y$ for any type a

We have now analyzed several classes of plural expressions found in cumulative sentences in terms of plural sets. The missing piece is an explicit formulation of the composition rule combining plural sets, which will replace predicate-level cumulation operators.

Adding plural projection to the compositional system Our goal is to define a generalized notion of cumulativity that can apply at any compositional step. For instance, to interpret the VP in (30), we want to combine a predicate conjunction of type $\langle e, \langle e, t\rangle\rangle^*$ with a plural object of type e^* and obtain a plural set of intransitive predicates (type $\langle e, t\rangle^*$).

(30) *Ada [$_{VP}$ fed and brushed the three pets].*

[7]Strictly speaking, $\bigoplus S$ is undefined if S contains a sum of two or more plural sets, since such sums are not in the domain of pl^{*-1}. Since this situation never occurs in our examples, it is irrelevant how the definition is extended to these cases.

In such cases, functional application should 'apply cumulatively' to a plural set of functions and a plural set of their arguments. But within existing theories of cumulativity, this doesn't make sense: cumulativity is only defined for operations with output type t, while functional application within a transitive VP returns a type $\langle e, t\rangle$ predicate. So what could cumulativity mean for operations with other output types? We will first reformulate the traditional notion of a binary cumulative relation using plural sets. The resulting definition will directly generalize to binary operations of other types.

On the standard view, a relation R holds cumulatively of two pluralities P and x iff each atomic part of P stands in relation R to some atomic part of x, and for each atomic part of x, at least one atomic part of P stands in relation R to it. Viewing R as a set of ordered pairs, this can be paraphrased as follows: Some subset C of R 'covers' all the atomic parts of P and x, in the sense that 1) the atomic parts of P are exactly the first components of the pairs in C, and 2) the atomic parts of x are exactly the second components of the pairs in C. The term **cover** is formally defined in (31-a); (31-b) gives some examples of covers for the verb conjunction and the plural object in (30).

(31) a. Let $P \in D_a, x \in D_b$. A relation $R \subseteq A_a \times A_b$ is a **cover** of (P, x) iff $+(\{P' \mid \exists x' : (P', x') \in R\}) = P$ and $+(\{x' \mid \exists P' : (P', x') \in R\}) = x$.

b. $P = \mathbf{fed} + \mathbf{brushed}$, $x = \mathbf{C} + \mathbf{D} + \mathbf{E}$
Covers: $\{\langle\mathbf{fed}, \mathbf{C}\rangle, \langle\mathbf{fed}, \mathbf{D}\rangle, \langle\mathbf{brushed}, \mathbf{E}\rangle\}, \{\langle\mathbf{fed}, \mathbf{C}\rangle, \langle\mathbf{brushed}, \mathbf{D}\rangle, \langle\mathbf{brushed}, \mathbf{E}\rangle\},$
$\{\langle\mathbf{fed}, \mathbf{C}\rangle, \langle\mathbf{fed}, \mathbf{D}\rangle, \langle\mathbf{brushed}, \mathbf{D}\rangle, \langle\mathbf{brushed}, \mathbf{E}\rangle\}, \ldots$
Not a cover: $\{\langle\mathbf{fed}, \mathbf{C}\rangle, \langle\mathbf{fed}, \mathbf{D}\rangle, \langle\mathbf{fed}, \mathbf{E}\rangle\}$
Not a cover: $\{\langle\mathbf{fed}, \mathbf{C}\rangle, \langle\mathbf{brushed}, \mathbf{E}\rangle\}$

The truth conditions of cumulative sentences like (30) can then be expressed *via* existential quantification over covers: R holds cumulatively of P and x iff there is a cover C of (P, x) such that each pair in C satisfies R. Now recall our idea that cumulative sentences denote plural sets of propositions (§ 2.1) and their truth conditions are derived by existential quantification over these sets. The generalization about covers brings us closer to an implementation of this idea if we can somehow map the covers to sums of propositions. Intuitively, we want a plural set whose elements correspond to the different covers of (P, x) – e.g., the plural set assigned to (30) will contain elements for the covers in (31-b), among others. If we have access to the relation R that applies cumulatively to the two pluralities, there is an obvious way of mapping a cover to a sum of propositions: For each pair (P', x') in the cover, the proposition $R(P', x')$ will be an atomic part of the sum. This generalization is stated in (32-a). Assuming that the cumulative relation between the predicate sum **fed** + **brushed** and the individual sum **C** + **D** + **E** is the one in (32-b), the covers in (31-b) correspond to the propositional pluralities in (32-c).

(32) a. A sentence in which a relation R applies cumulatively to two pluralities P and x denotes the plural set $\mathbf{[}\bigoplus(\{R(P', x') \mid (P', x') \in C\}) \mid C$ is a cover of $(P, x)\mathbf{]}$.

b. $R = \lambda P_{\langle e,\langle e,t\rangle\rangle}.\lambda x_e.P(x)(\mathbf{Ada})$

c. $\mathbf{[fed(C)(Ada)} + \mathbf{fed(D)(Ada)} + \mathbf{brushed(E)(Ada)}, \mathbf{fed(C)(Ada)} + \mathbf{brushed(D)(Ada)} + \mathbf{brushed(E)(Ada)}, \mathbf{fed(C)(Ada)} + \mathbf{fed(D)(Ada)} + \mathbf{brushed(D)(Ada)} + \mathbf{brushed(E)(Ada)} \ldots \mathbf{]}$

Simple cumulative sentences can therefore be mapped to plural sets of propositions, such that a cumulative sentence is true iff the corresponding set contains at least one sum of true propositions. This notion of truth is formally defined in (33).[8] Accordingly, (32-c) counts as true iff the relation R applies cumulatively to P and x.

(33) A plural set $p^* \in A_{t^*}$ of propositions is **true** in a world w iff there is a plurality $p \in pl^{*-1}(p^*)$

[8] We leave open the question how cumulativity interacts with presupposition projection, homogeneity and other potential cases of trivalence.

s.th. for all $q \leq_a p$, $q(w) = 1$, and **false** in a world w iff for all pluralities $p \in pl^{*-1}(p^*)$, there is a $q \leq_a p$ s.th. $q(w) = 0$.

This doesn't yet let us interpret cumulative sentences compositionally – (32-a) isn't a compositional rule, but merely a generalization about sentences that somehow involve a cumulative relation: Since no constituent of (30) denotes the relation R in (32-b), we cannot access this relation unless we posit a corresponding LF constituent. However, our reformulation of the predicate analysis in terms of plural sets will now enable us to leave the predicate analysis behind.

(32-a) has two notable properties. First, unlike our earlier characterization of relations applying cumulatively, it is not a direct statement of truth conditions. Truth conditions are now assigned by (33), while (32-a) simply provides a way of computing a plural set. Second, R in (32-a) is a binary relation, i.e. a function mapping its two arguments to a truth value, but nothing prevents us from dropping this restriction and defining cumulative versions of binary operations with arbitrary output types as in (34-a). (34-a) can be further generalized to cases where R combines with two plural sets, not two single pluralities (34-b): We simply consider all relations that are covers of *some* element of P^* and *some* element of x^*.

(34) For arbitrary types a, b, c and any binary operation $R_{\langle a,\langle b,c\rangle\rangle}$:

a. When R applies cumulatively to two pluralities P_a and x_b, the result is the plural set $[\bigoplus(\{R(P', x') \mid (P', x') \in C\}) \mid C$ is a cover of $(P, x)]$, of type c^*.

b. When $R_{\langle a,\langle b,c\rangle\rangle}$ applies cumulatively to two plural sets $P^*_{a^*}$ and $x^*_{b^*}$, the result is the plural set $[\bigoplus(\{R(P', x') \mid (P', x') \in C\}) \mid \exists P \in pl^{*-1}(P^*), x \in pl^{*-1}(x^*) : C$ is a cover of $(P, x)]$, of type c^*.

(34-b) tells us how to lift any binary operation of arbitrary type $\langle a, \langle b, c\rangle\rangle$ to an operation on plural sets, which has type $\langle a^*, \langle b^*, c^*\rangle\rangle$. If the binary operation returns truth values, this simply replicates the effects of the predicate analysis. For instance, applying the relation R in (32-b) cumulatively (in the sense of (34-b)) to the plural sets **[fed + brushed]** and **[C + D + E]** yields the plural set of propositions in (32-c). Given definition (33), this predicts the same truth conditions as the predicate analysis. But since (34-b) applies to operations of arbitrary type, we can now approach the problem differently: Our original goal was to directly obtain a plural set of type $\langle e, t\rangle^*$ from the plural sets **[fed + brushed]** and **[C + D + E]**. In plural-less sentences, transitive verbs combine with their objects via functional application. Since functional application is a binary operation, we can derive a cumulative version of it from definition (34-b). This cumulative functional application rule takes two plural sets of types $\langle a, b\rangle^*$ and a^* as arguments, returning a plural set of type b^* (35-a). We add this operation to our semantics as the composition rule 'Cumulative Composition' (35-b).

(35) Cumulative Composition (CC)

a. For any $P^* \in D_{\langle a,b\rangle^*}$ and $x^* \in D_{a^*}$:
$C(P^*, x^*) = [\bigoplus(\{P'(x') \mid (P', x') \in R\}) \mid \exists P \in pl^{*-1}(P^*), x \in pl^{*-1}(x^*) : R$ is a cover of $(P, x)]$

b. For any meaningful expressions ϕ of type $\langle a, b\rangle^*$ and ψ of type a^*, $[\phi\ \psi]$ is a meaningful expression of type b^*, and $[\![\phi\ \psi]\!] = C([\![\phi]\!], [\![\psi]\!])$.

The effect of (35-a) is that for any cover of some plurality in the functor set and some plurality in the argument set, we perform functional application for all pairs in the cover and sum up the results. We then collect the value pluralities corresponding to different covers into one plural set.

Let's apply this to the VP in (30). We construct all the relations that are covers of some element of

[fed + brushed] and some element of **[C + D + E]** – i.e., all covers of (**fed + brushed**, **C + D + E**). Some examples of such covers are repeated in (36-a). Cumulative Composition returns the plural set of unary predicates indicated in (36-b) (we only give the pluralities corresponding to covers in (36-a) – naturally, every cover of the verb conjunction and the plural object yields an element of the set).

(36) a. $\{\langle \mathbf{fed}, \mathbf{C}\rangle, \langle \mathbf{fed}, \mathbf{D}\rangle, \langle \mathbf{brushed}, \mathbf{E}\rangle\}, \{\langle \mathbf{fed}, \mathbf{C}\rangle, \langle \mathbf{brushed}, \mathbf{D}\rangle, \langle \mathbf{brushed}, \mathbf{E}\rangle\}, \{\langle \mathbf{fed}, \mathbf{C}\rangle, \langle \mathbf{fed}, \mathbf{D}\rangle, \langle \mathbf{brushed}, \mathbf{D}\rangle, \langle \mathbf{brushed}, \mathbf{E}\rangle\}, \ldots$

b. **[fed(C) + fed(D) + brushed(E), fed(C) + brushed(D) + brushed(E), fed(C) + fed(D) + brushed(D) + brushed(E) . . .]**

How does this plural set of type $\langle e,t\rangle^*$ combine with a semantically singular, type *e* subject? Since ordinary functional application won't work, we invoke our earlier assumption that non-plural denotations can be shifted to singleton plural sets: We shift the subject to the plural set **[Ada]**, which combines with (36-c) via another application of CC. Since **Ada** is semantically singular, every predicate sum in (36-b) induces a unique cover that relates **Ada** to each atomic part of the sum. The resulting set of propositional pluralities, (37), has the same structure as (36-c). Given (33), this set captures the cumulative truth conditions of (30).

(37) **[fed(C)(A) + fed(D)(A) + brushed(E)(A), fed(C)(A) + brushed(D)(A) + brushed(E)(A), fed(C)(A) + fed(D)(A) + brushed(D)(A) + brushed(E)(A) . . .]**

We can now analyze (36-a) without recourse to syntactically derived cumulative relations. The locus of cumulativity is neither the lexicon nor an operator modifying relation-denoting expressions, but a composition rule that is essentially a cumulative version of functional application. To see how this mechanism differs conceptually from the predicate analysis and why we call it 'plural projection', consider an example where the plurals are not syntactic sisters, like (38-a). The simplest version of the predicate analysis would associate (38-a) with the LF in (38-b), where the cumulative truth conditions arise from the cumulation operator ** modifying *fed* and each step involves regular functional application. In contrast, our approach doesn't attribute cumulativity to a single LF constituent: The lexical meaning of *fed* is a relation between atomic individuals. This is mapped to a singleton plural set by a type-shift called ⇑ in (38-c). This shift, crucially, doesn't turn ⟦*fed*⟧ into a cumulative relation. Rather, cumulativity results from two applications of CC at the nodes marked *C* in (38-c).

(38) a. *The two girls fed two pets.*

b. [*the two girls* [[** *fed*] [*two pets*]]]

c. [$_C$ [*the two girls*] [$_C$ [⇑ *fed*] [*two pets*]]]

Given the DP meanings from (23), composition proceeds as follows: Since there are no proper pluralities in **[fed]**, each plurality in the object plural set induces a unique cover, and CC reduces to applying **fed** 'pointwise' to each such pet plurality (39-a). Thus, ⟦*fed two pets*⟧ preserves the plural structure of ⟦*two pets*⟧: The parts of the predicate pluralities in (39-a) correspond to the parts of the pet pluralities in (23). This behavior is what we call 'projection'. In the next step, we apply CC to this predicate set and the set containing the sum of the girls, resulting in a plural set of propositions indicated in (39-b). Importantly, the covers considered at this step, which relate girls to atomic parts of the predicate sums in (39-a), stand in a one-to-one correspondence to the covers we would construct on the predicate analysis, which relate girls to pets.[9]

[9]Two reviewers of an earlier draft note that our proposal doesn't model cases where cumulativity seems to access 'subatomic' parts of individuals: (i) is true if the two girls shared one of the pizzas. This phenomenon interacts the more complex configurations discussed in this paper, like Schein-sentences (e.g.,Bayer 1997).

(39) a. $[\![$*fed two pets*$]\!] = C([\textbf{fed}], [\![$*two pets*$]\!])$
$= C([\textbf{fed}], [\textbf{C} + \textbf{D}, \textbf{C} + \textbf{E}, \textbf{D} + \textbf{E}])$
$= [\textbf{fed}(\textbf{C}) + \textbf{fed}(\textbf{D}), \textbf{fed}(\textbf{C}) + \textbf{fed}(\textbf{E}), \textbf{fed}(\textbf{D}) + \textbf{fed}(\textbf{E})]$

b. $[\![$(39-a)$]\!] = C([\![$*fed two pets*$]\!], [\![$*the two girls*$]\!])$
$= C([\textbf{fed}(\textbf{C}) + \textbf{fed}(\textbf{D}), \textbf{fed}(\textbf{C}) + \textbf{fed}(\textbf{E}), \textbf{fed}(\textbf{D}) + \textbf{fed}(\textbf{E})], [\textbf{A} + \textbf{B}])$
$= [\textbf{fed}(\textbf{C})(\textbf{A}) + \textbf{fed}(\textbf{D})(\textbf{B}), \textbf{fed}(\textbf{C})(\textbf{B}) + \textbf{fed}(\textbf{D})(\textbf{A}), \textbf{fed}(\textbf{C})(\textbf{A}) + \textbf{fed}(\textbf{D})(\textbf{A}) + \textbf{fed}(\textbf{D})(\textbf{B}), \ldots,$
$\textbf{fed}(\textbf{C})(\textbf{A}) + \textbf{fed}(\textbf{E})(\textbf{B}), \textbf{fed}(\textbf{C})(\textbf{B}) + \textbf{fed}(\textbf{E})(\textbf{A}) \ldots]$

In sum, by iterating CC, we can interpret cumulative sentences surface-compositionally without having to derive 'pluralized' versions of predicates. Configurations where the two plurals are not in a strictly local relation, like (38-a), can be reduced to multiple local steps, because the meaning of an expression containing a plural preserves certain aspects of the part structure of that plural.

2.3 Independent motivation for plural projection

The predictions of the PPA, with its much richer ontology, differ from those of the predicate analysis in at least two relevant respects.

Recall that in the predicate analysis, operators like ** syntactically modify relation-denoting constituents. If the cumulative relation doesn't correspond to a surface constituent, as in (14) above, covert movement of the two plurals is invoked to derive a suitable LF constituent. This produces several problems that our system avoids. Here we focus on one issue, the so-called **flattening effect**, which has immediate relevance for our analysis of ADUs (see Schmitt 2019 for additional arguments).

The core assumption behind the predicate analysis – that cumulative relations always correspond to object-language constituents – is challenged by cumulative readings of sentences where one plural expression syntactically contains another. For instance, (40-b) is true in scenario (40-a).

(40) a. Scenario: Ada owns a dog, Carl. Bea owns another dog, Dean, and a cat, Eric. Ada and Bea went on a trip and made Gene take care of their pets: Ada made Gene feed Carl, and Bea made Gene feed Dean and brush Eric.

b. *The two girls made poor Gene [[$_P$ feed the two dogs] and [$_Q$ brush Eric]].* **true** in (40-a) (Schmitt 2019)

The predicate conjunction *P and Q* has a cumulative reading relative to *the two girls*: In scenario (40-a), it is not the case that each girl made Gene brush Eric, as a distributive interpretation of predicate conjunction (e.g. Partee & Rooth 1983) would require. Rather, the relation [$\lambda P.\lambda x.x$ made Gene do P] intuitively applies cumulatively to the two girls and the two predicates P and Q. Since no surface constituent expresses this relation, the obvious solution within the predicate analysis would be an LF like (41).

(41) [[*the two girls*] [[*feed the two dogs and brush Eric*] [** [2 [1 [t_1 *made Gene* t_2]]]]]]

But crucially, scenario (40-a) also requires a cumulative relation between *the two dogs* and *the two girls*, since neither of the girls made Gene feed both dogs. There is no obvious way of interpreting *feed the*

(i) *The two girls ate two pizzas.*

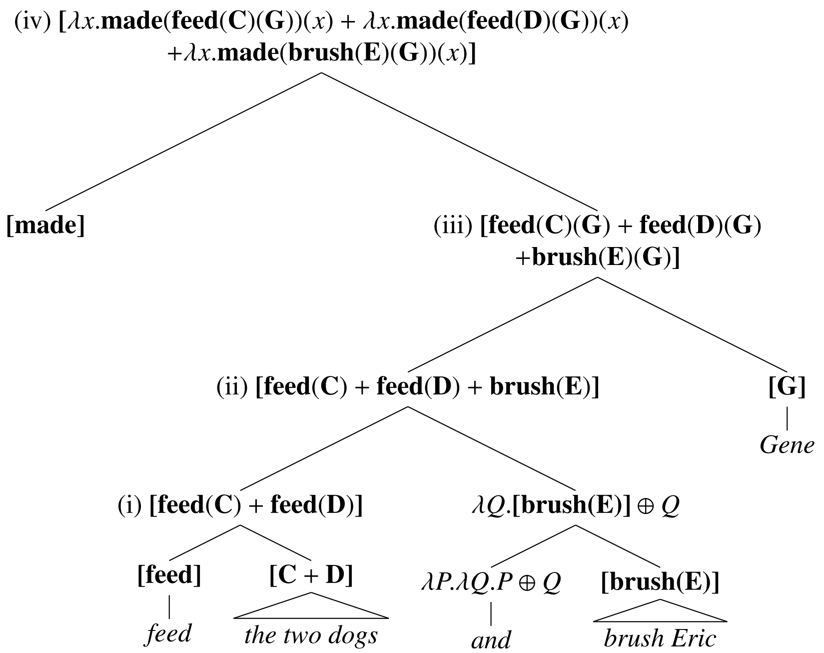
Our analysis could be extended to such cases if we enriched it with a material-part relation on individuals and liberalized the definition of a cover so that different material parts of the same pizza correspond to different pairs in a cover. We didn't attempt an implementation because we consider it an open (empirical) question how material parthood and the sensitivity of predicates to it should be modeled.

two dogs in (41) that accounts for this fact. In particular, interpreting it as being true of plural individuals that cumulatively feed both of the dogs won't solve the problem, since the semantic argument of *feed the two dogs* in (41) is the non-plural individual Gene.

Intuitively, three plurals participate in this cumulative reading – *the two girls*, *the two dogs* and the predicate conjunction – but there is no way of deriving a relation that could be modified by a cumulation operator, since one of the plurals syntactically contains another. If only two of the plurals move, as in (41), the resulting LF won't give us the right semantics. But moving *the two dogs* out of the predicate conjunction won't work either: The predicate conjunction would contain an unbound trace, yielding an uninterpretable structure. We thus have a cumulative reading that cannot be reduced to a single cumulative relation.

On our approach, there is no need to express the truth conditions of (40-b) in terms of such a relation: The derivations of cumulative sentences usually involve multiple applications of CC – one for each node on the 'path' between the relevant plurals. The relevant part of the derivation is sketched in (42), where CC applies at the nodes (i), (iii) and (iv). The structure of the plural set ⟦*the two dogs*⟧ = **[C + D]** 'projects' to the VP denotation in (i). In step (ii), the VPs are conjoined via the operation ⊕, which, when applied to two singleton plural sets, returns another singleton set containing the sum of their elements. The crucial property of our analysis that the predicate analysis cannot replicate is that the VP conjunction denotes a sum of *three* predicates, where the plural structure introduced by *the two dogs* and the plural structure introduced by conjunction are both preserved. Although one plural outscopes the other, we ultimately get a single 'flat' plurality with accessible parts corresponding to both plurals. We call this phenomenon the **flattening effect**.

(42)



In step (iii), each of these predicates applies to Gene. The resulting sum of three propositions combines with **[made]** via yet another application of CC. (Strictly speaking, this requires a cumulative version of intensional functional application, cf. Schmitt 2020) This step returns another sum of three predicates, which combines in the now familiar way with the plural matrix subject.

2.4 Interim summary

The above derivation illustrates two interesting properties of the PPA. First, it permits a surface-compositional treatment of non-lexical cumulative relations like [$\lambda P.\lambda x.x$ made Gene do P]. Second, it extends to cases where two plurals 'participate' in cumulativity although one outscopes the other. This is made possible by CC, a cumulative version of functional application. To derive this rule, we had to determine what it means for an operation with an arbitrary output type to apply cumulatively. Two nonstandard ontological assumptions were necessary for this notion to make sense: First, we distinguish ordinary unary predicates from plural sets, which are treated differently by semantic composition. Plural sets can be viewed as a generalization of Kratzer & Shimoyama's 2002 alternative-based semantics for indefinites. Second, we formulated a type-general sum operation, a move independently supported by analogies between conjunctions of arbitrary categories and plural definites.

3 A plural projection account of *every* DPs

But how does this machinery relate to ADUs? In § 1, we introduced two defining properties of ADUs. First, they cumulate with syntactically higher plurals, but not with syntactically lower plurals, (43) (= (1-b), (2)). Second, the mixed cumulative/distributive reading they have in Schein-sentences like (44) (= (6)) cannot be analyzed via a single cumulative relation between individuals.

(43) a. ***Every girl*** *in this town fed (the) two dogs.* **only distributive**
b. *(The) two girls fed* ***every dog*** *in this town.* **cumulative reading possible**

(44) a. SCENARIO: Two dogs, Carl and Dean. Ada taught Carl tricks 1 and 2. Ada taught Dean trick 3 and Bea taught Dean trick 2.
b. *Ada and Bea taught* ***every dog*** *two new tricks.* **true** in (44-a)

The latter property is what links the behavior of ADUs to the PPA. There is a structural analogy between Schein-sentences and the flattening effect: In our flattening example, one plural – the VP conjunction – outscopes another plural, but importantly, both plurals stand in a cumulative relation with a third plural higher up. Therefore their truth conditions cannot be expressed in terms of a single cumulative relation between individuals. Schein-sentences display exactly the same configuration: *two new tricks* occurs in the scope of the *every* DP, but still 'participates' in cumulativity in the sense that we need access to its part structure to formulate the correct cumulative truth conditions. In § 1.3, we sketched an intuitive take on this problem, namely that the subject in (44-b) must cumulatively satisfy at least one element of the plural set (45).

(45) $$\begin{aligned}&[\mathbf{taught}(\mathbf{T_1})(\mathbf{C}) + \mathbf{taught}(\mathbf{T_2})(\mathbf{C}) + \mathbf{taught}(\mathbf{T_1})(\mathbf{D}) + \mathbf{taught}(\mathbf{T_2})(\mathbf{D}),\\ &\mathbf{taught}(\mathbf{T_1})(\mathbf{C}) + \mathbf{taught}(\mathbf{T_2})(\mathbf{C}) + \mathbf{taught}(\mathbf{T_2})(\mathbf{D}) + \mathbf{taught}(\mathbf{T_3})(\mathbf{D}),\\ &\mathbf{taught}(\mathbf{T_1})(\mathbf{C}) + \mathbf{taught}(\mathbf{T_3})(\mathbf{C}) + \mathbf{taught}(\mathbf{T_1})(\mathbf{D}) + \mathbf{taught}(\mathbf{T_2})(\mathbf{D}), \dots]\end{aligned}$$

The predicate pluralities in (45) can be split into parts corresponding to the individual dogs, but also have accessible parts corresponding to the individual tricks they were taught. The denotation in (45) thus preserves the part structure introduced by *two new tricks* and treats it on a par with the part structure corresponding to the *every* DP. This is analogous to the 'flat' denotation for the VP in (40-b). The only missing component needed to derive (45) in our system is the lexical meaning for *every*. Together with the PPA, this meaning will account for cumulativity asymmetries and the behavior of *every* DPs in Schein-sentences. Thus our approach to the flattening problem illustrated by (40-b) will generalize to Schein-sentences.

3.1 The lexical meaning of *every*

In our analysis, *every* DPs are distributive regarding material in their scope, but the result of combining them with their scope argument is a plural set: Like conjunction and plural determiners, *every* directly manipulates plural sets of predicates, thus blocking application of CC. The atomic individuals satisfying the restrictor of *every* are matched up with pluralities of predicates from its nuclear scope. For instance, when we combine ⟦*every girl*⟧ with the plural set **[P + Q, R + S]**, each girl is associated with at least one predicate sum in this set, e.g. **Ada** with **P+Q** and **Bea** with **R+S**. This will yield the 'distributive effect'. By 'summing up' the results again, we obtain a plural set of values – in our example, $[\mathbf{P}(\mathbf{A}) + \mathbf{Q}(\mathbf{A}) + \mathbf{R}(\mathbf{B}) + \mathbf{S}(\mathbf{B})]$. Crucially, the pluralities in this set preserve the part structure of the scope argument of *every* and can be used for cumulation with syntactically higher plurals.

The operation in (46-a) (= (24)) takes a plural set of predicates and returns the set of all atomic individuals satisfying some part of a predicate sum in the set. (46-b) introduces another auxiliary operation, $\mathcal{D}$, which, given a plurality P of functions with argument type a and any object x of type a, applies each atomic part of P to x and sums up the values. (46-c) states the lexical entry for *every*: Its restrictor is a plural set of NP denotations, its nuclear scope a plural set of functions of type $\langle e, a\rangle^*$, for an arbitrary type a. Hence, *every* DPs can combine with predicates of any arity. The idea behind (46-c) is that when *every* combines with an NP and a plural set R^* of predicates, we consider different functions that map each atomic individual in the NP extension to an element of R^*. For each such function, we take every atomic NP individual, apply all the predicates in the corresponding predicate sum and then sum up the results over all the individuals. The sums obtained in this way are collected into a plural set.

(46) a. $\mathcal{A}(P^*_{\langle e,t\rangle^*}) = \lambda x_e.(\exists P_{\langle e,t\rangle}.P \in pl^{*-1}(P^*) \wedge \exists P'_{\langle e,t\rangle}.P' \leq_a P \wedge P'(x))$

b. For any $P_{\langle a,b\rangle}, x_a$: $\mathcal{D}(P, x) = +(\{Q(x) \mid Q \leq_a P\})$

c. $[\![every_{\langle\langle e,t\rangle^*,\langle\langle e,a\rangle^*,a^*\rangle\rangle}]\!] = \lambda P^*_{\langle e,t\rangle^*}.\lambda R^*_{\langle e,a\rangle^*}.[+(\{\mathcal{D}(f(x), x) \mid x \in \mathcal{A}(P^*)\})$
$\mid f$ is a function from $\mathcal{A}(P^*)$ to $pl^{*-1}(R^*)]$

3.2 Deriving cumulativity asymmetries

We start with the asymmetry in (47), a slightly simplified version of (43). (47-b) permits the cumulative reading, while (47-a) disallows it. We consider a scenario with two girls, Ada and Bea, and three pets, Carl, Dean and Eric.

(47) a. *Every girl fed two pets.*

b. *Two girls fed every pet.*

Given our assumptions from § 2, the VP in (47-a) denotes the plural set in (48): The structure of ⟦*two pets*⟧ projects via CC.

(48) $[\![\textit{fed two pets}]\!] = [\mathbf{feed}(\mathbf{C}) + \mathbf{feed}(\mathbf{D}), \mathbf{feed}(\mathbf{C}) + \mathbf{feed}(\mathbf{E}), \mathbf{feed}(\mathbf{D}) + \mathbf{feed}(\mathbf{E})]$

(49) gives the denotation of the *every* DP: *every* combines with its restrictor, a singleton plural set containing the predicate **girl**, yielding the function in (49-a), which reduces to (49-b).

(49) a. $[\![\textit{every girl}]\!] = [\![\textit{every}]\!]([\mathbf{girl}]) = \lambda R^*_{\langle e,a\rangle^*}.[+(\{\mathcal{D}(f(x), x) \mid x \in \mathcal{A}([\mathbf{girl}])\})$
$\mid f$ is a function from $\mathcal{A}([\mathbf{girl}])$ to $pl^{*-1}(R^*)]$

b. $\lambda R^*_{\langle e,a\rangle^*}.[+(\{\mathcal{D}(f(x), x) \mid x \in \{\mathbf{A}, \mathbf{B}\}\}) \mid f$ is a function from $\{\mathbf{A}, \mathbf{B}\}$ to $pl^{*-1}(R^*)]$
$= \lambda R^*_{\langle e,a\rangle^*}.[\mathcal{D}(f(\mathbf{A}), \mathbf{A}) + \mathcal{D}(f(\mathbf{B}), \mathbf{B}) \mid f$ is a function from $\{\mathbf{A}, \mathbf{B}\}$ to $pl^{*-1}(R^*)]$

$$= \lambda R^*_{\langle e,a\rangle^*}.[\mathcal{D}(P, \mathbf{A}) + \mathcal{D}(Q, \mathbf{B}) \mid P, Q \in pl^{*-1}(R^*)]$$

To apply this to the VP denotation in (48), we must consider all possible functions from $\{\mathbf{A}, \mathbf{B}\}$ – the set of atomic girls – to the plural set (48). (50) provides two examples.

(50) $\{\langle \mathbf{A}, \mathbf{fed}(\mathbf{C}) + \mathbf{fed}(\mathbf{D})\rangle, \langle \mathbf{B}, \mathbf{fed}(\mathbf{C}) + \mathbf{fed}(\mathbf{E})\rangle\}, \{\langle \mathbf{A}, \mathbf{fed}(\mathbf{C}) + \mathbf{fed}(\mathbf{E})\rangle, \langle \mathbf{B}, \mathbf{fed}(\mathbf{D}) + \mathbf{fed}(\mathbf{E})\rangle\}, \ldots$

The next steps required by (46-c) are as follows: (i) For each pair (P, x) in such an assignment, we sum up the values that result from applying atomic parts of P to x (51-a). (ii) For each assignment, we sum up these value pluralities for all pairs in the assignment (51-b). (iii) We collect these pluralities into a plural set (51-c).

(51) a. $\{\mathbf{fed}(\mathbf{C})(\mathbf{A}) + \mathbf{fed}(\mathbf{D})(\mathbf{A}), \mathbf{fed}(\mathbf{C})(\mathbf{B}) + \mathbf{fed}(\mathbf{E})(\mathbf{B})\}, \{\mathbf{fed}(\mathbf{C})(\mathbf{A}) + \mathbf{fed}(\mathbf{E})(\mathbf{A}), \mathbf{fed}(\mathbf{D})(\mathbf{B}) + \mathbf{fed}(\mathbf{E})(\mathbf{B})\}, \ldots$

b. $\mathbf{fed}(\mathbf{C})(\mathbf{A}) + \mathbf{fed}(\mathbf{D})(\mathbf{A}) + \mathbf{fed}(\mathbf{C})(\mathbf{B}) + \mathbf{fed}(\mathbf{E})(\mathbf{B})\}, \{\mathbf{fed}(\mathbf{C})(\mathbf{A}) + \mathbf{fed}(\mathbf{E})(\mathbf{A}) + \mathbf{fed}(\mathbf{D})(\mathbf{B}) + \mathbf{fed}(\mathbf{E})(\mathbf{B}), \ldots$

c. $[\mathbf{fed}(\mathbf{C})(\mathbf{A}) + \mathbf{fed}(\mathbf{D})(\mathbf{A}) + \mathbf{fed}(\mathbf{C})(\mathbf{B}) + \mathbf{fed}(\mathbf{E})(\mathbf{B}), \mathbf{fed}(\mathbf{C})(\mathbf{A}) + \mathbf{fed}(\mathbf{E})(\mathbf{A}) + \mathbf{fed}(\mathbf{D})(\mathbf{B}) + \mathbf{fed}(\mathbf{E})(\mathbf{B}), \ldots],$

We end up with (52) as our denotation for (47-a). This plural set counts as true iff at least one of the pluralities contains only true propositions – i.e., iff Ada and Bea each fed at least two pets.

(52) $[\![\textit{every girl}]\!]((48)) = [\mathbf{fed}(\mathbf{C})(\mathbf{A}) + \mathbf{fed}(\mathbf{D})(\mathbf{A}) + \mathbf{fed}(\mathbf{C})(\mathbf{B}) + \mathbf{fed}(\mathbf{E})(\mathbf{B}),$
$\mathbf{fed}(\mathbf{C})(\mathbf{A}) + \mathbf{fed}(\mathbf{E})(\mathbf{A}) + \mathbf{fed}(\mathbf{C})(\mathbf{B}) + \mathbf{fed}(\mathbf{D})(\mathbf{B}),$
$\mathbf{fed}(\mathbf{C})(\mathbf{A}) + \mathbf{fed}(\mathbf{E})(\mathbf{A}) + \mathbf{fed}(\mathbf{D})(\mathbf{B}) + \mathbf{fed}(\mathbf{E})(\mathbf{B}), \ldots]$

This correctly predicts the distributivity requirement of (47-a): Each atomic girl is assigned a separate predicate sum, which is applied to her 'distributively' via the operator $\mathcal{D}$. Since each girl must satisfy all the predicates in the sum assigned to her, and each predicate sum amounts to feeding a certain plurality of two pets, each girl is related to two pets. Thus, we don't get pluralities like $\mathbf{fed}(\mathbf{D})(\mathbf{A}) + \mathbf{fed}(\mathbf{C})(\mathbf{B})$, blocking the cumulative reading of (47-a): Although the entire sentence denotes a set of pluralities of propositions, there is no higher plural that could combine with this set via CC.

Next, we derive the cumulative reading of (47-b), where the *every* DP occurs in object position. First, we apply $[\![\textit{every pet}]\!]$ to the plural set $[\mathbf{fed}]$. Since this is a singleton, there is only one assignment of predicate sums to the individual pets. Definition (46) thus yields another singleton, (53-a). But since this set contains a plurality, it combines with the subject plurality via CC. The resulting plural set, sketched in (53-b), contains all sums of propositions of the form $\mathbf{fed}(x)(y)$ that 'cover' every pet and also 'cover' both Ada and Bea. This is exactly what we need for the cumulative reading.

(53) a. $[\![\textit{every pet}]\!]([\![\textit{fed}]\!]) = [\![\textit{every pet}]\!]([\mathbf{fed}]) = [\mathbf{fed}(\mathbf{C}) + \mathbf{fed}(\mathbf{D}) + \mathbf{fed}(\mathbf{E})]$

b. $C((53\text{-a}), [\mathbf{A} + \mathbf{B}]) = [\mathbf{fed}(\mathbf{C})(\mathbf{A}){+}\mathbf{fed}(\mathbf{D})(\mathbf{A}){+}\mathbf{fed}(\mathbf{E})(\mathbf{B}), \mathbf{fed}(\mathbf{C})(\mathbf{B})$
$+\mathbf{fed}(\mathbf{D})(\mathbf{A}){+}\mathbf{fed}(\mathbf{E})(\mathbf{A}), \mathbf{fed}(\mathbf{C})(\mathbf{B}){+}\mathbf{fed}(\mathbf{D})(\mathbf{A}){+}\mathbf{fed}(\mathbf{E})(\mathbf{B}), \ldots]$

Our analysis of *every* therefore predicts cumulativity asymmetries: Since *every* DPs directly take plural sets as their arguments, CC is blocked when they combine with their nuclear scope, and the distributive effect can be built into the lexical entry. Since combining *every* with its arguments produces a plural set, CC with *higher* plurals isn't blocked.

3.3 Deriving Schein-sentences

We now return to the interaction between distributivity and cumulativity in Schein-sentences. Recall that (54) (=(44-b)) is true in scenario (44-a).

(54) *Ada and Bea taught every dog two new tricks.* **true** in (44-a)

Due to CC and our analysis of plural indefinites, the predicate *taught two new tricks* denotes the plural set in (55-a). When combining this set with *every dog*, we must consider all possible functions mapping each dog to a sum in the set. Unlike in (53) above, the *every* DP now combines with a plural set with multiple elements. Therefore, Carl and Dean may be mapped to different elements of (55-a) – and since each dog is combined with all the predicates in his corresponding sum, we obtain a 'distributive' interpretation of *every dog* relative to *two new tricks*. For each assignment of predicate pluralities to the two dogs, the results of functional application are summed up, yielding the plural set in (55-b). This result matches our earlier intuitions: The predicate pluralities all encode 'distributivity' insofar as each dog is related to two tricks, but also have accessible parts corresponding to the individual dog-trick pairs, which reflect the part structure of *two tricks*.

(55) a. $[\![\textit{taught two new tricks}]\!] = C([\mathbf{taught}], [\![\textit{two new tricks}]\!]) =$
$[\mathbf{taught}(\mathbf{T1}) + \mathbf{taught}(\mathbf{T2}), \mathbf{taught}(\mathbf{T1}) + \mathbf{taught}(\mathbf{T3}), \mathbf{taught}(\mathbf{T2}) + \mathbf{taught}(\mathbf{T3})]$

b. $[\![[\textit{every dog}]\ [\textit{taught two new tricks}]]\!] = [\mathbf{taught}(\mathbf{T1})(\mathbf{C}) + \mathbf{taught}(\mathbf{T2})$
$(\mathbf{C}) + \mathbf{taught}(\mathbf{T2})(\mathbf{D}) + \mathbf{taught}(\mathbf{T3})(\mathbf{D}), \mathbf{taught}(\mathbf{T1})(\mathbf{D})$
$+ \mathbf{taught}(\mathbf{T2})(\mathbf{D}) + \mathbf{taught}(\mathbf{T2})(\mathbf{C}) + \mathbf{taught}(\mathbf{T3})(\mathbf{C}), \ldots]$

Finally, (55-b) combines with **[Ada + Bea]** via CC, resulting in the plural set of propositions in (56).

(56) $C([\![(55\text{-b})]\!]([\mathbf{A} + \mathbf{B}]) =$
$[\mathbf{taught}(\mathbf{T1})(\mathbf{C})(\mathbf{A}) + \mathbf{taught}(\mathbf{T2})(\mathbf{C})(\mathbf{A}) + \mathbf{taught}(\mathbf{T2})(\mathbf{D})(\mathbf{B}) + \mathbf{taught}(\mathbf{T3})(\mathbf{D})(\mathbf{A}),$
$\mathbf{taught}(\mathbf{T1})(\mathbf{D})(\mathbf{B}) + \mathbf{taught}(\mathbf{T2})(\mathbf{D})(\mathbf{B}) + \mathbf{taught}(\mathbf{T2})(\mathbf{C})(\mathbf{A}) + \mathbf{taught}(\mathbf{T3})(\mathbf{C})(\mathbf{A}), \ldots]$

Accordingly, the sentence is true if there is a predicate plurality P in (55-b) such that Ada and Bea each satisfy at least one atomic part of P, and each atomic part of P is satisfied by Ada or Bea. This matches our paraphrase for Schein-sentences from § 1.

3.4 Interim summary

We supplemented the PPA with a lexical entry for *every*: *every* DPs are distributive regarding material in their scope, but when they compose with their nuclear scope, the result is a plural set whose structure reflects the parts of pluralities in the scope of the *every* DP, making them accessible for further cumulative composition. This proposal correctly predicts cumulativity asymmetries and extends to the particular readings of Schein-sentences.[10]

[10] A reviewer of a previous draft wonders how the analysis extends to other distributivity markers, like *each* in (i-a). One analytical possibility, modeled on our entry for *every*, is given in (i-b). Its impact is illustrated in (i-d) using the VP-denotation in (i-c): $[\![\textit{each}]\!]$ takes the plural set containing all predicate pluralities of the kind **feed dog1** + **feed dog2**. It returns another plural set of predicates, whose elements each consist of atomic functions that map an individual to some predicate plurality in the argument set (one such atom would be the function $\lambda x.\mathbf{fed\ dog1}(x) + \mathbf{fed\ dog2}(x)$).

(i) a. *The girls each fed two dogs.*

b. $[\![\textit{each}]\!] = \lambda P^*_{\langle e,t\rangle^*}.[Q_{\langle e,t\rangle} \mid \forall Q' \leq_a Q.\exists P \in P^*.Q' = (\lambda x_e.\bigplus\{P'(x) | P' \leq_a P\})]$

c. $[\![\textit{fed two dogs}]\!] = [\mathbf{feed\ dog1} + \mathbf{feed\ dog2}, \mathbf{feed\ dog2} + \mathbf{feed\ dog3}, \mathbf{feed\ dog1} + \mathbf{feed\ dog3}]$

3.5 Revisiting the scope generalization

Before expanding the proposal to distributive conjunctions, we briefly revisit our assumption that cumulativity asymmetries are scope-related (following Champollion 2010). Our account is sensitive to scope if the latter is modeled as LF c-command, but it also makes the more specific prediction in (57). Since we assumed that cumulative sentences have no special syntactic properties, (57) connects the availability of cumulative readings to the scope options ADUs have on a *distributive* reading: If an ADU cannot scope below another plural on its distributive reading, a cumulative reading relative to that plural should be unavailable.

(57) An ADU can have a cumulative reading relative to another plural expression iff that plural expression may c-command the ADU at LF.

We now discuss some results of an informal survey of 30 speakers that we carried out to test this prediction for German *jed-* ('every') DPs in embedded subject position and in scrambling configurations.[11] They suggest that cumulativity asymmetries are indeed influenced by scope – it is the structural position rather than the thematic role of *jed-* DPs that matters – but also indicate that (57) is too strong: Certain configurations disallow cumulativity although their scopal properties should permit it. Further, the *base positions* of the plural expressions also seem to matter, an unexpected result under any existing analysis.

First, we tested examples with embedded infinitives where the embedded subject is an agent, but scopally lower than the matrix subject. Kratzer's 2003 description of the cumulativity asymmetries found with *every* DPs in terms of thematic roles predicts AGENT DPs to lack cumulative readings. Cumulativity should then also be blocked for *embedded* subjects that are agents. This prediction was not borne out: Most of our consultants accepted the cumulative reading for (5-b) above, but not for (58), as predicted by any analysis that appeals to the DPs' structural positions.

(58) *Gestern hat jeder Detektiv zwei von diesen Kriminellen Drogen verkaufen*
yesterday has every.NOM detective.NOM two.ACC of these criminals drugs sell
gesehen.
seen
'Yesterday, every detective saw two of these criminals sell drugs.' ***cumulative**

Certain scrambling data support this conclusion. Since scrambling can affect scope (Frey 1993, Beck 1996, Büring 1997, Pafel 2005, Wurmbrand 2008 a.o.), but not thematic roles, any effect of scrambling on cumulativity would support the scope-based account. We found such effects for examples like (59): Most consultants accepted the cumulative reading for a non-scrambled *jed-* DP in object position (59-a), as expected, and scrambling of the *jed-* DP affects the availability of this reading (59-b). The contrast, however, is less clear-cut than that in (58).

(59) a. *Gestern haben zwei Jäger jeden Hirsch in diesem Wald erschossen.*
yesterday have two.NOM hunters.NOM every.ACC stag.ACC in this forest shot
'Yesterday, two hunters shot every stag in this forest.'
NUM NP (SUBJ) > every NP (OBJ) **✓cumulative [some variation]**
b. *Gestern haben jeden Hirsch in diesem Wald zwei Jäger erschossen.*

---

d. ⟦*each fed two dogs*⟧ = [$(\lambda x_e.$**feed dog1**(x) + **feed dog2**$(x))$ + $(\lambda x_e.$**feed dog2**(x) + **feed dog3**$(x))$, $(\lambda x_e.$**feed dog1**(x) + **feed dog2**$(x))$ + $(\lambda x_e.$**feed dog2**(x) + **feed dog3**$(x))$ + $(\lambda x_e.$**feed dog1**(x) + **feed dog3**$(x))$, . . .]

[11] See Haslinger & Schmitt 2020b for a more detailed discussion.

every NP (OBJ) > NUM NP (SUBJ) **??cumulative [some variation]**

Yet, the judgments for examples with ADUs in subject position were unexpected: For a plural indefinite object, (57) predicts that scrambling of the indefinite should license a previously unavailable cumulative reading. We found no such effect: Both sentences in (60) were rejected in cumulative scenarios by almost all speakers.

(60) a. *Gestern hat jeder Jäger in diesem Ort fünf Hirsche erschossen.*
yesterday has every.NOM hunter.NOM in this town five.ACC stags.ACC shot
'Yesterday, every hunter in this village shot five stags.'
every NP (SUBJ) > NUM NP (OBJ) ***cumulative**

b. *Gestern hat fünf Hirsche jeder Jäger in diesem Ort erschossen.*
NUM NP (OBJ) > every NP (SUBJ) ***cumulative**

Thus, (57) makes incorrect predictions regardless of one's assumptions about scope reconstruction[12]: The judgments for (59-b) suggest that reconstruction is available, but dispreferred in this configuration, which does not explain the lack of a cumulative reading in (60-b). Given (57), the judgments for (60-b) entail that scrambled direct objects obligatorily reconstruct, which is demonstrably false in German and also wouldn't account for (59-b). Yet, the contrast in (59-a,b) shows that scope does affect cumulativity. We thus hypothesize that the restriction on cumulativity is sensitive to the base positions of the plurals (or, if scrambling involves A′-movement, their highest A-positions) as well as their surface positions and tentatively submit the assumptions in (61). While (61-a) has independent motivation (cf. Frey 1993, Beck 1996, Büring 1997, Wurmbrand 2008), our Plural Projection semantics would have to be extended to implement (61-b).

(61) a. Arguments in the German 'Mittelfeld' don't undergo QR. Scope ambiguities between co-arguments of a verb in German are the result of overt movement plus optional reconstruction. Reconstruction takes place in the syntax.

b. A plural expression α has a cumulative reading relative to an ADU only if the ADU doesn't c-command any element of α's chain at LF.

How does this predict the data? If there is no obligatory QR in German, the object in (59-a) remains *in situ* at LF. (61-b) then predicts a cumulative reading since the *jed*-DP fails to c-command the subject. In (59-b), overt movement of the *jed*-DP produces an additional LF with surface scope, which disallows cumulativity because the surface position of the *jed*-DP c-commands the subject. However, if scrambled phrases reconstruct, we generate a second LF with the *jed*- DP *in situ*, which permits cumulativity. If reconstruction of scrambled phrases is dispreferred, e.g. for information-structural reasons, the observed variation in judgments is predicted.

(61) also derives the lack of a clear contrast between (60-a) and (60-b): In both cases, the *jed*- DP c-commands the base position of the plural indefinite. Generalization (60-b) can never be met regardless of movement and both of the possible LFs should lack the cumulative reading.

Assuming some version of (61-b) holds, how should we implement the 'blocking' of cumulativity by antecedent-trace relations? In § 2, we intentionally ignored the role of traces in our framework, to sidestep technical problems that arise when combining Hamblin/Rooth-style Alternative Semantics with variable binding. But (61-b) suggests that the issue is not merely technical. Rather, cumulativity and trace binding interact in a previously unnoticed, non-trivial way: If the trace of a plural expression is c-commanded by an ADU, the ADU is apparently forced to distribute over pluralities 'introduced' by the

[12]Positions on the effect of scrambling on scope in German are surprisingly diverse (see e.g. Frey 1993, Beck 1996, Büring 1997, Pafel 2005, Wurmbrand 2008), possibly due to dialectal variation or to the understudied effects of prosody.

trace, blocking a non-distributive reading of the antecedent of the trace. This generalization has another empirical advantage: Chatain (2019) observes that (57) would wrongly generate cumulative readings for ADUs in subject position via covert movement of the object.[13] The presence of cumulativity asymmetries in English, which freely permits inverse scope with indefinite objects, is then unexplained. But the scrambling data provide independent motivation for a semantics of traces that blocks such 'derived' cumulative readings even if the relevant movement operations are permitted (but see Chatain 2019 for an event-based treatment of the English data).

Future research must investigate these theoretical consequences and also determine how stable the empirical pattern is. Here, we will continue using the simple scope-based view as it seems an adequate starting point for any more fine-grained analysis.

4 A plural projection account of D-conjunctions

We now address another type of ADU, the distributive conjunctions mentioned in § 1. After a brief data discussion, we show how our account extends to these cases.

4.1 Empirical background

Many languages have more than one conjunction strategy for type *e* conjuncts (Szabolcsi 2015, Flor et al. 2017 a.o.). German, for instance, has *A und B* (62-b), but also *sowohl A als auch B* (62-c) (= (4-b) above), Hungarian (discussed by Szabolcsi 2015) has *A és B* (63-a) and *A is és B is* (63-b), and Polish has *A i B* (64-b) alongside *i A i B* (64-c).

(62) German

a. SCENARIO: Two skiing races took place today. Ada competed in the downhill and won. Bea competed in the slalom and won.

b. *Heute haben* ***die Ada und die Bea*** *die zwei Rennen gewonnen!*
today have the Ada and the Bea the two races won
'Today, Ada and Bea won the two races.' **true** in (62-a)

c. *Heute haben* ***sowohl die Ada als auch die Bea*** *die zwei Rennen gewonnen!*
false in (62-a)

(63) Hungarian (Dóra Kata Takács, p.c.)

a. SCENARIO: Sára called 'Express Catering'. Marcsi called 'Star Catering'.

b. ***Sára és Marcsi*** *idöben felhívta a két kiszállító céget*.
Sára and Marcsi on-time called the two catering company.ACC
'Sára and Marcsi called the two catering companies ahead of time.' **true** in (63-a)

c. ***Sára is és Marcsi is*** *idöben felhívta a két kiszállító céget*. **false** in (63-a)

(64) Polish (Magdalena Roszkowski, Marcin Wągiel, p.c.)

a. SCENARIO: Sabina called 'Express Catering'. Magda called 'Star Catering'.

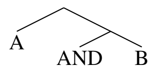
b. ***Sabina i Magda*** *dostatecznie wcześnie zadzwoniły do tych dwóch restauracji.*
Sabina I Magda enough early called to these two restaurants
'Sabina and Magda called these two restaurants early enough.' **true** in (63-a)

c. ***I Sabina i Magda*** *dostatecznie wcześnie zadzwoniły do tych dwóch restauracji.*
false in (63-a)

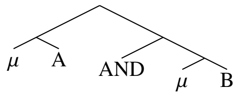
[13] A reviewer of a previous draft made the same point.

In each language, the two strategies differ semantically: While what we call the 'simple conjunction' strategies in the (b)-examples permit cumulative readings in the syntactic contexts provided, the strategies in the (c)-cases are restricted to a distributive reading in the same contexts. Following Szabolcsi's 2015 discussion of Hungarian, we call the latter 'distributive conjunctions'. In Hungarian and Polish, the D-conjunction strategies 'properly include' the simple conjunction strategies morphosyntactically: They contain additional 'conjunction particles', which seem to trigger the semantic contrast (Szabolcsi 2015, Mitrović & Sauerland 2016). Although German patterns differently – the two strategies are morphosyntactically unrelated – we assume cross-linguistically uniform underlying structures: While simple conjunctions have the structure in (65-a), (65-b) schematizes D-conjunctions, where an additional node 'μ' (spelled out as a conjunction particle) attaches to each conjunct (Mitrović & Sauerland 2016, Flor et al. to appear).

(65) a.



b.



To capture the plural-like behavior of simple conjunctions (see § 2.1), we analyze 'AND' as recursive sum formation (29). The different semantic behavior of D-conjunctions must then be due to the conjunction particles schematized as μ in (65-b).

It has gone unnoticed that even D-conjunctions sometimes permit cumulative construals.[14] As with singular universals, these readings are only available if another plural outscopes the D-conjunction. Thus, unlike (62-c), (63-c) and (64-c), the examples in (4-c) above from German, (66-b) and (67-b) can be true in 'cumulative' scenarios.

(66) Hungarian (Dóra Kata Takács, p.c.)

a. SCENARIO: Sára called Bálint. Marcsi called Péter.

b. *Szerencsére a két szervezö idöben felhívta* ***Bálintot is és Pétert is***.
fortunately the two organizers on-time called Bálint.ACC IS and Péter.ACC IS
'Fortunately, the two organizers called both Bálint and Péter ahead of time.'

true in (66-a)

(67) Polish (Magdalena Roszkowski, Marcin Wągiel, p.c.)

a. SCENARIO: Sabina called Adam. Magda called Piotr.

b. *Na szczęście dwie organizatorki dostatecznie wcześnie poinformowały* ***i Adama i Piotra***.
on the-luck two organizers enough early informed I Adam I Piotr
'Fortunately, the two organizers informed both Adam and Piotr early enough.'

true in (67-a)

For German and Polish D-conjunctions (but apparently not in Hungarian) we can also reproduce our argument against a thematic-role asymmetry on the basis of infinitival embedding data (see (5) above). Since (68-b) and (68-c) have cumulative readings, we can assume that the asymmetry is scope-related.

(68) a. SCENARIO: Yesterday, detectives Mia and Kai were observing two suspects, Peter and Anna. Mia saw Peter sell crack. Kai saw Anna sell pot.

b. *Die zwei Detektive haben* ***sowohl den Peter als auch die Anna*** *Drogen verkaufen*
The two detectives have PRT the Peter PRT also the Anna drugs sell

[14] Existing analyses of D-conjunctions with conjunction particles (Szabolcsi 2015, Mitrović & Sauerland 2016) fail to derive this.

gesehen.
seen
'The two detectives saw both Peter and Anna sell drugs.' German; **true** in (68-a)

c. *Wczoraj dwaj detektywi widzieli* ***i Petera i Anne*** *sprzedających*
yesterday two.NOM detective.NOM.PL see.PST.3PL I Peter.ACC I Anna.ACC sell.PTCP.ACC.PL
narkotyki.
drug.ACC.PL
Polish (Magdalena Roszkowski, p.c.); **true** in (68-a)

Finally, D-conjunctions occur in Schein-sentences: (69-b) from German, just like (6) above, is true in scenario (69-a).

(69) a. SCENARIO: Two dogs, Carl and Dean. Ada taught Carl tricks 1 and 2. Ada taught Dean trick 3 and Bea taught Dean trick 2.

b. *Die Ada und die Bea haben sowohl dem Carl als auch dem Dean zwei neue Tricks*
The Ada and the Bea have PRT the Carl PRT also the Dean two new tricks
beigebracht.
taught
'Ada and Bea taught both Carl and Dean two new tricks' **true** in (69-a)

Hence, D-conjunctions pose the same kind of analytical problem as *every* DPs, but introduce two additional complications. First, we want them to have the structure (65-b), where AND expresses recursive sum formation. Distributivity must thus be attributed to the semantics of the μ-particles. Second, *every* DPs required the application of predicate pluralities to the *atomic* individuals in the NP denotation, but in D-conjunctions, the 'units' of distribution are the denotations of the individual conjuncts, which may be pluralities. This is illustrated by (70-b) from German. As the sentence cannot express that the plurality made up of the boys and the girls fed two dogs in total, the predicate must distribute over the individual conjuncts, as expected for a D-conjunction in subject position. But, crucially, the sentence can be true in scenario (70-a). Here, it is not the case that each girl fed two dogs and each boy fed two dogs. Thus, the pluralities denoted by the individual conjuncts – ⟦*the girls*⟧ and ⟦*the boys*⟧ – can each be in a cumulative relation with the predicate.

(70) a. SCENARIO: Two girls: Ada and Bea. Two boys: Gene and Ivo. Three dogs: Carl, Dean, Pia. Ada fed Carl. Bea fed Dean. Gene fed Pia. Ivo fed Dean.

b. ***Sowohl die Mädchen als auch die Buben*** *haben zwei Hunde gefüttert*
PRT the girls PRT also the boys have two dogs fed
'Both the girls and the boys fed two dogs.' **true** in (70-a)

4.2 D-conjunctions: Analysis

The analysis of D-conjunctions should thus partially mirror that of *every* DPs: They should be distributive relative to scopally lower plurals, but when composed with their nuclear scope, the result should be a plural set which preserves the part structure of these plurals and makes it accessible for further cumulation. But their analysis must also deviate from that of *every* DPs: First, the semantic workload must be redistributed – we want the μ-particles to modify the individual conjuncts, which are then combined by means of $\oplus$. The challenge is that μ must block the cumulative reading that conjunctions involving $\oplus$ usually have. Second, (70) shows that the 'units' we distribute over are not always atomic individuals: If the conjuncts are plural, each conjunct may cumulate with syntactically lower plurals, although cumulative readings for the conjunction as a whole are unavailable.

We start with the denotation in (71) for conjunction particles. Like *every*, they directly manipulate plural sets, taking two arguments – a plural set of individuals (the conjunct denotation) and a plural set of predicates (the conjunction's nuclear scope). These arguments are composed via the Cumulative Composition operation C. This latter step permits us to model cumulative relations between the individual conjuncts and scopally lower plurals, as illustrated by (70).

(71) Conjunction particles
$[\![\mu_{\langle e^*,\langle\langle e,a\rangle^*,a^*\rangle\rangle}]\!] = \lambda x^*_{e^*}.\lambda P^*_{\langle e,a\rangle^*}.C(P^*, x^*)$

(72) illustrates how μ combines with the conjunct it modifies: A singular conjunct like *Carl* is first shifted to a singleton plural set (which also happens in standard conjunctions; see § 2.2). When applied to this set, $[\![\mu]\!]$ returns a function that takes a plural set P^* of predicates and composes it with **[Carl]** via CC.

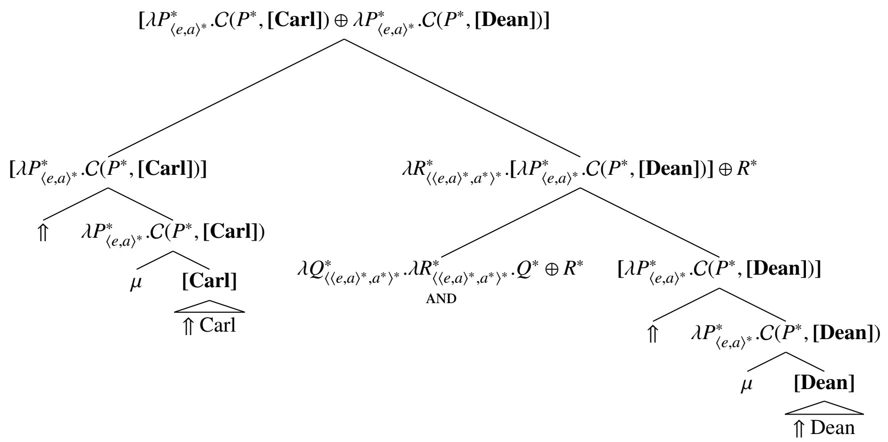
(72) $[\![\mu\ [\Uparrow \text{Carl}]]\!] = [\![\mu]\!](\textbf{[Carl]}) = \lambda P^*_{\langle e,a\rangle^*}.C(P^*, \textbf{[Carl]})$

Next, we consider the interpretation of full D-conjunctions like (73) from German.

(73) *sowohl Carl als auch Dean* ('both Carl and Dean')

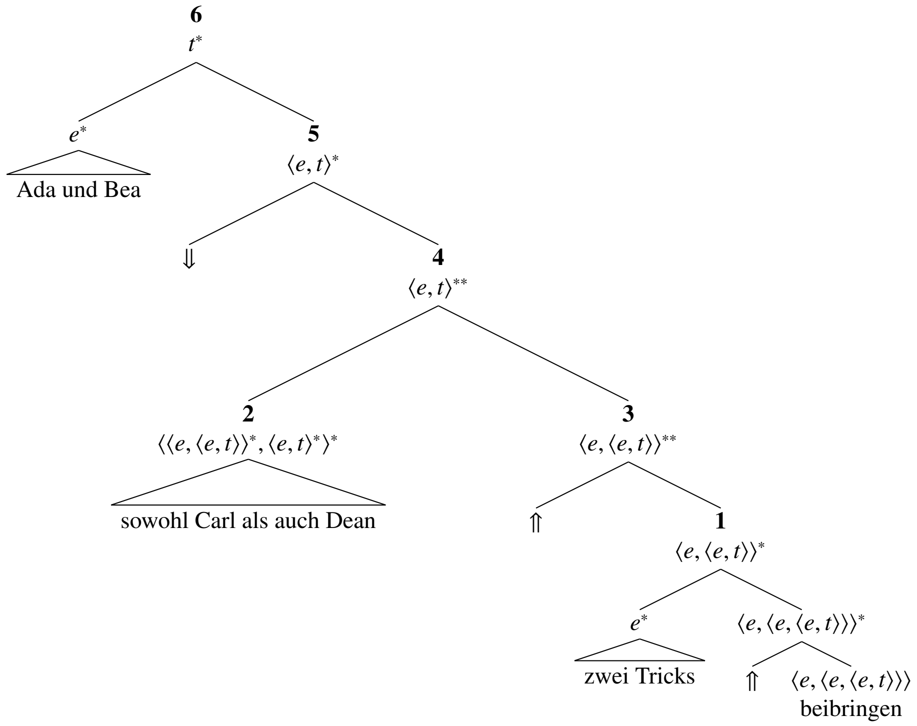
Given our conjunction meaning from § 2.2, each μ-affixed conjunct must be shifted to a plural set to combine with AND. This yields the LF and corresponding semantics in (74) for (73). The full D-conjunction denotes a plural set containing a single element, the sum of two functions on plural sets that perform cumulation with the sets **[Carl]** and **[Dean]**, respectively. Since each atom of this function plurality requires an argument of type $\langle e, a\rangle^*$ – a plural set – rather than an ordinary predicate of type $\langle e, a\rangle$, the conjunction as a whole can no longer cumulate with its nuclear scope argument.

(74)



To illustrate the further compositional steps, consider the Schein sentence in (69-b), which is true in scenario (69-a). The relevant reading corresponds to the LF in (75). (The role of the type-shifts $\Uparrow$ and $\Downarrow$ is explained below.)

(75)



For node **1**, CC returns a plural set of transitive predicates (76-a): Each predicate sum in this set corresponds to a plurality of two tricks. But this set cannot directly compose with the D-conjunction meaning in (76-b), since each atom of the plurality in (76-b) requires a *plural set* as its argument. Therefore, we lift (76-a) to a singleton plural set of plural sets via ⤊.[15] The result of this shift, shown in (76-c), combines with the D-conjunction meaning in (74) via CC. Since (76-c) is a singleton, there is only one cover, which relates the only element of (76-c) – a plural set of predicates – to each atom of the function plurality in (76-b). We must therefore apply each atom of the function plurality to the plural set of predicates, as in (76-d-i). Since these atomic functions involve the cumulation operation C, this amounts to cumulating the plural set of predicates with each of the conjunct denotations **[Carl]** and **[Dean]** (76-d-ii). For each conjunct, we obtain a plural set of one-place predicates (the first set encodes that Carl was taught two tricks, while the second set encodes that Dean was taught two tricks). Finally, these two plural sets are summed up using ⊕, (76-d-iii). We now have a plural set containing a plural set of one-place predicates. Each plurality in this latter set contains two atoms relating to Carl being taught a trick and two atoms relating to Dean being taught a trick. The type-shift ⇓ applies to 'reduce' this higher-type denotation to a simple plural set, shown in (76-e), which then cumulates with the subject plurality **[Ada + Bea]**, with the result in (76-f).

(76) a. $[\![\mathbf{1}]\!] = C([\![\textit{beibringen}]\!], [\![\textit{zwei Tricks}]\!]) = C([\mathbf{teach}], [\mathbf{t1}+\mathbf{t2}, \mathbf{t2}+\mathbf{t3}, \mathbf{t1}+\mathbf{t3}]) =$
$[\mathbf{teach(t1)}+\mathbf{teach(t2)}, \mathbf{teach(t2)}+\mathbf{teach(t3)}, \mathbf{teach(t2)}+\mathbf{teach(t3)}]$

b. $[\![\mathbf{2}]\!] = [\lambda P^*_{\langle e,t\rangle^*}.C(P^*, [\mathbf{Carl}]) \oplus \lambda P^*_{\langle e,t\rangle^*}.C(P^*, [\mathbf{Dean}])]$

c. $[\![\mathbf{3}]\!] = [\![⤊]\!]([\![\mathbf{1}]\!]) = [[\mathbf{teach(t1)}+\mathbf{teach(t2)}, \mathbf{teach(t2)}+\mathbf{teach(t3)}, \mathbf{teach(t2)}+\mathbf{teach(t3)}]]$

d. $[\![\mathbf{4}]\!] = C([\![\mathbf{2}]\!], [\![\mathbf{3}]\!])$

(i) $= [C([\mathbf{teach(t1)}+\mathbf{teach(t2)}, \mathbf{teach(t1)}+\mathbf{teach(t3)}, \mathbf{teach(t2)}+\mathbf{teach(t3)}], [\mathbf{Carl}]) \oplus$
$C([\mathbf{teach(t1)}+\mathbf{teach(t2)}, \mathbf{teach(t1)}+\mathbf{teach(t3)}, \mathbf{teach(t2)}+\mathbf{teach(t3)}], [\mathbf{Dean}])]$

(ii) $= [[\mathbf{teach(t1)(C)}+\mathbf{teach(t2)(C)}, \mathbf{teach(t1)(C)}+\mathbf{teach(t3)(C)}, \mathbf{teach(t2)(C)}+\mathbf{teach(t3)(C)}] \oplus$
$[\mathbf{teach(t1)(D)}+\mathbf{teach(t2)(D)}, \mathbf{teach(t1)(D)}+\mathbf{teach(t3)(D)}, \mathbf{teach(t2)(D)}+\mathbf{teach(t3)(D)}]]$

(iii) $= [[\mathbf{teach(t1)(C)}+\mathbf{teach(t2)(C)}+\mathbf{teach(t1)(D)}+\mathbf{teach(t2)(D)},$

[15] Since this shift permits us to derive distributive interpretations of standard conjunctions, which don't seem to be universally available (Flor et al. 2017), its use must be constrained. We assume that ⤊ can only apply to a node α if α couldn't compose with its sister without it.

teach(t2)(C) + teach(t3)(C) + teach(t1)(D) + teach(t2)(D),
teach(t1)(C) + teach(t2)(C) + teach(t2)(D) + teach(t3)(D), . . .]]

e. $[\![\mathbf{5}]\!] = [\![\Downarrow]\!]([\![\mathbf{4}]\!])$
= **[teach(t1)(C) + teach(t2)(C) + teach(t1)(D) + teach(t2)(D),**
teach(t2)(C) + teach(t3)(C) + teach(t1)(D) + teach(t2)(D),
teach(t1)(C) + teach(t2)(C) + teach(t2)(D) + teach(t3)(D), . . .]

f. $[\![\mathbf{6}]\!] = C([\![\mathbf{5}]\!], [\![\textit{Ada und Bea}]\!]) = C([\![\mathbf{5}]\!], \textbf{[Ada + Bea]})$
= **[teach(t1)(C)(A) + teach(t2)(C)(A) + teach(t1)(D)(B) + teach(t2)(D)(B),**
teach(t2)(C)(A) + teach(t3)(C)(A) + teach(t1)(D)(B) + teach(t2)(D)(A),
teach(t1)(C)(A) + teach(t2)(C)(A) + teach(t2)(D)(B) + teach(t3)(D)(A), . . .]

Accordingly, the sentence is true iff Carl and Dean were each taught two tricks and Ada and Bea cumulatively did the teaching. This correctly captures the truth conditions of the mixed cumulative/distributive reading of Schein-sentences.

Since our analysis of μ has cumulation built in, it extends to examples with plural conjuncts like (70-b). Such examples permit cumulative readings of the individual conjuncts, although the conjunction itself remains distributive. Given scenario (70-a), the D-conjunction has the denotation in (77-a). Combining this with the shifted VP-meaning in (77-b) via C yields (77-c): We again 'pair' parts of the functor plurality with the only element of (77-b), a plural set of predicates (77-c-i). Now the second application of C – lexically contributed by μ – kicks in, as shown in (77-c-ii): We form the sum for each cover of **[Ada + Bea]** and some element of the predicate plurality. Likewise, we form the sum for each cover of **[Gene + Ivo]** and some element of the predicate plurality. So in effect, each plural conjunct cumulates with *two dogs*. Finally, we 'sum up' the two resulting plural sets via $\oplus$ (77-c-iii). We end up with propositional pluralities that amount to the girls feeding two dogs between them and the boys feeding two dogs between them. This already indicates that while the individual conjuncts can cumulate with the predicate pluralities, the conjunction itself will remain distributive. Finally, $\Downarrow$ applies again to reduce this 'higher-order' plural set to a simple plural set of propositions.[16]

(77) a. $[\![[[\Uparrow [\mu \textit{ die Mädchen}]] [\textsc{and} [\Uparrow [\mu \textit{ die Buben}]]]]]\!]$
$= [\lambda P^*_{\langle e,t\rangle^*}.C(P^*, \textbf{[Ada + Bea]}) \oplus \lambda P^*_{\langle e,t\rangle^*}.C(P^*, \textbf{[Gene + Ivo]})]$

b. $[\![\Uparrow \textit{zwei Hunde gefüttert}]\!]$ = **[[feed(C) + feed(D), feed(C) + feed(P), feed(D) + feed(P)]]**

c. $C([\![(77\text{-a})]\!], [\![(77\text{-b})]\!])$
(i) = $[C($**[feed(C) + feed(D), feed(C) + feed(P), feed(D) + feed(P)], [Ada + Bea]**$) \oplus$
$C($**[feed(C) + feed(D), feed(C) + feed(P), feed(D) + feed(P)], [Gene + Ivo]**$)]$
(ii) = **[[feed(C)(A) + feed(D)(B), feed(C)(A) + feed(P)(B), feed(D)(A) + feed(P)(B),**
feed(C)(B) + feed(D)(A), . . .] $\oplus$
[feed(C)(G) + feed(D)(I), feed(C)(G) + feed(P)(I), feed(D)(G) + feed(P)(I), . . .]]
(iii) = **[[feed(C)(A) + feed(D)(B) + feed(C)(I) + feed(D)(G),**
feed(C)(A) + feed(P)(B) + feed(D)(G) + feed(P)(I),
feed(D)(A) + feed(P)(B) + feed(C)(G) + feed(D)(I), . . .]]

d. $[\![\Downarrow]\!]([\![(77\text{-c})]\!])$ = **[feed(C)(A) + feed(D)(B) + feed(C)(I) + feed(D)(G),**
feed(C)(A) + feed(P)(B) + feed(D)(G) + feed(P)(I),
feed(D)(A) + feed(P)(B) + feed(C)(G) + feed(D)(I), . . .]

Thus, we derive the two-faced behavior of D-conjunctions with plural conjuncts: The D-conjunction will be distributive, because each conjunct combines with the entire plural set of predicates in its nuclear scope by means of C. But since C encodes cumulativity, each individual conjunct can cumulate with this

[16] Assuming the analysis of *every* DPs from § 3, our proposal correctly predicts that simple conjunctions of two *every* DPs are always 'fully distributive' w.r.t. scopally lower plurals.

plural set.

5 Comparison to previous analyses of ADUs

We developed a new semantic analysis of ADUs that derives the peculiar truth conditions of Schein-sentences and relates the interpretation of ADUs to their structural position relative to other plurals. Two existing approaches share these features: the event-based approach pioneered by Schein (1993) and extended by Kratzer (2003), Ferreira (2005), Zweig (2008) a.o., and the predicate-based analysis in Champollion (2010). Like our approach, they build on Schein's 1993 insight that Schein-sentences cannot involve a unique cumulative relation – but they make different assumptions about the nature of the multiple cumulative operations at work: While our analysis requires repeated application of CC, Champollion (2010) posits an LF with two different cumulation operators, and the event-based approach uses cumulative thematic-role relations relating event pluralities introduced by the verb to plural individuals.

Both existing analyses make relatively conservative ontological assumptions, raising the question whether the expressive power of the PPA is needed. We will now argue that both existing approaches have problems with some of the more 'complex' instances of cumulativity discussed above.

5.1 The event-based approach

Several authors (e.g. Schein 1993, Kratzer 2003, Ferreira 2005, Zweig 2008) have explored the idea that cumulativity is inherently connected to a neo-Davidsonian event semantics, where verbs denote sets of events and combine with DP arguments *via* thematic-role relations of type $\langle e, \langle l, t\rangle\rangle$ (with l the type of events). The latter are modeled as cumulative relations connecting plural individuals to corresponding sums of events (cf. Kratzer 2003). As in our system, cumulativity is therefore built into the mechanism for function-argument composition, but since plural events play the role of our predicate pluralities, higher-type pluralities are not needed.

For concreteness, we introduce an event-based system inspired by Ferreira (2005). Thematic-role predicates map an individual to a one-place predicate of events (78) and lexically encode the effects of the ** operator: As (79-b) shows, the agent relation $[\![\text{AG}]\!]$ holds of an individual x and an event e iff e can be partitioned into disjoint subevents that the atomic parts of x are cumulatively agents of.[17]

(78) [[AG [*the two girls*]] [*fed* [TH [*the two dogs*]]]] (= (3))

(79) a. $\text{PARTITION}(P)(e)$ holds for a set P of events and an event e iff P is a set of disjoint subevents of e whose sum is e

b. $[\![\text{AG}]\!] = \lambda x_e.\lambda e_l.\exists P_{\langle l,t\rangle}[\text{PARTITION}(P)(e) \wedge \forall x'[x' \leq_a x \rightarrow \exists e'[P(e') \wedge \text{AGENT}(x')(e')]] \wedge \forall e'[P(e') \rightarrow \exists x'[x' \leq_a x \wedge \text{AGENT}(x')(e')]]]$

Each argument denotation thus maps to an event predicate that combines intersectively with the verb meaning (80). This predicts (78-a) to be true iff there is an event that (i) is a sum of disjoint feeding events, (ii) can be partitioned into disjoint subevents that cumulatively stand in the AGENT relation to **a** + **b**, and (iii) can be partitioned into disjoint subevents that cumulatively stand in the THEME relation to **c** + **d**.

(80) a. $[\![\textit{fed}]\!] = \lambda e_l.\exists P_{\langle l,t\rangle}[\text{PARTITION}(P)(e) \wedge \forall e'[P(e') \rightarrow \textbf{feeding}(e')]]$

[17] The requirement that each subevent in the partition must have an atomic agent is introduced here for simplicity, but not necessary.

b. $[\![\text{TH } [\textit{the two dogs}]]\!] = \lambda e_l.[\![\text{TH}]\!](\mathbf{c} + \mathbf{d})(e)$

c. $[\![\text{AG } [\textit{the two girls}]]\!] = \lambda e_l.[\![\text{AG}]\!](\mathbf{a} + \mathbf{b})(e)$

d. $[\![\textit{fed} \text{ [TH } [\textit{the two dogs}]]]\!] = \lambda e_l.\exists P_{\langle l,t\rangle}[\text{PARTITION}(P)(e) \wedge \forall e'[P(e') \rightarrow \mathbf{feeding}(e')]] \wedge [\![\text{TH}]\!](\mathbf{c}+\mathbf{d})(e)$

e. $[\![\text{[AG } [\textit{the two girls}]] \text{ [}\textit{fed} \text{ [TH } [\textit{the two dogs}]]]]\!]$
$= \lambda e_l.\exists P_{\langle l,t\rangle}[\text{PARTITION}(P)(e) \wedge \forall e'[P(e') \rightarrow \mathbf{feeding}(e')]] \wedge [\![\text{TH}]\!](\mathbf{c}+\mathbf{d})(e) \wedge [\![\text{AG}]\!](\mathbf{a}+\mathbf{b})(e)$

There is a clear parallel to the PPA: Since each event in (80-d) can be partitioned into a part whose THEME is **c** and a part whose THEME is **d**, these events 'preserve' the plural structure of **c** + **d** and are then cumulated with the atomic parts of the subject plurality. Thus, the denotations of complex expressions containing a plurality of individuals provide the part structure necessary to indirectly form a cumulative relation with these individuals, even if the complex expression is not individual-denoting. The event-based approach therefore implements the same intuition as the PPA and hence makes similar predictions about Schein-sentences. An LF for a Schein-sentence is given in (81). To model the distributive effect of *every* DPs, we assume that they don't compose with predicates of events in the way other plurals do: In (81),Ada and Bea should be the (cumulative) agent of an event which, for each dog, has a subevent of teaching that dog two tricks. An *every* DP therefore takes two semantic arguments, a thematic-role relation R and a predicate $\mathcal{P}$ of events (82). It returns the set of those events that can be partitioned into subevents satisfying $\mathcal{P}$ so that every individual in the NP denotation stands in relation R to some element of the partition.

(81) [[AG [*Ada and Bea*]] [[RE [*every dog*]] [*taught* [TH [*two new tricks*]]]]]

(82) a. $[\![\textit{every}]\!] = \lambda P_{\langle e,t\rangle}.\lambda R_{\langle e,\langle l,t\rangle\rangle}.\lambda \mathcal{P}_{\langle l,t\rangle}.\lambda e_l.\exists Q_{\langle l,t\rangle}.\text{PARTITION}(Q)(e) \wedge \forall e'[Q(e') \rightarrow \mathcal{P}(e')]$
$\wedge \forall x[P(x) \rightarrow \exists e'[Q(e') \wedge R(x)(e')]] \wedge \forall e'[Q(e') \rightarrow \exists x[P(x) \wedge R(x)(e')]]$

b. $[\![\text{RE } [\textit{every dog}]]\!] = \lambda \mathcal{P}_{\langle l,t\rangle}.\lambda e_l.\exists Q_{\langle l,t\rangle}.\text{PARTITION}(Q)(e) \wedge \forall e'[Q(e') \rightarrow \mathcal{P}(e')]$
$\wedge \forall x[\mathbf{dog}(x) \rightarrow \exists e'[Q(e') \wedge [\![\text{RE}]\!](x)(e')]] \wedge \forall e'[Q(e') \rightarrow \exists x[\mathbf{dog}(x) \wedge [\![\text{RE}]\!](x)(e')]]$

In (83-a), *two new tricks* combines with its thematic-role relation and the verb meaning, resulting in the set of all sums of teaching events whose theme consists of two new tricks. Composition with the *every* DP yields (83-b), the set of all events that can be partitioned into subevents so that every dog stands in the relation $[\![\text{RE}]\!]$ ('recipient') to one of the subevents, each subevent has a dog as its recipient and each subevent involves two tricks being taught. (81-a) is predicted true iff Ada and Bea are cumulatively agents of some such event.

(83) a. $[\![\textit{taught} \text{ [TH } [\textit{two new tricks}]]]\!] = \lambda e_l.\exists P_{\langle l,t\rangle}[\text{PARTITION}(P)(e) \wedge \forall e'[P(e') \rightarrow \mathbf{teaching}(e')]]$
$\wedge \exists x[\mathbf{trick}(x) \wedge |x| = 2 \wedge [\![\text{TH}]\!](x)(e)]$

b. $[\![\text{[RE } [\textit{every dog}]] \text{ [}\textit{taught} \text{ [TH } [\textit{two new tricks}]]]]\!] = \lambda e_l.\exists Q_{\langle l,t\rangle}.\text{PARTITION}(Q)(e) \wedge \forall e'[Q(e') \rightarrow$
$\exists P_{\langle l,t\rangle}[\text{PARTITION}(P)(e') \wedge \forall e''[P(e'') \rightarrow \mathbf{teaching}(e'')]] \wedge \exists y[\mathbf{trick}(y) \wedge |y| = 2 \wedge [\![\text{TH}]\!](y)(e')]]$
$\wedge \forall x[\mathbf{dog}(x) \rightarrow \exists e'[Q(e') \wedge [\![\text{RE}]\!](x)(e')]] \wedge \forall e'[Q(e') \rightarrow \exists x[\mathbf{dog}(x) \wedge [\![\text{RE}]\!](x)(e')]]$

(81) therefore involves several different cumulative relations: The THEME relation applies cumulatively to events and pluralities of two tricks. The lexical entry of *every* encodes another cumulative relation between individuals and events: Each dog is the recipient of a subevent in the partition, and each such subevent has a dog as its recipient. Finally, the 'plural' events with this property cumulatively stand in the AGENT relation to **ada** + **bea**.

This analysis therefore meets the challenge of Schein-sentences without recourse to higher-type pluralities, but crucially relies on the mereology of events. So does it differ empirically from our theory, on which cumulativity and events are unrelated?

Clearly, event-based accounts predict cumulative readings only for predicates with an event argument. If the presence of event arguments is connected to independent linguistic phenomena, a correlation between these phenomena and cumulativity is predicted. However, if a neo-Davidsonian semantics is assumed for all lexical predicates (see e.g., Schein 1993) the claim that cumulativity requires an event argument is not independently testable.

Yet, one class of cumulative sentences, exemplified for German in (84), is independently problematic for purely event-based analyses of cumulativity. Schmitt (2019) observes that cumulative relations can 'reach inside' complements of attitude verbs, which are standardly assumed to denote neither individuals nor events.

(84) a. CONTEXT: Abe's friends refuse to believe he was doing cocaine last night . . .
b. *Sie glauben nur, dass er gestern viel getrunken und gekifft hat.*
they believe only that he yesterday much drunk and smoked.weed has
'They only believe that he drank a lot and smoked weed yesterday.' (adapted from Schmitt 2019)
c. SCENARIO: Half of Abe's friends believe he drank a lot. The others believe he smoked weed.

Given a neo-Davidsonian verb semantics, we need a thematic-role relation relating propositional arguments of *believe* to belief states (cf. Kratzer 2006, Moulton 2015, Elliott 2017 a.o.).This relation must be cumulative: For (84), this means that Abe's friends are cumulatively the experiencers of a sum of belief-states containing some parts with the content ⟦*Abe drank a lot*⟧, and some parts with the content ⟦*Abe smoked weed*⟧. Assuming pluralities of belief-states is unproblematic – the crucial issue is how the parts of these pluralities receive their respective propositional content. If the embedded clause in (84-b) denotes a simple proposition, we cannot extract the 'parts' ⟦*Abe drank a lot*⟧ and ⟦*Abe smoked weed*⟧ from this proposition. Therefore, the notion of a cumulative relation between states and propositions only makes sense if the event-based analysis is supplemented with a notion of parthood and cumulativity for higher-type denotations. Although data like (84) don't falsify the use of events in theories of cumulativity, they show that event-based analyses must be supplemented with an additional mechanism. Such a 'mixed' theory would resemble the PPA in its empirical coverage, but with the disadvantage of being less uniform.[18]

But couldn't we let *believe* in (84-b) combine with a property of events rather than a proposition? For instance, one could use the property in (85), which maps every world w to a set of sums of events that reflect the part structure of the conjunction in (84-a). If we could cumulate the belief states of Abe's friends with the parts of these plural events, we wouldn't need pluralities of higher-type objects. The problem is that each belief state is 'made true' by many different events belonging to different possible worlds. The contents of non-factive attitudes like belief therefore cannot be encoded in a single event anchored to a particular world.[19] Thus, the truth conditions of (84-b) cannot be paraphrased in terms of a cumulative 'belief' relation holding between Abe's friends and a sum of events.

(85) $[\![\textit{that he drank and smoked weed yesterday}]\!] =$
$\lambda w.\lambda e.\exists e_1, e_2[\mathbf{drink}(\mathbf{abe})(e_1)(w) \wedge \mathbf{smoke\text{-}weed}(\mathbf{abe})(e_2)(w) \wedge e = e_1 + e_2 \wedge \mathbf{yesterday}(e)]$

[18] One could circumvent the problem by claiming that Abe's friends 'collectively believe' the proposition expressed by the embedded clause (see Kratzer 2003, Pasternak 2018) – however, Marty (2019), Schmitt (2020) give several counterarguments.

[19] A reviewer of a previous version of this paper mentions Schein (2016:sec. 5.1) as an analysis that encodes attitude contents in single eventualities, so that attitude predicates have no special status. It is unclear to us whether Schein's 2016 proposal meets the basic desiderata for an intensional semantics (like lack of existential commitment for indefinites embedded under attitude predicates): It appears to rely on a metalanguage expression 'that' whose truth-conditional contribution remains unspecified, and leaves open how the relevant eventuality arguments relate to familiar notions of intensional semantics like propositions or possible worlds. Further, Schein's examples involve the factive predicate *know*, raising the question whether the paraphrases generalize to the genuinely problematic cases with non-veridical attitudes.

5.2 Champollion (2010)

Both our approach and the event-based tradition attribute cumulativity asymmetries to the lexical semantics of ADUs. Champollion (2010) develops a fundamentally different account, where *every* DPs are run-of-the-mill plural expressions: *every boy* denotes $+\mathbf{boy}$, the sum of all boys. Cumulative readings relative to higher plural expressions are therefore expected regardless of the particular theory of cumulativity. The distributivity requirement of *every* DPs is encoded in additional assumptions about the syntax-semantics interface, which allow Champollion to maintain a predicate analysis of cumulativity: Cumulative relations between individuals are derived by syntactically adjoining a cumulation operator (**, ***, etc.). If there is no surface constituent denoting the required relation, movement creates a suitable LF constituent (see § 2). So, unlike the other approaches discussed here, Champollion's analysis does not build cumulativity into the basic mechanism for predicate-argument composition.

The LF Champollion (2010) assigns to the Schein sentence (81), given in (86), involves movement of *Ada and Bea* and *every dog* to derive the right relation for **.

(86) [[*Ada and Bea*] [[*every dog*] [** [2 [1 [[*two tricks*] [3 [t_1 [[*the*$_2$ *dog*] [[*** *taught*] t_3]]]]]]]]]]

What distinguishes Champollion's approach from other versions of the predicate analysis is the semantics of this relation, which blocks a cumulative reading of *every* relative to *two new tricks*: The trace of an *every* DP contains a copy of the singular NP, which combines with an indexed definite determiner (87) (Fox 1999) and expresses a predicate true only of atomic individuals. The values of the variable bound by the *every* DP must therefore be atomic individuals.

(87) $[\![the_2\ dog]\!]^g = g(2)$ if $g(2)$ is an atomic dog; undefined otherwise

This atomicity restriction holds even if *taught* itself bears a cumulation operator, as in (86): Given an assignment g, *two tricks* combines with a property true of those sums of tricks that were cumulatively taught to an atomic dog, $g(2)$, by a possibly plural individual, $g(1)$ (88). Abstraction over the indices corresponding to the subject and the *every* DP (88-b) creates a binary relation, but preserves the atomicity presupposition: If Ada and Bea cumulatively taught Dean trick 1 and trick 2, this relation holds of the pair $(\mathbf{a}+\mathbf{b}, \mathbf{d})$. However, if Ada taught Carl trick 1 and Bea taught Dean trick 2, the relation does not hold of $(\mathbf{a}+\mathbf{b}, \mathbf{c}+\mathbf{d})$, since $\mathbf{c}+\mathbf{d}$ is not atomic. Hence, every dog must have learned two tricks.

(88) a. $[\![3\ [t_1\ [[the_2\ dog]\ [[***\ taught]\ t_3]]]]]^g = \lambda z_e.[\![***]\!](\mathbf{taught})(z)(g(2))(g(1))$ if $\mathbf{dog}(g(2)) = 1$, undefined otherwise

b. $[\![2\ [1\ [[two\ tricks]\ [3\ [t_1\ [[the_2\ dog]\ [[***\ taught]\ t_3]]]]]]]\!] = \lambda x_e : \mathbf{dog}(x).\lambda y_e.\exists z_e.[\forall z' \leq_a z.\mathbf{trick}(z') \wedge |z| = 2 \wedge [\![***]\!](\mathbf{taught})(z)(x)(y)]$

To compose (88-b) with $[\![every\ dog]\!]$ despite its atomicity presupposition, another cumulation operator is attached. Importantly, Champollion's cumulation operators don't quantify over atomic parts, but just close a relation between individuals under 'pointwise sum' (89). This is crucial for (86), since ** attaches to a relation connecting each atomic dog to possibly plural individuals that cumulatively taught that dog two tricks (88-b).

(89) $[\![**]\!] = \lambda R_{\langle e,\langle e,t\rangle\rangle}.\lambda x_e.\lambda y_e.\exists R'[R' \subseteq R \wedge x = +\{x' \mid \exists y'.R'(x')(y')\} \wedge y = +\{y' \mid \exists x'.R'(x')(y')\}]$

Since the presupposition of (88-b) does not project above the ** operator, we obtain a relation between pluralities of dogs and individuals cumulatively responsible for teaching each dog two tricks.[20]

Champollion (2010) thus derives Schein-sentences without using events or higher-type pluralities. However, to derive scope-related cumulativity asymmetries, further syntactic assumptions are needed: Plurals must undergo obligatory LF movement – otherwise, we could interpret *every* DPs *in situ*, circumventing the atomicity requirement imposed by their traces. Further, we must exclude LFs like (90-b) for (90-a), which would yield the unattested cumulative reading: Since the atomicity presupposition doesn't project through **, the cumulative relation in (90-b) holds of any pair (x, y) where x is a girl *or plurality of girls* that cumulatively fed y.

(90) a. *Every girl fed the two dogs.*
b. [[*every girl*] [*the two dogs*] [** [2 [1 [[*the*$_1$ *girl*] [*fed* t$_2$]]]]]]

Champollion (2010) addresses this issue by requiring that *every* DPs are interpreted above a distributivity or cumulation operator, but doesn't specify the restrictions on its syntactic position. Forcing *every* DPs to minimally c-command the operator is insufficient, as it fails to block (91-a). If cumulation operators don't quantify over atomic parts of the pluralities, the * operator in (91-a) must be interpreted as in (91-b). Since the only effect of (91-b) is to close a predicate extension under sum, (91-a) receives a cumulative reading despite the position of the *every* DP.

(91) a. [[*every girl*] [* [*the two dogs*] [** [2 [1 [[*the*$_1$ *girl*] [*fed* t$_2$]]]]]]]
b. $[\![*]\!] = \lambda P_{\langle e,t\rangle}.\lambda x.\exists P'[P' \subseteq P \wedge x = +P']$

More generally, inserting * or ** between an *every* DP and another plural is insufficient to block cumulative relations between them. Rather, the other plural must be c-commanded by the binder index abstracting over the *every* DP's trace. If both the *every* DP and the other plural c-command that index due to 'tucking-in' movement, a cumulative reading is derived regardless of their relative syntactic positions. So, to derive the asymmetric behavior of *every* DPs, we would have to ban other plurals from 'tucking in' between an *every* DP and its binder index. This amounts to directly translating the scope-related restrictions on cumulativity into restrictions on syntactic movement. Nonetheless, at the cost of this complex LF syntax, Champollion (2010) analyzes Schein-sentences without events or Plural Projection. However, two classes of more complex examples cannot easily be accommodated in his framework.

The first case involves D-conjunctions. As noted in § 4, while a D-conjunction with plural conjuncts like (70-b) disallows a cumulative reading of the whole conjunction relative to a scopally lower plural, each conjunct can cumulate with that plural individually. In Champollion's system, the distributivity requirement of D-conjunctions would have to be modeled as a restriction on the trace of the conjunction, but it is unclear how. In (70-b) the trace should be permitted to range over pluralities (we don't want to require that every child fed two dogs). But it cannot range over *arbitrary* pluralities, since we would then lose the distributive effect of the D-conjunction. Intuitively, the predicate within the complex trace should be true of the conjunct denotations, and false of all other pluralities. Since (70-b) does not contain any overt subconstituent denoting this predicate, this requires either a nonstandard theory of traces or a more complex syntax for D-conjunction.

The second problem relates to our general criticism of the predicate analysis in § 3. (92) shows that the 'flattening effect' extends to ADUs within a conjunction: It has a reading where *die Nachbarinnen* cumulates both with the predicate conjunction and the DP headed by *jede* 'every', but this reading cannot

[20] In § 2 we claimed that Schein-sentences cannot be derived by cumulating a single relation between individuals. Our objection presupposed that the relation to be cumulated is restricted to atomic arguments, and no longer applies once we allow the second argument to be plural, while the first argument is restricted to atomic dogs.

be derived via movement for reasons discussed in § 3.

(92) *Die Nachbarinnen haben Gene jede von ihren Katzen füttern und den Hund bürsten lassen.*
the neighbours have Gene every of their cats feed.INF and the dog brush.INF let.PTCP
'The neighbours let Gene feed each of their cats and brush their dog.'

Summing up, Champollion (2010) shows that Schein-sentences don't require building cumulativity into the mechanism for function-argument composition. However, he needs certain assumptions about LF syntax that are problematic for more complex cases. While we agree with Champollion that cumulativity is not intrinsically connected to event semantics, we take these problems to strengthen the case for a purely semantic approach to Schein's puzzle.

6 Conclusion and open problems

In this paper, we studied expressions that permit cumulative readings in some, but not all syntactic contexts. In addition to English *every* DPs (Schein 1993, Kratzer 2003, Ferreira 2005, Zweig 2008, Champollion 2010) and German *jed-* DPs, this class of **asymmetrically distributive universals** (ADUs) was shown to contain distributive conjunctions in several languages. We further argued that this asymmetry is at least partially related to scope, rather than thematic roles (following Champollion 2010).

The behavior of ADUs imposes several constraints on theories of cumulativity: First, since cumulativity asymmetries correlate with syntactic asymmetries, they cannot be reduced to purely lexical properties. The data thus support Beck & Sauerland's 2000 claim that the mechanism behind cumulativity must be able to 'span' larger chunks of syntactic structure. In addition, the sensitivity to syntactic asymmetries challenges the traditional view that cumulativity is inherently symmetric, a point already made by Schein 1993: The behavior of ADUs in **Schein-sentences** shows that the part structure of plural expressions in the scope of ADUs must still be accessible for cumulative relations between the ADU and higher plurals. Consequently, Schein-sentences cannot be analyzed in terms of a single cumulative relation between all pluralities 'participating' in cumulativity.

To derive these observations, existing proposals either assume that cumulation targets thematic-role relations (Schein 1993, Kratzer 2003, Ferreira 2005, Zweig 2008), or introduce special restrictions on cumulative relations between individuals (Champollion 2010). We argued that both proposals undergenerate and presented a novel approach to ADUs and cumulativity that circumvents their problems. The core idea of this **plural projection** approach is that the notion of cumulativity applies to binary operations of arbitrary type, since all semantic domains contain pluralities. This allows us to formulate a compositional rule operating on so-called plural sets which applies at every node intervening between the plurals 'participating' in cumulativity. The parts of pluralities embedded in another plural expression correspond to parts of the denotation of the embedding plural expression. This captures the parallels between the so-called **flattening effect** and Schein-sentences with ADUs.

We then analyzed *every* DPs and distributive conjunctions as operators on plural sets. They combine elements of the plural set in their scope with (i) each atom of their restrictor (*every* DP) or (ii) each conjunct (D-conjunctions), which enforces distributivity. This operation returns a plural set which reflects the part structure of scopally dependent material and is available for cumulation with syntactically higher plurals.

Since we already mentioned several unresolved problems, we conclude by highlighting three open questions our proposal raises within a broader research context.

On the empirical side, existing compositional analyses of ADUs, including ours, don't immediately generalize to cumulative readings of non-upward-monotonic indefinites. Since our truth definition quantifies existentially over a plural set, the 'upper-boundedness' of *exactly two girls* in (93) remains unexplained.

(93) *Exactly two girls fed every dog in this town.*

A common response to this problem is to use a two-dimensional semantics, where modified numerals introduce a separate semantic dimension responsible for the upper-boundedness conditions (e.g. Krifka 1999, Landman 2000, Brasoveanu 2013; see Buccola & Spector 2016 for a dissenting view). Both dimensions are computed in parallel and combined at the sentence level. The upper-boundedness conditions can therefore 'take scope' over the cumulation operation while the component of modified numerals that introduces plurality is interpreted *in situ*, resulting in a 'split scope' effect. In Haslinger & Schmitt (2020a), we combine this approach with the plural projection system.[21]

Another question is how collectivity should be integrated into our system. Intuitively, collective predicates should block application of CC (like *every*), but the choice of implementation depends on an understudied empirical issue: Which configurations license collective readings of plural expressions? The relevant configurations are not exactly the same as those that permit cumulativity, as illustrated by German *jed-* DPs, which can occur in the object position of cumulative predicates but, for some speakers, not with object-collective predicates (94). Such data cast doubt on attempts to reduce cumulativity to collectivity or *vice versa*.

(94) *Ada hat alle Bücher / #jedes Buch verglichen.*
Ada has all books / every book compared
'Ada compared all books / %every book.'

On the theoretical side, one issue is particularly urgent (Buccola & Spector 2016, Haslinger & Schmitt 2017): We introduced non-classical meanings for logical expressions like conjunction or universal determiners, which are incompatible with the assumptions of all analyses relying on entailment relations between logical operators to derive phenomena like scalar implicatures (see e.g. Horn 1989, Sauerland 2004).

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[21] A reviewer of a previous draft worries that our proposal doesn't extend to Schein-sentences involving modified numerals outscoped by a distributivity operator. They assume we would analyze modified numerals as standard generalized quantifiers over pluralities scoping below the distributivity operator. This isn't true of Haslinger & Schmitt 2020a where modified numerals in Schein-sentences and other configurations have split scope (following Krifka (1999)) and do *not* denote generalized quantifiers over plural individuals. While we indeed fail to account for some of the examples in Schein 1993, these problems are arguably due to our treatment of distributivity operators, rather than a principled shortcoming of two-dimensional approaches to modified numerals. To our knowledge, all compositional analyses of modified numerals in Schein-sentences involve some form of multidimensionality.

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