

On truth-conditions for *if* (but not quite only *if*)*

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Folklore has it that there is no good way of assigning truth-conditions to ordinary, indicative *ifs* in such a way that they are bound from above by strict implication, from below by material implication, and *ifs* like

- (1) If Carl is away, then if Lenny is away then Sector 7G is empty

are rendered equivalent to

- (2) If Carl is away and Lenny is away, then Sector 7G is empty.

For – according to the lore – any attempt at pulling off this feat will reduce them to material implication, and that is decidedly not a good way of assigning truth-conditions to ordinary, indicative *ifs*.¹ But this ain't (quite) true and so the folklore needs revising.

1 The Argument

Fix a propositional language L to serve as an intermediate language – bits of natural language get assigned meanings by associating them with sentences of L , which themselves are interpreted. Assume that L is equipped with a set of atoms, the connectives \neg, \wedge , and the binary connective $(if \cdot)(\cdot)$.² Unless we say

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¹Some still insist that *if* is, indeed, just the material conditional. They are wrong, but that is an argument for another day.

²Some – e.g., Kratzer (1986) – deny that *if* is a connective in the first place, and take it instead to be a device whose only job is to mark the restriction of some other operator. I say that *ifs* restricting job is a job done best by taking it to be a connective. That, too, is an argument for another day – see Gillies (2006b).

otherwise, p, q, r, \dots range over the non-iffy fragment of L , and P, Q, \dots over arbitrary sentences of L .

One way of proceeding – Gibbard’s (1981) way – begins with a complex *if* like

$$(3) \quad (if\ p \supset q)((if\ p)(q))$$

The argument is that – given the constraints that *if* is bounded from above by strict implication, from below by material implication, and that the likes of (1) entail and are entailed by the likes of (2) – this is everywhere true and that it entails the corresponding material conditional $(p \supset q) \supset (if\ p)(q)$. Anything entailed by something everywhere true must be everywhere true, and a material conditional everywhere true is just one whose antecedent entails its consequent. Whence it follows $p \supset q$ entails the ordinary indicative $(if\ p)(q)$.³

But this way of putting things assumes more than is required by assuming that *if* can be scoped under arbitrary connectives. Although I see why a defender of truth-conditions for *if* often *does* say that, I see no reason why she *must*. So we should tell the legend in a way that does not assume it.⁴

Assume that – for any non-iffy p, q , and r – the following all hold:

(U)pper Bound: if p entails q then for any world i , $(if\ p)(q)$ is true at i

(L)ower Bound: if $p \wedge \neg q$ is true at i then $(if\ p)(q)$ can’t be

(I)mport: $(if\ p \wedge q)(r)$ entails $(if\ p)((if\ q)(r))$

Then the ordinary, indicative *if* is just the material conditional after all. Precisely: for any world i the ordinary, indicative $(if\ p)(q)$ is true at i iff either p is false at i or q is true at i .

Proof. The left-to-right direction is not in dispute. The other direction is secured by two rather dull entailments. Suppose p is false at i . Since $(\neg p \wedge p)$ entails q – and since by (U) *if* is bounded from above strict implication – then for any world whatever

$$(4) \quad (if\ \neg p \wedge p)(q)$$

³See Gibbard (1981, pp. 234–235). This particular way of telling the story has been retold often – see, e.g., Bennett (2003); Edgington (1995, 2006); Kratzer (1986).

⁴A similar version of this argument can be found in Veltman (1985). Indeed, if indicatives hook up with entailment to support a deduction theorem – r and p conspire to entail q just in case r alone entails $if(p)(q)$ – then we are in trouble enough, for then the equivalence between (1) and (2) follows straightaway.

is true at that world, hence at i . So, by (I), the complex *if*

$$(5) \quad (if \neg p)((if p)(q))$$

must also be true at i . Since $\neg p$ is true at i , the embedded conditional $(if p)(q)$ cannot be false at i – else *if* would not be bounded from below by material implication, violating (L). And so it must be true at i .

Suppose q is true at i . Since $(q \wedge p)$ entails q – and since by (U) *if* is bounded from above strict implication – then for any world whatever

$$(6) \quad (if q \wedge p)(q)$$

is true at that world, hence at i . So, by (I), the complex *if*

$$(7) \quad (if q)((if p)(q))$$

must also be true at i . Since q is true at i , the embedded conditional $(if p)(q)$ cannot be false at i – else *if* would not be bounded from below by material implication, violating (L). And so it must be true at i . QED

It thus appears that there is no good way of assigning truth-conditions to ordinary, indicative *ifs* – not, at least, if we constrain them in the ways we have. But appearances can mislead.

I favor a view according to which ordinary, indicative *ifs* are strict conditionals over the set of relevant possibilities in a context. Roughly put: they say that all the antecedent possibilities compatible with the context are also consequent possibilities. But my aim here is not to defend this hypothesis.⁵ Instead I have a less ambitious goal: tell enough of the strict conditional story to show that the folklore needs rewriting. I will tell two versions of that story.

2 First Version

The first version begins in familiar territory. Sentences get truth values at an index – a world – with respect to a context. Contexts determine the set of possibilities not yet ruled out. These contexts are idle when it comes to interpreting sentences of L with no *ifs* since they are, *ex hypothesi*, context

⁵Some of that is done elsewhere – see, e.g., Gillies (2004, 2006b). I favor a similar line on counterfactuals, though – naturally enough – the domains over which the two sorts of conditionals are strict tend to differ (Gillies, 2006a). There are, to be sure, other strict conditional stories for indicatives on the market, though none that tell the story in just the way I do.

invariant: a negation is true at a world (in a context) iff the negated claim isn't true at that world (in that context); a conjunction is true just in case its conjuncts are. But the iffy bits depend on what has been ruled out. Thus – assuming that *if* is indeed a strict conditional over the possibilities not yet ruled out – the meaning of *if* is context dependent. But it is not unruly.

Contexts determine the sets of possibilities relevant at worlds. Those sets are the domains over which *if* is a strict conditional. I will assume that the functions supplied by context that do the determining are well-behaved:

Constraint (WELL-BEHAVEDNESS). For any s_c , i , and j :

- $i \in s_c(i)$
- if $j \in s_c(i)$ then $s_c(i) \subseteq s_c(j)$

Thus the facts at i are always relevant, though perhaps not decisive, to the truth of a conditional at i . Together the constraints straightaway imply that the set of relevant worlds in a context is *closed*: if $j \in s_c(i)$, then $s_c(i) = s_c(j)$. That means that possibilities relevant in a context do not vary between worlds compatible with it.

The schoolyard version of the Ramsey test says that an indicative conditional is true in a context just when adding the information carried by its antecedent to that context leaves us in a situation in which the consequent is true. Since truth depends on both an index and a context, the *if*-clause thus has two jobs to do. First: it restricts the set of indices throughout which we check for the consequent's truth. Second: it contributes to the relevant context for figuring out whether – at an index in that set – the consequent is true. If the consequent is context-invariant, then this second job is trivially done. But not otherwise.

That is very nearly the strict conditional story I want to tell. An indicative $(if\ P)(Q)$ is true – at i , with respect to c – iff all the worlds in $s_c(i)$ at which P is true – with respect to c – are all worlds at which Q is true – with respect to c -plus-the-information-that- P . The shift in context for evaluating Q allows the *if*-clause to do both jobs assigned to it by the schoolyard version of the Ramsey test. Officially:

Analysis (FIRST VERSION). For any i , c , P , Q :

- $c + P = \lambda i. s_c(i) \cap \llbracket P \rrbracket^c$
- $\llbracket (if\ P)(Q) \rrbracket^{c,i} = 1$ iff $s_c(i) \cap \llbracket P \rrbracket^c \subseteq \llbracket Q \rrbracket^{c+P}$

This is enough to disrupt the legend.

Proof. (U): Suppose p entails q . Then all the p -worlds are q -worlds. But then – no matter the world i or context c – $s_c(i) \cap \llbracket p \rrbracket \subseteq \llbracket q \rrbracket$, and so $(if\ p)(q)$ is bound to be true at i in c . (L): Suppose that $(p \wedge \neg q)$ is true at i . Then – no matter the context – i is one of the worlds relevant to an *if* at i . And so there is a $(p \wedge \neg q)$ -world in $s_c(i)$, and thus $(if\ p)(q)$ is bound to be false at i . (I): Suppose – for arbitrary c and i – that $\llbracket (if\ p \wedge q)(r) \rrbracket^{c,i} = 1$. Then all of the $(p \wedge q)$ -worlds in $s_c(i)$ are worlds at which r is true – with respect to c . To see that all the p -worlds in $s_c(i)$ are worlds at which $(if\ q)(r)$ is true – with respect to $c + p$ – consider any p -world in $s_c(i)$. Call it j , and note that

$$\begin{aligned} \llbracket (if\ q)(r) \rrbracket^{c+p,j} = 1 &\text{ iff } s_{c+p}(j) \cap \llbracket q \rrbracket \subseteq \llbracket r \rrbracket \\ &\text{ iff } (s_c(j) \cap \llbracket p \rrbracket) \cap \llbracket q \rrbracket \subseteq \llbracket r \rrbracket \\ &\text{ iff } (s_c(i) \cap \llbracket p \rrbracket) \cap \llbracket q \rrbracket \subseteq \llbracket r \rrbracket \\ &\text{ iff } \llbracket (if\ p \wedge q)(r) \rrbracket^{c,i} = 1 \end{aligned}$$

Whence it follows that if $(if\ p \wedge q)(r)$ is true – at i , in c – then so must be $(if\ p)((if\ q)(r))$. Indeed, they are equivalent.

And yet: *if* is not the material conditional. For let $s_c(i)$ contain two worlds, i and j , the first a $(p \wedge q)$ -world and the second a $(p \wedge \neg q)$ -world. Then $p \supset q$ is true – at i , in c – and $(if\ p)(q)$ false. QED

3 Intermezzo

There are two noteworthy features of this strict conditional story. First: it treats *if* as doubly context dependent. Second: it treats *if* as a doubly shifty operator.

It is a well-worn fact that, at least for some sentences, truth values at an index depend also on features of context. A sentence of our language L is *locally context dependent* just in case it is possible that its truth value at a given index i varies across contexts. There is nothing especially noteworthy in that. But perhaps there are sentences whose truth values in a context are required to co-vary no matter the choice of index: whatever truth value it gets – at a world, in a context – it gets in all worlds compatible with that context. Such context dependence would be an all or nothing affair – any sentence so dependent would either be true at any admissible world with respect to the context or true at none with respect to that context. That would make the truth values of such sentences *globally* dependent on features of context.

According to the first way of telling the strict conditional story, *if* is context dependent, both locally and globally. It is locally context dependent since an *if* at i can go from false to true by reducing the set of worlds not yet ruled out, ruling out all counterexamplifying possibilities in the context. And globally: an ordinary, indicative $(if\ p)(q)$ is true – at i , in c – just in case all of the p -worlds in $s_c(i)$ are q -worlds; s_c is closed, so if j is in $s_c(i)$ then $s_c(j) = s_c(i)$; and hence $(if\ p)(q)$ must also be true at j , with respect to c .

There is an argument – championed by Edgington (1995, 2006) – that no non-truth-functional assignment of truth-values to *if* can be right because the barest information supporting the material conditional seems also and always to be sufficient for the ordinary, indicative *if*. What makes a non-truth-functional story non-truth-functional is that the truth value of $(if\ p)(q)$ – at i , in c – is not fixed by the truth values of p and q at i . And so if p is false and q is true at i then perhaps the conditional is true at i and perhaps it isn't. Both are possible. But then the barest information supporting the material conditional – e.g., just the information that $(\neg p \vee q)$ – would not also and always be enough to support $(if\ p)(q)$. The variability in truth value of an *if* with a false antecedent and true consequent that is required by non-truth-functionality is just what is ruled out by the supporting facts. No non-truth-functional theory can get this right.

Not quite! This is true: if p is false and q is true at i then perhaps the conditional is true at i and perhaps it isn't. But this variability may well be a variation in truth value across contexts, not within any one context. And variability like that does nothing to preclude the supporting facts. For any given c it is open to have $(if\ p)(q)$ get the same truth value at any two worlds compatible with c , and so at any two at which p is false. That is just what the strict conditional story does. It delivers a truth value uniformly throughout the set of worlds compatible with c at i : iff $s_c(i)$ has any $(p \wedge \neg q)$ -worlds is the indicative false throughout. But things may be different in different contexts: a context minimally characterizing that $(\neg p \vee q)$ has no $(p \wedge \neg q)$ -worlds – and so the indicative is true therein. The argument misfires because it overlooks the possibility that the non-truth-functional truth conditions of *ifs* might be both locally and globally dependent on features of context.

It is a well-worn fact that the truth values – at an index, in a context – of some sentences depend on the truth values of their embedded constituents at different indices. Such sentences are *index-shifty*. Thus tense operators:

(8) Jimbo washed the dishes

is true at an index (in a context) just in case *Jimbo washes the dishes* is true at a (recent-ish) earlier index. And modal operators:

(9) Jimbo has to wash the dishes

is true at an index (in a context) just in case *Jimbo washes the dishes* is true at all the indices compatible with the house rules.

We might imagine another kind of shiftiness: the truth values – at an index, in a context – of some sentences might depend on the truth values of their embedded constituents at ever so slightly different contexts. Such sentences would be *context-shifty*.

According to the first way of telling the strict conditional story, *if* is certainly index-shifty. Whether

(10) If Carl is at the party, then Lenny is at the party

is true – at i , in c – depends on whether *Lenny is at the party* is true at various other worlds: those antecedent-worlds compatible with the context. But it is also context-shifty since the consequent is evaluated in a subordinate or derived context: *Lenny is at the party* is evaluated not in c but in c -plus-the-information-that-Carl-is-at-the-party. This is a shift that makes no difference if – as in this case – the consequent has no context sensitive bits in it.

But only if. That is how it can be that *if* is a strict conditional that validates the equivalence between $(if\ p \wedge q)(r)$ and $(if\ p)((if\ q)(r))$. Taking *if* to be a merely index-shifty strict conditional, given well-behavedness of the s_c 's, could not do that. Mere index-shiftiness would require only that all of the antecedent-worlds compatible with the context be worlds at which the consequent is true – true, that is, with respect to that context. As follows:

$$(11) \quad \llbracket (if\ P)(Q) \rrbracket^{c,i} = 1 \text{ iff } s_c(i) \cap \llbracket P \rrbracket^c \subseteq \llbracket Q \rrbracket^c$$

No such story about *if* can render the likes of (1) equivalent to the likes of (2).

Example: take any p, q , and r . It is established in c that either all of them are true, or just one is. Then $(if\ p \wedge q)(r)$ is true at a world compatible with c – say i – since all of the $(p \wedge q)$ -worlds compatible with c are indeed r -worlds. But assuming mere index-shiftiness, $(if\ p)((if\ q)(r))$ is not true at i , in c . There is a $(q \wedge \neg r)$ -world compatible with the context. Whence it follows that at no world not yet ruled out is $(if\ q)(r)$ true – true, that is, with respect to c . A

fortiori at no p -world not yet ruled out is $(if\ q)(r)$ true with respect to c . So more than index-shiftiness is required.

4 Second Version

The first version of the strict conditional story began in familiar territory and proceeded by assigning to the doubly shifty *ifs* doubly context dependent propositions. That makes the truth of an *if* – at a world, in a context – sensitive to both the context and a subordinate context got by hypothetically adding some information to it.

But we might begin elsewhere.⁶ Rather than assigning truth values to sentences of L at indices (in contexts) let's assign them truth values in contexts, full-stop. (As before, assume that the actual world is always among the worlds compatible with the context; that is, the facts are always relevant, though perhaps not decisive, to the truth of an indicative.) A sentence is true in a context, we will say, just in case an assertive utterance of it in the context adds no information to that context – for then the information carried by the sentence is already present. We can still talk about truth-at-a-world as a special case, if we like: truth-at- i is just truth at the singleton context $\{i\}$.⁷ To say all of this properly we will have to say what it means for a sentence to add its information to a context.

Before s_c was a function from worlds to sets of worlds compatible with c . Now let us simply identify a context s with the set of worlds compatible with it.⁸ The semantic values of sentences of L are their *context change potentials*: how they change the contexts in which they are successfully issued. Or, if you prefer: the meanings of declarative sentences are like the meanings of recipes and programs.

Some of these changes are straightforward: atomic sentences remove worlds from the context in which the atoms are not true, negation is relative complementation, and conjunction is functional composition. But *ifs* induce a different sort of change. Some programs carry instructions whose point is to have non-null effect: **set the value of x to 1**. And some programs have as their point null effects: **check whether the value of x is 1**. Non-iffy constructions are more like the former, *ifs* more like the latter. They test

⁶See Gillies (2004).

⁷For atoms truth-at-world is taken to be primitive.

⁸The space of contexts thus forms a lattice with W and \emptyset as the limit cases and \cap as join.

to see whether – in s – the information carried by the antecedent brings the consequent in its wake. If so, the conditional is true; otherwise, not.

Let the behavior of negation and conjunction, and the idea that truth at a context is information saturation, constrain the analysis. As follows:

Constraint (NON-IFY UPDATES). For any s , atom p , and arbitrary P, Q :

- $s[p] = \{i \in s : i(p) = 1\}$
- $s[\neg P] = s \setminus s[P]$
- $s[P \wedge Q] = s[P][Q]$
- P is true in s iff $s[P] = s$

The constraint is sensible since it means that for the fragment of L with no *ifs* whatever there is no difference between truth at a context and truth at an index with respect to a context.⁹ The strict conditional story can then be told this way:

Analysis (SECOND VERSION). For any s , and arbitrary P, Q :

- $s[(if P)(Q)] = \{i \in s : Q \text{ is true in } s[P]\}$

Thus an indicative is true in a context s iff its consequent is true in the derived or subordinate context got by adding the information carried by its antecedent to s . It follows straightaway that an ordinary indicative $(if P)(Q)$ is true in a context just in case all the worlds surviving an update with P also survive an update with Q , and so it is a strict conditional analysis properly so-called.¹⁰ And, as before, this is enough to disrupt the legend.

Proof. Take any non-iffy p, q , and r . (U) If p -worlds are all q -worlds, then since – no matter the context s – p is true in $s[p]$, it follows that q is true in $s[p]$. Hence $s[(if p)(q)] = s$ and the conditional is true in s . Since it is true in any context whatever, it is true at every singleton context and so at every world. (L) If p is true at i , and q false at it, then at no context with which i is compatible is the conditional true – indeed, its negation is. (I) The conditional so analyzed supports a deduction theorem: if, and only if, Q is true in $s[P]$ is the conditional $(if P)(Q)$ true in s . Now suppose that $(if (p \wedge q))(r)$ is true in

⁹Suppose P has no *if*. Then $W[P]$ is just the set of P -worlds and – for any context s whatever – $s[P]$ is just $s \cap \llbracket P \rrbracket$. Thus, P is true in s iff $s \cap \llbracket P \rrbracket = s$ – which is just to say iff $s \subseteq \llbracket P \rrbracket$. Reducing this to truth-at-a-world talk, we then have: P is true at i iff $i \in \llbracket P \rrbracket$.

¹⁰Equivalently: add a diamond to our language, and interpret it as a possibility test in this way:

- $s[\diamond P] = \{i \in s : s[P] \neq \emptyset\}$

Let \Box abbreviate $\neg \diamond \neg$. Then it follows from the second version of the strict conditional analysis that $(if P)(Q)$ induces just the changes to a context that $\Box(P \supset Q)$ does.

s . Hence r is true in $s[p \wedge q]$, and so in $s[p][q]$. And so $(if\ q)(r)$ is true in $s[p]$, and $(if\ p)((if\ q)(r))$ in s .

And yet: *if* is not the material conditional. For they induce different changes to contexts, and thus have different semantic values, and so behave differently under other operators. Example: $\neg(p \supset q)$ is true in s iff, for each i in s , it is true in $\{i\}$. Not so for the indicative. Let s contain just two worlds – i and j – the first a $(p \wedge q)$ -world, the second a $(p \wedge \neg q)$ -world. Then $\neg(if\ p)(q)$ is true in s but not at $\{i\}$. QED

5 Dolci

The first version of the strict conditional analysis takes *if* to be a doubly shifty operator and assigns doubly context-sensitive propositions to it. These two features conspire, letting *if* behave in some respects like the material conditional while still acting as a strict conditional over the set of possibilities compatible with the context.

The second version of the strict conditional story also takes *if* to be context dependent, and doubly so. First: because whether or not a context passes the test posed by an indicative depends on what possibilities are not yet ruled out. Second: because the context relevant for the evaluation of a conditional's consequent is a subordinate context got from the original context by hypothetically updating it. But it also insists – as did the first analysis – that the truth value of an indicative in a context depends on global facts about that context. An indicative induces an all or nothing test on a context in which it is issued, and that test is not a continuous function of the values it would have on the singleton contexts got from the possibilities that make that context up.¹¹ That is why $\neg(if\ p)(q)$ and $\neg(p \supset q)$ have different truth conditions. These two features conspire, letting *if* behave in some respects like a material conditional while still acting as a strict conditional over the set of possibilities compatible with the context.

Although these two versions of the strict conditional analysis start in very different territory, they come to much the same thing, and that is because they rely on the same sort of shifty behavior and global context dependence.

Fix a context c and world i , and suppose that we have a context s such that $s_c(i) = s$. And take an indicative $(if\ P)(Q)$. Then it turns out that if j is in s

¹¹That is: it is not in general so that $s[(if\ P)(Q)]$ is the same as $\bigcup_{i \in s} \{i\} [(if\ P)(Q)]$.

– if j is compatible with c – then $(c+P)(j) = s[P]$. The two stories exploit this in pretty much the same strict-conditional way, using this subordinate context as relevant for checking the truth of Q . Whence it follows that the indicative $(if P)(Q)$ is true – at i , in c – according to the first analysis iff, according to the second analysis, Q is true in $s[P]$ and so iff $(if P)(Q)$ is true in s , full-stop.

To put things the other way around: assume the first analysis, and suppose that the characteristic change induced by an assertion of P in a context is to intersect the worlds compatible with that context with the proposition expressed by P in it. Then the characteristic change of an indicative is just the context change potential assigned to it by the second analysis. For consider an indicative $(if P)(Q)$ at i , in c . If it is true at i , it is true everywhere in $s_c(i)$; if it is false, it is false everywhere in $s_c(i)$. So the proposition expressed by it at i in c is either the whole of $s_c(i)$ or none of it; and adding that proposition to the worlds compatible with the context will return either the whole lot of them or none at all. And that is precisely the profile of the context change potential $[(if P)(Q)]$: at s – tht is, at $s_c(i)$ – it returns either the whole of it or none of it at all.

But wait – don't you smell a rat? Consider the first version of the strict conditional story. I said that it meets the demands of (L) and thereby treats indicatives as at least as strong as material conditionals. True enough it meets the demands of (L), but that requirement says less than what we might have thought. It requires that, for non-iffy p 's and q 's, the truth of $(p \wedge \neg q)$ rules out the truth of $(if p)(q)$. But suppose – as the folklore argument requires – we have an embedded indicative in the consequent instead. Then (L) is silent. But we might wonder whether: does the truth of $(P \wedge \neg Q)$ rule out the truth of $(if P)(Q)$?

Not according to the first version of the strict conditional story. For suppose that p is false at i but that $s_c(i)$ has some $(p \wedge \neg q)$ -worlds. Then $(if p)(q)$ is false at i , in c – there are $(p \wedge \neg q)$ -worlds compatible with the context. But since no worlds are both p -worlds and $\neg p$ -worlds, $(if \neg p)((if p)(q))$ is true at i , in c . So we have an *if* true at i , in c , with a true antecedent and false consequent (at i , in c). Things only seem to get worse: for if indicatives are not, in general, bound from below by the corresponding material conditionals then they also do not, in general, go in for modus ponens. That would be an embarrassment than which none greater can be imagined.

I agree that it would be an embarrassment for the first version of the strict conditional story if, according to it, an *if* together with its antecedent did not

bring the truth of its consequent in their wake. But what does that mean, *bring the truth of its consequent in their wake*? Let c be any context and i a world compatible with it. Consider any complex *if* with antecedent P and consequent Q . And suppose that both P and $(if P)(Q)$ are true at i , in c . Then according to the first way of telling the strict conditional story Q is also true at i , in $((c + P) + (if P)(Q))$. Since the complex *if* is true at i in c , adding $(if P)(Q)$ to $(c + P)$ idles. So Q is true at i in $(c + P)$.

Therefore the truth of an *if* and its antecedent at a world in a context brings with it the truth of its consequent at that world in the context as it would be after adding the antecedent to it. That is as good a reason as any to say that the *if* together with its antecedent entails its consequent, and that is as good a reason as any to say that *if* does in general go in for modus ponens after all.

The picture generalizes. Entailment is not mere preservation of truth-at-point. That would be fine if truth values never were sensitive to context-shiftiness. But we know better. So entailment must also keep track of changes in contexts as we go along, adding the information carried by premises to the evolving context. Precisely:

Definition (ENTAILMENT, v.1).

- P_1, \dots, P_n entails Q iff, for any i and c , if $\llbracket P_1 \rrbracket^{c,i} = 1$ and ... and $\llbracket P_n \rrbracket^{((c+\dots+P_{n-2})+P_{n-1}),i} = 1$ then $\llbracket Q \rrbracket^{((c+\dots+P_{n-1})+P_n),i} = 1$

We preserve truth at i , but increment just which context is at play by adding the information of them premises. (If Q is context-invariant, then this is a difference that makes no difference.) All of this is very reminiscent of Stalnaker's (1975) pragmatic surrogate for entailment – *reasonable inference* – except that this is the real thing. And we will find no counterexamples to modus ponens here.

But we might have started in different territory, as does the second version of the strict conditional story. That analysis assigns truth values to sentences not at indices but assigns truth values in contexts, full-stop. Iff a sentence would not change a context s at all were we to add its information to s is that sentence true in s . If we like, we can then stick to the characterization of entailment as preservation of truth:

Definition (ENTAILMENT, v.2.0). *text*

- P_1, \dots, P_n entail Q iff, for any context s , if P_1, \dots, P_n are all true in s then so is Q

And we can show that if an indicative $(if\ P)(Q)$ – even one with embedded *ifs* figuring in it – is true in a context and P is also true there, then Q must also be true in that context. For if the conditional is true in s , then $s[(if\ P)(Q)] = s$ – that is, Q is true in $s[P]$. But, *ex hypothesi*, P is also true in s . Whence it follows that $s[P] = s$ and thus that Q is true in s .

So we could insist that entailment is preservation of truth, and leave it at that. That would be to opt, in the lingo of dynamic semantics, for a *test-to-test* characterization of entailment. Better, I think, to treat entailment not as preservation of truth but as preservation of information. Suppose that – no matter the context – adding the information carried by P and the information carried by R always results in a context in which Q is true. That is as good a reason as any to say that the former sentences entail the latter. More generally:

Definition (ENTAILMENT, v.2.1).

- P_1, \dots, P_n entail Q iff, for any context s , if $s[P_1] \dots [P_n] = s'$ then Q is true in s'

This is to opt, in the lingo of dynamic semantics, for an *update-to-test* characterization of entailment.¹² It is again straightforward to show that $(if\ P)(Q)$ and P entail Q . For if $(if\ P)(Q)$ is true in s , then Q is true in $s[P]$. (If $(if\ P)(Q)$ is not true in s , then the entailment is vacuous.) But $s[(if\ P)(Q)]$ just is s . Hence Q is true in $s[(if\ P)(Q)][P]$.

If we are happy enough with the second version of the strict conditional story, then I say we plump for this characterization of entailment. Modus ponens – and, in fact, a deduction theorem – come with it. And so plumping closes the gap between the two analyses since they then agree on when indicatives are true and on the entailments involving them.

¹²See Veltman (1996); update-to-test entailment is coupled with the strict conditional story in Gillies (2004).

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