

Two alternatives for disjunction: an inquisitive reconciliation*

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Abstract

Traditionally, disjunction is taken to express an operator that takes two propositions A and B , and yields their *join*, $A \cup B$. In more recent work, however, it has been argued that disjunction should rather be treated as generating a *set* of propositions, $\{A, B\}$. Each of the two approaches has certain vantage points that the other one lacks. Thus, it would be desirable to reconcile the two, combining their respective strengths. This paper shows that this is indeed possible, if we adopt a notion of meaning that does not just take informative content into consideration, but also inquisitive content.

1 Introduction

There is a long tradition in natural language semantics that analyzes disjunction words like English *or* as expressing a *join* operator, which delivers the least upper bound of the two disjuncts with respect to entailment (e.g., Montague, 1970; Partee and Rooth, 1983; Keenan and Faltz, 1985; Winter, 2001). However, there is also a substantial body of work in which disjunction is treated differently, namely as generating sets of alternative propositions (e.g., von Stechow, 1991; Simons, 2005; Alonso-Ovalle, 2006; Aloni, 2007). This alternative treatment of disjunction yields better predictions for a range of phenomena. However, the more traditional treatment also has important advantages. In particular, it provides a lucid explanation for the cross-linguistic ubiquity of disjunction words, and the fact that these words, across a wide range of typologically unrelated languages, all share a very specific semantic function.

Ideally, one would like to reconcile the two approaches, incorporating the benefits of both. The aim of this paper is to show that this is indeed possible. The key to this result is to generalize the traditional notion of meaning, which captures only informative content, to one that encompasses both informative and inquisitive content. Such a generalized notion of meaning has been developed in recent work on *inquisitive semantics* (e.g., Ciardelli, Groenendijk, and Roelofsen, 2013a), building on earlier work on the semantics of questions (e.g., Hamblin, 1973; Karttunen, 1977; Groenendijk and Stokhof, 1984). If we enrich our notion of meaning in this way, entailment

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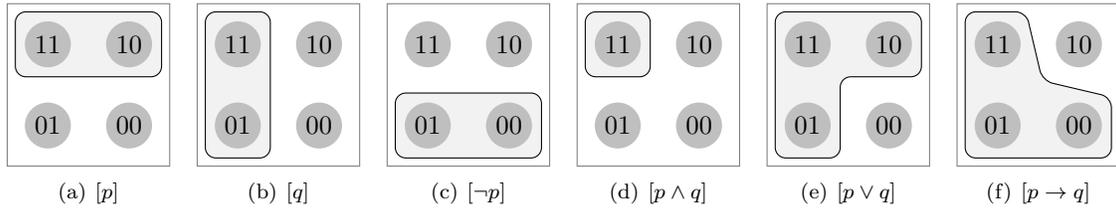


Figure 1: Propositions expressed by some simple sentences in classical logic.

also becomes sensitive to both informative and inquisitive content, and if we treat disjunction as a *join* operator with respect to this more fine-grained entailment order, it automatically generates the desired alternatives.

The paper is organized as follows. Section 2 briefly reviews the traditional treatment of disjunction as a *join* operator, and points out what makes this treatment particularly attractive from a linguistic perspective. Section 3 reviews the treatment of disjunction as generating sets of alternatives, and summarizes one particular argument that has been given in favor of it, involving counterfactual conditionals with disjunctive antecedents (Alonso-Ovalle, 2006). Section 4 shows how the two approaches can be reconciled, and Section 5 concludes, placing the obtained result in a somewhat broader context.

2 The classical treatment of disjunction

2.1 Classical logic

In classical logic, the meaning of a sentence—the *proposition* that it expresses—is construed as a set of possible worlds, namely, those worlds in which the sentence is true. Thus, the meaning of a sentence in classical logic embodies its *truth conditions*. This notion of meaning forms the basis for most formal semantic analyses of natural language. Intuitively, a proposition carves out a region in the space of all possible worlds. In asserting a sentence, a speaker is taken to provide the information that the actual world is located within this region, i.e., that the actual world is one in which the uttered sentence is true. Thus, the proposition expressed by a sentence captures the *informative content* of that sentence, i.e., the information that is provided by a speaker in uttering the sentence.

Some simple classical propositions are depicted in Figure 1. In these examples, it is assumed that there are just two elementary sentences, p and q , which means that there are just four distinct possible worlds: one in which both p and q are true, which is labeled 11, one in which p is true but q is false, labeled 10, etcetera. Figure 1(a) depicts the classical proposition expressed by p , denoted $[p]$, which is the set of worlds where p is true, and Figure 1(b) does the same for the proposition expressed by q , denoted $[q]$. The *connectives* in classical logic are treated as expressing simple operations on propositions: for any two sentences φ and ψ , $[\varphi \wedge \psi]$ is the *intersection* of $[\varphi]$ and $[\psi]$, $[\varphi \vee \psi]$ is the *union* of $[\varphi]$ and $[\psi]$, $[\varphi \rightarrow \psi]$ is the largest set of worlds whose intersection with $[\varphi]$ is contained in $[\psi]$, and $[\neg\varphi]$ is the largest set of worlds whose intersection with $[\varphi]$ is empty, i.e., the set of all worlds that are not in $[\varphi]$. This is depicted for some concrete examples in Figures 1(c)-1(f).

2.2 Linguistic relevance

Even though virtually any introduction to formal semantics starts out with classical logic, the linguistic relevance of the classical treatment of the connectives is not so often explicitly argued for. What makes this particular treatment of the connectives so special? Why is it called the *classical* treatment? Is this terminology just a historical coincidence, or does it carry some real substance?

To answer these questions, it is helpful to take an *algebraic* perspective. In a nutshell, what is characteristic for classical logic is that it takes each connective to express one of the most basic algebraic operations on propositions. This is what makes the classical treatment of the connectives special among other possible treatments. Moreover, this also explains why this specific treatment is of particular interest from a linguistic point of view. After all, it is to be expected that the basic algebraic operations on propositions that are associated with the connectives in classical logic will generally be expressible in natural languages as well, just like one would expect, for instance, that the basic algebraic operations in arithmetic—addition, subtraction, multiplication, and division—are generally expressible in natural languages as well.

2.3 An algebraic perspective

Let us briefly review the algebraic perspective on classical logic, then. The basic observation underlying this perspective is that propositions in classical logic are naturally *ordered* in terms of their informative strength. Recall that the proposition expressed by a sentence can be thought of as carving out a region in the set of all possible worlds, and that in asserting a sentence, a speaker is taken to provide the information that the actual world is located within this region. One proposition is more informative than another, then, if it carves out a smaller region, i.e., if it locates the actual world more precisely. This order on propositions is referred to as *entailment*. One proposition A entails another proposition B , $A \models B$, just in case $A \subseteq B$.

Now, every ordered set has a certain algebraic structure and comes with certain basic algebraic operations. The set of classical propositions, ordered by entailment, forms a so-called *Heyting algebra*, which comes with four basic operations: *join*, *meet*, and *relative/absolute pseudo-complementation*.

The *join* of two propositions A and B is the *least upper bound* of A and B with respect to entailment, i.e., the strongest proposition that is entailed by both A and B . As depicted in Figure 2(a), this least upper bound amounts to the *union* of the two propositions: $A \cup B$.

The *meet* of A and B on the other hand, is the *greatest lower bound* of A and B with respect to entailment, i.e., the weakest proposition that entails both A and B . As depicted again in Figure 2(a), this greatest lower bound amounts to the *intersection* of the two propositions: $A \cap B$.

The *pseudo-complement* of a proposition A relative to another proposition B , which we will denote as $A \Rightarrow B$, can be thought of intuitively as the ‘difference’ between A and B : it is the weakest proposition C such that A and C together contain at least as much information as B . More formally, it is the weakest proposition C such that $A \cap C \models B$. This is visualized in Figure 2(b). The shaded area in the figure is the set of all propositions C which are such that $A \cap C \models B$. The weakest among these, i.e., the topmost one, is the pseudo-complement of A relative to B . This proposition consists of all possible worlds which, if contained in A , are also contained in B :

$$A \Rightarrow B = \{w \mid \text{if } w \in A \text{ then } w \in B \text{ as well}\}$$

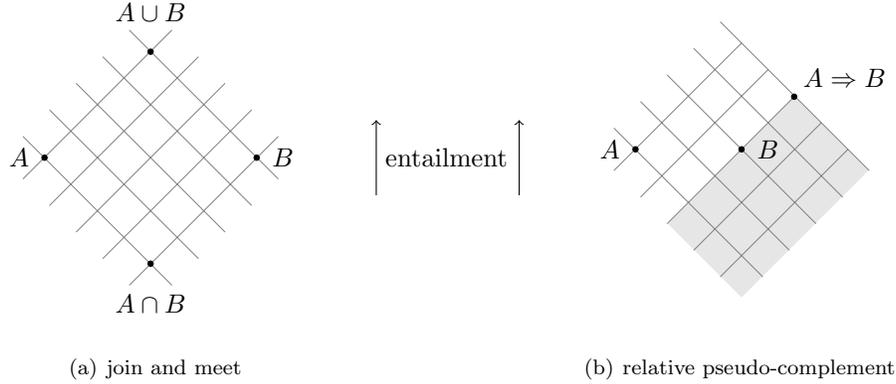


Figure 2: Join, meet, and relative pseudo-complement.

Absolute pseudo-complementation is a limit case of relative pseudo-complementation. The *absolute pseudo-complement* of a proposition A , which will be denoted as A^* , is the weakest proposition C such that $A \cap C$ entails *any* other proposition. Since the only proposition that entails any other proposition is the *empty* proposition, denoted as \perp , A^* can be characterized as the weakest proposition C such that $A \cap C = \perp$. It consists simply of all worlds that are not in A itself:¹

$$A^* = \{w \mid w \notin A\}$$

Now, with these four basic algebraic operations in place, we are ready to return to the classical treatment of the connectives. This treatment consists in associating each connective with one of the basic algebraic operations: disjunction expresses *join*, conjunction expresses *meet*, implication expresses *relative pseudo-complementation*, and negation expresses *absolute pseudo-complementation*.²

$[\varphi \vee \psi]$	=	$[\varphi] \cup [\psi]$	<i>join</i>
$[\varphi \wedge \psi]$	=	$[\varphi] \cap [\psi]$	<i>meet</i>
$[\varphi \rightarrow \psi]$	=	$[\varphi] \Rightarrow [\psi]$	<i>relative pseudo-complementation</i>
$[\neg \varphi]$	=	$[\varphi]^*$	<i>absolute pseudo-complementation</i>

As mentioned above, since these algebraic operations are so basic, it is to be expected that natural languages will generally have ways to express them. And indeed, in language after language

¹In classical logic it does not only hold for any proposition A that $A \cap A^*$ is the *strongest* of all propositions, i.e., the proposition that entails all other propositions (this is directly enforced by the definition of A^*), but also that $A \cup A^*$ amounts to the *weakest* of all propositions, i.e., the proposition that is entailed by all other propositions. Because it has this special property, A^* is called the *Boolean complement* of A in classical logic, and the set of all classical propositions ordered by entailment forms a *Boolean algebra*, a special kind of Heyting algebra. Anticipating what is to come, this special property will be lost when we generalize the classical notion of meaning so as to incorporate inquisitive as well as informative content. Everything else, however, will be exactly the same.

²In classical *predicate* logic, the existential and the universal quantifier are also treated as *join* and *meet* operators, respectively, applied to a possibly infinite set of propositions. We will restrict our attention to propositional logic here, but everything we will say about disjunction and conjunction applies just as well to existential and universal quantification.

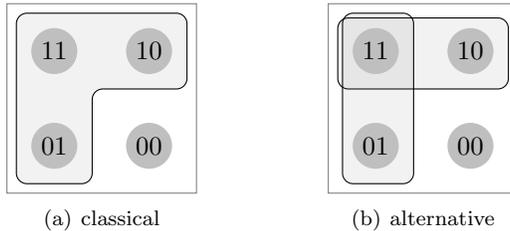


Figure 3: $[p \vee q]$ in classical logic and in alternative semantics.

words or morphemes have been found that may be taken, at least at first sight, to fulfill exactly this purpose (e.g., [Haspelmath, 2007](#); [Gil, 2013](#)). The algebraic perspective on meaning provides a simple explanation for the cross-linguistic ubiquity of such words. This makes the treatment of the connectives in classical logic attractive from a linguistic point of view. And indeed, it has for a long time been the standard account. However, in the case of disjunction, a number of compelling arguments have recently been put forth in favor of an alternative treatment. To this we turn next.³

3 Disjunction in alternative semantics

3.1 Alternatives

There is a substantial body of work (e.g., [von Stechow, 1991](#); [Simons, 2005](#); [Alonso-Ovalle, 2006](#); [Aloni, 2007](#); [Groenendijk, 2009](#); [Mascarenhas, 2009](#); [Groenendijk and Roelofsen, 2009](#); [Ciardelli, 2009](#); [AnderBois, 2011](#); [Biezma and Rawlins, 2012](#)) in which disjunctions are not taken to express single propositions, but rather *sets* of propositions. For instance, as depicted in Figure 3, $[p \vee q]$ is not taken to amount to the union of $[p]$ and $[q]$ as in classical logic, but rather to $\{[p], [q]\}$, i.e., the set consisting of $[p]$ and $[q]$. These propositions are referred to as the *alternatives* that the disjunction introduces.

The treatment of disjunction as introducing sets of alternatives is part of a broader line of work, which analyzes various other kinds of constructions as introducing sets of alternatives as well, including questions ([Hamblin, 1973](#)), focus ([Rooth, 1985](#)), indeterminate pronouns (e.g., [Kratzer and Shimoyama, 2002](#)) and indefinites (e.g., [Kratzer, 2005](#); [Menéndez-Benito, 2005](#); [Aloni, 2007](#)). The general framework that has emerged from this work is referred to as *alternative semantics*.

³The linguistic adequacy of the classical treatment of the other connectives has also been subject to much debate. For instance, it has been debated whether the word *and* can be treated as involving a *meet* operator in sentences involving collective predication, like *John and Sue got married* (see [Winter, 2001](#); [Champollion, 2013](#), and references therein). Similarly, it has been debated whether the word *if* can indeed generally be treated as a relative pseudo-complementation operator (e.g., [Kratzer, 1986](#)). Since our focus is on disjunction, these debates are beyond the scope of this paper. Moreover, note that they do not directly affect the argument that was given here for the linguistic relevance of classical logic. The underlying algebraic perspective only leads us to expect that natural languages will generally have the means to express the given algebraic operations. It may very well be that the words that are used to express these operations are, in some or all languages, used for other purposes as well. This may be the case for English *and* and *if*. Moreover, these words may not be the only means available in English to express meet and relative pseudo-complementation; other words that may be used for this purpose arguably include *as well as* and *implies*, respectively. Finally, even if English had no way of expressing the basic algebraic operations on propositions at all, this would of course not by itself disprove the expectation that *in general* these operations *are* expressible.

The arguments that have been given in favor of the treatment of disjunction in alternative semantics involve a number of constructions, ranging from modals and conditionals to disjunctive questions and imperatives. To illustrate the nature and force of these arguments, let us briefly review one of them, due to [Alonso-Ovalle \(2006\)](#).

3.2 Illustration: counterfactuals with disjunctive antecedents

Consider the following scenario. Sally had a birthday party at her house, and some friends brought their instruments. One of Sally's friends, Bart Balloon, who has a terrible musical taste, fortunately forgot to bring his trumpet. Now consider the following sentence:

- (1) If Bart Balloon had played the trumpet, people would have left. $\varphi \rightsquigarrow \psi$

According to the classical treatment of counterfactuals due to [Stalnaker \(1968\)](#) and [Lewis \(1973\)](#), $\varphi \rightsquigarrow \psi$ is true in a world w just in case, among all worlds that make φ true, those that *differ minimally* from w also make ψ true. This correctly predicts that (1) is true in the given scenario.

But now consider a counterfactual with a *disjunctive* antecedent:

- (2) If Bart Balloon or Louis Armstrong had played the trumpet, people would have left.

The Stalnaker/Lewis treatment of counterfactuals, together with the classical treatment of disjunction, wrongly predicts that (2) is true in the given scenario. This is because all worlds that make the antecedent true and differ minimally from the actual world are ones where Bart Balloon played the trumpet, and not Louis Armstrong. The second disjunct in the antecedent is thus effectively disregarded.

Initially, this observation was presented as an argument against the Stalnaker/Lewis treatment of counterfactuals ([Fine, 1975](#); [Nute, 1975](#); [Ellis *et al.*, 1977](#); [Warmbröd, 1981](#)). But another way to approach the problem is to pursue an alternative treatment of disjunction ([Alonso-Ovalle, 2006, 2009](#); [Van Rooij, 2006](#)). Indeed, if the disjunctive antecedent is taken to generate two alternatives and if verifying the counterfactual involves separately checking every alternative generated by the antecedent, the problem is avoided.⁴

3.3 Impasse

This is just a brief illustration of one of the arguments that have been put forth in favor of the alternative treatment of disjunction. It seems that in a wide range of constructions alternative semantics provides a better account of the behavior of disjunction than the traditional treatment. Unfortunately, however, adopting an alternative semantic treatment of disjunction comes at a significant cost. Namely, it forces us to give up the classical analysis of disjunction as expressing one of the basic algebraic operations on propositions. This means in particular that we no longer have a uniform treatment of disjunction, conjunction, negation, and implication, and moreover, perhaps most importantly, we no longer have an algebraic explanation for the cross-linguistic ubiquity of disjunction words.

Thus, we seem to have reached an impasse. Both treatments of disjunction have clear vantage points, but the two approaches seem to rest on very different, incompatible assumptions. Ideally we

⁴See also [Franke \(2009\)](#), [Klinedinst \(2009\)](#), and [Van Rooij \(2010\)](#) for pragmatic approaches to the problem.

would be able to bring the two closer together and obtain an analysis that shares their respective virtues. But how could this be achieved?

4 Reconciliation

I will suggest that the two approaches can be reconciled after all. In order to achieve this, however, a rather fundamental step needs to be taken. Namely, we need to move from the traditional notion of meaning, which captures only informative content, to a more general notion of meaning that encompasses both *informative* and *inquisitive* content. Such a generalized notion of meaning has been developed in recent work on *inquisitive semantics* (e.g., Ciardelli, Groenendijk, and Roelofsen, 2012, 2013a). This work is rooted in the seminal work of Hamblin (1973), Karttunen (1977), and Groenendijk and Stokhof (1984) on the semantics of questions. However, it involves some innovations that are essential for our purposes here. Most importantly, inquisitive semantics comes with a well-behaved notion of entailment, which is sensitive to both informative and inquisitive content. This is crucial to preserve the algebraic treatment of the connectives.⁵ Indeed, disjunction can be treated in this framework as a *join* operator, just like in classical logic but now with respect to a more fine-grained notion of entailment. And indeed, if we do this, disjunction automatically generates the desired alternatives. Let us see how this works in more detail.

4.1 Propositions in inquisitive semantics

We start by recalling some standard notions. As before, we assume a set of possible worlds, W . Each possible world encodes a possible way the world may be. A set of possible worlds $s \subseteq W$ can be thought of as encoding a certain *body of information*, namely the information that the actual world corresponds to one of the elements of s . Similarly, a set of possible worlds $s \subseteq W$ can also be taken to encode the *information state* of one of the conversational participants, or the body of information that is shared among the conversational participants at a given point. Following Stalnaker (1978), we will refer to the latter body of information as the *common ground* of the conversation.

In classical logic, the proposition expressed by a sentence is also construed as a set of possible worlds. As remarked above, a classical proposition intuitively carves out a region in the space of all possible worlds, and in asserting a sentence, a speaker is taken to provide the information that the actual world is located within this region. In this way, the proposition expressed by the sentence captures the *informative content* of the sentence.

Now, if we want propositions to be able to capture both informative and inquisitive content, they cannot simply be construed as sets of possible worlds. However, there is a very natural generalization of the classical notion of propositions that does suit this purpose. Namely, propositions can be construed as *sets of information states*. In uttering a sentence, a speaker can then be taken to steer the common ground of the conversation towards one of the states in the proposition ex-

⁵The lack of an appropriate notion of entailment was one of the main criticisms that Groenendijk and Stokhof (1984) brought up against Hamblin (1973), Karttunen (1977), and other early work on the semantics of questions. Groenendijk and Stokhof's partition framework partly overcomes this problem; however, as noted by Groenendijk and Stokhof themselves, while the resulting notion of entailment yields a suitable treatment of conjunction as a *meet* operator, it does not yield an appropriate treatment of disjunction and the other connectives. Inquisitive semantics can be seen as a generalization of the partition framework, which fully regains the classical algebraic treatment of the connectives (see Roelofsen, 2013a; Ciardelli, 2014b, for more elaborate discussion).

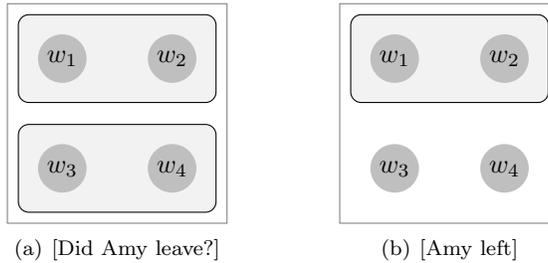


Figure 4: Some simple propositions in inquisitive semantics.

pressed, while at the same time providing the information that the actual world must be contained in one of these states.

We say that a proposition A is *settled* in a state s if and only if $s \in A$, and we assume that if A is settled in a certain state s then it remains settled in any more informed state $s' \subset s$. This means that propositions are *downward closed*: if $s \in A$ and $s' \subset s$, then $s' \in A$ as well. Finally, we will assume that the *inconsistent* state, \emptyset , settles any proposition. This means that any proposition has \emptyset as an element and is therefore non-empty. These considerations lead to the following characterization of propositions:

Definition 1 (Propositions in inquisitive semantics).

A proposition in inquisitive semantics is a non-empty, downward closed set of information states.

Among the states that settle a proposition A , the ones that are easiest to reach are those that contain least information. The states in A that contain least information are those that contain most possible worlds, i.e., they are the *maximal elements* of A . These maximal elements are referred to in inquisitive semantics as the *alternatives* in A . In depicting a proposition, we will generally only depict the alternatives that it contains.

Finally, since in uttering a sentence that expresses a proposition A a speaker is taken to provide the information that the actual world is contained in one of the elements of A , i.e., in $\bigcup A$, we refer to $\bigcup A$ as the *informative content* of A , and denote it as $\text{info}(A)$.

To illustrate these notions, consider the following two sentences:

- (3) Did Amy leave?
- (4) Amy left.

These sentences may be taken to express the propositions depicted in Figure 4, where w_1 and w_2 are worlds where Amy left, w_3 and w_4 are worlds where Amy didn't leave, and the shaded rectangles are the alternatives contained in the given propositions (by downward closure, all substates of these alternatives are also contained in the given propositions).

The proposition in Figure 4(a) captures the fact that in uttering the polar interrogative *Did Amy leave?*, a speaker (i) provides the trivial information that the actual world must be w_1 , w_2 , w_3 , or w_4 (all options are open) and (ii) steers the common ground towards a state that is either within $\{w_1, w_2\}$, where it is known that Amy left, or within $\{w_3, w_4\}$, where it is known that Amy didn't leave. Other conversational participants are requested to provide information in order to establish such a common ground.

On the other hand, the proposition in Figure 4(b) captures the fact that in uttering the declarative *Amy left*, a speaker (i) provides the information that the actual world must be either w_1 or w_2 , i.e., one where Amy left, and (ii) steers the common ground of the conversation towards a state in which it is commonly accepted that Amy left. In order to reach such a common ground, it is sufficient for other participants to accept the information that the speaker herself already provided in uttering the sentence; no further information is needed.

4.2 Informative and inquisitive propositions

Notice that in the case of the polar interrogative *Did Amy leave?*, the information that is provided is *trivial* in the sense that it does not exclude any possible world as a candidate for the actual world. A proposition A is called *informative* just in case it does exclude at least one possible world, i.e., iff $\text{info}(A) \neq W$.

On the other hand, in the case of the declarative *Amy left*, the inquisitive component of the proposition expressed is trivial, in the sense that in order to reach a state where the proposition is settled, other conversational participants only need to accept the informative content of the proposition expressed, and no further information is required. A proposition A is called *inquisitive* just in case settling A requires more than just mutual acceptance of $\text{info}(A)$, i.e., iff $\text{info}(A) \not\subseteq A$.

Given a picture of a proposition it is easy to see whether it is inquisitive or not. This is because, under the assumption that there are only finitely many possible worlds—and this is a safe assumption to make for all the examples to be considered in this paper—a proposition is inquisitive just in case it contains at least two alternatives. For instance, the proposition in Figure 4(a) contains two alternatives, which means that it is inquisitive, while the proposition in Figure 4(b) contains only one alternative, which means that it is not inquisitive.

4.3 Entailment and algebraic operations

In order for one proposition A to entail another proposition B in inquisitive semantics, two natural conditions need to be fulfilled: (i) A has to be at least as informative as B , and (ii) A has to be at least as inquisitive as B . Whether the first condition is satisfied can be checked by comparing $\text{info}(A)$ to $\text{info}(B)$. Just as in classical logic, A is at least as informative as B iff $\text{info}(A) \subseteq \text{info}(B)$. But what does it mean for A to be at least as *inquisitive* as B ? This can be made precise by comparing what it takes to settle A with what it takes to settle B . More specifically, A is at least as inquisitive as B just in case every state that settles A also settles B . But this just means that $A \subseteq B$. And moreover, if $A \subseteq B$, then it automatically holds that $\text{info}(A) \subseteq \text{info}(B)$ as well. Thus, at a formal level entailment in inquisitive semantics just amounts to inclusion, exactly as in classical logic.

Definition 2 (Entailment in inquisitive semantics). $A \models B$ iff $A \subseteq B$

At this point we are back on familiar ground: we have a generalized notion of propositions and a corresponding generalized notion of entailment which induces a partial order on the set of all propositions. The next thing to do is to consider the algebraic structure of this ordered set. And what we find is that, just as in classical logic, the set of propositions ordered by entailment forms a Heyting algebra. This means in particular that we have the same four basic algebraic operations: *join*, *meet*, and *relative/absolute pseudo-complementation*. In fact, the *join* and the *meet* of two

propositions can still be constructed simply by taking their union and intersection, respectively.⁶

Thus, to obtain the equivalent of classical logic in the inquisitive realm, we can associate these operations with disjunction, conjunction, implication, and negation, respectively, obtaining the familiar clauses repeated below, though now the propositions involved are not sets of worlds but rather sets of states:

$$\begin{array}{lll}
 [\varphi \vee \psi] & = & [\varphi] \cup [\psi] & \textit{join} \\
 [\varphi \wedge \psi] & = & [\varphi] \cap [\psi] & \textit{meet} \\
 [\varphi \rightarrow \psi] & = & [\varphi] \Rightarrow [\psi] & \textit{relative pseudo-complementation} \\
 [\neg\varphi] & = & [\varphi]^* & \textit{absolute pseudo-complementation}
 \end{array}$$

Thus, while the notion of meaning has been enriched, the essence of the classical treatment of the connectives has been preserved. The most basic inquisitive semantics system, InqB , which is defined for the language of first-order logic, consists precisely of the four clauses above, plus two clauses for existential and universal quantification, analogous to disjunction and conjunction, respectively, as well as a clause for atomic sentences, which says that the proposition expressed by an atomic sentence p is the set of all states consisting of worlds where p is true: $[p] = \{s \mid \forall w \in s : w(p) = 1\}$.⁷

Now, let us look at a number of examples to see how this system behaves. For easy comparison, let us consider the simple sentences that we also considered above when discussing classical logic (see Figure 1). The propositions expressed by these sentences in InqB are depicted in Figure 5. Notice that most of these propositions contain only one alternative, which means that they are not inquisitive. Moreover, notice that in each of these cases the unique alternative corresponds exactly to the proposition that the sentence at hand expresses in classical logic. More generally, it holds in InqB that for any sentence φ , $\text{info}(\varphi)$ always coincides precisely with the proposition expressed by φ in classical logic.

Now consider the proposition expressed by $p \vee q$, depicted in Figure 5(e). This is the only proposition among the ones that are depicted here that is inquisitive. Indeed, it contains two alternatives, and these alternatives are precisely the ones that are associated with $p \vee q$ in alternative semantics (see Figure 3(b)). Thus, while disjunction is treated as a *join* operator, as in classical logic, it also has the alternative-generating behavior that is assumed in alternative semantics.

Our aim, then, has been accomplished: the two competing analyses of disjunction have been reconciled. This means that the empirical phenomena involving disjunction that have been successfully dealt with in alternative semantics can be accounted for without giving up the idea that disjunction expresses one of the basic algebraic operations on meanings. One concrete consequence of this result is that the Stalnaker/Lewis treatment of counterfactuals turns out to be compatible with the algebraic treatment of disjunction as a *join* operator after all (contra the general assumption since Fine, 1975; Nute, 1975; Ellis *et al.*, 1977). Moreover, perhaps most importantly, we retain the algebraic explanation of the cross-linguistic ubiquity of disjunction words. Finally, the fact that the *join* operator in inquisitive semantics is a source of inquisitiveness forms a natural basis for an explanation of the observation that in many languages, disjunction words are also used as question markers (see, e.g., Jayaseelan, 2008; Cable, 2010; AnderBois, 2011; Slade, 2011; Szabolcsi, 2014).

⁶For proofs of these claims, see Roelofsen (2013a).

⁷For a basic exposition of InqB , see Ciardelli *et al.* (2012). In other work on inquisitive semantics, the logic that InqB gives rise to has been investigated in detail (see, e.g., Ciardelli, 2009; Ciardelli and Roelofsen, 2011), the basic notion of meaning that InqB relies on has been further enriched (see, e.g., Ciardelli *et al.*, 2013c, 2014; Groenendijk and Roelofsen, 2014; Punčochář, 2014; Roelofsen and Farkas, 2014), and different logical languages have been considered, e.g., making an explicit distinction between declaratives and interrogatives (Ciardelli *et al.*, 2013b), adding modal operators (Ciardelli and Roelofsen, 2014b; Ciardelli, 2014a), and moving from a first-order language to a full-fledged type-theoretic system (Ciardelli and Roelofsen, 2014a; Theiler, 2014).

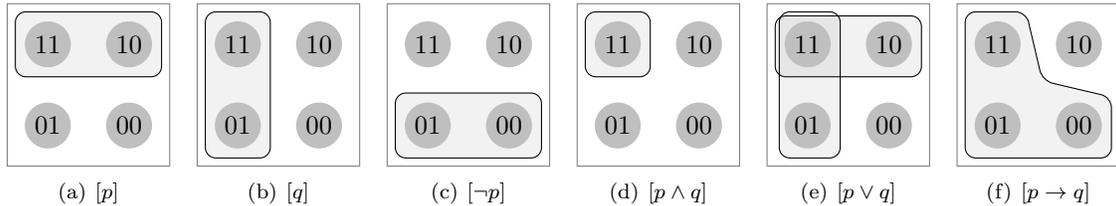


Figure 5: Propositions expressed by some simple sentences in InqB.

5 Concluding remarks

The obtained result raises a number of issues for further exploration. We will briefly highlight four of these issues, with pointers to other work where they are addressed in more detail.

Disjunction cross-categorically. First, while we have restricted our attention to sentential disjunction in this paper, disjunction can of course apply to expressions of other syntactic categories (e.g., noun phrases and verb phrases) as well. An important feature of the classical treatment of sentential disjunction, conjunction, and negation is that it can be generalized in a principled way to apply to expressions of all suitable categories (Gazdar, 1980; Partee and Rooth, 1983; Keenan and Faltz, 1985). This generalization can also be carried out in inquisitive semantics, in essentially the same way. To do so, the basic InqB system considered here has to be extended to a full-fledged type theoretical framework (Ciardelli and Roelofsen, 2014a; Theiler, 2014). Interestingly, this enterprise also shows how another non-standard feature of alternative semantics, namely its *compositional* architecture, can be reconciled with the classical Montagovian approach.

Interaction with clause type marking and intonation. The meaning of sentences containing disjunction depends, of course, on all kinds of factors that interact with the disjunction operator itself. Among such factors are clause type marking (e.g., declarative/interrogative word order) and intonation (e.g., pitch contour, prosodic phrase structure). This is illustrated in (5)-(8) below, where \uparrow and \downarrow indicate rising and falling pitch contours, respectively, and hyphenation and commas indicate the absence and presence of prosodic phrase breaks, respectively:

- (5) Igor speaks English-or-French \downarrow .
- (6) Does Igor speak English-or-French \uparrow ?
- (7) Does Igor speak English \uparrow , or French \uparrow ?
- (8) Does Igor speak English \uparrow , or French \downarrow ?

While all these disjunctive sentences contain exactly the same lexical material, they each receive a different interpretation. Clearly, a treatment of disjunction that takes both informative and inquisitive aspects of meaning into consideration is necessary to account for these different interpretations in a uniform way, i.e., without stipulating lexical ambiguity of the disjunction word *or*. However, such a treatment of disjunction is not sufficient; it needs to be complemented by a semantic account of the different clause type markers and intonational features. Thus, the inquisitive treatment of disjunction as characterized in the present paper should not be taken to make any direct predic-

tions about disjunctive sentences in English or other natural languages. Such predictions only arise when the semantic contribution of the other relevant features is specified (see [Roelofsen, 2013c](#); [Roelofsen and Farkas, 2014](#), for a proposal that assumes the InqB treatment of disjunction).

Expressive power. A third issue that arises from our discussion concerns the *expressive power* of inquisitive semantics versus that of alternative semantics. Recall that in InqB , propositions are downward closed sets of states, and the alternatives in a proposition are its maximal elements. This means that one alternative can never be contained in another (otherwise it would not be a maximal element). By contrast, in alternative semantics one alternative may very well be contained in another. Thus, there is a difference in expressive power between the two frameworks: in alternative semantics there are more distinct meanings than in InqB . The question is whether this is a vice or a virtue.

One empirical phenomenon that sheds light on this issue is the infelicity of disjunctive sentences where one disjunct implies another, e.g., *John lives in Paris or in France* (see, e.g., [Hurford, 1974](#); [Chierchia et al., 2009](#); [Katzir and Singh, 2013](#); [Meyer, 2014](#)). It can be argued that the notion of meaning adopted in InqB , which is incompatible with nested alternatives, offers an advantage in accounting for the oddness of such sentences, essentially because the stronger disjunct is predicted to be *redundant* under this notion of meaning. For more detailed discussion, see [Ciardelli and Roelofsen \(2015\)](#).

The attentive route. Finally, it is interesting to examine whether the general strategy that we adopted here to reconcile alternative semantics with classical semantics can be fruitfully separated from the concrete proposal that we made. The general strategy was (i) to look for a sensible notion of meaning that is more fine-grained than the classical notion, i.e., one that goes beyond plain informative content, (ii) to construe a notion of entailment that matches this more fine-grained notion of meaning, and then (iii) to explore the algebraic structure of the new, richer space of meanings under the corresponding entailment order. Our concrete proposal was to adopt a notion of meaning which, besides informative content, also takes *inquisitive* content into account. We saw that this particular choice allowed us to preserve the essence of the classical algebraic treatment of the logical connectives, and more specifically, that treating disjunction as a *join* operator in this setting yields the alternative-generating behavior that is assumed in alternative semantics. However, it is not necessarily the case that this is the *only* way to obtain these results. One could also think of other ways to enrich the classical notion of meaning, and these might yield similar results, either in general or only specifically when it comes to disjunction.

Such an approach has been explored in [Roelofsen \(2013b\)](#). In this work, the classical notion of meaning is not enriched with inquisitive content but rather with *attentive* content (see also [Yalcin, 2008](#); [Brumwell, 2009](#); [de Jager, 2009](#); [Ciardelli et al., 2009, 2014](#); [Westera, 2013](#), among others). The attentive content of a sentence can be modeled as a set of alternatives—the alternatives that the sentence draws attention to—and the informative content of the sentence can be seen as the union of these alternatives. A corresponding notion of entailment can be defined, one that is sensitive to both informative and attentive strength. However, it does not really make sense to treat disjunction as a *join* operator in this setting, because, while a disjunction is always *at most* as informative as each individual disjunct, it will always be *at least* as attentive. The sensible thing to do, then, is to tease informative entailment and attentive entailment apart and to let disjunction express an operator which, for any two sets of alternatives A and B , delivers a new set of alternatives that is the *join* of A and B w.r.t. informative entailment, and the *meet* of A and

B w.r.t. attentive entailment. Remarkably, it can be shown that this yields precisely in the *union* of A and B . So, again, we obtain the treatment of disjunction that was posited in alternative semantics, though now through a rather different route. A detailed comparison with the inquisitive route taken in the present paper is left for another occasion.

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