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DYNAMIC SITUATIONS: ACCOUNTING FOR DOWTY'S INERTIA NOTION USING DYNAMIC SEMANTICS

Master's Thesis

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CHAPTER 1
INTRODUCTION, OR: WHY BOTHER READING ON

The Problem

Imagine this: a zebra is pasturing on the endless plains of the Serengetti savanna. It is finishing off a patch of greenery. According to a semantic analysis of the progressive offered back in 1979 by the linguist David Dowty (Dowty 1979), this more or less means that there are still some sprouts to eat but if all goes on normally as it did up until now they will soon be gobbled up by the zebra. Take another look at the zebra's surroundings. Note the feline silhouette crouching but a few meters away. A hungry lioness is in the final stages of hunting down the zebra. What this means, according to that same linguist, is that the zebra is still alive right now but if all goes on normally as it did up until now it will shortly be served as lunch to the lioness' cubs.

Intuitively it seems reasonable enough to interpret a progressive sentence like (1) as (2), and to interpret (3) as (4).

(1) The zebra is finishing off the patch of greenery

(2) There are still some sprouts but if all goes on normally as it did until now then the zebra will shortly eat every one of them.

(3) The lioness is hunting down the zebra

(4) The zebra is still alive but if all goes on normally as it did until now then the lioness will shortly kill it.

Unfortunately, while (1) and (3) may both be true of the same scenario, (2) and (4) are very hard to reconcile. Moreover, usage of intuitively appealing phrases like 'all goes on normally as it did up until now', or to be more precise '..the future course of events develops in ways most compatible with the past course of events'[1] still leaves a lot to be desired from an analytic theory of meaning.

The problem examined in this thesis then is not Dowty's renowned 'imperfective paradox' of the progressive aspect in general. Accepting Dowty's initial and in my mind very intuitive reasoning about the progressive, I will be concerned here with the inconsistencies that it brings about, and with trying to formalize the primitive notion of normality that remains as a rather large chunk of unanalyzed intuition right at the core of Dowty's theory. Both issues will be resolved through highlighting the crucial part played by partial information in the semantics of the progressive.

The Theory Propounded

The progressive aspect has served as troubled waters testing the craftsmanship of many a theory faring the semantic seas. This has been particularly true in the wake of Dowty's proposed analysis of it. It is fascinating to observe the multitude of morbid examples that are associated with these theories: people get run over by trucks, they get hit by lighting, they drown, they are eaten by bears, they walk into minefields, etc, etc. Apart from telling us something about the theorists involved, this also points out something about the progressive itself. Death, as we all know, comes ever uninvited, unexpected. The progressive aspect, I maintain, is very sensitive to the relation between what is expected and what is unexpected.

The theory I advocate is three fold. First, while trying to follow closely in the footsteps of Dowty's intuitively appealing concept of inertia (the idea of 'things going on in a normal fashion'), I hold that the modal basis for this concept is epistemic and not ontological. This may seem to be in line with Dowty's own theory, at least with that fuzzy part about things going on normally. But I will show that Dowty's modality is either completely ontological, in which case it does not provide the required results, or else is an inconsistent mix up of an ontological and an epistemic base.

Second, I hold that the notion of partiality plays a critical role in the semantics of the progressive. I think that at the intuitive level this too is an enticing conviction. The

[1] (Dowty 1979) page 148

progressive appears to be a kind of commonsensical projection of what we know on to the parts of reality of which we do not know. Thus the zebra may truly be said to be finishing off the greenery if its (or our) partial knowledge does not include data about the approaching feline death. In trying to analytically bite off a chunk from the vague notion of normality I will take partiality a step further and use it to formally explain what it means for nothing unexpected or out of the ordinary to happen. This is a particularly difficult notion to catch formally because of the double use of negation: not only are we after those 'things' which are *un*-expected, but also are we interested in those cases where they *don't* happen.

This leads us to the third pillar on which this thesis rests. Partiality will give us an explanation of what the unexpected happenings are, and my third point is that built into the progressive operator is a kind of minimality constraint. Being interested only in those cases where nothing unexpected happens means throwing away all those cases where something superfluous does happen if we can also imagine a similar case where it does not. Once again, my aim is to crystallize this intuition in a formal way.

So the theory brought forth here is a formal theory of partial information, with a proposed application to the progressive aspect. In theories of knowledge it is often the case that what is known is structured using a discourse context or common ground. The novelty in my approach to partial information is that it also offers us a chance to structure what is only speculated but not asserted. This domain of the doxastic stands midway between the common ground, to which we are fully committed, and the completely uncontemplated, which is irrelevant for our reasoning processes.

What Follows Next

Chapter 2 closely investigates the modality involved in Dowty's proposed semantics for the progressive. The chronological evolvement of the theory hints at the problem of inconsistency associated with it, because the demand for normality was only added at a later stage of the theory. This inconsistency is brought out more clearly in chapter 3, which also proposes the epistemic modal base as a possible solution to it. Turning to an epistemic modality also beckons us to try and further analyze normality itself, and an initial analysis is indeed offered. Having proposed a different approach, but merely by waving of hands and other forms of intimidation, we turn to formalize a framework which will support the required modality and offer a chance to define normality. Chapter 4 gives a summary of dynamic semantics, an epistemic framework which is particularly friendly to notions of partial knowledge. This same framework is adopted, and further adapted in chapter 5. There the notion of a *situation*, a partially defined possible scenario, is defined. Situations are the basic discernibles of my information state, and so their existence and distinctness from one another depends on the fine grainedness of the linguistic (or conceptual) context in which I operate. Chapter 5 is where the bulk of formal development takes place in this work. Chapter 6 is where it all comes together. The progressive operator is given a dynamic semantics analysis which crucially depends on the concept of situation defined earlier in order to formalize the notion of minimality that is required. Finally, chapter 7 takes on a bunch of formidable puzzles of the progressive and pits the theory advocated in this thesis against them. Many of the puzzles are handled by the theory, and this without need to resort to a primitive notion of normality. The one challenge that it does not rise to meet is when a sentence seems to simultaneously incorporate multiple contrasting viewpoints. It does however point the way toward a proper treatment of even such sentences through multiple agent epistemic systems.

CHAPTER 2
DOWTY'S MODAL ACCOUNT OF THE PROGRESSIVE

This chapter provides a brief summary of Dowty's theory of the progressive as outlined in (Dowty 1979) and (Dowty 1977). The impefective paradox is described, and in examining Dowty's ways out of it, special emphasis is laid on the ontological nature of the modality involved. Tracing the chronological evolvement of the theory hints at a possible inconsistency, which will be brought out fully in chapter 3.

Dowty's Imperfective Paradox

What is the imperfective paradox? Even after it has been identified and christened by Dowty, the exact phrasing of the paradox is not at all clear. Both (Landman 1992) and (Vlach 1981) phrase the paradox as the problem of explaining the difference in truth conditions for the progressive between accomplishments and activities. Hence the paradox is exemplified by asking how can it be that (5) entails (6), but (7) does not entail (8).

(5) John was pushing a cart.
(6) John pushed a cart.
(7) John was drawing a circle.
(8) John drew a circle.

By itself this comparison does not seem very paradoxical. But the paradox does not just apply to this comparison, rather it takes into account also Dowty's semantic modeling of the distinction between the verb classes, his 'Aspect Calculus'. This calculus reduces the various verb classes, as set out by the seminal (Vendler 1967), to different combinations of stative predicates and special aspectual operators such as DO, BECOME, AT and CAUSE. In this calculus accomplishments are analyzed as a structure containing process and result, connected by causality. Only now, having accepted this basic intuition, does the aspect calculus run into the trouble that Dowty calls paradox. Because from the calculus we can infer the validity of (9), and yet (7) together with (9) fails to entail that (10) is also valid.

(9) To draw a circle means to bring about the existence of a circle.

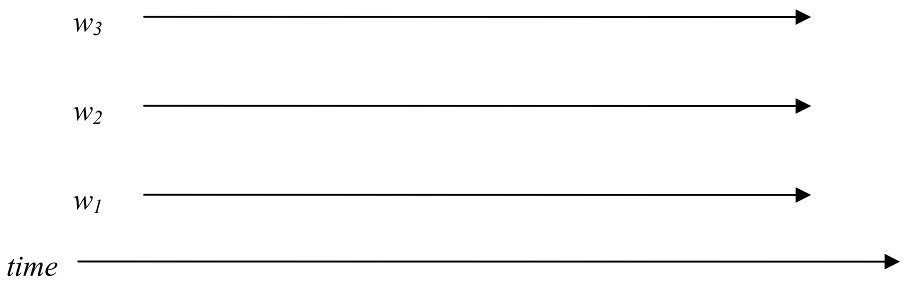
(10) John brought a circle into existence.

Combining Modality and Temporality

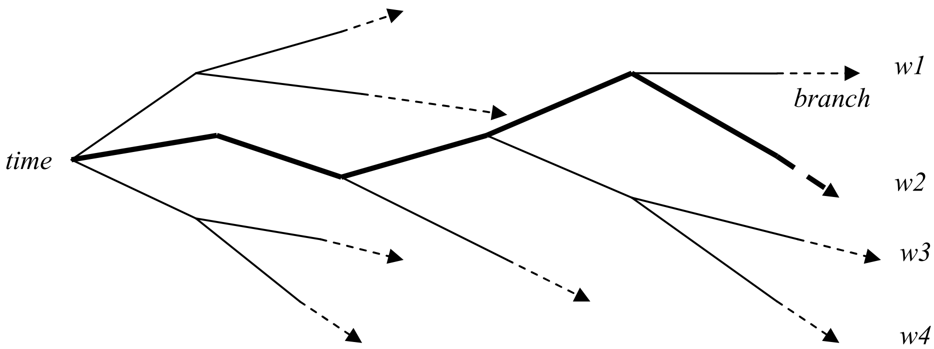
Dowty solves the problem by adding a modal element into the semantics. Referring back to (7), (9) and (10), he dispelled the paradox by claiming that in fact John's circle does come into existence eventually, but not necessarily in the actual world. Rather, it is in a special group of possible worlds, called the inertia worlds, that this happens. Although this approach is fully accepted in the current work, it is important to note down that a different approach to the paradox, one that does not involve modality, has been offered by (Parsons 1989). Parsons evades modality by extending his ontology to include partial entities. Thus drawing a circle means that there is a partial circle being drawn, and a partial drawing event in the denotation of 'drawing a circle'. (Landman 1992) convincingly argues against Parsons for the intensionality of the progressive.[2]

Now we will soon turn our full attention to the set of inertia worlds, but first let us examine how modality is combined with temporality. Two structures are offered by Dowty as an ontological infrastructure for a logic that combines both tense and modality. The first structure is the Cartesian product of a non-empty set of worlds and a non-empty, linearly ordered, set of times. Assignment takes place relative to a world and a time, which means that time itself is not world dependent. This is the possible world approach. Every world advances through time independently of the other worlds, and no assumption is made on possible identities between worlds.

[2] See also (Zucchi 1999) and (Szabo 2004) for further comparisons between the two approaches.



The second structure is more intriguing, and involves a non-linear structure on the set of times which may be envisioned as 'treelike' in its tendency to branch toward the future. From any point *t* in the set of times one can trace only one path leading backward in time, but numerous possible paths leading forward. Formally the structure is characterized as transitive ($\forall t_1, t_2, t_3 \in T$: if $t_1 < t_2$ and $t_2 < t_3$ then $t_1 < t_3$) and backward linear ($\forall t_1, t_2, t_3 \in T$: If $t_1 < t_3$ and $t_2 < t_3$ then $t_1 < t_2$ or $t_1 > t_2$ or $t_1 = t_2$). Worlds in this structure are represented as maximal chains on the structure of times, or more vividly as branches on the tree of time. Assignment takes place relative only to time, but time itself is different in the different worlds (from the point where any two worlds diverge and onwards toward the future).



As shown by (Thomason 1984), the branching time structure can be constructed as a special case of the possible world structure, where each point of (universal) time determines an equivalence relation on the set of worlds, and where the assignment function obeys the following restriction: if φ is a non-tensed sentence and *h()* an assignment function then if $w \approx_t w'$ and $t' \leq t$ then $<t', w> \in h(\varphi) \leftrightarrow <t', w'> \in h(\varphi)$.

For every point t in time, two worlds sharing the equivalence class in the equivalence relation determined by t also share the assignment values for all atomic formulas at all times up to and including the time *t*. More than that, they share the same assignments also for complex sentences which do not reach beyond time *t* in their future. More informally then, the set of equivalence relations on the worlds can be understood as a criterion for identity up to a point between worlds.

The branching time structure embodies the age old intuition that what has gone on before is by now settled and can not now have happened in any other way, while what is still to come is unsettled and there exist numerous possible futures. Thus from any point in time there is always only one path extending towards the past, but there are multiple paths extending into the future. But the ontological structure does not fully determine the semantics of tensed sentences, and specifically the semantics of the future tense operator had been debated by logicians since antiquity.

Initial Formalization of the Progressive Operator

In (Dowty 1977) the semantics for the progressive, apart from incorporating the switch inaugurated by (Bennett and Partee 1972) from the use of temporal instants to intervals, is built about the modality explicitly provided by the branching time structure. Each interval, or stretch of time, provides as a modal base the set of branches, or histories, passing through it in the structure. Formally the semantics is given using both the possible world and the branching time structures, with the assumption that a criterion of identity up to a point (IUP), which makes it possible to reduce the former into the latter, is given in advance.

With the modal base thus given, the truth conditions for the progressive could be defined either as a requirement on all of the modally relevant histories, or as a requirement on one of them. The ontological structure does not provide any means of limiting the choice of histories based just upon their past and present, so there is no third option. A constraint on all of the histories would result in a semantics resembling (Thomason 1970)'s semantics of the future. As the progressive is to have weaker entailment power than the future, (Dowty 1977) chose a constraint on any of the histories. This in fact made his semantics for the progressive into a combination of Bennett and Partee's progressive and (Prior 1967)'s weaker notion of future called the Okhamist future. Prior's Okhamist considers that for the value of a proposition like Future(φ) to be true at interval t, it is enough that there be a single branch passing through t for which Future(φ) is true at t. This means though that not all future propositions considered true at t will actually come to pass, as the actual world branch may turn out to be quite another branch from the one validating Future(φ) at t. Thus the Okhamist ends up rejecting the enticing proposition that every true proposition concerning the past is necessary. Exactly what is needed to break up the inference from (7) to (8).

(11) [PROG φ] is true at *I* and *w* iff there is an interval *I'* such that $I \subset I'$ and there is a world *w'* for which φ is true at *I'* and *w',* and *w* is exactly like *w'* at all times preceding and inclining *I.*

We can see in the proposed semantics the use made of Bennett and Partee's requirement for truth relative to a larger, including, interval. We can also see the IUP constraint being made use of to constrain the set of possible worlds. Finally, note the weak Okhamist demand that any such world will do to validate the sentence.

Later Formalization of the Progressive Operator

In his revised theory, (Dowty 1979) has strengthened the both the IUP requirement on the modal base and the Okhamist requirement that any member of this base will do. This, according to him, was done due to the following counter intuitive result. Suppose an untampered coin is being flipped in the air. There are worlds where the coin ends up heads and also worlds where it's tails, so according to the given semantics both 'The coin is coming up heads' and 'The coin is coming up tails' are true, and therefore also 'The coin is coming up heads and the coin is coming up tails'.

The modal base was reduced by adding a further requirement on it besides the IUP. The set of worlds chosen now had to comply also with a demand that its members realize a natural course of events from the reference time interval onwards. Using this further limited set of worlds the Okhamist approach that any world will do could be abandoned in favor of a demand that the enclosing interval I' validate the tenseless sentence in all modally chosen worlds.

Combining the identity up to a point (IUP) constraint with the natural course of events constraint (NCE) into one primitive notion of an inertia function (*Inr*) which chooses the set of relevant worlds, Dowty got the following revised semantics.

(12) [PROG ϕ] is true at $<I,w>$ iff for some interval I' such that $I \subset I'$ and I is not a final subinterval for I', and for all w' such that $w' \in Inr(<I,w>)$, ϕ is true at $<I,w'>$.

The problem of the flipping coin was now overcome because neither 'The coin is coming up heads' nor 'The coin is coming up tails' could be said to be true in all the worlds chosen by the inertia function. However, this achievement was won at the price of incorporating into the semantics the very fuzzy notion of a natural course of events. Dowty himself 'reluctantly concludes' that the inertia function must be added as a non-reducible primitive.

CHAPTER 3
THE INERTIA CONSTRAINT

According to Dowty the inertia function embodies a primitive, non-reducible, intuition. And yet this intuition is reducible, if only into the two constraints of IUP and NCE, and this by its very first exposition in (Dowty 1979). I maintain that this union of two constraints is not as harmonious as it appears. The conflicts between the constraints may be solved if we switch to an epistemic modal base. This would naturally transform 'identity up to a point' (IUP) into 'knowledge up to a point' (KUP), which would go smoother with the natural course of events constraint. Switching to an epistemic modal base also gives us a start on further analysis of the NCE constraint itself. The characterizations informally provided at the end of the current chapter will be made formal in chapter 6 once we have the formal apparatus up and running.

Inertia as a Compound

What exactly is meant by the inertia function? Do we really have to give up in exasperation on the effort of sharpening the intuition behind this notion? We would if it appeared that the cognitive understanding behind it is monolithic or in any case non-contradictory. But even if we forgo formal exactness and make do with intuition, inertia is made of two complementary but distinct notions.

The Inr function is described as providing those worlds which are identical to the given world w up to interval I and which develop from I onward according to the most natural course of events given their history. Dowty tries to attribute to the Inr function a sort of contained vagueness which supposedly reflects a primitive cognitive vagueness. The technique is reminiscent of the (Lewis 1973) notion of similarity. On one hand the notion in question is said to capture the root of a cognitive fuzziness and nothing more, so that there is no way to further analyze it. On the other hand it can be outwardly defined as a formal black box, and so be made to fit in within a formal explanation of a wider scope without the contained fuzziness 'contaminating' the rest of the explanation.

Suppose that, again making use of ‘contained vagueness’, we have at our disposal a primitive criterion for judging the ‘normality’ of possible worlds. Assume that given a world and interval, we are able to decide for each world whether it develops according to the natural course of events or not. Can the inertia function be defined based on this criterion and the criterion for identity up to a point, which gives us the branching tree structure? Dowty’s initial description of inertia seems to assert just this, and so already at the onset inertia is not really as primitive his formal semantics would have us believe.

Conflicts between the Two Constituents of Inertia

It is clear that the intended definition of the set of inertia worlds is just as a further sharpening of the set of possible futures. The set of inertia worlds can be derived by intersecting the possible futures at $<w,I>$ with those that develop normally based on $<w,I>$. The normality criterion is implicitly assumed to be somehow affiliated with the criterion of identity up to a point, as there is no mentioning and no need to worry about the worlds which are lost during the intersection between the two. It may be that the normality criterion is always assumed to bring in a subset of the identity up to a point criterion. This definition of inertia works well to save the progressive theory from such counter intuitive results as those of the coin flipping scenario mentioned above. It is also minimalistic in its requirements. Although we need to assume two previously given criteria notions, each in its turn seems commonsensical and so no conflict between them is to be expected.

Problems with Dowty’s formulation of the inertia function had been pointed out by many, including (Landman 1992), (Vlach 1981), (Asher 1992) and (Lascarides 2001). But it is interesting to note that Dowty himself seems to be implicitly parting from his formal account in his discussion of the crackpot professor Jones. Take the following claim, uttered by one of Jones’ colleagues at an academic convention:

(13) Jones is ruining his academic reputation by publishing all those crackpot papers on politeness rules in Pre-Indo-European.

Dowty maintains that even if we are quite certain that Jones will eventually be persuaded to come to his senses before his reputation is ruined, and will get back to his 'really profound ideas on Austronesian morphophonemics', we can still uphold the utterance as true. Jones' being persuaded to steer away from his current course, even though it is highly probable, would be considered as a breach of the natural course of events. The utterance is judged using only those possible futures where no such persuasion takes place. It may not seem like a different inertia function is being used here, but the striking contrasts between the majority of the possible futures (Jones will come to his senses) and the futures in the inertia sets (he will continue as he is now) can be pushed even further. Vlach's example brings home the point.

(14) Max is in the middle of the street, heading towards the other side, in short - crossing the street. While unknown to him a speeding bus is just inches away from his body.

The fact that Max literally has no possible future at all does not deter us from choosing a set of worlds in which he gets to the other side as the inertia set (even with the bus being there, we had no problem agreeing that he was crossing the street). The definition suggested above for the inertia function is clearly not what is in use here, because the set of inertia worlds by which the progressive was evaluated, no matter how it was arrived at, does not have any future in common with the ontologically determined set of futures.

It may seem that the worlds of the inertia set and the ontologically available worlds differ only in their futures, that we are here faced with yet another complication deriving from the future's unsettledness. But the following example suffices to show that the inertia and the ontological worlds may also differ in their pasts.

(15) Max is at it again. He is in the middle of the street, heading towards the other side, in short - crossing the street. Whilst unknown to him a carefully hidden time bomb is just seconds away from exploding.

As the bomb was set a while ago, the ontologically available worlds differ from the inertia worlds not only in their grim futures but also in their subversive pasts. This in fact may also be said to be true of the bus example. In our inertia worlds Max is not hit by a bus, and it is therefore most likely that there is no bus and never was. Otherwise we need to incorporate a small miracle into the scenario, but miraculous accommodations do not go unnoticed and there is no feeling of accommodation here.

Further Support for the Epistemic Modal Base

Our intuition of the set of worlds chosen by the inertia function seems good enough to indicate that in many cases Dowty's formulation of it is simply inadequate, and not because of the vague concept of normality. The inertia function seems to produce a set of worlds that are in conflict with the set initially provided by the ontological modal base. The modal base that is more appropriate for the last examples, including the one about Professor Jones, is an epistemic one which takes into account the knowledge state of the agents involved. Jones is unaware of the ruinous implications to his career, nor is Max aware of the calamities that are about to befall him. In every case the set of inertia worlds seems somehow derived from a set of worlds that reflects this ignorance.

Supposing we tried to reapply the semantics on a different modal base, what would we stand to gain by giving up the ontological modal base in favor of an epistemic one? Now an epistemic base may be fine for conscious agents such as Jones and Max, but the progressive may also be applied with inanimate agents. Whose information state should be taken into account for the choice of inertia worlds in a case like Thomason's flipping coin brought at the beginning of the chapter? To answer to this question it must be noted that an epistemic viewpoint is like a coordinate in the space of possible information states. An agent's knowledge state is related to a specific epistemic viewpoint just like an object is associated with a physical coordinate in space. Conversely, an epistemic viewpoint need not be associated with any available agent's state any more than a coordinate in space needs to be associated with an existing object. We need not associate with the flipping coin scenario any knowing agent, though we could if we wanted associate our own information state with the epistemic viewpoint that presents futures where the coin comes up heads and also futures where it comes up tails.

The optionality for multiple contradicting results, which Dowty tried to get rid off by introducing the natural course of events and switching to a constraint on all modal alternatives, can actually be endorsed in an epistemic framework by keeping contradictory results to different viewpoints, as in the following example due to (Thomason 1995).

(16) I was climbing the tree and the bear was pulling me off

One epistemic viewpoint ignores the wishes of the bear to bring me down, while the other viewpoint ignores my own efforts to save my skin by getting to the treetop. We could have multiple epistemic viewpoints in a sentence just like we could have multiple times in one.

A switch to an epistemic modal base may seem like an unwarranted blow to the cause of objectivity, but reanalyzing the examples brought in the previous chapter we can see that it is unavoidable. In all the cases involving conscious agents, we had in fact used two different viewpoints: one that could easily be attributed to the agent involved and another one, all encompassing, to us as bystanders. The viewpoint need not necessarily be ‘ours’ or ‘Max’s’. In keeping with the abstractness of the notion of viewpoint I think that it is in fact associated with an event, and only through the event with its agent if one exists, or with us in general if the event is a state. Identifying viewpoints with agents though gives us an intuitive grasp of the scope which the viewpoint allows, so I will continue to use this inexact, though mostly harmless, phrasing.

Keeping all that was said above in mind, and going back to the case of Max and the speeding bus, if our own viewpoint did not include the speeding bus we would gladly agree with Max about his prospects for crossing the road. If Max was also aware of the bus, he would no longer be engaged in crossing the street. He would either running for his life, or committing suicide (one can think of other options, but ‘crossing the street’ is not among them if the verb’s lexical definition demands intent). Finally, it would obviously be senseless trying to adhere to the ontological modality and at the same time allow multiple perspectives by allowing multiple ontological viewpoints.

Switching to an epistemic base would help us bring together the NCE and the IUP constraints, or their look alikes under the new modal base. It would also provide a natural answer to the problem of the multiple viewpoints that are often implicitly assumed in the progressive.

An Intuitive Comparison of the Ontological and Epistemic Bases

In the ontological modality the past and present were identified with the necessary, the future with the possible. The branching time structure gave shape to this intuition. The various semantics of tense which are often proposed on top of the structure differ in their treatment of the seemingly determinist transitivity of the necessary from the past into the future. In an epistemic modality it is what is known 'for sure' that is identified with the necessary, and what is speculated about that is identified with the possible. The passage of time in the ontological modality is a process that moves propositions from the possible into the necessary or the impossible. In an epistemic modality it is knowledge growth that has this effect on propositions. Pushing the comparison a little bit further, we can imagine a sort of branching knowledge structure. At any point of knowledge on that structure, a knowledge state, there is one path toward the 'past' of ignorance but there are many paths into the doxactical future of total knowledge. Although this last simile already suggests that Dowty's semantics could be given using an epistemic modality, I think that the metaphor just about runs dry with the construction of the knowledge structure.

Leaving aside the impossible, with an epistemic modality propositions about both the past and the future can be either known, and therefore necessary, or unknown and therefore possible. The fact that we are usually more sure of past propositions than of future ones is made manifest only indirectly by our knowledge growth being based on the past more often than not. With the simple events like Max's crossing, it is often the case that the part of the event that has already come to pass is known to us, while the event's completion is as yet unsure. This puts the progressive as much in the realm of knowledge change as it is in the realm of temporal change.

To translate Dowty's semantics into the epistemic modality we must at the very least be able to define what an information state is, and what are the possibilities that are still open given an information state. We would also need to explain what constitutes the dynamic process of knowledge growth, which is to serve as the equivalent of the passage of time in terms of moving us from the possible into the necessary. Once we have these we can reformulate Dowty's semantics. In the new epistemic semantics, the notion of identity up to a point is replaced with a clearer notion of consistency with what we know, or knowledge up to a point (KUP). That is, the multiple branches passing through a knowledge state are those 'knowledge histories' which are consistent with the body of information that we have so far collected.

In the next chapter I will describe the dynamic semantics framework of (Groenendijk, Stokhof and Veltman 1996) which offers a formalization of such required notions as information states, possibilities and the process of information growth. And yet before doing that, could we also get a better grip on normality by switching to an epistemic base?

On the whole, if we can achieve the above mentioned move to an epistemic base then our semantics will no longer be susceptible to the kind of conflicting intuitions which plague the ontological version. Let us now aim even higher and cautiously enquire whether normality, the last bastion of vagueness in the semantics, could also be reanalyzed. The notion of normality smacks of the subjective and the relative, and the move from the ontological to the epistemic may seem like a move in the same direction. But in truth there is nothing subjective about epistemic semantics, nor are we interested in the subjective in connection with possible analysis of normality. If we can reduce the subjectivist element in normality to yet another notion of contained vagueness then that too will count as advancement.

Normality Revisited

In what follows I have tried to characterize normality in such a way that will make it possible, once we have set up the formal epistemic basis, to bite a few chunks off its perimeter.

Normality of the relevant and yet non-biasing of the irrelevant. Switching to an epistemic modal base, we give up the requirement that everything up to the ongoing event be identical with the real world, in favor of a looser requirement that everything be identical with what we know about the (presumably) real world. As our knowledge is finite, many of the infinite facts true of the world are left out of the requirement. This means that our choice of worlds which are identical with respect to our knowledge up to a point is not affected in any way by them. The same should be true of the normality constraint, except that normality is not about what is known, but about what is yet unknown. We no longer need to require that everything going on be normal. We require only that all those aspects of the course of events which are relevant to the ongoing event be so. Conversely, those aspects which we do not consider relevant for the event should not interfere, or otherwise bias, our judgment of the normal.

Knowledge versus assumptions, estimations, and the generally doxastic. So what is relevant with regard to normality? Normality is not judged by the things we know and are certain about. These facts are accounted for by the KUP constraint. Whether a world is normal or not depends in the ontological modality on the future happenings, and in the epistemic modality it depends on what is yet uncertain. Will the bomb blow up? Will it malfunction? Is there a chance for the SWAT team to get there in time? At the state of information relative to which sentence (15) is evaluated, these scenarios are speculative and uncertain. They concern propositions which may be relevant to the event but are as yet unknown to be either true or false. We can tell if a world passes the KUP constraint by simply checking if it is consistent with our knowledge. In order to do the same for the NCE constraint we need to compare the world with our assumptions, estimations, imaginations, and other notions of dubious epistemic standing. This domain includes what is known, but extends beyond it. In order to give shape to the NCE constraint we need to provide a formal domain where all these entities may be stacked.

Relativity of the notion of normality. Is the normality criterion an absolute criterion or a relative one? It is an absolute criterion if given a course of events we are able to say whether the course is normal or not. It is relative when we can only say, when given two possible courses, that one course of events is more normal than the other. I do not have a straight answer to this question. That is why I suggest that we must assume a relativistic criterion of normality. Or rather, the framework within which the epistemic modality will be defined must be able to accommodate a *relativistic* notion of normality. The reason for this is that one can always define an absolute notion using a relativistic predicate, but not the other way around. A relativistic notion of normality is more generic than an absolute one.

Minimal happenings of the unexpected. We know that normality is about the uncertain. But what do we require of our normal worlds in terms of these uncertainties? Dowty does not have much to say about what being normal is all about, except that it is concerned with “...all worlds...in which nothing out of the ordinary or unexpected happened.” That’s really not much to go on, and yet it is uncanny how this short description seems to give us what we need to know. I suspect that the main reason why the concept of normality is so easy to understand but so hard to formulate is that its contents involve negation, and not once but twice over. First, normality is characterized as a course of events where things do not happen. It is a course with *minimal happenings*. The logic behind this minimality goes like this. We expect that if an event does not reach its described result stage, then it must be because something else had happened that had affected its projected course. It is kind of like Newton’s second law of motion: events keep on evolving towards their natural final states unless they are stopped by something else. A normal course of events would then be such that nothing happens to stop the events from traversing their projected course. But if we stop all events from happening we will be left with no course of events at all, and this is where a second occurrence of negation takes place. Not all happenings are to not-happen, just those which are *unexpected*. Now this occurrence seems even more enigmatic than the previous one. What exactly are the unexpected events? If they are unexpected, how can we evade them? I think that the issues of relevancy and uncertainty discussed above may give us a shove in an interesting direction. As normality only concerns those aspects of the course of events which are relevant to the ongoing event, we need not be able to say what an unexpected event is in the general case, but only what an unexpected relevant event is. And this makes it easier because we do expect to be able to tell what the relevant aspects are. Now of all relevant aspects of the course of events some are considered to be known, while others are, well, uncertain. I therefore urge that we identify the relevant unexpected events as a part of the relevant uncertainties.

Summing up and adding a consideration of minimality and relativity as well, a normal course of events will be such that when compared to any other course and all things being otherwise equal will contain fewer occurrences of uncertainties.

CHAPTER 4
DYNAMIC SEMANTICS

The context in which dynamic semantics is located is briefly discussed. The structural properties of the framework are then introduced, followed by the dynamic process of information growth. The object language semantics close this brisk exposition. Partial information is very convincingly modeled by the dynamic semantics framework. I will therefore adopt it as formal infrastructure for the required progressive operator. The framework will be expanded in chapter 5 to define situations, and finally made use of to provide the epistemic progressive operator when we reach chapter 6.

Dynamic Semantics in Context

The dynamic semantics framework offered in (Groenendijk, Stokhof and Veltman 1996) follows a relatively recent turn in formal semantics from sentential truth value semantics to discursive change potential semantics. Heralding this change within the linguistic tradition were (Kamp 1981) and (Heim 1982), while at the same time the evolving field of programming language semantics has also had a great influence on this evolving approach through, for example, (Harel 1984) . (Groenendijk, Stokhof and Veltman 1996) differs from these predecessors not in expressibility, but in being able to maintain compositionality whilst providing that very same expressibility.

Discourse semantics demands that bits of knowledge acquired at earlier stages of the discourse be accessible at the later stages too. This is most often achieved by the introduction of some sort of structure representing an epistemic scoreboard, or state of knowledge, that is continuously updated. Sentence meanings are then interpreted as homomorphic functions on the space of possible information states.

In dynamic semantics (henceforth DS) information change in an agent is modeled as a series of transformations from one information state to another. Accumulated information comes in two flavors: there is information about the actual world, and there is information about the discourse itself. Discourse information is kept in the *referent system*, a structure that keeps track of all discourse referents introduced so far. Information about the world is only indirectly maintained in an information state. The information state is comprised of *possibilities*, which are interpreted as possible ways for the actual world to be. As more information is gained, supposedly fewer possibilities remain in the information state because many possibilities which were previously entertained are no longer deemed possible. Thus information about the world is encoded in the set of possibilities: all of the possibilities in the state comply with the information so far gained, either validating or invalidating the same propositions, and may differ from each other with respect to propositions which are not a part of the current information.

I say supposedly, because the necessary interaction between world information and discourse information complicate matters. A possibility is not simply a possible world, but is also a mapping from the set of discourse referents kept in the referent system into the possible world. Thus two possibilities which share the same referent system and the same possible world are distinct if their mappings from the former into the latter are not the same too.

During the process of information growth the number of possibilities in the information state can either grow or shrink. It grows whenever a new discourse referent is added, because this creates a multitude of new possible ways to map from the referent system into the possible worlds, and each such mapping is added to the new information state as a new possibility. The number of possibilities can also shrink, whenever facts about the world are asserted. Such facts filter out of the new information state all those possibilities which do not validate them. Let us see how all this is made formally possible.

Structures : Worlds, Referent Systems, Possibilities, Information States

Discourse information is kept as a referent system. This system is but a function from variables to pegs, which are intermediate formal entities standing between the variable and its denotations. Pegs are best understood as a bunch of distinct natural numbers (this promises that we will always have more pegs when we need them, and that each is distinct and can be told from the others).

(17) A **referent system** is a function *r* from a finite set of variables to a finite initial sequence of the natural numbers (these are the pegs).

(18) Let *r* be a referent system with domain *v* and range *{1,...,n-1}* (we will say that this is range *n-1*). *r[x/n]* is the referent system *r'* which is like *r* except that the domain is $v \cup \{x\}$, its range is *n*, and *r'(x)=n*.

(19) Let *r,r'* be two referent systems with domain *v,v'* and range *n,n'* respectively. ***r' is an extension of r,r*** $\leq$ ***r',*** iff

a. $\mathrm{v} \subseteq \mathrm{v'}$

b. $\mathrm{n} \leq \mathrm{n'}$

c. if $\mathrm{x} \in \mathrm{v}$ then $\mathrm{r'(x)=r(x)}$ or $r'(x) > n$

d. *if* $x \notin v$ and $\mathrm{x} \in \mathrm{v'}$ then $r'(x) > n$

The use of pegs makes it possible to re use a quantifier. In the extended referent system the previous peg associated with the variable will remain in the range but will no longer be referenced by any variable. The variable previously associated with it will now be re-associated with a new peg.

Turning now to world information, each possible world consists of a set of objects, the domain of discourse, and an interpretation function which assigns denotations to the non logical vocabulary of the object language. It is assumed that all the worlds share the same domain of discourse, so that worlds are here identified with their interpretation functions. The basic building block for information states is the possibility. The possibility incorporates a unique link between world information and discourse information.

(20) Let **D**, the domain of discourse, and **W,** the set of possible worlds, be two disjoint non empty sets.

(21) ***I*** *,*the set of possibilities based on D and W, is the set $\{<r,g,w>|$ where *r* is a referent system, *g* a function from the range of *r* into D, and $w \in \mathrm{W}\}$.

The referent system *r*, along with the assignment function *g*, together determine a denotation for every defined variable, since $g(r(x)) \in D$ for every variable *x* in the referent system. As for constants and n-place predicates, their denotations are defined by the interpretation function of the world. These functions are assumed to be independently given and, in our case, to fully characterize the world because the domain of individuals is shared by all worlds. For this reason worlds and interpretation functions are interchangeably symbolized by *w* (*w* being the interpretation function of world *w*). So the most basic expressions in the object language are given an interpretation in each possibility.

(22) Let α be a basic expression; $i=<r,g,w>$, $i \in I$, with *v* the domain of *r* and *I* based on D and W. The denotation of α in *i*, *i(α)*, is defined as:

a. If α is an individual constant, then $i(\alpha) = w(\alpha) \in D$.
b. If α is an n-place predicate, then $i(\alpha) = w(\alpha) \subseteq D^n$.
c. If α is a variable such that $\alpha \in v$, then $i(\alpha) = g(r(\alpha)) \in D$. Else *i(α)* is not defined.

Information states are sets of possibilities which are all based on the same discourse information. Note that they need not be maximally such, that is – there is no assurance that an information state contains all of the possibilities that can be defined based on some discourse information. As the referent system is shared between the possibilities in the state, it could have equally been placed at the level of the information state itself.

The notation introduced in the next definition is a sort of semi-formal projection function, $<A>^x$ being the Xth element in the A n-tuple, only instead of a numeric index I will use the element's typical meta variable symbol. Thus $<i>^r$ means the *r* element of the possibility $i=<r,g,w>$. I will use this notation throughout the work.

(23) Let *I* be the set of possibilities based on D and W. The set of information states based on *I* is the set S such that

$s \in S$ iff $s \subseteq I$, and $\forall i,i' \in s: <i>^r = <i'>^r$

Information Growth

The process of information growth through discourse can now be sketched out. We start out with an initial state, a state containing only possibilities belonging to the following set $\{<\emptyset,\emptyset,w> | w \in W\}$. These are the possibilities available to us when no discourse information has yet been collected. The initial state containing all of this set of possibilities is called the state of ignorance, but an initial state may also contain just a subset of this set of possibilities in case some information about the world is available at the outset, canceling out certain worlds as potential realities.

Supposing we now need to update our information with the utterance that there exists an object with property P, $\exists xP(x)$. In our discourse information the variable *x* will be associated with a new peg *n*, and in each possibility this peg will be assigned with a denotation for which *P(x)* holds in its related world. If more than one object in the related world may be assigned as such, then the possibility will spawn several new possibilities, one for each possible assignment function. This operation brings out clearly the difference between a world and a possibility. A possibility is a way to imprint our current discourse information upon a possible world.

The next utterance may curb our enthusiasm about the possible denotations for x, by claiming that x also has the property Q, Q(x). This utterance does not extend our discourse information as it uses an already defined variable, moreover it prunes our set of possibilities by posing an extra constraint on the denotation of x. Only possibilities whose worlds support x's having the property Q will be left in our information state.

We can see how the set of possibilities in the current information state can either shrink or grow. However, the set of worlds indirectly associated with our information states monotonically shrinks, as even the existential quantifier does not cause new worlds to be introduced, while repeated pruning of the set of possibilities may cause a world to be dropped out by filtering all of the possibilities related to it. Once this happens, such a world can not be re-introduced into the following information states.

In the above mentioned discourse, one of DS's strongest features went almost unnoticed as it was so intuitively easy to accept. The existential quantifier in the former utterance was binding the variable which appeared in the latter utterance. This feature is made available by having the referent system keep tabs of available variables instead of forcing the logical form to do that.

Continuing with the formal definitions, we first get a definition for assignment. Assignment in a possibility includes extending the referent system with a variable and peg pair, and then extending also the assignment function from pegs to the possible world by relating the new peg to an object in that world. Assignment in an information state is just a pointwise operation on all the possibilities in the state.

(24) Let i=<r,g,w>, $i \in I$; n in the range of r; $d \in D$; $s \in S$

i. $i[x/d] = < r[x/n], g[n/d], w >$

ii. $s[x/d] = \{ i[x/d] | i \in s \}$

With the extension relation possibilities and states part ways. Assignment to a possibility forces an extension to that possibility (a real extension, ignoring weak ordering for now). Although it is not explicitly stated, there is no way to extend a possibility other than by assignment. An assignment to an information state extends the state, but so does any other type of sentence in the object language, as we shall see.

In extending a possibility we may add variables and pegs, but we maintain the same world as before. Also, extending a possibility may cause a variable to be reassigned to a different peg, but pegs are never reassigned themselves to another object in the world. State extension is a more complex relation. State extension holds when every possibility in the extended state is an extension of a possibility in the extended state. This leaves room for various operations on states to still count as extensions, including removal of possibilities, adding of possibilities, and running tests on possibilities.

(25) Let i=<r,g,w>, i'=<r',g',w'> $i,i' \in I$; $s,s' \in S$

i. i' is an extension of i, $i \le i'$ iff $r \le r', g \subseteq g'$, and $w = w'$

ii. s' is an extension of $s, s \le s'$ iff $\forall i' \in s' : \exists i \in s : i \le i'$

State *s'* may be an extension of *s* even though some of the possibilities in *s* do not have any extended counterparts in *s'*, just as long as every possibility in *s'* is an extension of some possibility in *s*.

Based on the extension relations, a few other notions follow through. One can look for the descendants of a state's possibility in the extending states. These would be the extending possibilities. If indeed such descendants are found in a state's extension then the possibility may be said to subsist in the extension. Recall that this need not always hold between state extensions because sometimes the extension is arrived at by possibility reductions. If every one of a state's possibilities subsists in its extension, then the state itself is said to subsist in its extension.

(26) Let $s, s' \in S$, $s \leq s'$; $i \in s$, $i' \in s'$

i. i' is a descendant of i in s' iff $i \leq i'$

ii. i subsists in s' iff i has one or more descendants in s'

iii. s subsists in s' iff all $i \in s$ subsist in s'

If *s* subsists in *s'* then not only is *s'* an extension of *s*, but also no possibility in *s* had been eliminated during the update, or transition. This means that the only way by which information growth could have occurred between *s* and *s'* is through new variable assignments. It could be that pruning filters, such as *Q(x)*, were also applied – but they did not constitute new information as all the possibilities were inherently supportive of this constraint to begin with (or they would have been eliminated).

Object Language Semantics

Let us now take a look at how sentences are interpreted as partial transition functions from the set of states into itself. The syntax of the object language is the ordinary syntax of predicate logic, with a unary modal operator ◊ on top.

The update of an atomic formula eliminates those possibilities in which the objects denoted do not stand in the specified relation. If one of the terms used includes a variable not available in the referent system then the formula as a whole can not be interpreted, nor can any compound formula enclosing it, and the interpretation process is stopped. Negation is treated by taking the complement of a hypothetical update with the inner formula. Conjunction is a composition of the updates associated with the parts. As is often the case with state maintaining systems, order does matter: the left conjunct is interpreted before the right one is, and so discourse information collected in the first update may be used in the second one. We can now observe how the existential operator 'spreads out', trying to extend every available possibility by assigning the peg newly associated with the quantified variable to every object in the domain. The crucial bit to take note of here is that the update with the embedded formula takes place for each individual assignment, and not at the end of all assignments. This choice is of considerable consequence when coreference and modality are simultaneously employed. The modal operator ◊ corresponds intuitively to 'might'. Its semantics amount to testing the information state to see whether or not the embedded formula could be applied as a further update without landing us in the absurd state.

(27) Let $s \in S$ be an information state, and φ a formula in the object language. The update of s with φ is defined as follows:

i. s[R$t1,...,tn$] = $\left\{ i \in s \middle| < i(t_1),...,i(t_n) > \in i(R) \right\}$.

ii. s[$t1=t2$] = $\{i \in s \mid i(t_1) = i(t_2)\}$.

iii. s[¬φ] = $\{i \in s \mid i$ does not subsist in $s[\varphi]\}$.

iv. s[φ ∧ ψ] = s[φ][ψ]

v. s[∃ x φ] = $\bigcup_{d \in D} (s[x/d][\varphi])$.

vi. s[◊φ] = $\{i \in s \mid s[\varphi] \neq \varnothing\}$.

It may be worthwhile to take another look at the definitions for negation and for existential quantification , by examining two examples. Suppose we are to update state s with the sentence 'nobody is coming in' (i.e. it is not true that somebody is coming in). The update consists of throwing away possibilities in s which would subsist in an update with the inner negated sentence (somebody is coming in). A possibility would subsist with the latter update if the expression 'somebody', translated as a variable, could be assigned a denotation in the possibility's world. That would mean that there is indeed in the possibility's world someone who is coming in. Such a possibility would not pass the real update with the matrix sentence 'nobody is coming in'. On the other hand, if no object could be assigned to 'somebody' when hypothetically updating with the inner sentence, that would mean that in the possibility's world nothing can be found to be coming in. The possibility would pass the update with the original matrix sentence.

Turning to existential quantification, suppose *s* is to be updated with 'there is a blue whiskered cat'. The update with an existential quantifier consists in collecting together the possibilities which result in pointwise application of the following procedure to each object in the domain of individuals. For each object the state is extended (that is, each possibility in the state) through the introduction of a new variable and peg in the referent system and the assignment of the peg to that object. Then the requirements on the individual, in this case that it be a blue whiskered cat, are applied. If the individual is indeed such a cat then the extended possibility gets to be a member of the updated state, if not, then the possibility is removed. This procedure, as I said, is applied again and again until every individual has been tried with each of the state *s* possibilities. At the end of this complex update we get a state in which the referent system has been extended with a new variable / peg pair, and the maximal set of extended possibilities which are able to support the new variable by tying it to an object in their domain without achieving inconsistency. The result of the update might also be the absurd state, if no possibility can be augmented with the required individual.

By using the operators defined above along with the usual logical relations between operators, the following further operators can be defined:

(28)

vii. $s[\varphi \rightarrow \psi] = \left\{ i \in s \,\middle|\, \begin{array}{l} \text{if } i \text{ subsists in } s[\varphi] \text{ then all descendents of } i \\ \text{in } s[\varphi] \text{ subsist in } s[\varphi][\psi] \end{array} \right\}$

viii. $s[\varphi \vee \psi] = \{ i \in s \mid i \text{ subsists in } s[\varphi] \text{ or } i \text{ subsists in } s[\neg\varphi][\psi] \}$

ix. $s[\forall x \, \varphi] = \{ i \in s \mid \text{for all } d \in D\text{: } i \text{ subsists in } s[x/d][\varphi] \}$

x. $s[\Box\varphi] = \{ i \in s \mid s \text{ subsists in } s[\varphi] \}$

We have already mentioned in passing the existential quantifier's ability to bind variables outside of its formal scope. This feature offers compositional translations into the logical language even for cases of cross sentential anaphora like the familiar donkey anaphora sentences. With the predicate calculus, the correct anaphoric analysis of both (29) and (30) yields the non compositional translation into (31). In DS on the other hand, we get (32) and (33) as respective translations.

(29) If a farmer owns a donkey he beats it.

(30) Every farmer who owns a donkey beats it.

(31) $\forall x \forall y[[farmer(x) \wedge donkey(y) \wedge owns(x,y)] \rightarrow beats(x,y)]$

(32) $\exists x[farmer(x) \wedge \exists y[donkey(y) \wedge owns(x,y)]] \rightarrow beats(x,y)$

(33) $\forall x[farmer(x) \wedge \exists y[donkey(y) \wedge owns(x,y)] \rightarrow beats(x,y)]$

Information states do not wear their collected world information on their sleeve, so to speak, like they do their discourse information. An information state that is the result of updating with formula φ does not contain explicit information that it was updated with φ. It may well be that the same resultant state could have been achieved without updating with φ. So not only can we not say that φ is true, we are also prevented from saying that it is explicitly known. What we can say is that our current state *s* has the potential for being updated with φ - that it is consistent with it, or in a stronger sense of knowing, that accepting φ would not change anything about our information on the world – that *s* supports φ . This seems to be as close as we come to a notion of truth. One can not be said to assert an utterance truthfully if its very acceptance would cause a change in her information structure. Entailment is also defined in terms of support, along with the same sensitivity to order that was also exhibited in the semantics of conjunction.

(34) Let *s* be an information state:

i. φ is consistent with *s* iff $s[\varphi]$ exists and $s[\varphi] \neq \varnothing$
ii. φ is supported by *s* iff $s[\varphi]$ exists and *s* subsists in $s[\varphi]$
iii. $\varphi_1, \ldots \varphi_n \models \psi$ iff for all information states *s* such that $s[\varphi_1], \ldots [\varphi_n] [\psi]$ exists, it holds that $s[\varphi_1], \ldots [\varphi_n]$ supports $[\psi]$

Equivalence is supposed to tell when two sentences can be exchanged in a meaning preserving way. In DS, a part of meaning is the potential created for further updates. The usual definition for equivalence as mutual entailment can not guarantee meaning preservation because it does not guarantee equal potential. For example, $\exists xP(x)$ and $\exists yP(y)$ mutually entail each other but create quite different discourse information upon being updated. Equivalence is therefore defined in terms of mutual similarity. Similarity does away with differences in the discourse information that result only from a different order in the bindings of variables to pegs and of pegs to individuals. It as if pegs were altogether removed from the system by joining together the referent system and the assignment function. Two sentences are equivalent if a state with either one of them results in similar states.

(35) Let $i,i' \in I$, $i=\langle r,g,w\rangle$, $i'=\langle r',g',w'\rangle$; $s,s' \in S$

1. *i* is similar to *i'* iff *Dom(r)=Dom(r'),w=w',* and
   $\forall x \in Dom(r):[g \circ r](x)=[g' \circ r'](x)$
2. s is similar to s' iff
   $\forall i \in s: \exists i' \in s': i$ is similar to i' and $\forall i' \in s': \exists i \in s: i'$ is similar to i
3. $\psi \equiv \varphi$ iff for all $s \in S$: s[ψ] is similar to s[φ]

We will have use for the notion of similarity later on when we define situations by grouping together sets of possibilities which are in essence – similar.

CHAPTER 5
DYNAMIC SITUATIONS

I introduce a theory of dynamic situations, which is an extension of the dynamic semantics framework. Situations are conceived of as sets of indiscernible possibilities which form a partition on the information state. Indiscernibility between entities is a relative notion, it depends on the fine grainedness, or granularity, with which we inspect these entities. Under the granularity determined by the discourse context itself, all possibilities are indiscernible, as they all validate every one of the discourse utterances. Motivated by the need to distinguish between knowledge and uncertainty, I introduce a finer grained context, the linguistic context, and partition the possibilities according to this context. Situations thus account for a slippery kind of doxastic half-knowledge: the agent's knowledge of the things he is uncertain of. The extended framework is shown to be isomorphic to the original dynamic semantics under the object language update relations.

Why We Need Situations

The epistemic structures defined in DS make manifest different forms of partiality. Possibilities are partial mappings from discourse referents to worlds. This partiality is echoed by the extension partial order which is defined on them. It gives formal shape to our always only partial ability to directly reference objects in the real world. States are also partial, and in more than one way because partiality depends on what we take to be the whole. The extension relation formally ordering information states has as its maximal element the absurd state, which suggests a pessimistic outlook on the process of information growth. If we conceive of information growth as a process unhindered by the possibility of inconsistency then our whole becomes any of the total states, states that contain just one possibility. This partiality then embodies our limited ability to tell what the world is truly like.

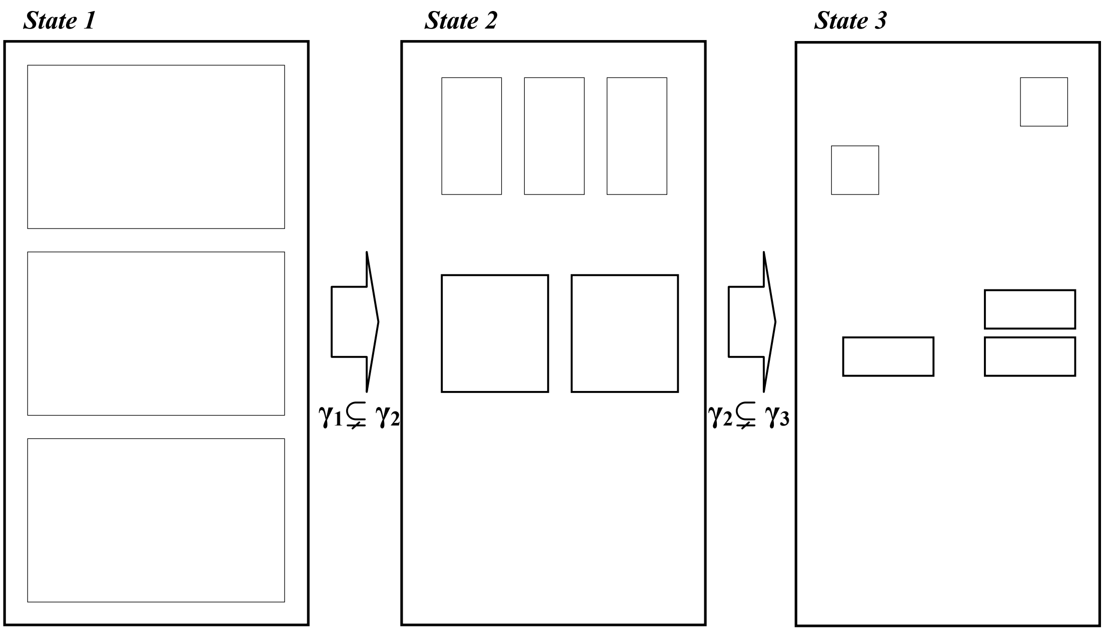
There is however a mode of partiality which is not supported by the DS structures. In DS the information that we have is partial, but the means to express it, the language, is always complete. A mode of partiality missing here then is partiality of linguistic context, of concepts that take part in our models of the actual world. This is a second order of partiality. Even if you have reached total information in one of its senses, you still only know about what could be said using your salient subset of the language. Enriching this linguistic (and conceptual) context would take you right back into a state of partial information. For example you may know that you are now sitting down in front of your desk. This may be a complete characterization of you physical situation in a rather crude linguistic context. But when you start thinking of your body in terms of bodily organs the same description no longer fully characterizes you. You need to fill in further information about your arms and legs, head, torso, etc. A special case of this process of linguistic context enriching is already modeled in DS with the expansion of the discourse information through the referent set growth. However the language itself is always considered as static.

In the dynamic situations framework, which I will often refer to as EDS (Extended Dynamic Semantics), the concept of a linguistic context is formally defined and its growth pattern accounted for. The linguistic context is a set of sentences that always includes the actual discourse utterances but is most likely to extend beyond them. We thus simultaneously maintain two contexts: one of actual utterances and another one of salient conceptions, vague uncertainties, or otherwise partially cognizant propositions. All of the possibilities in the information state are by definition indiscernible under the discourse context, but the extended linguistic context allows us to make finer distinctions and partition this unified body into subsets. These are the situations, discernible from each other only in terms of the extended linguistic context, and they provide us with the formal apparatus needed to for bringing the minimality of unexpected happenings into play, as we shall see later.

In DS states are, in principle, sets of possibilities. In the framework developed here this coarse definition is taken up by situations. We must now apply changes to the information state in order to accommodate these new structures. The obvious change is that states will now become sets of situations instead of sets of possibilities. Previously, updating the information state with a new utterance would only cause a shift in the population of possibilities, sometimes possibilities being rejected, at other times new possibilities created. Now another process takes place as well: the linguistic context grows, and through its growth situations, groups of previously indiscernible possibilities, may now become discernible. This causes situations to multiply as they split into new, finer grained, situations, in a way reminiscent of amoeba.

The following diagram attempts to give an intuitive grasp of the dynamics of partitioning into situations as the linguistic context grows. The movement depicted is orthogonal to the process in which the population of possibilities grows or shrinks. One can imagine each situation box as containing a substantial number of possibilities. Occasionally though, a situation is rejected during the state's update, and with it go all of the possibilities which were located under it.

As the state gets updated, the state's situations keep splitting due to the finer grained resolution of the growing linguistic context (the contexts of states 1,2 and 3 are shown here as γ1, γ2 and γ3 respectively).



So much for intuition. We will now proceed to give formal shape to EDS, the dynamic semantics framework which incorporates the required notions of linguistic context and situations.[3]

Preliminary Definitions

Syntactic Properties of Sentences and Syntactic Sets

Two useful syntactic characterizations of sentences, which appeared first in (Groenendijk and Stokhof 1991), are carried over into DS, and through it into EDS as well. The operator *Aq* recursively collects occurrences of active quantifiers in a sentence. An active quantifier is a quantifier which will be able to bind occurrences of the corresponding variable further along. Thus the set of active quantifiers of a formula is equivalent to the set of variables which will be added to the referent system if the current state is updated with the formula. The operator *Fv* calculates the free variables in a formula. As both *Aq* and *Fv* are sets whose members are syntactic particles of the logical form, such particles will be represented surrounded by apostrophes.

(36) *Aq(φ),* the set of active quantifiers of a formula φ, and *Fv(φ),* the set of free variables of a formula φ are recursively defined as follows (I have added the rules for the modal operator):

$Aq(\mathrm{R}t1,...,tn) = \varnothing$

$Fv(\mathrm{R}t1,...,tn) = \{t_i \mid t_i \text{ is a variable}\}$

$Aq(t1{=}t2) = \varnothing$

$Fv(t1{=}t2) = \{t_i \mid t_i \text{ is a variable}\}$

$Aq(\neg\varphi) = \varnothing$

$Fv(\neg\varphi) = Fv(\varphi)$

$Aq(\varphi \wedge \psi) = Aq(\psi) \cup \{'\exists x' \in \varphi \mid \neg('\exists x' \in \psi)\}$

[3] See (Portner 1998) and (Bonomi 1997) for alternative theories of the progressive which also define some sort of epistemic structure based on a contextually determined set of sentences. In Portner both the modal base and its ordering source are contextually based. In Bonomi it is the course of concomitant facts.

$Fv(\varphi \wedge \psi) = Fv(\varphi) \cup \{'x' \in Fv(\psi) | '\exists x' \text{ is not a member of } Aq(\varphi)\}$

$$Aq(\exists x\ \varphi) = \begin{cases} aq(\varphi) \cup \{'\exists x'\} & \text{if } '\exists x' \text{ does not belong to } aq(\varphi) \\ aq(\varphi) & \text{else} \end{cases}$$

$Fv(\exists x\ \varphi) = Fv(\varphi)$ minus the occurrence of x in φ

$Aq(\Diamond\varphi) = \varnothing$

$Fv(\Diamond\varphi) = Fv(\varphi)$

I have extended the original scope of the *Aq* and *Fv* predicates by adding the modal operator as well. As the operator is a test, it does not project outward any active quantifier defined within its scope, but it is transparent with regard to the free variables that appear within that scope. In the following definitions some syntactic meta language predicates are used:

(37) a. FORMULAS – the set of well formed formulas, or sentences, in the (object) language.

b. PREDICATES(γ) – the set of atomic predicates used in the set of formulas γ.

c. FREE-VARS(γ) - the set of free variables used in the set of formulas γ. This is just pointwise application of *Fv* on every formula in γ.

d. ACTIVE-QUANTIFIERS(γ) - the set of active quantifiers used in the set of formulas γ. This is just pointwise application of *Aq* on every formula in γ.

Notational Conventions

As mentioned before, I will be using the following semi formal notational shortcut. We are dealing excessively with n-tuples, from which it is often required that a certain sub-structure be extracted. As each structure type is characterized by its own meta variable (states use *s, s'*, possibilities use *i, i'*, referent systems use *r*, etc.), I will write $< i >^r$ meaning 'the referent system (symbolized by *r*) of possibility *i*. Moreover, sometimes a structure fully determines a sub-structure without directly containing it as one of its elements. I will still use the same notation, as if the sub structure was in fact such an entity. For example, $< s >^r$ means 'the referent system of state *s*'. Even though states do not contain referent systems directly, they fully determine a related referent system, which is the reference system shared by all of the state's possibilities. It is that referent system which is to be retrieved in the case of $< s >^r$.

The Linguistic Context

As situations are highly context sensitive, we need to formalize not only the information states but also the context that surrounds them. A discourse will be a finite sequence of utterances, ignoring the possibility for multiple agents. The first utterance in the discourse is d^1. The last utterance made is d^n.

(38) ***Dis*** ,the set of discourses, is the set $\left\{ d \middle| d \in < FORMULAS >^n, n \in \mathbb{N} \right\}$

Situations are all about dynamic language use. The linguistic context is a key concept which represents the currently available fraction of a language, or partial language, within which our information state is defined.

(39) ***Γ,*** the set of linguistic contexts, is the set of subsets of *FORMULAS*:
$\Gamma = \wp(FORMULAS)$

Evolving hand in hand with the discourse, each utterance in the discourse may add to the linguistic context a set of salient formulas. The linguistic context being just a set of sentences, what is more interesting to describe is the pattern by which the context changes throughout discourse. We will assume that all contexts grow according to the following three constraints. The first constraint is that the linguistic context monotonically grows. Salient linguistic formulas do not become stale and get removed from the context. The second constraint is that at the very least, the actual utterance made is included in the linguistic context. The third constraint is that only referentially meaningful sentences be added to the context. A sentence is referentially meaningful if all of the free variables appearing in it are assigned in the current states' referent system. This constraint would seem to involve an unhealthy mix of semantic and syntactic notions, but in truth the assigned variables can be syntactically deducted from the set of previous utterances in the discourse using the *Aq* predicate described above.

(40) Function *f* is a **growth pattern** function iff :

i. $f \in \Gamma \times FORMULAS \longrightarrow \Gamma$

ii. For every $\gamma \in \Gamma$ and for every $\varphi \in$ FORMULAS the following hold:

(a) $f(\gamma,\varphi) \supseteq \gamma$

(b) $\varphi \in f(\gamma,\varphi)$

(c) $FREE-VARS(f(\gamma,\varphi)) \subseteq ACTIVE-QUANTIFIERS(\gamma \cup \{\varphi\})$

Growth patterns are functions that move us from one linguistic context to its extension based on an utterance. Requirement (a) makes sure that the new context is an extension of the previous one. Requirement (b) insures that the utterance itself is added to the linguistic context. This also insures along the way that $FREE-VARS(f(\gamma,\varphi)) \supseteq ACTIVE-QUANTIFIERS(\gamma \cup \{\varphi\})$. Requirement (c) takes care that all free variables in the linguistic context are defined in the current information state. This means that the sentences in the linguistic context are always 'defined' in the sense given in DS: their interpretation process does not come to a premature halt due to undefined variables being used. Note that while FREE-VARS contains occurrences of variables, ACTIVE-QUANTIFIERS contains occurrences of quantifiers, so that applying set inclusion between them is not precisely possible. I have used this notation for purposes of readability, and the reader should keep in mind that within such comparisons a variable 'x' in FREE-VARS is said to be the equal of the quantifier '$\exists$x' in ACTIVE-QUANTIFIERS.

The growth pattern and the actual discourse together determine for us what the linguistic context is at every stage of the discourse.

(41) Let f be a growth pattern function, $d = < \phi_1, .., \varphi_n > \in Dis$.
Then γ_d, the linguistic context defined by discourse d
(under f which we'll consider as a meta level parameter) is defined in the following way:
We first define a series of growing linguistic contexts,
$\gamma_d^1 = f(\varnothing, \phi_1),\ \gamma_d^2 = f(\gamma_d^1, \phi_2), ..., \gamma_d^n = f(\gamma_d^{n-1}, \phi_n)$

We can now define $\gamma_d : \gamma_d = \gamma_d^n \ (= \bigcup_{i=1..n} \gamma_d^i)$

There are some typical growth patterns that can be characterized. Going from the coarsest one possible, through the more fine grained, to the maximally fine grained which is no longer language dependent as a result, we get

(42)

$f_{discourse}(\gamma, \varphi) = \gamma \cup \{\varphi\}$

$f_{predicates}(\gamma, \varphi) = \gamma \cup P$
where P is the maximal subset of FORMULAS
such that $PREDICATES(P) = PREDICATES(\gamma \cup \{\varphi\})$
and $FREE-VARS(P) = ACTIVE-QUANTIFIERS(\gamma \cup \{\varphi\})$

$f_{full-language}(\gamma, \varphi) = \gamma \cup P$
where P is the maximal subset of FORMULAS
such that $FREE-VARS(P) = ACTIVE-QUANTIFIERS(\gamma \cup \{\varphi\})$

The first growth pattern provides the minimal linguistic context, which means that only the actual utterances of the discourse are added. The linguistic context is then trivially reduced to the discourse itself. The second pattern accumulates the predicates and variables defined through the referent system, and brings in at every step all those formulas which can be produced using a partial language that includes just those predicates and variables. The third pattern is not sensitive to the predicates used, just to the variables which are defined, and so it adds all those formulas for which a meaning will be provided in the possibilities. Each growth pattern extends the previous one in terms of the formulas that it brings into the linguistic context at every step.

Non-Discernibility

I would like to define situations by partitioning the set of possibilities in each information state into equivalence classes of indiscernible possibilities. But for this we must clarify what would make two possibilities non-discernible. Intuitively, two possibilities are non-discernible when there is no possibility to tell them apart by means of the linguistic granularity that we currently posses. There is however more than one possible way to define this notion.

Discernibility by Abstracting on Information States

Fleshing out the notion of discernibility by abstracting on information states, we start out by first defining consistency between possibility and sentence. In essence sentence φ is consistent with possibility *i* if the information conveyed in the sentence is already implicitly true in the possibility. That is, no matter what information state *i* is a part of, the state resultant from updating it with φ will contain *i*'s descendants. Note that consistency is not defined for *i* and φ if $\forall s \in S$ s.t $i \in s : s[\varphi]$ is undefined :

(43) Let $i \in I$; $\varphi \in$ FORMULAS.
φ is consistent with *i*, *Cons(i,φ)*, iff
$\forall s \in S$ s.t $i \in s : \exists i^* \in s[\varphi] : i^*$ is similar to an extension i' of i.

The requirement on the updated possibility is slightly weaker than could have been expected. Instead of a demand that it actually extend the original possibility, the demand is that it will be similar to such an extended possibility. The similarity relation was previously defined to hold between possibilities whose only difference lies in the set of pegs that they contain. Two similar possibilities will subsist in exactly the same updates so the relation is meaning preserving. This weakening of the consistency requirement will help us later in showing possibility consistency and possibility denotation to be equivalent.

It is interesting to note that for modal sentences of the form $\Diamond\varphi$, a possibility *i* is consistent with $\Diamond\varphi$ iff it is consistent with φ, because of the abstraction over information states. *i*'s consistency with $\Diamond\varphi$ means that in every information state that contains *i* one can find a possibility $i*$ as required in (43). Because the state could also contain just *i* alone, we can deduct that *i* itself could also count as the required $i*$. For similar reasons, with the $\Box$ operator *i* will never be consistent with $\Box\varphi$ unless φ is a tautology consistent with every possibility.

Based on the notion of possibility consistency we can define non discernibility in a linguistic context. Two possibilities are non discernible in a linguistic context if there doe not exist a sentence in that context such that the one is consistent with it while the other is not. Recall that if the linguistic context is changed according to a growth pattern, any sentence in the context will be defined.

(44) Let i,i' $\in$ I; $\gamma \in \Gamma$. i and i' are non discernible$_1$ in γ iff

$$\forall \varphi \in \gamma: \ Cons(i,\varphi) \leftrightarrow Cons(i',\varphi).$$

Discernibility Based on Possibility Truth Values

Another way to define non discernibility would be by using the well defined notions of truth and falsity in the possibility's world, instead of reaching up for information states. There are three difficulties involved with this approach.

The first one is that modality in DS is not defined in terms of conditions on possibilities, but on states. This makes it hard to account for it within a possible world's object language. The second difficulty is that in the DS framework no recursive definitions for the interpretation of a sentence within the context of a possible world are explicitly provided. Recall that only the interpretation rules for basic expressions were explicitly given (see chapter 4, on the denotations of basic expressions in a possibility). The third and final complication is that while in DS quantifiers can bind outside of their syntactic scope, this does not also happen in the classical predicate logic interpretation that we are using for truth in a possibility. Due to this complication many potentially meaningful sentences in a possibility, such as $\exists xP(x) \wedge Q(x)$, would not be given a defined truth value due to the second conjunct's using a variable out of its quantifier's scope.

Each of the enumerated problems needs to be individually solved before we arrive at an interpretation function for DS sentence denotations in a possibility. The formal apparatus for arriving at such a result is provided for in appendix A. The following definition just gives the final resultant definition of the interpretation function for sentence denotations in a possibility. Based on the interpretation function a second notion of non discernibility falls into place. Note that the following formulation contains syntactic functions *Nbf* (short for 'normal binding form') and *NonModal* which are applied to the original sentence *α* as a part of the interpretation. The functions are defined in the appendix A but in essence what the former does is shift around the variables in the sentence so that each appears only within the syntactic scope of its quantifier, and the latter simply removes the modal operator from the sentence.

(45) Let α be an expression; $i=\langle r,g,w\rangle$, $i\in I$, with *v* the domain of *r* and *I* based on D and W. The denotation of α in *i*, *i(α)*, is defined based on previous rules and now also the following:

If *α* is a sentence then the denotation of *α* in *i*, *i(α)* is defined as follows:

$$i(\alpha)=\begin{cases}1 & [\![Nbf(NonModal(\alpha))]\!]_{<i>^{w}}^{<i>^{g}(<i>^{r})}=1\\ 0 & [\![Nbf(NonModal(\alpha))]\!]_{<i>^{w}}^{<i>^{g}(<i>^{r})}=0\\ undefined & \text{otherwise}\end{cases}$$

(46) Let $i,i'\in I$; $\gamma\in\Gamma$. *i* and *i'* are non discernible$_2$ in *γ* iff

$$\forall\varphi\in\gamma:\ i(\varphi)\ and\ i'(\varphi)\ are\ both\ undefined,\ or\ i(\varphi)=\ i'(\varphi).$$

Our first notion of non discernibility was defined in terms of information state semantics, only the states themselves were abstracted away. Our second notion of non discernibility was defined in terms of classical first order predicate logic truth value in the possible worlds. The question of which notion to use in the upcoming definition for situations is solved by the following fact. The proofs of all theorems presented in this chapter appear in Appendix B.

(47) **Theorem 1**: Let $i \in I, i = \langle r, g, w \rangle, \gamma \in \Gamma$. For every $\varphi \in \gamma$: $i(\varphi) \leftrightarrow cons(i, \varphi)$

This leads straightforward to the equivalence of the two discernibility notions.

(48) **Theorem 2:** Let i,i' $\in$ I; $\gamma \in \Gamma$.
i and *i'* are non discernible$_1$ in γ iff *i* and *i'* are non discernible$_2$ in γ.

The following example illustrates possibility indiscernibility. Suppose the world is populated by cops and crooks, in the following way (each world is characterized by the objects it contains and by its interpretation function).

$$\text{W}=\{\text{w}_1, \text{w}_2\}$$

$$\text{w}_1=\left\{\{\text{Harry, Bugsy}\}, \begin{Bmatrix}\text{Cop}=\{\text{Harry}\}\\ \text{Crook}=\{\text{Bugsy}\}\\ \text{Handsome}=\{\text{Harry}\}\end{Bmatrix}\right\}$$

$$\text{w}_2=\left\{\{\text{Jake, Don Corrleone}\}, \begin{Bmatrix}\text{Cop}=\{\text{Jake}\}\\ \text{Crook}=\{\text{Don Corrleone}\}\\ \text{Handsome}=\{\text{Jake, Don Corrleone}\}\end{Bmatrix}\right\}$$

We start out from the state of ignorance only to update it with the fact that there is a handsome someone who is either cop or crook, as in sentence (49)-a. Our information state *S* now contains the three possibilities i1, i2 and i3.

(49) a. $\{\exists x Handsome(x) \wedge (Cop(x) \vee Crook(x))\}$

b. $\begin{Bmatrix}1.\exists x Handsome(x) \wedge (Cop(x) \vee Crook(x))\\ 2.Cop(x)\\ 3.Crook(x)\end{Bmatrix}$

(50) $\text{i1}=\langle \{x \to 1\}, \{1 \to Harry\}, w_1 \rangle$

$\text{i2}=\langle \{x \to 1\}, \{1 \to Jake\}, w_2 \rangle$

i3= $\langle \{x \rightarrow 1\}, \{1 \rightarrow \mathit{Don\ Corleone}\}, w_2 \rangle$

Discernibility is measured relative to the linguistic context. If the linguistic context γ contains just the explicit utterances (or utterance, as it is in the current case), then of course all of the possibilities in the information state are indiscernible because they all satisfy the explicit utterances. What happens when the actual discourse is only a proper subset of γ? Suppose our context growth function is such that when the uttered sentence is a disjunction, it also adds each of the disjuncts to the linguistic context as a separate sentence. This seems to me to be a reasonable enough pattern, and as a result we get (49)-b as the linguistic context instead of just (49)-a.

Nothing changes in the information state. No possibility is removed because of failure to satisfy a sentence from the extended linguistic context. However, we can now discern between either of possibilities i1 and i2, which satisfy the first two sentences of (49)-b, and possibility i3 which satisfies the first and the third sentences. We still can not discern between i1 and i2 though. We see that (non)discernibility is oblivious to the possible worlds associated with possibilities.

New Formal Structures

Situations

Finally we can use either of the non discernibility notions to define situations, a context dependent set of distinct options within our information state. Situations are sets of possibilities which share the same referent system and which conform to certain constraints under a given linguistic context.

First, the set can not be empty. Second, the possibilities share the same referent system. Third, the set of variables which forms the domain of the referent system is the same as the set of free variables in the linguistic context. These constraints maintain a sort of consistency between the linguistic context and the situations that can be defined in it. In particular we want to be able to assign truth values to sentences in a situation, which means that denotations must exist for the existing variables. The fourth constraint gives the essence of a situation: its possibilities must be indiscernible under the given linguistic context. They all assign the same truth values to every one of the linguistic context's sentences. The fifth and final constraint provides a sort of closure property to the situation.

(51) **Sit**, the set of situations, is the set

$$\left\{ <J,\gamma> \left| \begin{array}{l} \gamma \in \Gamma, J \subseteq I \\ 1.\ J \neq \varnothing, \\ 2. \forall i,i' \in J: <i>^r = <i'>^r \\ 3. \forall i,i' \in J: Dom(<i>^r) = FREE\text{-}VARS(\gamma) \\ 4. \forall i,i' \in J:\ i \text{ and } i' \text{are indiscernible in } \gamma \\ 5. \forall i \in J: \forall i' \in I:\ \text{if } <i>^r = <i'>^r, <i>^w = <i'>^w \text{ and } i \text{ and } i' \\ \text{are indiscernible in } \gamma \text{, then } i' \in J \end{array} \right. \right\}$$

Each situation includes a non empty set of possibilities and a linguistic context. The possibilities are such that not only do they share the same referent system (like possibilities in a DS state do), but they are also indiscernible in the situation's context. Of course this does not mean, and it is not to be expected, that the different possibilities in the situation be identical: they can vary tremendously from each other – but not in ways which can be told using the linguistic context.

Intuitively we would want the situation to include all of the possibilities which are indiscernible under the linguistic context. Indiscernibility ought to prevail only within situations, while situations as such ought, again intuitively, to be discernible. This intuition needs to be replaced by the somewhat weaker closure constraint that ensures us only of the inclusion of possibilities whose possible world is already contained by some possibility in the situation. The reason for replacing the more intuitive closure property for the weaker one is that in some discourses not all possibilities in *I* can even potentially participate. Sometimes a discourse starts with only a fragment of the set of possible worlds as possibilities. If situations only contained maximal subsets of *I*, they would be of no use in such discourses. The weaker sort of closure that is used helps us evade this unwelcome scenario.

The denotation of a sentence in situation *m* is only defined for sentences in which every possibility in the situation gives the same truth value. That truth value is passed on as the denotation of the situation as a whole. Other sentences may be undefined in the context of the possibilities (if more variables appear than the referent system defines), or maybe defined but with different truth values given by different possibilities in the situation. Either way the situation does not have a denotation for them. The sentences for which the situation's truth value is defined include all those sentences of the linguistic context. This is so because you can not have a linguistic context sentence which has undefined variables, and because you can not have the one situation containing different possibilities with contradicting truth values for some linguistic context sentence, by definition of a situation.

(52) Let φ ∈FORMULAS, m∈Sit, $[\![\varphi]\!]_m$, the denotation of φ in situation m, is partially defined as follows: $[\![\varphi]\!]_m = \begin{cases} 1 & if \quad \forall i \in \langle m \rangle^J : i(\varphi) = 1 \\ 0 & if \quad \forall i \in \langle m \rangle^J : i(\varphi) = 0 \\ undefined & otherwise \end{cases}$

(53) Let *m,m'*∈Sit, *m*=<*J*, γ> , *m'*=<*J'*, γ>. *m* and *m'* are said to be indiscernible in γ iff $\forall \varphi \in \gamma : [\![\varphi]\!]_m = [\![\varphi]\!]_m$.

(54) **Theorem 3:** Let *m,m'*∈Sit, *m*=<*J*, γ> , *m'*=<*J'*, γ>. *m* and *m'* are indiscernible in γ iff $\exists i \in m : \exists i' \in m' : i$ and i' are indiscernible possibilities in γ.

As the possibilities within the situation are all indiscernible from each other, it is enough to have a pair of possibilities, one from each situation, which are indiscernible for the whole lot to be so too.

Let us reexamine our previous cops and crooks example with the extended linguistic context given in (49)-b. We can now see that our initial partition of the possibilities into the sets {i1,i2} and {i3} is actually a distinction between two situations. The pair <{i3}, γ> trivially satisfies all of the prescribed conditions for situations. For the pair <{i1,i2}, γ> it is no longer trivial because i1 and i2 must be indiscernible, as they are indeed in γ. However, taking the same three possibilities, one could also define each as a separate situation: $<\{i1\}, \gamma>$, $<\{i2\}, \gamma>$, $<\{i3\}, \gamma>$. This is possible because situations are only closed under indiscernibility within the same world. As i1 and i2 do not share the same possible world they need not belong to the same situation even though they are indiscernible. This however would cause the situations $<\{i1\}, \gamma>$ and $<\{i2\}, \gamma>$ to be indiscernible situations. As mentioned, with the redefinition of information states such cases will be evaded within the scope of a single information state. Information states will now include the constraint that no two situations in them are indiscernible.

To summarize, in EDS situations are to fill the cognitive part that was played by possibilities in DS. They are the distinct cognitive optional realities that we recognize as possible given our information state and linguistic context, uniting sets of distinct but indiscernible possibilities. Formally a situation is a pair consisting of a set of possibilities and a linguistic context. Under this context, all of the situation's possibilities are non discernible. If the linguistic context gets larger, the same set of possibilities may no longer be non discernible, and the set of possibilities will be further partitioned into smaller, sharper, situations. To the relation between situation and state, and to the dynamic process of refining the partitioning we now turn.

Information States are Redefined

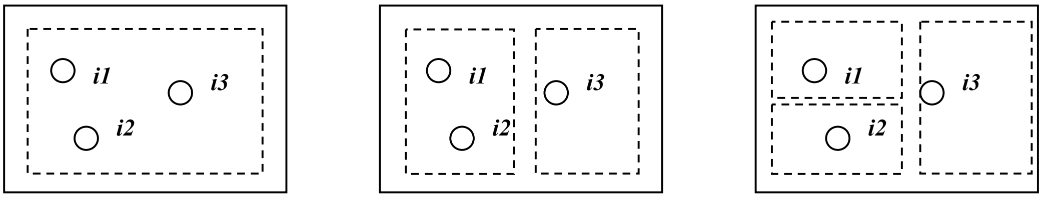
We will now proceed to redefine the information states. We will define the set of situation based states, S_{EDS}, and later show its relation to the DS set of states, S_{DS}. Note that although it is possible for two distinct situations to be indiscernible under their linguistic context, we do not allow for this within the information state. A technical note: starting with the following definition I will try to adhere to the following new notational convention: while information states in the set S_{DS} will be denoted by the letter *s*, information states in the set S_{EDS} will be denoted by the letter *t*.

(55) $\mathbf{S_{EDS}}$,the set of information states based on Sit, is the set

$$\left\{ t \subseteq Sit \;\middle|\; \begin{array}{l} \forall m, m' \in t: \\ <m>^{\gamma} = <m'>^{\gamma}, <m>^{r} = <m'>^{r}, \\ m \text{ and } m' \text{ are discernible in } <m>^{\gamma} \end{array} \right\}$$

An information state consists of a collection of situations which share the same linguistic context and through their possibilities also share the same referent system. The discernibility clause makes sure that the state contains only maximal sets of indiscernible possibilities. This solves the problematic case mentioned above where two indiscernible possibilities, i1 and i2, were not merged into the same situation. Indeed although <{i1},γ>, <{i2}, γ> and <{i3}, γ> are each a situation by its own right, <{i1},γ> and <{i2},γ> can not exist in the above mentioned state *S* because they are not discernible in γ. The relations between possibilities, situations and states is graphically given in the following three examples for the three cases discussed in relation to the cops and crooks example: (a) where only (49)-a is the linguistic context, (b) where (49)-b is the linguistic context and (c) which also has (49)-b as context but provides a non-possible state in which indiscernible situations m1 and m2 coexist.

Two possible and one impossible configurations for situation based states



(a): *Context is only (49a). All three possibilities are indiscernible and so they are in the same situation m. The only one in the state.*

(b): *Context is (149b). Two situations exist, one where the handsome man is a crook and one where he's a cop.*

(c): *Same as (b) except that two situations exist in which the handsome man is a cop. As the situations are indiscernible, they can not coexist in the same state.*

The granularity of the situations depends on the context, which in turn depends on the growth pattern employed. Let us further analyze the patterns already described. The first growth pattern, $f_{discourse}$, reduces the context into the discourse itself. As the possibilities in the current state are those that are consistent with the set of previous utterances, they will all be grouped under one situation. So with this growth pattern situations are trivially reduced into states: each state contains only one situation, which in turn contains all of the possibilities. The second pattern, $f_{predicates}$, provides a significantly richer context. As a result we may expect more than one situation per state, with the number of situations in each state constantly growing because more and more predicates are introduced into the discourse. The last pattern, $f_{full\text{-}language}$, has a full blown linguistic context to begin with. As a result the situations are as fine grained as is possible within the language, with more or less one situation per possibility.

Situation Extension

Let us now define a partial order of extension on the set of situations. As always, the special status of situations as intermediates between information states and possibilities gives two equivalent notions for this.

(56) **Theorem 4:**

Let m,m' $\in$ Sit; m=<J,γ>, m'=<J',γ'>:

Def1: m' is an extension$_1$ of m, m $\leq_1$ m', iff

$$\gamma \subseteq \gamma'; \langle m\rangle^r \leq \langle m'\rangle^r; \forall i' \in J' : \exists i \in J : \langle i'\rangle^w = \langle i\rangle^w;$$
$$\forall \varphi \in \gamma : [\![\varphi]\!]_m = [\![\varphi]\!]_{m'}$$

Def2: m' is an extension$_2$ of m, m $\leq_2$ m', iff

$$\gamma \subseteq \gamma'; \forall i' \in J' : \exists i \in J : i \leq i'$$

Theorem: $m \leq_1 m' \Leftrightarrow m \leq_2 m'$

One situation is an extension of another if its linguistic context includes that of the other, if it does not introduce any new worlds, and if they share sentence denotations on the smaller of the linguistic contexts, which is shared among them. The sharing of denotations on the common ground context along with the non introduction of new worlds is equivalent to the requirement that every possibility in the extended situation be a descendent of a possibility in the other one.

Based on situation extension we can now redefine state extension and related notions such as subsistence. It can be seen in the following definitions that situations are to be used by information states in exactly the same way as possibilities were in DS. This will become even more apparent when we next redefine the object language in terms of situations.

(57) Let t,$t' \in S_{EDS}$.t' is an extension of t,$t \leq t'$,iff $\forall m' \in t' : \exists m \in t : m \leq m'$

(58) Let $t, t' \in S_{EDS}, t \leq t', m \in t, m' \in t'$:

i. m' is a descendent of *m* in t' iff $m \leq m'$

ii. m subsists in *t'* iff *m* has one or more descendents in *t'*.

iii. t subsists in *t'* iff $\forall t \in s : m\ subsists\ in\ t'$

The above definition for state extension recounts, along formal lines, the initial intuition that situations 'split' as the discourse evolves and the linguistic context becomes more fine grained. Every new situation in t' is descendent from only one parent situation in t, while every parent situation in t can have more than one descendent in t'.

Entailment is defined in the exact same manner as it was defined in DS, based on the notions of consistency (in the information states) and support.

(59) Let $t \in S_{EDS}$:

i. φ is consistent with t iff $t[\varphi]$ exists and $t[\varphi] \neq \emptyset$.

ii. φ is supported by *t* iff $t[\varphi]$ exists and *t* subsists in $t[\varphi]$.

iii. $\varphi_1, \ldots \varphi_n \models \psi$ iff for all information states *t* such that $t[\varphi_1], \ldots [\varphi_n] [\psi]$ exists, it holds that $t[\varphi_1], \ldots [\varphi_n]$ supports $[\psi]$.

Dynamics of Linguistic Context Updates

By now we can give a formal characterization of the dynamics of linguistic context updates. Such a characterization will come in handy for the next parts where we define the object language semantics and show that the information state structures in the two frameworks DS and EDS are isomorphic under the update relation. Recall that the intuition behind linguistic context updates is that with the context's growth each situation may ‘split’ into several situations. These situations are not discernible under the initial linguistic context, but they become so in the new context due to one or more of the new sentences that enter the context during its update.

A linguistic context update on a situation then is a function from the set of situations and the set of formulas to the power set of situations. Each of the resultant situations holds the newly updated linguistic context, each of them is discernible from the others, and most importantly, no possibility is either removed or added when we compare the possibilities in the source situation and the possibilities in the union of the target situations. As each of the situations in the resultant set is discernible from the others, we get that the function’s range is in fact the set of situation based information states S_{EDS}.

Let m=$\langle J,\gamma\rangle \in$ Sit, $\varphi \in$ FORMULAS, and CG the context growth pattern.
m{φ}, the result of linguistic context update with φ to the situation m,
(60) is defined as follows:

$$m\{\varphi\} = \{m_1, m_2, ..., m_n\} \; where: \begin{cases} \langle m\rangle^J = \bigcup_{l=1..n} \langle m_l\rangle^J \\ \forall l = 1..n : \langle m_l\rangle^\gamma = CG(\gamma,\varphi) \\ \forall l \neq k \in \{1..n\} : m_l \text{ and } m_k \text{ are discernible in } CG(\gamma,\varphi) \end{cases}$$

Of course the linguistic context update operation can not just be stipulated. The following theorem assures us that it is well defined. Not only is linguistic context update a well defined function, but it also applies a partition on the set of possibilities in the original situation, $\langle m\rangle^J$.

(61) **Theorem 5**:

The linguistic context update operation {} is a well defined function from the set of situations and the set of formulas to the set of situation based information states. The function provides a partition on the set of possibilities in the original situation.

Linguistic update occurs on each situation independently of the other situations in the state. We can nevertheless define the linguistic update of a whole information state as the pointwise application of the update on each of its situations.

(62) *Let* $t \in S_{Sit}, \varphi \in FORMULAS$, *and CG the context growth pattern.*
$t\{\varphi\}$, *the result of linguistic context update with* φ *on the state* t, *is defined as follows*:
$$t\{\varphi\} = \bigcup_{m \in t} m\{\varphi\}$$

It is through consequent linguistic context updates that the 'amoeba splitting' effect of situations is achieved. As we shall see next, the semantics of the object language in EDS make sure to keep the linguistic context in the situations constantly up to date.

Object Language Semantics

Variable assignment

Before stating the semantics for the object language of situation based dynamic semantics, we need to cope with the definition of variable assignment under these semantics. Variable assignment in EDS is a pretty straight forward adaptation of DS assignment. Recall that in DS assignment of an individual to a variable in the information state takes place by taking each possibility in the state and extending it with a relation between the variable and the individual (by extending the referent system). In EDS situations were added as intermediate sets in between the information state and its possibilities, but variable assignment is defined so as to 'pass' through them: information state assignment of a variable is defined as pointwise application of situation assignment, which in turn is defined as pointwise application of possibility assignment to each one of the situation's possibilities.

(63) Let $i \in I$, $m \in Sit$, $t \in S_{EDS}$, $d \in D$. The effect of assignment to variables:

i. $i[x/d] = < r[x/n], g[n/d], w >$

ii. $m[x/d] = \left\{ i[x/d] \middle| i \in \langle m \rangle^{J} \right\}$

iii. $t[x/d] = \left\{ m[x/d] \middle| m \in t \right\}$

I had initially devised EDS as a framework in which each world has its own distinct domain of individuals, as I find the idea of a shared domain quite unintuitive. However, I finally decided to leave multiple domains out of the picture when I became fully convinced that it is a notion which is completely independent of the notion of situation. One can add multiple domains to DS also, or define EDS without multiple domains (as I have done here).[4] Now, as we will see in the next chapter situations are comparable and may even be ordered based on existence or non-existence of individuals in them. Multiple domains would help to accentuate this characteristic, but are not essential for it.

The Object Language

We are now encumbered not with one but with two dynamic processes taking part in the semantic update of utterances. One involves the constant repartitioning of situations as a result of linguistic context growth. The other is about semantic interpretation, the adding and eliminating situations (previously possibilities) in accordance with the semantic content of the discourse utterances. These two processes are fused together in the new framework's object language semantics.

There is one substantial difference between updating through linguistic context growth and updating through semantic interpretation. The former is not a recursive process but the latter process is. In the process of semantic interpretation every complex formula's interpretation depends upon the interpretation of its constituent formulas. This is not the case when updating the linguistic context. There I only intend to add the complex formula itself and whichever other formulas are indicated by the growth pattern.

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[4] I had originally made use in EDS of individual concepts as devised by (Carnap 1947) and (Hintikka 1969) to accommodate multiple domains. Suffice it to say here that once individual concepts are defined quantification in effect ranges over them and not over concrete individuals in the possible worlds. The benefit of EDS over Hintikka's treatment of concepts is that no primitive cross world identity is needed to make quantification work.

This difference is echoed in the definition of the update function []: this function will now be defined as the subsequent application first of the linguistic context update function {} and then of the recursive semantic interpretation function which will be marked as ⌜⌝.

(64) Let $t \in S_{EDS}$, $\phi \in FORMULAS$. t[ϕ], the update of *t* with ϕ, is defined as the subsequent application of linguistic context update and then semantic interpretation update: t[ϕ]=t{ϕ}⌜ϕ⌝.

The semantic interpretation update function ⌜ϕ⌝ is recursively defined as follows:

$t\ulcorner Rq1,...,qn \urcorner = \{m \in t \,|\, [\![R(q1,...,qn)]\!]_m = 1\}$

$t\ulcorner q1=q2 \urcorner = \{m \in t \,|\, [\![q1 = q2]\!]_m = 1\}$

$t\ulcorner \neg\varphi \urcorner = \{m \in t \,|\, m \textit{ does not subsist in } t[\varphi]\}$

$t\ulcorner \varphi \wedge \psi \urcorner = t\ulcorner \varphi \urcorner \ulcorner \psi \urcorner$

$t\ulcorner \exists x\, \varphi \urcorner = \bigcup_{d \in D} (t[x/d][\varphi])$

$t\ulcorner \Diamond\varphi \urcorner = \{m \in t \,|\, t[\varphi] \neq \varnothing\}$

$t\ulcorner \varphi \rightarrow \psi \urcorner = \left\{ m \in t \,\middle|\, \begin{array}{l} \textit{if m subsists in } t[\varphi] \textit{ then all descendents of m} \\ \textit{in } t[\varphi] \textit{ subsist in } t[\varphi][\psi] \end{array} \right\}$

$t\ulcorner \varphi \vee \psi \urcorner = \{m \in t \,|\, \text{m subsists in t}[\varphi] \text{ or m subsists in t}[\neg\varphi][\psi]\}$

$t\ulcorner \forall x\, \varphi \urcorner = \{m \in t \,|\, \textit{for all } d \in D\textit{: m subsists in } t[x/d][\varphi]\}$

$t\ulcorner \Box\varphi \urcorner = \{m \in t \,|\, \text{t subsists in t}[\varphi]\}$

It is quite obvious that the semantic interpretation update function ⌜⌝ in S_{EDS} pretty much resembles the update function [] in S_{DS}. The one major difference is that no reference is made to possibilities, the interpretation is entirely defined in terms of situations. For further explanations of the semantics of the operators I refer the reader to chapter 4, where the object language for S_{DS} was discussed. Note that the semantic interpretation update function $\ulcorner\urcorner$ makes use of values given by the update function [] as whole (utilizing both linguistic context and semantic interpretation) for structurally simpler sentences.

Compatibility with Dynamic Semantics

Discourse Based Information States

Information states in both DS and EDS are defined independently of the discourse which brings about their existence. In order to show an isomorphism between the frameworks I will confine myself to those states, in either framework, which are discourse derived. The following definitions carve out from each set of information states the subset of states which can actually be arrived at from an initial state of ignorance and a sequence of uttered sentences, that is, a discourse. Once confined to these subsets, which obviously form the 'interesting' information states in each framework, it will be shown that there is an isomorphism function between them that is maintained under the operations of object language update.

First, starting from the total set of DS states, here repeated, we carve out the subset of discourse based states.

(65) $\mathbf{S_{DS}}$, the set of information states based on *I*, is the set

$$\left\{ s \subseteq I \,\middle|\, \forall i, i' \in s : \langle i \rangle^r = \langle i' \rangle^r \right\}$$

(66) $\mathbf{S_{DS}^{Discourse}}$, the set of DS information states which can be arrived at by a discourse,

$$\text{is the set } \left\{ s \in S_{DS} \,\middle|\, \begin{array}{l} \exists s_0 \in S_{DS} : \forall i \in s_0 : \langle i \rangle^r = \varnothing \text{ and } \langle i \rangle^g = \varnothing \\ \exists d \in Dis, d = < \phi_1, .., \varphi_n > \\ s = s_0[\phi_1]...[\phi_n] \end{array} \right\}$$

Recall that a state of ignorance is such that no discourse information has yet been collected in it, and therefore the referent sets in its possibilities (and consequently the assignment functions too) are empty. A state is discourse based if there exist such an initial state of ignorance from which it can be arrived at by some discourse, which is nothing more than a sequence of utterances.

Each discourse based information state implicitly defines the current linguistic context, in the way that was shown in (41). The discourse based states of DS can also be characterized in terms of possibility indiscernibility, in a way which is similar to situations. For every such state, if it contains possibility *i*, then it also contains all of the possibilities which are indiscernible from *i* and share the same world with it.

(67) **Theorem 6:**

Let $d = <\phi_1,..,\varphi_n> \in Dis, s = s_0[\phi_1]...[\phi_n] \in \mathrm{S}_{\mathrm{DS}}^{\mathrm{Discourse}}$.

$\mathrm{s} \supseteq \left\{ \mathrm{i} \in \mathrm{I} \middle| \exists \mathrm{i'} \in \mathrm{s}: \langle i \rangle^w = \langle i' \rangle^w, \langle i \rangle^r = \langle i' \rangle^r \text{ and i and i' are indiscernible in d} \right\}$

The set of EDS information states which can be arrived at by discourse, $\mathbf{S}_{\mathbf{EDS}}^{\mathbf{Discourse}}$, is defined in a similar manner to $\mathbf{S}_{\mathbf{DS}}^{\mathbf{Discourse}}$.

(68) $\mathbf{S}_{\mathbf{EDS}}^{\mathbf{Discourse}}$, the set of situation based information states which can be arrived at by a discourse, is the set

$$\left\{ s \in S_{Sit} \middle| \begin{array}{l} \exists s_0 \in S_{Sit} : \forall m \in s_0 : \langle m \rangle^\gamma = \varnothing \text{ and } \forall i \in m : \langle i \rangle^r = \varnothing \text{ and } \langle i \rangle^g = \varnothing \\ \exists d \in Dis, d = <\phi_1,..,\varphi_n> \\ s = s_0[\phi_1]...[\phi_n] \end{array} \right\}$$

Note that in the definition provided for the initial states of ignorance here, there is always only situation in the state. This is due to the linguistic context's being empty, as a result of which any two possibilities are indiscernible there. One could also think about initial states of ignorance where even the initial context is not empty, but I leave this slightly more complicated case unattended here.

Transition Function Between DS States and Situation Based States

Before moving on to the isomorphism theorem we need to prove some lemmas that will bring out the relations that hold between possibilities and situations over object language updates. The first lemma shows that in the DS framework, each possibility has exactly one ancestor (that is, it is the descendent of exactly one possibility). The second lemma does the same but for situations in the EDS framework. The third lemma shows that if one situation does not extend the other then neither do their possibilities form an extension relation. (the exclamation mark ! signifies that there exists exactly one).

(69) a. **Lemma 1**: Let $\phi \in$ FORMULAS and let $s \in S_{DS} : \forall i' \in s[\phi] : \exists! i \in s : i \leq i'$

b. **Lemma 2**: Let $\phi \in$ FORMULAS and let $t \in S_{Sit} : \forall m' \in t[\phi] : \exists! m \in t : m \leq m'$

c. **Lemma 3**: Let $\phi \in$ FORMULAS and let $t \in S_{Sit}$:
$\forall m \in t$ and $m' \in t[\phi] : m \not\leq m' \rightarrow \forall i \in \langle m \rangle^{J} : \forall i' \in \langle m' \rangle^{J} : i \not\leq i'$

The following result shows that the two discourse derived sets are isomorphic under object language updates.

(70) **Theorem 7:** There exists a function *F* between $\mathbf{S}_{\mathbf{DS}}^{\mathbf{Discourse}}$ the set of DS discourse derived states, and $\mathbf{S}_{\mathbf{EDS}}^{\mathbf{Discourse}}$ the set of EDS ones. This function is 1-1, onto, and is structure preserving on the update relations defined by the object language.

Let $s = s_0[\phi_1]...[\phi_n] \in S_{DS}^{Discourse}$.

$d =< \phi_1,..,\varphi_n >\in Dis$ and γ_d is the linguistic discourse defined by d.

Let $T_s \subseteq S_{EDS}^{Discourse}$ be $\left\{ t \in S_{EDS}^{Discourse} \middle| \begin{array}{l} for\ every\ m,m' \in t\ the\ following\ hold: \\ \langle m \rangle^{\gamma} = \gamma_d \\ \forall i \in \langle m \rangle^{J} : i \in s \end{array} \right\}$

$F(s) : S_{DS}^{Discourse} \xleftrightarrow{\text{1-1}} S_{EDS}^{Discourse} = Max(T_s)$

F is structure preserving on the update relations defined by the object language

We see that at any point in the discourse one can switch back and forth between the DS information states structure and the EDS one.

With this we conclude our investigation into the formal aspects of EDS, the dynamic situations extension to dynamic semantics. By maintaining the linguistic context as a super set of the actual discourse we can partition the seemingly monolithic information state into discernible situations. These situations are distinguished from each other through linguistic means, that is by at least one of the sentences available in the linguistic context. As the sentences of the extended context are not asserted, an existential sentence could be true in one situation yet false in another, in effect allowing us to embed one situation within another in terms of recognized entities that reside within them. We will use this ability in order to implement our previously specified characterizations of the normality criterion in the next chapter.

CHAPTER 6
THE PROGRESSIVE IN A DYNAMIC SEMANTICS ENVIRONMENT

Chapter three advocated a move to an epistemic modal base for the semantics of the progressive. Chapter five offered a framework designed to model a dynamic process of information growth in two concentric domains, that of the discourse and that of the extended linguistic context. We now combine the two in order to reapply Dowty's (Dowty 1979) semantics for the progressive in the epistemic framework. Further assumptions and considerations about both the temporal relations between events and the compositional structure of Dowty's aspectual classes are spelled out. Finally the two components of the inertia notion are defined in epistemic terms.

Further Assumptions and Articulations

Basic Assumptions on Event Temporal Relations

Recall Dowty's later formulation of the progressive operator's semantics, repeated below. The inertia function is of course at the heart of this formulation, and the main incentive for translating the semantics into an epistemic framework. But I am also interested in switching to an event based semantics, and this necessitates further assumptions, or clarifications.

(71) [PROG ϕ] is true at $<I,w>$ iff for some interval I' such that $I \subset I'$ and I is not a final subinterval for I', and for all w' such that $w' \in Inr(<I,w>)$, ϕ is true at $<I',w'>$.

Following (Link 1987) and (Landman 1992) I will assume the existence of a temporal trace function τ, a function from the set of events into a set of underlying intervals. Unlike Landman I assume that each event belongs to one world only and so there is no need to parameterize the function by a world as well as an event. I do not contend that the notion of event is metaphysically dependent on that of an interval. The trace function is merely a convenience, because whatever temporal notion we deem most basic must have some sort of primitive order defined on it. My interest is in having events as ontological individuals, and as Dowty already assumes an ordering on intervals, we can use this ordering for events too through the trace function.

Another assumption is that we have a reference time interval at our disposal when we come to analyze the progressive. This interval is the result of a more general analysis of the relative temporal positions of the events in the discourse. This interval will be denoted as *RT*, and is the equivalent of Dowty's *I* interval.

Articulations Regarding the Compositional Structure of Dowty's Aspectual Classes

As was already mentioned in chapter 2, the imperfective paradox is only coherent when we assume that the different aspectual classes are compositionally composed of basic stative predicates glued together using special aspectual operators such as DO, BECOME, AT and CAUSE. Our main concern here being the relative compositional complexity of accomplishments relative to activities, suffice it to say that activities are taken to be composed of a single predicate, while in contrast the accomplishment class is analyzed as a structure containing an activity and a resulting state, connected by causality. Both the activity and the result predicates will be seen as plain object language sentences indexed only by the event they are to apply to.

Furthermore, Dowty's meaning postulate for activities is also assumed, namely that for an activity A to be true at an interval *i* it must also be true at every reasonably sized subinterval *j* of *i*. This meaning postulate is often referred to as the subinterval property of activities, conferring a certain type of homogeniety on them.

Knowledge Up to a Point

We now move on and concentrate on the inertia function. Reverse engineering Dowty's conception of it, I redivide inertia into two distinct constraints on the set of worlds chosen. A constraint requiring identity of known facts up to the reference time point (KUP), and a constraint requiring that the course of events continue normally from the reference time onward (NCE). I have argued in chapter three for a move into an epistemic modal base. Here the KUP constraint is turned into a consistency constraint. The possibilities chosen must be consistent with what is known and has been learnt up to the reference time point.

It must be pointed out that the set of worlds consistent with an information state need not coincide, nor even overlap, the set of worlds identical with the real one up to a point in time, even when the information state is accurate for that point. In the epistemic framework we can not assume to have objective acquaintance with the real world. Our knowledge state need not contain the real world as a basis for some of its possibilities. In fact formally there is no specifically marked 'real world' in the DS framework.

Lucky for us, the demand of consistency with the current knowledge state is trivially achieved in DS. The set of possibilities which make up the information state is precisely the set of those possibilities which are consistent with it. We get the first half of the inertia notion 'for free' when we use DS.

It must be acknowledged though that in the DS framework, and consequently in EDS too, only a single agent's information state can be kept track of. An information state models knowledge *given a certain viewpoint*. We get in trouble when we try to account for more than one viewpoint here. When Max is crossing the road and a speeding bus is right behind him we can define an information state easily identifiable with Max's state (one in which there is no bus), or we can define a state that is more naturally associated with us as bystanders (here the bus exists). But we can not define both simultaneously. We can not even formulate the sort of function that would move us from one viewpoint to the other, because such a move may necessitate the re-introduction of worlds already removed from the information state: if we wanted to move from the bystander viewpoint to Max's viewpoint we would have to 'forget' about the bus. We could of course remove any variable that refers to the bus, but that would not be enough. We would also need to re-introduce back into the information state all of the worlds which were previously removed from it only because they contained no bus. As mentioned in chapter 4, in DS such a change in information states is not possible: worlds can only be removed from an information state, not added.

One possible way to deal with multiple viewpoints within the semantics of an epistemic framework may be to expand it into a multi-modal epistemic framework, such as the frameworks proposed by (Fagin, Halpern, Moses & Vardi 1995) and (Gerbrandy and Groeneveld 1997). Such an extension however is well beyond the scope of my current work. I will have to assume then that when a progressive event description is analyzed, a certain viewpoint has already been chosen based on pragmatic means, and so the starting information state for the analysis echoes the extent of the chosen viewpoint's knowledge scope.

Normal Course of Events

Dowty found the NCE a lot harder to formalize than the IUP, and this hardly comes as a surprise. We are often confronted with difficulty judging the normality of even more mundane things as behavior, height, taste. There seems to be a subjective element lurking at the bottom of this concept, but that does not mean that we have to leave it all unanalyzed. I have earlier suggested that we can nibble at its perimeter, striving to isolate that subjective element as the only truly primitive notion involved in inertia.

Let us have another look at the list of characterizations previously brought up at the end of chapter two, and see how they might translate into more formal notions based on EDS.

Normality of the relevant and yet non-biasing of the irrelevant. In DS what is relevant is the set of actually uttered sentences, and they are relevant for complying with the KUP constraint. Any information state will only contain possibilities consistent with these relevant facts of knowledge. For modeling the NCE we must extend the set of relevant sentences to ones which, although salient in the context, have not been actually asserted as utterances. This extended sphere is provided for by the linguistic context. The actual contents of this set are dependent on the utterances and on the context growth pattern, as discussed in chapter five. As all of these sentences have not been actually asserted (except the subset of discourse utterances, which is also included in the linguistic context), the possibilities in the state are not restricted to those which validate them. However, we can tell the possibilities apart based on these sentences even if they were not explicitly asserted, because they are at the base of the state's partitioning into situations. Sentences which are not part of the actual discourse, nor even of the linguistic context, are considered as irrelevant for the information state. Even within a situation we can have two possibilities such that one of them validates such an irrelevant sentence and the other does not, or it may be a sentence undefined in the current referent system. All this shows that indeed that irrelevance does not bias in any way the creation of possibilities or the partition into situations.

Knowledge versus assumptions, estimations, and the generally doxastic. In EDS we maintain simultaneously two concentric but distinct contexts, one of discourse and the other of a more generally linguistic nature. The inner discursive context models sure knowledge. Every sentence in it has been explicitly asserted and is consistent with all the possibilities in the state. The outer context is only indirectly determined by the discourse, as each utterance is applied to the context growth pattern, and the resulting set of sentences is added to it. This sphere thus models a collection of contextually relevant unuttered thoughts, images, or anything else made salient though not explicitly said. As mentioned above, these sentences are considered as non-committed speculations, no possibility is rejected from the information state for not complying with a linguistic context speculation. Nevertheless, the sentences in the linguistic context structure uncertainty in the information state through the situations: the relation between the sentences in the linguistic context and a situation is similar to that which holds between the sentences of the discourse context and the information state. Just as all the possibilities in the information state hold the same truth value for every discourse sentence, so do all the possibilities in a situation hold the same truth value for every sentence in the linguistic context. The difference is of course that with the discourse context there is only one allowed value: truth.

Relativity of the notion of normality. Up until now we have been discussing the structures with which normality will be concerned, namely the linguistic context and situations. Relativity is about normality itself, how it is modeled using the above mentioned building blocks. Normality's being essentially a relative notion means that basically, a course of events can only said to be *more* normal, and that when compared to another. Absolute normality then is a derived notion. A course of events is normal when it is more normal than any other comparable course. Our natural formal candidates for being compared, the formal equals of courses of events, are situations. We will see how their being linguistically derived helps us when we come to compare them. The relativity of normality is fully brought to play when we consider the next characteristic.

Minimal happenings of the unexpected. Minimality has already been hinted at with regard to relativity, but the more difficult question is minimality of what. What are the unexpected happenings? This too has been mentioned earlier in chapter three, but we now have a stronger formal framework to back us up. The task before us seems initially impossible, to take into account those happenings which by definition are unexpected. Indeed, I do not know how to express the class of unexpected events, but this cuts both ways. The same inability confronts me when I come to actually asses a progressive sentence, and yet I do asses such sentences on a daily basis. The job at hand must then be somehow simplified. It is the other characteristics of normality that come to our help here. The unexpected events which need be explored must also be relevant, and so what we are looking for are unexpected relevant events. Such events would not be asserted as part of the discourse, because then they would not be unexpected. Thus the events we are concerned with here are those which are speculated about at the outer linguistic context without being asserted. Such events enjoy a strange existential status. They exist partially, i.e. in only some of the possibilities, but are indirectly mentioned (which is not the same as being directly referred to) by a sentence that concerns the information state as a whole. Returning to minimality, a possible course of events formalized as a situation in the information state will be considered normal when it contains less unexpected events than any other comparable situation. This notion of minimality I will call minimal eventiveness, but before formally giving shape to this notion let us reflect on the mode of partiality of being hinted at above.

Partiality of Being

Dynamic semantics (DS) makes a distinction between two notions of an entity, be it event or object. The primitive notion of an entity is that of the individual. These are the entities that populate the possible worlds. The other notion of an entity combines the variables in the referent system, the pegs, and the individuals they refer to in the underlying possibilities. The first notion of an entity may be said to be ‘real’ or ontological, in the sense that it does not depend on the information state in any way. Each such individual is a fully determined instance whether anything is known about it or not. The second notion is epistemic. The properties of the entity depend on the information state. The entity is not a true instance, more precisely it defines a unique type to which all related instances belong.

The event entities we will be interested in, when comparing situations, are of an epistemic kind, but not quite the same as those epistemic entities just mentioned. The epistemic event entities discussed above can be identified by and referenced through a variable in the state's referent system. The extended dynamic situations framework (EDS) introduces an epistemic entity of a different nature from that just discussed. In DS every epistemic entity which is said to exist in one of the discourse utterances and is therefore assigned a variable must posses an instance in every possibility. Let us see what happens in EDS, when we add the linguistic context and partition the possibilities by it. Notwithstanding sentences which are also actual discourse utterances, the sentences in the linguistic context need not be true in every possibility in the information state. What happens when we have an existential sentence like (72) in a linguistic context which is not an actual utterance?

(72) $\exists e Event(e) \wedge Walk(e) \wedge Ag(Mary, e)$

Some possibilities may have such an event individual in their world and therefore validate the sentence, others may not and in them the sentence is false. We will get situations in which the sentence is true (hosting possibilities of the former kind) and situations in which it is false (containing the latter). The sentence defines an epistemic event, but one that is not referable through a variable. Moreover, the epistemic entity need not have an instance in every possibility in the state. Does it describe something that exists? Well, not entirely. The event of Mary's walking is not explicitly asserted in the discourse, although it is contextually relevant. Some situations have it, others don't. I call such entities partially existing. They lead a shadowy existence, they do not have any formal identifier, and their instances can not be referenced in any way. And when we are looking for situations where fewer things happen, it is events of this type that we are counting and comparing.

Minimal Eventiveness

Without further ado let us turn to see how minimal eventiveness can be made formally possible. Intuitively we would like to say that one situation is less eventive or equal to another one in the same state if every relevant event in it also happens in the other situation. In truth we only want to compare non-discursive events, events which were not asserted in discourse. Shouldn't we then try to filter out those events which *are* discursive, before we go on to order the situations? The answer is simple: there is no need to filter out the discursive events. Notice that as these events appear in every possibility, they will not affect the minimality ordering one way or the other.

Now obviously, we can not get a direct handle on events, and entities in general, which are not discursively marked in the referent system. In order to circumvent this problem we must indirectly compare situations by the linguistic descriptions which set them apart from each other. These are the sentences of the linguistic context.

Schematically, minimal eventiveness can be defined in the following way. First, we must identify those sentences in the linguistic context that proclaim the existence of an entity in general. From these we must choose those sentences in which the entity is an event. Next we can sift our flour even further and specifically choose those events which answer our specific requirements. In the case of the progressive the events we are interested in are those that happen within a specified time frame (this will become clearer when give the full semantics for the progressive in the next section). Finally, taking the carefully selected subset of sentences, we can compare any two situations in the state by checking if either one of the situations validate all of the sentences which are also validated by the other situation. We use the linguistic context, and the fact that every one of its sentences has a denotation in each situation, to evade having to actually come up with embeddings of one situation in another, which would require that we be able to reference entities outside the scope of the referent system.

Let us advance formally by devising a general method for existentially based comparisons of situations. Let t be an information state with m and m' situations in it, and let $\alpha(x)$ be a wff with possibly many free variables but only one which is not defined in the referent system of the information state t. In every sentence belonging to the linguistic context we can identify those entities which are existentially quantified by using again the *Aq()* syntactic predicate used before in chapter four. We then check on each of those existentially quantified variables whether they conform to whatever strict properties are required of them in the *α*-sentence. In order for *m* to be *α*-less than *m'* it must be that every existentially proclaimed entity in it which also conforms to the *α* requirements also does so in *m'*.

(73)

$$\text{Let } t \in S_{Sit}; m, m' \in s; \alpha(x) \in \text{FORMULAS } : m \text{ is } \alpha\text{-less than } m', m \preccurlyeq_{\alpha} m', \textit{ iff}$$
$$\forall \phi \in \langle t \rangle^{\gamma} : \forall q \in \ Aq(\phi) : [\![\phi \wedge \alpha(q)]\!]_m = 1 \Rightarrow [\![\phi \wedge \alpha(q)]\!]_{m'} = 1$$

It may be that the general definition is slightly confusing due to the use of the *α*-sentence. The following definition for *α* in the case of our required 'least eventiveness' should clarify its use and also show that this scheme is indeed usable for our purposes.

(74) Assuming a referent system in which event *e* which is in the progressive is defined, and two primitive predicates on intervals: StartTime() and EndTime(), the α sentence for minimal eventiveness is the following, with x being a free variable not defined in the referent system:

$$event(x) \wedge StartTime(\tau(x)) \geq StartTime(\tau(e)) \wedge EndTime(\tau(x)) \leq EndTime(\tau(e))$$

This *α* sentence makes sure that we only 'count' for our minimality purposes such events that occur during the progressive event *e.*

Combining the above scheme for situation comparisons with the specific *α* sentence required here we get definitions for being *less eventive* and then also *minimally eventive*.

Let $t \in S_{Sit}; m, m' \in t, e \in \langle t \rangle^{r} : m \textit{ is less eventive than } m' \textit{ relative to } e, m \preccurlyeq_{ev(e)} m', \textit{ iff}$

(75) $\forall \phi \in \langle t \rangle^{\gamma} : \forall q \in Aq(\phi):$

$$\left[\!\!\left[\begin{array}{l}\phi \wedge event(q) \wedge \\ StartTime(\tau(q)) \geq StartTime(\tau(e)) \wedge \\ EndTime(\tau(q)) \leq EndTime(\tau(e))\end{array}\right]\!\!\right]_{m} = 1 \Rightarrow \left[\!\!\left[\begin{array}{l}\phi \wedge event(q) \wedge \\ StartTime(\tau(q)) \geq StartTime(\tau(e)) \wedge \\ EndTime(\tau(q)) \leq EndTime(\tau(e))\end{array}\right]\!\!\right]_{m'} = 1$$

Let $t \in S_{Sit}; m \in t, e \in \langle t \rangle^{r} : m \textit{ is minimally eventive in } t$

(76) $\textit{relative to } e, \; minimal(m, e), \textit{ iff}$

$\forall m' \in t: \; m \preccurlyeq_{ev(e)} m' \textit{ or } m \not\preccurlyeq_{ev(e)} m' \textit{ and } m' \not\preccurlyeq_{ev(e)} m$

The partial order defines chains of situations in the information state based on the events they validate. The ordering is not total because two situations are only comparable if every one of the events in one appears also in the other. Thus it may also happen that we get a singleton chain of one situation. Such a situation would obviously be minimally eventive. The events are cross situationally identified only through their description in the sentence ϕ. A situation is minimally eventive if it is the minimal member of a chain. There could easily be more than one minimally eventive situation in an information state.

In order to make use of the minimal eventiveness order within the EDS framework, we need to define an object language operator that will embody it in its semantics. For it to serve as a sort modal test similar to the *might* operator, we need to pass it as argument not only the event relative to which minimally eventive situations are picked, but also a result sentence which must be tested on each of these situations. The following clause should be added to the object language semantics already given at the end of chapter 5.

(77) t[*normally(e, φ)*] $= \left\{ m \in t \,\middle|\, \forall m' \in t : minimal(m', e) \rightarrow [\![\varphi]\!]_{m'} \right\}$

Compare this operator with the other object language modal operators. Applying a modal operator on an information state t may result either with the same state t or with the absurd state. Here too the same choice applies. If every minimal situation m’ (minimal with respect to e) also validates φ then we get our state t back by getting the set of all situations m in t. Otherwise we get the absurd state. Also, like the other modal operators, *normally* is simply transparent when we come to evaluate truth value at the level of the individual possibility.

Dynamic Semantics of the Progressive

At last we have all the ingredients ready at our disposal, to be mixed together into a formal semantics for the progressive. Let us review them: an epistemically modal framework of information states, an operator for minimal eventiveness in an information state, an aspectual calculus model for accomplishments that ties activity and result through causality, a reference time interval, and finally the temporal trace function from events to intervals.

The formalization of the progressive will be refined in several steps. First, assuming that accomplishment description φ is composed of process part ξ and result part ψ, we get

(78) $$t[prog(\varphi)] = t\begin{bmatrix}\text{there is an event } e \text{ of type } \xi \\ \text{going on relative to } RT\end{bmatrix}\begin{bmatrix}\psi \text{ is the result description of } e \\ \text{in every inertia situation}\end{bmatrix}$$

Updating with the progressive operator is actually done in two phases, the activity part and the result part. This is done because the two parts will function in different ways. The activity part must be true at *s*. The result part will function as a test, a filter applied to the updated state resulting either with the same state or with the absurd state. That the activity part must be formalized as an actual change of state results from the fact that activities do entail the truth of the past simple from the past progressive. This also allows us to infer the simple past of activity part of the accomplishment from the progressive past of the accomplishment as a whole. Thus we get ‘she ran’ from ‘she was running a mile’.

The activity part requires us to formalize the 'going on' relation, but we will use the exact same formalization used by Dowty. Apart from that it must introduce in the referent system the event upon which the inertia predicate in the result part hinges.

(79) $$t[prog(\varphi)] = t\begin{bmatrix}\exists e(event(e) \wedge \xi(e) \wedge RT \subset \tau(e) \\ \wedge EndTime(RT) < EndTime(\tau(e))\end{bmatrix}\begin{bmatrix}\psi \text{ is the result description of } e \\ \text{in every inertia situation}\end{bmatrix}$$

The progressive is true if the result of e is truly described by ψ in every one of the inertia worlds. This is exactly the semantics that the modal *normally* operator gives us. It would be tempting to use the following formulation.

(80) $$t[prog(\varphi)] = t\begin{bmatrix}\exists e(event(e) \wedge \xi(e) \wedge RT \subset \tau(e) \\ \wedge EndTime(RT) < EndTime(\tau(e))\end{bmatrix}\begin{bmatrix}normally(e,\ \psi) \\ \end{bmatrix}$$

Unfortunately, this formulation does not take into account the causal relation that is asserted between the activity part ξ and the result part ψ. Writing *normally(e, ψ)* does not preclude the possibility that ψ was true from the beginning regardless of the event's taking place. I therefore resort to making use of φ, the event description as a whole, as the argument for the normality operator. This allows us to leave causality completely unanalyzed: remember, this work is about the progressive.

(81) $$t[prog(\varphi)] = t\begin{bmatrix}\exists e(event(e) \wedge \xi(e) \wedge RT \subset \tau(e) \\ \wedge EndTime(RT) < EndTime(\tau(e))\end{bmatrix}\begin{bmatrix}normally(e,\ \varphi) \\ \end{bmatrix}$$

Some Concrete Examples

OK, let's take it all in again, one step at a time. This time starting with the dynamic semantics in general and only later focusing our attention on minimal eventiveness. Dowty's accomplishments are compound structures made of activity and result. This structure is mirrored in the semantics of their progressive as a sequence of two independent updates. It is crucial that we make this partition, because of the two updates only the first alters the information state, and the second update is merely a test. This reflects the intuition that for it to be true that John is crossing the road, it must be true that John is engaged in some sort of activity now (whatever would count as the activity part of 'crossing the road') but it need not always be the case that John gets to the other side at the end (the result part). The partition into two separate updates is then a formal method for treatment of the imperfective paradox along the lines laid out by Dowty himself.

Indeed, let us formalize the road crossing example to see how the two updates interact. The following lexical semantics stipulation gives a Dowty style compositionality to the accomplishment verb 'cross the road'.

(82) $$\text{Cross(x,y,e)} = \begin{cases} \text{process part } (\xi)\text{:} \begin{cases} WalkDirOtherSide(e) + \\ Ag(x,e) + Theme(y,e) \end{cases} \\ \text{result part } (\psi)\text{:} \quad GetToOtherSide(x,y) \end{cases}$$

(83) i. John is crossing the road

ii. $$\exists e \exists x \exists y \begin{pmatrix} event(e) \wedge object(x) \wedge object(y) \wedge \\ WalkDirOtherSide(e) \wedge John(x) \wedge \\ Ag(x,e) \wedge Road(y) \wedge Theme(y,e) \end{pmatrix} \wedge normally\big(e, Cross(x,y,e)\big)$$

(84) $$t\begin{bmatrix} \exists e \exists x \exists y (event(e) \wedge object(x) \wedge object(y) \wedge \\ WalkDirOtherSide(e) \wedge John(x) \wedge Ag(x,e) \\ \wedge Road(y) \wedge Theme(y,e)) \\ \wedge RT \subset \tau(e) \wedge EndTime(RT) < EndTime(\tau(e)) \end{bmatrix}\begin{bmatrix} normally\big(e, Cross(x,y,e)\big) \end{bmatrix}$$

Turning to minimal eventiveness, I think that enough has been said on its possible applicability as formalization for the NCE constraint in an epistemic framework. Let us now see how it functions in choosing the most 'normal' scenario in the case of John's crossing of the road. Starting with a state of ignorance, suppose we are being told explicitly that John is crossing the road, and suppose further that our linguistic context contains the following thoughts, or snatches of possible events: that John may scratch his ear as he walks across the street, that a truck may come, and that if a truck comes, it may break down before nearing John. This gives us the following context, with '*' signifying explicit utterances. Using a sort of semi formal notation and for simplicity's sake assuming only the existence of events we get:

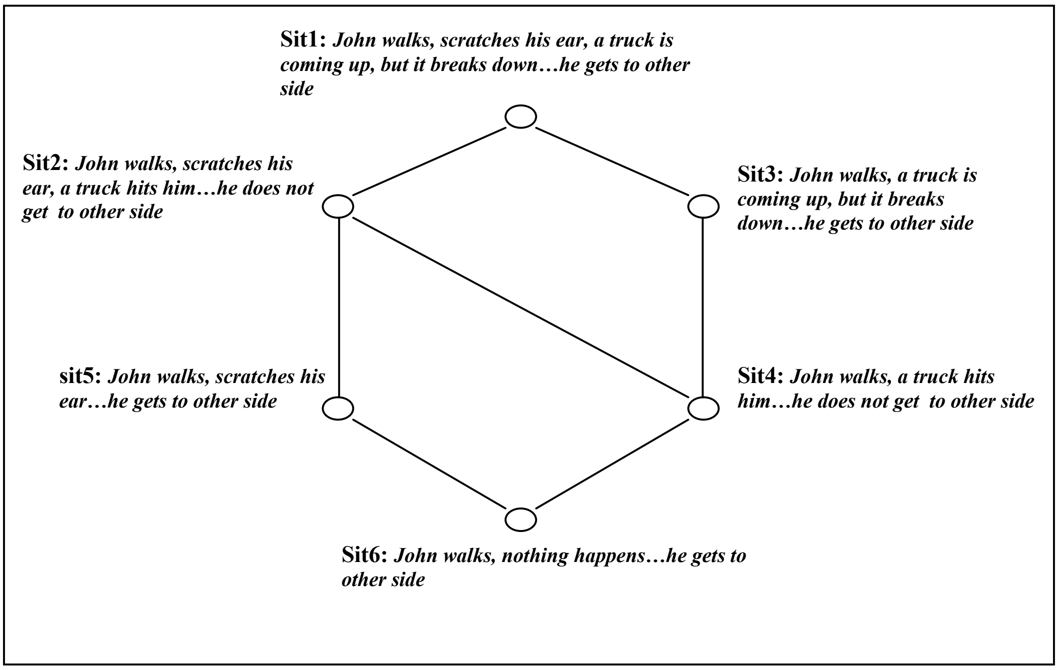
(85)
$$
\begin{aligned}
&a.\exists e_{JohnWalkingDirOtherSideRoad} \wedge time(e) \supseteq RT \ * \\
&b.normally(e, JohnCrossRoad(e)) \ * \\
&c.\exists e'_{JohnScratchHisEar} \wedge time(e') \subset time(e) \wedge time(e') > RT \\
&d.\exists e''_{TruckFlyingDownTheRoad} \wedge time(e'') \subset time(e) \wedge time(e'') > RT \\
&e.\exists e'''_{TruckBreakDown} \wedge time(e''') \subset time(e) \wedge time(e''') > RT
\end{aligned}
$$

Note that the progressive sentence is broken into the two separate updates. Suppose that we now see John steadfastly heading toward the other side of the road, and our update with the activity part of the progressive is successful (i.e. we are not at the absurd state). We now have the following situations in the state, each possibly uniting quite a large number of possible worlds. I have already reduced the possibility of there being a truck breaking down event without there being a truck in the first place.

	John is walking	Truck is coming	John scratches	Truck breaks down	John crosses road
Situation1	T	T	T	T	T
Situation2	T	T	T	F	F
Situation3	T	T	F	T	T
Situation4	T	T	F	F	F
Situation5	T	F	T	F	T
Situation6	T	F	F	F	T

It is important to keep in mind which kinds of information are given to us and which are not. We need not be aware of all the possible worlds participating in the state, but we do know about every situation in it. We need not know of every event that happens in any of the situations, but we do know whether the progressive event, the crossing in our case, is successful in the situation. Moreover, as discussed in chapter 3 when analyzing what inertia may mean, if the event isn't successful then we expect that it is because of another event that has disrupted its projected path, an event that must itself be known.

In terms of the 'degree of eventiveness', if we may call it so, the above situations form the following graph, with the upper situations being more eventive than the lower ones connected to them. Situation 6, where nothing really happens during John's crossing, is the least eventive situation. It is also a situation in which John manages to cross the road, as should be expected: we do not posses any positive knowledge of anything preventing John from crossing[5].



Now if it was positively known that the truck is coming then only situations 1, 2, 3 and 4 would remain in the state, with situation 4 being the least eventive. As it is also a situation where John gets splattered, the progressive sentence fails in such an information state.

[5] (Portner 1998) offers a similar, if less articulated structure of the 'unexpected'. The 'modal base' in his theory is here comprised of the uttered sentences. The 'ordering source' is somewhat equivalent to the partial order here provided by the eventivess order on situations.

Finally, if it was known that something will happen, but not known what, then only situation 6 would get eliminated, and we would get two least eventive situations: one where that something is John’s scratching his ear (situation 5), and the other is one where he meets the truck (situation 4). In this case too the progressive fails, this time because there was no unison decision within the minimally eventive situations jury. It is, admittedly, rather weird. I tend to find Dowty’s original semantics at fault here, because it allows multiple inertia worlds but demands unity between them.

CHAPTER 7
CASE STUDIES, COMPARISON AND CONCLUSION

A host of progressive related puzzles is examined. They are solved uniformly by paying close attention to the available viewpoints in each example. This section concludes with a note on sentences which utilize several different viewpoints simultaneously. Next a limited comparison with circumscription, a form of non-monotonic logic is offered. A short conclusion gives the final words.

Case studies

Treatment of activities

Accomplishments no doubt pose a major obstacle for a theory of the progressive, but no theory will be let off the hook without assuring that activities are also handled properly. Moreover, in a Dowty style theory this should come out ‘for free’ once accomplishments have been handled because of the compositionality relations between the two verb classes. Within the current theory this is indeed so. Since activity verbs are not compounds, our two part update scheme works in the following way. Given an activity verb φ, in the first part a check is made for the existence of an event e encompassing the reference time such that φ is true in e. Once in the second update, a trivial check is made whether the event indeed exists in every inertia world. The check is trivial because the previous update made sure that this will be the case.

Problem of creation verbs

Creation verbs are accomplishments that pose a particularly difficult case for the analysis of the progressive. The reason for this lies with the unclear existential status of the object of creation. Take the following sentence.

(86) Mary is drawing a circle. It will be at least four inches across.

On the one hand, the object of Mary's drawing already exists: it can be referred to and even described. On the other hand it does not yet exist or the imperfective paradox would not come into being. I will term the sense of existence that is entailed by the truth of sentence (86) linguistic existence, because it is most clearly manifest in the linguistic roles that are fulfilled by the object. The sense of existence that is not entailed by (86), the exact same kind of existence that would be entailed had we replaced 'drawing' by 'filling in' or some other kind of activity instead, I will call ontological existence. The names convey a certain intuition, but I would accept any other names just as long as the differences between the two senses of existence are kept in mind.

It appears then that when the progressive aspect is used with a creation verb, linguistic existence is entailed already at reference time, while ontological existence is entailed only upon successful culmination of the verb. Dowty solves this tension by entailing ontological existence together with linguistic existence right from reference time, but only in inertia worlds. The equivalent of this treatment in terms of the current theory would be to stipulate the following lexical semantics for, say, build a house:

(87) $$\text{BuildHouse(x,e)} = \begin{cases} \text{process part } (\xi)\text{: } NailBoards(e) \wedge Ag(x,e) \\ \text{result part } (\psi)\text{: } \exists y Obj(y,e) \wedge House(y) \end{cases}$$

The activity part of building is here trivialized into nailing boards together. Note though that the existence of the event's theme, or object, is not asserted within the process part, but only in the result part. An update with sentence (88) gets interpreted as (89). These semantics resemble those offered by Dowty with respect to the fact that both kinds of existence of the object are only entailed in the inertia worlds.

(88) John was building a house

(89) $$t\begin{bmatrix}\exists e(event(e)\wedge NailBoards(e)\\ \wedge Ag(John,e))\wedge RT\subset\tau(e)\\ \wedge EndTime(RT)<EndTime(\tau(e))\end{bmatrix}\begin{bmatrix}normally\begin{pmatrix}e, process\ part+\\ \exists y(Theme(y,e)\wedge House(y))\end{pmatrix}\end{bmatrix}$$

The second update contains both process and result parts of BuildHouse(x,e). I used the above notation to clarify what exactly is included in the result part. Such a semantics may not be satisfactory because the linguistic existence should be entailed in every world, not just in the inertia worlds. One can continue to refer in discourse to the house that John is building, but with these semantics no related discourse referent exists for it. (Roberts 1996) accounts for similar cases of what she calls 'modal subordination', where the intensional object can only be referred to under the scope of a similar modal operator. She does it by nesting any further use of the discourse referent in a modal environment similar to the one in which it was introduced. Thus, augmenting (88) with (90) would be possible because (90) would be formally interpreted as (91), which re-introduces the referent to the house.

(90) It would have a red roof.

(91) $$t\left[normally\left(e,\begin{Bmatrix}process\ part+\\ \exists y(Theme(y,e)\wedge House(y))\end{Bmatrix}\wedge\exists xRoof(x,y)\wedge Red(x)\right)\right]$$

The solution proposed above coincides then with existing theories. Its account of the ontological non-existence of the theme in a creation verb is plainly satisfactory, but explaining why we still get linguistic existence is a bit of a problem.

I would like to propose a possible alternative to the above explanation. One in which linguistic existence is well accounted for while it is the ontological existence that provides more of a problem. I think that the above semantics is not completely satisfactory because the linguistic existence should be entailed in every world, not just in the inertia worlds, while the ontological existence is only entailed in the inertia worlds. This is what would allow us to refer without any problem to an object whose ontological existence we are not yet willing to accept. Also, filling in the Theme role of many such events is, to my knowledge, obligatory. But in contrast we get a semantics where the theme role is only conditionally provided. I therefore prefer the following formulation. (92) gives the lexical semantics for house building, while (93) shows an update with sentence (88) using the new semantics.

(92) BuildHouse(x,e) = $\begin{cases} \text{process part } (\xi)\text{: } \exists y NailBoards(e) \wedge Ag(x,e) \wedge Theme(y,e) \\ \text{result part } (\psi)\text{: } \; House(y) \end{cases}$

(93) $t\begin{bmatrix} \exists e \exists y(event(e) \wedge Theme(y,e) \wedge NailBoards(e) \\ \wedge Ag(John,e)) \wedge RT \subset \tau(e) \\ \wedge EndTime(RT) < EndTime(\tau(e)) \end{bmatrix}\begin{bmatrix} normally\begin{pmatrix} e, process\ part + \\ House(y) \end{pmatrix} \\ \end{bmatrix}$

What the latter formulation means is that upon a successful update with sentence (88), there exists a discourse referent bound to y, the theme of the event *e.* Nothing is known of this referent in general other than the fact that in every normal course of events it will end up as a house by the building event's end. The update with (93) then further partitions every previously existing situation into at most two new situations: one which is minimal and in it y is a house upon the event's completion, and one which is not minimal and in it y can be anything. In the latter kind of situation we get a multitude of new descendent possibilities: one for each binding of y with an object in the domain of individuals. But they all just have to crowd within their non-minimal situations.

So the proposed explanation does indeed commit itself to a certain type of ontological existence already at the process stage, but it is the minimal commitment possible: that something exists. This commitment pretty much serves as a stub to fill in so that the linguistic existence can be legitimized. Keep in mind that in possible world semantics, the domain of individuals is always static: nothing is created or destroyed in a way that will affect the domain of individuals. We may shift around the dynamic part of our semantics in order to get a closer approximation of language use but as long as we build the dynamic part on top of classical model theory there will always be an ultimate barrier to our explanations: at some very low level of our semantics there is a model with a domain of individuals in which John's house exists eternally. This is true of both of the above accounts for creation verbs.

Problem of interruption

A good number of challenges for any theory of the progressive that takes a Dowty like approach are brought up by (Landman 1992). We start though by repeating Vlach's example of Max's street crossing, first mentioned in chapter 3.

(94) Max is in the middle of the street, heading towards the other side. While unknown to him a speeding bus is just inches away from his body.

(95) Max is crossing the street

The problem of interruption, as it is called by Landman, is that given the scenario portrayed in (94), (95) would be evaluated as true even though it would definitely be normal for the truck to squash Max before he gets to the other side. The current theory, adopting an epistemic base, explains this by making a distinction between the epistemic viewpoint most easily associable with Max as he is crossing and that which is naturally related to all-knowing reader. In the current example the difference is of course that Max's viewpoint does not include the speeding truck. Max's minimal situations therefore are such that nothing happens to him during the cross, and this brings about the truth of (95). We get the correct truth value because of the two viewpoints it is the viewpoint that I call 'Max's' that is more pragmatically salient (more on this below), and so the sentence is interpreted according to it.

Problem of non-interruption

The next puzzle is the problem of non-interruption. It concerns our strong intuition that the simple past entails the past progressive. Thus, given scenario (96), (97) is also true at some reference time during the ordeal.

(96) Mary took off her trainings, did some warm-ups, and jumped to sea at Normandy. She started swimming and eventually dragged herself on shore, in Coney Island.

(97) Mary was crossing the ocean.

Now assuming that crossing the ocean by swimming is next to impossible, how could Mary's successful crossing be considered as a normal course of events? And if it isn't, and a normal course has her eventually drowning, then (97) should come out false – but it doesn't. Once again we can distinguish (at least) two different epistemic viewpoints, or knowledge states, relating to the reference interval. The first may be termed 'Mary's viewpoint at the time'. It does not contain the knowledge of her eventual success. The second viewpoint is, once again, associated with the reader, and does include this knowledge. The truth of (97) is evaluated relative to the reader's knowledge state. As this knowledge state contains the explicitly stated bit of news that Mary had arrived on shore, every possibility must conform to that and therefore so must every situation. In particular those which are minimally eventive.

By now the reader might be questioning the legitimacy of the explanations offered. For the problem of interruption we chose the less informed knowledge state associated with the agent, but for the problem of non-interruption we chose that of the more informed reader. Recall that in chapter 6 we had to resort to pragmatic (extra theoretic) means in order to account for switches between viewpoints. I think that in these particular two cases there were contextual clues that 'suggested', or made salient, a specific viewpoint. In the first case the example is in the present, and the grim future is only implied (though very strongly so). This, I believe, makes it easier to pick the agent's viewpoint. In the second case the whole event has already happened and is done. We have clear knowledge of the outcome, and even the surrounding theoretic explanation suggests that the progressive's truth is based on the simple past – which is only available to the reader and not to the agent involved. All this helps to make the reader's viewpoint a more natural choice. Admittedly, these are rather ad-hoc explanations. I will touch upon this subject again in the next section.

Problems of continuation I

We now come to problems of continuation. The first case of a continuation scenario brought up by Landman is not really different from the interruption problem already discussed, when examined using the current theory. Scenario (98) resurrects Max only to overrun him once again. The difference between (98) and (94) is relevant to theories of the progressive in which normality is accounted for by switching, upon an unexpected or un-normal happening, to the closest possible world in which the unexpected event does not exist. Such a theory would find it hard to account for the truth of (99) because the existence of the second bus suggests that Max will get run over even in the relatively safer possible world.

(98) Max is in the middle of the street, heading towards the other side. While unknown to him a speeding bus is just inches away from his body, and another one is also coming up along the next lane.

(99) Max is crossing the street

With the theory suggested here though, the truth of (99) is assured for as long as we keep taking Max's ignorant viewpoint, as Max does not know about any of the prowling bus drivers. Within the current scenario the quantity of immanent dangers, when they can be counted, does not influence the theory's predictions. This is because from Max's point of view they do not exist, while the reader expects Max to get run over in any case. And this brings us to the second problem of continuation.

Problems of continuation II

The second continuation case is more perplexing. It has been suggested that theories of the progressive can be saved from cases like (98) by repeatedly switching to safer worlds until no unexpected event halts the coming about of the progressive event. This solution however is challenged by scenario (100), which seems to bring about the truth of (101) even though it should come out false.

(100) Mary was just your average teen aged girl, who found herself battling the whole Roman army. She killed off one soldier. She killed off another. At the time of reference she was biting the third soldier's ear. (Extra bit: eventually she was pierced and she died).

(101) Mary was wiping out the Roman army.

If we just keep on removing obstacles by switching possible worlds whenever Mary is about to get killed, we will eventually remove every single soldier in the Roman army, leaving Mary alive and kicking. This would wrongfully make (101) into a true sentence and that, supposedly, is the wrong result.

Within the current theory we can, and should, distinguish between different possible viewpoints before deciding on the required truth value for (101). I think it quite possible that (101) should come out true under certain circumstances, when considering from Mary's own viewpoint. I suspect that in the following sentence Mary's epistemic view is naturally suggested, and is unbalanced enough to support her supposed wiping of the Romans. I also tend to think that it is about as silly as the original sentence (101).

(102) Mary was always a fighter for lost causes. When I last saw her, one summer morning in 100 BC, she was wiping out the Roman army. Needless to say, she was dead by noon.

It must be admitted though that Mary's viewpoint hardly suggests itself in the original (101). The viewpoint that does seem most appropriate is that of the reader, and when taking it, it does seem that (101) is a blatant falsity. It is obviously so when we know that extra bit about her death, but it is also false when we don't. We expect that Mary will never make it through. Part of the obvious falsity of (101) stems, I think, from 'wiping' lexically indicating not only a type of result (great victory) but also a type of activity (continuously being far ahead of your adversary). This comes out clearly when we examine (103), where I added Mary's best friend Sue on Mary's side so that they can be wiped out together as a plural object. If wiping was only about the result there would be no problem with it, but it sounds to me that it is actually infelicitous.

(103) Mary and Sue were wiping out the Roman army when they were wiped out by it.

But even if we stipulate that 'wiping out' is only about the result, i.e. wipe out = balanced battling activity + win result, explaining why (101) is false from the reader's viewpoint is a real challenge. Within a given viewpoint, the progressive is only falsified if the least eventive scenarios are ones where the event fails to complete. This interpretation is inherited from Dowty's original 'least unexpectance' formulation. In contrast we expect that Mary's death will come as an extra event on top of her fighting the soldiers, and as a result we expect that in the least eventive situations Mary will live through. This, alas, goes against our very strong intuition as reasonable bystanders that Mary is about to die.

The reason that we don't get a reasonable answer in this case is that the scenario seems to leave open an information gap that needs to be filled in. It does not insist on Mary's upcoming death but it doesn't leave her much of a chance either, taking into account her being 'just your average teen aged girl'. I contend that this gap does not need to be filled as a part of the operator's semantics, but that it is a gap that pertains between our intuition and the structuring that we (did not) apply to the state's situations. If it is common knowledge that a single person can not win the Roman army then Mary's death is indeed an extra event but it is evenly spread over all of the situations. If you can not envision Mary's exact death but you know that it must be either by crucifixion, by sword or by being dragged around by a speeding chariot then that's ok too. Whatever it is that makes Mary's death into a 'default' must be included in the linguistic context, and through that must affect the structure of the situations so that even the least eventive ones will echo it.

Problems of perspective

It is a little out of place to pose a section thus named when perspective, or viewpoint, plays such a key role in every one of the problems of progressive examined thus far according to my theory. But in the literature it is often brought up as an independent problem, with the following case (originally offered by Landman) shows.

(104) Max is on the plane to Boston, at midair, during reference time. A little while later the plane is hijacked and lands in Bismarck, North Dakota.

(105) Max was flying to Boston

(106) Max was flying to Bismarck, North Dakota

The problem raised is how to explain the fact that both (105) and (106) seem to be true, while their conjunction seems like a contradiction. By now the answer offered by the theory I propose must be obvious. Both (105) and (106) can be true, depending of the knowledge state relative to which they are analyzed. A state where (105) comes out true is one where no knowledge of the hijacking exists. There may be some situations in which the hijacking occurs, but they are not minimally eventive. The minimal situations show a peaceful flight landing at Logan International Airport. States where (106) is true are ones where it is known that the plane landed in Bismarck and every situation, minimally eventive or not, supports this datum.

The problem of the lioness and the zebra, which served as ‘appetizer’ in the first chapter, is of a similar flavouring. Both the lioness and the zebra may be truthfully described as being engaged in contradicting accomplishments, because the sentences describing them are analyzed relative to different knowledge states.

A note on the issue of multiple viewpoints

It is widely acknowledged that sentences in the progressive depend for their truth value on the perspective being taken. In the current theory the notion of viewpoint is most naturally integrated by the use of the epistemic dynamic semantics framework. Equating viewpoint with epistemic knowledge state provides utmost flexibility. I have contended that viewpoints are critical to the semantic evaluation even in the cases shown above, which were not originally construed to bring out the issue of perspective.

But just how flexible and allowing is this mechanism of multiple viewpoints? We have already seen cases where two viewpoints are contextually based on the same scenario but giving different truth values to the progressive. In each case there were at most two views offered, they were associated with the cognizant agents involved, and once a viewpoint was chosen it seemed to cancel out the possibility of another one simultaneously applying too. The following examples show that none of these restrictions is universal.

(107) Max was driving home unattentively and Mary was crossing the road blindfolded when they crashed together.

(108) The wheel was rolling across the road when it was knocked over by the falling rock.

(109) Max was climbing the tree while the bear was pulling him off

In the first example the viewpoints of Max, Mary and the reader clearly differ from each other. In the second example, originally pointed out by (Asher 1992), there is no cognizant agent to whom we can attribute the assumption that the wheel will roll across the road. Even the reader can not be considered as suitable candidate because she already knows that it will be knocked over by the rock.

As mentioned, my theory offers flexibility in choosing the relevant viewpoint by equating epistemic states with perspective. First a viewpoint is chosen and then the progressive sentence is analyzed according to the chosen view. The last example, adapted from (Thomason 1995) , shows however that sometimes two contradicting viewpoints must simultaneously co appear in the same sentence, under the same evaluation. Choosing only Max's viewpoint or only the bear's would not enable us to ever treat (109) as a true sentence, contrary to intuition. This is also brought out in the first example, but I find that the bear example makes for a very convincing scenario. The usage of multiple viewpoints in the very same sentence could, in principle, be accommodated into the theory if we allowed reference to explicit viewpoints within our object language.[6] As I have already mentioned the DS framework can be extended into a multimodal framework, and this sort of extension seems to me to be very promising for semantic treatment of multiple viewpoints.

Comparison with circumscription

The principle of minimality which was employed to achieve the set of inertia situations no doubt brings to mind the non-monotonic circumscription technique introduced in (McCarthy 1980) and (McCarthy 1986). Indeed I suspect that the exact notion of minimality that is employed in my theory could also be expressed as a specific case of circumscription. Nevertheless how exactly to formulate minimal eventiveness by using circumscription is not obviously apparent to me, and of course even if we had the equivalent circumscriptive formulation, it would still be dependent on the situation structures.

Circumscription is defined as a syntactic operation on a second order formula A(P) where P is a predicate constant in the language, which transforms it the formula into the following formula CIRC[A,P], which is also second order.

(110) $CIRC[A,P] := A(P) \wedge \forall p[A(p) \wedge [p \rightarrow P] \rightarrow [p \equiv P]]$

The lower case p stands for a second order variable on predicates, and the equivalence sign as mutual entailment. Circumscription brings about minimization by adding a clause which ensures that the extension of P will not include objects not necessitated by A(P). The above formulation is actually a simplified version of

[6] There are two problems involved. One is to exactly predict which viewpoints would be contextually salient for a given sentence. The other is being able to keep track of the existing viewpoints within the semantics of the progressive theory. The theories of (Asher 1992), (Glasbey 1996), (Bonomi 1997) and (Portner 1998) all offer the notion of viewpoints but do not tackle the above mentioned problems.

circumscription. The complete formalization introduces varying parameters and, more importantly, circumscription on a formula.

(111) $CIRC[A,E(P,x)] := A(P) \wedge [\forall x E(p,x) \rightarrow E(P,x)] \rightarrow [\forall x E(p,x) \equiv E(P,x)]]$

E(P,x) is a second order formula with P being a free predicate symbol in E (it can also signify a tuple of such predicates) and x being a tuple of free first order variables in E. Circumscription proceeds in much the same way except that the specified variables are allowed to vary in value under the entailed interpretations, and of course it is the formula as a whole and not just the predicate that must have a minimal extension.

Predicate extensions are a precursor to the model theoretic interpretation of circumscription. In the simplified case of a predicate P being circumscribed, a partial order is defined on the models of A(P) such that $M_1 \leq^P M_2$ iff the domains of M_1 and M_2 are identical, the interpretations that they give to every constant C which is not P are identical, and $[\![P]\!]_{M_1} \subseteq [\![P]\!]_{M_2}$. It can be shown that the M is a model of CIRC[A,P] iff it is a minimal element in the partial order $\leq^P$ on the models of A(P). I do not know how exactly the model theoretic interpretation generalizes to the more complicated case of formula circumscription, given in (111).

Now one evident difference between model theoretic circumscription and the minimalization offered in my theory is that the minimalized structures here are situations and not models. This makes it difficult to define the ordering relation in circumscriptive terms. What exactly is the ‘domain’ of a situation, needed in order to define the partial order? Next, I think that we could use for our ‘theory’ A the conjunction of all explicitly uttered sentences, but what of the second order formula E? It would somehow be related to the set of linguistic context sentences and the minimal eventiveness definition, but I am not sure how to precisely define it.

Conclusion

We have come to the end of our long road. In this thesis three separate topics were discussed. First, I argued in favor of switching from the ontological modal base on which Dowty's theory rests into an epistemic one. The motivation for this switch was shown to be the inner tension that exists between the two demands that make up the intuitive notion of inertia: identity up to a point (IUP) and normal course of events (NCE). Not only does the epistemic base resolve these tensions by allowing partial epistemic viewpoints but it also makes it possible to further investigate the up until now amorphous NCE. A key trait of this demand was said to be context dependence, which defines the granularity of our concepts and envisioned scenarios.

Second, trying to cash in on this understanding we moved from philosophical debating to formal logic. Using the framework of dynamic semantics introduced by (Groenendijk, Stokhof and Veltman 1996) as infrastructure I introduced situations as sets of indiscernible possibilities in the information state. Situations are built from the start to be context dependent. They structure our uncertainty, embodied in the multitude of possibilities that reside under the information state, by partitioning these possibilities into maximal groups of possibilities which are indiscernible under the current context. Thus the more detailed the context, the finer grained the situations become. Not only does the context define the situations which exist, but it also provides us with cross situation identity. This identity is utilized in formalizing the last topic which is discussed here.

In the last point brought up in this thesis I argued that an important factor in the notion of normality, the idea that in a normal world nothing unexpected happens, can be accounted for formally by ordering situations under a certain type of partial order: the order of their eventiveness. The minimal situations under this ordering, the least eventive situations, thus provide us with the set of 'inertia worlds' initially sought by Dowty. And the partial order of eventiveness is licensed by that same sort of cross situation identity that is given with the very notion of situation. Using this account of normality I proceeded to provide a Dowty style semantics of the progressive within the dynamic semantics context.

As for the relative success of the theory presented to resolve the progressive puzzles, the epistemic approach taken in this theory can handle the puzzles but provides results that are different from those of (Landman 1992). It is perhaps closer in its results to those suggested in (Asher 1992). On the one hand, the use of viewpoints provides uniform analysis for many of the puzzles, but on the other it is very hard to determine a progressive sentence as false if one only has to come up with a different viewpoint in order to justify its truth. The task of determining which viewpoints are salient has been relegated to the pragmatic context and this goes against the intuitions of some language users.

In this thesis I attempted not so much to give answers to as yet unsolved linguistic puzzles as to provide the very same explanatory power based on fewer assumptions. And with 'fewer assumptions' I principally mean getting rid of Dowty's primitive normality function, which recurs under various guises in other works too. Instead of postulating the existence of such a primitive, I claim to have derived normality from the current linguistic context. Of course there still exists an element of fuzziness in the theory. It is in the exact characterization of the linguistic context within which normality is calculated that an element of primitive fuzziness has been left. Indeed, it seems natural enough to me to suppose that that is where such fuzziness belongs.

The theory presented here makes allowances for the existence of more than one viewpoint within the same sentence, but like other modern theories it does not attempt to predict which viewpoints will be salient at a given point in the discourse. Nor can the epistemic structures defined here maintain more than one viewpoint simultaneously. I think though that the dynamic epistemic paradigm allows for much flexibility in this respect and that by extending the formal backbone into a multi modal framework it may be possible to incorporate multiple viewpoints within the semantics. Such an extension would be a major advancement as no theory of the progressive known to me can achieve such a task today.

It was the extension of DS with situations, to create EDS, which allowed me to reduce normality to context dependence. As mentioned, this is done by ordering the situations based on their degree of eventiveness. I strongly believe that situations can be used to explore and to provide explanations in other areas of semantics as well. I have already mentioned in passing the issues of cross world identity, which spring up with multiple domains. Another natural area for investigation would be counterfactual reasoning. Still a third possibility for further investigation may be the phenomenon of modal subordination as presented in (Roberts 1996).

APPENDIX A
DISCERNIBILITY BASED ON POSSIBILITY TRUTH VALUES

Another way to define non-discernibility would be by using the well defined notions of truth and falsity in the possibility's world, instead of reaching up for information states. There are three difficulties involved with this approach.

The first one is that modality in DS is not defined in terms of conditions on possibilities, but on states. So there really is no way to translate the DS modal operator into our sub-world predicate logic semantics. Yet as our objective is to define situations as sets of non-discernible possibilities, this same fact also helps us out: two possibilities are not to be modally discernible. What we need to make sure of though is that situations will be sensitive, or discernible, based on the phrases embedded within the modal sentences of the linguistic context. So the first problem is solved brutally, by simply filtering all but the modal operators when we move from state semantics to possibility semantics. The only clause of importance in the following definition is the last one, which erases the modal operator.

(112) The non modal form of a sentence $\varphi \in$ FORMULAS, NonModal(φ), is a one place operation on FORMULAS, recursively defined as follows:
NonModal(Rt1,...,tn) = Rt1,...,tn
NonModal(t1=t2) = (t1=t2)
NonModal($\neg\varphi$) = $\neg$ NonModal(φ)
NonModal($\varphi \wedge \psi$*) = NonModal(*φ*)* $\wedge$ *NonModal(*ψ*)*
NonModal($\exists x\ \varphi$*) =* $\exists x$ *NonModal(*φ*)*
NonModal($\Diamond\psi$) = NonModal(ψ)

The second difficulty is that in the DS framework no recursive definitions for the interpretation of a sentence under a possibility are explicitly provided. Recall that only the interpretation rules for basic expressions were explicitly given. But as worlds are just first order models, this problem has a natural solution if we extend the interpretation rules with the usual recursive definitions for truth in a model under an assignment function. A sentence may be said to be true or false in a possibility if it is true or false in the possibility's world under the assignment function defined by its referent set and peg assignment functions. Extending the original dynamic semantics definition for denotation in a possibility we get a temporary definition for the interpretation function.

(113) Let α be an expression; i=<r,g,w>, $i \in I$, with v the domain of r and I based on D and W. The denotation of α in i, $i(\alpha)$, is defined based on previous rules and now also the following:
If α is a sentence then the denotation of α in i, $i(\alpha)$ is defined as follows:

$$i(\alpha)=\begin{cases}1 & [\![\alpha]\!]_{<i>^w}^{<i>^g(<i>^r)}=1\\ 0 & [\![\alpha]\!]_{<i>^w}^{<i>^g(<i>^r)}=0\\ \textit{undefined} & \text{otherwise}\end{cases}$$

The third looming complication is that while in DS quantifiers can bind outside of their syntactic scope, this does not also happen in the classical predicate logic interpretation that we are using for truth in a possibility. Due to this complication many potentially meaningful sentences in a possibility, such as $\exists x P(x) \wedge Q(x)$, would not be given a defined truth value due to the second conjunct's using a variable out of its quantifier's scope. To overcome this we can adapt from (Groenendijk and Stokhof 1991) a syntactic form that merges into one the binding scope and the syntactic scope of quantifiers, while maintaining truth value.

(114) The normal binding form of a sentence $\varphi \in FORMULAS$, $Nbf(\varphi)$, is a one place operation on *FORMULAS*, recursively defined as follows:
Nbf(Rt1,...,tn) = Rt1,...,tn
Nbf(t1=t2) = (t1=t2)
Nbf(¬φ) = ¬Nbf(φ)

$$Nbf(\varphi \wedge \psi) = \begin{cases} Nbf(\chi \wedge (\phi \wedge \psi)) & \varphi = \chi \wedge \phi \\ (\exists x Nbf(\chi \wedge \psi)) & \varphi = \exists x \chi \\ Nbf(\varphi) \wedge Nbf(\psi) & \text{otherwise} \end{cases}$$

$$Nbf(\varphi \rightarrow \psi) = \begin{cases} Nbf(\chi \rightarrow (\phi \rightarrow \psi)) & \varphi = \chi \wedge \phi \\ (\forall x Nbf(\chi \rightarrow \psi)) & \varphi = \exists x \chi \\ Nbf(\varphi) \rightarrow Nbf(\psi) & \text{otherwise} \end{cases}$$

$Nbf(\forall x\ \varphi) = \forall x\ Nbf(\varphi)$

$Nbf(\exists x\ \varphi) = \exists x\ Nbf(\varphi)$

The normal binding form that results from applying Nbf on any sentence in the language of DS is again a sentence in that language. Only now every variable that is bound to a quantifier appearing in the sentence, itself appears only within the syntactic scope of the quantifier. Thus $Nbf(\exists x P(x) \wedge Q(x))$ is $\exists x(P(x) \wedge Q(x))$, and $Nbf(\exists x P(x) \rightarrow Q(x))$ is $\forall x(P(x) \rightarrow Q(x))$.

We would not have any use of these transformations were it not for the equivalence that is maintained by them.

(115) **Theorem A1:** Truth value equivalence of sentences and their normal binding form counterparts:

For any non modal sentence φ in $FORMULAS$: $s[\varphi] \equiv s[Nbf(\varphi)]$

Proof:
The proof is by induction on the length of the sentence, with only conjunction and implication providing any difficulty (by providing a difference between the the source and target sentence).

In conjunction the equivalence rests on the following two basic equivalences:

i. $(\varphi \wedge \psi) \wedge \chi \equiv \varphi \wedge (\psi \wedge \chi)$

$s[(\varphi \wedge \psi) \wedge \chi] = s[\varphi \wedge \psi][\chi] = s[\varphi][\psi][\chi] = s[\varphi][\psi \wedge \chi] = s[\varphi \wedge (\psi \wedge \chi)]$

ii. $\exists x\varphi \wedge \psi \equiv \exists x(\varphi \wedge \psi)$

$s[\exists x\varphi \wedge \psi] = s[\exists x\varphi][\psi] = (\bigcup_{d \in D} s[x/d][\varphi])[\psi] \equiv \bigcup_{d \in D} (s[x/d][\varphi][\psi])$ (because an update with a non modal sentence is just a pointwise application of updates on its possibilities) $= \bigcup_{d \in D} (s[x/d][\varphi \wedge \psi]) = s[\exists x(\varphi \wedge \psi)]$

In implication the equivalence rests on these two basic equivalences:

iii. $(\varphi \wedge \psi) \rightarrow \chi \equiv \varphi \rightarrow (\psi \rightarrow \chi)$

$s[(\varphi \wedge \psi) \rightarrow \chi] = \{i \in s \mid \text{i does not subsist in } s[\varphi \wedge \psi]\} \cup \{i \in s \mid \text{i subsists in } s[\varphi \wedge \psi][\chi]\}$

$= \{i \in s \mid \text{i does not subsist in } s[\varphi]\} \cup \{i \in s \mid \text{i subsists in } s[\varphi] \text{ but not in } s[\varphi][\psi]\} \cup$
$\{i \in s \mid \text{i subsists in } s[\varphi][\psi][\chi]\}$

$= \{i \in s \mid \text{i does not subsist in } s[\varphi]\} \cup \{i \in s \mid \text{i subsist in } s[\varphi][\psi \rightarrow \chi]\}$

$= \{i \in s \mid \text{i subsists in } s[\varphi \rightarrow (\psi \rightarrow \chi)]\} = s[\varphi \rightarrow (\psi \rightarrow \chi)]$

iv. $\exists x\varphi \rightarrow \psi \equiv \forall x(\varphi \rightarrow \psi)$

$s[\exists x\varphi \rightarrow \psi] = \{i \in s \mid \text{i does not subsist in } s[\exists x\varphi]\} \cup \{i \in s \mid \text{i subsists in } s[\exists x\varphi][\psi]\}$

$= \{i \in s \mid \forall d \in D\text{: i[x/d] does not subsist in } s[\varphi]\} \cup \{i \in s \mid \forall d \in D\text{: i[x/d] subsists in } s[\varphi \wedge \psi]\}$
(due to equivalence ii)

$= \{i \in s \mid \forall d \in D\text{: i[x/d] subsists in } s[\varphi \rightarrow \psi]\} = s[\forall x(\varphi \rightarrow \psi)]$

We can now provide a final version of the interpretation function for sentence denotations in a possibility. The next function incorporates the modal stripping and the translation into normal binding form. Based on the interpretation function a second notion of non-discernibility falls into place.

(116) Let α be an expression; *i=<r,g,w>, i*∈ *I*, with *v* the domain of *r* and *I* based on D and W. The denotation of α in *i*, *i(α)*, is defined based on previous rules and now also the following:

If *α* is a sentence then the denotation of *α* in *i*, *i(α)* is defined as follows:

$$i(\alpha) = \begin{cases} 1 & [\![Nbf(NonModal(\alpha))]\!]_{<i>^{w}}^{<i>^{g}(<i>^{r})} = 1 \\ 0 & [\![Nbf(NonModal(\alpha))]\!]_{<i>^{w}}^{<i>^{g}(<i>^{r})} = 0 \\ \mathit{undefined} & \text{otherwise} \end{cases}$$

APPENDIX B
PROOFS OF THEOREMS IN CHAPTER 5

Theorem 1

Theorem:
Let $i \in I$ *be a possibility*, $i = \langle r, g, w \rangle$, *and let* $\gamma \in \Gamma$ *be linguistic context.*
For every $\varphi \in \gamma$: $i(\varphi) \leftrightarrow cons(i,\varphi)$

Proof:
The proof proceeds in stages. First I will show that the two notions are defined for exactly the same set of sentences. Then I will prove equivalence for the set of non modal, Nbf sentences which forms a subset of FORMULAS, after which I will generalize for non-Nbf, and finally for modal non-Nbf.

Range of defined sentences:
$i(\varphi)$ *is undefined iff* $\exists v \in Fv(\varphi)$ *s.t* $v \notin Dom(r)$. *Which is true iff* $\forall s \in S$ *s.t* $i \in s$, $s[\varphi]$ *is undefined. Which is true iff consistency is not defined for* $\langle i, \varphi \rangle$

For non modal, Nbf sentences: by induction on sentence length

$\varphi = Rt_1,...,t_n : i(\varphi) = 1$ *iff* $[\![Rt_1,...,t_n]\!]_w^{g \circ r} = 1$. *By def. true iff* $\langle g \circ r(t_1),...g \circ r(t_n) \rangle \in w(R)$.
Itself true iff $\forall s \in S$ *s.t* $i \in s : i \in s[Rt_1,...,t_n]$. *Which is true iff* $cons(i,\varphi)$

$\varphi = (t_1 = t_2) : i(\varphi) = 1$ *iff* $[\![t_1 = t_2]\!]_w^{g \circ r} = 1$. *By def. true iff* $g \circ r(t_1) = g \circ r(t_n)$.
Itself true iff $\forall s \in S$ *s.t* $i \in s : i \in s[t_1 = t_2]$. *Which is true iff* $cons(i,\varphi)$

$\varphi = \neg\phi : i(\varphi) = 1$ *iff* $[\![\neg\phi]\!]_w^{g \circ r} = 1$. *By def. true iff* $[\![\phi]\!]_w^{g \circ r} = 0$. *By induction hyp. this is true iff it is not the case that* $cons(i,\phi)$. *Which is true iff* $\exists s \in S$ *s.t* $i \in s$ *for which* $\neg\exists i^* \in s[\phi] : i^*$ *is similar to an extension* i' *of* i.
(*this means that* i *does not have any descendent* i' *in* $s[\phi]$ *or its descendent would serve as the required* i^*). *Which is true, in other words, iff* $\forall s \in S$ *s.t* $i \in s : \neg\exists i^* \in s[\phi] : i^*$ *is similar to an extension* i' *of* i.
But this is true iff $\forall s \in S$ *s.t* $i \in s : \exists i^* \in s[\neg\phi] : i^*$ *is similar to an extension* i' *of* i.
Which, in turn, is true iff $cons(i,\varphi)$.

$\varphi = \phi \wedge \psi : i(\varphi) = 1$ *iff* $[\![\phi]\!]_w^{g\circ r} = 1$ *and* $[\![\psi]\!]_w^{g\circ r} = 1$. *Which is true, by induction hyp., iff* $cons(i,\phi)$ *and* $cons(i,\psi)$. *Note however, that this is true iff* $\forall s \in S$ *s.t* $i \in s$: $\exists i^* \in s[\phi]$ *s.t* i^* *is similar to an extension* i' *of* i, *and* $\exists i^{**} \in s[\psi]$ *s.t* i^{**} *is similar to an extension* i'' *of* i. *But in Nbf the set of active quantifiers of* $\phi = \varnothing$ *so so the extension is trivial and* i^* *is actualy similar to* i. *We get then that* $\forall s \in S$ *s.t* $i \in s$: $\exists i^* \in s[\phi]$ *s.t* i^* *is similar to* i, *and* $\exists i^{**} \in s[\psi]$ *s.t* i^{**} *is similiar to an extension* $i^{*\prime}$ *of* i^*. *This, in turn, is true iff* $\forall s \in S$ *s.t* $i \in s : \exists i^{**} \in s[\phi \wedge \psi] : i^* \leq i^{**}$ *but* i^{**} *is similar to an extension* i'' *of* i. *This being true iff* $cons(i,\varphi)$.

$\varphi = \exists x \phi : i(\varphi) = 1$ *iff* $\exists d \in D$ *s.t.* $[\![\phi]\!]_w^{g\circ r_x^d} = 1$. *By induction hyp. this is true iff* $cons(i[x/d],\phi)$. *This, by def, is true iff* $\forall s \in S$ *s.t* $i \in s : \exists i^* \in s[x/d][\phi]$ *s.t* i^* *is similar to an extension* i' *of* i. *And this, in turn, is true iff* $\forall s \in S$ *s.t* $i \in s : \exists i^* \in \bigcup_{d \in D} s[x/d][\phi]$ *s.t* i^* *is similar to an extension* i' *of* i. *Which is true iff* $cons(i,\varphi)$.

For non modal sentenes of a general form:
$i(\varphi) = 1$ *iff* $[\![Nbf(\varphi)]\!]_w^{g\circ r} = 1$ (*by definition of* $i(\varphi)$). *The latter being true iff* $\exists i^* \in s[Nbf(\varphi)]$ *s.t* i^* *is similar to an extension* i' *of* i (*by above proof for Nbf sentences*). *This in turn is true iff* $\exists j^* \in s[\varphi]$ *s.t* j^* *is similar to* i^* (*by equivalence of* j *and* $Nbf(j)$). *Which is true iff* $\exists j^* \in s[\varphi] : j^*$ *is similar to an extension* i' *of* i (*by similarity being an equivalence*). *Which is true iff* $cons(i,\varphi)$.

For any sentence in FORMULAS:
By induction on sentence length, the only case which is not trivial is the following:
$\varphi = \Diamond \phi$:
$i(\varphi) = 1$ *iff* $[\![Nbf(NonModal(\varphi))]\!]_w^{g\circ r} = 1$ (*by definition of* $i(j)$). *Which is true iff* $[\![NonModal(\varphi)]\!]_w^{g\circ r} = 1$ (*by equivalence of Nbf and regular notation (Appendix A)*). *And that is true iff* $[\![\phi]\!]_w^{g\circ r} = 1$ (*by the* $\Diamond$ *clause in the definition of NonModal*). *Which is true iff* $cons(i,\phi)$ (*by induction hyp.*). *Now* $cons(i,\phi)$ *iff* $\forall s \in S$ *s.t* $i \in s$: $\exists i^* \in s[\phi]$ *s.t* i^* *is similar to an extension* i' *of* i (*by def of cons*). *The latter bein true iff* $\forall s \in S$ *s.t* $i \in s : s[\phi] \neq \varnothing$, *which is true iff* $\forall s \in S$ *s.t* $i \in s : s[\Diamond \phi] = s$ (*by semantics of* $\Diamond$). *Which is true iff* $\forall s \in S$ *s.t* $i \in s : \exists i^* \in s[\Diamond \phi]$ *s.t* i^* *is similar to an extension* i' *of* i (*the only way for it not to be so would be if* $s[\phi] = \varnothing$, *but this option is cancelled*). *The last sentence being true iff* $cons(i,\varphi)$.

Theorem 2

Theorem:
Let $i, i' \in I,\ \gamma \in \Gamma$. *Then*
i *and* i' *are non discernible*$_1$ *in* $\gamma \iff i$ *and* i' *are non discernible*$_2$ *in* γ.

Proof :
$indicernible_1 \Rightarrow indicernible_2$:
$i(\varphi) = 1 \Leftrightarrow (by\ th.\ 1)\ Cons(i, \varphi) \Leftrightarrow (by\ assumption)\ Cons(i', \varphi) \Leftrightarrow (by\ th.\ 1)\ i'(\varphi) = 1$

$indicernible_2 \Rightarrow indicernible_1$:
$Cons(i, \varphi) \Leftrightarrow (by\ th.\ 1)\ i(\varphi) = 1 \Leftrightarrow (by\ assumption)\ i'(\varphi) = 1 \Leftrightarrow (by\ th.\ 1)\ Cons(i', \varphi)$

Theorem 3

Theorem :
Let $m, m' \in Sit,\ m = < J,\ \gamma >, m' = < J',\ \gamma >$.
m *and* m' *are indiscernible in* γ *iff* $\exists i \in m : \exists i' \in m'$: i *and* i' *are indiscernible in* γ

Proof :
$\Rightarrow$ (m *and* m' *are indiscernible in* γ):
$J \neq \varnothing$ *so* $\exists i \in J$. $J' \neq \varnothing$ *so* $\exists i' \in J'$. m *and* m' *are indiscernible in* γ,
so $\forall \varphi \in \gamma : [\![\varphi]\!]_m = [\![\varphi]\!]_{m'}$. *For the possibilities* i *and* i' *this means that*
$\forall \varphi \in \gamma : i(\varphi) = [\![\varphi]\!]_m = [\![\varphi]\!]_{m'} = i'(\varphi)$. *Which gives us the second definition for possibility indiscernibility.*
$\Leftarrow$ ($\exists i \in m$ *and* $\exists i' \in m'$ *such that* i *and* i' *are indiscernible in* γ):
in m *we have* $\forall \varphi \in \gamma : \forall i^* \in m : i^*(\varphi) = i(\varphi)$, *so* $\forall \varphi \in \gamma : [\![\varphi]\!]_m = i(\varphi)$
and the same goes for m': $\forall \varphi \in \gamma : \forall i^{**} \in m' : i^{**}(\varphi) = i'(\varphi)$, *so* $\forall \varphi \in \gamma : [\![\varphi]\!]_{m'} = i'(\varphi)$
Put them together and you get :
$\forall \varphi \in \gamma : [\![\varphi]\!]_m = i(\varphi) = i'(\varphi) = [\![\varphi]\!]_{m'}$, *which gives us situation indiscernbility.*

Theorem 4

Let m,m’ $\in$ Sit; m=<J,γ>, m’=<J’,γ’>:

Def1: m' is an extension$_1$ of m, $m \leq_1 m'$, iff
$\gamma \subseteq \gamma'; \langle m \rangle^r \leq \langle m' \rangle^r; \forall i' \in J' : \exists i \in J : \langle i' \rangle^w = \langle i \rangle^w;$
$\forall \varphi \in \gamma : [\![\varphi]\!]_m = [\![\varphi]\!]_{m'}$
Def2: m' is an extension$_2$ of m, $m \leq_2 m'$, iff
$\gamma \subseteq \gamma'; \forall i' \in J' : \exists i \in J : i \leq i'$

Theorem: $m \leq_1 m' \Leftrightarrow m \leq_2 m'$
Proof:

$\Rightarrow$:

$\gamma \subseteq \gamma'$ *by def.*
Let $i' \in J'$ $(i' = <r', g', w'>)$. *By def. of* $\leq_1$ $\exists i \in J : \langle i \rangle^w = \langle i' \rangle^w$ *(let* $i = <r', g, w'>$*).*
If $g \subseteq g'$ *then we are done because* $i \leq i'$. *Else let* $g^* = g' \upharpoonright Rng(r)$ *and let* $i^* = <r', g^*, w'>$.
This means that $i^* \leq i'$ *(note* $g^* \subseteq g'$*). Now* $\forall \varphi \in \gamma$, $i(\varphi)$ *is defined (*γ *is the linguistic context of situation m containing i), and therefore* $i^*(\varphi)$ *is also defined (same r).*
Moreover $\forall \varphi \in \gamma : i^*(\varphi) = i'(\varphi)$ *(*φ *uses the subset of variables also defined for i*).*
Hence i^* *and i are indiscernible in* γ, *and therefore* $i^* \in \langle m \rangle^J$ *by def. of J. So we get that* $\exists i^* \in J : i^* \leq i'$.

$\Leftarrow$:

$\gamma \subseteq \gamma'$ *by def.*
Let $i' \in J'$ $(i' = <r', g', w'>)$. *By def of* $\leq_2$ $\exists i \in J$ $(i = <r, g, w>)$ *s.t* $i \leq i'$. *This means that* $w = w'$, $r \leq r'$ *and* $g \subseteq g'$ *in such a way so that* $g' \upharpoonright Rng(r) = g$. *Hence* $\forall v \in Dom(r) : i(v) = i'(v)$, *which means that* $\forall \varphi \in \gamma : i(\varphi) = i'(\varphi)$ *(*$\varphi \in \gamma$ *uses only variables defined in r). Therefore* $\forall \varphi \in \gamma : [\![\varphi]\!]_m = i(\varphi) = i'(\varphi) = [\![\varphi]\!]_{m'}$

Theorem 5

$Let\ m = \langle J, \gamma \rangle \in Sit,\ \varphi \in FORMULAS,\ and\ CG\ the\ context\ growth\ pattern.$
$m\{\varphi\},\ the\ result\ of\ linguistic\ context\ update\ with\ \varphi\ to\ the\ situation\ m,$
$is\ defined\ as\ follows:$

$$m\{\varphi\} = \{m_1, m_2, ..., m_n\}\ where: \begin{cases} \langle m \rangle^J = \bigcup_{l=1..n} \langle m_l \rangle^J \\ \forall l = 1..n : \langle m_l \rangle^J = CG(\gamma, \varphi) \\ \forall l \neq k \in \{1..n\} : m_l\ and\ m_k\ are\ discernible\ in\ CG(\gamma, \varphi) \end{cases}$$

Theorem: The linguistic context update operation {} is a well defined function.
The function provides a partition on the set of possibilities in the original situation.

Proof
Existence: we can construct such a sequence of situations that will comply with the requirements.
First, we create of each possibility i in $\langle m \rangle^J$ *a singleton situation* $\langle \{i\}, CG(\gamma, \varphi) \rangle$.
Now we compare every pair of situations to see if they are discernible under $CG(\gamma, \varphi)$.
If there are no indiscernible pairs then we are done because every possibility in $\langle m \rangle^J$ *is accounted for in one of the situations, all the situations have* $CG(\gamma, \varphi)$ *as context, and they are pairwise discernible.*
If we do find a pair of indiscernible situations, say $\langle \{i\}, CG(\gamma, \varphi) \rangle$ *and* $\langle \{j\}, CG(\gamma, \varphi) \rangle$, *we merge them onto one situation* $\langle \{i, j\}, CG(\gamma, \varphi) \rangle$. *Note that by this operation we can still account for every possibility in* $\langle m \rangle^J$, *and that every situation still has* $CG(\gamma, \varphi)$ *as context.*
We continue comparing and merging until no two situations are indiscernible. The process is guaranteed to end because we start out with situations $\left| \langle m \rangle^J \right|$ *and the number of situations drops by one at each merging operation, so after at most* $\left| \langle m \rangle^J \right|$ *steps we will be left with only one situation, which surely can no longer be merged with anything.*

uniqueness: *suppose there exist* $\{m_1,m_2,...,m_n\} \neq \{l_1,l_2,...,l_k\}$ *such that* $m\{\varphi\}$ *can be either* $\{m_1,m_2,...,m_n\}$ *or* $\{l_1,l_2,...,l_k\}$. *From* $\{m_1,m_2,...,m_n\} \neq \{l_1,l_2,...,l_k\}$ *we can derive WLOG that* $\exists m' \in \{m_1,m_2,...,m_n\}$ *s.t.* $m' \notin \{l_1,l_2,...,l_k\}$.
m' *is a situation so* $\exists i \in \langle m' \rangle^J$. *If* $\exists l' \in \{l_1,l_2,...,l_k\}$ *s.t.* $i \in \langle l' \rangle^J$ *then* $\forall i' \in \langle m' \rangle^J$, $i' \in \langle m \rangle^J$ *so there must be some* l'' *s.t.* $i' \in l''$. *Because* i *and* i' *are indiscernible* (*they both belong to the same situation* m') *then* l' *is identical with* l'' (*or we would have two indiscernible situations in* $\{l_1,l_2,...,l_k\}$). *So we get* $\exists l' : \forall i \in \langle m' \rangle^J : i \in l'$.
And the same goes the other way around (*once we choose* l'): $\forall i \in \langle l' \rangle^J : i \in m'$.
This means that $\langle m' \rangle^J = \langle l' \rangle^J$, *and by def* $\langle m' \rangle^\gamma = \langle l' \rangle^\gamma = CG(\gamma,\varphi)$, *which gives us* $m' = l'$. *Contradiction to assumption*, *and therefore* $\{m_1,m_2,...,m_n\} = \{l_1,l_2,...,l_k\}$.

Partition:
That $\langle m \rangle^J = \bigcup_{l=1..n} \langle m_l \rangle^J$ *is already given us by the definition of* $\{\}$.
That for no $1 \leq l \leq n$ *is* $\langle m_l \rangle^J = \varnothing$ *is also given us*, *by the definition of situations*.
We only need to further show that for every $1 \leq l \leq k \leq n$ $\langle m_l \rangle^J \cap \langle m_k \rangle^J = \varnothing$:
But this too is easily derived from the definition of $\{\}$, *as we know from it that* m_l *and* m_k *are discernible. From theorem* 3 *we know that* $\langle m_l \rangle^J \cap \langle m_k \rangle^J = \varnothing$.

Theorem 6

Theorem:
Let $d_n = <\phi_1,..,\phi_n> \in Dis, s = s_0[\phi_1]...[\phi_n] \in \mathrm{S}_{\mathrm{DS}}^{\mathrm{Discourse}}$.
$\mathrm{s} \supseteq \left\{ \mathrm{i} \in \mathrm{I} \middle| \exists i' \in s : \langle i \rangle^w = \langle i' \rangle^w, \langle i \rangle^r = \langle i' \rangle^r \text{ and i and i' are indiscernible in } \mathrm{d}_\mathrm{n} \right\}$

Proof :

By double induction on the length of the discourse and the length of the last utterance. The inductions proceed in parallel, because of the special case of $\phi \wedge \psi$ (*it is like showing first for a language without conjunction and then showing again with conjunction also allowed*).

For brevity, say

$Ind_{n,s} = \left\{ i \in I \,\middle|\, \exists i' \in s : \langle i \rangle^w = \langle i' \rangle^w, \langle i \rangle^r = \langle i' \rangle^r \text{ and } i \text{ and } i' \text{ are indiscernible in } d_n \right\}.$

The special case of $n = 0$:

$d = <>,\ s \subseteq \{ < \varnothing, \varnothing, w > | w \in W \}.$

For every $w \in W$, *there is only one possibility with an empty referent system and world* w. *So* $s = Ind_n$.

Suppose true for $< \phi_1,..,\phi_n >$. *Show for* $d =< \phi_1,..,\phi_n,\phi_{n+1} >$:

$s \supseteq Ind_{n,s}$. *Show that* $s' = s[\phi_{n+1}] \supseteq Ind_{n+1,s[\phi_{n+1}]}$

Note that for any $s \in S : Ind_{n,s} \supseteq Ind_{n+1,s}$ *so* $s \supseteq Ind_{n,s}$. *Which means that* $s \supseteq Ind_{n+1,s}$.

If $s[\phi_{n+1}] = \varnothing$ *then there are no possibilities in* $s[\phi_{n+1}]$. *In which case* $Ind_{n+1} = \varnothing$, *and therefore* $s[\phi_{n+1}] \supseteq Ind_{n+1}$. *Else, by induction on sentence length of* $\varphi = \phi_{n+1}$:

$\varphi = Rt_1,...,t_n$: $s \supseteq Ind_{n+1,s}$

$s \supseteq s[\varphi]$, *so* $Ind_{n+1,s} \supseteq Ind_{n+1,s[\varphi]}$. *Which means that* $s \supseteq Ind_{n+1,s[\varphi]}$.

$\{ i \in s | i \text{ does not subsist in } s[\varphi] \} \cap Ind_{n+1,s[\varphi]} = \varnothing$

because $\forall i \in Ind_{n+1,s[\varphi]} : (\exists i' \in s[\varphi] : i \text{ and } i' \text{ are indiscernible } \Rightarrow cons(i,\varphi))$

$s' = s - \{ i \in s | i \text{ does not subsist in } s[\varphi] \}$, *hence* $s' \supseteq Ind_{n+1,s[\varphi]}$

same proof for all tests : $\varphi = (t_1 = t_2),\ \varphi = \neg\phi, \varphi = \Diamond\phi$.

$\varphi = \phi \wedge \psi : s' = s_0[\phi_1]...[\phi_n][\phi \wedge \psi] = s_0[\phi_1]...[\phi_n][\phi][\psi]$

For the dicourse $d = \ < \phi_1,..,\phi_n,\phi,\psi >$:

$s_0[\phi_1]...[\phi_n][\phi] \supseteq Ind_{n+1,s[\phi]}$ *by the induction h. on sentence length*

$s_0[\phi_1]...[\phi_n][\phi][\psi] \supseteq Ind_{n+2,s[\phi][\psi]}$ *by the induction h. on both sentence length and discourse length.*

Note that for any $s \in S : Ind_{n+2,s[\phi][\psi]} = Ind_{n+1,s[\phi\wedge\psi]}$ *because for any* $i \in s$, $cons(i,\phi \wedge \psi)$ *iff* $cons(i,\phi)$ *and* $cons(i,\psi)$.

Hence from $s' \supseteq Ind_{n+2,s[\phi][\psi]}$ *we get* $s' \supseteq Ind_{n+1,s[\phi\wedge\psi]}$

$\varphi = \exists x\phi$: *For any individual* $d \in D$,

$$s[x/d] \supseteq \left\{ i \in I \left| \begin{array}{l} \exists i' \in s[x/d] : \langle i \rangle^w = \langle i' \rangle^w, \langle i \rangle^r = \langle i' \rangle^r \\ \text{and } i \text{ and } i' \text{ are indiscernible in } d_n \end{array} \right. \right\} \text{ by the definition}$$

of state assignment. Hence $s[x/d] \supseteq Ind_{n,s[x/d]}$, *which means that* $s[x/d][\phi] \supseteq Ind_{n+1,s[x/d][\phi]}$ *by the induction hyp. on sentence length. We get that* $\bigcup_{d \in D} s[x/d][\phi] \supseteq \bigcup_{d \in D} Ind_{n+1,s[x/d][\phi]} = Ind_{n+1,s[\exists x\phi]}$, *and hence*

$$s' = \bigcup_{d \in D} s[x/d][\phi] \supseteq Ind_{n+1,s[\exists x\phi]}$$

Lemma 1

Let $\phi \in FORMULAS$ *and let* $s \in S_{DS} : \forall i' \in s[\phi] : \exists! i \in s : i \leq i'$

($\exists!$ *means that there exists exactly one entity with the specified qualities*)

Proof :

By induction on sentence structure.

$\phi \in \{R(p_1,...p_n), p_1 = p_2, \neg\varphi, \Diamond\varphi\}$:

$\forall i' \in s[\phi] : i' \in s$ *so obviously* $\exists i \in s : i \leq i'$. *Now suppose* $\exists i^* \neq i \in s : i^* \leq i'$. *From* $i, i^* \in s$ *we derive* $\langle i \rangle^r = \langle i^* \rangle^r$. *From* $i \leq i'$ *and* $i^* \leq i'$ *we derive* $\langle i \rangle^w = \langle i^* \rangle^w$. *So it must be that* $\langle i \rangle^g \neq \langle i^* \rangle^g$. *But* $\langle i \rangle^g = \langle i' \rangle^g = \langle i^* \rangle^g$. *Contradiction, hence* $\forall i^* \neq i \in s : i^* \not\leq i'$.

$\phi = \varphi \wedge \psi$:

$\forall i' \in s[\phi] : i' \in s[\varphi][\psi]$. *By induction hypothesis* $\exists! i^* \in s[\varphi] : i^* \leq i'$. *Again by induction hypothesis* $\exists! i \in s : i \leq i^*$. *So we get* $\exists! i \in s : i \leq i'$.

$\phi = \exists x\varphi$:

$\forall i' \in s[\phi] : \exists i \in s$ *and* $\exists d \in D : i' = i[x/d]$. *So obviously* $i \leq i'$. *Now suppose* $\exists i^* \neq i \in s : i^* \leq i'$. *From* $i, i^* \in s$ *we derive* $\langle i \rangle^r = \langle i^* \rangle^r$. *From* $i \leq i'$ *and* $i^* \leq i'$ *we derive* $\langle i \rangle^w = \langle i^* \rangle^w$. *So it must be that* $\langle i \rangle^g \neq \langle i^* \rangle^g$. *But* $\langle i \rangle^g = \langle i' \rangle^g \restriction rng(\langle i \rangle^r) = \langle i^* \rangle^g$. *Contradiction, hence* $\forall i^* \neq i \in s : i^* \not\leq i'$.

Lemma 2

Let $\phi \in FORMULAS$ *and let* $t \in S_{Sit} : \forall m' \in t[\phi] : \exists! m \in t : m \leq m'$

Proof :
By induction on sentence structure.
$\phi \in \{R(p_1,...,p_n), p_1 = p_2, \neg\varphi, \Diamond\varphi\}$:
$\forall m' \in t[\phi] : \exists m \in t : m' \in m\{\phi\}$ *so* $\exists m \in t : m \leq m'$. *Now suppose* $\exists m^* \neq m \in t : m^* \leq m'$.
Looking at $m' \langle m' \rangle^J \neq \emptyset$, *which means that* $\exists i' \in \langle m' \rangle^J$. *Moreover*, $m \leq m'$ *so* $\exists i \in \langle m \rangle^J$ *s.t* $i \leq i'$. *Also* $m^* \leq m'$, *so* $\exists i^* \in \langle m^* \rangle^J$ *s.t* $i^* \leq i'$. *From* $i \leq i'$ *and* $i^* \leq i'$ *we get* $i^* = i$ (*by lemma* 1). *Hence* $\langle m \rangle^J \cap \langle m^* \rangle^J \supseteq \{i\}$, *but* $m \neq m^* \in t$ *so they must be discernible, which means that* $\langle m \rangle^J \cap \langle m^* \rangle^J = \emptyset$. *Contradiction, hence* $\forall m^* \neq m \in t : m^* \not\leq m'$.

$\phi = \varphi \wedge \psi$:
$\forall m' \in t[\phi] : m' \in t[\varphi][\psi]$. *By induction hypothesis* $\exists! m^* \in t[\varphi]$ *s.t* $m^* \leq m'$. *Again by induction hypothesis* $\exists! m \in t : m \leq m^*$. *So we get* $\exists! m \in t : m \leq m'$.

$\phi = \exists x \varphi$:
$\forall m' \in t[\phi] : m' \in \bigcup_{d \in D} (t[x/d][\varphi])$. *So* $\exists d \in D$ *s.t* $m' \in t[x/d][\varphi]$. *By induction hypothesis* $\exists! m^* \in t[x/d]$ *s.t* $m^* \leq m'$. *Now by def of information state assignment* $\exists m \in t$ *s.t* $m[x/d] = m^*$. *We get* $m \leq m^* \leq m'$ *which means that* $m \leq m'$.
Now suppose $\exists m'' \neq m \in t$ *s.t* $m'' \leq m'$. $m' \in t[x/d][\varphi]$ *and* m^* *is the only situation in* $t[x/d]$ *s.t* $m^* \leq m'$, *so it must be that* $m'' \leq m^*$. *We get* $m''[x/d] = m^* = m[x/d]$. *But by def of situation assignment this means* $\left\{ i[x/d] \middle| i \in \langle m'' \rangle^J \right\} = \left\{ i[x/d] \middle| i \in \langle m \rangle^J \right\}$. *Hence* $\langle m'' \rangle^J = \langle m \rangle^J$.
At the same time $m, m'' \in t$, *so* $\langle m'' \rangle^\gamma = \langle m \rangle^\gamma$. *Put the two together and we get* $m'' = m$, *in contradiction to* $m'' \neq m$. *Hence* $\exists! m \in t : m \leq m'$.

Lemma 3

$Let\ \phi \in FORMULAS\ and\ let\ t \in S_{Sit}:$

$\forall m \in t\ and\ m' \in t[\phi] : m \not\lesssim m' \rightarrow \forall i \in \langle m \rangle^J : \forall i' \in \langle m' \rangle^J : i \not\lesssim i'$

proof :

$From\ lemma\ 2\ we\ get\ \exists! m^* \in t\ s.t\ m^* \leq m'.\ So\ \forall i' \in \langle m' \rangle^J : \exists i^* \in \langle m^* \rangle^J\ s.t\ i^* \leq i'.$

$Suppose\ \exists i \in \langle m \rangle^J\ and\ \exists i' \in \langle m' \rangle^J\ s.t\ i \leq i'.\ \langle i \rangle^r = \langle i^* \rangle^r\ (because\ m, m^* \in t),\ \langle i \rangle^w = \langle i^* \rangle^w$

$(because\ i'\ is\ an\ extension\ of\ both),\ and\ \langle i \rangle^g = \langle i' \rangle^g \upharpoonright rng(\langle i \rangle^r) = \langle i^* \rangle^g,\ therefore\ i = i^*.$

$But\ if\ i = i^*\ then\ m\ and\ m^*\ are\ indiscernible,\ and\ since\ they\ both\ belong\ in\ t,\ m = m^*.$

$In\ contradiction\ to\ m \not\lesssim m'\ and\ m^* \leq m'. So\ \forall i \in \langle m \rangle^J\ and\ \forall i' \in \langle m' \rangle^J : i \not\lesssim i'$

Theorem 7

Theorem:

There exists a function F between $S_{DS}^{Discourse}$ *the set of DS discourse derived states, and* $S_{Sit}^{Discourse}$ *the set of situation based ones. This function is* 1-1, *onto, and is structure preserving under the update relation.*

$Let\ s = s_0[\phi_1]...[\phi_n] \in S_{DS}^{Discourse}.$

$d =< \phi_1,..,\phi_n > \in Dis\ and\ \gamma_d\ is\ the\ linguistic\ discourse\ defined\ by\ d.$

$$Let\ T_s \subseteq S_{Sit}^{Discourse}\ be\ \left\{ t \in S_{Sit}^{Discourse} \middle| \begin{array}{l} for\ every\ m \in t\ the\ following\ hold: \\ \langle m \rangle^{\gamma} = \gamma_d \\ \forall i \in \langle m \rangle^J : i \in s \end{array} \right\}$$

$F(s) : S_{DS}^{Discourse} \xleftrightarrow{1\text{-}1} S_{Sit}^{Discourse} = Max(T_s)$

$And\ \forall \phi \in FORMULAS : F(s[\phi]) = F(s)[\phi]$

Proof :

Existence:

We need to show that for every $s \in S_{DS}^{Discourse}$, T_s contains at least one maximal element.

Now T_s can not be empty because $\varnothing$ itself trivially belongs to it ($\varnothing \in S_{DS}^{Discourse}$).

But suppose then that $T_s \neq \varnothing$ and that there is no maximal element, i.e that $\forall t \in T_s : \exists t' \in T_s$ s.t $t' \supset t$.

We choose by induction a series $\left\langle t_\alpha \middle| \begin{array}{l} \alpha \text{ is an ordinal number} \\ t_\alpha \in T_s \end{array} \right\rangle$ in the following way:

$\alpha = 0:\ t_0 = \varnothing \in T_s$

$\alpha = \beta + 1:$ By assumption $\exists t_\alpha \in T_s : t_\alpha \supset t_\beta$

$\alpha = \bigcup_{\beta<\alpha} \beta :\ t_\alpha = \bigcup_{\beta<\alpha} t_\beta .$

In the last case $t_\alpha \in T_s$ because otherwise there exist $m \neq m' \in t_\alpha$ s.t one of the following holds:

1. m is indiscernible from m' in γ_d.
2. $\langle m \rangle^\gamma \neq \gamma_d$
3. $\exists i \in \langle m \rangle^J : i \notin s$

But $m, m' \in t_\alpha$ means that there exist some $\beta, \delta < \alpha$ s.t $m \in t_\beta$ and $m' \in t_\delta$.

WLOG suppose $\beta < \delta$ then $m, m' \in t_\delta$. Therefore also $t_\delta \notin T_s$, contradiction.

Using the series $\left\langle t_\alpha \middle| \begin{array}{l} \alpha \text{ is an ordinal number} \\ t_\alpha \in T_s \end{array} \right\rangle$ we can define a 1-1 function from the class of ordinals Ord to T_s ($f(\alpha) = t_\alpha$).

Therefore $|Ord| \leq |T_s|$. Contradiction.

So there must be some $t^* \in T_s$ s.t $\forall t' \in T_s$: it is not the case that $t' \supset t^*$.

Hence $Max(T_s) = t^*$.

Uniqueness:

We have to show that the set T_s has exactly one maximal element.

Suppose there is more than one. Then there exist $t \neq t' \in S_{Sit}^{Discourse}$ such that both are the result of applying F to some $s \in S_{DS}^{Discourse}$. WLOG $\exists m \in t,\ m \notin t'$. By def $\exists i \in \langle m \rangle^J$ s.t $i \in s$.

If $\forall m' \in t' : i \notin \langle m' \rangle^J$ then $< \{i\}, \gamma_d >$ is discernible from all the situations in t' under γ_d and $t' \cup < \{i\}, \gamma_d >$ is also a result of applying F to s.

But $t' \cup < \{i\}, \gamma_d > \supset t'$, which contradicts with the maximality of the set t'.

On the other hand, if $\exists m' \in t'$ s.t $i \in \langle m' \rangle^J$ then m' and m are indiscernible. Contradicting the definition. Hence $t = t'$.

$1-1$:

Suppose not. Let $s \neq s' \in S_{DS}^{Discourse}$, and $t = F(s) \in S_{Sit}^{Discourse}, t' = F(s') \in S_{Sit}^{Discourse}, t = t'$.

From $s \neq s'$ it follows WLOG that $\exists i \in s : i \notin s'$.

We know that $\exists m \in t : i \in \langle m \rangle^J$ (otherwise t is not maximal). Therefore $m \in t'$ $(t = t')$, and hence $i \in s'$. Contradiction to $i \notin s^*$.

structure preserving under the update relation:

Let $d =< \phi_1,..,\phi_n > \in Dis, s = s_0[\phi_1]...[\phi_n] \in S_{DS}^{Discourse}$ as in def. for $S_{DS}^{Discourse}$.

Let $t \in S_{Sit}^{Discourse}$, such that $t = F(s)$.

We need to show that $\forall \phi \in FORMULAS : F(s[\phi]) = t[\phi]$. \

proof:

Let us define $I_t = \left\{ i \in \langle m \rangle^J \middle| m \in t \right\}$ for every $t \in S_{Sit}^{Discourse}$.

Lemma: Let $\phi \in FORMULAS$ and let $t \in S_{Sit}^{Discourse}$ and $s \in S_{DS}^{Discourse}$ such that $F(s) = t$. Then $I_{t[\phi]} = s[\phi] \Leftrightarrow t[\phi] = F(s[\phi])$.

proof of lemma:

First, $\forall m \in t : \langle m \rangle^{\gamma} = \gamma_d$, so $\forall m \in t[\phi] : \langle m \rangle^{\gamma} = CG(\gamma_d, \phi) = \gamma_{d+\phi}$ = the linguistic context of $s[\phi]$.
Next, we show that the sets of possibillities on either side of the equations are the same.

$\Rightarrow$: $t[\phi] \in T_{s[\phi]}$ because $I_{t[\phi]} \subseteq S[\phi]$ and by def of T_s. Suppose next that $t[\phi] \neq max(T_{s[\phi]})$. This means that $\exists t' \in T_{s[\phi]} : t' \supset t[\phi]$. Hence $\exists m' \in t' : m' \notin t[\phi]$ and therefore $\exists i' \in m'$ such that $\forall m \in t[\phi] : i' \notin \langle m \rangle^{J}$ (if $i' \in \langle m \rangle^{J}$ then m and m' are indiscernble situations in t'). Hence $I_{t[\phi]} \cup \{i'\} \subseteq S[\phi]$, contradiction to supposition. So $t[\phi] = max(T_{s[\phi]}) = F(s[\phi])$.

$\Leftarrow$: $t[\phi] \in T_{s[\phi]}$ so $\forall m \in t[\phi] : \forall i \in \langle m \rangle^{J} : i \in s[\phi]$, hence $I_{t[\phi]} \subseteq s[\phi]$.
$t[\phi] = Max(T_{s[\phi]})$, so $\neg\exists i \in s[\phi] : \forall m \in t[\phi] : i \notin \langle m \rangle^{J}$ (if there was, $t[\phi] \cup \langle \{i\}, \gamma_{d+\phi} \rangle$ would also belong in $T_{s[\phi]}$, and $t[\phi]$ would not be maximal). So $I_{t[\phi]} \supseteq s[\phi]$.
Altogether we get $I_{t[\phi]} = s[\phi]$.

proof of structure preservation under update relation:

By induction on sentence structure.

$\phi = R(q_1, \ldots q_n)$:

$\forall i \in I : i \in I_{t[\phi]}$ iff $i \in \left\{ i \in \langle m \rangle^{J} \middle| m \in t\{\phi\} \text{ and } [\![R(q_1, \ldots q_n)]\!]_m = 1 \right\}$ (by the semantics of $R(q_1, \ldots q_n)$). This in turn is true iff $i \in \left\{ i \in \langle m \rangle^{J} \middle| m \in t\{\phi\} \text{ and } \forall i' \in m : [\![R(q_1, \ldots q_n)]\!]_{i'} = 1 \right\}$ (by def of situation denotation). Which is true iff $i \in I_{t\{\phi\}}$ and $[\![R(q_1, \ldots q_n)]\!]_i = 1$, itself true iff $i \in I_t$ and $[\![R(q_1, \ldots q_n)]\!]_i = 1$, but $I_t = s$ (because $F(s) = t$ and because of lemma) so the latter is true iff $i \in s$ and $[\![R(q_1, \ldots q_n)]\!]_i = 1$, which is true iff $i \in s[\phi]$.

$\phi = (q_1 = q_2)$: same as the above proof for $R(q_1, \ldots q_n)$

$\phi = \neg\varphi$:

$$\forall i \in I : i \in I_{t[\neg\varphi]} \text{ iff } i \in \left\{ i \in \langle m \rangle^{J} \middle| \begin{array}{l} m \in t\{\neg\varphi\} \text{ and} \\ \forall m' \in t[\varphi] : m \not\lesssim m' \end{array} \right\} \text{ (by semantics of } \neg).$$

$$\text{By lemma 3, this is true iff } i \in \left\{ i \in \langle m \rangle^{J} \middle| \begin{array}{l} m \in t\{\neg\varphi\} \text{ and} \\ \forall i' \in I_{t[\varphi]} : i \not\lesssim i' \end{array} \right\}, \text{ which is true iff}$$

$i \in s$ and $\forall i' \in s[\varphi] : i \not\lesssim i'$ (by ind. hyp $I_{t[\varphi]} = s[\varphi]$). Itself true iff $i \in s[\neg\varphi]$.

$\phi = \Diamond\varphi$:

If $s[\varphi] = \varnothing$ then $s[\Diamond\varphi] = \varnothing$ *but also* $t[\varphi] = \varnothing$, *which means that* $t[\Diamond\varphi] = \varnothing$.

So $I_{t[\Diamond\varphi]} = \varnothing = s[\Diamond\varphi]$.

If $s[\varphi] \neq \varnothing$ then $s[\Diamond\varphi] = s$ *but also* $t[\varphi] \neq \varnothing$, *which means that* $t[\Diamond\varphi] = t\{\Diamond\varphi\}$.

Giving us that $I_{t[\Diamond\varphi]} = I_{\tau\{\Diamond\varphi\}} = I_t = s = s[\Diamond\varphi]$.

$\phi = \exists x\varphi$:

$\forall i \in I : i \in I_{t[\phi]}$ *iff* $i \in \bigcup_{d \in D}(t[x/d][\varphi])$ (*by semantics of* $\exists x\varphi$). *Which is true iff*

$i \in \bigcup_{d \in D}((\bigcup_{m \in t} m[x/d])[\varphi])$ (*by def of assignment in situations*). *The latter being true iff* $i \in \bigcup_{d \in D}((\bigcup_{m \in t} \bigcup_{i' \in \langle m \rangle^J} i'[x/d])[\varphi])$ (*by def of possibility assignment*). *But*

$\bigcup_{m \in t} \bigcup_{i' \in \langle m \rangle^J} i' = I_t$ by definition, and $I_t = s$ (because F(s)=t and because of lemma),

so $\bigcup_{m \in t} \bigcup_{i' \in \langle m \rangle^J} i'[x/d] = \bigcup_{i' \in s} i'[x/d]$.

This all brings about that $i \in I_{t[\phi]}$ *iff* $\bigcup_{d \in D}(\bigcup_{i' \in s} i'[x/d][\varphi])$, which is equivalent to

$i \in \bigcup_{d \in D}(s[x/d][\varphi])$.

onto :

We need to show that for every $t \in S_{Sit}^{Discourse}$ *there exists some* $s \in S_{DS}^{Discourse}$ *such that* $t = F(s)$. *We will show this by induction on discourse length n.*

Let $d = \langle \phi_1, .., \phi_n \rangle \in Dis$ *the discourse by which t was derived.*

Let us define $I_t = \{i \in m \mid m \in t\}$

If $n = 0$:

$d = \varnothing$, *which means that* $\gamma_d = \varnothing$, *so* $\forall m \in t : \langle m \rangle^\gamma = \gamma_d = \varnothing$ *and* $\forall i \in m : \langle i \rangle^r = \varnothing$ *and* $\langle i \rangle^g = \varnothing$. *Hence* $\exists m^* \in Sit$ *s.t* $t = \{m^*\}$ (*t is a singleton set*). *Now let* $s_0 = I_t$. *We get that* $\forall i \in s_0 : \langle i \rangle^r = \varnothing$ *and* $\langle i \rangle^g = \varnothing$. *Let* $s = s_0$. *As d is empty* $s \in S_{DS}^{Discourse}$. *Because the linguistic context is empty, every member in* T_s *is a singleton set of one situation and so* $t \in T_s$. *Also, because* $\forall i \in s : i \in m^*$ $t = max(T_s)$ *so* $F(s) = t$.

Suppose true for some discourse length n, *let us show for* $n+1$:

$t = t'[\phi]$ *for some* $t' \in S_{Sit}^{Discourse}$ *and* $\phi \in FORMULAS$ (*by def of* $S_{Sit}^{Discourse}$).

t is derived by discourse of length $n+1$, *so* t' *is derived by discourse of length* n,

so by induction hypothesis $\exists s' \in S_{DS}^{Discourse}$ *such that* $F(s') = t'$.

By the structure preserving property of F under the update relation we get

$t = t'[\phi] = F(s')[\phi] = F(s'[\phi]) = F(s)$, *and specifically*, $F(s) = t$.

As for any $t \in S_{Sit}^{Discourse}$ *t is derived by some finite discourse*, *we get that F is onto* $S_{Sit}^{Discourse}$.

BIBLIOGRAPHY

Asher, Nicholas: 1992, "A default, truth conditional semantics for the progressive", *Linguistics and Philosophy* 15: 463-508.

Bennett, Michael & Barbara Partee: 1972, "Toward the logic of tense and aspect in English", IULC, Bloomington.

Bonomi, Andrea: 1997, "The progressive and the structure of events", *Journal of Semantics* 14: 173-205.

Carnap, Rudolf: 1947, *Meaning and Necessity*, U. of Chicago Press.

Dowty, David. 1977: "Towards a semantic analysis of verb aspect and the English 'imperfective progressive'", *Linguistics and Philosophy* 1: 45-78.

Dowty, David: 1979, *Word Meaning and Montague Grammar*, Kluwer, Dordrecht.

Fagin, Ronald, Joseph Y. Halpern, Yoram Moses, and Moshe Y. Vardi: 1995, *Reasoning about Knowledge*, The MIT Press, Cambridge, Massachusetts.

Gerbrandy, Jelle and Willem Groeneveld: 1997, "Reasoning about Information Change", *Journal of Logic, Language and Information* 6: 147–169.

Glasbey, Sheila: 1996, "The progressive: A channel-theoretic analysis", *Journal of Semantics* 13: 331-361.

Groenendijk, Jeroen & Martin Stokhof: 1991, "Dynamic Predicate Logic", *Linguistics and Philosophy* 14: 39-100.

Groenendijk, Jeroen, Martin Stokhof & Frank Veltman: 1996, "Coreference and Modality", in: Shalom Lappin (ed), *The Handbook of Contemporary Semantic Theory*, Blackwell, Oxford: 179-216.

Harel, David: 1984, "Dynamic logic", in D. Gabbay and F. Guenther (eds), *Handbook of Philosophical Logic*, volume 2,. Reidel, Dordrecht, Holland: 497-604.

Heim, Irene: 1982, *The Semantics of Definite and Indefinite Noun Phrases*, PhD thesis, UMass, Amherst, 1982. Appeared in 1989 at Garland, New York

Hintikka, Jaakko: 1969, "Semantics for Propositional Attitudes", in: J. W. Davis et al.(eds.), *Philosophical Logic*, Dordrecht: 21–45.

Kamp, Hans: 1981, “A Theory of Truth and Semantic Representation”, in: Jeroen Groenendijk, Theo Janssen & Martin Stokhof (eds), *Formal Methods in the Study of Language*, Mathematisch Centrum, Amsterdam: 1-41.

Landman, Fred: 1992, “The progressive”, *Natural Language Semantics* 1: 1-32.

Lascarides, Alex: 2001, “The progressive and the imperfective paradox”, *Synthese* 87: 401-447.

Lewis, David: 1973: *Counterfactuals*, Cambridge, MA, Harvard University Press.

Link, Godehard: 1987, “Algebraic Semantics for Event Structures,” in Jeroen Groenendijk, Martin Stokhof, and Frank Veltman (eds.), *Proceedings of the 6th Amsterdam Colloquium*, University of Amsterdam, Institute for Language, Logic, and Information: 243–262.

McCarthy, John: 1980, “Circumscription: A form of non-monotonic reasoning”, *Artificial Intelligence* 13: 27-39.

McCarthy, John: 1986, “Applications of circumscription to formalizing common sense knowledge”, *Artificial Intelligence* 13: 27-39.

Parsons, Terence: 1989: “The progressive in English: Events, States and Processes”, *Linguistics and Philosophy* 12: 213-241.

Portner, Paul: 1998 “The Progressive in modal semantics”, *Language* 74: 760-787.

Prior, Arthur: 1967, *Past, Present and Future*. Oxford University Press, Oxford.

Roberts, Craige: 1996, “Anaphora in Intensional Contexts”, in Shalom Lappin, ed., *The Handbook of Contemporary Semantic Theory*, Oxford, Blackwell: 215–247.

Szabo, Zoltan: 2004, “On the Progressive and the Perfective”, *Noûs* 38: 29-59.

Thomason, Richmond: 1970, “Indeterminist time and truth-value gaps”, *Theoria* 36: 264-281.

Thomason, Richmond: 1984, “Combinations of tense and modality”, in D. Gabbay and F. Guenthner, (eds), *Handbook of Philosophical Logic. Volume II: Extensions of Classical Logic*, volume 146 of *Synthese Library*, chapter II.3, D. Reidel: 89–134.

Thomason, Richmond: 1995, ``Logicism in Formalizing Common Sense and in Natural Langugae Semantics”, available via http://www.eecs.umich.edu/~rthomaso/

Vendler, Zeno: 1967, "Verbs and Times", in Zeno Vendler, *Linguistics in Philosophy*, Cornell University Press, Ithaca.

Vlach, Frank: 1981 "The Semantics of the progressive" in P. Tedeschi & A. Zaenen (eds.) *Tense and Aspect* (Syntax and Semantics 14) Academic Press New York et al.: 271-292.

Zucchi, Sandro: 1999, "Incomplete events, intensionality and imperfective aspect", *Natural Language Semantics* 7: 179-215.