The existential/uniqueness presupposition of *wh*-complements projects from the answers

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Abstract

The projection patterns of the existential/uniqueness presupposition of a *wh*-complement vary depending on the predicate that embeds it. This variation poses problems for existing accounts that treat the presupposition as a semantic contribution of an operator merging with the *wh*-complement (Dayal, 1996) or of the embedding predicate (Uegaki, 2015). Rather, the patterns suggest an analysis that treats the presupposition as contributed by the propositions corresponding to the answers of the embedded question.

1 Introduction

In recent years, there has been a renewed interest in the semantic analysis of embedded questions (e.g., Spector and Egré, 2015; Uegaki, 2015; Cremers, 2016; Xiang, 2016; Theiler et al., 2018), following earlier pioneering works (e.g., Karttunen, 1977; Groenendijk and Stokhof, 1984; Heim, 1994; Dayal, 1996; Lahiri, 2002). One of the primary goals of the investigation in this domain is to provide a *unified* analysis of the interpretations of pairs of sentences of the form in (1), where Q is an interrogative and p is a declarative complement, across different predicates V:

(1) a. *x Vs Q* (e.g., *Max knows who danced.*)
  b. *x Vs that p* (e.g., *Max knows that Pat danced.*)

This paper aims to further advance the investigation into the semantics of these constructions, focusing on an issue concerning their *presuppositions*, i.e., how the presupposition of a *wh*-complement is projected by different embedding predicates.

At least since Katz and Postal (1964), it has been observed that *wh*-questions in general presuppose that at least one of its Hamblin answers (Hamblin, 1973) is true. The following sentences exemplify this observation (see also Keenan and Hull 1973 for an early discussion):

(2) a. Who smokes? $\overset{\text{presup}}{\Rightarrow}$ Someone smokes. ($\overset{\text{presup}}{\Rightarrow}$: 'presupposes')

b. Which semanticists danced? $\overset{\text{presup}}{\Rightarrow}$ Some semanticist danced.

In addition, it is well-known that a singular-*which* question of the form ⌜*Which NP* φ*?*⌝ presupposes that exactly one NP φ (e.g., Dayal 1996; cf. also Groenendijk and Stokhof 1984):

(3) Which semanticist danced? $\overset{\text{presup}}{\Rightarrow}$ Exactly one semanticist danced.

What is less investigated is what happens to the presupposition when the *wh*-question is embedded by different predicates,[1] as in the following examples:

(4) a. Max knows which semanticist danced.
b. Max is certain (about) which semanticist danced.
c. Max agrees with Kim on which semanticist danced.

As will be argued below, the projection patterns of the presupposition of the *wh*-complement in sentences like (4) pose problems for existing accounts. The goal of the paper is thus to provide an alternative analysis of the presupposition of *wh*-complements that correctly captures its projection patterns. Specifically, I will argue that the data require a view where the existential/uniqueness presupposition is contributed by *the propositions corresponding to the answers of the question*, rather than an operator that merges with the *wh*-complement (Dayal, 1996) or the lexical semantics of the embedding predicate (Uegaki, 2015).

It is also important to note that the analysis is intended as a part of the overall semantics of question-embedding, as envisioned in the beginning of the paper. For this reason, our account of the presupposition of ⌜*x Vs Q*⌝ has to be compatible with the presupposition of ⌜*x Vs that p*⌝, given a unified account of the interpretations of ⌜*x Vs Q*⌝ and ⌜*x Vs that p*⌝. That is, the ultimate account of the presuppositions of (4) should also account for the presuppositions of the following kind of sentences:

(5) a. Max knows that Ash danced.
b. Max is certain that Ash danced.
c. Max agrees with Kim that Ash danced.

[1]Though, see Karttunen and Peters (1976) for a pioneering work, which discusses the projection of the existential presupposition of interrogative complements embedded under *know*, *wonder* and *matter*. They treat what we call the existential presupposition of embedded questions as a 'conventional implicature' and formulate a two-dimensional compositional analysis where both the denotation (or the at-issue content in modern terms) and the conventional implicature are calculated recursively (Karttunen and Peters, 1979). In this system, how the existential presupposition (or their implicature) is projected by each embedding predicate is stipulated in the form of meaning postulates. The current paper departs from Karttunen and Peters (1976) in two important respects. First, rather than stipulating the projection behavior of UP/EP for each question-embedding verb, I will aim to *derive* it, based on the general theory of question-embedding and what we independently know about the presupposition-projection behavior of each verb. Second, I take the sentences as in (4) to semantically presuppose, rather than conventionally implicate, the relevant uniqueness implications.

The structure of the rest of the paper is as follows. In Section 2, I lay out the basic data and discuss why they pose problems for existing accounts: Dayal (1996), a possible extension of Dayal (1996) based on Spector and Egré (2015) and Uegaki (2015). Section 3 presents the proposal, where the existential/uniqueness presupposition of *wh*-complements is contributed by the answer propositions, together with two concrete implementations of the analysis: (i) encoding the relevant presupposition to each answer within the question denotation and (ii) modifying the answerhood operator to return a presuppositional proposition. Section 4 discusses how the present analysis can be extended to data involving matrix questions and rogative predicates (e.g., *wonder*), and provides arguments for favoring the first implementation of the analysis. Section 5 summarizes the proposal and discusses several open issues.

2 Data and problems for existing accounts

In this section, I present data concerning the projection of the presupposition of *wh*-complements, and discuss why they are problematic for the existing accounts by Dayal (1996) and Uegaki (2015), as well as for a possible extension of Dayal (1996) based on Spector and Egré (2015). Here and in the rest of the paper, I will mostly present data involving the uniqueness presupposition (UP) of singular-*which* questions. However, the analyses to be discussed will be compatible with the existential presupposition (EP) of plural-*which* and simplex *wh*-questions, as they are based on Dayal's (1996) analysis that uniformly treats both UP and EP as the maximality presupposition, as discussed at the end of Section 2.1.

2.1 Embedding under veridical predicates and Dayal (1996)

When a singular-*which* question is embedded under a veridical predicate, such as *know* and *surprise*, the uniqueness presupposition (UP) projects to the matrix level. This can be seen in the following examples:

(6) a. Max doesn't *know* which student smokes.
 b. Does Max *know* which student smokes?
 c. If Max *knows* which student smokes, he will tell us about it.
 $\overset{\text{presup}}{\Rightarrow}$ 'Exactly one student smokes.'

(7) a. Max isn't *surprised* (about) which student smokes.
 b. Is Max *surprised* (about) which student smokes?
 c. If Max is *surprised* (about) which student smokes, he will tell us about it.
 $\overset{\text{presup}}{\Rightarrow}$ 'Exactly one student smokes.'

Dayal's (1996) analysis of the UP employing her answerhood operator straightforwardly captures this matrix projection pattern. In Dayal (1996), *wh*-complements obligatorily merge with the answerhood operator ANS defined below.[2,3]

(8) a. $\textsc{Ans}_w = \lambda Q_{\langle st,t\rangle}\colon \underline{\exists p \in Q[p = \textsc{Max}_{\text{inf}}(Q, w)]}.\ \textsc{Max}_{\text{inf}}(Q, w)$
b. $\textsc{Max}_{\text{inf}}(Q, w) = p$ iff $w \in p \wedge \forall q \in Q[w \in q \rightarrow p \subseteq q]$

ANS roughly acts as a definite determiner over propositions. It carries the presupposition that there is a maximally informative true answer in the set of propositions it combines with, and picks out such a maximally informative true answer. Hereafter, I will refer to the presupposition of ANS as the MAXIMALITY PRESUPPOSITION, and the proposition that the ANS-operator returns from a question as the DAYAL-ANSWER of the question. Given that a singular-*which* question denotes a set of mutually-independent 'atomic' answers, as in (9) (cf. Hamblin, 1973), this treatment captures the UP associated with it.[4,5]

(9) *which student smokes* $\rightsquigarrow \left\{ \begin{array}{l} \lambda w'.\mathbf{student}_{w'}(a)\wedge\mathbf{smoke}_{w'}(a), \\ \lambda w'.\mathbf{student}_{w'}(b)\wedge\mathbf{smoke}_{w'}(b), \\ \lambda w'.\mathbf{student}_{w'}(c)\wedge\mathbf{smoke}_{w'}(c) \end{array} \right\}$ $(= Q)$

This is so because, for every w and every set Q of mutually-independent propositions, $\textsc{Ans}_w(Q)$ is defined only if exactly one of Q's members is true in w. Sentences like (6-7) have a semantic representation like (10), where $\textsc{Ans}_w(Q)$ (with the matrix evaluation world w) serves as an argument of the embedding predicate.

(10) *Max knows which student smokes* $\rightsquigarrow \mathbf{know}_w(\mathbf{m}, \textsc{Ans}_w(Q))$

The meaning in (10) is defined only if $\textsc{Ans}_w(Q)$ is defined, which holds just in case exactly one student smokes in w, the matrix evaluation world.

The ANS-operator further enables a uniform treatment of the UP of singular-*which* questions and the EP of plural/simplex *wh* questions, under the assumption that plural/simplex *wh*-phrases are number-neutral. This is so since the maximality presupposition is satisfied for proposition-sets that are closed under conjunction as long as there is a true answer in the set.[6]

[2]The formulation using the predicate $\textsc{Max}_{\text{inf}}$ is from Fox and Hackl (2007).

[3]Following Heim and Kratzer (1998), I model presuppositions in terms of partial functions with the notation $\lambda x\colon \pi(x).\varphi(x)$. Here, $\pi(x)$ is the domain specification corresponding to the presupposition.

[4]I assume that a linguistic expression is translated into a formula in an intermediate logical language similar to Ty2 (Gallin, 1975), which then receives a model-theoretic interpretation. I notate '$S \rightsquigarrow \varphi$' to indicate that the sentence S is translated into the formula φ.

[5]Here, I represent the question denotation as having the 'de dicto' reading, with the world index of the NP-part of the *which*-phrase bound by the lambda introducing the world-dependence. See Section 3.3.3 for the compositional derivation of 'de re' and 'de dicto' readings of which-questions following Beck and Rullmann (1999).

[6]Though, see Elliott et al. (2018) for recent discussion about the difference between simplex *wh* and *which*-questions.

2.2 Embedding under non-veridical predicates

However, the treatment along the lines of Dayal (1996) cannot be directly extended to embedding under *non-veridical* predicates.[7] The examples in (11) illustrate that, when *be certain* embeds a singular *which*-complement, the UP does not necessarily project to the matrix level, but rather projects into the subject's beliefs.

(11) a. Max isn't *certain* (about) which student smokes.
b. Is Max *certain* (about) which student smokes?
c. If Max *is certain* (about) which student smokes, he will let us know.
$\overset{\text{presup}}{\not\Rightarrow}$ 'Exactly one student smokes.'
$\overset{\text{presup}}{\Rightarrow}$ 'Max believes that exactly one student smokes.'

The contrast between *know* and *be certain* with respect to the relevant presupposition projection behavior is also clear in the following minimal pair.[8]

(12) No student smokes. But, Max believes that there is a student smoker. Only, he {isn't certain / #doesn't know} which student smokes.

The following is an attested example where, given the context, it is obvious that the presupposition of the *which*-question under *be certain* does not project to the matrix level, but only to the beliefs of the subject.

(13) Every blinking light, every bell and every Christmas carol has a beauty to a child which we are unable to remember. So, what if the lights blow a fuse? Does it really matter if the bells are a little bit dented or not in tune? Who cares if you've heard the same song 500 times? Even if you see a dozen Santas in a day, every new meeting with Santa Claus is special. Santa Claus can't be everywhere so he has helpers (of course!), but <u>how can a child be certain which Santa is real?</u> It isn't possible, so he or she treats them all with equal awe and love.[9]
$\overset{\text{presup}}{\not\Rightarrow}$ 'Exactly one Santa is real.'
$\overset{\text{presup}}{\Rightarrow}$ 'A child believes that exactly one Santa is real.'

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[7]I do not intend this to be a criticism of Dayal's (1996) analysis *per se*, as the analysis is not originally intended to cover non-veridical question-embedding predicates. In Section 2.3, I will argue that an analysis in terms of Dayal's answerhood cannot capture the projection pattern of the UP/EP even if it is extended to non-veridical predicates employing a recent analysis of non-veridical predicates by Spector and Egré (2015).

[8]M. Erlewine (p.c.) pointed out to me that there seems to be a contrast between the projection patterns of the existential implication and the 'less than two' implication in (11). That is, although (11) sounds felicitous when there is in fact no student smoker (as long as Max believes that there is a student smoker), it sounds infelicitous when there are more than one student smokers. If it turns out that this judgment can be replicated systematically, the discussion in this paper should be viewed as concerning only the existential presupposition of *wh*-complements. At the same time, we would need an independent analysis of the 'less than two' implication that predicts matrix projection.

[9]'The Lafayette' Vol. 114, No. 12. URL: `http://digital.lafayette.edu/collections/newspaper/19871127/pdf` (accessed on July 17, 2018)

Another non-veridical predicate, *agree*, exhibits a slightly different presupposition projection behavior, as illustrated in the following examples:

(14) a. Max doesn't *agree* with Kim on which student smokes.
b. Does Max *agree* with Kim on which student smokes?
c. If Max *agrees* with Kim on which student smokes, he'll let her know.
$\overset{\text{presup}}{\not\Rightarrow}$ 'Exactly one student smokes.'
$\overset{\text{presup}}{\Rightarrow}$ 'Max and Kim believes that exactly one student smokes.'

These examples show that the UP of the embedded question is projected not only to the belief state of the subject (Max in (14)) but also to the belief state of the individual denoted by the *with*-phrase (henceforth the *with*-ARGUMENT; Kim in (14)).[10] The presupposition, however, is not necessarily projected to the matrix level. The examples in (14) can be felicitous even if there is in fact no, or more than one, student smokers.

Dayal's (1996) analysis in terms of ANS outlined in the previous subsection is not designed to deal with embedding under non-veridical predicates. A simple extension of the analysis to the non-veridical cases would have ANS interpreted with respect to the matrix evaluation world, meaning that the predicted interpretation would involve the 'actual' answer (relative to the evaluation world) regardless of the embedding predicate. Among other issues, such an analysis would incorrectly predict the UP to project to the matrix level even when the embedding predicate is non-veridical. For example, the embedding under *be certain* would be analyzed as follows:

(15) *Max is certain about which student smokes* $\rightsquigarrow \mathbf{certain}_w(\mathbf{m}, \textsc{Ans}_w(Q))$

Just as in the case with veridical predicates, the predicted meaning in (15) is defined only if $\textsc{Ans}_w(Q)$ is defined, which in turn holds just in case exactly one student smokes in the matrix evaluation world w. Thus, this treatment fails to capture the lack of the matrix projection of the UP as well as its projection to the subject's beliefs observed in (11). Similarly, the projection behavior of *agree* would be problematic. The analysis would treat question-embedding under *agree* as follows:

(16) *Max agrees with Kim on which student smokes* $\rightsquigarrow \mathbf{agree}_w(\mathbf{m}, \mathbf{k}, \textsc{Ans}_w(Q))$

Just as in (15), the analysis predicts the matrix projection, and no projection to the subject's or to the *with*-argument's beliefs.

In sum, the projection pattern of the UP varies across veridical and non-veridical predicates. This variation is not captured by a simple extension of Dayal's (1996) analysis, as such an analysis predicts the presupposition triggered by ANS to project to the matrix level regardless of the embedding predicate.

[10]More precisely, the presupposition concerning Kim's beliefs is stronger than what is stated in (14). That is, the sentences presuppose that there is exactly one student such that Kim believes that they smoke. The same does not hold for the presupposition concerning Max's beliefs. Since the weaker formulation of the presupposition in (14) suffices in the illustration of relevant problems for Dayal's (1996) and Uegaki's (2015) analysis to be given below, I will stick to the weaker formulation in this section for the sake of simplicity. I will argue in Section 3.3.2 that my ultimate analysis captures the asymmetric presuppositions with respect to Max's beliefs and Kim's beliefs in the examples in (14).

2.3 An extension of Dayal (1996) based on Spector and Egré (2015)

At this point, it is worth considering Spector and Egré's (2015) analysis for question-embedding, as it is designed to address issues surrounding non-veridical question-embedding predicates, such as *be certain*, although Spector and Egré (2015) do not specifically discuss the issues of the projection of the UP/EP.

In contrast to the Dayal-style analysis, where the embedded question is analyzed in terms of its unique true answer, Spector and Egré (2015) analyze question-embedding as involving existential quantification over possible answers. Making use of Dayal's answerhood operator, this analysis can be schematically stated as a lexical rule converting a proposition-taking denotation, $\mathbf{V}_{decl}$, of a predicate V into its question-taking counterpart, $\mathbf{V}_{int}$, as follows:[11]

(17) $\mathbf{V}_{int} = \lambda Q_{\langle st,t\rangle}\lambda x_e \lambda w_s\colon \underline{\exists w'[\mathbf{V}_{decl}(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))(x)(w) \text{ is defined}]}.$
$\exists w'[\mathbf{V}_{decl}(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))(x)(w) \text{ is defined} \wedge \mathbf{V}_{decl}(\textsc{Ans}_{w'}(Q))(x)(w)]$
(after Spector and Egré, 2015: 1767)

(18) $\textsc{Exh}_Q(p) := \lambda w.[\textsc{Ans}_w(Q) = p]$ (after Spector and Egré, 2015: 1747)

According to this analysis, roughly, $\ulcorner x$ *Vs* $Q\urcorner$ asserts that there is a world w' such that '$x\ Vs\ \textsc{Ans}_{w'}(Q)$' is true, while presupposing that the same world satisfies the presupposition of '$x\ Vs\ \textsc{Exh}_Q(\textsc{Ans}_{w'}(Q))$'.[12]

For instance, sentences involving *know*/*be certain* and a singular *which*-question are analyzed as follows:

(19) a. *Max knows which student smokes* $\rightsquigarrow$
$\lambda w_s\colon \underline{\exists w'[\mathbf{know}_w(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))(\mathbf{m}) \text{ is defined}]}.$
$\exists w'[\mathbf{know}_w(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))(\mathbf{m}) \text{ is defined} \wedge \mathbf{know}_w(\textsc{Ans}_{w'}(Q))(\mathbf{m})]$

b. *Max is certain (about) which student smokes* $\rightsquigarrow$
$\lambda w_s\colon \underline{\exists w'[\mathbf{certain}_w(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))(\mathbf{m}) \text{ is defined}]}.$
$\exists w'[\mathbf{certain}_w(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))(\mathbf{m}) \text{ is defined} \wedge \mathbf{certain}_w(\textsc{Ans}_{w'}(Q))(\mathbf{m})]$

This analysis makes different predictions regarding the projection of the UP from the Dayal-style analysis sketched in the previous section. To see what the predicted presuppositions underlined in (19) amount to, we have to consider both the presupposition

[11]Spector and Egré (2015) also posit a variant of this lexical rule where answer in the assertive condition is also exhaustified, in order to capture the strongly-exhaustive reading. Also, here I glosses over issues concerning the presuppositional monotonicity and the 'sensitivity to false answers'. These issues are discussed in Sections 6 and 7 of Spector and Egré (2015). This simplification does not affect the argument here.

[12]Spector and Egré's (2015) lexical rule requires that the answerhood in the presupposition always involves Exh, i.e., it is the 'strongly-exhaustive' one, even if the answerhood in the assertion is a 'weakly-exhaustive' one. This is based on their observation that emotive veridicals like *surprise*, which is typically associated with a weakly-exhaustive answer in their assertion involves a strongly exhaustive reading in the presupposition (Spector and Egré, 2015: 1762-1764).

triggered by $\textsc{Ans}$ and that triggered by the predicates **know**/**certain**. The latter can be modeled after the presuppositional behaviors of *know*/*be certain* when they embed declarative complements, as follows:

(20) a. Max knows that Ash smokes. $\overset{\text{presup}}{\Rightarrow}$ Ash smokes.

b. Max is certain that Ash smokes.
$\overset{\text{presup}}{\Rightarrow}$ It is compatible with Max's beliefs that Ash smokes.

Given these presuppositional behaviors of *know* and *be certain*, we assume that **know** and **certain** have the following presuppositions:

(21) For all w, p and x (which themselves do not have presuppositions),

a. $\mathbf{know}_w(p)(x)$ is defined iff $p(w)$ (factivity)

b. $\mathbf{certain}_w(p)(x)$ is defined iff $p \cap \mathbf{Dox}^x_w \neq \varnothing$

With this in mind, the predicted presuppositions in (19) can be rewritten as follows:

(22) a. $\exists w'[\mathbf{know}_w(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))(\mathbf{m})$ is defined$]$
$\Leftrightarrow \exists w'[\textsc{Ans}_{w'}(Q)$ is defined $\wedge\, \textsc{Exh}_Q(\textsc{Ans}_{w'}(Q))(w)]$

b. $\exists w'[\mathbf{certain}_w(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))(\mathbf{m})$ is defined$]$
$\Leftrightarrow \exists w'[\textsc{Ans}_{w'}(Q)$ is defined $\wedge\, \textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)) \cap \mathbf{Dox}^{\mathbf{m}}_w \neq \varnothing]$

Let us consider (22a) and (22b) in turn. First, the predicted presupposition for *Max knows which student smokes* in (22a) states that 'there is a world such that the Dayal-answer in that world is defined and the exhaustification of that answer is true in the evaluation world (w)'. This correctly predicts that the UP is satisfied in the evaluation world because the exhaustification of $\textsc{Ans}_{w'}(Q)$ for any w' states that exactly one student smokes.

Turning to (22b), i.e., the predicted presupposition of *Max is certain (about) which student smokes*, we have 'there is a world such that the Dayal-answer in that world is defined and the exhaustification of that answer is compatible with Max's beliefs in the evaluation world (w)'. This does not predict matrix projection of the UP, unlike the simple extension of Dayal's analysis. This is empirically correct. However, it does not predict that Max *believes* the UP. Crucially, the UP triggered by $\textsc{Ans}$ does not project to Max's belief state in (22b) since the argument of **certain**, i.e., $\textsc{Ans}'_w(Q)$, *itself* is not partial when it is defined. Instead, (22b) only predicts that Max's beliefs are *compatible* with the UP. This predicted presupposition is too weak in view of the empirical pattern we observed in the previous subsection.

Essentially the same problem arises with *agree*. A sentence with *agree* embedding an interrogative complement would be analyzed as follows in the S&E-style analysis:

(23) *Max agrees with Kim on which student smokes* $\rightsquigarrow$
$\lambda w_s\colon \underline{\exists w'[\mathbf{agree}_w(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))(\mathbf{k})(\mathbf{m}) \text{ is defined}]}.$
$\exists w'[\mathbf{agree}_w(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))(\mathbf{k})(\mathbf{m})$ is defined $\wedge\, \mathbf{agree}_w(\textsc{Ans}_{w'}(Q))(\mathbf{k})(\mathbf{m})]$

The presupposition predicted in (23) can be rewritten as follows, given that $\mathbf{agree}_w(p)(y)(x)$ $\overset{\text{presup}}{\Rightarrow}$ $\mathbf{Dox}^y_w \subseteq p$, that is, ⌜*x agrees with y that p*⌝ presupposes that y believes that p.

(24) $\exists w'[\mathbf{agree}_w(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))(\mathbf{k})(\mathbf{m})$ is defined$]$
$\Leftrightarrow \exists w'[\textsc{Ans}_{w'}(Q)$ is defined $\wedge\, \mathbf{Dox}^{\mathbf{k}}_w \subseteq \textsc{Exh}_Q(\textsc{Ans}_{w'}(Q))]$

Again, this does not amount to a matrix projection of the UP. To this extent, the prediction is correct. However, the account fails to capture the fact that the UP projects to the attitude holder *Max*'s beliefs, as (24) only predicts that the UP projects to *Kim*'s beliefs.

In sum, although S&E's analysis is designed to address some of the issues surrounding non-veridical question-embedding predicates, it fails to completely capture the projection pattern of the UP with non-veridical predicates. In particular, the analysis fails to capture the fact that the UP projects to the attitude holder's beliefs with non-veridical predicates.

2.4 Uegaki (2015)

Uegaki (2015) provides a solution to the problem pointed out above regarding the contrast between *know* and *be certain*, by letting the question-embedding predicates relate the subject's attitude representation to the Dayal-answer of a question in different ways. This is done by treating Ans as a part of the predicate meaning. Specifically, *know* and *be certain* are analyzed as follows:

(25) a. *know* $\rightsquigarrow \lambda Q_{\langle st,t\rangle}\lambda x_e.\mathbf{know}_w(x, \textsc{Ans}_w(Q))$
b. *be certain* $\rightsquigarrow \lambda Q_{\langle st,t\rangle}\lambda x_e.\forall v[v \in \mathbf{Dox}^x_w \rightarrow \mathbf{certain}_w(x, \textsc{Ans}_v(Q))]$

Let us consider the predictions of this analysis in turn. In (25a), *know* is analyzed as a question-taking predicate that relates an individual to the Dayal-answer of a question. Thus, this treatment derives the same interpretation for a sentence like *Max knows which student smokes* as the Dayal-style analysis does in (10). Thus, the analysis preserves Dayal's correct prediction that UP projects to the matrix level.

On the other hand, *be certain* in (25b) predicts something different from the Dayal-style analysis. According to (25b), ⌜*x is certain (about) Q*⌝ is true iff for all worlds compatible with x's beliefs, x is certain that the Dayal-answer of Q *in that world* is true. The following exemplifies the treatment of the embedding of singular-*which* complement according to this analysis:

(26) *Max is certain which student smokes* $\rightsquigarrow$
$\forall v[v \in \mathbf{Dox}^{\mathbf{m}}_w \rightarrow \mathbf{certain}_w(\mathbf{m}, \textsc{Ans}_v(Q))]$

The interpretation given in (26) roughly states that Max believes that he knows which student smokes. This matches the intuitive interpretation of *certain-wh*. In particular, it captures the obligatory strong exhaustivity of a question embedded under *be certain*, as Uegaki (2015) points out.[13]

What is important for our purposes is the fact that the analysis captures the projection pattern of the UP with *be certain*. Assuming universal projection out of universal

[13]See Theiler et al. 2018 for a similar treatment of *be certain* in terms of the notion of introspection.

quantification, (26) is defined only if $\forall v[v \in \mathbf{Dox}^{\mathbf{m}}_w \rightarrow \textsc{Ans}_v(Q)$ is defined]. This holds only if Max believes that exactly one student smokes. What is crucial here is the fact that the world with respect to which Ans is evaluated is not the matrix evaluation world, but is bound by the universal quantification over the subject's belief worlds. This is made possible in Uegaki (2015) by letting Ans be part of the lexical semantics of the embedding predicate, rather than an independent operator that feeds a propositional argument to the embedding predicate, as in Dayal (1996).

The presupposition projection behavior of *agree* discussed in the previous section remains to be a problem for Uegaki (2015) since it is not straightforward to define a plausible lexical entry for *agree* along the lines of (25) that would derive the projection behaviors. Furthermore, as we will see in the next subsection, even if such a lexical entry were possible, the analysis would face problems when it is extended to declarative complements.

Uegaki's (2015) analysis faces further problems when viewed as a part of the general semantic theory of question-embedding, which would encompass a *unified* account of the presuppositions of ⌜x *Vs* Q⌝ and ⌜x *Vs that* p⌝ across different predicates V, as envisioned in Section 1. This is so since extending the analysis to the declarative-embedding cases would make empirically incorrect predictions. To illustrate the problems, we first have to make it explicit how Uegaki's (2015) treatment of interrogative complements embedded under *know* and *be certain* can be integrated with an analysis of declarative complements.

2.4.1 Uniform semantics of complementation

Uegaki's (2015) analysis is based on the UNIFORM SEMANTICS OF COMPLEMENTATION, where declarative and interrogative complements share the same semantic type, i.e., a set of propositions, which is selected by clause-embedding predicates such as *know* and *be certain* (see also Theiler et al. 2018).[14] In particular, Uegaki (2015) analyzes declarative complements as denoting the singleton set consisting of the proposition it traditionally denotes. For instance, the declarative complement *that Ash smokes* is analyzed as follows, where A is the proposition that Ash smokes.

(27) *that Ash smokes* $\rightsquigarrow \{A\}$

Given this, declarative-embedding under *know* is analyzed as follows:

(28) *Max knows that Ash smokes* $\rightsquigarrow \mathbf{know}_w(\mathbf{m}, \textsc{Ans}_w(\{A\}))$

The interpretation in (28) presupposes that Ash smokes, and asserts that Max knows that Ash smokes. Here, the presupposition of Ans boils down to the factivity presupposition that Ash smokes.[15] This is an empirically correct prediction.

[14] Selectional restrictions of predicates like *believe* and *wonder* are analyzed as arising from the semantic incompatibility between their lexical semantics and the particular complement type. See e.g., Ciardelli and Roelofsen (2015); Uegaki (2015); Theiler et al. (2017).

[15] Note that it is reasonable to assume that the predicate **know** in the intermediate language also

2.4.2 Problem 1: *be certain that*

A problem arises when we consider declarative-embedding under *be certain*. The following is the interpretation predicted by Uegaki (2015) for a sentence with *be certain* and a declarative complement.

(29) *Max is certain that Ash smokes* $\rightsquigarrow \forall v[v \in \mathbf{Dox}^{\mathbf{m}}_{w} \rightarrow \mathbf{certain}_{w}(\mathbf{m}, \textsc{Ans}_{v}(\{A\}))]$

Assuming a universal projection of presuppositions out of universal quantification (just as in the case of (26) above), we have that (29) presupposes $\forall v[v \in \mathbf{Dox}^{\mathbf{m}}_{w} \rightarrow \textsc{Ans}_{v}(\{A\})$ is defined]. Since $\textsc{Ans}_{v}(\{A\})$ is defined only if $A(v)$, this presupposition amounts to $\forall v[v \in \mathbf{Dox}^{\mathbf{m}}_{w} \rightarrow A(v)]$, i.e., Max believes that Ash smokes. Empirically, this presupposition seems too strong for (29). Rather than presupposing Max's belief that Ash smokes, (29) seems to presupposes that it is merely *compatible* with Max's beliefs that Ash smokes.

2.4.3 Problem 2: *agree that*

The second problem concerns *agree*. As discussed in Section 2.2, the presupposition of a *which*-question under *agree* projects both to the subject's and to the *with*-argument's beliefs, as illustrated below:

(30) *Does Max agree with Kim on which student smokes?*

1. $\overset{\text{presup}}{\Rightarrow}$ 'Max believes that exactly one student smokes.'
2. $\overset{\text{presup}}{\Rightarrow}$ 'Kim believes that exactly one student smokes.'

An analysis along the lines of Uegaki (2015) would capture this projection behavior by defining a lexical entry for *agree* in terms of Ans that derives the following:

(31) ⌜*x agrees with y on Q*⌝ presupposes:

1. x believes that $\textsc{Ans}_{w}(Q)$ is defined.
   $\Leftrightarrow$ x believes that Q has a strongest true member.
2. y believes that $\textsc{Ans}_{w}(Q)$ is defined.
   $\Leftrightarrow$ y believes that Q has a strongest true member.

As briefly mentioned above, it is not straightforward to define a plausible lexical entry for *agree* that derives these presuppositions. What is more crucial is that, regardless of whether such a lexical entry can be defined, (31) would make incorrect predictions about the presuppositions of the declarative-embedding under *agree*. This is so since we would have the following as the declarative-embedding case where Q is a singleton set:

presupposes the truth of the propositional argument. In such a case, the factivity is redundantly stated in (28), once by the factivity of **know** and once by Ans. See Uegaki (2015) for a formulation where the attitude predicate in the intermediate language corresponding to **know** is non-veridical, and thus the factivity is solely contributed by Ans.

(32) $\ulcorner$*x agrees with y that p*$\urcorner$ presupposes:

1. x believes that $\{p\}$ has a strongest true member. $\Leftrightarrow$ x believes p.
2. y believes that $\{p\}$ has a strongest true member. $\Leftrightarrow$ y believes p.

This is an incorrect prediction, as an *agree-that* sentence does *not* presuppose that the subject believes the complement, as can be seen in the following example:

(33) Does Max **agree** with Kim that Ash smokes?
$\overset{\text{presup}}{\Rightarrow}$ Kim believes that Ash smokes.
$\overset{\text{presup}}{\not\Rightarrow}$ Max believes that Ash smokes.

Taking a step back, both with *be certain* and with *agree*, we have seen that the UP projecting to the subject's belief poses a problem for Uegaki (2015). The core of the problem is the same across the two predicates. Even if the analysis correctly predicts the projection behavior in the interrogative case, it incorrectly predicts that a similar pattern would hold for the declarative case. This is by virtue of the fact that the presupposition is encoded in the lexical semantics of the embedding predicate and that declarative complements are treated as singleton proposition-sets.[16]

2.5 Diagnosing the problems

So far, I have considered three existing analyses concerning the uniqueness/existential presupposition of *wh*-complements, i.e., Dayal (1996), an extension of Dayal (1996) based on Spector and Egré (2015) and Uegaki (2015). I have argued that none of them can fully capture the projection patterns with different embedding predicates. In this section, I will state the problem in more general terms.

Abstractly, we can understand the difference between Dayal (1996) and Spector and Egré (2015) on the one hand and Uegaki (2015) on the other as the difference in the locus of the presupposition carrier, i.e., which lexical item is defined as a partial function. In Dayal (1996) and its extension following Spector and Egré (2015), the relevant presupposition is carried by the ANS-operator. On the other hand, in Uegaki

[16]It is in principle possible to define a lexical entry for the relevant predicates that avoids this issue, by making the entry sensitive to the cardinality of the proposition-set in the first argument. For example, the following lexical entry for *be certain* captures the presupposition projection behavior in the interrogative-embedding case while avoiding the problematic prediction in the declarative-embedding case:

(1) $$\textit{be certain} \rightsquigarrow \lambda Q_{\langle st,t\rangle}\lambda x_e. \begin{bmatrix} |Q| > 1 \rightarrow \forall v[v \in \mathbf{Dox}_x^w \rightarrow \mathbf{certain}_w(x, \textsc{Ans}_v(Q))] \\ |Q| = 1 \rightarrow \mathbf{certain}_w(x, \bigcup Q) \end{bmatrix}$$

However, it is plausible to assume that there is a general constraint against a lexical entry that is sensitive to the cardinality of the proposition-set. This is so, since allowing such lexical entries leads to the danger of ruling in various empirically implausible lexical entries. One such lexical entry is **shknow* discussed by Spector and Egré (2015), which would mean 'know' with declarative complements and 'wonder' with interrogative complements. See George (2011: §4.5.2), Theiler et al. (2018: §5) and Uegaki (2019: §6) for related discussion.

(2015), it is carried by the embedding predicate (since the ANS-operator is part of the predicate's lexical semantics). This is schematically represented in (34), where the boxes mark the items that carry the presupposition.

(34) $\ulcorner x\ Vs\ Q \urcorner \rightsquigarrow$

(i) $V(x, \boxed{\textsc{Ans}_w}(Q))$ (Dayal, 1996)

(ii) $\exists w'[V(x, \boxed{\textsc{Ans}_{w'}}(Q))]$ (Spector and Egré, 2015)

(iii) $\boxed{V}(x, Q)$ (Uegaki, 2015)

In (i), ANS is defined as a partial function that triggers the maximality presupposition. The application of ANS to Q is defined if this presupposition is met with respect to the matrix evaluation world and Q. Crucially, the proposition resulting from the application does not carry the presupposition. The problem with this treatment is that it incorrectly predicts the presupposition to project to the matrix level regardless of the embedding predicate V.

In (ii), the presupposition of ANS is not automatically projected to the matrix level because of the existential quantification into the world in which it is evaluated. It is projected to the matrix level in the case where V is a factive predicate like *know*, by virtue of the factivity presupposition. However, this is not the case with non-veridical predicates like *be certain* and *agree*. Although the lack of the matrix projection with non-veridical predicates is a correct prediction, the account fails to predict that the presupposition of ANS projects to the attitude holder's beliefs. This is so because $\textsc{Ans}_{w'}(Q)$ itself does not carry the presupposition, in the sense that it is not a partial function.

In (iii), the maximality presupposition is encoded in the lexical semantics of the predicate in ways that vary across predicates, deriving lexically-specific projection patterns. As such, this treatment overcomes the problem with the Dayal-style analysis in (i). However, it encounters another problem, i.e., it makes an incorrect prediction with respect to declarative complements. Since the presupposition is encoded in the predicate, the analysis incorrectly predicts that the presupposition shows up with declarative complements as well, assuming that the same lexical entry is used for both interrogative and declarative complementation. The prediction with respect to the declarative complements is not a problem for the Dayal-style analysis schematized in (i) and the S&E-style analysis in (ii) since ANS appears only in interrogative complements in these analyses.

At this point, another analytic possibility presents itself: The presupposition can in principle be carried by the complement meaning. We can schematize this possibility as follows:

(35) $\ulcorner x\ Vs\ Q \urcorner \rightsquigarrow V(x, \boxed{f(Q)})$

Here, f is a (possibly vacuous) operator such as ANS that takes the question denotation as an input and passes it to the predicate meaning. What is crucial in this schema is that the semantic argument of the predicate corresponding to the interrogative complement carries the presupposition. Note that this possibility is different from that in (34-i/ii), since $f(Q)$ *as a whole* in (35) carries the relevant presupposition while $\textsc{Ans}(Q)$ in (34-i/ii) doesn't.

It is clear at this point that the analysis schematized in (35) does not run into the same problem as the Uegaki-style analysis in (34-iii), i.e., the incorrect prediction with declarative complements, since the presupposition would not be triggered in the case of declarative complementation. The remaining questions are whether the analysis can overcome the problem for the analyses in (34-i/ii), i.e., the variation in the projection patterns across predicates, and whether it can correctly capture the projection of the UP to the attitude holder's beliefs in the case of non-veridical predicates. In the next section, I will argue that the line of analysis in (35) provides a straightforward account of the variation in the projection patterns, once we analyze the answers in the question denotation as carrying the UP/EP.

3 Proposal: UP/EP comes from the answers

I propose that it is the complement meaning that carries the UP/EP of *wh*-complements, rather than an operator that outputs an answer proposition (which itself is devoid of the UP/EP) (Dayal, 1996) or a question-embedding predicate (Uegaki, 2015). Taking Spector and Egré's (2015) theory of question-embedding as a backdrop, I will present two ways of fleshing out this proposal: one is to encode the UP/EP in *each answer* of the question denotation (cf. Rullmann and Beck, 1998), and the other is to modify the Ans-operator so that it returns a presuppositional answer as its output.

Once the proposal is implemented in either of these two ways, the projection pattern of the UP/EP can be straightforwardly accounted for. In this section, I will demonstrate this with the now-familiar three predicates: *know*, *be certain* and *agree*. In doing so, I will assume two things as given: (i) presuppositions of sentences involving the predicates embedding a *declarative* complement containing a presupposition and (ii) Spector and Egré's (2015) existential semantics for question-embedding, which we have already touched on in Section 2.3. Below, I will discuss these assumptions in turn.

3.1 The presuppositions of declarative-embedding sentences

The presuppositions of sentences involving *know*, *be certain* and *agree* embedding a presuppositional declarative complement can be tested by considering the following kind of examples:

(36) a. Max knows that the unicorn danced.
$\overset{\text{presup}}{\Rightarrow}$ There is a unique unicorn & that it danced & Max believes that there is a unique unicorn.

b. Max is certain that the unicorn danced.

$\overset{\text{presup}}{\Rightarrow}$ Max believes there is a unique unicorn & it is compatible with Max's beliefs that it danced.

c. Max agrees with Kim that the unicorn danced.
$\overset{\text{presup}}{\Rightarrow}$ Both Max and Kim believe that there is a unique unicorn & Kim believes that it danced.

Schematically, we can write the presuppositions observed in (36) as follows, where the operator π retrieves the presupposition from a proposition, as defined in (37).

(37) $\pi(p) := \lambda w.[p(w) = 1 \vee p(w) = 0]$

(38) **Presuppositions of $\ulcorner$*x Vs that p*$\urcorner$**

For all x, w and p,

a. $\mathbf{know}_w(x, p)$ is defined iff $p(w) \wedge \mathbf{Dox}^x_w \subseteq \pi(p)$

b. $\mathbf{certain}_w(x, p)$ is defined iff $\mathbf{Dox}^x_w \subseteq \pi(p) \wedge \exists w'[w' \in \mathbf{Dox}^x_w \wedge p(w')]$

c. $\mathbf{agree}_w(x, y, p)$ is defined iff
$\mathbf{Dox}^x_w \subseteq \pi(p) \wedge \mathbf{Dox}^y_w \subseteq \pi(p) \wedge \forall w'[w' \in \mathbf{Dox}^y_w \rightarrow p(w')]$

From a theoretical perspective, the presuppositions in (38) can be understood as the combination of (a) the projection of the presupposition of the complement and (b) the presuppositions triggered by the predicates themselves. The former is constant across *know*, *be certain* and *agree*, and arguably universal across all attitude predicates: $\ulcorner$*x Vs p*$\urcorner$ presupposes that x believes $\pi(p)$ (Karttunen, 1973; Heim, 1992). This is stated below.

(39) For any attitude predicate $\mathbf{V}$ and for all x, w and p,
$\mathbf{V}_w(x, p)$ is defined iff $\mathbf{Dox}^x_w \subseteq \pi(p)$ (Karttunen, 1973)

The latter is lexically-dependent presuppositions triggered by the embedding predicates. Specifically, *know* has the factivity presupposition that the complement is true; *be certain* has the presupposition that the complement is compatible with the subject's beliefs; $\ulcorner$*x agrees with y that p*$\urcorner$ presupposes that y believes p. These presuppositions are formally represented as follows:

(40) for all x, w, and presupposition-free p,

a. $\mathbf{know}_w(x, p)$ is defined iff $p(w)$

b. $\mathbf{certain}_w(x, p)$ is defined iff $\exists w'[w' \in \mathbf{Dox}^x_w \wedge p(w')]$

c. $\mathbf{agree}_w(x, y, p)$ is defined iff $\forall w'[w' \in \mathbf{Dox}^y_w \rightarrow p(w')]$

The presuppositions schematized in (38) can be derived from the lexically-specific presuppositions in (40) and the general presupposition projection pattern in (39).[17]

[17] The second conjunct of (38c) is the result of the universal presupposition projection from the consequent of $\forall w'[w' \in \mathbf{Dox}^y_w \rightarrow p(w')]$.

3.2 The existential semantics for question-embedding

The semantics of question-embedding I will adopt is the existential semantics by Spector and Egré (2015). For concreteness, I will assume the general lexical rule by S&E that converts a proposition-taking denotation $\mathbf{V}_{decl}$ to a question-taking denotation $\mathbf{V}_{int}$, repeated below from Section 2.3:

(41) **The lexical rule generating question-embedding predicates**

$$\mathbf{V}_{int} = \lambda Q_{\langle st,t \rangle} \lambda x_e \lambda w_s \colon \underline{\exists w'[\mathbf{V}_{decl}(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))(x)(w) \text{ is defined}]}. \\ \exists w'[\mathbf{V}_{decl}(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))(x)(w) \text{ is defined} \wedge \mathbf{V}_{decl}(\textsc{Ans}_{w'}(Q))(x)(w)]$$

I will adopt this formulation since it is shown by Spector and Egré (2015) to be compatible with the detailed empirical considerations concerning aspects of the question-embedding phenomena other than the UP/EP. However, my analysis of the projection patterns of the UP/EP only relies on the *presuppositional* part of this formulation. Furthermore, the analysis does not rely on the presence of the Exh-operator in the presupposition. That is, the derivations of concrete examples presented below will go through regardless of the presence of Exh. Thus, as long as the semantics of the question-embedding predicts the existentially quantified presupposition (over the answers in the question denotation, or the worlds with respect to which Ans is evaluated), the analysis makes the same predictions. See Uegaki (2018) for a version of the analysis that makes use of a existential quantification over answers.

See the appendix for the discussion of how the semantics of question-embedding in (41) can be made compatible with the various levels of exhaustivity in the interpretation of embedded questions and with the question-oriented semantics (rather than proposition-oriented semantics) for question-embedding predicates (e.g., Uegaki, 2015; Theiler et al., 2018).

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3.3 Implementation I: UP/EP in each answer

Now that we have established the two background assumptions, i.e., the presuppositions of declarative-embedding sentences and the existential semantics for question-embedding, we are in a position to illustrate the analysis of the projection of UP/EP. In this section, I will present an implementation of the proposal where the UP/EP is encoded in each answer of the question denotation. To motivate the analysis, we start by observing the projection behavior of non-UP/EP presuppositions triggered within an interrogative complement.

3.3.1 Presupposition triggers within an interrogative complement

Consider example (42), a *wh*-clause involving a singular definite DP inside it:

(42) who caught the unicorn

Here, it is reasonable to think that each answer in the Hamblin-denotation of this clause carries a presupposition about the unique existence of a unicorn. That is, the Hamblin-denotation of (42) would look like the following:

(43) $$\left\{\begin{array}{c} \lambda w' : \underline{\exists! x[\mathbf{unicorn}_{w'}(x)]}.\ \mathbf{caught}_{w'}(a,x), \\ \lambda w' : \underline{\exists! x[\mathbf{unicorn}_{w'}(x)]}.\ \mathbf{caught}_{w'}(b,x), \\ \lambda w' : \underline{\exists! x[\mathbf{unicorn}_{w'}(x)]}.\ \mathbf{caught}_{w'}(c,x) \end{array}\right\}$$

What is crucial for us is that the projection patterns of this presupposition under various embedding predicates behaves exactly like the projection patterns of the UP of singular-*which* questions observed in the previous section:

(44) a. Max knows who caught the unicorn.
$\overset{\text{presup}}{\Rightarrow}$ There is a unique unicorn.

b. Max is certain who caught the unicorn.
$\overset{\text{presup}}{\Rightarrow}$ Max believes that there is a unique unicorn.

c. Max agrees with Kim on who caught the unicorn.
$\overset{\text{presup}}{\Rightarrow}$ Max and Kim believe that there is a unique unicorn.

As can be seen above, the presupposition that there is a unique unicorn projects to the matrix level with *know*, to the subject's beliefs with *be certain*, and both to the subject's and to the *with*-argument's beliefs with *agree*.

The parallel in projection patterns in (44) and the UP of singular-*which* questions is straightforwardly explained if the latter is encoded in each proposition in the Hamblin denotation, just as in (43). That is, *which student smokes* has the Hamblin-denotation that looks like (45), where each proposition in the set carries the proposition that exactly one student smokes.

(45) *which student smokes* $\rightsquigarrow$

$$\left\{\begin{array}{c} \lambda w' : \underline{\exists! x[\mathbf{student}_{w'}(x) \wedge \mathbf{smoke}_{w'}(x)]}.\ \mathbf{student}_{w'}(a) \wedge \mathbf{smoke}_{w'}(a), \\ \lambda w' : \underline{\exists! x[\mathbf{student}_{w'}(x) \wedge \mathbf{smoke}_{w'}(x)]}.\ \mathbf{student}_{w'}(b) \wedge \mathbf{smoke}_{w'}(b), \\ \lambda w' : \underline{\exists! x[\mathbf{student}_{w'}(x) \wedge \mathbf{smoke}_{w'}(x)]}.\ \mathbf{student}_{w'}(b) \wedge \mathbf{smoke}_{w'}(c) \end{array}\right\}$$

3.3.2 Application of the analysis to particular predicates

Once the UP/EP is encoded in each proposition in the Hamblin denotation of a *wh*-complement, its projection pattern can be straightforwardly accounted for. In the rest of this section, I will demonstrate this with the three predicates: *know*, *be certain* and *agree*.

As stated above, my analysis takes two things as given: the presuppositions of declarative-embedding sentences and the existential semantics for question-embedding along the lines of Spector and Egré (2015). These two assumptions are repeated below for easier reference.

(38) **Presuppositions of ⌜*x Vs that p*⌝**

For all x, w and p,

a. $\mathbf{know}_w(x,p)$ is defined iff $p(w) \wedge \mathbf{Dox}^x_w \subseteq \pi(p)$

b. $\mathbf{certain}_w(x,p)$ is defined iff $\mathbf{Dox}^x_w \subseteq \pi(p) \wedge \exists w'[w' \in \mathbf{Dox}^x_w \wedge p(w')]$

c. $\mathbf{agree}_w(x,y,p)$ is defined iff
$\mathbf{Dox}^x_w \subseteq \pi(p) \wedge \mathbf{Dox}^y_w \subseteq \pi(p) \wedge \forall w'[w' \in \mathbf{Dox}^y_w \rightarrow p(w')]$

(41) **The lexical rule generating question-embedding predicates**

$$\mathbf{V}_{int} = \lambda Q_{\langle st,t\rangle}\lambda x_e \lambda w_s\colon \underline{\exists w'[\mathbf{V}_{decl}(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))(x)(w) \text{ is defined}]}.$$
$$\exists w'[\mathbf{V}_{decl}(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))(x)(w) \text{ is defined} \wedge \mathbf{V}_{decl}(\textsc{Ans}_{w'}(Q))(x)(w)]$$

Also, in the illustrations to follow, I will repeatedly use the following fact:

(46) **Fact**. Let Q be a set of propositions such that there is a proposition p that each member of Q presupposes p and nothing else. Then, for all worlds w such that $\textsc{Ans}_w(Q)$ is defined, $\textsc{Exh}_Q(\textsc{Ans}_w(Q))$ presupposes p and nothing else (i.e., for all worlds w', $\textsc{Exh}_Q(\textsc{Ans}_w(Q))(w')$ is defined iff $p(w')$).

More concretely, this means that, if the UP/EP is encoded in each proposition in Q, $\textsc{Exh}_Q(\textsc{Ans}_w(Q))$ also presupposes the UP/EP for all w that makes $\textsc{Ans}_w(Q)$ defined. It is easy to see why this is the case, as $\textsc{Ans}$ is defined to pick out a certain member of Q and $\textsc{Exh}_Q(\textsc{Ans}_w(Q))$ has the same presupposition as $\textsc{Ans}_w(Q)$.

Know The matrix projection of the UP with *know* can be derived as in (48), where I use the shorthands in (47):

(47) a. *which student smokes* $\rightsquigarrow Q$

b. $\mathbf{up} := \lambda w.\exists! x[\mathbf{student}_w(x) \wedge \mathbf{smoke}_w(x)]$

(48) *Max knows which student smokes.* $\overset{\text{presup}}{\Rightarrow}$

$\exists w'[\mathbf{know}_w(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))(\mathbf{m}) \text{ is defined}]$ [by (41)]

i. $\Leftrightarrow \exists w'[\textsc{Ans}_{w'}(Q) \text{ is defined} \wedge \textsc{Exh}_Q(\textsc{Ans}_{w'}(Q))(w) \wedge \mathbf{Dox}^{\mathbf{m}}_w \subseteq \pi(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))]$
[by the presupposition of Ans and (38a)]

ii. $\Leftrightarrow \exists w'[\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q))(w) \wedge \mathbf{Dox}^{\mathbf{m}}_w \subseteq \pi(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))]$
[the 1st conj. of (i) is entailed by the 2nd conj.]

iii. $\Leftrightarrow \exists w'[\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q))(w) \wedge \mathbf{Dox}^{\mathbf{m}}_w \subseteq \mathbf{up}]$ [by Fact (46)]

iv. $\Leftrightarrow \exists w'[\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q))(w)] \wedge \mathbf{Dox}^{\mathbf{m}}_w \subseteq \mathbf{up}$

The derivation can be roughly paraphrased as follows. First of all, the sentence has the presupposition predicted by the lexical rule in (41). This can be rewritten as in line (i) given the projection of the presupposition of $\textsc{Ans}$ itself and that of the *know-that* sentence in (38a). Line (i) can be paraphrased as 'there is a defined Dayal-answer of Q that is true and whose presupposition is believed by Max'. Since '$\textsc{Ans}_{w'}(Q)$ is defined' (i.e., 'the Dayal-answer is defined') is entailed by '$\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q))(w)$' (i.e., 'the exhaustification of the Dayal-answer is true'), the former conjunct can be dropped as in line (ii). By Fact (46), this can be rewritten as in line (iii), i.e., 'there is a Dayal-answer of Q that is true and Max believes **up**'. Finally, since the variable w' does not appear in the second conjunct of (iii), it is equivalent to (iv).

Now, it can be proved that the first conjunct of (iv) is equivalent to **up**:

(49) $\exists w'[\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q))(w)] \Leftrightarrow \mathbf{up}(w)$

This is so because of the following reasoning: the left-hand side of (49) states that the exhaustification of some Dayal-answer is true in w. Furthermore, this Dayal-answer presupposes **up** by assumption. This happens if and only if **up** is true in w. Note that this reasoning holds regardless of the presence of the Exh-operator in the left-hand side.

All in all, the predicted presupposition of (48) can be paraphrased as follows:

(50) 'Exactly one student smokes & Max believes that exactly one student smokes.'

The first conjunct amounts to the UP projected to the matrix level, which we have observed in the previous section. On the other hand, the belief presupposition in the second conjunct has not been mentioned in connection to *know* up to this point. In fact, native speakers report that the belief presupposition seems quite weak with *know*, compared with *be certain*.

(51) John doesn't know which student smokes.
?? $\overset{\text{presup}}{\Rightarrow}$ John believes that exactly one student smokes.

Below, I suggest a solution to this potential problem.

One possible explanation for why the belief presupposition is felt weak in the case of *know* and other veridical predicates, compared to non-veridical predicates, stems from the pragmatics of sentence verification, especially that of truth-value-less sentences. As von Fintel (2004) and Abrusán and Szendrői (2013) show, speakers may judge a sentence as false even if it is semantically a presupposition failure when the sentence entails a piece of information serving as a 'foothold' for verification. To see this, consider the following pair of sentences:

(52) a. The king of France isn't bold.
b. The king of France hasn't visited Australia.

Both of these sentences are semantically presupposition failures. However, (52a) sounds odd to most English speakers whereas (52b) tend to be felt true (Abrusán and Szendrői, 2013). One way to account for the contrast is to say that, in (52b), the object DP *Australia* denotes an entity that serves as a foothold for verification. That is, when speakers try to verify (52b), they base the judgment on Australia, an existing entity in the actual world, and determines the sentence's truth value depending on whether the set of its visitors include the king of France or not. Since the visitors of Australia do not include the king of France, (52b) is felt as true. On the other hand, (52a) does not mention any existing entity that can serve as the foothold for sentence verification in a similar manner.

Something similar can be said about the following kind of sentences:

(53) a. Max knows that the elevator in the South Building is out of service.

b. Max doesn't know that the elevator in the South Building is out of service.

$\overset{\text{presup}}{\Rightarrow}$ (i) there is exactly one elevator in the South Building; and (ii) Max believes that there is exactly one elevator in the South Building.

(54) a. Max knows that Kim stopped smoking.

b. Max doesn't know that Kim stopped smoking.

$\overset{\text{presup}}{\Rightarrow}$ (i) Kim used to smoke; and (ii) Max believes that Kim used to smoke.

These sentences have two kinds of presupposition: the presupposition of the complement and the presupposition that Max, the attitude holder, believes the presupposition of the complement. In the context where the first presupposition is met but the second presupposition is violated, the sentences are presupposition failures, semantically speaking. However, it is plausible that the first presupposition serves as the foothold for verification in the following way. The presupposition of the complement represents a preliminary fact that an attitude holder must know in order for them to be considered as knowing the information represented by the complement. If they don't know this preliminary fact, positive knowledge sentences, such as (53a,54a), are rejected as false. Conversely, their negative counterparts, such as (53b,54b), are judged as true.

A parallel argument holds for (48, 51). Here, the matrix UP of (48, 51) serves as a foothold for sentence verification. To see this, consider a context in which it is known that exactly one student smokes, but Max does not believe that there is a unique student smoker. In this context, the semantic analysis predicts (48, 51) to be presupposition failures, as the context violates the belief presupposition. However, once it is established that there is *actually* a unique student smoker, (48) can be rejected as false since Max does not know this preliminary fact. Conversely, (51), i.e., the negation of (48), can be judged true. To wrap up, the judgment that the UP belief presupposition (the second conjunct of (50)) is felt weak with veridical predicates can be explained in terms of the pragmatics of sentence verification.

Be certain We can derive the projection of the UE with *be certain* in a similar fashion.

(55) *Max is certain (about) which student smokes.* $\overset{\text{presup}}{\Rightarrow}$

$\exists w'[\mathbf{certain}_w(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))(\mathbf{m}) \text{ is defined}]$ [by (41)]

i. $\Leftrightarrow \exists w'[\textsc{Ans}_{w'}(Q) \text{ is defined} \wedge \mathbf{Dox}^{\mathbf{m}}_{w} \subseteq \pi(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q))) \wedge \exists w''[w'' \in \mathbf{Dox}^{\mathbf{m}}_{w} \wedge \textsc{Exh}_Q(\textsc{Ans}_{w'}(Q))(w'')]]$ [by the presup. of Ans and (38b)]

ii. $\Leftrightarrow \exists w'[\mathbf{Dox}^{\mathbf{m}}_{w} \subseteq \pi(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q))) \wedge \exists w''[w'' \in \mathbf{Dox}^{\mathbf{m}}_{w} \wedge \textsc{Exh}_Q(\textsc{Ans}_{w'}(Q))(w'')]]$ [the 1st conj. of (i) is entailed by the 3rd conj.]

iii. $\Leftrightarrow \exists w'[\mathbf{Dox}^{\mathbf{m}}_{w} \subseteq \mathbf{up} \wedge \exists w''[w'' \in \mathbf{Dox}^{\mathbf{m}}_{w} \wedge \textsc{Exh}_Q(\textsc{Ans}_{w'}(Q))(w'')]]$ [by Fact (46)]

iv. $\Leftrightarrow \mathbf{Dox}^{\mathbf{m}}_{w} \subseteq \mathbf{up} \wedge \exists w'[\exists w''[w'' \in \mathbf{Dox}^{\mathbf{m}}_{w} \wedge \textsc{Exh}_Q(\textsc{Ans}_{w'}(Q))(w'')]]$

v. $\Leftrightarrow \mathbf{Dox}^{\mathbf{m}}_{w} \subseteq \mathbf{up}$ [the 1st conj. of (iv) entails the 2nd conj.][18]

Paraphrasing, (i) says that there is a defined Dayal-answer of Q the presupposition of whose exhaustification is believed by Max and whose exhaustification is compatible with Max's beliefs. After dropping the first conjunct as it is entailed by the other conjuncts as in (ii), we can use Fact (46) to rewrite (ii) as in (iii), i.e., Max believes **up** and there is a Dayal-answer whose exhaustification is compatible with Max's beliefs. Since the latter conjunct is entailed by the first conjunct (see fn. 18 for a proof), we end up with the following presupposition predicted for (55).

(56) 'Max believes that exactly one student smokes.'

This is exactly what we observed earlier, i.e., the projection to the subject's beliefs.

Agree Finally, here is the derivation in the case of *agree*:

(57) *Max agrees with Kim on which student smokes.* $\overset{\text{presup}}{\Rightarrow}$

$\exists w'[\mathbf{agree}_{w}(\textsc{Exh}_{Q}(\textsc{Ans}_{w'}(Q)))(\mathbf{k})(\mathbf{m})$ is defined$]$ [by (41)]

i. $\Leftrightarrow \exists w'[\textsc{Ans}_{w'}(Q)$ is defined $\wedge\ \mathbf{Dox}^{\mathbf{m}}_{w} \subseteq \pi(\textsc{Exh}_{Q}(\textsc{Ans}_{w'}(Q))) \wedge \mathbf{Dox}^{\mathbf{k}}_{w} \subseteq \pi(\textsc{Exh}_{Q}(\textsc{Ans}_{w'}(Q))) \wedge \forall w''[w'' \in \mathbf{Dox}^{\mathbf{k}}_{w} \rightarrow \textsc{Exh}_{Q}(\textsc{Ans}_{w'}(Q))(w'')]]$
[by the presup. of ANS and (38c)]

ii. $\Leftrightarrow \exists w'[\mathbf{Dox}^{\mathbf{m}}_{w} \subseteq \pi(\textsc{Exh}_{Q}(\textsc{Ans}_{w'}(Q))) \wedge \mathbf{Dox}^{\mathbf{k}}_{w} \subseteq \pi(\textsc{Exh}_{Q}(\textsc{Ans}_{w'}(Q))) \wedge \forall w''[w'' \in \mathbf{Dox}^{\mathbf{k}}_{w} \rightarrow \textsc{Exh}_{Q}(\textsc{Ans}_{w'}(Q))(w'')]]$
[the 1st conj. of (i) is entailed by the 4th conj.]

iii. $\Leftrightarrow \exists w'[\mathbf{Dox}^{\mathbf{m}}_{w} \subseteq \mathbf{up} \wedge \mathbf{Dox}^{\mathbf{k}}_{w} \subseteq \mathbf{up} \wedge \forall w''[w'' \in \mathbf{Dox}^{\mathbf{k}}_{w} \rightarrow \textsc{Exh}_{Q}(\textsc{Ans}_{w'}(Q))(w'')]]$
[by Fact (46)]

iv. $\Leftrightarrow \mathbf{Dox}^{\mathbf{m}}_{w} \subseteq \mathbf{up} \wedge \mathbf{Dox}^{\mathbf{k}}_{w} \subseteq \mathbf{up} \wedge \exists w'[\forall w''[w'' \in \mathbf{Dox}^{\mathbf{k}}_{w} \rightarrow \textsc{Exh}_{Q}(\textsc{Ans}_{w'}(Q))(w'')]]$

v. $\Leftrightarrow \mathbf{Dox}^{\mathbf{m}}_{w} \subseteq \mathbf{up} \wedge \exists w'[\forall w''[w'' \in \mathbf{Dox}^{\mathbf{k}}_{w} \rightarrow \textsc{Exh}_{Q}(\textsc{Ans}_{w'}(Q))(w'')]]$
[the 3rd conj. of (iv) entails the 2nd conj.][19]

Again paraphrasing, (i) says that there is a defined Dayal-answer of Q the presupposition of whose exhaustification is believed by both Max and Kim and whose exhaustification is believed by Kim. After dropping the first conjunct and using Fact (46), it can be rewritten as in (iv), i.e., Max and Kim believes **up** and there is a Dayal-answer of Q whose exhaustification is believed by Kim. Since the latter conjunct entails that Kim believes **up** (see the proof in (19)), we end up with the following presupposition predicted for (57):

---

[18]This entailment holds for the following reason. If $\mathbf{Dox}^{\mathbf{m}}_{w} \subseteq \mathbf{up}$, we can find an answer $p* \in Q$ and a world $w*$ such that Max believes $p*$ and $p* = \textsc{Exh}_{Q}(\textsc{Ans}_{w*}(Q))$. Since Max believes $p*$, $\exists w''[w'' \in \mathbf{Dox}^{\mathbf{m}}_{w} \wedge p*(w'')]$ holds. This means $\exists w''[w'' \in \mathbf{Dox}^{\mathbf{m}}_{w} \wedge \textsc{Exh}_{Q}(\textsc{Ans}_{w*}(Q))(w'')]$ since $p* = \textsc{Exh}_{Q}(\textsc{Ans}_{w*}(Q))$. Hence, $\exists w'[\exists w''[w'' \in \mathbf{Dox}^{\mathbf{m}}_{w} \wedge \textsc{Exh}_{Q}(\textsc{Ans}_{w'}(Q)(w'')]]$.

[19]This entailment holds for the following reason. Since $\exists w'[\forall w''[w'' \in \mathbf{Dox}^{\mathbf{k}}_{w} \rightarrow \textsc{Exh}_{Q}(\textsc{Ans}_{w'}(Q))(w'')]]$ holds, $\mathbf{Dox}^{\mathbf{k}}_{w} \subseteq \textsc{Exh}_{Q}(\textsc{Ans}_{w'}(Q))$ for some world w'. For any world w, $\textsc{Exh}_{Q}(\textsc{Ans}_{w}(Q)) \subseteq \mathbf{up}$. Thus, $\mathbf{Dox}^{\mathbf{k}}_{w} \subseteq \mathbf{up}$ given the transitivity of $\subseteq$.

(58) 'Max believes that exactly one student smokes & there is an answer of Q such that Kim believes that its exhaustification is true.'

Not only does this capture the projection of the UP both to the subject's and to the *with*-argument's beliefs, but it also accounts for the asymmetry in strength between the subject's presupposed belief and the *with*-argument's presupposed belief, discussed earlier in footnote 10.

Summing up, once we assume that each answer in the question denotation carries the UP/EP, its projection patterns with *know*, *be certain* and *agree* can be accounted for, given independently motivated presupposition projection behaviors of the predicates as well as the existential semantics for question-embedding.

3.3.3 Internal composition of *wh*-complements

We now move on to the internal composition of *wh*-complements, i.e., how to compositionally derive the denotation of the complements where the answers carry the UP/EP. Broadly speaking, there are two approaches to this. One is to posit an additional operator that adds the Dayal-style maximality presupposition to the answers in the Hamblin denotation. The other is to treat *which*-NPs as a kind of definite description and derive a set of partial propositions through presupposition projection. The former approach is considered in Uegaki (2018). The latter approach has been adopted by Rullmann and Beck (1998) and Champollion et al. (2017) in their treatment of a presupposition associated with *which*-questions (to be detailed immediately below), and has been recently applied to the Dayal-style maximality presupposition by Hirsch and Schwarz (to appear).

As stated in the introduction, my goal in this paper is to investigate the projection patterns of UP/EP in the context of the general theory of question-embedding. Thus, the issue concerning the internal semantic composition of *wh*-complements is of secondary nature, as long as the composition guarantees that the individual answers in the question denotation carry the UP/EP. For this reason, I will keep my discussion of the internal composition of *wh*-complements minimal, and sketch a simple analysis in the latter approach mentioned above, i.e., deriving a set of partial propositions through the projection of presupposition from a definite-like semantics for *which*-NPs.

The analysis follows the insights of Rullmann and Beck (1998), who roughly treat the semantics of *which*-complements as follows:

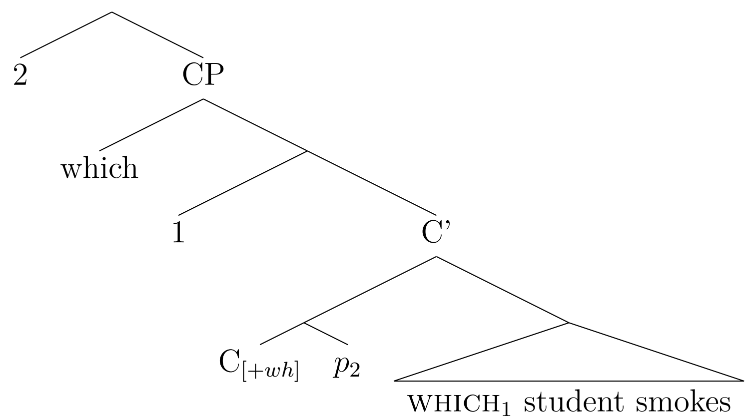
(59) *which student smokes*
$\rightsquigarrow$ {'the student a smokes', 'the student b smokes', 'the student c smokes',...}

R&B compositionally derive this using a definite-like semantics for *which*-NPs. As a result of the definiteness, each answer in (59) presupposes existence of a student. Note that the UP/EP we are after is stronger than the presupposition captured in (59). Rather than 'there is a student', we want each answer of the complement denotation to presuppose that there is a unique student *smoker*. To achieve this, I roughly treat the denotation of a *wh*-complement as follows, i.e., a set of propositions each identifying an individual with the student smoker.

(60) *which student smokes* $\rightsquigarrow$ {a is the student who smokes, b is the student who smokes, c is the student who smokes,...}

Formally, this is derived by assuming an LF that looks like the following:

(61)



This structure follows the LF-based rendition of Karttunen's (1977) analysis of questions by Heim (2016) and Dayal (2016), together with the additional assumption that the lower copy of *which* is realized in the LF as the operator WHICH, defined shortly below.[20] Furthermore, following Beck and Rullmann (1999), the NP-part of a *which*-phrase is left downstairs to allow both the *de re* and *de dicto* readings via world-indexing on the NP. (See Hirsch and Schwarz to appear for an analysis along the same lines.)

The translations of lexical items in (61) are given below, with the definition of the operator **the** in (63), akin to Link's (1983) σ-operator.

(62) a. which $\rightsquigarrow \lambda P_{\langle e,t\rangle}.\exists x[P(x)]$
b. $\mathrm{C}_{[+wh]} \rightsquigarrow \lambda p_{\langle s,t\rangle}[\lambda q_{\langle s,t\rangle}.p = q]$
c. $\textsc{which}_i \rightsquigarrow \lambda P_{\langle e,t\rangle}\lambda Q_{\langle e,t\rangle}.\ i = \mathbf{the}(\lambda x.P(x) \wedge Q^*(x))$

(63) $\mathbf{the} := \lambda P_{\langle e,t\rangle} \colon \underline{\exists x[P(x) \wedge \forall y[P(y) \rightarrow y \leq x]]}.\ \iota x[P(x) \wedge \forall y[P(y) \rightarrow y \leq x]]$

Hence, the structure in (61) is translated as in (64), and each answer of the question (64) represents has the presupposition in (65) projected from **the**.

(64) (61) $\rightsquigarrow \lambda p.\exists z[p = \lambda w.[z = \mathbf{the}(\lambda x.\ \mathbf{student}_w(x) \wedge \mathbf{smoke}^*_w(x))]]$

(65) $\exists x[\mathbf{student}_w(x) \wedge \mathbf{smoke}^*_w(x) \wedge \forall y[\mathbf{student}_w(y) \wedge \mathbf{smoke}^*_w(y) \rightarrow y \leq x]]$

Since **student** only ranges over singular individuals, (65) effectively states that there is only one student smoker.

The analysis sketched above concerns *which*-complements, and its prediction with respect to complements involving a simplex *wh*-phrase (e.g., *who smokes*) depends on

[20]In order for WHICH to take scope over the materials in the TP, it has to be assumed that a *wh*-phrase internally merges to a projection below CP before internally merging to the specifier of CP. Furthermore, WHICH has to be the realization of this intermediate copy rather than the lowest copy.

the definition of the *wh*-operator, WHO, in the lower copy position. Following Dayal's (1996) insight, the fact that simplex *wh*-complements only presuppose existence and not uniqueness is captured by treating WHO as number-neutral, as follows:

(66) $\text{WHO}_i \rightsquigarrow \lambda P_{\langle e,t\rangle}.\ i = \mathbf{the}(\lambda x.\mathbf{human}^*_w(x) \wedge P^*(x))$

3.4 Implementation II: Modifying the Ans-operator

In the previous subsection, I provided an implementation of the analysis where *each* answer of the embedded question caries the UP/EP. This, however, is not the only possible implementation of the analysis. An alternative way to formulate the analysis is to redefine Dayal's ANS-operator so that the relevant answer involved in the semantics of question-embedding always caries UP/EP. A version of ANS redefined in such a way, which I call ANS′, is given in (67b) below, together with the original definition of ANS in (67a).

(67) a. $\text{ANS}_w = \lambda Q_{\langle st,t\rangle} : \underline{\exists p \in Q[p = \text{MAX}_{\text{inf}}(Q,w)]}.\ \text{MAX}_{\text{inf}}(Q,w)$

b. $\text{ANS}'_w = \lambda Q_{\langle st,t\rangle} : \underline{\exists p \in Q[p = \text{MAX}_{\text{inf}}(Q,w)]}.$
$\lambda w' : \underline{\exists p \in Q[p = \text{MAX}_{\text{inf}}(Q,w')]}.\ \text{MAX}_{\text{inf}}(Q,w)(w')$

The ANS′-operator presupposes that the 'answer' that it returns presupposes that the input Q contains a maximally strong true answer.[21] For example, when $Q = \{A, B, C\}$ (where each proposition is independent from each other) and only A is true in w, we have the following:

(68) $\text{ANS}'_w(Q) = \lambda w' : \underline{\exists p \in \{A, B, C\}\,[p = \text{MAX}_{\text{inf}}(\{A, B, C\}, w')]}.\ A(w')$

That is, the answer A with the presupposition that the original proposition-set contains a maximally true answer.

Once we have this redefined version of the answerhood operator, the projection behavior of UP/EP can be captured in exactly the same ways as in the first implementation of the analysis. The other components of the analysis needed are the same as in the case of the first implementation: the existential semantics for question-embedding (though defined in terms of ANS′ instead of ANS) and the presuppositions of declarative-embedding sentences:

(69) **The lexical rule generating question-embedding predicates**
$\mathbf{V}_{int} = \lambda Q_{\langle st,t\rangle}\lambda x_e \lambda w_s : \underline{\exists w'[\mathbf{V}_{decl}(\text{EXH}_Q(\text{ANS}'_{w'}(Q)))(x)(w) \text{ is defined}]}.$
$\exists w'[\mathbf{V}_{decl}(\text{EXH}_Q(\text{ANS}'_{w'}(Q)))(x)(w) \text{ is defined} \wedge \mathbf{V}_{decl}(\text{ANS}'_{w'}(Q))(x)(w)]$

(38) **Presuppositions of ⌜*x Vs that p*⌝**
For all x, w and p,
a. $\mathbf{know}_w(x,p)$ is defined iff $p(w) \wedge \mathbf{Dox}^x_w \subseteq \pi(p)$

[21] The operator preserves the presupposition on Q (i.e., $\exists p \in Q[p = \text{MAX}_{\text{inf}}(Q,w)]$) from the original definition in order to guarantee that $\text{MAX}_{\text{inf}}(Q,w)$ is defined.

b. $\textbf{certain}_w(x,p)$ is defined iff $\textbf{Dox}^x_w \subseteq \pi(p) \wedge \exists w'[w' \in \textbf{Dox}^x_w \wedge p(w')]$

c. $\textbf{agree}_w(x,y,p)$ is defined iff

$$\textbf{Dox}^x_w \subseteq \pi(p) \wedge \textbf{Dox}^y_w \subseteq \pi(p) \wedge \forall w'[w' \in \textbf{Dox}^y_w \rightarrow p(w')]$$

The projections of the UP/EP with *know*, *be certain* and *agree* are derived in exactly the same manners as in (48, 55, 57) in Section 3.3.2. This is so because every time we used Fact (46) to replace $\pi(\text{Exh}_Q(\text{Ans}_{w'}(Q)))$ with **up** in the derivations above, we can simply use the fact that $\text{Exh}_Q(\text{Ans}'_{w'}(Q))$ presupposes **up** (given the redefined Ans') to replace $\pi(\text{Exh}_Q(\text{Ans}_{w'}(Q)))$ with **up**.

More informally, the reason why the first and the second implementation don't make distinct predictions regarding the projection of UP/EP can be stated as follows. In both implementations, the presupposition of an interrogative-embedding sentence is analyzed in terms of Spector and Egré's (2015) lexical rule for question-embedding, which picks out a certain answer from the question denotation employing the answerhood operator. In both implementations, the answer picked out by the lexical rule caries the UP/EP, which projects to the sentential level in different ways depending on the embedding predicate. The difference between the two implementations lies in how the answer picked out by the lexical rule is guaranteed to carry the UP/EP. In the first implementation, the answer caries the UP/EP since *all* answers in the question denotation do. On the other hand, in the second implementation, it does because of the redefined answerhood operator Ans′.

3.5 Section summary

In this section, I have shown that the projection pattern of UP/EP observed in Section 2 can be properly captured, once we assume that the answer(s) to the embedded question caries UP/EP. In addition to this central proposal, my analysis is based on two independently motivated components: the existential semantics for question-embedding by Spector and Egré (2015) and the presuppositional behaviors of relevant embedding predicates with respect to declarative complements. I have provided two implementations of the central proposal. Under one implementation, each answer in the question denotation caries the UP/EP (due to an additional operator or a definite-like semantics for the *wh*-item). In the other implementation, the answerhood operator (using which the semantics of question-embedding is formulated) is redefined to return a proposition that carries the UP/EP.

4 Matrix questions and rogative predicates

In the previous section, I have demonstrated that, once we assume that the UP/EP comes from the answers, we can correctly capture the projection patterns of the UP/EP in sentences involving questions embedded under predicates such as *know*, *be certain* and *agree*. However, as I will discuss in detail below, the account so far does not readily explain the presence of UP/EP in *matrix* questions as in (70), or the projection from under *rogative predicates* (those predicates that embed only interrogative complements; Lahiri 2002) as in (71).

(70) Which student smokes? $\overset{\text{presup}}{\Rightarrow}$ 'Exactly one student smokes.'

(71) a. Max wonders which student smokes.
  b. Max investigated which student smokes.
  c. Max is curious which student smokes.
  $\overset{\text{presup}}{\Rightarrow}$ 'Max believes that exactly one student smokes.'

In this section, I discuss how the current analysis can capture the data as in (70-71) with the help of additional assumptions.

Another goal of the current section is to elucidate the difference between the two implementations discussed in the previous section. Importantly, the nature of the additional assumptions required by the data in (70-71) differ depending on which implementation we use for the analysis. More specifically, both under Implementation I (i.e., assigning UP/EP to each answer) and II (i.e., modifying the Ans-operator), we need an additional existential presupposition (pragmatically derived in the case of matrix questions and lexically triggered by the rogative predicates) to capture the data. These presuppositions can be stated by simply referring to the propositions in the question denotation under Implementation I, but they have to be stated in terms of the Ans′-operator in Implementation II. In Section 4.3, I will give two arguments to prefer the analysis within Implementation I to the one within Implementation II.

4.1 UP/EP of matrix questions

Matrix questions, as exemplified in (72) below, seem to carry the UP/EP. (In examples involving matrix questions, I will hereafter indicate the UP/EP-like inferences with the symbol $\overset{\text{presup}}{\Rightarrow}$, but I will refine this empirical description later and actually argue that it is an inference based on the speaker's expectation, rather than a semantic presupposition.)

(72) a. Which student smokes? $\overset{\text{presup}}{\Rightarrow}$ 'Exactly one student smokes.'
  b. Which students smoke? $\overset{\text{presup}}{\Rightarrow}$ 'Some student smokes.'
  c. Who smokes? $\overset{\text{presup}}{\Rightarrow}$ 'Someone smokes.'

This fact cannot be immediately accounted for in the current proposal, where UP/EP comes from the answers. The reason for this is that the analysis only states that the *answers* carry the relevant presuppositions, and does not state that the *question* having such answers do.

Note that the presupposition does not follow from either implementation of the proposal. In Implementation I, it is the answers rather than the question itself that carry the UP/EP. In Implementation II, the UP/EP is derived only after the application of the ANS′-operator, but a matrix question as in (72) does not involve the ANS′-operator in its semantic representation. Below, I will discuss possible solutions to this problem each within Implementation I and II.

4.1.1 Account within Implementation I

Under Implementation I of the current proposal, the UP/EP-like inferences of matrix questions can be captured by assuming that information-seeking matrix questions in general come with the speaker's expectation that at least one of their possible answers is defined. This can be stated as a pragmatic principle, as follows:

(73) **Ask only those questions that you believe have a defined answer (I)**
When a speaker utters an interrogative sentence φ to seek information, for all worlds w compatible with the speaker's beliefs, there is $p \in [\![\varphi]\!]$ such that $p(w)$ is defined.

Given the UP/EP encoded in each possible answer of a matrix singular *which*-question, the principle in (73) is satisfied only if the speaker of such a question believes that the UP/EP is met. Hence, the data in (72) is captured as the combination of two factors: the answer-level UP/EP and the question-level speaker expectation in (73).

The account presented here can be illustrated using a matrix question involving a presupposition trigger *within* the clause, as follows:

(74) Who caught the unicorn? $\overset{\text{presup}}{\Rightarrow}$ 'There is a unique unicorn'

We cannot capture the inference in (74) that there is a unique unicorn by the presupposition triggered by the definite DP *the unicorn* alone. This is so because, it is the possible answers to this question that carry the presupposition triggered by the definite, and we need a further mechanism to account for the fact that the question itself seems to imply that there is a unique unicorn. My claim here is that the pragmatic principle in (73) provides such a mechanism, and exactly the same explanation applies to the matrix effect of the UP/EP.

The account of the matrix EP/UP-like effect presented above is similar to the account of a matrix-level presupposition of *which*-questions discussed by Rullmann and Beck (1998: 226). According to R&B, the existential presupposition carried by each answer of a *which*-question projects to the matrix level due to the question-level presupposition, which requires that there be a true answer to a question. One crucial difference between R&B and my analysis (other than the content of the presupposition carried by each answer, discussed in Section 3.3.3 above) is the nature of the question-level presupposition. R&B treat the question-level presupposition as a semantic definedness condition. On the other hand, the principle above treats the question-level 'presupposition' as a pragmatic phenomenon. Consequently, the two accounts differ in whether the common ground or the speaker belief is required to entail the existence of a true answer.

I argue that the latter view—the pragmatic view based on speaker expectation—is empirically more plausible, in light of the following kind of contrast between the *inter-speaker* and *intra-speaker* denial of the existence of a true answer, pointed out in Dayal (2016: 51) (cf. Karttunen and Peters, 1976: 355):

(75) A: Which student does Mary like?
  B: No one.

(76) #I'm not sure whether Mary likes any student. Which student does she like?

As (75) shows, an interlocutor other than the questioner can overtly deny the existence of a true answer in the question denotation. In contrast, as shown in (76), it is odd for the questioner themself to overtly acknowledge the possibility that the question denotation does not contain a true answer.[22] This contrast is expected under the pragmatic view based on speaker expectation while it is hard to capture under the semantic presupposition view, as far as I can see.

It is also worth noting that a pragmatic principle similar to (73) is assumed by Groenendijk and Stokhof (1984: 30-37) to account for what they call the existential 'suggestion' of matrix questions. However, their account is crucially different from mine in lacking the answer-level UP/EP. This has led to two empirical shortcomings in Groenendijk and Stokhof's treatment of the presuppositions of questions. First, since the UP/EP is treated entirely as a pragmatic phenomenon, it is not straightforward how to account for the projection patterns of UP/EP from under different embedding predicates. Second, the existential suggestion alone cannot account for the contrast between the UP triggered by singular-*which* questions and the EP triggered by plural-*which* and simplex-*wh* questions.

4.1.2 Account within Implementation II

Under Implementation II, the UP/EP-like inferences of matrix questions cannot be derived by the principle in (73) since, under Implementation II, the propositions in the question denotation do not carry the UP/EP. Rather, according to this version of the analysis, the UP/EP is triggered by the answer resulting from the application of

[22]It is possible for the questioner to suspend the existence expectation using *if any*, as follows:

(i) Which student, *if any*, does Mary like?

I argue that this construction involves a conditional question, as follows:

(ii) If Mary likes any student, which student does she like?

Following Isaacs and Rawlins (2008), I assume that conditional questions involve temporary contextual update. With respect to a temporary context in which Mary indeed likes some student, the speaker expectation that the question in the consequent contains a true answer is satisfied. Thus, (ii) does not involve any inconsistency in the speaker's expectations. On the other hand, (76) above involves a genuine inconsistency. For, the first sentence states that the speaker considers it possible that Mary likes no student whereas the question in the second sentence invokes the speaker expectation (in the actual, non-temporary, context) that Mary likes some student.

ANS$'$. For this reason, in order to capture the matrix effect, we have to modify the pragmatic principle governing the utterance of information-seeking matrix questions. The modified version of the principle is given in (77), which is defined to refer to the ANS$'$-operator, repeated below from (67b).

(77) **Ask only those questions that you believe have a defined answer (II)**
When a speaker utters a question φ to seek information, for all worlds w compatible with their beliefs, $\textsc{Ans}'_w[\![\varphi]\!]$ is defined.

(78) $$\textsc{Ans}'_w = \lambda Q_{\langle st,t\rangle} : \underline{\exists p \in Q[p = \textsc{Max}_{\text{inf}}(Q,w)]}.\ \lambda w' : \underline{\exists p \in Q[p = \textsc{Max}_{\text{inf}}(Q,w')]}.\ \textsc{Max}_{\text{inf}}(Q,w)(w')$$

Since $\textsc{Ans}'_w[\![\varphi]\!]$ is defined only if $[\![\varphi]\!]$ has a strongest true answer in w, the principle entails that the speaker believes that $[\![\varphi]\!]$ contains a strongest true answer, i.e., that the speaker believes the UP/EP with respect to $[\![\varphi]\!]$.[23]

4.2 Projection from under rogative predicates

Another issue left open by the analysis presented in Section 3 is the treatment of cases involving rogative predicates, as exemplified in the following:

(79) Max wonders which student smokes.
$\overset{\text{presup}}{\Rightarrow}$ 'Max believes that exactly one student smokes.'

To account for this data within the current proposal, we have to state how the UP/EP carried by the answer(s) of the embedded question is projected by the rogative predicate *wonder*. The analysis laid out in Section 3 does not automatically extend to rogative predicates. Here is why: the analysis in Section 3 employs the existential semantics for question-embedding by Spector and Egré (2015), which states the meaning of $\ulcorner x\ Vs\ Q\urcorner$ in terms of the proposition-taking denotation of the predicate V. This strategy does not extend to rogative predicates since rogative predicates do not straightforwardly have proposition-taking denotations.[24]

However, this simply means that the existential semantics for question-embedding employed in Section 3 cannot be used to analyze the presupposition-projection behavior of rogative predicates, and does not mean that the data in (79) is incompatible with the current central proposal. In fact, we will see below that making a plausible assumption about the definedness condition of the *question-taking* denotation of rogative predicates

[23]An assumption along the lines of (77) is in fact required in Dayal's (1996) analysis of the matrix UP/EP in terms of the original ANS-operator. Dayal (2016) states the assumption as follows: "In asking a question, we can assume that the speaker takes it to be answerable. That is, she expects [ANS] to be defined." (Dayal, 2016: 51-52)

[24]There are attempts to analyze rogative predicates using proposition-taking semantics. For example, following suggestions in Karttunen (1977) and Guerzoni and Sharvit (2007) among others, Uegaki (2015) proposes a proposition-taking semantics for *wonder*, where is semantically decomposed into 'want to know'. However, it is unclear if such a decompositional strategy is available for rogative predicates in general, including e.g., *be curious*, *investigate* and *ask*.

enables us to account for their projection behavior, seen in (79). Furthermore, we will see that the structure of the additional assumption mirrors that of the pragmatic principle assumed in the account of the matrix effect above. Again, we will discuss the accounts in each of Implementation I and II, in turn.

4.2.1 Account within Implementation I

Under Implementation I, where each answer in the question denotation carries the UP/EP, the projection of the UP/EP in sentences involving *wonder* can be accounted for by assuming the following presupposition for $\ulcorner x$ *wonders* $Q\urcorner$:

(80) **Wondering presupposes believing the quest. has a defined answer (I)**
For all $Q \in D_{\langle st,t\rangle}$, $x \in D_e$ and $w \in D_s$,
$\mathbf{wonder}_w(x, Q)$ is defined only if $\forall w' \in \mathbf{Dox}^x_w[\exists p \in Q[p(w') \text{ is defined}]]$

When each proposition in Q carries the UP/EP, this presupposition is satisfied only if the attitude holder x believes the UP/EP. As one can easily see, the structure of this presupposition mirrors the pragmatic principle in (73) above. Above, we required that the questioner believe that the question has a defined answer. Here, we require that the wonderer believe that the question has a defined answer.

Just as in the case of the matrix effect discussed in the previous subsection, the analysis can be illustrated using an interrogative complement that involves a presupposition trigger inside the complement, such as the following:

(81) Max wonders who saw the unicorn.
$\overset{\text{presup}}{\Rightarrow}$ 'Max believes that there is a unique unicorn.'

Each possible answer of the interrogative complement in this example carries the presupposition that there is a unique unicorn. The condition in (80) predicts that this presupposition is projected by *wonder* to the attitude holder's belief state, which is exactly the pattern we see in (81).

I argue that the condition along the lines of (80) is general to rogative predicates, such as *investigate*, *be curious* and *inquire*. This predicts the projection behavior concerning the UP/EP similar to the case of *wonder* for these rogative predicates. I submit that this is an empirically correct prediction.

4.2.2 Account within Implementation II

The account within Implementation II again mirrors the account of the matrix case within Implementation II. Since the UP/EP under Implementation II is the result of the application of $\textsc{Ans}'$, the relevant definedness condition for *wonder* is restated as follows:

(82) **Wondering presupposes believing the quest. has a defined answer (II)**
For all $Q \in D_{\langle st,t\rangle}$, $x \in D_e$ and $w \in D_s$,
$\mathbf{wonder}_w(x, Q)$ is defined only if $\forall w' \in \mathbf{Dox}^x_w[\textsc{Ans}'_{w'}(Q) \text{ is defined}]$

Given the definition of $\textsc{Ans}'$, this condition predicts that ⌜*x wonders Q*⌝ presupposes that x believes that Q contains a strongest true answer, i.e., that the UP/EP is satisfied.

4.3 Teasing apart the two implementations

In the previous subsections, I presented accounts of the UP/EP-like effect in matrix questions and the projection of the UP/EP with rogative predicates, each within Implementations I and II of my central proposal. Under both implementations, accounting for the data requires additional assumptions about the pragmatics of (information-seeking) questions and the lexical semantics of rogative predicates. In this section, I focus on the difference between the two implementations in light of this discussion. As it has already been made clear, the difference between the accounts in the two implementations boils down to whether the relevant conditions are stated in terms of the $\textsc{Ans}'$-operator (Implementation II) or not (Implementation I). In this section, I will present two arguments that favor the accounts within Implementation I over those within Implementation II. As it turns out, because of the structural similarity between the two lines of accounts, both of the arguments I will present below are not particularly robust in the sense that they don't come without methodological/theoretical assumptions. This said, I believe the arguments require serious attention as they illuminate the contrast between the two implementations of the proposal.

4.3.1 Simplicity

My first argument concerns the comparative simplicity of the additional assumptions about the pragmatics of matrix questions and the lexical semantics of rogative predicates required in the two implementations. For easy reference, I repeat below the relevant assumptions under both implementations:

(83) **Ask only those questions that you believe have a defined answer**

I: When a speaker utters an interrogative sentence φ to seek information, for all worlds w compatible with the speaker's beliefs, there is $p \in [\![\varphi]\!]$ such that $p(w)$ is defined.

II: When a speaker utters a question φ to seek information, for all worlds w compatible with their beliefs, $\textsc{Ans}'_w[\![\varphi]\!]$ is defined.

(84) **Wondering presupposes believing the question has a defined answer**

I: For all $Q \in D_{\langle st,t\rangle}$, $x \in D_e$ and $w \in D_s$,
$\textbf{wonder}_w(x, Q)$ is defined only if $\forall w' \in \textbf{Dox}^x_w[\exists p \in Q[p(w')$ is defined$]]$

II: For all $Q \in D_{\langle st,t\rangle}$, $x \in D_e$ and $w \in D_s$,
$\textbf{wonder}_w(x, Q)$ is defined only if $\forall w' \in \textbf{Dox}^x_w[\textsc{Ans}'_{w'}(Q)$ is defined$]$

Again, the difference between the two versions of (83) and (84) concerns whether they are stated in terms of $\textsc{Ans}'$ or not.

These assumptions are indeed what is needed to account for the behavior of UP/EP in matrix questions and sentences involving rogative predicates, under the respective implementation of the proposal. It also turns out that they are both compatible with empirical observations that are independent of UP/EP. The relevant empirical consideration here is the projection pattern of presuppositions other than the UP/EP, as exemplified in the following, repeated from above.

(85) Who caught the unicorn? $\overset{\text{presup}}{\Rightarrow}$ 'There is a unique unicorn'

(86) Max wonders who saw the unicorn.
$\overset{\text{presup}}{\Rightarrow}$ 'Max believes that there is a unique unicorn.'

As discussed above, the assumptions under Implementation I, i.e., (83-**I**) and (84-**I**), account for these data. When the speaker believes that some answer of (85) is defined, they believe that there is a unique unicorn. When Max believes that some answer of the embedded question in (86) is defined, Max believes that there is a unique unicorn. Similarly, the assumptions under Implementation II, i.e., (83-**II**) and (84-**II**), also capture the data. When the speaker believes that the $\textsc{Ans}'$-answer of (85) is defined, it follows that they believe that there is a unique unicorn. When Max believes that the $\textsc{Ans}'$-answer of the embedded question in (86) is defined, it follows that Max believes that there is a unique unicorn. All in all, both versions are compatible with the projection behavior of presuppositions in questions. Thus, available empirical considerations do not seem to distinguish the two versions of the assumptions.

However, the fact that both versions are empirically adequate does not imply that we cannot make *conceptual* arguments to favor one over the other. In particular, we can rely on Ockham's Razor and compare the accounts in terms of the *simplicity* of the required assumptions. As we compare the two versions in (83-84), it is evident that the versions under Implementation I are simpler than those under Implementation II. This point can be argued on the basis of the fact that (83/84-**II**) rest on the definition of the $\textsc{Ans}'$-operator but (83/84-**I**) don't. In other words, the explanation of (85-86) under Implementation II requires the principles in (83/84-**II**) *as well as* the definition of $\textsc{Ans}'$. On the other hand, the explanation of (85-86) under Implementation I only requires the principles in (83/84-**I**). Given Ockham's Razor, we thus have a reason to prefer the explanation under Implementation I over the one under Implementation II.

One might ask if this simplicity-based argument holds if (83/84-**II**) are rewritten as equivalent statements without the mention of $\textsc{Ans}'$, as follows:[25]

(87) **Ask only those questions that you believe have a defined answer (II)**
When a speaker utters a question φ to seek information, for all worlds w compatible with their beliefs, $\exists p \in [\![\varphi]\!][p = \textsc{Max}_{\text{inf}}([\![\varphi]\!], w)]$.

(88) **Wondering presupposes believing the quest. has a defined answer (II)**
For all $Q \in D_{\langle st,t\rangle}$, $x \in D_e$ and $w \in D_s$,
$\textbf{wonder}_w(x, Q)$ is defined only if $\forall w' \in \textbf{Dox}^x_w[\exists p \in Q[p = \textsc{Max}_{\text{inf}}(Q, w')]]$

[25]The statements in (87-88) still mention the operator $\textsc{Max}_{\text{inf}}$, but they can of course be rewritten without recourse to it.

I argue that these principles are still conceptually less attractive than those in (83/84-**I**). This is so since the former principles are logically stronger than the corresponding principles in the latter.[26] When comparing two theories that differ only in the logical strength of an assumption, everything else being the same, the theory with a logically weaker assumption should be preferred over the other.[27]

In sum, the assumptions required by Implementation I to account for the data involving matrix questions and rogative predicates are simpler/weaker than those required by Implementation II. Assuming that the two implementations have equal empirical and explanatory adequacy in other respects, this motivates methodological preference for Implementation I over Implementation II.

4.3.2 Polar questions

Another argument for Implementation I concerns the treatment of polar questions, though it should be noted that the argument rests on a particular theoretical assumption about the semantics of polar questions. The relevant assumption is that the semantic value of a polar question is a *singleton* proposition-set, as exemplified below:

(89) a. *Is it raining?* $\rightsquigarrow \{\lambda w.\mathbf{raining}_w\}$
b. *whether it is raining* $\rightsquigarrow \{\lambda w.\mathbf{raining}_w\}$

The singleton analysis of polar questions has been put forth by Roberts (1996), Biezma and Rawlins (2012) and Westera (2017) among others. In particular, Biezma and Rawlins (2012) empirically motivate the analysis based on the behavior of dubitative predicates (e.g., *doubt*), the interpretation of disjoined polar questions (e.g., *Is it a bird or is it a plane?*) and answer particles.

If the singleton analysis of polar questions is correct, polar questions present a challenge for Implementation II while not for Implementation I. This is so because of the following fact:

(90) For all w and p, $\textsc{Ans}'_w(\{p\})$ is defined iff $p(w)$

More specifically, this means that Principle (83-**II**) is true with respect to the polar question *Is it raining?* only if the speaker believes that it is raining; Principle (84-**II**) is true with respect to *Max wonders whether it is raining* only if Max believes that it is raining. Clearly, these are incorrect predictions.

On the other hand, the principles in (83/84-**I**) do not make problematic predictions with respect to singleton polar questions. Given the polar question *Is it raining?*,

[26] The statement in (ia) is true whenever (ib) is, but not vice versa.

(i) a. $\exists p \in Q[p(w) \text{ is defined}]$
b. $\exists p \in Q[p = \textsc{Max}_{\text{inf}}(Q, w)]$

[27] We can justify this methodological principle in a way similar to the probabilistic justification of Ockham's Razor (Baker, 2016). A theory that relies on a weaker assumption is better because it has better likelihood to be true.

Principle (83-**I**) simply states that the speaker believes that 'it is raining' (the unique proposition in the set) is defined. Given the sentence *Max wonders whether it is raining*, Principle (84-**I**) states that Max believes that the proposition 'it is raining' is defined. Both of these predictions are quite weak and unproblematic. Hence, the singleton analysis of polar questions would provide an argument to favor Implementation I over II.

The force of this argument is somewhat restricted since the validity of the singleton analysis of polar questions is under debate. Roelofsen and van Gool (2010) and Roelofsen and Farkas (2015) propose to employ the notion of *highlighting* to capture the phenomena that have motivated Biezma and Rawlins (2012) to adopt the singleton analysis, while treating polar questions as having the size-2 bipolar denotations, as follows:

(91) a. *Is it raining?* $\rightsquigarrow \{\lambda w.\mathbf{raining}_w, \lambda w.\neg\mathbf{raining}_w\}$

b. *whether it is raining* $\rightsquigarrow \{\lambda w.\mathbf{raining}_w, \lambda w.\neg\mathbf{raining}_w\}$

If polar questions have the bipolar denotations as exemplified in (91), the principles in (83/84-**II**) turn out to be unproblematic. Principle (83-**II**) applied to *Is it raining* states that the speaker believes that one of the two propositions in (91a) is true. Similarly, Principle (84-**II**) applied to *Max wonders whether it is raining* states that Max believes that one of the two propositions in (91a) is true. Neither of these predictions is problematic.

4.4 Section summary

In this section, I have investigated how my central proposal that the UP/EP comes from the answers can be extended to the behavior of UP/EP in matrix questions and sentences involving rogative predicates. Although the account presented in Section 3 cannot directly account for the data, it can do so by assuming plausible assumptions about the pragmatics of (information-seeking) matrix questions and the semantics of rogative predicates.

Furthermore, the discussion in this section highlighted the difference between the two implementations of the proposal presented in Section 3 (Implementation I: encoding UP/EP to each answer; Implementation II: modifying the answerhood operator). I have pointed out that the formulations of the assumptions required to account for matrix questions and rogative predicates differ depending on the implementation. Importantly, this consideration has revealed potential advantages for Implementation I over Implementation II for the following reasons. First, the assumptions required by Implementation I are simpler (or logically weaker) than those required by Implementation II, thus making Implementation I more preferable on the basis of Ockham's Razor. Furthermore, the assumptions required by Implementation II (but not those required by Implementation I) make incorrect empirical predications about polar questions, assuming the singleton semantic analysis of polar questions.

5 Conclusions and open issues

In this paper, I have pointed out that the projection behavior of the uniqueness/existential presupposition (UP/EP) of *wh*-questions under various question-embedding predicates poses problems for existing accounts of the UP/EP. Specifically, Dayal's (1996) account based on the Ans-operator incorrectly predicts that the UP/EP projects to the matrix level even with non-veridical predicates. Amending Dayal's (1996) account based on Spector and Egré's (2015) analysis of non-veridical predicates avoids the incorrect prediction that the UP/EP projects to the matrix level with non-veridical predicates. However, it fails to capture the fact that the presupposition projects to the attitude holder's beliefs. Finally, although Uegaki's (2015) analysis, which encodes the relevant presupposition to the embedders, correctly captures the behavior of non-veridical predicates, it makes incorrect predictions about the cases when the predicates embed *declarative* complements.

I have proposed a solution to the problems based on the idea that the UP/EP is carried by the *answers* in the question denotation. Once the UP/EP is encoded in the answers, the relevant projection behaviors are naturally accounted for, given two independently motivated mechanisms, i.e., existential semantics of question-embedding following Spector and Egré (2015) and the presupposition-projection behavior of individual predicates with respect to declarative complements.

There are two possible implementations of this central proposal: (i) assigning the UP/EP to each answer in the question denotation and (ii) modifying the answerhood operator so that it outputs a presuppositional proposition. Given considerations concerning matrix questions and rogative predicates, I have suggested arguments that favor the first implementation.

Taking a step back, the current paper can be considered as contributing to the larger project that aims to construct a uniform semantics for question-embedding, currently being undertaken by a number of researchers working in the domain (e.g., George, 2011; Spector and Egré, 2015; Uegaki, 2015; Xiang, 2016; Theiler et al., 2018). Although few of the existing works specifically discuss the projection of the UP/EP, the discussion in this paper reveals that it can be properly analyzed without any significant modification to the overall semantics of question-embedding, once we assume that the answers carry the UP/EP. In the body of the paper, this is concretely shown by taking Spector and Egré's (2015) theory of question-embedding. Thus, the current paper can be considered as providing a (heretofore) missing piece of the overall semantics of question-embedding.

As mentioned in Section 3.3.3, I have largely left open the issue concerning the internal composition of *wh*-complements, as I have focused on the analysis of the projection of the UP/EP within the context of the investigation of the semantics of question-embedding. However, it goes without saying that a more detailed investigation of the internal composition of the *wh*-complement provides a clearer view of the source of the UP/EP, and can provide further arguments to tease apart the two implementations of the proposal I have discussed in this paper. In fact, in a recent paper, Hirsch and

Schwarz (to appear) argue that the UP of singular *which*-questions originates from the semantics of *which* (in a way similar to the picture I have suggested in Section 3.3.3) based on an observation that uniqueness can take scope below a position where the answerhood operator would normally take scope. It is possible that evidence from the embedding phenomena (as discussed in this paper) and that from the internal composition of *wh*-complements (as discussed by Hirsch and Schwarz to appear) converge to suggest that the UP/EP indeed originates from the answers, more specifically, from the semantics of the *wh*-element.

A related open issue is the analysis of the presupposition of non-constituent questions, such as alternative questions. If the UP/EP is attributed to the semantics of the *wh*-element, the presupposition of alternative questions (that exactly one of the alternative is true) has to be accounted for in a separate fashion, possibly in terms of its intonation (e.g., Westera, 2017). On the other hand, if the UP/EP is attributed to an operator (ANS′ or otherwise) that attaches to interrogative complements in general, a unified analysis of the presupposition of *wh*-complements and alternative questions would be possible. We need further empirical and theoretical investigation to properly compare the relative advantages of these two lines of analysis.

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A Exhaustivity and question-oriented semantics

A.1 Exhaustivity

The rule in (41) only predicts the so-called WEAKLY EXHAUSTIVE (WE) reading of embedded questions, under which $\ulcorner x\ Vs\ Q \urcorner$ would be paraphrased as 'x Vs that all true answers to Q are true' (e.g., Karttunen 1977; Dayal 1996). However, it is known that embedded questions in principle allow interpretations with other various strengths of exhaustivity. These include STRONG EXHAUSTIVITY (SE; 'x Vs that all true answers to Q are true and all false answers to Q are false'; Groenendijk and Stokhof 1984) and MENTION-SOME (MS; 'For some true answer p to Q, x Vs that p.'; e.g., van Rooij 2003; George 2013; Xiang 2016).

Furthermore, it has been argued more recently that there exists another reading (in addition to, or in place of, the WE reading) that is 'intermediate' in strength between weak exhaustivity and strong exhaustivity. Such a reading is sometimes said to involve INTERMEDIATE EXHAUSTIVITY (IE; Klinedinst and Rothschild 2011) or 'sensitivity to false answers' (Spector 2005; cf. also Berman 1991; Preuss 2001), and can be paraphrased as 'x Vs that all true answers to Q are true and x does not V all false answers to Q'.[28] Mention-some readings are also argued to involve sensitivity to false answers (George, 2011). How can these different levels of exhaustivity be accounted for in the current analysis?[29]

Strong exhaustivity and intermediate exhaustivity I assume that strong exhaustivity is derived by an optional point-wise exhaustification of the Hamblin denotation (Nicolae, 2013; Fox, 2018), which projects the presupposition of the answers. Further, following Spector and Egré (2015: 1771-6), I assume that 'intermediate exhaustivity' is captured by the additional condition in the lexical rule, as in the box below:

(92) $\mathbf{V}_{int} = \lambda Q_{\langle st,t\rangle}\lambda x_e \lambda w_s\colon\ \underline{\exists w'[\mathbf{V}_{decl}(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))(x)(w) \text{ is defined}]}.$
$\exists w'[\mathbf{V}_{decl}(\textsc{Exh}_Q\overline{(\textsc{Ans}_{w'}(Q)))(x)(w) \text{ is defined} \wedge \mathbf{V}_{decl}(\textsc{Ans}_{w'}(Q))(x)(w)}$

[28] This paraphrase is only approximate since the second occurrence of V has to be devoid of factivity in case V is originally a veridical predicate (see e.g., Klinedinst and Rothschild 2011; Spector and Egré 2015).

[29] For a recent overview of the issue, see Theiler et al. (2018). See also Cremers and Chemla (2016) for an experimental study on the empirical status of various levels of exhaustivity of questions embedded under *know* and *predict*.

$$\wedge \boxed{\neg\exists w''[\textsc{Ans}_{w''}(Q) \subset \textsc{Ans}_{w'}(Q) \wedge \mathbf{V}'_{decl}(\textsc{Ans}_{w''}(Q))(x)(w)]}]$$

(where $\mathbf{V}'_{decl}$ is the 'non-veridical counterpart' of $\mathbf{V}$)
(after Spector and Egré 2015: (127))

For example, when $\mathbf{V}$ = **know**, the additional conjunct in the box amounts to the condition that there is no possible Dayal-answer which is (a) stronger than the one the subject knows and (b) merely believed (rather than known) by the subject. This successfully captures the sensitivity to false answers. If the subject believes a false answer in addition to the true answer, the boxed condition would be violated because the conjunction of the two answers would constitute a possible Dayal-answer that the subject believes.[30]

It is important to note that assuming any of the WE, SE and IE readings in this setup does not make any difference to the prediction regarding the projection of the UE. This is so because the presuppositional components of the lexical rules in (41-92) do not differ, and it stays the same even when the question-denotation Q has undergone point-wise exhaustification. For this reason, in the discussion of the proposal below, I will illustrate the predictions only with the basic lexical rule in (41).

Mention-some I largely set aside the analysis of mention-some in this paper, as it presents a separate contested issue which requires an article-length discussion. Here, I will keep the discussion minimal by simply mentioning the locus of the issue and how it affects the current proposal, with references to existing works on the topic.

There are largely two views on the mention-some readings of questions: pragmatic and semantic. According to the pragmatic view (e.g., Groenendijk and Stokhof, 1984; van Rooij, 2003), the semantic content of a question like (93) does not differ from non-mention-some questions in terms of the exhaustivity it licenses, but the pragmatics of the question licenses a non-exhaustive answer, in view of the relevant conversational goal that can be inferred from the question.

(93) Who can chair the committee?

For example, in the case of (93), even though there are several candidates for the chair, mentioning one such candidate can suffice the relevant conversational goal.

According to the semantic view (e.g., George, 2011; Fox, 2013; Xiang, 2016), on the other hand, a question like (93) has a specific semantic property that licenses the mention-some reading. Specifically, Fox (2013) and Xiang (2016) argue that a mention-some reading of a question arises if the semantics of the question licenses multiple maximally-strong true answers. For example, (93) may have the following as the set of maximally-strong true answers.

[30]This account is applicable only to cases where the question denotation is closed under conjunction. In the case where *know* embeds a singular-*which* complement, whose denotation is not closed under conjunction, belief in a false answer (in addition to the true one) either falsifies the sentence under the SE reading or makes it a presupposition failure given the projection of the UE to the subject's beliefs, which I will account for below.

(94) {Ann can chair the committee, Bill can chair the committee}

Crucially, this is made possible by the fact that the conjunction of these two answers is *not* in included in the question denotation (if it was, it would be the unique maximally-strong true answer). Fox's (2013) and Xiang's (2016) compositional semantics guarantees the exclusion of the conjunctive answer by virtue of the presence of the weak modal *can* or other existential quantification scoping over the distributive operator in the structure, following a similar suggestion in George (2011).

As both Fox (2013) and Xiang (2016) note, this view is in conflict with the analysis of the UP/EP in terms of Dayal's (1996) Ans-operator. For, the presupposition of Ans would be violated if a question denotation has multiple maximally-strong true answers. Fox (2013) and Xiang (2016) propose separate solutions to this dilemma by essentially proposing a modified version of the Ans-operator with a weakened presupposition that preserves Dayal's (1996) predictions regarding non-mention-some questions (i.e., those that do not involve a weak modal or an existential quantification in the structure) but is satisfied by a question like (93) with multiple maximally strong true answers.

Where does this leave us with respect to the goal of the current paper, i.e., the analysis of the projection of UP/EP? If the pragmatic view is correct, the analysis of mention-some is orthogonal to the current goal, as the projection of the UP/EP is a *semantic* issue whereas whether a particular question receives a mention-some reading would depend on pragmatic considerations. On the other hand, if the semantic view is correct, an issue arises as to how the weakened presupposition predicted by the modified Ans (by Fox 2013; Xiang 2016) in a question like (93) would project from interrogative complements. I would like to leave the discussion of this issue for future occasions, and limit the focus of this paper to those questions that do not involve weak modal or existential quantification in the structure. My analysis of the projection of UP/EP is stated in terms of how the UP/EP triggered by the answers project, given Spector and Egré's (2015) schema in (41) involving Ans. Since Fox's (2013) and Xiang's (2016) modified Ans preserve the predictions of Dayal's (1996) Ans within our limited empirical scope, our analysis, which assumes Dayal's (1996) definition of Ans, would carry over to setups involving alternative definitions of Ans by Fox (2013); Xiang (2016).

A.2 Question-oriented semantics for the embedding predicates

Spector and Egré's (2015) semantics for question-embedding assumes a *proposition-oriented semantics* for question-embedding predicates. That is, it assumes that the basic denotations of question-embedding predicates are proposition-taking ones, and that a lexical rule is required to convert them into question-taking counterparts. Uegaki (2015, 2016) and Theiler et al. (2018) offer a *question-oriented semantics* for question-embedding predicates, which goes against Spector and Egré's (2015) assumption. According to Uegaki's and Theiler et al.'s view, the basic denotation of a question-embedding predicate takes a question (qua a set of propositions), and a declarative complement is analyzed as providing a singleton proposition-set (or a downward-closed set of propositions with only one maximal set, in the case of Theiler et al.). See Uegaki (2019) for a review of existing empirical arguments for preferring the question-oriented semantics over the proposition-oriented semantics for clause-embedding predicates.

Although our analysis will be stated in terms of Spector and Egré's (2015) lexical rule, the analysis is perfectly compatible with a question-oriented formulation. This is so since the question-taking denotation of a predicate resulting from the lexical rule can be considered as the basic denotation of the predicate, as exemplified in the case of *know* below:

(95) *know* $\rightsquigarrow \lambda Q_{\langle st,t\rangle}\lambda x_e \lambda w_s\colon \underline{\exists w'[\mathbf{know}_w(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))(x) \text{ is defined}]}.$
$\exists w'[\mathbf{know}_w(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))(x) \text{ is defined} \wedge \mathbf{know}_w(\textsc{Ans}_{w'}(Q))(x)]$

Once declarative complements to *know* are analyzed as having the singleton-set denotation, as in (96), declarative complementation is straightforwardly analyzed as in (97).[31]

(96) *that Ann smokes* $\rightsquigarrow \{A\}$

(97) *know that Ann smokes* $\rightsquigarrow$
$\lambda x_e \lambda w_s\colon \underline{\exists w'[\mathbf{know}_w(\textsc{Exh}_{\{A\}}(\textsc{Ans}_{w'}(\{A\})))(x) \text{ is defined}]}.$
$\exists w'[\mathbf{know}_w(\textsc{Exh}_{\{A\}}(\textsc{Ans}_{w'}(\{A\})))(x) \text{ is defined} \wedge \mathbf{know}_w(\textsc{Ans}_{w'}(\{A\}))(x)]$
$\equiv \lambda x_e \lambda w_s\colon \underline{A(w)}.\ \mathbf{know}_w(A)(x)$

One potential advantage of adopting the question-oriented formulation concerns the analysis of intermediate exhaustivity. If one takes the proposition-oriented semantics, the lexical rule needed to capture intermediate exhaustivity has to refer to the 'non-veridical counterpart' of the predicate **V**, rewritten as **V′** in the lexical rule repeated below:

(92) $\mathbf{V}_{int} = \lambda Q_{\langle st,t\rangle}\lambda x_e \lambda w_s\colon \underline{\exists w'[\mathbf{V}_{decl}(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))(x)(w) \text{ is defined}]}.$
$\exists w'[\mathbf{V}_{decl}(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))(x)(w) \text{ is defined} \wedge \mathbf{V}_{decl}(\textsc{Ans}_{w'}(Q))(x)(w)$
$\wedge \boxed{\neg\exists w''[\textsc{Ans}_{w''}(Q) \subset \textsc{Ans}_{w'}(Q) \wedge \mathbf{V}'_{decl}(\textsc{Ans}_{w''}(Q))(x)(w)]}]$

To ensure that the non-veridical counterpart of **V** is accessible for the lexical rule, Spector and Egré resort to decomposition of presuppositional embedding predicates into 'the presuppositional part' and 'the assertive part' (see Klinedinst and Rothschild 2011:17 for a similar suggestion). However, it is not obvious if such decomposition can be independently motivated for the predicates that allow intermediate exhaustivity in general. On the other hand, under the question-oriented formulation, there is no need

[31] The equivalence between the two formulae in (97) is guaranteed by the following fact:

(i) a. For all w and p, $\textsc{Ans}_w(\{p\})$ is defined iff $p(w)$.
If defined, $\textsc{Ans}_w(\{p\}) = p$

b. For all w and p, $\textsc{Exh}_{\{p\}}(\textsc{Ans}_w(\{p\}))$ is defined iff $p(w)$.
If defined, $\textsc{Exh}_{\{p\}}(\textsc{Ans}_w(\{p\})) = p$

for an extra assumption such as lexical decomposition, as the meaning of the 'non-veridical counterpart' can be directly encoded in the lexical semantics of embedding predicates, as follows:

(92) *know* $\rightsquigarrow \lambda Q_{\langle st,t\rangle}\lambda x_e \lambda w_s\colon\ \underline{\exists w'[\mathbf{know}_w(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))(x) \text{ is defined}]}.$
$$\exists w'[\mathbf{know}_w(\textsc{Exh}_Q(\textsc{Ans}_{w'}(Q)))(x) \text{ is defined} \wedge \mathbf{know}_w(\textsc{Ans}_{w'}(Q))(x) \wedge \boxed{\neg\exists w''[\textsc{Ans}_{w''}(Q) \subset \textsc{Ans}_{w'}(Q) \wedge \mathbf{believe}_w(\textsc{Ans}_{w''}(Q))(x)]}]$$

In sum, although Spector and Egré's (2015) semantics of question-embedding I will adopt in the analysis assumes the proposition-oriented semantics for embedding predicates, the analysis is also compatible with the question-oriented semantics for the relevant predicates, along the lines of Uegaki (2015, 2016) and Theiler et al. (2018).