Partition by Exhaustification: towards a solution to Gentile and Schwarz's puzzle

Abstract: This paper presents evidence in favor of Partition by Exhaustifiction, a constraint that requires pointwise exhaustification of a question's denotation to partition the context set. This constraint captures data that has been used to argue in favor of the presupposition that Dayal (1996) attributes to an obligatory *Answer* operator. But PbE also makes a few different predictions from Dayal's, which this paper attempts to corroborate. First, we present evidence that a question is admissible even when its denotation does not have a maximally informative true member (which Dayal demands), as long as it has a member which is mapped to a cell by exhaustification (as PbE demands). Second, we present evidence that a question is not admissible even when its denotation does have a maximally informative true member, as long as it has a member which is not mapped to a cell by exhaustification (which PbE prohibits).

Keywords: *Exh*, *Ans*, negative-islands, uniqueness presupposition, maximal-informativity.

In a recent paper (Fox, 2018) I argued for a new perspective on the admissibility conditions imposed by questions, which I called Partition by Exhaustification, PbE. My goal in this paper is to argue that PbE can account for a puzzle identified by Gentile & Schwarz (2017). But I would also like to take this opportunity to introduce evidence for PbE which does not depend on the analysis of so-called *mention some* readings, a major focus of Fox (2018). I will, thus, begin by presenting a simplified version of PbE, one which will not be intricate enough to accommodate mention some readings but will, nevertheless, suffice for current purposes (section 1). In section 2-4, I will go over some evidence for PbE that can be appreciated even when we focus on the simplified version and ignore mention some readings. Then I will introduce G&S's puzzle in section 5 and will outline my solution in section 6. Section 7 will compare my proposal to a competing proposal made in Gentile & Schwarz (2019). Sections 8-10 will deal with various challenges and open questions.

1. Partition by Exhaustification

Theories of the pragmatics of questions often assume that questions must partition a space of possibilities (Groenendijk & Stokhof, 1984), as partitions seem to be crucial for understanding the role of a question in thought and communication.[1] PbE is the demand that questions fulfill this pragmatic function (of partitioning the relevant space of possibilities) through the use of exhaustification, a demand that, in turn, imposes presuppositional requirements on the conversational context, i.e. on a Stalnakerian Context-Set. More specifically, point-wise exhaustification of the question denotation (viewed as a Hamblin/Karttunen set) must partition the context-set:

[1] With a partition one can understand when a communicative move is *relevant* or *informative* (partially or completely) and define important relationships that might hold between questions/topics (see also Lewis, 1988).

(1) **Partition by Exhaustification (PbE)**: A question with denotation Q is acceptable given a context-set A only if

$\{[Exh(Q)(p)]_A\text{: } p\in Q\}$ is a partition of A.[2]

The function *Exh* applies to every proposition in the question denotation and the result of this point-wise application must yield a partition, i.e., a set of mutually exclusive non-contradictory propositions (non-empty sets) such that every world not excluded by the common ground is a member of one of the propositions. The function *Exh*, in turn, is tailored to its role in partitioning the context-set (along the relevant dimension). Specifically, it determines the truth value of every proposition in Q: every member of Q which is *innocently excludable* given p and Q (Fox, 2007a) is determined to be false and the remaining propositions are determined to be true.

(2) **Exhaustivity as Cell-identification**[3]

$Exh(Q_{st,t})(p) = \lambda w.\ \forall q\in Q[q(w)=1 \text{ iff } q\notin IE(Q,p)]$

[Simplification of Bar-Lev & Fox, 2017]

Where IE(Q,p) is defined (as in Fox, 2007a) to be the intersection of all maximal consistent exclusions (that is

$IE(Q,p)= \bigcap\{A\text{: A is a maximal subset of Q, such that } \{p\}\cup\{\neg q\text{:}q\in A\} \text{ is a consistent set of propositions}\}$)

A different way of saying the same thing, which will turn out to be useful, begins with a traditional way of defining a partition given a Hamblin/Karttunen denotation, as in (3). The admissibility conditions on questions can then be stated as in (4).[4] Since they are virtually equivalent, I will sometimes use PbE and QPM interchangeably, though as we will see, QPM, given its two component, namely *Cell Identification* and *Non Vacuity*, will allow us to be more precise about the nature of the empirical evidence (as each piece of evidence we will consider will support one of the components and will be silent about the other).[5]

---

[2] I write $[\phi]_A$ for the result of updating the context-set, A, with ϕ, i.e. $A\cap\phi$.

[3] (2) Involves a simplification of the operator proposed in Bar-Lev and Fox (2017), a simplification sufficient for our current purposes. Specifically, (2) yields contradictions that are avoided by Bar-Lev and Fox (2017) through the use of *innocent inclusion*. However, this is not needed for current purposes, since every case in which (2) yields a contradiction will anyway violate PbE; in other words, if we restrict ourselves to context-sets in which (1) is satisfied, (2) is equivalent to Bar-Lev and Fox's operator (as Bar-Lev & Fox (in press), discuss, with reference to the function *Cell* (their (20)).

[4] In Fox (2018) I started with the second statement and entertained the statement in (1), suggested by Roger Schwarzschild (p.c.), at the very end of the paper.

[5] QPM is equivalent to the requirement that $\{[Exh(Q)(p)]_A\text{: } p\in Q\}$ = Partition(A, Q), and *Exh(Q)(p)*, whenever it is non-contradictory, is a member of Partition(A, Q). This means that as long as *Exh(Q)* is a bijective function on the members of Q, PbE will be equivalent to QPM. To ensure full equivalence, it might be a good idea to strengthen PbE and demand a bijection:

(i) **Bijective Partition by Exhausitfication (BPbE)**: A question with denotation Q is acceptable given a context-set A only if Exh(Q) is a bijective function on Q and

$\{[Exh(Q)(p)]_A\text{: } p\in Q\}$ is a partition of A.

(3) ***Partition induced by a Question*** (definition)

Let Q be a set of propositions, A a Stalnakerian context-set:

Partition(A, Q), the partition of A induced by Q, is the partition of A to cells that agree with each other on the truth value of every member of Q, i.e., the set of equivalence classes under $\sim_Q$, where $w \sim_Q w'$ iff

$$\{p: p\in Q \ \&\ p(w)=1\}=\{p: p\in Q \ \&p(w')=1\}$$

(4) **Question Partition Matching (QPM)**: Q is acceptable given a context-set A only if

a. $\forall C\in Partition(A,Q)\ \exists p\in Q$

$[Exh(Q)(p)]_A = C$ **[Cell Identification]**

b. $\forall p\in Q \exists C\in Partition(A,Q)$

$[Exh(Q)(p)]_A = C$ **[Non Vacuity]**[6]

The argument for PbE in Fox (2018) was based on the observations (a) that PbE provides an account for phenomena commonly attributed to the presuppositions of Dayal's *Answer* operator (Dayal, 1996), (b) that PbE explains counter-examples to Dayal's presupposition, and (c) that PbE extends to account for phenomena that are outside the reach of Dayal's presupposition. Section 2 will focus on (a), while (b) and (c) will be discussed in sections 3 and 4 (though this time with no reference to mention some readings).

Note about de-dicto readings: To simplify things, I assumed (here and in Fox (2018)) that the denotation of a question is constant across members of the context set, but this need not be the case, as is often pointed out (see e.g., (George, 2013)).[7] To deal with situations where the question denotation varies across members of the context-set, we need to think of the Hamblin/Karttunen *intension*, a function from worlds to sets of propositions. We can then restate PbE as follows.

(5) **Partition by Exhaustification (PbE)**: A question with intension $Q_{<s,stt>}$ is acceptable given a context-set A only if

$\{[Exh(Q_{s,stt})(p)(w)]]_A: \exists w\in A[p\in Q(w)]\}$ is a partition of A.

where $Exh(Q_{s,stt})(p) = \lambda w.\ p\in Q(w) \& [\forall q\in Q(w)[q(w)=1 \text{ iff } q\notin IE(Q(w),p)]$

(6) ***Partition induced by a Question*** (definition)

Let Q be a function from worlds to sets of propositions, A a Stalnakerian context-set:

Partition(A, Q), the partition of A induced by Q, is the set of equivalence classes under $\sim_Q$, where $w \sim_Q w'$ iff $\{p: p\in Q(w)\&p(w)=1\}=\{p: p\in Q(w')\&p(w')=1\}$

[6] A precursor to Non-Vacuity and the arguments in its favor can be found in Fox, 2010: (46). (46) in Fox, 2010 is empirically equivalent to a combination of Non-Vacuity with a version of *Exh* as defined in (11b).

[7] Consider e.g. the question *which applicants were admitted?* This question can be entertained in a context where the identity of the applicants is not determined by the common ground. In such a context, the semantic object that is used to form a partition of the context-set should not be thought of as a set of propositions, but rather as a non-constant function from worlds to sets of propositions (a Hamblin/Karttunen intension).

(7) **QPM**: Q is acceptable given a context-set A only if

a. $\forall C \in \text{Partition}(A,Q)\ \exists w \in A \exists p \in Q(w)$

$$[\text{Exh}(Q)(p))]_A = C \qquad \textbf{[Cell Identification]}$$

b. $\forall p[\exists w \in A[p \in Q(w)] \rightarrow \exists C \in \text{Partition}(A,Q)$

$$[\text{Exh}(Q)(p))]_A = C \qquad \textbf{[Non Vacuity]}$$

2. Dayal's Presupposition

Dayal assumes that an answer operator is obligatory in every interrogative construction. The operator takes a question denotation, Q, i.e., a set of propositions, and a world, w, and returns the strongest proposition in Q that is true in w:

(8) a. $\text{Ans}_D(Q) = \lambda w{:}\exists p[p = \text{Max}_{inf}(Q,w)].\ \text{Max}_{inf}(Q,w)$

b. $\text{Max}_{inf}(Q,w)=p$ iff $p \in Q\ \&\ w \in p\ \&\ \forall q \in Q[w \in q \rightarrow p \subseteq q]$

iff $p \in Q\ \&\ w \in p\ \&\ p = \bigcap\{q \in Q{:}\ w \in q\}$

In other words, Ans_D, like the answer operator that Heim (1994) attributes to Karttunen (1977) returns the conjunction of all the true members of Q (what is sometimes called weak exhaustivity), but is different from Karttunen's operator in demanding that this conjunction be, itself, a member of Q (Dayal's presupposition, DP). DP explains the uniqueness and existence inferences sometimes associated with questions (Dayal, 1996) as well as patterns of acceptability (in particular, negative islands: Abrusán & Spector, 2011; Abrusán, 2007; Abrusán, 2014; Fox, 2007b; Fox & Hackl, 2006; Schwarz & Shimoyama, 2011). In this section, I will go over some of this evidence for DP and explain why it can be equally seen as providing evidence for one component of PbE/QPM, namely, for Cell Identification in (4a).

2.1. Existence and Uniqueness

Consider the sentences in (9). Utterance of (9a) leads to the inference that exactly one girl came to the party – uniqueness – while utterance of (9b) leads simply to the inference that some girl came to the party – existence. (The inference in both cases pertains to the girls in the quantificational domain, consisting – if material in parenthesis is part of the question – of three individuals.)

(9) a. Which girl (among a, b and c) came to the party?

Question denotation: $Q = \{\lambda w.\text{came}(a,w), \lambda w.\text{came}(b,w), \lambda w.\text{came}(c,w)\}$

b. Which girls (among a, b and c) came to the party?

Question denotation: $Q = \{\lambda w.\text{came}(a,w), \lambda w.\text{came}(b,w), \lambda w.\text{came}(c,w), \lambda w.\text{came}(a+b,w), \lambda w.\text{came}(a+c,w), \lambda w.\text{came}(b+c,w), \lambda w.\text{came}(a+b+c,w)\}$

These inferences follow from DP. The question denotation in the case of (9a) is a set consisting of three logically independent propositions. This set can have a maximally informative true member only if it has a unique true member, hence the uniqueness presupposition. The question denotation in the case of (9b) consists of the same three logically independent propositions, but this time the set is closed under conjunction, since the domain of quantification in (9b) consists of plural individuals (and the relevant predicate is distributive). So now the set will have a maximally informative true member iff it has some true member, hence the existence presupposition.[8]

The same inferences follow from PbE. To see this consider for each of the two cases the point-wise exhaustifications of the question denotations provided in (10). The resulting set can partition the context-set only if, in every world in the context set, one of the propositions in the set is true, and this, of course, entails uniqueness in the case of (10a) and existence in the case of (10b).

(10) **Point-wise exhaustification of the question denotations in (9):**

a. $PWE(Q_{(9a)}) = \{\lambda w.came(a,w)\ \&\ \neg came(b,w)\ \&\ \neg came(c,w),$
$\lambda w.came(b,w)\ \&\ \neg came(a,w)\ \&\ \neg came(c,w),$
$\lambda w.came(c,w)\ \&\ \neg came(a,w)\ \&\ \neg came(b,w)\}$

b. $PWE(Q_{(9b)}) = \{\lambda w.came(a,w)\ \&\ \neg came(b,w)\ \&\ \neg came(c,w),$
$\lambda w.came(b,w)\ \&\ \neg came(a,w)\ \&\ \neg came(c,w),$
$\lambda w.came(c,w)\ \&\ \neg came(a,w)\ \&\ \neg came(b,w),$
$\lambda w.came(a+b,w)\ \&\ \neg came(c,w),$
$\lambda w.came(a+c,w)\ \&\ \neg came(b,w),$
$\lambda w.came(b+c,w)\ \&\ \neg came(a,w),$
$\lambda w.came(a+b+c,w)\}$

More generally, Ans_D can be restated with reference to a rather standard exhaustivity operator (that of Krifka, 1995):

(11) **Restatement of Dayal**

a. $Ans_D(Q) = \lambda w{:}\exists p{\in}Q[Exh_{stnd}(Q)(p)(w)=1].\ (\iota p{\in}Q)[Exh_{stnd}(Q)(p)(w)=1]$

b. $Exh_{stnd}(Q)(p)(w) = 1$ iff $p=Max_{inf}(Q,w)$

It follows that, whenever *Exh* as defined in (2) yields the same truth conditions for the members of Q as the standard *Exh* defined in (11b) (whenever the two versions of *Exh* agree on Q), Dayal's presuppositions are entailed by PbE.[9] Since the two versions of *Exh* agree on the set of denotations in (9), PbE also derives the uniqueness and existence presuppositions.[10]

---

[8] Karttunen (1977) observes that a question such as *who came to the party* is not associated with an existence presupposition. This can be captured by DP (or PbE) if the domain of *who*, as apposed to that of *which girl*, contains that "zero individual" argued for by Buccola & Spector (2016) and Bylinina & Nouwen (2018).

[9] PbE demands that point-wise exhaustification yields a partition of the context-set, A. If this holds, every world in A, w, belongs to a cell in the partition, C_w, which is identical to $[Exh(Q)(p)]_A$ for some $p{\in}Q$. Since $Exh(Q)(p)(w)=1$, Dayal's presupposition is met. In fact, it is easy to see that whenever the two definition of *Exh* agree on Q, Dayal's presupposition is equivalent to Cell Identification as defined in (7).

[10] *Exh(Q)(p)* as defined in (2) is identical to $Exh_{stnd}(Q)(p)$ as defined in (9), whenever $IE(Q, p) = \{q{\in}Q{:}\ \neg[p{\subseteq}q]\}$ (see Spector, 2016). The two operators agree on Q, whenever this identity holds for all p in Q.

2.2. Negative Islands (Fox and Hackl's perspective)

The second type of evidence for DP, which likewise argued for PbE, comes from Negative Islands. Consider the contrast between (12) and (13). This contrast has been assumed by various researchers to follow from the presuppositions of Ans_D. Specifically, it was claimed that the presuppositions are always going to be satisfied in the case of (12) and never in the case of (13).

(12) Guess how fast John drove
$Q = \{\lambda w.\ \mathrm{Speed}(J,w) \geq d: d \in D_d\}$
$\mathrm{Ans}_D(Q)(w^0)$ is *always* defined
$= \lambda w.\ \mathrm{Speed}(J,w) \geq \mathrm{Speed}(J,w^0)$

(13) *Guess how fast John didn't drive
$Q = \{\lambda w.\ \mathrm{Speed}(J,w) < d: d \in D_d\}$
$\mathrm{Ans}_D(Q)$ is *never* defined: $\neg\exists w \exists p \in Q[p=\mathrm{Max}_{inf}(Q,w)]$.

Here, for presentational purposes (see note 11), I will present the proposal made by Fox and Hackl (2006). F&H adopt the assumption made by various researchers that degree predicates are *downward scalar* (see Heim, 2001). In other words, they assume that if John drove d fast it follows (unless exhaustification applies) that his driving speed was greater or equal to d (from which it follows that John drove d' fast for all d'<d). So when Ans_D applies to the question denotation in (12) the result is always defined, i.e. it is always the proposition that John drove d* fast, where d* is John's actual driving speed (his driving speed at the world of evaluation). However, in the case of (13) the result is never defined, since there is never a degree, d, that yields the strongest true proposition of the form *John did not drive d fast*; if there were such a degree, it would be the minimal degree greater than d* (d*'s successor), but F&H assume that the domain of degrees is dense, from which it follows that the there is no such minimal degree (that the successor function is not defined).

F&H argue for this account of unacceptability with the observation that the phenomena of modal obviation illustrated in (14) follows. Specifically, while there is never a minimal degree d such John did not drive d fast, there is sometimes (in some worlds) a minimal degree d such that John is not *allowed* to drive d fast.[11]

[11] As shown in Fox (2007b), the account of modal obviation does not depend on F&H's account of the contrast between (12) and (13). Specifically, any assumption about the denotation of Q in (13) from which it would follow that the presupposition of Ans_D cannot be satisfied (on logical grounds) would predict this not to be the case in (13). And, indeed, alternative accounts of the contrast between (12) and (13) have been provided, all based on Ans_D but differing in their assumptions about the denotation of the questions yet yielding modal obviation based on the logic outlined in Fox (2007b). Hence the argument for the presuppositions of Ans_D are rather strong.

(14) Guess how fast John is not allowed to drive
$Q = \{\lambda w.\ \neg\exists w'\in Allowed_w\ Speed(J,w') \geq d: d\in D_d\}$

$Ans_D(Q)$ is *sometimes* defined, if, e.g., the rules in w^0 specify that John must drive slower than 90 kilometers per hour. In such a case,

$$Ans_D(Q)(w^0) = \lambda w.\ \neg\exists w'\in Allowed(w)[Speed(J,w') \geq 90\ \text{kmh}]$$

Once again, it is easy to see that the facts also follow from PbE. Specifically in (13) every $p\in Q$ is such that *exh(Q)(p)* is contradictory, independently of which definition of *Exh* we use (the one in (2) or the one in (11b)).[12] Hence PbE cannot be satisfied in (13), whereas it can be satisfied in (12) and (14).[13] We have, so far, seen two arguments that support DP and likewise one of the components of QPM/PbE. We now turn to considerations that are problematic for DP but support PbE.

3. Modification of Dayal – #1 (Cell Identification)

Consider the restatement of Dayal's answer operator in (11) repeated below. As mentioned in note 9, the presupposition is equivalent to Cell Identification of QPM in (4) whenever the exhaustivity operators defined in (11)b and the one in (2) yield the same results.

(11) **Restatement of Dayal**
a. $Ans_D(Q) = \lambda w{:}\exists p\in Q[Exh_{stnd}(Q)(p)(w)=1].\ (\iota p\in Q)[Exh_{stnd}(Q)(p)(w)=1]$
b. $Exh_{stnd}(Q)(p)(w) =1$ iff $p=Max_{inf}(Q,w)$

So the first modification I would like to propose would make Dayal's presupposition fully equivalent to Cell Identification. Specifically, I would like to suggest that we replace (11)b with (2).

(15) **Modification of Dayal** (with presupposition equivalent to Cell Identification, (4a))
a. $Ans(Q) = \lambda w{:}\exists p\in Q[Exh(Q)(p)(w)=1].\ (\iota p\in Q)[Exh(Q)(p)(w)=1]$
b. $Exh(Q)(p)(w)=1$ iff $\forall q\in Q[q\in IE(Q,p)\rightarrow q(w)=0]\ \&$
$\forall q\in Q[q\notin IE(Q,p)\rightarrow q(w)=1]$.

---

[12] In fact, the two definition of *Exh* agree on Q in (12-14), since these sets of propositions are totally ordered by entailment, see Gajewski (2011). So again Dayal's presupposition here is equivalent to Cell Identification.

[13] PbE also demands that every proposition in Q be mapped by *Exh* to a cell (Non Vacuity). For this to be satisfied in any realistic context, Q in (14) would have to be pruned quite extensively, by implicit domain restriction. [For any realistic context-set A there will be a small enough degree, d, such that $Exh(Q)(\lambda w.\ \neg\exists w'\in Allowed_w\ Speed(J,w') \geq d)$ would be false in every world in A.] Violations of Non Vacuity yield unacceptability when there is a vacuous proposition in Q if that proposition cannot be pruned because it is in the Boolean closure of the propositions that are not pruned. When this is the case, pruning violates basic constraints on pruning because relevance is closed under Boolean operations (Fox & Katzir, 2011). See 4.2 below.

The argument I will present here for this modification is based on the cases that motivated (2) in the domain of scalar implicatures, namely cases of free choice disjunction where the necessary strengthening of disjunction to conjunction does not follow from (11)b, but does follow from (2).

Consider the question in (16a) and the answer provided in (16b).

(16) a. Question: What are we allowed to eat for dessert?
b. Answer: Cake or ice cream. (Ambiguous)

As pointed out in Spector (2008) the answer is ambiguous and can be understood either conjunctively, i.e. as a statement that we are allowed to eat cake *and* allowed to eat ice cream (the FC reading) or disjunctively, suggesting that the speaker is ignorant about which of the options is allowed. Spector has argued extensively that this ambiguity is already present in the question denotation. This position and some arguments for it will be presented shortly (in section 4 below), but now I will simply accept the conclusion. Specifically, I will assume that the disjunctive interpretation results from the standard interpretation of the question, where the *wh*-phrase ranges over ordinary individuals, and the FC interpretation results from a higher order interpretation where *wh*-phrase ranges over generalized quantifiers. My argument in favor of the modification in (15) is based on the higher order interpretation. In (17) I provide the relevant LF, a suggestive paraphrase, and the resulting denotation:

(17) **LF for the FC reading (before application of *Ans*)**
λp. What^{R} (books) $\lambda\mu$ C_p λw. allowed$_w$ μ λx. [you to read x for this class]?

Where μ is a variable ranging over GQs and *what*^{R} is an existential quantifier over Upward Monotone quantifiers over the domain of *what (books)*.

Paraphrase: What is the upward monotone quantifier that lives on the set of books, μ, such that the requirements are compatible with the proposition λw. μ(λx. you read x for this class in w)

Question Denotation
Q= {$\lambda w \exists w' \in$Allowed(w)[μ(λx. you read x for this class in w')]: μ is an upward monotone quantifier that lives on the set of books}

The argument for (15) is based on situations where we are allowed to have cake and allowed to have ice cream, and are not allowed to have both (nor anything else) for dessert. In such worlds there is no maximally informative true member in the question denotation, so Dayal's presupposition (in (11)) is not satisfied. However, the presupposition of the modified operator in (15) is satisfied.

To see this, consider the true members of the question denotation in the worlds we characterized. Among these propositions are the proposition that we are allowed to have cake, $\Diamond C$, the proposition that we are allowed to have ice cream, $\Diamond IC$, and the proposition that we are allowed to have cake or ice cream, $\Diamond[C \vee IC]$, all non-exhaustified. Since these propositions are not exhaustified, none of them entails all the others. In particular, the disjunctive member of Q is not interpreted conjunctively prior to exhaustification ($\Diamond[C \vee IC]$ is equivalent to $\Diamond C \vee \Diamond IC$; for extensive arguments see Bar-Lev, 2018; Bar-Lev & Fox, in press). So if Dayal's presupposition were to be satisfied, there would have to be a proposition in the question denotation that entails all three. But there isn't. Any generalized quantifier that could be plugged under the scope of $\Diamond$ such that the resulting proposition would entail $\Diamond C$ and entail $\Diamond IC$ would be at least as strong as the conjunction, and would consequently be false in the relevant worlds (since $\Diamond[C \wedge IC]$ is false).

We conclude that Dayal's presupposition makes the wrong prediction for the higher order reading of the question.[14] The modification in (15) makes the right prediction: although the question denotation does not contain a maximally informative true member, it does contain a member whose exhaustification is true. Specification Exh(Q)($\Diamond[C \vee IC]$) is the proposition that cake is allowed, ice cream is allowed and nothing else is allowed, precisely the cell that the relevant world belong to in the partition induced by the question.

4. Modification of Dayal – #2 (Non Vacuity)

Spector's argument for the higher order interpretation centered on questions that contain universal modals rather than existential modals. Consider the question answer pairs in (18) and (19). The answers (all quantificational fragments like in (16) above) show an ambiguity that can, once again, be explained if higher order quantification is available in addition to standard quantification over individuals. If the *wh* trace ranges over simple individuals, the quantifier in the fragment answer will have scope over the modal *required*. If the trace ranges over generalized quantifier, the modal will have wide scope.[15]

(18) What are you required to read for this class?
War and Peace or Brothers Karamazov. (*required*>*or*; *or* >*required*)

(19) Which books are you required to read for this class?
a. The Russian books or the French books. (*required*>*or*; *or* >*required*)
b. Three Russian books. (*required*>*3*; *3* >*required*)
c. [MB or SE] and [W&P or BK] (*required*>*or*; *or* >*required*)

[14] This conclusion, of course, is based on the assumption we made about the question denotation. One possible response would involve a modification of the assumption (Gentile & Schwarz, 2019; Spector, 2008). Specifically, we could assume that the question nucleus is strengthened by exhaustification so that the disjunction is strengthened to conjunction and Dayal's presupposition is met (see (29'') below). However, this would predict that question embedding will be disambiguated in favor of the so called strongly exhaustive reading, which is problematic in light of the following, which I think can have a higher order reading which is not strongly exhaustive:

(i) Mary correctly predicted what we are allowed to eat, namely cake or ice cream.
(ii) It surprised Mary what we are allowed to eat.

[15] Of course, a full account of the connection between the representation of the question and of the fragment requires specific assumptions about the analysis of fragment answers. For concreteness, we can adopt the assumption that fragments involve ellipsis, which must satisfy a Parallelism condition of the sort argued for in Rooth, 1992. This would probably require the assumption that *wh*Ps have a landing site above the subject and below the interrogative complementizer (Romero, 1998).

The unavailability of narrow scope for the quantifier in the fragment answer in (20) can be seen to demonstrate that the construction is sensitive to negative islands. This is further supported by the observation of modal obviation in (21), corresponding to what we've seen in (14c).

(20) What did you not read for this class?
War and Peace or Brothers Karamazov. (*not>or; *or >not*)[16]

(21) What are you not allowed to read for this class?
War and Peace or Brothers Karamazov. (?*not*>*or*; *or >not*)

In the case of degree expressions, modal obviation provided support for the claim that Dayal's presupposition is involved in the account. So one would hope that DP will provide an account here as well. But this is not the case, as pointed out by Spector. If you read everything but War and Peace and brothers Karamzov, the proposition that you didn't read War and Peace or Brothers Karamzov (*not*>*or*) would be the most informative true member of the question denotation. Thus, DP would be satisfied with higher order quantification and we would expect the fragment answer in (20) to be acceptable on the *not*>*or* representation.[17]

So here is where we are now. In section 3, the FC interpretation of the answer was taken to reveal an area where Ans_D demands too much from a question (under-generation).[18] Here we see a place where it arguably demands too little (over-generation). As mentioned, DP is equivalent to the demand that every cell in the partition of the context-set be identifiable (via *Exh*, as defined in (11b)) by a member of Q. We have seen that the problem of under-generation is resolved the moment we move from the view of *Exh* in (11b) to an alternative that derives the FC meaning of disjunction, e.g. that in (2), equivalently, the moment we adopt Cell Identification from (4), repeated below. We will now see that the problem of over-generation, exemplified by the unacceptability of *not*>*or* in (20), is resolved the moment we add to Cell Identification the converse requirement of Non Vacuity, (4b), yielding the requirement of QPM (the moment we think of Cell Identification as one of the consequences of PbE):

(4) **Question Partition Matching (QPM)**: Q is acceptable given a context-set A only if
a. $\forall C \in \text{Partition}(A,Q)\ \exists p \in Q: [\text{Exh}(Q)(p)]_A = C$ **[Cell Identification]**
b. $\forall p \in Q \exists C \in \text{Partition}(A,Q): [\text{Exh}(Q)(p)]_A = C$ **[Non Vacuity]**

[16] A way to see that *not*>*or* is ruled out is to track "ignorance inferences" – to observe the obligatory inference that the speaker does not know whether W&P was read (and likewise for BK). If *not*>*or* were possible, the fragment (when exhaustified) could provide a complete answer to the question, which would be associated with no ignorance inferences.

[17] One might suggest to rule out higher order quantification for (20) based on the observation that the resulting interpretation is not sufficiently distinct from the basic interpretation. Specifically, higher order quantification yields the same partition as the one induced by the more basic semantic type. This could account for the restriction along with an appropriately stated economy condition (along the lines of e.g., Reinhart, 1983 or Fox, 2000). See (Fox, 2018) for relevant discussion.

[18] The problem of under generation for Dayal is more dramatic when we consider mention some readings (see Fox, 2018).

For *not* to outscope *or* in the fragment answer in (20), the antecedent question would require higher order quantification. In other words, the antecedent question in (20) would have to receive the LF representation and semantic denotation in (20'):

(20') **Higher-order quantification**:
LF: Wh shift(restrictor) $\lambda\mu_{ett}$ Not μ λx. we read x for this class?
Denotation: $Q = \{p: \exists\mu\in UGQ(R)\ [p=\lambda w.\ \{x: \text{We read}_w\ x \text{ for this class}\}\notin\mu\}$,
Where UGQ(R) is the set of upward entailing generalized quantifiers that live on R.

The question denotation is guaranteed to have a maximally informative true member, thus satisfying DP as well as Cell Identification. However, it does not satisfy Non Vacuity, at least if there are two or more objects in R. To see this, assume that $b_1\in R$ and $b_2\in R$. The conjunction of (the Montague-Lift of) the two is a member of UGQ(R), and the proposition in (22) is in the question denotation. But this proposition is never mapped to a cell by *Exh*, hence Non Vacuity cannot be satisfied.[19]

(22) $\lambda w.\ \neg$ ([We read$_w$ b_1 for this class] $\wedge$ [We read$_w$ b_2 for this class])

A weak proposition of this sort ($\neg > \wedge$) is always in the question denotation.[20] And applying *Exh* to this proposition will never yield a cell in the partition, hence Non Vacuity will never be satisfied.[21,22] The moment additional quantificational expressions are introduced above the *wh*-trace, as in (21), things change (for certain context sets) for the reasons discussed in Fox, 2007b – generalizing observations in Fox and Hackl, 2006.[23] For example, the corresponding proposition to (22) in the case of (21) will be the following:

(23) $\lambda w.\ \neg$ Allowed$_w$ ([We read b_1 for this class] $\wedge$ [We read b_2 for this class])

[19] Applying *Exh*, as defined in (2), to this proposition yields a contradiction. Note that nothing changes if we move to a theory of *Exh* that does not yield contradictions in cases of this sort (e.g. Bar-Lev & Fox, in press; Fox, 2007a; Groenendijk & Stokhof, 1984). In all of these theories, *Exh* yields a meaning that is necessarily weaker than a cell in the partition.

[20] There is of course the worry that this vacuous proposition would be pruned – by implicit domain restriction – from the question denotation (see note 13). This worry is addressed in 4.2.

[21] NV also predicts no higher order quantification for simple questions such as *what did you read?*, as apposed to *what are we required to read?* I haven't figured out a way to test this prediction directly, though I should note that, on the one hand, it is supported by the existence presuppositions of the sentences in (9), and, on the other hand, it might be disconfirmed by the proposal in Elliott, Nicolae, & Sauerland (2017).

[22] As it stands, the proposal can't be right. The contradictory GQ is an UE GQ living on R, and the resulting proposition cannot be in the question denotation if QAP is a requirement. So we need to change the type shift rule slightly, so that the contradictory GQ is not in the domain of the *wh*P. See Fox 2018, note 48 for one proposal.

[23] Let *Op* be a universal modal (type <s,<st,t>>). The relevant logical fact is that for every set of propositions Q, and contingent proposition $p\in Q$, it is possible for $\lambda w.OP(w)(p)$ to be the maximally informative true member in $\{\lambda w.OP(w)(q): q\in Q\}$.

And this proposition can be the most informative true proposition in the Hamblin set associated with higher order quantification, hence can be mapped by *Exh* to a cell in the partition. So QPM (and likewise PbE) accounts for the negative island and its obviation by appropriately placed modals.

In the next section, I will introduce the puzzle discovered by Gentile & Schwarz (2017). But before I get there, I'd like to touch on a minor point of implementation and on a more substantial worry pertaining to contextual restriction and its possible influence on PbE.

4.1. A revised *Answer* operator

PbE (and the equivalent QPM) was stated as an admissibility condition and was not encoded in the presuppositions of a lexical item like Ans_D. But this difference is, as far as I can tell, of no real consequence: it is trivial to use PbE (or QAM) in the definition of an answer operator, as long as we allow that relevant lexical item to take a set of worlds as an argument, i.e. a *local context*. We can simply write a relation PbE (equivalently QAM) that holds between a question Q and context-set A iff Q is admissible given A according to (1) (or equivalently (4)), and write an operator that returns the weakly exhaustive answer whenever the presupposition is met.[24]

(24) **PbE encoded in *Ans***

$$\text{Ans} = \lambda Q_{stt}.\ \lambda A_{st}.\ \lambda w\text{: } w \in A \,\&\, \text{PbE}(Q,A).\ (\iota p \in Q)[\text{Exh}(Q)(p)(w)=1]$$

4.2. PbE and constraints on domain restriction

Non Vacuity rules out the higher order Q in (20') based on the observation that for any set of worlds A there is a p∈Q that is not mapped by *Exh* to a cell in Partition(Q,A). We have seen this by focusing on a particular p∈Q (the disjunctive proposition in (22)) and observing that it cannot be mapped to a cell by *Exh*. An obvious question that should be raised at this stage is whether the question denotation can be contextually restricted (or pruned), by, a covert domain restrictor, C, conjoined with the restrictor of the *wh*P (UGQ(R)∩C). The worry is that such contextual restriction might "prune" (22) (and all other propositions that lead to a violation of Non Vacuity) from the question denotation leading to an acceptable result.[25]

To rule this pruning out, I would like to appeal to constraints on pruning introduced in the context of work on exhaustivity (Moshe E. Bar-Lev, 2018; Crnič, Chemla, & Fox, 2015; Fox & Katzir, 2011; Katzir, 2014; Magri, 2009, 2011). For example, assume with Fox and Katzir, that a proposition p can be pruned from a set of propositions Q only if the resulting question [Q – {p}] makes p irrelevant, i.e. leads to a different partition of the context set. This constraint on pruning would rule-out pruning of (22) given that it is the disjunction of two non-pruned alternatives (and the fact that relevance, defined by reference to partitions, is closed under Boolean operators).

[24] The argument from mention some in Fox (2018) was based on the observation that with PbE it is possible to write a different operator, one that returns the set of propositions that entail (ιp∈Q)[Exh(Q)(p)(w)=1]

[25] See Fox and Hackl, 2006 and subsequent work on negative islands where a very similar issue arises.

So Non Vacuity together with this constraint on pruning predicts Q to be unacceptable whenever there is a p∈Q that cannot be mapped to a cell in the partition induced by Q *and cannot be pruned*. We considered one reason why p cannot be pruned. But PbE would make additional predictions if we could identify additional reasons. Consider from this perspective the claim that domain restriction is impossible when domains are explicitly enumerated, e.g. in partitive constructions: *everyone of these three children attended the party* cannot be understood as a weak statement involving universal quantification over a proper subset of the set containing the three individuals. If this claim is correct, PbE explains the oddness of the following:

(25) a.#We both know that Mary wasn't at the party, but tell me who among Mary, Sue, and Jane was there.
b.#We both know that Mary was at the party, but tell me who among Mary, Sue, and Jane was there.

The first conjunct in both (25a) and (25b) leads to a context set with fewer cells than what we get from pointwise exhaustification. This situation leads to a violation of Non Vacuity. For example the proposition that Mary was at the party which is in the denotation of Q will be a vacuous proposition in (25a) – not mapped to a cell by *Exh* – and likewise for the proposition that Sue+Jane were at the party in (25b).

5. Gentile and Schwarz's Puzzle

We are now ready to turn to the new puzzle introduced by G&S, which, as we will see, is quite confusing from the perspective of Dayal's presupposition. But we will also see that a solution to the puzzle falls out without further ado from the assumptions we just introduced. Consider the question (26), which G&S argue comes with a uniqueness presupposition.

(26) **Uniqueness presupposition for *how many* questions with collective predicates:**
How many students solved the problem together?
(Presupposition: only one group of students solved the problem together or at least every group of students that solved the problem together has the same cardinality)
$Q = \{\lambda w\ \exists X\ |X|=n\ \&\ \mathrm{Students}(X)\ \&\ \mathrm{SPT}(X, w): n \in N\}$

To see the argument, consider first the situation in (27), where it is common ground that exactly one group of students solved the problem together.

(27) **Context where (26) is natural**: A class is divided into two groups. One group is assigned a math problem: the members of the groups are asked to solve it together. The other group is assigned some reading material. All of this is common ground. At the end of the day we learn that the problem was solved. The person who asks the question in (26) wants to know how many students were in

the first group (perhaps because she wants to have an estimate of how many people can work together).

We note that the question is natural to ask in this context. Imagine, for example, that the addressee of (26) knows that the relevant group consists of 9 students. The question would be very clear to her and *Nine!* would be acceptable as a short answer. Consider now the context in (28), where it is common ground that two groups of students (not necessarily of equal size) solved the problem. Now the question seems unacceptable and cannot receive a conjunctive short answer, e.g. *Nine and Eleven*.[26]

(28) **Context where (26) is unacceptable**: A class is divided into two groups (not necessarily evenly). Each of the two groups is assigned the same math problem: in both cases the members of the group are asked to solve the problem together. At the end of the day we learn that the problem was solved. The person who asks the question in (26) wants to know how many students were in each group (perhaps because she wants to have an estimate of how many people can work together).

G&S point out that this contrast follows from Ans_D. Specifically, the members of Q in (26) are logically independent (just like in (9a)). This independence stems from the presence of a collective predicate (Buccola & Spector, 2016). Specifically, if members of a group of students, X, solved the problem together, it doesn't follow that members of subsets of X solved the problem together. Hence Max_{inf} can apply to (26) (just like to (9a)) only if Q has exactly one true member.

This is, all, very good. The problem identified by G&S is that the uniqueness presupposition is absent in (29), as we can see by inspecting our intuitions about the short answers in parenthesis.[27]

(29) **Modal circumvention of Uniqueness Presupposition:**
How many students are allowed to solve the problem together?
(*between 2 and 4 children*; *up to eight*; *between three and eight*; *any multiple of three*,…)
$Q = \{\lambda w\ allowed_w\ (\lambda w' \exists X\ |X|=n\ \&\ Students(X)\ \&\ SPT(X, w'): n \in N\}$

Members of Q in (29) are logically independent (just like the members of Q in (26) and (9a)). To see this, note that Q in (29) results from pointwise application of an upward entailing operator, *allowed*, to the members of Q in (26) (or more concretely by observing that the existence of a group that solved the problem together in an allowed world does not guarantee that there is a group of a different cardinality that solved the problem

[26] G&S report a disagreement between their informants pertaining to situations in which it is presupposed that multiple groups are involved and *furthermore it presupposed that that they are of equal cardinality*, e.g. if it is common ground that the class is divided to two groups with the same number of members. On the face of it, Dayal's presupposition (and PbE) predicts the question to be admissible in such a context, though see the discussion in section 10.

[27] Examples of this sort are discussed in Beck & Rullmann, 1999, and Fox, 2010.

together in an allowed world). Hence (29) is incorrectly predicted by Ans_D to have a uniqueness presupposition.

6. The Solution

PbE, just like Ans_D, makes the wrong prediction for the Q in (29). Specifically, when *Exh* applies to any member of this Q, the resulting proposition states that there is a group of students with cardinality n that solved the problem together and no other proposition in Q is true. PbE requires point-wise exhaustification to partition the context-set. It, thus, follows that in every world in the context-set one of the exhaustified propositions is true (Cell Identification), i.e. that there aren't two groups of different cardinality that solved the problem together. More generally, *Exh* as defined in (2) and *Exh* as defined in (11b) agree on the Q in (29) and thus Cell Identification is equivalent to Dayal's presupposition. The difference between Ans_D and Cell Identification emerges when *Exh*, as defined in (2), and *Exh* as defined in (11b) are no longer equivalent, and this, as we will see, happens when (29) receives a higher order LF. More specifically, we will see:

a. that PbE (in contrast to Ans_D) no longer predicts a uniqueness presupposition for (29), given the general availability of higher order quantification, and,
b. that higher order quantification is impossible in (26), as it necessarily violates PbE (in particular Non Vacuity) for the same reason that higher order quantification is subject to negative islands. Hence the introduction of higher order quantification doesn't affect the predictions of PbE for (26).

6.1. Higher Order Quantification eliminates uniqueness in (29)

Consider the higher order denotation of (29) provided below:

(29') Q-higher-order =

$$\{\lambda w\ \exists w' \in Allowed(w)[\mu(\lambda n.\ \exists X\ |X|=n\ \&\ Students\ (X)\ \&\ SPT(X, w')]:$$

μ is an upward entailing quantifier that lives on the set of degrees}

Imagine that 2, 3, and 4 students are allowed to solve the problem together and that no other number is allowed. The Q in (29) will not contain a proposition such that its exhaustification is true, and subsequently Cell Identification will be violated for this denotation. But the situation with the higher-order Q in (29') is different. (29') still has no maximally informative true member. It has three true member – $\Diamond(2)$, $\Diamond(3)$, $\Diamond(4)$ – and various weaker propositions. However, just like in (17), there is now a true member – $\Diamond(2 \vee 3 \vee 4)$ – such that also its exhaustification is true. So, again, just like in (17), Dayal's presupposition is not met, but Cell Identification (and more generally PbE) is met.

6.2. Higher Order Quantification is impossible in (26)

If higher order quantification were possible in (26), it could have the denotation in (26'). This question denotation would contain within it the conjunction of the two true propositions in a scenario such as (28), so Cell Identification – or equivalently the presupposition of the minimal modification of Dayal in (15) – would be satisfied.

(26') $Q = \{\lambda w\ \mu\ (\lambda n.\ \exists X\ |X|=n\ \&\ Students(X)\ \&\ SPT(X, w))$:
μ is an upward entailing quantifier over degrees$\}$

So what accounts for the uniqueness presupposition of (26)? The answer comes from Non Vacuity: (26)' will always contain propositions whose exhaustification cannot possibly be true, such as (30) below – the proposition that we get when μ in replaced with the disjunction (of the Montague lift of) two degrees.

(30) $\lambda w.\ \exists X\ |X|=3\ \&\ Students(X)\ \&\ SPT(X, w)$
$\vee\ \exists X\ |X|=5\ \&\ Students(X)\ \&\ SPT(X, w)$

A weak proposition of this sort will always be in the question denotation. And applying *Exh* to this proposition will never yield a cell in the partition; hence NV will never be satisfied (see note 19). In other words, the fact that a uniqueness presupposition cannot be circumvented in (26) receives the same explanation that we provided for the sensitivity of higher order quantification to negative islands (exemplified in (22)) – only a non higher order quantification construal is admitted, and this construal yields uniqueness by QPM/PbE. The fact that a modal circumvents the uniqueness effect receives the same explanation that we gave to the modal obviation of negative island – higher order quantification construal is admitted and this construal does not yield uniqueness by QPM/PbE.

7. Gentile and Schwarz 2019

Gentile & Schwarz (2019) have, independently, made a somewhat similar proposal to my own, but one that, interestingly, does not depend on PbE. Specifically, like me they assume that higher order quantification is responsible for the contrast between (26) and (29). But, unlike me, they propose that $Ans_{\underline{D}}$ is responsible for uniqueness and existence presuppositions in (26) (and elsewhere, e.g. in (9)). While I find their proposal interesting, I would like to argue that it ultimately fails and that understanding this failure allows us to better appreciate the nature of the argument for PbE.

Consider again Q-higher-order in (29'). I have pointed out that the presuppositions of $Ans_{\underline{D}}$(Q-higher-order) are not satisfied in a particular type of circumstance where the question in (29) is acceptable, e.g. when $\Diamond(2)$, $\Diamond(3)$, and $\Diamond(4)$ are the most informative true members of Q-higher-order. The acceptability of (29) when such circumstances are compatible with the common ground [e.g. when the answers in parenthesis in (29) are provided] was presented as an argument that $Ans_{\underline{D}}$ needs to be modified. The proposed modification in (15), with presupposition equivalent to Cell Identification, explained the acceptability of the question based on the observation that (29') does contain a proposition whose exhaustification is a cell in the partition induced by the question, $\Diamond(2 \vee 3 \vee 4)$ in the example we considered. In other words, Cell Identification overcame the challenge that (29) poses for Dayal in exactly the same way that it overcame the parallel challenge posed by (16).

But G&S (2019) propose a different approach, one that keeps $Ans_{\underline{D}}$ in tact. Specifically, they propose that (29) has another higher order parse, one that involves grammatical exhaustification within the question-nucleus (rather than in the statement of the presuppositions of the answer operator):

(29'') Q-higher-order-with-*exh* =
$$\{\lambda w\ Exh(\exists w' \in Allowed(w)[\mu(\lambda n.\ \exists X\ |X|=n\ \&\ Students(X)\ \&\ SPT(X, w')]):$$
$$\mu \text{ is an upward entailing quantifier that lives on the set of degrees}\}$$

To fully spell-out the proposal one needs to specify the Q argument of *Exh* – the set of alternatives for exhaustification (left out in the above formula). So let us assume what is ultimately needed, namely that the Q argument of *Exh* ends up being Q-higher-order in (29'), perhaps through associating with a focused trace. Under these assumptions, the question denotation ends up having a most informative true member (as long as it has a true member to begin with). In particular, under the circumstances described above *Exh(Q-higher-order)(◊(2 v3 v4))* would be the most informative true member in (29''), as long as we adopt a theory of *Exh* that delivers FC readings. So while PbE resolves the problem posed by (29) with simple higher order quantification – given the existence of a proposition in the higher order denotation of (29) whose exhaustification is true – G&S (2019) propose that we keep Dayal's presupposition in its unmodified format and instead modify our assumptions about the nature of higher order quantification – so that the exhaustified proposition be in the question denotation.

I think there are reasons to be skeptical about this proposal, which I have mentioned in note 14. However, I'd like to put such considerations aside here and move to the more important question which is how to avoid an over-application of higher order readings, one that would get rid of the uniqueness presuppositions in the first place, e.g. in the basic case where the modal is absent, exemplified in (26). To answer this question, G&S (2019) propose a modification of Spector's assumptions about the nature of higher order quantification. While Spector suggested that higher order quantification quantifies over upward entailing generalized quantifiers, G&S (2019) proposed that only a subset of such quantifiers are quantified over, namely existential quantifiers (or disjunction) over subsets of the relevant domain. So, specifically, (26') is not a possible denotation for (26). Instead, (26'') is the higher order denotation, and this denotation does not contain stronger propositions than the propositions that were in the basic denotation (before type-shift can apply, i.e. Q in (26)).

(26'') $Q = \{\lambda w\ \mu\ (\lambda n.\ \exists X\ |X|=n\ \&\ Students(X)\ \&\ SPT(X, w)):$
$$\mu \text{ is an } \textit{existential quantifier} \text{ over a subset of the domain of degrees}\}$$

With this move we would lose the account of the negative island provided in section 4. But I think that the move is problematic even on the basis of a more limited set of concerns. Consider, first, examples like (31) from Spector (2008). On the face of it, the short answers in (31a) and (31b) suggest that the GQs quantified over are not limited to existential quantifiers: conjunctions, universal and proportional quantifiers appear to be in the domain as well.

(31) *Which books are we required to read?*
(a) More than three Russian books **and** more than 2 French books
(b) **Every** Russian book on the list or **half of** the French books.

The alternative suggested to me by Bernhard Schwarz capitalizes on a property of the domain of individuals that the quantifiers *live on*. Specifically, this domain includes pluralities, and the quantifiers in (a) and (b) can be paraphrased as the disjunction of (the Montague-lift of) plural individuals (with appropriately placed distributivity operators). While I don't necessarily object to this idea (see Fox 2018, note 48),[28] I don't think it is helpful in this context.

Specifically, G&S's move would be helpful only if we could bar plural degrees from populating (26') with conjunctive propositions that would obviate the uniqueness effect. The problem is that, as far as I can see, there is no obvious way of meeting this demand. Specifically, I can think of two assumptions that would have the desired effect but they both lead to dead ends. One possible assumption is that the domain of degrees simply does not contain degree pluralities to begin with, contrary to Beck, 2014; Dotlačil & Nouwen, 2016. But this would make it impossible to account for questions such as (32) from Fox (2010).

(32) How fast are we not allowed to fly?
Below 50 meters (too low) **and** above 2000 meters (too high)

An alternative would accept plural degrees but impose restrictions on the placement of distributivity operators, so that they would not appear between μ and λn in (26''). I think this, too, is not a reasonable way out, in light of (33):

(33) How many students should solve this problem together
between 5 and 7 (the small group) **and** between 8 and 10 (the large group)

These examples teach us, I propose, that the absence of conjunctive propositions in (26) cannot be explained by assumptions about the nature of the higher order denotation: conjunctive propositions must be derivable through higher order quantification (either directly because they are quantified over by the shifted *wh*-Phrase, as assumed by Spector, or through some other means, e.g. the presence of pluralities, as assumed by G&S (2019)). In light of this, I would like to suggest that the contrast between (26) and (29) teaches us that the modal (present in (29) and absent in (26)) is somehow crucial for higher order quantification, as predicted by Non Vacuity, and as I've suggested when discussing the negative island in (20). I conclude that Ans_D is not sufficient in an account of G&S's effect.

In the next sections, we will discuss some challenges to the account proposed here, but before we get there, I think a short recap might be useful. We began this paper with two problems for Ans_D that come from the higher order readings of questions. On the one hand, we've seen that Ans_D makes demands that are too strong when existential modals

[28] In fact I think the observation of Hirsch & Schwarz (2020) can be seen to argue in favor of this position. See (Xiang, 2020).

are involved, e.g. (16). In such cases, questions are acceptable also when a maximally informative true proposition is not presupposed. What I suggested is that a possible conceptual underpinning for *Ans*$_D$ can nevertheless be maintained. This was made transparent, I hope, by the friendly amendment of *Ans*$_D$ provided in (15), the presupposition of which is equivalent to Cell Identification.

On the other hand, we've seen that *Ans*$_D$ makes demands that are too weak when negation is involved (negative island): introducing negation above a *wh*-trace blocks the higher order readings, (20), although a maximally informative true proposition is guaranteed. We also saw that the relevant cases share formal properties with other cases of unacceptability that argue in favor of *Ans*$_D$, strongly suggesting that a generalization is being missed. I suggested that the generalization is captured by QPM/PbE for which there is clear conceptual motivation. (A partition, the object needed for language use, needs to be derived from the object generated by grammar; if pointwise exhaustification is the method for this derivation, PbE is a direct consequence.) If this view is correct, Non Vacuity can be assumed in addition to Cell Identification (as yet another consequence of PbE) and the generalization is captured.

With this at hand we moved to G&S's puzzle, which was resolved based on the two arguments for PbE that we considered. On the one hand, G&S observed a case where *Ans*$_D$ demands too much. Specifically, the acceptability of (29), when no uniqueness is presupposed, raised exactly the same kind of challenge to *Ans*$_D$ that was raised by (16), a challenge which was resolved by Cell Identification in the same way (or equivalently by the friendly amendment of *Ans*$_D$ in (15)). On the other hand, G&S observed a case where *Ans*$_D$ demands too little. Specifically, the moment higher order quantification is allowed *Ans*$_D$ does not account for the uniqueness presupposition of (26). We have seen that this can be viewed as an identical challenge to the one raised by negative islands with higher order quantification, (20). In both cases, things are understood the moment we realize that there are too many members in the question denotation for Non Vacuity to be satisfied.

8. Higher-order Pluralities

Xiang (2016, 2020) raises a very interesting challenge for Dayal's presupposition that comes from simple questions with collective predicates. This challenge is particularly interesting in light of G&S's observation that Dayal's presupposition makes the correct predictions in very similar constructions, exemplified in (26). We will see that Xiang's proposal cannot easily account for G&S's observation, leading to a dilemma. The dilemma takes exactly the same shape under PbE: PbE, just like Dayal's presupposition, can account for G&S's observation in (26) but not for Xiang's observation (and Xiang's proposal is incompatible with the PbE account of G&S's observation). I will argue for a resolution of this dilemma based on higher order pluralities and an independently justified constraint on the meaning of number words.

8.1. Xiang's challenge and the resulting dilemma

Consider (34) and Xiang's observation that the question should come with a uniqueness presupposition given the obvious question denotation. Specifically, if *Ans*$_D$ is obligatory, the question should presuppose that Q in (34) has a maximally informative true member, i.e. that only one group of students solved the problem together, an incorrect prediction as illustrated in (35).

(34) **Uniqueness presupposition incorrectly predicted for *which* questions with collective predicates:**
Which students solved the problem together?
(**Predicted presupposition**: only one group of students solved the problem together)
$Q = \{\lambda w\ SPT(X, w)$: x is a plurality of students$\}$

(35) I know which students solved the problem together. Mary and Sue; John, Bill, Jane and Fred; and the eight kids over there (no uniqueness presupposition)

This observation is particularly striking in light of G&S's observation in (26) repeated below. In (26), too, the question should be associated with a uniqueness presupposition, given the obvious question denotation. But this time the prediction is correct. Specifically, if Ans_D is obligatory, the question should presuppose that Q in (26) has a maximally informative true member, i.e. that there cannot be two groups of students (with different cardinalities) that solved the problem together, this time a correct result as we have seen in (28), and as illustrated again in (36), with a minimal variant of (35).

(26) **Uniqueness presupposition for *how many* questions with collective predicates:**
How many students solved the problem together?
(Presupposition: only one group of students solved the problem together or at least every group of students that solved the problem together has the same cardinality)
$Q = \{\lambda w\ \exists X\ |X|=n\ \& Students(X)\ \&\ SPT(X, w): n \in N\}$

(36) #I know how many students solved the problem together. Two; Four; and Eight. (uniqueness presupposition)

So for (26) Ans_D makes the correct predictions, and for (34) it makes wrong predictions. In order to accommodate (34), Xiang proposes the higher order denotation in (34)', which obviates the uniqueness effect by closing Q in (34) under conjunction.

(34') $Q = \{\lambda w\ \mu\ (\lambda X.\ SPT(X, w))$:
μ is an upward entailing quantifier over pluralities of students$\}$

But, as we've seen, such a move needs to be blocked for (26): the parallel higher order denotation in (26') needs to be excluded.

(26') $Q = \{\lambda w\ \mu\ (\lambda n.\ \exists X\ |X|=n\ \&\ Students(X)\ \&\ SPT(X, w))$:
μ is an upward entailing quantifier over degrees$\}$

We have also seen that this exclusion is achieved by PbE, based on the fact that (26') is not only closed under conjunction, but also under disjunction and thus contains propositions that would not be mapped to cells in the relevant partition by exhaustification, for example the disjunctive proposition in (30).

(30) $\lambda w.\ \exists X\ |X|=3\ \&\ \text{Students}(x)\ \&\ \text{SPT}(X, w)$
$\lor\ \exists X\ |X|=5\ \&\ \text{Students}(x)\ \&\ \text{SPT}(X, w)$

Moreover we supported this perspective by the observation that the higher order reading is possible when the reason for exclusion is eliminated, i.e., with the introduction of a modal, as in (29). So this was all very good. The problem is that PbE rules out the higher-order denotation in (34') for exactly the same reason: (34'), just like (29), contains disjunctive propositions whose exhaustification will not be a cell.

So there is a clear dilemma here. If we adopt Xiang's proposal for (34), we need a new account for (26), and in particular for the contrast between (26) and (29). Conversely, if we adopt our account of the contrast between (26) and (29), PbE, we need an alternative account for (34). My goal will, naturally, be to provide an alternative account of (34).

8.2. Higher Order Pluralities

I would like to propose, following a suggestion made to me by Križ and by Spector (p.c.), that (34) involves higher order pluralities (rather than higher order quantification). Landman, 1989a, 1989b invoked higher order pluralities for various purposes and suggested that they are unavailable when numerals are introduced. This proposal turns out to have exactly the right components to account for the contrast between (33) and (25).[29]

A straightforward way to implement Landman's proposal is to begin with a domain of singular/atomic individuals, D_e, and to form the different levels of plurality with a function *pl* which applies to a set and returns its power set: so the domain of pluralities is the power set of D_e (without the empty set),[30] the domain of 2^{nd} order pluralities is the power set of the domain of pluralities, etc. If we assume, further, that plural morphology can be associated with any order of plurality (n-order with $n \geq 1$), the question in (34) will have, among its possible denotations, one that is closed under conjunction.

For concreteness, assume that (34) can have the LF in (37), in which *dist* is a distributivity operator that yields (in this case) universal quantification over the sets in the higher order plurality that the trace ranges over.

(37) [which [pl pl student]] t dist solved this problem together?

With the assumptions in (38), (37) would have the denotation of Q in (39). Since this denotation is closed under conjunction, it is guaranteed to have a maximally informative true member (as long as it has one true member to begin with), thus satisfying Dayal's

[29] Kriz & Spector (2019) provide new evidence for higher order pluralities which I will not go over.
[30] If Buccola & Spector (2016) or Bylinina & Nouwen (2018) are right the empty set will not be excluded from the domain (see note 8).

presupposition.

(38) a. $[\![pl]\!](A) = \lambda X.\forall y{\in}X[A(y)=1]$
b. $[\![pl]\!]([\![pl]\!](A)) = \lambda \mathrm{X}.\forall Y{\in}\mathrm{X}\forall z{\in}Y\ [A(z)=1]$
c. Spell-out (*pl+N) = Spell-out (pl+N)

(39) Q={$\lambda w.\ \forall Y{\in}\mathrm{X}$[Y solved$_w$ the problem together]: X is a set of sets of students}

In particular, in the case of (35), the most informative true proposition in Q is the proposition that every plurality/set in (40) consists of individuals that solved the problem together.

(40) a. {Mary, Sue}
b. {John, Bill, Jane, Fred}
b. {x: x is one of the eight kids that the speaker is referring to ostensively}

PbE will likewise be satisfied. Cell Identification is equivalent to Dayal's presupposition because the definition of *Exh* in (2) and (11b) agree on Q. Non Vacuity is also satisfiable, given that (39), in contrast to Xiang's higher order denotation, is not closed under disjunction: Non Vacuity will be satisfied as long as for every subset of the set of pluralities of students, there is a world in the context set where this set consists of all the pluralities that solved the problem together in the relevant world.[31]

8.3. Higher-order pluralities in *how many* questions, take 1

Higher order plurality can account for Xiang's observation that (34) is not associated with a uniqueness presupposition. But in order for this to capture the contrast between (26) and (34), we would need to understand why a parallel move is not available for (26). Specifically, we need something that would block the LF in (41) with the denotation in (42). If (42) were a possible denotation, (26) would no longer presuppose that only one number of students solved the problem together. For example, in the scenario alluded to in (35), in which there are three different groups of students who solved the problem together, the answer should be three. In other words, (36) is still correctly predicted to be bad, but, if (42) is a possible denotation for (26), (43) is incorrectly predicted to be good.

(41) [how$_n$ [n many pl pl student]] t dist solved this problem together?

(42) Q={$\lambda w.\ \exists \mathrm{X}|\mathrm{X}|$=n & X is a set of sets of Students &
$\forall Y{\in}\mathrm{X}$[Y solved$_w$ the problem together]: n∈N}

(43) #I know how many students solved the problem together. Three. (Namely, Mary and Sue; John, Bill, Jane and Fred; and the eight kids over there.)

So it looks like we need to block (41) and (42). I suggest we do this by appeal to Landman (1989a, 566-567; 589-590), who made very basic observations teaching us that

[31] And again pruning/domain-restriction will be needed in certain cases. See 4.2.

a system with higher order plurality must indeed limit counting to first order plurals. Specifically, we must assume that only sets of singular individuals (first-order pluralities) are counted by number words, not sets of plural individuals (not higher order pluralities), or else we would predict a sentence such as *3 students solved the problem together* to be true if 3 groups of students each solved the problem together. This restriction, which we will revisit in 8.5. below, is encoded right now – by brute force – in the meaning of *many* (overt or covert), which presupposes that the NP denotation it combines with is a first order plurality:

(44)[32] $\text{many}(n)=\lambda X{:}\forall y\in X[y\in D_e].\ |X|=n$

To summarize, if we adopt Landman's higher order pluralities, we can account for Xiang's observation in (34). (34) is compatible with PbE given the higher order plurality parse in (37), which closes the question denotation under conjunction. The higher-order quantification parse that Xiang proposes also closes the denotation under conjunction, but the problem is that it contains two many members, closing the denotation under disjunction as well, thus violating Non Vacuity. The advantage of higher order plurality is that it accounts for the contrast between Xiang's case in (34) and G&S's (26). G&S's case involves *how many* questions and thus cannot have the higher order plurality parse, given Landman's observations, encoded in the lexical entry in (44). G&S's case can have a higher order quantification parse, but only when a modal is introduced above the higher order trace obviating the violation of PbE.

8.4. Independent evidence for higher-order pluralities

Is there any independent evidence for higher order plurality?[33] Here I will outline one piece of evidence that I have identified together with Luka Crnič.[34] Consider first (45), versions of which have been discussed in the literature on plurality (Link, 1983; Roberts, 1987, section 3.2.4.). This sentence receives a very interesting interpretation that can be analyzed without higher order pluralities and is characterized by the approximate paraphrase provided below.

(45) All letters (on the blackboard) that are on the same line are different.
Possible Paraphrase:
Each of the lines on the blackboard (which has letters written on it) consists of letters that are different from each other.

Specifically the plural predicate *letters* is (a distributive predicate) true of all the first-order sets/pluralities of letters. The relative clause *that are on the same line* is a non-distributive plural predicate true of first-order sets of objects the members of which are

[32] This lexical entry will need to be revised if UDM is assumed. I think that Haida and Trinh's (2016) could be adapted for this purpose, https://drive.google.com/file/d/0B0oeGQ78K8a1bkFjaHBPQVBrQlk/view.

[33] Landman proposed higher order pluralities to deal with a variety of phenomena that have since received an alternative account (Schwarzschild, 1996).
[34] Many thanks also to Benjamin Spector. For further evidence for higher order pluralities, see Kriz & Spector (2019).

on the same line. When the two predicates are intersected, the result is the non-distributive predicate, *letters that are on the same line*, which is true of all the first-order sets of letters the members which are on the same line, which serves as the restrictor of the quantifier *all*. The scope of the quantifier is the predicate *are different* which is a non-distributive plural predicate true of first-order sets such that each of their members is different from every other member. The entire sentence, thus, asserts that every first-order set of letters the members which are all on the same line is a set the members all of which are each different from all others. We can, thus, explain the fact that (45) is judged as true if the relevant letters are arranged on the blackboard, as in (46). In this blackboard there are many sets of letters the members of which are on the same line and every one of these sets consists of letters all of which are different from all others.

(46) Blackboard: J, K, L, P
J, K, R
L, M, P, Z

The argument for higher order plurality that Crnič and I would like to consider is based on a minor variation on (46), namely (47). (47) is a partitive version of (46). In order for it to be interpretable, the definite article must apply to the predicate *letters that are on the same line* and return the unique maximal set that the predicate is true of. The problem is that the predicate is true of three maximal sets. (For each of the three lines, a maximal set in the predicate extension consists of all of the letters on that line.) This problem is resolved if we allow for higher order pluralities. Specifically, (47) can have the LF in (47') in which the argument of the definite article is plurlalized – pl(A). The result of this pluralization is the set of subsets of A. And the set of subsets of any set always has a maximal member, namely the set itself, as stated in (48).

(47) All of the letters (on the blackboard) that are on the same line are different.
Possible Paraphrase:
Each of the lines on the blackboard (which has letters written on it) consists of different letters.

(47') *All [of [the pl ([pl letter] [that are on the same line])]]*
are different

(48) ⟦the pl A⟧ = {x: ⟦A⟧(x) = 1}, for any *A*.

So the parse in (47'), which involves higher-order plurality, yields the same meaning that we derived for (45) with first order plurality. Following Landman we note that numerals are unacceptable: *10 letters that are on the same line are different* cannot mean that there are 10 sets of letters (say each with a very small cardinality) the members of which are on the same line and each of which is different from all others.

8. 5. Higher-order pluralities with *how many*-questions, take 2

In 8.3 we've discussed Landman's constaint according to which only first order pluralities can be counted by number words. This blocked the higher-order-plurality LF in (37) for (26) with the denotation in (39), leaving us only with the simple denotation, Q in (26), from which uniqueness follows under PbE (or Dayal's presupposition). Here I would like to suggest a different perspective on Landman's constraint according to which higher order pluralities can be counted, but only if an appropriate unit of measurement is provided, perhaps an appropriate *classifier*.

Compare (43), repeated below, to (49) with its three variants. (49) seems to violate the uniqueness presupposition. I would like to suggest that this is possible due to an LF such as that in (37), which I argued before needs to be blocked.

(43) #I know how many students solved the problem together. Three, Four and Eight.

(49) ? I know how many students solved the problem together.

a. Five pairs of students
b. 3 groups of five students.
c. Five pairs of students and one group of five.

What this teaches us, if I am correct, is that G&S's effect comes to light only because of the lack of appropriate measure terms for higher order pluralities. Once these are made salient, higher order pluralities become available and the uniqueness presupposition is circumvented, just as in Xiang's case. And here is a minimal pair arguing for same conclusion:

(50) a. #I know how many students solved the problem together. Two and Four.
b. ?I know how many students solved the problem together. One group of two and one group four.

If these judgments are correct, they support the account provided here for the distinction between the two cases (G&S's and Xiang's). Of course, a complete account of what goes on in (49) would require a theory of units of measurements and an engagement with the rich literature on classifiers, something which I will have to leave for another occasion.

9. *Require* vs. *allowed*[35]

G&S contrast (29) with a minimal variant where the existential modal is replaced by a universal modal, (51):

(29) How many students are allowed to solve the problem together?

(51) How many students are required to solve the problem together?

They claim that (51) differs from (29), and like (26) is associated with a uniqueness presupposition (that there is a unique number n such n students are required to solve the problem together).[36] This is unexpected given that questions with universal modals served as Spector's prime examples of higher order quantification in questions. It is also not predicted by PbE, since universal modals, just like existential modals, circumvent the

[35] The response to G&S's challenge here was suggested to me by Benjamin Spector.

[36] There is a potential scope ambiguity here, the resolution of which is critical for the predictions of PbE as we will see shortly.

violation of Non Vacuity that arises from higher order quantification, and, thus, should not yield a judgment of uniqueness.

I do not have an interesting response to this empirical observation. However, I would like to point out conflicting data which conforms with the predictions of PbE, namely the contrast in (52), as well as the data in (53) and (54) [=(33)].

(52) a. How many soldiers are standing in square formation?
[Uniqueness presupposition: #30 and 50]
b. How many soldiers are allowed to stand in square formation?
[No uniqueness, between 30 and 50]
c. How many soldiers are required to stand in square formation?
[No uniqueness: between 30 and 50, between 30 and 50 and more than 60]

(53) How many students are required to solve this problem together for the school to be eligible for extra funding under our new regulations?
[No uniqueness: between 30 and 50]

(54) How many students should solve the problems together?
[No uniqueness: between 30 and 50]

Given this data, I would like to suggest, following a proposal made by Benjamin Spector, p.c., that G&S's observation is an indication of a preference for a parse in which *many students* outscopes *require* (a parse which would rule out higher-order readings for the same reasons that such a reading is ruled-out for (26)).

10. Weakly distributive predicates

G&S point out that the same judgments reported for (26) are attested with what Buccola and Spector call "weakly distributive predicates" such as *have the same name*. This is illustrated in (55) and (56).

(55) How many students have the same name?
[Uniqueness presupposition,
5 (cannot mean that 5 is the number of the largest group of students with the same last name)
3 and 5,
#at most five]

(56) a. How many students are allowed to have the same name?
[No uniqueness: between 3 and 5]
b. How many students should have the same name?
[No uniqueness: between 3 and 5]
c. How many students are required to have the same name for any teacher to be absolutely confused?
[No uniqueness: more than 5]

But as G&S point out, this is not predicted by Dayal's presupposition: If 5 student have the same name, it follows (logically) that 3 students do. Ans_D when applied to (55) should thus yield the proposition corresponding to the largest number, contrary to fact. The same holds for PbE, as the reader can verify.

While I cannot currently meet this challenge, I would like to suggest a possible strategy. First, I would like to point out that *how many* questions involve pied-piping: the phrase that moves, e.g. *how many students*, a DP, is larger than the *wh*P, a degP (see Hackl, 2000; Heycock, 1995 and much subsequent work). Next I would like to suggest, following Elliott & Sauerland (2019), that every question involving pied-piping must also satisfy the presuppositions of a different question, one where the moved phrase (which properly dominates a *wh*P) is itself a *wh*P. If this is the case, in *how many students have the same name*, the presuppositions of *who*/*which-students have the same name* has to be satisfied as well. This will capture the uniqueness presupposition of (55) and its circumvention in (56): the Dayal/PbE presupposition of the *who*/*which-students* question can only be satisfied if there is a unique maximal set of students that have the same name.

12. Conclusions

This paper provided further evidence for the view that a question denotation must be mapped to a partition by point-wise exhaustification, an idea proposed in Fox (2018), but defended there on other grounds (primarily on the basis of the distribution of mention some readings). If the arguments for PbE are successful, we can ask whether the partitions formed by point-wise exhaustification are generated within syntax/semantics or at the interface with pragmatics. The obvious reason to assume the latter is the inadequacy of partitions for question embedding, as pointed out in Heim (1994) and much subsequent work. In work in progress I revisit this question, in light of a richer view of exhaustification argued for in Bassi, Del Pinal, & Sauerland (2019), one in which exhaustification yields a trivalent proposition that distinguishes positive from negative information.

References

Abrusán, M., & Spector, B. (2011). An interval-based semantics for degree questions: negative islands and their obviation. *Journal of Semantics.*, *28(1)*, 107–147.

Abrusán, Marta. (2007). *Wh -Islands in Degree Questions : A Semantic Approach*. MIT.

Abrusán, Márta. (2014). *Weak Island Semantics*. Oxford University Press.

Bar-Lev, Moshe E. (2018). *Free choice, homogeneity, and innocent inclusion.* Hebrew University of Jerusalem Israel.

Bar-Lev, Moshe Elyashiv, & Fox, D. (n.d.). Free Choice, Simplification and Innocent Inclusion. *Natural Language Semantics*.

Bar-Lev, Moshe Elyashiv, & Fox, D. (2017). Universal Free Choice and Innocent Inclusion. In D. Burgdorf, J. Collard, S. Maspong, & B. Stefánsdóttir (Eds.), *SALT 27* (pp. 95–115).

Bassi, I., Del Pinal, G., & Sauerland, U. (2019). *Presuppositional Exhaustification*. Retrieved from https://ling.auf.net/lingbuzz/004821

Beck, S. (2014). Plural predication and quantifier 'than'-clauses. In L. Crnič & U. Sauerland (Eds.), *The art and craft of semantics: A Festschrift for Irene Heim*.

Beck, S., & Rullmann, H. (1999). A Flexible Approach to Exhaustivity in Questions. *Natural Language Semantics*, *7*(3), 249–298.

Buccola, B., & Spector, B. (2016). Modified numerals and maximality. *Linguistics and Philosophy*, *39*(3), 151–199.

Bylinina, L., & Nouwen, R. (2018). On 'zero' and semantic plurality. *Glossa: A Journal of General Linguistics*, *3*(1), 98.

Crnič, L., Chemla, E., & Fox, D. (2015). Scalar implicatures of embedded disjunction. *Natural Language Semantics*. https://doi.org/10.1007/s11050-015-9116-x

Dayal, V. (1996). *Locality in {Wh}-Quantification: Questions and Relative Clauses in {H}indi*. Dordrecht: Kluwer Academic Press.

Dotlačil, J., & Nouwen, R. (2016). The comparative and degree pluralities. *Natural Language Semantics*, *24*, 45–78.

Elliott, P. D., Nicolae, A., & Sauerland, U. (2017). Who and what do who and what range over cross-linguistically? *Theoretical and Experimental Approaches to Presuppositions*. Genoa, Italy.

Elliott, P., & Sauerland, U. (2019). *Nuclear intervention: Deriving Beck effects via cyclic scope and local exhaustification.*

Fox, D. (2007a). Free Choice and the Theory of Scalar Implicatures. In *Presupposition and Implicature in Compositional Semantics*. https://doi.org/10.1057/9780230210752_4

Fox, D. (2007b). "Too many alternatives: density, symmetry and other predicaments". *SALT 17*.

Fox, D. (2010). *Negative Islands and Maximization Failure*. Retrieved from http://lingphil.mit.edu/papers/fox/MIT_Colloq_2010.pdf

Fox, D. (2018). Partition by exhaustification : comments on Dayal 1996. *Proceedings of Sinn Und Bedeutung*.

Fox, D., & Hackl, M. (2006). The universal density of measurement. *Linguistics and Philosophy*. https://doi.org/10.1007/s10988-006-9004-4

Fox, D., & Katzir, R. (2011). On the characterization of alternatives. *Natural Language Semantics*, *19*(1), 87–107.

Gajewski, J. (2011). *Innocent Exclusion is not Contradiction Free*. Retrieved from https://jon-gajewski.uconn.edu/wp-content/uploads/sites/1784/2016/08/IEnotCF.pdf

Gentile, F. P., & Schwarz, B. (2017). A uniqueness puzzle: how many -questions and non-distributive predication 1. *Proceedings of Sinn Und Bedeutung*.

Gentile, F. P., & Schwarz, B. (2019). *Uniqueness and higher order quantification in how many questions*.

George, B. R. (2013). Which judgments show weak exhaustivity? (And which don't?). *Natural Language Semantics*, *21*, 401–427.

Groenendijk, J., & Stokhof, M. (1984). *Studies on the Semantics of Questions and the Pragmatics of Answers*. University of Amsterdam, Amsterdam.

Hackl, M. (2000). *Comparative Quantifiers*. Massachusetts Institute of Technology.

Heim, I. (1994). Interrogative Semantics and Karttunen's Semantics of Know. *IATL 1*, 128–144.

Heim, I. (2001). Degree Operators and Scope. In C. Fery & W. Sternefeld (Eds.),

Audiatur Vox Sapientiae. A Festschrift for {A}rnim von {S}techow (pp. 214–239). Berlin: Akademie Verlag.
Heycock, C. (1995). Asymmetries in reconstruction. *Linguistic Inquiry*, *26*(4), 547–570.
Hirsch, A., & Schwarz, B. (2020). Singular which, mention-some, and variable scope uniqueness. In K. Blake, F. Davis, K. Lamp, & oseph Rhyne (Eds.), *SALT 29* (pp. 748–767).
Karttunen, L. (1977). Syntax and Semantics of Questions. *Linguistics and Philosophy*, *1*(1), 3–44.
Katzir, R. (2014). On the roles of markedness and contradiction in the use of alternatives. In S. Pistoia Reda (Ed.), *Semantics, pragmatics, and the case of scalar implicatures* (pp. 40–71). Palgrave-Macmillan.
Krifka, M. (1995). The semantics and pragmatics of polarity items. *Linguistic Analysis*, *25*, 209–257.
Kriz, M., & Spector, B. (2019). *Interpreting Plural Predication: Homogeneity and Non-Maximality*.
Landman, F. (1989a). Groups I. *Linguistics and Philosophy*, *12*(5).
Landman, F. (1989b). Groups II. *Linguistics and Philosophy*, *12*(6).
Lewis, D. (1988). Relevant Implication. *Theoria*, *54*, 162–174.
Link, G. (1983). The Logical Analysis of Plurals and Mass Terms: A Lattice-Theoretical Approach. In R. Baeuerle, C. Schwarze, & A. von Stechow (Eds.), *Meaning, Use and Interpretation of Language*. DeGruyter.
Magri, G. (2009). A theory of individual-level predicates based on blind mandatory scalar implicatures. *Natural Language Semantics*. https://doi.org/10.1007/s11050-009-9042-x
Magri, G. (2011). Another argument for embedded scalar implicatures based on oddness in downward entailing environments. *Semantics and Pragmatics*, *4*, 1–51.
Roberts, C. (1987). *Modal Subordination, Anaphora and Distributivity*. University of Massachusetts, Amherst.
Romero, M. (1998). *Focus and reconstruction effects in wh-phrases*. University of Massachusetts, Amherst.
Rooth, M. (1992). Ellipsis redundancy and reduction redundancy. In S. Berman & A. Hestvik (Eds.), *Proceedings of the Stuttgart Ellipsis Workshop*. Stuttgart.
Schwarz, B., & Shimoyama, J. (2011). Negative Islands and Obviation by wa in Japanese Degree Questions. In N. Li & D. Lutz (Eds.), *SALT 20* (pp. 702– 719). CLC Publications.
Schwarzschild, R. (1996). *Pluralities*. Dordrecht ; Boston: Kluwer Academic.
Spector, B. (2008). An Unnoticed Reading for Wh-Questions: Elided Answers and Weak Islands. *Linguistic Inquiry*, *39*(4), 678–694.
Spector, B. (2016). Comparing exhaustivity operators. *Semantics and Pragmatics*, *9*, 1–33.
Xiang, Y. (2020). *Higher Order Readings of Wh-questions*.