The treatment of ‘now’ as a 1-place sentential operator*

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Let L be a pragmatic language the operators of which are P, F, N (all 1-place).†

Def 1. A triple $\langle A, F, R\rangle$ is a 10-standard interpretation for L iff

1) $\langle A, F, R\rangle$ is a possible interpretation for L
2) $\mathbf{D}A = \mathbb{R} \otimes \mathbb{R}$
3) (i) $R_{\mathsf{P}} = \{\langle\langle i, j\rangle, J\rangle : i, j \in \mathbb{R},\ J \subseteq \mathbb{R} \otimes \mathbb{R}$ and there is a $k < i$ such that $\langle k, j\rangle \in J\}$
   (ii) $R_{\mathsf{F}} = \{\langle\langle i, j\rangle, J\rangle : i, j \in \mathbb{R},\ J \subseteq \mathbb{R} \otimes \mathbb{R}$ and there is a $k > i$ such that $\langle k, j\rangle \in J\}$
   (iii) $R_{\mathsf{N}} = \{\langle\langle i, j\rangle, J\rangle : i, j \in \mathbb{R}, J \subseteq \mathbb{R} \otimes \mathbb{R}$ and $\langle j, j\rangle \in J\}$
4) (i) For all $i, j, k \in \mathbb{R}, A(\langle i, j\rangle) = A(\langle i, k\rangle)$
   (ii) For all $i, j, k \in \mathbb{R}$ and $G \in \mathbf{D}F$, $F_G(\langle i, j\rangle) = F_G(\langle i, k\rangle)$

A sentence ϕ of L is 10-logically true if ϕ is $(\mathtt{K}, \mathtt{J})$-valid where

(i) $\mathtt{K}$ is the class of all 10-standard interpretations

(ii) $\mathtt{J}$ is the function with domain $\mathtt{K}$ such that for all $\mathfrak{A} \in \mathtt{K}$, $\mathtt{J}(\mathfrak{A}) = \{\langle i, i\rangle : i \in \mathbb{R}\}$

∗ This is a typeset version of Hans Kamp’s 1967 hand-written notes (“Philosophy, Fall 1967, Notes by H. Kamp” is written on the top of the page). As a graduate student Kamp presented this material to Richard Montague’s UCLA seminar on pragmatics in the Fall of 1967. The material is an ancestor to Kamp’s famous 1971 “Formal properties of ‘now”’ (*Theoria* 37, 227-274), where he introduces a doubly-indexed semantics and the key concepts of “two-dimensional” semantics. Kamp sent the notes to A.N. Prior, which led to a subsequent correspondence, and greatly influenced Prior’s thinking (see Blackburn and Jørgensen, “Arthur Prior and ‘Now”’, *Synthese*. forthcoming). Prior’s intense engagement with Kamp’s notes resulted in his 1968, “Now”, (*Noûs*, 2(2), 101-119). The present version is based on the copy of Kamp’s notes that is stored in the Prior Archives at the Bodleian Library, Oxford (Box 15).

† [For the relevant background definitions see Montague, (1970) “Pragmatics and intensional logic”, *Synthese*, 22(1-2), 68-94. The choices for the typesetting of symbols are based on a combination of the notational conventions used in Montague’s papers on pragmatics as well as Kamp’s “Formal properties of ‘now”.]

Remark: If ϕ is a sentence of L and $\mathfrak{A}$ a 10-standard interpretation, then

(a) $\mathsf{P}\phi$ is true$_{\langle i,j\rangle,\mathfrak{A}}$ iff there is a $k < i$ such that ϕ is true$_{\langle k,j\rangle,\mathfrak{A}}$

(b) $\mathsf{F}\phi$ is true$_{\langle i,j\rangle,\mathfrak{A}}$ iff there is a $k > i$ such that ϕ is true$_{\langle k,j\rangle,\mathfrak{A}}$

(c) $\mathsf{N}\phi$ is true$_{\langle i,j\rangle,\mathfrak{A}}$ iff ϕ is true$_{\langle j,j\rangle,\mathfrak{A}}$

Comment: The problem with 'now' is that it always refers to *the moment of utternace of the sentence in which it occurs* and that this moment stands in general in no direct relationship to the moment referred to by the tense operator in the immediate scope of which this particular occurrence of 'now' stands. Therefore we can not develop the semantics of 'now' together with e.g. the past and future tenses, by means of interpretations, which have simply the real numbers as their points of reference. Whatever we make R_{N} in such interpretations the result will be wrong. Indeed, since the schema

$$\phi \leftrightarrow \mathsf{N}\phi$$

should of course be logically valid, we should have $\langle i,J\rangle \in R_{\mathsf{N}}$ iff $i \in J$. But then the schema $\phi \leftrightarrow \mathsf{PN}\phi$ would also come out logically valid, whereas the scheme $\mathsf{P}\phi \leftrightarrow \mathsf{PN}\phi$ would not; and these two facts conflict with the behaviour of 'now' and the past tense in English.

The reason for the inevitable failure of this approach is that, as soon as we take 'now' into account, what we need is not just a definition of the notion:

'ϕ is true at i'

but of the more complex notion:

'ϕ is true at *i when occurring in a sentence uttered at j*'

So our points of reference should be pairs $\langle i,j\rangle$ of moments of time, rather than single moments of time. Of course our main interest is in the question whether a sentence is true at a given moment of utterance—i.e. whether it is true at the moment of utterance *of itself*, or, formally, whether it is true at a pair $\langle i,i\rangle$; and a sentence should be called logically true if it is true at any moment of utterance in any interpretation. This may explain our definition of '10-logically true'.

It is the first members of our points of reference that 'represent' the moments of time—in roughly the same way as did the real numbers in the earlier development; of course the truth of an atomic formula at a given point of reference $\langle i,j\rangle$ should be independent of the moment of utterance of the sentence of which it is a part; which explains condition 4) of our definition.

As an example may serve the sentence discussed earlier, viz.,

(2) c will remember (at some particular moment) everyone now alive.

The obvious symbolization of (2) in L is

(3) $\mathsf{F}(\forall x)(\mathsf{N}(\mathsf{al}(x)) \to \mathsf{c\,rem}\,x))$

(under the assumption, of course, that L contains the predicate constants ‘al’ and ‘rem’).

To check the semantics developed above let us express the truth of (3) at an index $\langle i,i\rangle$ in a 10-standard interpretation $\mathfrak{A} = \langle A,F,R\rangle$ for L, in terms of satisfaction of its atomic subformulae:‡

$$\begin{aligned}
\text{(3) is true}_{\langle i,i\rangle,\mathfrak{A}} \quad &\text{iff} \quad \text{there is a } k > i \text{ such that } (\forall x)(\mathsf{N}(\mathsf{al}(x)) \to \mathsf{c\,rem}\,x)) \text{ is true}_{\langle k,i\rangle,\mathfrak{A}} \\
&\text{iff} \quad \text{there is a } k > i \text{ such that for all } a \in \cup \mathbf{\Game} A, \\
&\qquad\qquad \text{if } a \text{ sat}_{\langle k,i\rangle,\mathfrak{A}}\ \mathsf{N}(\mathsf{al}(x)), \text{ then } a \text{ sat}_{\langle k,i\rangle,\mathfrak{A}}\ \mathsf{c\,rem}\,x \\
&\text{iff} \quad \text{there is a } k > i \text{ such that for all } a \in \cup \mathbf{\Game} A, \\
&\qquad\qquad \text{if } a \text{ sat}_{\langle i,i\rangle,\mathfrak{A}}\ \mathsf{al}(x), \text{ then } a \text{ sat}_{\langle k,i\rangle,\mathfrak{A}}\ \mathsf{c\,rem}\,x
\end{aligned}$$

So in any 10-standard interpretation (3) is true whenever a correct symbolization of (2) ought to be.

A similar treatment applies to the expressions ‘today’, ‘tomorrow’, ‘yesterday’. Let $\mathbb{I}$ be the set of integers. Let L′ be a pragmatic language the operators of which are N, Y, T, V, L (all 1-place).

Def 2. A 10′-standard interpretation for L′ is a triple $\mathfrak{A} = \langle A,F,R\rangle$ such that

1) $\mathfrak{A}$ is a possible interpretation for L′

2) $\mathbf{D}A = \mathbb{I} \otimes \mathbb{I}$

3) (i) $R_{\mathsf{V}} = \{\langle\langle i,j\rangle,J\rangle : i,j \in \mathbb{I},\, J \subseteq \mathbb{I}\otimes\mathbb{I},\, \langle i-1,j\rangle \in J\}$

‡ [The backwards-D notation, i.e “$\mathbf{\Game}A$” is used for the image of the function A, in the same way that $\mathbf{D}A$ is use for the domain of A. Thus, $\cup\mathbf{\Game}A = \{x : \forall_{i,j\in\mathbb{R}}(A(\langle i,j\rangle) = x)\}$, the set of all possible individuals.]

(ii) $R_{\mathsf{L}} = \{\langle\langle i,j\rangle,J\rangle : i,j \in \mathbb{I}, J \subseteq \mathbb{I}\otimes\mathbb{I}, \langle i+1,j\rangle \in J\}$

(iii) $R_{\mathsf{N}} = \{\langle\langle i,j\rangle,J\rangle : i,j \in \mathbb{I}, J \subseteq \mathbb{I}\otimes\mathbb{I}, \langle j,j\rangle \in J\}$

(iv) $R_{\mathsf{Y}} = \{\langle\langle i,j\rangle,J\rangle : i,j \in \mathbb{I}, J \subseteq \mathbb{I}\otimes\mathbb{I}, \langle j-1,j\rangle \in J\}$

(v) $R_{\mathsf{T}} = \{\langle\langle i,j\rangle,J\rangle : i,j \in \mathbb{I}, J \subseteq \mathbb{I}\otimes\mathbb{I}, \langle j+1,j\rangle \in J\}$

Comment: N stands for ‘today’, Y for ‘yesterday’, T for ‘tomorrow’, V for ‘the day before’, and L for ‘the day after’ (V and L are the initials of the substantives in the corresponding French expressions ‘la veille’ and ‘le lendemain’, respectively). Note that our operators V and L correspond to what Scott calls (not quite correctly, it seems) ‘yesterday’ and ‘tomorrow’.

Problem: A $10''$-standard interpretation for L is a triple $\mathfrak{A} = \langle A,F,R\rangle$, where

1) as in definition 1, and

2′) **D**A is the Cartesian square of any linearly ordered set $\langle S,<\rangle$

3′) as 3) of def. 1 with $\mathbb{R}$ replaced by S

4′) as 3) of def. 1 with $\mathbb{R}$ replaced by S

Give an elegant axiom system which is complete with respect to $10''$-logical truth.