Determiners, conservativity, witnesses

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A cherished semantic universal is that determiners are conservative (Barwise & Cooper 1981, Keenan & Stavi 1986). Well-known problem cases are *only* (if it has determiner uses) and certain uses of proportional determiners like *many* (Westerståhl 1985). Fortuny 2017, in a retracted contribution to this journal, proposed a new constraint (the Witness Set Constraint) to replace Conservativity. He claimed that his constraint is satisfied by *only* and the Westerståhl-*many*, thus correctly allowing the existence of these non-conservative determiners, while it is not satisfied by unattested non-conservative determiners (such as *allnon*). In fact, we show here that *only* does not satisfy Fortuny's Witness Set Constraint (nor does Westerståhl-*many*, which we leave to the readers to convince themselves of). Upon reflection, it turns out that the reason is simple: the Witness Set Constraint is in fact equivalent to Conservativity. There simply cannot be non-conservative determiners that satisfy the Witness Set Constraint. We consider further weakening of the Witness Set Constraint but show that this would allow unattested determiners.

1 Background

Barwise & Cooper 1981 introduced the notion of a set that a generalized quantifier lives on:

(1)   A generalized quantifier Q (a set of sets of individuals) *lives on* a set A iff $\forall B\colon B \in Q \Leftrightarrow A \cap B \in Q$.

When a quantifier lives on a set A, then to see whether B is an element of the quantifier, we need only check whether the intersection of A and B is in the quantifier. The part of B that's not in A is irrelevant. Quantifiers trivially live on the set of all individuals U since $B = B \cap U$. But quantifiers can have smaller sets that they live on. Particularly interesting is the smallest set a quantifier lives on. The generalized quantifier denoted by *John*, the set of sets

of individuals that contain John, has a smallest set that it lives on: {John}. The smallest set that *John or Mary*, conceived of as a generalized quantifier, lives on is {John, Mary}. The smallest set that *two doctors* lives on is the set of doctors. The smallest set that *no doctor* lives on is the set of doctors.[1]

Barwise & Cooper 1981 observed that all determiners that they were considering give rise to generalized quantifiers that live on the set denoted by the nominal predicate that is the sister of the determiner. They conjectured that this is a semantic universal: "It is a universal semantic feature of determiners that they assign to any set A a quantifier (i.e. family of sets) that lives on A" (p.170). The proposed universal is better known in its guise due to Keenan & Stavi 1986, who define the notion of conservativity:

(2)   A determiner denotation D is *conservative* iff
$$\text{for all sets } A, B\colon\ B \in D(A) \Leftrightarrow A \cap B \in D(A).$$

Then obviously, (3a) and (3b) are equivalent:

(3)   a.   Every natural language determiner is conservative.

      b.   Every natural language determiner lives on its first argument.

In what follows, we shall refer to (3) as the Conservativity Constraint (CONS).

It helps to look at two determiner meanings that would violate Conservativity. One classic example is from Chierchia & McConnell-Ginet 2000: the determiner *allnon*, which is defined as follows:

(4) $[\![\text{allnon}]\!](A)(B)$ iff $\overline{A} \subseteq B$.

Allnon students smoke would mean that all non-students smoke. *Allnon* would be non-conservative because $\overline{A} \subseteq B$ is not equivalent to $\overline{A} \subseteq (A \cap B)$. The latter only holds if $\overline{A}$ is the empty set, while the former holds in many more situations. Since *allnon* doesn't exist as a determiner, we can see the universal CONS at work.

Writing $|X|$ for the cardinality of the set X, the non-conservativity of the cardinal equality quantifier D defined by: $B \in D(A)$ iff $|B| = |A|$, is noted in Keenan 1996. That of the *greater than* quantifier ($B \in D(A)$ iff $|A| > |B|$)

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[1] Szabolcsi 1997: p.12, fn.2 remarks: "E. Keenan (p.c.) notes that the notion of a smallest live-on set is unproblematic as long as the universe is finite or at least our GQ does not crucially rely on infinity. But e.g. the intersection of the sets which $[\![\text{all but finitely many stars}]\!]$ lives on in an infinite universe is itself not a live-on set." We expand on this in the Appendix, where we show that this kind of quantifier has many live-on sets and many witness sets, none minimal.

is noted in Keenan & Westerståhl 1997. We illustrate with *blik*, defined as follows:

(5) $[\![\text{blik}]\!](A)(B)$ iff $|A| < |B|$.

Blik doctors speak French would mean that there are strictly fewer doctors than individuals who speak French. An entirely plausible meaning to want to express, but there's no determiner in any language that can be used to express this meaning. Again, *blik* would violate Conservativity because $|A| < |B|$ is not equivalent to $|A| < |A \cap B|$. The latter is trivially false. But the former has truth-conditions that are non-trivial.

Apart from the apparent empirical success of the Conservativity universal, Barwise & Cooper also point out that when determiners are conservative, a semantic processing strategy becomes possible that limits what needs to be done to verify a quantificational statement. They introduce the notion of a witness set:

(6) A *witness set* for a quantifier Q living on A is any subset w of A such that $w \in Q$.

Then, they show that for conservative determiners, looking at witness sets can determine the truth of any quantificational statement:

(7) Let w range over witness sets for the quantifier $D(A)$ living on A.

   (ii)   If $D(A)$ is mon↑ then for any X,
          $X \in D(A)$ iff $\exists w [w \subseteq X]$

   (iii) If $D(A)$ is mon↓ then for any X,
          $X \in D(A)$ iff $\exists w [(X \cap A) \subseteq w]$

(The notations "mon↑" and "mon↓" here abbreviate the monotonicity properties of determiners with respect to their second argument.) In the appendix (Barwise & Cooper 1981: p.212f, Proposition C11), they prove that (7) holds for conservative determiners.

Based on this observation, they suggest the following procedure for verifying a quantificational statement:

(8) To evaluate $X \in D(A)$ do the following:

1. Take some subset w of A which you know to be in $D(A)$.
2. (i) For mon↑ $D(A)$, check $w \subseteq X$.
   (ii) For mon↓ $D(A)$, check $(X \cap A) \subseteq w$.
3. If there is such a w, the sentence is true. Otherwise it is false.

For non-monotonic determiners, there are no shortcuts: the best we can do is use conservativity and check whether $X \cap A$ is in $D(A)$.[2]

The connection that Barwise & Cooper drew between Conservativity and the witness-set based verification procedure in (8) was the jumping-off point in Fortuny 2017.[3] We should also mention another attempt at grounding Conservativity: the idea that under the Copy Theory of Movement, non-conservative determiners would reliably lead to triviality of some sort and that such lexical entries would therefore not be stable (see in particular Romoli 2015, building on earlier discussion by Fox 2002: p.67,fn.8 and others).

2 Two problem cases

So far so good then for the universal CONS. It rules out certain unattested determiners. And it seems to suggest a constrained perspective on semantic processing.

But, unfortunately, there are well-known potential counterexamples: *only* in its determiner-like uses and certain occurrences of *many*. Consider first *only*:

(9) Only students smoke.

The traditional view of such sentences is that *only* is the converse of *all*, so (9) is equivalent to *All smokers are students*. In both cases, we need to deal with the existence inference (that there are smokers). Fortuny 2017 discusses it as a presupposition but builds it in as an entailment:

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2 This clause is added without comment by Szabolcsi 1997: p.17.
3 We will not explore the relation between the armchair speculation about quantifier verification in Barwise & Cooper 1981 and bona-fide empirical work (some prominent recent work includes Hackl 2009, Szymanik & Zajenkowski 2010.)

(10) $[\![only]\!](A)(B)$ iff $B \neq \emptyset$ & $B \subseteq A$.   [or in Fortuny's notation: $\emptyset \neq B \subseteq A$]

The non-conservativity of *only* is obvious: *only students smoke* is not equivalent to *only students are students who smoke*. The latter is trivial while the former is not.

A common response is that *only* simply isn't a determiner (of the type $D_{\langle et,ett \rangle}$): rather, it is an adverbial operator that operates in a semantically different form. When it combines with a nominal like *students*, so the story goes, appearances are deceiving. It in fact combines with a bare plural which itself is already a full-fledged DP that doesn't need a determiner anymore. A story along those lines is explored in von Fintel 1997. For explorations of the opposite tack, that one should bite the bullet and admit that there are non-conservative determiners of the *only*-kind, see de Mey 1991, who argues that the putatively adverbial nature of DP-*only* is paralleled by the uncontroversial (?) determiner *all*, and Zuber 2004, who argues for a complex expression in Polish that it is a determiner and shows that it is non-conservative.

The other well-known potential counterexample to CONS comes from certain uses of the determiner *many*, first pointed out by Westerståhl 1985:

(11)   Many Scandinavians have won the Nobel Prize in literature.

It is easy to judge (11) as true in a situation such as the one reported by Westerståhl as of 1984: 14 out of a total of 81 Nobel Prize winners in literature had been Scandinavians.[4] One perspective on this kind of reading is that it is "reverse proportional". A proportional reading of *many students smoke* would say that a large proportion of students are smokers: $D(A)(B)$ iff $|A \cap B|$ is large compared to $|A|$. The reverse proportional reading instead would say that $D(A)(B)$ iff $|A \cap B|$ is large compared to $|B|$. It should be obvious that the proportional reading is conservative (replacing B with $A \cap B$ makes no difference), while the reverse proportional reading is not conservative. Based on work by Cohen 2001, Fortuny 2017 adopted a different semantics for Westerståhl's *many*, but still one that is non-conservative.

Again, one can explore ways of dismissing this use of *many* as a counterexample to CONS. The most promising line of attack may be the idea that *many* should be treated as a gradable adjective which has a POS operator in

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4 Westerståhl's observation seems to have had a chilling effect: since 1984, only one of the 33 further Nobel Prize winners has been Scandinavian.

its degree argument position, that POS can take sentential scope and associate with focus. See Romero 2015a,b for explorations of ideas in this direction.[5,6]

If we do take these to be genuine counterexamples to CONS, because these uses of *only* and *many* are genuine occurrences of determiners, the next question is whether there is a reasonable weakening of CONS. One idea comes from the observation that *only* is "conservative with respect to its second argument": $only(A)(B)$ iff $only(A \cap B)(B)$. So, instead of restricting its second argument to its intersection with its first argument, *only* (as the converse of *all*) can be seen as restricting its first argument to its intersection with its second argument. Similarly, reverse proportional *many* (but not the Cohen/Fortuny interpretation) is also conservative with respect to its second argument. As Keenan 1996 puts it, "the whiff of generality is in the air". Keenan 2002, like de Mey 1991 before him, explores the notion of conservativity on either argument. Zuber 2004: 164, as well, concludes that "even though natural languages have non-conservative determiners, the class of such determiners is still very restricted." The basic idea in all three works is that the only non-conservative determiners we find are the converses of conservative determiners.

3   Fortuny's Witness Set Constraint

The core idea of Fortuny's article was clear and compelling: we need to find a condition that is judiciously weaker than CONS, one that allows non-conservative *only* and *many* but still rules out unattested non-conservative determiners such as *allnon* or *blik*. His proposal was that the condition that Barwise & Cooper had shown to follow from CONS is exactly what we need. The idea was that determiners need to satisfy a Witness Set Constraint: their meaning needs to be equivalent to the existence of an appropriate witness set. We reproduce here the core definition of *witness set*:

(12)   **Definition 9. Witness Set**   (Fortuny 2017: p.17)
       For all U and all $A \subseteq U$, a set w is a witness set for a generalized
       quantifier $D(A)$ iff w is a subset of A such that $w \in D(A)$.

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5 Another approach worth considering is the one proposed in Bastiaanse 2013.

6 In a recent paper, Ahn & Sauerland 2017 discuss the apparently non-conservative construal of proportional measure phrases as in "The company hired 75% women". They argue that at a relevantly abstract level, there is no non-conservative determiner present.

So, a witness set w for any quantifier $D(A)$ has to satisfy two conditions: (i) w has to be a subset of the first argument A, (ii) w has to be an element of $D(A)$. This is the same as Barwise & Cooper's definition but eliminates the condition that $D(A)$ lives on A. Consequently, non-conservative determiners like *only* can have witness sets, but what it means for a set w to be a witness set for a quantifier $D(A)$ when D is not conservative is subject to the very same set of conditions as before: $w \subseteq A$ and $w \in D(A)$.

And here is the proposed Witness Set Constraint:

(13) *Witness set Constraint*   ([Fortuny 2017](): p.17, (27))
      For all U, all functions $D\colon \wp(U) \to \wp(\wp(U))$ expressed by a linguistic category, all $A, B \subseteq U$:

      a. if $D(A)$ is monotone increasing, then
      $$B \in D(A) \Leftrightarrow (\exists w)(w \in W[D(A)] \wedge w \subseteq B)$$
      b. if $D(A)$ is monotone decreasing, then
      $$B \in D(A) \Leftrightarrow (\exists w)(w \in W[D(A)] \wedge (A \cap B) \subseteq w)$$
      c. if $D(A)$ is non-monotonic, then
      $$B \in D(A) \Leftrightarrow (\exists w)(w \in W[D(A)] \wedge (A \cap B) = w)$$

Here, $W[D(A)]$ is the set of witness sets of $D(A)$.

The way to understand this constraint is that it requires of any natural language determiner that its truth-condition be equivalent to the existence of an appropriate witness set. What the appropriateness condition is depends on the monotonicity properties of the determiner. In a very precise sense, the idea was to replace CONS with a weaker condition satisfied by all conservative functions as well as a few non-conservative ones. This would include *only* and *many* but still exclude *allnon* and *blik*.

We will now show that Fortuny's claim was incorrect: *only* and *many* do not satisfy his (27). The deeper reason for this is that (27) is in fact not weaker than CONS but strictly equivalent to it. And so, non-conservative determiners cannot satisfy (27). The constraint can't play the role Fortuny wanted it to play.

4   Applying the Witness Set Constraint to the problem cases

Fortuny correctly states that the relevant determiners, with the meanings he gives for them, are non-monotonic. This means that clause (27c) of the Witness Set Constraint should apply to them (repeated here):

(27c)   For all U, all functions $D\colon \wp(U) \to \wp(\wp(U))$ expressed by a linguistic category, all $A, B \subseteq U$: if $D(A)$ is non-monotonic, then
$$B \in D(A) \Leftrightarrow (\exists w)(w \in W[D(A)] \land (A \cap B) = w)$$

We will now show that, contrary to Fortuny's claim, *only* does not satisfy this constraint. We will do so in several different ways to make it crystal clear that the account is incorrect at its core.

First, a simple counterexample. Consider two sets: $A = \{a, b, c, e\}$ and $B = \{d, e\}$. Obviously, *only*$(A)(B)$ is false: $B \notin D(A)$. Why? Well, *only*$(A)(B)$ is true iff $B \subseteq A$ (we omit the non-emptiness condition on B, since it's irrelevant), and that is not the case here: $\{d, e\} \nsubseteq \{a, b, c, e\}$. So, the left hand side (LHS) of the condition in (27c) is false in this case. Therefore, the RHS should be false too, since (27c) requires the truth of $B \in D(A)$ to stand and fall with the existence of an appropriate witness set. But there is in fact a set that satisfies the RHS: $A \cap B = \{e\}$ is a witness set for $D(A)$, since it is a subset of A and it is an element of $D(A)$ (which is simply the set of (non-empty) subsets of A). Therefore, the equivalence fails to hold in this case: the LHS is false but the RHS is true. One counterexample is enough to refute a claim of equivalence. So, *only* does not satisfy the Witness Set Constraint.

More generally, we can note that the RHS is very weak in the case of *only*. Recall that $B \in only(A)$ iff $B \neq \emptyset \land B \subseteq A$. Now, what is the set of witness sets for *only*(A)? To be a witness set for *only*(A) a set has to be (i) a subset of A, and (ii) it has to be an element of *only*(A). To be an element of *only*(A), a set has to be a non-empty subset of A. Thus, the set of witness sets for *only*(A) is the set of non-empty subsets of A. Now, what do A and B have to be like for the RHS of (27c) to be true? $(\exists w)(w \in W[D(A)] \land (A \cap B) = w)$ says that there is a witness set for *only*(A), which further is identical to $A \cap B$. That is the same as saying that $A \cap B$ has to be a witness set for *only*(A). For that to hold, $A \cap B$ has to be a non-empty subset of A. And clearly that will always be the case as long as $A \cap B$ is non-empty (it is trivially a subset of A, so the non-emptiness condition is the only substantial condition). In other words, the RHS of (27c) is true for any A, B that have a non-empty intersection. That is a much weaker condition than the LHS (that B is a non-empty subset of A). So, we see again, on general grounds, that *only* does not satisfy (27c).

We will leave to the readers to convince themselves that Westerståhl's *many* also fails to satisfy the Witness Set Constraint. We also omit pointing out the precise places where Fortuny's proofs were flawed. Instead, we will now provide a proof that the Witness Set Constraint is in fact precisely equiv-

alent to Conservativity, and thus cannot possibly play the role Fortuny wanted it to play: it makes no space for further determiners than Conservativity did.

5 Witness Set Constraint = Conservativity

Given a domain of objects U (we mostly adopt Fortuny's notation to facilitate easy comparison), a possible Det denotation D is a function from $\wp(U)$, the set of subsets of U, to $\wp(\wp(U))$, the set of sets of subsets of U. We sometimes refer to subsets of U as properties. So Dets, such as *every*, *some*, *most*, *most but not all*, *most of Ted's*, etc. denote functions from properties to sets of properties. CONS and WSC are given below.

(14) CONS
    For all U, a D from $\wp(U)$ to $\wp(\wp(U))$ is *conservative* iff for all $A, B \subseteq U$, $B \in D(A)$ iff $B \cap A \in D(A)$

The WSC is stated in terms of generalized quantifiers (sets of properties) and witness sets. The latter is defined as follows (Fortuny p. 17):

(15) *Witness Set* (p. 17):
    For all domains U and all $A \subseteq U$, a set w is a witness set for a generalized quantifier $D(A)$ iff $w \subseteq A$ and $w \in D(A)$. We write $W[D(A)]$ for the set of witness sets for $D(A)$. (Assumed here is that D is an arbitrary function from properties to sets of properties).

(16) WSC
    For all U, all functions $D\colon \wp(U) \to \wp(\wp(U))$ expressed by a linguistic category, all $A, B \subseteq U$,

    a. if $D(A)$ is monotone increasing then
$$B \in D(A) \text{ iff } \exists w\,(w \in W[D(A)] \,\&\, w \subseteq B)$$
    b. if $D(A)$ is monotone decreasing then
$$B \in D(A) \text{ iff } \exists w\,(w \in W[D(A)] \,\&\, (A \cap B) \subseteq w)$$
    c. if $D(A)$ is non-monotonic then
$$B \in D(A) \text{ iff } \exists w\,(w \in W[D(A)] \,\&\, (A \cap B) = w)$$

We add some comments on the definition:

C1: Worth noting is that for any U, any set Q of properties, there is some property A and some function D from $\wp(U)$ to $\wp(\wp(U))$ such that $Q = D(A)$. This remains true if we restrict D to ones that are conservative (and so

satisfy the WSC by Fortuny's reckoning (p.18)).[7] It fails to be true for more severe restrictions. For example, logical Dets — ones that are automorphism (permutation) invariant (see van Benthem 1983) — severely limit what sets can be their $D(A)$s.

C2: The three conditions cover all the cases in the sense that any $D(A)$ satisfies at least one of the conditions. Any $D(A)$ is either monotonic or not. If it isn't it satisfies condition (c) and fails conditions (a) and (b). If it fails condition (c) then it satisfies either (a) or (b), and (just) two $D(A)$ satisfy both: the function 1, where $1(A)$ maps all A to $\wp(U)$, and dually for the function 0 which maps all A to $\emptyset$. Fortuny should have mentioned this as the right hand sides of the biconditionals in (a) and (b) are not logically equivalent, so the specter looms that for one of these "extremal" $D(A)$'s one of the right hand sides might hold and the other fail, in which case WSC would not be well defined. In fact each of $\wp(U)$ and $\emptyset$ satisfy both biconditionals in the consequents of (a) and (b).

(17)   *Theorem*
    For all U, all $D \colon \wp(U) \to \wp(\wp(U))$, D satisfies CONS iff D satisfies WSC.

Proof: Let U be arbitrary (assumed non-empty, as usual).

$\Rightarrow$ Suppose that D is CONS. We show D satisfies WSC (in agreement with Fortuny, (p. 18)). Let A, B arbitrary subsets of U. We give a proof in three cases:

case 1:   Assume $D(A)$ is increasing. Show $B \in D(A)$ iff $\exists w\,(w \subseteq A, w \in D(A)\ \&\ w \subseteq B)$.
    $\Rightarrow$ Assume $B \in D(A)$. Then by CONS, $B \cap A \subseteq D(A)$. Since $B \cap A \subseteq B$ and $B \cap A \subseteq A$ so there is a $w \subseteq A, w \subseteq B$ and $w \in D(A)$. $\square$
    $\Leftarrow$ Let w be the witness. Then $w \subseteq A \cap B\ \&\ w \subseteq D(A)$, so by the increasingness of $D(A)$, $A \cap B \in D(A)$, whence by the conservativity of D, $B \in D(A)$. $\square$

case 2: $D(A)$ is decreasing.
    $\Rightarrow$ Assume $B \in D(A)$. Show $\exists w\,(w \subseteq A\ \&\ w \in D(A)\ \&\ (A \cap B) \subseteq w)$. Choose $w = A \cap B$ $\square$.
    $\Leftarrow$ Assume $\exists w\,(w \subseteq A\ \&\ w \in D(A)\ \&\ (A \cap B) \subseteq w)$. Then since $D(A)$ is decreasing and $w \in D(A)$ then, since $A \cap B \subseteq w$, $A \cap B \in D(A)$, whence by CONS of D, $B \in D(A)$.$\square$

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7 $D_Q XY = 1$ iff $X = U$ and $Y \in Q$. Exercise: show that D_Q is CONS.

case 3: Assume $D(A)$ is non-monotonic. Show $\exists w(w \subseteq A \,\&\, w \in D(A) \,\&\, (A \cap B) = w)$

⇒ Assume $B \in D(A)$. Then $B \cap A \in D(A)$ so $B \cap A$ is our desired w.
⇐ Assume $\exists w(w \subseteq A \,\&\, w \in D(A) \,\&\, (A \cap B) = w)$. So $A \cap B \in D(A)$, so by CONS, $B \in D(A)$.

This exhausts the cases completing the ⇒ proof. □

⇐ Suppose D satisfies WSC. Show that D satisfies CONS. Let $A, B \subseteq U$. There are 3 cases:

case 1: Assume $D(A)$ is increasing. Then by condition (a),

$$
\begin{aligned}
B \in D(A) \quad &\text{iff } \exists w(w \subseteq A \,\&\, w \in D(A) \,\&\, w \subseteq B) \\
&\text{iff } \exists w(w \subseteq B \cap A \,\&\, w \in D(A)) \\
&\text{iff } B \cap A \in D(A)
\end{aligned}
$$

case 2: Assume $D(A)$ is decreasing. Then by condition (b),

$$
B \in D(A) \text{ iff } \exists w(w \subseteq A \,\&\, w \in D(A) \,\&\, (A \cap B) \subseteq w)
$$

⇒ assume $B \in D(A)$. Show $A \cap B \in D(A)$. By condition (b) there is a $w \in D(A)$ which includes $B \cap A$, hence by the decreasingness of $D(A)$, $B \cap A \in D(A)$.

⇐ assume $B \cap A \in D(A)$. Show $B \in D(A)$. Assume leading to a contradiction that $B \notin D(A)$. Then there is no witness w of $D(A)$ which includes $(A \cap B)$, that is, no $w \subseteq A$ with $w \in D(A)$ and $(A \cap (A \cap B)) = (A \cap B) \subseteq w$. But that is false, as $(A \cap B)$ itself is such a witness since $(A \cap B) \subseteq A$, a member of $D(A)$ and a superset of itself. Thus $B \in D(A)$.

case 3: Assume $D(A)$ is not monotonic. Then by condition (c),

$$
\begin{aligned}
B \in D(A) \quad &\text{iff } \exists w(w \subseteq A \,\&\, w \in D(A) \,\&\, (A \cap B) = w) \\
&\text{iff } (A \cap B) \subseteq A \text{ and } (A \cap B) \in D(A) \\
&\text{iff } (A \cap B) \in D(A)
\end{aligned}
$$

This exhausts the cases, so if D satisfies WSC then D is CONS. Both directions having been proven, Theorem (17) is established. The Witness Set Constraint is equivalent to Conservativity. □

In sum, rather than being a useful replacement for Conservativity, the Witness Set Constraint is in fact equivalent to it. Therefore, it cannot be used to explain the existence of non-conservative determiners.

6 Now what?

Fortuny's idea had been to replace CONS with a weaker processing-based condition that would allow in attested non-conservative determiners like *only* and *many* but continue to rule out unattested non-conservative determiners like *allnon* or *blik*. It turns out however that the Witness Set Constraint, which Barwise & Cooper had shown to follow from Conservativity is not just entailed by Conservativity but equivalent to it. Thus, it does rule out *only* and *rpmany*, contrary to Fortuny's claim and contrary to our hopes. Can we modify Fortuny's proposal to achieve its stated goals?

One idea might be to weaken (27c) a bit so that it does in fact impose a weaker condition than Conservativity. For example, could we replace the equivalence requirement with an entailment requirement? Could the following do the trick?[8]

(27c") For all U, if a function $D \in \mathcal{D}_U$ is expressed by a linguistic category, then, for all $A, B \subseteq U$: if $D(A)$ is non-monotonic, then
$$B \in D(A) \Rightarrow (\exists w)(w \in W[D(A)] \wedge (A \cap B) = w)$$

Note that as soon as we move to a one-way entailment, we would give up the processing motivation for the constraint because the witness set calculation would not anymore be a way of establishing the truth of a quantificational claim; it could only be used to falsify a claim (since if the witness set condition is false, the quantificational would have to be false as well; but the truth of the witness set condition wouldn't guarantee the quantificational claim). But be that as it may, does the weakened constraint actually rule in the right determiners and rule out unattested determiners?

Well, it turns out that *only* is in fact ruled in. As we established above, the RHS just requires that $A \cap B$ is non-empty. So, yes, the LHS (which says that B is a non-empty subset of A) entails the RHS (that $A \cap B$ is non-empty). That's promising.

But it turns out that (27c") is too weak. It also rules in unattested non-conservative determiners. Consider the converse of *blik*, call it *kilb* (it's the determiner defined by Fortuny in his Definition 16):

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8 This possibility was suggested by Maribel Romero (pc to the editor Rick Nouwen).

(18) $[\![\text{kilb}]\!](A)(B) \Leftrightarrow |A| > |B|$

As Fortuny discusses, the witness sets for *kilb(A)* are all the proper subsets of A. And since $A \cap B$ is always a proper subset of A unless $A = B$, the RHS of (27c') is always true if $A \neq B$. And the LHS ensures that $A \neq B$, because otherwise A couldn't have more elements than B. So, the LHS entails the RHS for *kilb*. And since *kilb* is an unattested non-conservative determiner, the attempt at weakening (27) to give the desired result is unsuccessful.

At this point, we do not know whether there is any other way of making space for *only* and *rpmany* as non-conservative determiners than the approaches considered by de Mey 1991 or Keenan 2002: somehow unifying left and right conservativity. One might be more attracted by the other line of attack, however: that these items are not in fact determiners (they do not have the type $\langle et, ett \rangle$). Or, in our time of desperation, we might consider an analysis of *only women* as *no individuals other than women*, which would have the conservative discontinuous determiner *no ... other than women*, which might be another interesting bullet to bite.

Appendix

We note that there are Q that live on many sets, but none minimal. Examples: *All but finitely many English expressions (contain more than ten words), all but finitely many natural numbers.* Let the universe be $\mathbb{N}$, the set $\{0, 1, 2, \dots\}$ of natural numbers. Then for D the denotation of *all but finitely many* we see that a property $X \in D(\mathbb{N})$ iff X is a subset of $\mathbb{N}$ that lacks at most finitely many numbers.

Fact: $D(\mathbb{N})$ lives on each set $X \in D(\mathbb{N})$.

Proof: let $X \in D(\mathbb{N})$. Show: for all $A \subseteq \mathbb{N}$, $A \in D(\mathbb{N})$ iff $A \cap X \in D(\mathbb{N})$.

$\Leftarrow$ Assume first, for A arbitrary, that $A \cap X$ lacks at most finitely many numbers. So then does A as $A \cap X \subseteq A$. (Of course, A could lack zero numbers). Thus if $A \cap X \in D(\mathbb{N})$ then $A \in D(\mathbb{N})$.

$\Rightarrow$ If both A and X lack at most finitely many numbers then $A \cap X$ lacks just those which A lacks union those which X lacks, and the union of two finite sets is finite, so $A \cap X \in D(\mathbb{N})$. $\square$

But $D(\mathbb{N})$ has no minimal live-on set (= a live-on set that is a subset of every set $D(\mathbb{N})$ lives on).

Proof: Assume, leading to a contradiction, that W is a minimal live-on set, so W is a subset of every set $D(\mathbb{N})$ lives on. $W \neq \emptyset$, as $\mathbb{N} \in D(\mathbb{N})$ as it lacks only finitely many numbers (zero in fact) but $\mathbb{N} \cap \emptyset = \emptyset \notin D(\mathbb{N})$ as it does not lack just finitely many numbers. And if some $k \in W$ then $W \nsubseteq \mathbb{N}-\{k\}$, a set which $D(\mathbb{N})$ lives on, so W is not a minimal live-on set, contradiction. □

($\mathbb{N}$ may be replaced by any infinite set K, and the universe any set that includes K).

There is another sense in which the notion minimal live-on set might be construed. Namely, a set M that Q lives on which is such that it is minimal with respect to the subset relation, that is: there is no proper subset M' of M s.t. Q lives on M'. In this sense, $D(\mathbb{N})$ above has no minimal live-on set either, for any set it lives on is $\mathbb{N}-K$, for some finite subset K of $\mathbb{N}$. Let $k \in \mathbb{N}-K$ and consider $\mathbb{N}-(K \cup \{k\})$. It is a proper subset of $\mathbb{N}-K$ and $D(\mathbb{N})$ lives on it so $\mathbb{N}-K$ is not minimal. Since K was arbitrary, no $\mathbb{N}-K$ is a minimal live on set.

Further, if U is infinite then *all but finitely many*(U) has as members just the sets it lives on. And as these are subsets of U, these sets are just the witness sets for *all but finitely many*(U). So this property set has no minimal witness sets.

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