Another look at superlative modifiers as modified superlatives[1]

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Abstract. This paper studies the morpho-semantic puzzle of superlative modifiers (SMs) where in many languages SMs typically involve a quantity adjective and a degree morpheme. This paper takes English *at least* as a case study and offers a decompositional analysis connecting its semantic properties with its morphological components. One central proposal is that SMs like *at least* can be structurally decomposed into three morphological pieces: a quantity adjective, a superlative and an existential operator. It is shown that the current analysis not only explains the role of the quantity adjective and degree morphology in the semantics of *at least*, but also captures two types of semantic parallels suggested in the literature on SMs: the parallel with disjunction and the parallel with epistemic indefinites.

Keywords: superlative modifier, superlative, focus, degree, alternative

1. Introduction

Superlative modifiers (SMs) such as English *at least* and *at most* have posed a longstanding and intriguing morpho-semantic puzzle: Why do SMs morphologically involve a quantity adjective and the superlative morpheme? What is the role of quantity adjective and superlative morpheme inside SMs? How are these morphological pieces connected with the semantics of SMs? (1) and (2) below illustrate the relevant facts in English, where the same morphological components *least* and *most* are involved in SMs and superlatives.

(1) Superlative modifiers (SMs)
  a. John bought at **least** $[\text{three}]_F$ apples.
  b. John bought at **most** $[\text{three}]_F$ apples.

(2) Quantity superlatives (QSs)
  a. John drank the **least** water.
  b. John climbed the **most** mountains.

This paper is dedicated to the morpho-semantic puzzle of SMs by taking English *at least* as a case study, and offers a decompositional analysis connecting the semantics of *at least* with its degree morphology. One central proposal is that SMs like *at least* can be structurally decomposed into three pieces: a quantity adjective, a superlative and an existential operator.

The rest of this paper proceeds as follows. Section 2 introduces some semantic properties of *at least* (that have been observed in previous studies). Section 3 discusses the semantics of superlatives and points out one crucial semantic difference between SMs and superlatives. Section 4 spells out the decompositional analysis and shows how the semantic properties discussed in section 2 follows from the current proposal. Section 5 concludes the paper.

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2. Basic facts

This section introduces three empirical properties of English *at least* (some of them have been observed in previous studies, see e.g., Krifka, 1999; Geurts and Nouwen, 2007; Nouwen, 2010; Coppock and Brochhagen, 2013; among others): **(a)** it is focus-sensitive; **(b)** it is compatible with various types of scales and respect the monotonicity constraint; **(c)** it makes the prejacent the lower bound among the set of focus alternatives. First of all, it has been observed that *at least* is focus-sensitive: the semantic contribution of *at least* depends on its focus associate and different positions of the associate lead to truth-conditional differences. (3) and (4) (borrowed from Coppock and Brochhagen, 2013: 12) illustrates the point.[2]

(3) We should at least invite [the postdoc]$_F$ to lunch.

(4) We should at least invite the postdoc to [lunch]$_F$.

(3) and (4) are truth-conditionally different: the former conveys that we should invite the postdoc to lunch or some other people (higher-ranked than the postdoc in the context, say, assistant professor for example) to lunch; in contrast, the latter conveys that we should invite the postdoc to lunch or to some other occasions (higher-ranked than lunch in the context, say, dinner for example). Moreover, as discussed in Coppock and Brochhagen (2013), (3) and (4) are related to different discourse questions: the former addresses the question "Who should we invite to lunch?", while the latter the question "What should we invite the postdoc to?".

Second, *at least* is compatible with various scales (see Mendia, 2016a, b), as shown below.

(5) **Numeral Scales** (a contextual ranking: $4 \succ 3 \succ 2$)
John **at least** wrote [three]$_F$ novels.

(6) **Plurality Scales** (a contextual ranking: adam⊕bill⊕chris ≻adam⊕bill ≻adam)
John **at least** hired [Adam and Bill]$_F$.

(7) **Lexical Scales** (a contextual ranking: gold medal≻silver medal≻bronze medal)
John **at least** got a [silver]$_F$ medal.

(8) **Pragmatic Scales** (a contextual ranking: cherries ≻apples ≻bananas)
John **at least** bought [apples]$_F$.

Note that the numerical scale and plurality scale are based on semantic strength (entailment relation). Therefore, writing four novels *entails* writing three novels, and hiring Adam and Bill *entails* hiring Adam and hiring Bill. In contrast, the lexical scale and pragmatic scale are based on pragmatic strength (non-entailment relation). Thus, winning a gold medal does *not* entail winning a silver medal, and buying apples does *not* entail buying bananas.

Moreover, a novel observation here is that the ranking of the alternatives seems to respect the monotonicity property (cf. Schwarzschild, 2006). By manipulating the context, it seems easy

[2] To my knowledge, Krifka (1999) is the first study pointing out that English *at least* is focus-sensitive.

enough to reverse the ordering between the alternatives in the case of pragmatic scales.[3] In contrast, however, it does not seem possible to reverse the ordering in the case of numerical scales or plurality scales, even with some contextual effort.

(9) Context: John and his friends plan to have a party. John is responsible for buying some fruit for the party. There are three types of fruit available to them: cherries, apples and bananas. However, they are poor and do not have enough money to buy everything. For them, bananas are the optimal because they are the cheapest; apples are less optimal but acceptable because they are still cheaper than cherries.
The contextual ranking (in terms of price): <u>bananas ≻apples ≻cherries</u>

(10) Context: John is planning to hire some people. There are three applicants in the discourse: Adam, Bill and Chris. But the budget is limited. If three people are all hired, John needs to pay a great amount of money for their salary. If only Adam and Bill are hired, the situation is better, but John still pays more than he does in hiring only Adam. The best situation is simply to hire just one person while getting all the work done.
The intended contextual ranking: <u>only adam ≻only adam&bill ≻only adam&bill&chris</u>

Under the context (9), the utterance with *at least* in (8) is understood to convey that John bought apples or bananas (given the contextual ranking: bananas ≻apples ≻cherries). This means that the original ranking (cherries ≻apples ≻bananas) in (8) is now **reversed**. In contrast, the utterance with *at least* in (6) cannot be understood to be that *Liubei* hired only Adam and Bill, or hired only Adam, even with the contextual massage in (10). This indicates that the original ranking (adam&bill&chris ≻adam&bill ≻adam) in (6) **cannot** be reversed. The same observation applies numeral scales. I leave it for readers to verify the case of numerical scales. In short, these facts indicate that there is an intrinsic discrepancy between scales based on semantic strength and those based on pragmatic strength.

Third, in each sentence (5) − (8), the associate of *at least* is made as the lower bound among the set of focus alternatives. Take (5) for example, the sentence is judged true if and only if John wrote three novels or John wrote more than three novels, where the number *three* (the focus associate) is the lower bound on the quantity of novels that John wrote. Similarly, in (7), a silver medal is the lower bound on the type of medals that John won.

To sum up, I have shown that *at least* demonstrates three semantic properties: **(a)** it is focus-sensitive; **(b)** it is compatible with various types of scales and respect the monotonicity constraint; **(c)** it makes the prejacent the lower bound among the set of focus alternatives. With these in mind, the task then is how to account for those semantic properties of *at least* while connecting with the contribution of its superlative morphology *least*. To be concrete, the empirical facts to be captured in this paper are summarized in (11).

(11) The morpho-semantic properties of English *at least*
   a. **The morpho-semantic puzzle**: The same morphological components *least* are involved in superlative modifiers (*at least*) and quantity superlatives (*the least*).
   b. **Focus-sensitivity**: The semantic contribution of *at least* depends on the position of its focus associate.

[3] For lexical scales, although context manipulations are not impossible, they are harder because the ordering is based on our common world knowledge. Instances of lexical scale are *gold medal ≻silver medal ≻bronze medal*, and *full professor ≻associate professor ≻assistant professor*.

c. **Scale types and their discrepancy**: *At least* is compatible with various scales (based on semantic strength or pragmatic strength). However, in contrast to lexical scales and pragmatic scales, the ordering between the alternatives cannot be reversed in numerical scales and plurality scales.
d. **The bounding property**: Under sentences with *at least*, the prejacent is set up as the lower bound among the set of focus alternatives.

Before spelling out the decompositional analysis of *at least*, I would like to briefly discuss the semantics of superlatives and point out one crucial semantic difference between superlative modifiers and superlatives: the former conveys **the non-strict comparison relation**, but the latter encodes **the strict comparison relation**.

3. The semantics of superlatives

A sentence containing a superlative expression, such as *the highest mountain* in (12), can receive different interpretations depending on how the comparison class is specified with respect to different constituents of the sentence (e.g., Heim, 1985, 1999; Hackl, 2009). When the comparison class is determined with respect to the superlative DP itself, the absolute reading arises. In contrast, the relative reading arises when the comparison class is established with respect to one of the constituents in the sentence, such as Adam.

(12) Adam climbed the highest mountain.
Absolute reading: Adam climbed the mountain that is higher than any other (relevant) mountain.
Relative reading: Adam climbed a mountain that is higher than any other (relevant) individual did.

Heim (1999) propose that the absolute-relative ambiguity of a superlative sentence is derived by allowing the superlative morpheme *–est*, with the semantics in (13), to take different scope within the clause. Under this movement approach, the ambiguity of a superlative sentence is actually a case of structural ambiguity. The computation of the relevant pieces is illustrated below, with the absolute reading in (14) and the relative reading in (15).

(13) a. $[\![\text{-}est]\!] = \lambda C_{<e,t>} \lambda G_{<d,et>} \lambda x_{<e>}.\forall y[y \in C \wedge y \neq x \rightarrow \max(\lambda d.G(x, d)) > \max(\lambda d.G(y, d))]$
b. Presuppositions: $x \in C$, $\forall y[y \in C \wedge y \neq x \rightarrow \exists d[G(y, d)]]$

(14) Absolute Reading
a. $[_{DP}$ the $[_{NP}$ [-***est*** (*C*)]$[_{NP}$ *d*-high mountain]]]
b. $[\![d\text{-high mountain}]\!] = \lambda d.\lambda x.\text{mountain}(x) \wedge \text{high}(x) \geq d$
c. $C = \{x: \exists d.\text{mountain}(x) \wedge \text{high}(x) \geq d\}$
d. $[\![\text{DP}]\!] = \iota x \forall y[y \in C \wedge y \neq x \rightarrow \max(\lambda d.\text{mountain}(x) \wedge \text{high}(x) \geq d) > \max(\lambda d.\ \text{mountain}(y) \wedge \text{high}(y) \geq d)]$

(15) Relative Reading
a. $[_{IP}$ Adam [-***est*** (*C*)] $\lambda d.\ \lambda x.$ $[_{vP}$ *x* climbed a *d*-high mountain]]
b. $C = \{x: \exists d \exists z[\text{mountain}(z) \wedge \text{high}(z) \geq d \wedge x \text{ climbed } z]\ \}$
c. $[\![\text{IP}]\!] = 1$ iff

$\forall y[y \in C \wedge y \neq \text{adam} \rightarrow \max(\lambda d.\exists z[\text{mountain}(z) \wedge \text{high}(z) \geq d \wedge \text{adam climbed } z])$
$> \max(\lambda d.\exists z[\text{mountain}(z) \wedge \text{high}(z) \geq d \wedge y \text{ climbed } z])$

Under the absolute reading in (14), the superlative morpheme takes scope within the DP and the comparison class *C* is a set of relevant mountains. In contrast, under the relative reading shown in (15), the superlative morpheme takes scope outside the DP (specifically, -*est* scopes over the VP) and the comparison *C* is a set of relevant mountain-climbers.

Alternatively, some researchers purse an in-situ approach (e.g., Farkas and Kiss, 2000; Sharvit and Stateva, 2002), where the superlative morpheme never moves out of the DP, and the relative reading is derived from domain restriction. Consider (16), where the bolded part indicates the additional contextual restriction on the value of *C*.

(16) Relative Reading (an in-situ approach)
  a. $[_{DP}$ the $[_{NP}$ [-***est*** (*C*)]$[_{NP}$ *d*-high mountain]]]
  b. $[\![d\text{-high mountain}]\!] = \lambda d.\lambda x.\text{mountain}(x) \wedge \text{high}(x) \geq d$
  c. $C = \{x: \exists d \exists z.\text{mountain}(x) \wedge \text{high}(x) \geq d \wedge z \text{ climbed } x\}$
  d. $[\![\text{DP}]\!] = \iota x \forall y[y \in C \wedge y \neq x \rightarrow \max(\lambda d.\text{mountain}(x) \wedge \text{high}(x) \geq d)$
  $> \max(\lambda d.\ \text{mountain}(y) \wedge \text{high}(y) \geq d)]$

At this point, it is worth noting that superlatives are focus-sensitive (e.g., Heim, 1999). For example, (17) and (18) are truth-conditionally different; on the relative reading, (17) conveys that John bought more apples on Sunday than any other day, while (18) conveys that John bought more apples than anyone else did on Sunday.

(17) John bought the most apples on $[\text{Sunday}]_F$.

(18) $[\text{John}]_F$ bought the most apples on Sunday.

To synthesize the scope-taking property of the superlative morpheme and the contribution of focus, Heim (1999) provides another possible entry as defined in (19). The idea behind (19) is that focus helps set the contextual value of the domain *C*. Put differently; focus restricts the domain of the superlative operator.

(19) a. $[\![\text{-}est]\!] = \lambda C_{<<d,t>,t>} \lambda P_{<d,t>} \forall Q[Q \in C \wedge Q \neq P \rightarrow \max(\lambda d.P(d)) > \max(\lambda d.Q(d))]$
  b. Presuppositions: $P \in C$, $\exists Q[Q \in C \wedge Q \neq P]$

With (19) in mind, the relevant computation of (17) is shown in (20), and the relevant computation of (18) is shown in (21).

(20) a. The LF of (17): [-***est*** (*C*)] [λd. $[_{IP}$ John bought *d*-apples on $\text{Sunday}_F]$~C]
  b. $C = \{x: \lambda d.$ John bought *d*-apples on $x\}$
  c. $[\![(17)]\!] = 1$ iff $\forall Q[Q \in C \wedge Q \neq (\lambda d.$ John bought *d*-apples on Sunday)
  $\rightarrow \max(\lambda d.$ John bought *d*-apples in Sunday) $> \max(\lambda d.Q(d))]$

(21) a. The LF of (18):[-***est*** (*C*)] [λd. $[_{IP}$ John_F bought *d*-apples on Sunday]~C]
  b. $C = \{x: \lambda d.\ x$ bought *d*-apples on Sunday$\}$
  c. $[\![(18)]\!] = 1$ iff $\forall Q[Q \in C \wedge Q \neq (\lambda d.$ John bought *d*-apples on Sunday)

$\rightarrow$ max(λd. John bought d-apples in Sunday) > max($\lambda d.Q(d)$)]

In both cases; the superlative operator plus its domain restrictor ***-est*** (*C*) takes scope over the whole sentence. Recall that the crucial difference between (17) and (18) lies in the position of focus. Crucially, the effect of focus is captured by different contextual values of the domain *C*: in (20b), the set of degree properties vary with respect to the days John bought apples, while in (21b) the set of degree properties vary with respect to the individuals who bought apples on Sunday.[4]

The choice between a movement approach and an in-situ approach is an ongoing debate in the literature on superlatives. However, it may well be that both approaches are needed (see Tomaszewicz, 2015 for a comparative perspective on the correlation between definiteness marking and different types of relative readings).[5] The debate between the two approaches will not concern us in this paper. What is crucial here is that under either approach, the semantics of superlatives, like that of comparatives, encode a strict comparison relation: a greater-than relation symbolized by > (or a less-than relation symbolized by <).

In a sharp contrast to the strict comparison relation in the semantics of superlatives, a non-strict comparison relation (e.g., the greater-than/ less-than or equal-to relation) has been assigned to the semantics of *at least* in the previous studies, as shown in (22).[6]

(22) The non-strict comparison relation of *at least*
  a. The degree-based approach (e.g., Kennedy 2015)
  $[\![at\ least]\!] = \lambda m_{<d>} \lambda P_{<d,\ t>}.\ max\{n \mid P(n)\} \geq m$
  b. The discourse-based approach (e.g., Coppock and Brochhagen 2013)
  $[\![at\ least\ (C)]\!]^{w,\ g} = \lambda p_{<s,\ t>}.\exists q\ [q \in C \wedge q(w) \wedge q \succeq_{i} p]$

Although the non-strict comparison relation may correctly characterize the truth-conditions of sentences with *at least* (see section 2), it raises a non-trivial question in connecting with the semantics of superlatives: Where does the "equal-to" relation (i.e., =) come from? From our discussion above, superlatives encode the strict-comparison, but not the non-strict one.

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[4] In explicating the role of focus, the three-place superlative operator requires the movement of the focus-marked constituent to serve as its third argument. Heim (1999) discusses this point and explicitly expresses her doubt that multiple LFs actually go with the *relative prominence* on focus-marked constituents at PFs. Readers are referred to Heim (1999) and Sharvit and Stateva (2002) for discussion on the role of focus in superlatives.

[5] A crucial assumption under the movement approach is that the definite article *the* in superlatives is optionally interpreted as an indefinite (Heim,1999). This assumption has been a soft spot for the movement approach. However, couched in the framework of dynamic semantics, Bumford (2017) recently argues that the definite article *the* can be semantically decomposed into two components: one builds a set of witness that satisfies the restricting noun phrase and the other imposes the uniqueness test. The former amounts to the meaning of an indefinite in dynamic semantics. Under the relative reading, the superlative morpheme *–est* takes a parasitic scope (in the sense of Barker, 2007) between the first and the second component of *the*. Bumford's analysis elegantly removes the long-standing soft spot for the movement approach. Readers are referred to his paper for details. See also Coppock and Beaver (2014) for discussion of definiteness marking in superlatives.

[6] Although there have been various proposals for the semantics of *at least*, they can be generally classified into two approaches, depending on what kind of scales SMs are thought to make reference to: a degree-based approach and a discourse-based approach. The degree-based approach considers SMs as degree operators and invokes a scale of degrees (Nouwen, 2010; Kennedy, 2015). In contrast, the discourse-based approach invokes scales of pragmatic strength, which are not restricted to numerals and may not even respect entailment (e.g., Krifka, 1999; Geurts and Nouwen, 2007; Büring, 2008; Coppock and Brochhagen, 2013).

Despite the issue of the strict comparison vs. non-strict comparison, both superlatives and superlative modifiers do convey a similar bounding property. (23) and (24) illustrate the point.

(23)	a.	John drank the least coffee.	Superlative
	b.	John ate at least $[\text{three}]_F$ apples.	Superlative modifier
(24)	a.	John ate the most apples.	Superlative
	b.	John ate at most $[\text{three}]_F$ apples.	Superlative modifier

In (23), sentence (a) conveys that the quantity of coffee that John drank is the *lowest*, and sentence (b) that the *lowest* quantity of apples that John ate is three. Similarly, in (24), sentence (a) conveys that the quantity of apples that John ate is the *greatest*, and sentence (b) that the *greatest* quantity of apples that John ate is three. This semantic parallel on the bounding property suggests that superlatives and superlative modifiers should be intrinsically connected in the semantics, beyond the level of surface morphology.

To sum up, we have seen that one crucial aspect in the interpretation of superlatives lies in how the content of the comparison class is determined; in this respect, focus plays an important role in shaping the comparison class. Furthermore, we have also seen that superlatives semantically encode the strict comparison, while superlative modifiers convey the non-strict comparison. Finally, both constructions similarly convey a bounding property on the relevant scale. Given our discussion so far, anyone who wants to connect the semantics of superlatives with that of superlative modifiers, she has to find a way maintaining the strict comparison of superlatives while deriving the non-strict comparison of superlative modifiers somewhere else. The next section provides one such proposal.

4. The proposal

The section proceeds as follows. Section 4.1 proposes that *at least* can be structurally decomposed into three major components: a quantity adjective (Q-adjective), a superlative SupP and an existential operator E-OP. Also, it demonstrates how the three components are semantically composed in the internal structure of *at least*. Section 4.2 first illustrates how the current decompositional analysis captures the semantic properties of *at least* (discussed in section 2) and then discusses how the two types of semantic parallels (the parallel with disjunction and that with epistemic indefinites) are captured.

Below, I briefly introduce two core assumptions in this paper. First, I assume Rooth (1992)'s focus semantics. Briefly put, every expression φ has an ordinary semantic value and a focus semantic value. For an unfocused constituent, its focus semantic value is a singleton set containing the ordinary value of that expression. For a focused constituent, its focus semantic value is a set of alternatives: a set of objects that have the same semantic type as the focused constituent. The set of alternatives induced by focus is computed recursively (essentially as in Rooth, 1992). Furthermore, the semantic contribution of a focus-sensitive operator depends on the focus semantic value of its sister. The set of focus alternatives projects until they meet the focus operator where they are interpreted by a squiggle operator ~ and restricted by a contextual variable C. The definition of ~ in (25) is drawn from Rooth (1996: (20)).

(25) Where φ is a syntactic phrase and C is a syntactically covert semantic variable, $\varphi \sim C$ introduces the presupposition that C is a subset of $[\![\varphi]\!]^f$ containing $[\![\varphi]\!]^o$ and at least one

another element.

Second, I assume Bobaljik (2012: 4)'s Containment Hypothesis, as shown in (26).

(26) Containment Hypothesis (Bobaljik, 2012: 4)
The representation of the superlative properly contains that of the comparative.

The next section presents a decomposition of English *at least*.

4.1. Decomposing *at least*

To begin with, I propose that English SMs can be structurally decomposed into three major components: a quantity adjective (Q-adjective), a superlative SupP and an existential operator E-OP. Assuming Bobaljik (2012)'s Containment Hypothesis, I propose that in English SMs, the superlative construction structurally embeds a comparative construction, as illustrated in (27). Note that CompP represents the embedded covert comparative construction.

(27) $[_{\text{SupP}}$ *-est* $[_{\text{CompP}}$ Comp^0 $[_{\text{AdjP}}$ *much*$]]]$ SupP

Regarding the semantic details, let's start in a bottom-up fashion. For the quantity adjective *mu*, I propose that it encodes a measure function μ_c, which maps the elements induced by focus to their corresponding positions along a contextually given dimension, as defined in (28). Note that η is a meta-variable, intended to capture the fact that the semantic type of focused elements may vary.

(28) $[\![much]\!]^c = \lambda\alpha.\mu_c(\alpha)$ $<\eta, d>$

Moreover, I propose that in the case of SMs, the measure function μ_c respects the monotonicity constraint, but crucially not restricted to it (cf. Wellwood, 2014, 2015). This has two consequences immediately. First, English SMs such as *at least* (and their cross-linguistic counterparts) can apply to the alternatives whose domain is not structured (e.g., by the part-of relation), as in the case of lexical scale and pragmatic scale. Second, when the domain of the alternatives is structured, as in the case of numerical scale and plurality scale, the structure-preserving mapping guarantees that the output ranking between the alternatives cannot be altered. The definition of monotonicity is offered below.

(29) Monotonicity
A measure function μ: $D_{\preceq Part} \rightarrow D_{\preceq Deg}$ is **monotonic** iff
for all $\alpha, \beta \in D_{\preceq Part}$, if $\alpha \prec^{Part} \beta$, then $\mu(\alpha) \prec^{Deg} \mu(\beta)$.

Given the current analysis, the discrepancy between different scale types is now derived: When the input focus alternatives have their own internal structure (e.g., a partial ordering or a total ordering), through a structure-preserving mapping, the output ranking between the alternatives cannot be altered despite contextual effort (e.g., numerical scale and plurality scale). In contrast, when the set of focus alternatives is NOT structured by entailment relation or part-of relation, the output ranking between the alternatives is subject to contextual factors and thus is not constant across contexts (e.g., lexical scale and pragmatic scale).

Next, the comparative morpheme Comp$^+$ (cf. English *-er*), after taking the Q-adjective *much* as its first argument, returns a comparison relation between the alternatives as in (30). The semantics of the superlative morpheme *-est* offered in (31) is like the entry traditionally assigned to English *–est* (Heim, 1999), except for the additional comparison relation and its type-flexibility. The semantics of SupP is obtained in(31), when the pieces are put together.

(30) $[\![\text{Comp}^{+}\text{P}]\!]^{c} = \lambda\alpha\lambda\beta.\mu_c(\alpha) > \mu_c(\beta)$ $\quad <\eta, <\eta, t>>$

(31) $[\![\textit{-est}]\!]^{c} = \lambda COM_{<\eta, <\eta, t>>}\lambda C_{<\eta, t>}\ \lambda\alpha_{<\eta>}.\forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow COM(\alpha, \beta)]$

Following the decompositional analysis of English *little* and *less* along the line in Heim (2006a, b) and Büring (2007), I propose that the quantity adjective *little* contributes to two semantic components at LF: a negative feature NEG and a covert *much* (see also Solt, 2009, 2015). The covert comparative morpheme Comp$^+$ combined with the negative feature NEG is reanalyzed as a covert comparative morpheme Comp$^-$ (with the opposite comparison relation), as defined in (32). The connection between Comp$^+$ and Comp$^-$ is reminiscent of Heim's and Büring's analyses of English *less* as a reanalyzed result from the combination of a negation contributed by adjectives with negative polarity (glossed as LITTLE in their analyses) and the comparative morpheme *–er*.

(32) NEG-Comp$^+$ is reanalyzed as Comp$^-$ (Heim, 2006a,b; Büring, 2007)

The semantics of Comp$^-$ (cf. English *less*) takes the Q-adjective *much* as its first argument and returns a comparison relation between the alternatives along a contextually-valued scale, as defined in (33). Combining with the meaning pieces of *much* and *–est* above, the semantics of SupP (i.e., *least*) involved in *at least* is obtained, as shown in (34).

(33) $[\![\text{Comp}^{-}\text{P}]\!]^{c} = \lambda\alpha\lambda\beta.\mu_c(\alpha) < \mu_c(\beta)$ $\quad <\eta, <\eta, t>>$

(34) $[\![[_{\text{SupP}}\ \textit{-est}\ [_{\text{Comp-P}}\ \text{Comp}^{-}\ [_{\text{AdjP}}\ \textit{much}]]]]\!]^{c}$
$= \lambda C_{<\eta, t>}\ \lambda\alpha_{<\eta>}.\forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) < \mu_c(\beta)]$ $\quad$ SupP

Finally, the existential operator (propositional version), the third component, is offered below.

(35) $[\![\text{E-OP}]\!]^{w, c} = \lambda SUP_{<<st, t>, <st, t>>}\lambda C_{<st, t>}\ \lambda\alpha_{<st>}.\exists\gamma[\gamma\in C \wedge \gamma_w \wedge SUP\ (C, \alpha)]$

Putting together the semantics of SupP and that of E-OP, the semantics of *at least* (propositional version) is obtained in (36). [7] The overall internal structure of *at least* is in (37).

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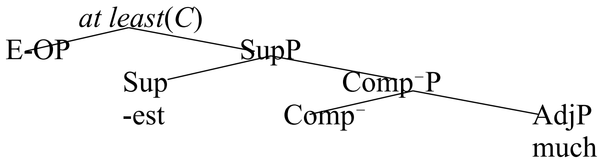
[7] To capture the fact that SMs can be syntactically adjoined to constituents of non-propositional meanings, one may assume that English *at least* also has the following non-propositional lexical entries, which are derived by type-shifting (e.g., the Geach rule and the backward Geach rule).

(i) a. A non-propositional version (by the Geach rule)
$[\![\textit{at least}\ (C)]\!]^{w, c} = \lambda\alpha_{<\eta, st>}\lambda P_{<\eta>}.\exists\gamma[\gamma\in C \wedge \gamma_w(P) \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha(P)) < \mu_c(\beta(P))]]$

b. A non-propositional version (by the backward Geach rule)
$[\![\textit{at least}(C)]\!]^{w, c} = \lambda\alpha_{<\eta>}\lambda P_{<\eta, st>}.\exists\gamma[\gamma\in C \wedge P_w(\gamma)\wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(P(\alpha)) < \mu_c(P(\beta))]]$

(36) $[\![\textit{at least}(C)]\!]^{w,c} = \lambda\alpha_{<st>}.\exists\gamma[\gamma\in C \wedge \gamma_w \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) < \mu_c(\beta)]]$

(37) The internal structure of English SM *at least* at LF



According to (36), *at least* takes the prejacent proposition α and conveys that there is one proposition γ in the domain C such that γ is true. Moreover, the superlative SupP (i.e., $\forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) < \mu_c(\beta)]$) requires all the alternative propositions that are contextually relevant and non-identical to the prejacent α be higher ranked than the prejacent α. In other words, SupP excludes all the lower alternatives while keeping the higher alternatives. Given the focus presuppositions imposed by the squiggle operator ~ in (25), repeated as (38) below, the prejacent α has to be an element of the domain *C*. Taken together, an assertion with *at least* amounts to saying that there is one element γ in the domain C which is a set consisting of the prejacent α and its higher alternatives β such that the element γ is true. The apparent non-strict comparison in the truth-conditions of sentences with *at least* is thus derived, more specifically, from the interaction between the focus presuppositions imposed by the squiggle operator ~ and the semantics of the superlative SupP.

(38) $\alpha \sim C$ is defined iff $[\![\alpha]\!]^o \in C \wedge \exists\alpha'[\alpha'\neq\alpha \wedge [\![\alpha']\!]^o \in C] \wedge C \subseteq [\![\alpha]\!]^f$

Notice that E-OP is assumed to be covert in English and the preposition *at* is treated as semantically vacuous in the compositional process. One alternative view would be to say that the preposition *at* is actually an overt realization of the existential operator E-OP in English. On this view, it is important to note that English may not be the only language where the E-OP is overtly realized. Two other candidates are observed: the morpheme ***al*** in Italian SMs (*al massimo* 'at most' and *al meno* 'at least') and the morpheme ***au*** in French SMs (*au plus* 'at most' and *au moins* 'at least'). Seen in this light, I leave open this alternative view; further studies on the morpho-semantics of SMs are needed to verify whether the realization of the E-OP is limited to the family of Indo-European languages.

Before leaving this section, let me briefly highlight several important features of the current decompositional analysis of SMs: **(a)** Q-adjectives play a crucial role in encoding a measure function mapping the alternatives to their corresponding positions ordered along a contextually-valued scale; this provides the foundation of the scalarity of SMs; **(b)** the superlative SupP has dual roles: it not only serves as a domain restrictor of SMs but also crucially introduces the scalarity of SMs, which is a comparison relation between the prejacent and its alternatives; **(c)** the internal structure of *at least* contains a superlative, which in turn structurally embeds a comparative (Containment Hypothesis); **(d)** the existential operator E-OP might be overtly realized in some languages, while covert in others.

4.2. Deriving the semantic properties of *at least*

This section illustrates how the current analysis captures the semantic properties of *at least* (discussed in section 2). Recall that English *at least* demonstrates three semantic properties: **(a)** it is focus-sensitive; **(b)** it is compatible with various types of scales and there is a discrepancy between scales based on semantic strength (numerical scales and plurality scales) and those based on pragmatic strength (lexical scales and pragmatic scales); **(c)** it makes the prejacent the lower bound among the set of focus alternatives.

First, under the current analysis, information focus induces a set of elements serving as the input to the measure function in the semantics of Q-adjective. In this sense, focus determines the input domain of Q-adjective and it thus follows that *at least* is focus-sensitive.

Second, as mentioned in section 4.1, depending on whether the set of focus elements has its internal structure, a discrepancy between scales on semantic strength and those on pragmatic strength is expected. More specifically, when the set of the input focus elements stands in a partial ordering or a total ordering, through a structure-preserving mapping, the output ranking between the elements cannot be altered despite contextual effort. This is the case of numerical scales and plurality scales. In contrast, when the set of the input focus elements is NOT structured by the entailment relation or a part-of relation, the output ranking between the elements is subject to contextual factors and is thus not constant across contexts. This is the case of lexical scales and pragmatic scales. The discrepancy between scales is derived.

Third, under the current analysis, the bounding property of *at least* results from the fact that the prejacent is set up as <u>the comparative standard</u> in the strict comparison relation encoded in the semantics of the superlative SupP (see (34)). Below, the sentence (39) and its computation (40) illustrate the point.

(39) Adam is **at least** an $[\text{associate}]_F$ professor.

(40) a. LF: $[_{vP}$ **at least**(C) $[_{vP}$ $[_{vP}$ Adam be an $[\text{associate}]_F$ professor] ~C]][8]

b. $[\![\textit{at least}(C)]\!]^{w,c}$
$= \lambda\alpha_{<st>}.\exists\gamma[\gamma\in C \wedge \gamma_w \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) < \mu_c(\beta)]]$

c. $\alpha \sim C$ is defined iff $[\![\alpha]\!]^o \in C \wedge \exists\alpha'[\alpha'\neq\alpha \wedge [\![\alpha']\!]^o \in C] \wedge C \subseteq [\![\alpha]\!]^f$

d. $[\![(39)]\!]^{w,c} = 1$ iff
$\exists\gamma[\gamma\in C\wedge \gamma_w \wedge\forall\beta[\beta\in C\wedge \beta\neq\lambda w.$Adam is an associate professor in w
$\rightarrow\mu_c(\lambda w.$Adam is an associate professor in $w) < \mu_c(\beta)]]$

Because of the presuppositions introduced by the ~ squiggle operator (Rooth, 1992), the prejacent proposition α must be an element in the domain *C*. Furthermore, because of the domain restrictor SupP, all the elements *β* non-identical to the prejacent α are ranked above the prejacent α. Put differently, the semantics of SupP removes the lower alternatives from the domain *C*. Taken together; the domain *C* combined with the contribution of SupP now denotes a set consisting of the pejacent and its higher alternatives. According to (40), (39) is judged true if and only if there is one element in the domain (i.e., in the set consisting of the prejacent and its higher alternatives) such that the element is true. Assume that the set of propositional alternatives induced by focus is {*λw.Adam is a full professor in w*, *λw.Adam is an associate professor in w*, *λw.Adam is an assistant professor in w*}, given the ranking of the

[8] For simplicity and illustration, I assume that the preverbal *at least* is syntactically adjoined to vP and the subject *Adam* reconstructs back to Spec, vP (where it is originally generated). Nothing crucial in my proposal hinges on these assumptions.

alternatives is: *λw.Adam is a full professor in w* ≻ *λw.Adam is an associate professor in w* ≻ *λw.Adam is an assistant professor in w*, the truth-conditions in (40) require (39) to be true if and only if Adam is a full professor or Adam is an associate professor.

Note that a natural consequence of the current analysis is that both superlative modifiers and superlatives convey a similar bounding property, as we have seen in (23), because both constructions involve a superlative. Crucially, the apparent non-strict comparison of *at least* results from the focus presuppositions (i.e., the prejacent must be an element in the domain) and the semantics of SupP (i.e., it removes the lower alternatives while keeping the higher alternatives in the case of *at least*).

Before leaving this section, I would like to point out that the current analysis additionally captures two types of semantic parallels: **(a)** the parallel to disjunction (Büring, 2008); **(b)** the parallel to epistemic indefinites (Nouwen, 2015). Let's first consider the semantic parallel between *at least* and disjunction. Simply put, an existential claim over a set amounts to a disjunction of each element in the set. This is informally schematized in (41).

(41) The parallel with disjunction
$\exists\gamma\ [\ \gamma\in\{\alpha, \beta\}\ \wedge\ \gamma$ is true$]\ \Leftrightarrow\ \alpha$ is true $\vee\ \beta$ is true

Suppose that α, β and γ are propositions; α is the prejacent and β is the higher alternative. The claim that there is a proposition γ in the set $\{\alpha, \beta\}$ such that γ is true is equivalent to the disjunctive claim that the prejacent α is true or the proposition β is true. In this respect, under the current analysis, English *at least* is not only parallel with disjunction for the pragmatic calculation of ignorance inferences (as argued in Büring, 2008 and others), but also parallel with disjunction from the viewpoint of their semantics.

Finally, let's consider the parallel between *at least* and epistemic indefinites (EIs). Both SMs such as *at least* and EIs such as Spanish *algún* lead to ignorance inferences (e.g., Büring, 2008 on SMs and Alonso-Ovalle and Menéndez-Benito, 2010 on Spanish *algún*). For example, (42) illustrates the relevant Spanish example and (43) shows the semantics of *algún* proposed in Alonso-Ovalle and Menéndez-Benito (2010).

(42) Epistemic Indefinites: Spanish *algún*
María se caso´ con **algún** estudiante del departamento de lingüística.
María SE married with ALGÚN student of the department. of linguistics
'María married a linguistics student.'

(43) $[\![algún]\!] = \lambda f_{<<e,t>, <e, t>>}$: anti-singleton ($f$). $\lambda P_{<e, t>}.\lambda Q_{<e, t>}.\exists x_{<e>}[f(P)(r) \wedge Q(r)]$

According to Alonso-Ovalle and Menéndez-Benito, by using *algún* in (42), the speaker is ignorant about the linguistics student who Maria married. As shown in (43), an important aspect in their analysis of *algún* is that it imposes an anti-singleton presupposition on the domain. In this respect, under the current analysis, the domain of *at least* will always be non-singleton, consisting of the prejacent (obtained by focus presuppositions) and the higher alternatives (obtained by the superlative SupP). In other words, *at least* and *algún* both require a non-singleton domain. This seems to be one common core of those expressions leading to ignorance inferences. For comparison, the semantics of *at least* is repeated below.

(44) $[\![at\ least(C)]\!]^{w,c} = \lambda\alpha_{<st>}.\exists\gamma[\gamma\in C \wedge \gamma_w \wedge \forall\beta[\beta\in C \wedge \beta \neq \alpha \rightarrow \mu_c(\alpha) < \mu_c(\beta)]]$

Note that the ignorance inference induced by *algún* seems obligatory (Alonso-Ovalle p.c.); in contrast, the ignorance inference associated with *at least* seems defeasible (e.g., Mendia, 2016b; among others), thus not obligatory.[9] Seen in this light, there is crucial semantic difference between *algún* and *at least* on the non-singleton requirement: In a contrast to *algún* where the anti-singleton requirement is a presupposition, the non-singleton domain of *at least* is part of the truth-conditions. This semantics of *at least* together with the pragmatics of focus result in the sensitivity of their ignorance inferences to different discourse questions (i.e., Questions-Under-Discussions in the sense of Roberts, 1996/ 2012).

5. A comparison with Coppock (2016)

This section presents a brief comparison between my decompositional analysis of SMs with Coppock (2016). Building on Penka (2010)'s and Solt (2011)'s insights on the compositional derivation of SMs, Coppock (2016) presents a decompositional analysis of English *at least* couched in Coppock and Brochhagen (2013)'s discourse-based analysis. There are three core ingredients in Coppock (2016). First, she assumes some sort of alternative semantics where natural language expressions are translated into expressions of a formal logic that denote a set of intensions. (45) shows how the adjective *tall* is translated under her semantic framework.

(45) $[\![tall]\!]^w = \{\lambda w\lambda d\lambda x.\ tall_w(d)(x)\}$

Next, she assumes the Containment Hypothesis, where a superlative structurally embeds a comparative. She assigns the comparative morphemes *–er* and *less* under phrasal comparatives the semantics in (46). Specifically, *-er* denotes a singleton set containing a function that expects, besides the world argument, a gradable predicate *G*, a comparative standard *s*, and a comparative target *t*, and returns true if the comparative target is *G* to a greater extent than the comparative standard. The entry of *less* encodes the opposite comparison relation.

(46) a. $[\![\text{-}er]\!]^w = \{\lambda w.\ \lambda G_{<d,\tau t>}\lambda s_{<\tau>}\lambda t_{<\tau>}.\ \max(\lambda d.G_w(d)(t)) > \max(\lambda d.G_w(d)(s))\}$
b. $[\![less]\!]^w = \{\lambda w.\ \lambda G_{<d,\tau t>}\lambda s_{<\tau>}\lambda t_{<\tau>}.\ \max(\lambda d.G_w(d)(t)) < \max(\lambda d.G_w(d)(s))\}$

Second, for the superlative morpheme -*t*, she assumes the following entry.

(47) $[\![\text{-}t]\!]^w = \{\lambda w.\ \lambda R_{<\tau,\tau t>}\lambda C_{<\tau>}\lambda x_{<\tau>}.\forall x'_\tau \in C.\ x \neq x' \rightarrow R_w(x, x')\}$

In order to compose meanings, she assumes a point-wise, intension-friendly version of functional application, as illustrated below. With the compositional rule, the meaning of *least* is offered in (49), where **m** is a contextually-specified gradable predicate.

(48) **Functional Application**
Let α and β be the only sub-trees of the tree γ. If

[9] For purposes of this paper, I do not discuss how the ignorance inference associated with *at least* arises. Given the proposed semantics of *at least*, the current analysis is compatible with the pragmatic approach where the ignorance inference of *at least* arises pragmatically from certain mechanism of implicature calculation (see e.g., Cummins and Katsos, 2010; Kennedy, 2015; Mendia, 2016a, b and Schwarz, 2016a, b for proposals and discussion).

(a) $[\![\alpha]\!]^w = \alpha'$, where α' is of type $<s, <\sigma, \tau>>$
(b) $[\![\beta]\!]^w = \beta'$, where β' is of type $<s, \sigma>$
Then: $[\![\gamma]\!]^w = \{\lambda w.f(w)(a(w)) \mid f \in \alpha' \wedge a \in \beta'\}$

(49) $[\![least]\!]^w = \{\lambda w.\ \lambda C_{<\tau>}\ \lambda x_{<\tau>}.\forall x' \in C \wedge x' \neq x \rightarrow \max(\mathbf{m}_w(x)) < \max(\mathbf{m}_w(x'))\}$

Finally, the most important ingredient from Coppock's analysis (I believe) is that the famous *at*-operator is actually meaningful: it introduces the alternatives and scales. The entry of *at* is provided in (50), with a sketchy compositional derivation in (51).

(50) $[\![at]\!]^w = \{\lambda w.\lambda S_{<\tau, <\tau, t>>}\ \lambda x_{<\tau>}\ \lambda y_{<\tau>} \mid y \in C \wedge S_w(C)(x)\}$
'the set of things *y* in a comparison class *C* such that *x* is *S* [least/ most] in *C*'

(51) The case of *at least three*
a. The LF: [**at** [**-t** [**less** [**m**]]]] three]
b. $[\![at\ least]\!]^w = \{\lambda w.\lambda x_{<\tau>}\ \lambda y_{<\tau>} \mid y \in C \wedge \forall x' \in C \wedge x' \neq x \rightarrow \max(\mathbf{m}_w(x)) < \max(\mathbf{m}_w(x'))\}$
c. $[\![at\ least\ three]\!]^w = \{\lambda w.\ \lambda y_{<\tau>} \mid y \in C \wedge \forall x' \in C \wedge x' \neq 3 \rightarrow \max(\mathbf{m}_w(3)) < \max(\mathbf{m}_w(x'))\}$

At this point, it is worth noting that under Coppock's implementation, the job of *at*-operator not only introduces the alternatives and scales, but also *encodes* a non-strict comparison relation (see the meaning of *at* above). Specifically, the meaning derived for *at least three* under Coppock's analysis is a set consisting of the prejacent (the comparative standard) and the relevant higher-ranked elements: {3, 4, 5,…}, ignoring intensionality.

Although Coppock's analysis elegantly explains the morpho-semantic puzzle of SMs, several important questions are raised by her treatment of *at*: **(a)** Why doesn't focus introduce the alternatives in the first place, as in Rooth (1985, 1992)? **(b)** What is role of focus in the analysis after all? How to incorporate the contribution of focus in the analysis? **(c)** Why do SMs have a morpheme specialized in introducing alternatives and scales? **(d)** Shall we expect such alternative-introducing or scale-introducing morpheme to occur somewhere else in natural language? Maybe also in other focus particles like *only* and *even*?

Given these considerations, the decompositional analysis proposed in this paper differs from Coppock (2016) in several crucial respects. First, unlike Coppock's analysis, the current decompositional analysis is deeply connected with Rooth (1985, 1992)'s focus semantics. Second, assuming focus is anaphoric to some QUDs in a discourse (Rooth, 1992; Roberts, 1996/ 2012; among others), just like the discourse function of other focus particles (e.g., *only*), SMs imposes further restrictions on the set of (possible) answers, despite their internal complexity and an "anti-specific" domain. Third, as discussed in section 3, many discourse-based analyses assign a non-strict comparison relation to the semantics of SMs. An example lexical entry of *at least* following that approach is presented below.

(52) $[\![\textbf{at least (C)}]\!]^g = \lambda p_{<s, t>}.\lambda w_{<s>}.\exists q\ [q \in C \wedge q(w) \wedge q \succeq_i p]$
Some alternative q in C and as strong as the prejacent p is true in w

However, an entry like (52) raises a non-trivial question pertaining to the morpho-semantic puzzle of SMs: While *at least* involves an **existential** quantifier and a **non-strict comparison**; superlatives typically involve a **universal** quantifier and a **strict comparison**, as in (53).

(53) a. Adam drank the **least** coffee.
b. $[\![(53a)]\!] = 1$ iff $\forall y[y \neq \text{Adam} \wedge y \in C \rightarrow \max(\lambda d.\ \text{Adam ate } d\text{-many coffee}) < \max(\lambda d.\ y \text{ ate } d\text{-many coffee})]$

In Coppock (2016)'s analysis, the non-strict comparison is obtained through the defined semantics of the *at*-operator. In contrast, under the current analysis, the non-strict comparison of SMs is only an illusion: it is derived from the SupP collaborating with focus presuppositions. There is no need to hard-wire a non-strict comparison relation into the semantics of SMs.

Finally, it is worth pointing out that that despite the several differences discussed above, the current analysis, in fact, is largely inspired by Coppock (2016). In particular, the two analyses share two significant assumptions: **(a)** both analyses adopt Bobaljik's Containment Hypothesis in their decomposition of SMs; **(b)** both analyses adopt the view that the internal structure of SMs is an instantiation of phrasal comparatives.

6. Concluding remarks

In this paper, I have proposed a decompositional analysis of SMs connecting their degree morphology with their semantic properties, by taking English *at least* as a case study. One central proposal is that SMs like *at least* can be structurally decomposed into three pieces: a quantity adjective, a superlative and an existential operator. There are several important features of the current decompositional analysis of SMs: **(a)** Q-adjectives play a crucial role in encoding a measure function mapping the alternatives to their corresponding positions ordered along a contextually-valued scale; this provides the foundation of the scalarity of SMs; **(b)** the superlative SupP has dual roles: it not only serves as a domain restrictor of SMs but also crucially introduces the scalarity of SMs, which is a comparison relation between the prejacent and its alternatives; **(c)** the internal structure of *at least* contains a superlative, which in turn structurally embeds a comparative (Containment Hypothesis); **(d)** both superlatives and SMs convey a similar bounding property because both involve a superlative. **(e)** the apparent non-strict comparison of *at least* is derived from the focus presuppositions combined with the semantics of the superlative SupP. **(f)** the existential operator E-OP might be overtly realized in some languages, while covert in others.

One important issue not touched in this paper concerns how the current analysis connects with an insightful distinction between comparative quantifiers (CQs) and SMs suggested by Nouwen (2010). To the best of my knowledge, Nouwen (2010) is the first study suggesting an insightful distinction between CQs and SMs. In his terminology, the former belongs to the group of Class A modifiers and the latter the group of Class B modifiers, as shown in (54). Nouwen (2010) further proposes that Class A modifiers are degree quantifiers while Class B modifies are not; the latter introduce a bounding property. Although the two classes are internally heterogeneous, Kennedy (2015: (4)) suggests that the distinction between the two groups of modifiers can be understood as in (55):

(54) a. Class A modifiers: *more/ fewer/ less than n*, *over n*, *between n and m*, etc…

b. Class B modifiers: *at least*, *at most*, *up to*, *maximally*, *minimally*, etc…

(55) a. Class A modifiers express **exclusive (strict) orderings** relative to the modified numeral.
b. Class B modifiers express **inclusive (non-strict) orderings** relative to the modified numeral.

Seen in this light, the underlying distinction between CQs and SMs can be understood as pointing to what kind of ordering relation is employed in the semantics. Crucially, the current analysis is not only compatible with the suggested distinction but also decompose the non-strict ordering relation further. More specifically, in the case of SMs, the non-strict ordering is not a semantic primitive, but derived from focus presuppositions together with the semantic contribution of a superlative component. However, it remains to be seen exactly how the different ordering relation (strict vs. non-strict) is connected with the contrast between CQs and SMs in the robustness of ignorance inferences. One possible line of thought would be that the inclusion of the prejacent (due to focus presuppositions) makes the domain of SMs always non-singleton and thus their resulting ignorance inferences would be more robust than those given by CQs, where their domain may not be always non-singleton (cf. Schwarzschild, 2002's idea on singleton indefinites). Overall, the current analysis of *at least* adds to Schwarz et al. (2012)'s insight that Class B modifiers are not homogeneous, particularly with respect to whether the inclusive (non-strict) ordering is a semantic primitive.

Finally, it is important to note that the current analysis cannot be the whole story for the morpho-semantic puzzle of SMs because more than one possible morpho-semantic mapping is attested in natural languages. More studies are needed to see how the morpho-semantic mapping works in (56) and (57).

(56) Quantity adjectives plus even-if (e.g., Japanese and Korean)
a. *ooku-temo* 'at most' Japanese
many-even.if
b. *sukunaku-temo* 'at least'
few-even.if

(57) Quantity adjectives plus comparatives (e.g., Magahi, Hindi, Russian)
a. *jaadaa se aadaa* 'at most' Magahi
more than more
b. *kam se kam* 'at least'
less than less

References

Alonso-Ovalle, L. and P. Menéndez-Benito. (2010). Modal indefinites. *Natural Language Semantics* 18: 1−31.
Barker, C. (2007). Parasitic scope. *Linguistics and Philosophy* 30(4): 407−444.
Bobaljik, J. (2012). *Universals in comparative morphology*. Cambridge, MA: MIT Press.
Bumford, D. (2017). Split-scope definites: Relative superlatives and Haddock descriptions. *Linguistics and Philosophy* 40(6): 549−593.
Büring, D. (2007). Cross-polar nomalies. In *Proceedings of SALT 17*, 37–52.
Büring, D. (2008). The least *at least* can do. In *Proceedings of West Coast Conference on*

Formal Linguistics (WCCFL) 26, 114–120.
Coppock, E. (2016). Superlative modifiers as modified superlatives. In *Proceedings of Semantics and Linguistic Theory 26*, M. Moroney, C.-R. Little, J. Collard, and D. Burgdorf (eds.), pp. 471–488. University of Texas at Austin.
Coppock, E. and D. Beaver. (2014). A superlative argument for a minimal theory of definiteness. In *Proceedings of SALT 24*, 177−196.
Coppock, E. and T. Brochhagen. (2013). Raising and resolving issues with scalar modifiers. *Semantics and Pragmatics* 6: 1–57.
Cummins, C. and N. Katsos. (2010). Comparative and superlative quantifiers: Pragmatic effects of comparison type. *Journal of Semantics* 27(3): 271–305.
Farkas, D. and Katalin É. Kiss. 2000. Comparative and absolute readings of superlatives. *Natural Language and Linguistic Theory* 18: 417−455.
Geurts, B. and R. Nouwen. (2007). *At least* et al.: The semantics of scalar modifiers. *Language* 83(3): 533–559.
Hackl, M. (2009). On the grammar and processing of proportional quantifiers: most versus more than half. *Natural Language Semantics* 17: 63−98.
Heim, I. (1985). Notes on comparatives and related matters. Ms., University of Texas, Austin.
Heim, I. (1999). Notes on superlatives. Ms., MIT, Cambridge.
Heim, I. (2006a). Little. In *Proceedings of SALT 16*, 35−58.
Heim, I. (2006b). Remarks on comparative clause as generalized quantifiers. Ms., MIT, Cambridge.
Kennedy, C. (2015). A "de-fregean" semantics (and neo-gricean pragmatics) for modified and unmodified numerals. *Semantics and Pragmatics* 8: 1–44.
Krifka, M. (1999). *At least* some determiners aren't determiners. In K. Turner (Ed.) *The semantics/ pragmatics interface from different points of view*, pp. 257–291. Leiden: Brill.
Mendia, J. A. (2016a). Focusing on Scales. In *Proceedings of North East Linguistic Society (NELS) 46*, Concordia University, Montreal.
Mendia, J. A. (2016b). *Known unknowns*. Ms., University of Massachusetts at Amherst.
Nouwen, R. (2010). Two kinds of modified numerals. *Semantics and Pragmatics* 3, 1–41.
Nouwen, R. (2015). Modified numerals: The epistemic effect. In L. Alonso-Ovalle and P. Menéndez-Benito (eds.), *Epistemic Indefinites: Exploring modality beyond the verbal domain*, pp. 244–266. Oxford: Oxford University Press.
Penka, D. (2010). A superlative analysis of superlative scalar modifiers. Handout from Sinn und Bedeutung 15.
Rooth, M. (1992). A theory of focus interpretation. *Natural Language Semantics* 1(1):75–116.
Rooth, M. (1996). Focus. In S. Lappin (Ed.), *The Handbook of Contemporary Semantic Theory*, pp. 271–297. Oxford, UK: Blackwell Publishers.
Roberts, C. (1996/ 2012). Information structure in discourse: Towards an integrated formal theory of pragmatics. *Semantics and Pragmatics* 5, 1–69.
Schwarz, B., B. Buccola and M. Hamilton. (2012). Two types of numeral B modifiers: a reply to Nouwen 2010. *Semantics and Pragmatics* 5: 1–25.
Schwarz, B. (2016a). Consistency preservation in Quantity implicature: the case of *at least*. *Semantics and Pragmatics* 9: 1–47.
Schwarz, B. (2016b). At least and ignorance: a reply to Coppock and Brochhagen 2013. *Semantics and Pragmatics* 9: 1–17.
Schwarzschild, R. (2002). Singleton indefinites. *Journal of Semantics* 19: 289–314.
Schwarzschild, R. (2006). The role of dimensions in the syntax of noun phrases. *Syntax* 9: 67–110.

Sharvit, Y. and P. Stateva. (2002). Superlative expressions, context, and focus, *Linguistics and Philosophy* 25: 453−504.
Solt, S. (2009). *The semantics of adjectives of quantity*. Ph.D. Dissertation, City University of New York.
Solt, S. (2011). How many most's? In I. Reich, E. Horsch & D. Pauly (eds.), *Proceedings of Sinn und Bedeutung* 15, pp. 565–580. Saarland University Press.
Solt, S. (2015). Q-adjectives and the semantics of quantity. *Journal of Semantics* 32: 221–273.
Tomaszewicz, B. (2015). Superlative ambiguities: a comparative perspective. Ph.D. Dissertation, University of Southern California.
Wellwood, A. (2014). *Measuring predicates*. Ph.D. Dissertation, University of Maryland, College Park.
Wellwood, A. (2015). On the semantics of comparison across categories. *Linguistics and Philosophy* 38: 67–101.