# P-HYPE: A monadic situation semantics for hyperintensional side effects[1]

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**Abstract.** P-HYPE is a hyperintensional situation semantics in which hyperintensionality is modelled as a 'side effect', as this term has been understood in natural language semantics, Charlow (2014); Shan (2007), and in functional programming. We use monads from category theory in order to 'upgrade' an ordinary intensional semantics to a possible hyperintensional counterpart. Hyperintensional side effects are analysed as a special type of perspective sensitivity. We combine Asudeh and Giorgolo's (2016) perspective sensitive semantic theory with a hyperintensional situation semantics, HYPE (Leitgeb, 2018), a logic with connections of Barwise and Perry's (1983) situation semantics, truthmaker semantics (Fine, 2017) and data semantics (Veltman, 1985). P-HYPE builds on the account of Asudeh and Giorgolo (2016), by carving out a notion of perspectives as special sets of situations which can be combined together via a fusion relation. In addition, we are able to capture the utterer's perspective on other people's perspectives, a phenomenon that plays a role in Asudeh and Giorgolo (2016) but which is not formally defined by them.

**Keywords:** Attitude verbs, monads, side effects, perspective, hyperintensionality, situation semantics.

## 1. Introduction

Semantic theories based on possible worlds that treat sentence meanings as sentence intensions (functions from worlds to truth values) have been plagued by the following problem since their inception (Hintikka, 1962; Montague, 1974): if, as presumably paradigm cases of necessary truths, we treat mathematical and logical truths as true in all worlds in every standard model, then they have the same intension (they are *intensionally equivalent*) in all standard models, and are thus logically equivalent–this despite the intuitive difference in meaning between certain mathematical and logical truths.[2] But consider (3) and (4) (see Cresswell, 1985: p.82), where 'the prime numbers' denotes the set of prime numbers:

(1) Kim proved that the prime numbers are infinite.

(2) Kim proved that the prime numbers are not inductive.

(3) The prime numbers are infinite.

(4) The prime numbers are not inductive.

In Cresswell (1985: p.82) a set is defined to be 'finite' iff it cannot be put into a one-one correspondence with a proper subset of itself, and a set is 'inductive' iff it can be put into a one-one correspondence with a proper initial segment of the natural numbers.[3] Given the assumption that the axioms of Zermelo–Fraenkel set theory with the axiom of Choice (ZFC) are true in all worlds of every standard model, (1) and (2) have the same intension, as do (3)

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[2]We understand 'logically true/equivalent' or 'intensionally equivalent' with respect to classical logic. Later, when discussing HYPE we may if necessary refer to HYPE logical truths and equivalences.

[3]Readers may be more familiar with the terms 'Dedekind finite' for the former term and 'finite' for the latter (Cameron, 2012).


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and (4), since the inductive sets and the finite sets are provably equivalent in ZFC.[4] But (1), (2) intuitively differ in meaning, as do (3) and (4).

So-called 'hyperintensional' semantic theories allow us to block substitution of intensionally equivalent sentences (see Fox and Lappin, 2008 and references therein). But there are two troubling features of contemporary discussions of hyperintensionality.[5] First: whilst hyperintensional logics abound, compositional accounts of how hyperintensionality works at the subsentential level are not always provided as a matter of course (Jago, 2014; Fine, 2017; Yablo, 2014). In some compositional accounts (Muskens, 2007; Pollard, 2015), it is not entirely clear what identity criteria for hyperintensional semantic values are assumed and furthermore, the subjective element of meaning (Haas-Spohn, 1995) that agents attach to co-intensional predicates is not captured.[6] Second: hyperintensional semantic theories generally focus on mathematical and logical truths embedded under attitude verbs, and often (Égré, 2014; Cresswell, 1985; Jago, 2014) don't transparently and straightforwardly apply to unembedded mathematical and logical truths, such as (3) and (4) above, even though they intuitively differ in meaning.

The intuition behind P-HYPE, which extends HYPE (Leitgeb, 2018) to the subsentential level, is that logically equivalent sentences sometimes differ in meaning relative to the perspective of interlocutors or agents. Developing the account of Asudeh and Giorgolo (2016) ('AG' from now on), attitude verbs thus require the constituents of their complement sentences to be interpreted from a particular perspective, and shifts in perspective are determined pragmatically via the context of utterance, much as in certain hyperintensional semantic theories in Cresswell and Von Stechow (1982) and Égré (2014) in which hyperintensionality is modelled by means of *de re* interpretations and it is up to pragmatics to select an appropriate *de re* reading.[7] In addition, a special designated perspective, the *enlightened* perspective allows constituents to receive their ordinary intensional interpretation. Monads enter the picture by allowing us to include perspective sensitive semantic values without revision of compositional rules. However, they offer an additional, compelling lens with which to view hyperintensionality itself, as a 'side-effect' of semantic computations.

In section **2** we propose to analyse hyperintensionality as a side effect, and then discuss (**2.1**) Asudeh and Giorgolo's (2016)'s semantic theory. In section **3** we introduce HYPE (**3.1**) and P-HYPE (**3.2**). We then discuss some toy lexical entries (**3.3**), before giving an example of how P-HYPE analyses (**3.4**) hyperintensionality. Finally (**3.5**) we discuss how our semantics might deal with unembedded mathematical/logical truths.

## 2. Hyperintensionality as a side effect

To capture certain linguistic phenomena compositionally, non-deterministic, intensional or state-changing operators are introduced (Charlow, 2014), in addition to extensions. Shan (2002)

[4]This example could, of course, be replaced by any other example on which two mathematical predicates are necessarily co-denoting, and which does not involve the assumption that the axioms of ZFC are valid in every model.

[5]We restrict our discussion throughout to hyperintensionality in natural language semantics and ignore other domains in which the concept may be applicable.

[6]Greg Kobele points out that certain algorithmic accounts of hyperintensionality (Moschovakis, 2006; Muskens, 2005) might be able to model certain forms of subjectivity as differences between reduction sequences that are acceptable to different agents.

[7]However, the pragmatics of these perspective shifts will not be studied in this paper.

had the intuition that we can model many of these phenomena–which might naively seem non-compositional–as 'side effects' of computing the main value of an expression. Shan (2007) includes amongst so-called linguistic side effects certain types of referential opacity and certain expressions whose meaning and compositional contribution is not pre-theoretically transparent. Hyperintensionality is arguably a good example of a linguistic side effect, since it is not clear what distinction to make between the semantic contribution of logically equivalent statements, how their meanings relate to their truth conditions, and how to characterise their behaviour compositionally. But from both Charlow (2014) and Shan's list of linguistic side effects, hyperintensionality is conspicuous by its absence. We propose that hyperintensionality be added to the list of linguistic side-effects, and try to study it from this vantage point.[8]

Monads have been used to model linguistic side effects (see Giorgolo and Unger, 2009; Van Eijck and Unger, 2010; Asudeh and Giorgolo, 2012; Unger, 2011; Charlow, 2014; Barker and Shan, 2014; Bumford, 2015; Charlow, 2017) and, in particular, to obviate continual revision of compositional rules, as different types of semantic value are added to a semantic theory (for motivation along these lines, see Shan, 2002 and Charlow, 2014). Monads map operations and values in a given type-space with operations and values in an enriched type-space (which might include such exotic semantic values as intensions, focus-sensitive and judge-sensitive semantic values) whilst preserving ordinary extensional function application as the main compositional principle and without generalising to the worst case. Normally, to compositionally combine intensions we need intensional function application (Heim and Kratzer, 1998), in addition to extensional function application. Monads allow us to do forego this additional compositional rule.

Both P-HYPE and the semantics of AG, enrich the typed lambda calculus with a reader monad (Shan, 2002) defined on $P$, the set of perspective indices. We will discuss how perspective indices are employed later. For now we can just say that to every agent in a discourse there corresponds a perspective index, and that certain terms which are perspective sensitive are interpreted relative to perspective indices. If an expression has type $\alpha$, a perspectivally sensitive expression has type $P \rightarrow \alpha$. The reader monad is a triple $(\Diamond, \eta, \bigstar)$. $\Diamond : TYPE \rightarrow TYPE$, is a type-constructor, which behaves as a special modal operator in Lax logic (Fairtlough and Mendler, 1997).[9] $\Diamond$ maps any type $\tau$ to $P \rightarrow \tau$ and, for all $a, b$, maps a function $f : a \rightarrow b$ to a function $\Diamond f : \Diamond a \rightarrow \Diamond b$, such that $(\Diamond f)(x) = \lambda i. f(xi)$.[10] $\eta : \tau \rightarrow \Diamond \tau$ is a value-constructor that takes a non-monadic value $x : \tau$ and trivially upgrades it to monadic values by forming a constant function from perspective indices to $x$. It is called the *unit* of the monad:

**Definition 1** : $\eta(x) =_{def} \lambda i.x : P \rightarrow \alpha$

Finally, $\bigstar$ (called *bind*) is a polymorphic binary infix operator acting as a sort of functional

[8]The null hypothesis would be that hyperintensionality is a phenomenon which exhibits one sort of side-effect as opposed to a multiplicity of side effects. This hypothesis is not invalidated by the the idea (p.c Daniel Rothschild), that there is not a single phenomena of hyperintensionality for which we need to go beyond or develop the resources of standard intensional framworks, but rather a number of different cases, lumped together as 'hyperintensional'. A uniform account of these often-lumped-together cases as exhibiting a certain kind of side effect, might actually reveal certain interesting commonalities between the cases.

[9]It is in fact an endofunctor, as this is understood in Category theory; that is, a functor that maps a category to itself (in this case the category of types).

[10]Throughout '$x : \alpha$' is read '$x$ is of type $\alpha$'.

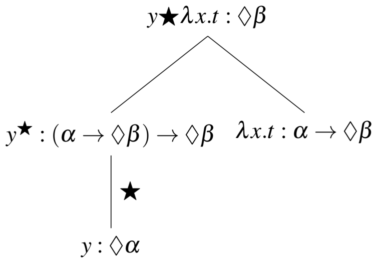
application:

(5) ★ (‘bind’): $\Diamond \to (\tau \to \Diamond\delta) \to \Diamond\delta$

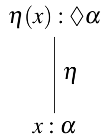
**Definition 2** : $a \bigstar f =_{def} \lambda i.f(a(i))(i)$ where $a : \Diamond\tau, f : \tau \to \Diamond\delta$

We will present ★ as a type-shifting operation as in (6), which shifts something of type $\Diamond\alpha$, to a generalised quantifier of type $(\alpha \to \Diamond\beta) \to \Diamond\beta$ which is then able to take a predicate abstract of type $\alpha \to \Diamond\beta$ as argument. The $\eta$ operator instead will be presented via a non-branching tree (see (7)):

(6)




(7)




### 2.1. AG’s semantic theory

AG aim to model the behaviour of co-referring names in attitude reports via a special form of perspective relativity. Consider the sentence (8a), uttered in the scenario (8b)

(8) a. Mary Jane loves Spiderman.
b. *Scenario:* Mary Jane does not know Peter Parker’s secret identity and loves the man she calls ‘Peter Parker’. A speaker $\sigma$ who knows or is ‘enlightened’ (Zimmermann, 2005) about Peter Parker’s secret identity utters (8a)

According to AG, there is a sense in which (8a) is true, from the perspective of an enlightened utterer, but false from Mary Jane’s perspective. (Asudeh and Giorgolo, 2016) model this by making certain names perspective relative, so that Mary Jane can associate a distinct denotation with the names ‘Spiderman’ and ‘Peter Parker’. Thus names denote certain people’s mental representations, which they call perspectives. We can then imagine a sort of private mental lexicon for each person, consisting of the set of perspectives that a given person associates with terms of her language, which we call that person’s perspective or their mental model. We use ‘perspective’ ambiguously–both to denote a semantic value in someone’s mental model, and that person’s mental model–with the context serving to disambiguate which notion we have in mind. Consider the lexicon (**Table 1**) of the enlightened speaker $\sigma$ of (8a). Plain names are subscripted with $\sigma$ to indicate that this is the denotation of that name for $\sigma$. The names which are type $\Diamond e$ have different denotations, depending on what perspective index they are interpreted relative to. AG suppose that certain names vary in perspective but others do not. Those which do not vary in perspective have something like a default status, in the following sense: if someone becomes enlightened, and learns, for example that Spiderman and Peter Parker are one and the same thing, then they will, by and large, just use plain ‘Peter Parker’, and this name will thence have default status, with ‘Spiderman’. Notice the $\kappa$ operator in the

| WORD | DENOTATION | TYPE |
|---|---|---|
| *Mary Jane* | $\mathbf{mj}_\sigma$ | $e$ |
| *Peter Parker* | $\mathbf{pp}_\sigma$ | $e$ |
| *believe* | $\lambda c.\lambda s.\mathbf{B}(s, c(\kappa(s)))$ | $\Diamond t \to e \to t$ |
| *love* | $\lambda o.\lambda s.\mathbf{love}(s, o(\kappa(s)))$ | $\Diamond e \to e \to t$ |
| *Spider-Man* | $\lambda i.\begin{cases}\mathbf{sm}(\mathbf{i}) & \text{if } i = \kappa(mj)\\ \mathbf{pp}(\mathbf{i}) & \text{if } i = \kappa(\sigma)\end{cases}$ | $\Diamond e$ |

**Table 1** Lexicon of $\sigma$, the enlightened speaker

denotation of *believe* and *love*. $\kappa$ has the following interpretation (where $D_e$ is the domain of individuals of a model):

(9) $\forall x \in D_e\ (\kappa(x) \in P)$

Perspective sensitive expressions that scope below $\kappa$ are interpreted relative to the perspective index corresponding to the subject of the attitude report (see **Table 1**). Expressions that scope above $\bigstar$, are interpreted relative to the default perspective of the utterer. Let us consider the two readings of (8a).

The false reading of (8a) is represented by (10),which $\beta$-reduces to (11), and the true reading is represented by (12), which $\beta$-reduces to (13):

(10) $$love(mj_\sigma, \lambda i.\begin{cases}\mathbf{sm}(\mathbf{i}) & \text{if } i = \kappa(mj)\\ \mathbf{pp}(\mathbf{i}) & \text{if } i = \kappa(\sigma)\end{cases}(\kappa(mj)))$$

(11) $love(mj_\sigma, sm(\kappa(mj)))$

(12) $$\left(\lambda i.\begin{cases}\mathbf{sm}(\mathbf{i}) & \text{if } i = \kappa(mj) : P\\ \mathbf{pp}(\mathbf{i}) & \text{if } i = \kappa(\sigma) : P\end{cases} \bigstar \lambda z.\eta\,(love(mj, z)))\right)(\kappa(\sigma))$$

(13) $love(mj_\sigma, sm(\kappa(\sigma)))$

Since *Spiderman* in (12) scopes above $\bigstar$ and above $\kappa$, it is interpreted relative to the default perspective index, which is the index of the speaker, who they assume in their model to be enlightened. They thus stipulate that the speaker's perspective is the one fed to an expression of the form $a\bigstar f$, which by definition denotes $\lambda i.f(a(i))(i)$, and thus, if the speaker's index is $j$, we always evaluate some expression of the form $a\bigstar f$ at $j$.[11] When, however, a perspective

[11]The technical stipulation they make is grounded in certain claims about perspective relativity, such as the claim that sentences or expressions which are perspective relative are usually relative to the perspective of the utterer of them, and if they are relative to other perspectives, they are relative either to individuals salient in some group within a given context, or are relative to the perspective of the subject of the sentence. We won't assess these claims here, but suffice to say that they have been discussed and broadly endorsed by researchers working on

relative expression scopes below the function $f$ in $a\bigstar f$, it is caught by the $\kappa$ operator.

AG make a certain assumption not reflected in their formalisation: that the utterer of (8a) never *really* interprets *Spiderman* from Mary Jane's perspective, since it is only really accessible to her. Rather, in this case, the denotation of *Spiderman* for the utterer is something in the perspective of the utterer which represents what the utterer takes to be the denotation of the expression relative to the perspective of Mary Jane. And, in general, according to AG, an expression occurring in a sentence only really ever receives a denotation in the mental model of the utterer of that sentence. Whatever the strengths of this assumption, one feature of P-HYPE is that we are able to capture their idea formally, or abandon it entirely, if desirable.

## 3. P-HYPE: a combination of HYPE and monads

### 3.1. HYPE

HYPE (Leitgeb, 2018) is a logic which employs states/situations.[12] States may be like classical possible worlds, but may also be partial (or *gappy*)–verifying neither a formula nor its negation– and inconsistent (or *glutty*)– verifying a formula and its negation. One nice feature of HYPE is that it behaves entirely classically at a subset of states; as such, linguistic analyses couched in classical logics can be transferred to HYPE. The language of First order HYPE is that of Classical Predicate logic: a countably infinite set of individual variables $x, x', x'''$... and predicates of finite arity $P, P'$..., alongside the logical symbols $\forall, \exists, \neg, \wedge, \vee, \supset, \top$. But HYPE incorporates special incompatibility $\perp$ and fusion operators $\circ$ in the satisfaction clauses for negation and the conditional, somewhat like Veltman (1985: pp. 202–7) and truthmaker semantics.

A First-order HYPE model is a structure $\mathfrak{M} = \langle S, D, V, \circ, \perp \rangle$, such that:[13]

- $S \neq \emptyset$ is a set of states or situations. $S = S_c \uplus S_n$, where $S_c$, $S_n$ are the set of classical and nonclassical states, respectively, where a nonclassical state is a glutty or gappy state.
- $D \neq \emptyset$ is the domain of individuals. We can define the local domain of a world if necessary (Leitgeb, 2018).
- $V : S \mapsto \mathscr{P}(SoA)$ is a valuation function from $S$ to the power set of the set of states of affairs, where $SoA$ is defined as follows:

  **Definition 3** The set of states of affairs, *SoA*, relative to a given domain $D$ and vocabulary, is the set of all tuples $P(d_1, ..., d_n)$ and $\overline{P}(d_1, ..., d_n)$, where $P$ is an $n$-ary predicate $(n \geq 0)$ such that $\overline{P}$ is its negation, $P = \overline{\overline{P}}$ and each $d_i \in D\,(1 \leq i \leq n)$.

- $\circ$ and $\perp$ are the fusion and incompatibility operators, respectively, such that:
  1. $\circ : S \times S \rightarrow S$ is a partial commutative, associative binary function (called *fusion*), such that:
     - Either $s \circ s'$ is undefined, or $s \circ s'$ is defined (and hence in $S$) in which case it is required that $V(s \circ s') \supseteq V(s) \cup V(s')$.

perspective relativity.

[12] We will use the words 'states' and 'situations' interchangeably.

[13] HYPE also incorporates a special star operator. See (Leitgeb, 2018) for more details.

- $s \circ s$ is defined, and $s \circ s = s$.

2. $\perp$ is a binary symmetric relation on $S$ (the incompatibility relation), such that:
   - If there is a $v$ with $v \in V(s)$ and $v \in V(s')$, then $s \perp s'$.
   - If $s \perp s'$ and both $s \circ s''$ and $s' \circ s'''$ are defined, then $s \circ s'' \perp s' \circ s'''$.

$\circ$ gives rise to a partial order $\leq$, such that, for all $s, s' \in S$, $s \leq s'$ iff $s \circ s'$ is defined and $s \circ s' = s'$. Importantly, truth is monotonic under fusion extension: for all $s$, if $s \models A$ and $s \circ s'$ is defined, then $s \circ s' \models A$.

Variable assignments $\rho$ and their modified variants $\rho(d/x)$ behave as in Classical Predicate logic. Satisfaction of a formula $\phi$ is defined relative to a state and a variable assignment (written: $s, \rho \models \phi$), and the clauses for the logical symbols are as usual, except for $\neg$ and $\supset$, which have a distinctly modal flavour: $s, \rho \models \neg A$ iff for all $s'$ : if $s' \models A$ then $s \perp s'$ and $s, \rho \models A \supset B$ iff for all $s'$ : if $s' \models A$ and $s \circ s'$ is defined, then $s \circ s' \models B$.

### 3.2. Introducing P-HYPE: a combinination of HYPE and AG's perspective-sensitive semantics

In P-HYPE, we require the usual hierarchy of typed domains familiar from (Gallin, 1975), whose elements correspond to different kinds of entities. To this end, let $TYPE$ be the smallest set such that:

1. $e, t, P \in TYPE$

2. If $\alpha, \beta \in TYPE$, then $\alpha \to \beta \in TYPE$

Let a frame based on $D$ and $\mathscr{P}(S)$ be a set $\mathscr{D} = \{D_\alpha \mid \alpha \in TYPE\}$ such that $D_e = D, D_t = \mathscr{P}(S)$, $D_P = P$ and $D_{\alpha\to\beta} \subseteq \{f \mid f : D_\alpha \to D_\beta\}$ for each type $\alpha \to \beta$. The basic idea behind P-HYPE is that we add a set $P$ of perspective indices, relative to which perspective sensitive expressions are interpreted. These are in the image of a function, $\kappa : D_e \times S \mapsto P$, much like AG's $\kappa$ function, which assigns perspective indices to agents at states. We then provide a function $\pi : P \times S \to \mathscr{P}(S)$ which maps perspective indexes and states to a set of states which we call the *perspective set (p-set) of an agent* at that state (this is the *perspective* or *mental model* of the agent, in the sense discussed above). Following AG, we can then, if we desire, enforce the distinctness requirement amongst perspectives they impose by requiring that the perspective set of two distinct agents is always distinct. This models their intuition that perspectives are entirely private. The semantic values of perspective-sensitive expressions then pick out subsets of the perspective set of agents. In this sense a name and predicate denotation interpreted relative to a perspective is something which only inhabits that perspective.

Let $VAR_\alpha$, and $CON_\alpha$ be countably infinite sets of variables and constants, for each $\alpha \in TYPE$, and let $\{\rho_\alpha \mid \rho_\alpha : VAR_\alpha \to D_\alpha\}$, for all $\alpha \in TYPE$ be the set of assignments. The semantic value $V_\rho(A_\alpha)$, of $A_\alpha$ with respect to assignment $\rho$ in a model $\mathfrak{M}$ is then defined so as to include quantification at all levels of the type-hierarchy and clauses for both function application and lambda abstraction:

- $V_\rho(x : \alpha) = \rho(x : \alpha)$

- $V_\rho(c:\alpha) \in D_\alpha)$
- $V_\sigma(\forall x:\alpha\, A) = \bigcap_{d\in D_\alpha} V_{\sigma(d/x)}(A)$
- $V_\sigma(\exists x:\alpha\, A) = \bigcup_{d\in D_\alpha} V_{\sigma(d/x)}(A)$
- $V_\sigma(A_{\alpha\to\beta}B_\alpha) = V_\sigma(A_{\alpha\to\beta})V_\sigma(B_\alpha)$
- $V_\sigma(\lambda x_\alpha A_\beta) =$ the function $f$ on $D_\alpha$ whose value at $d \in D_\alpha$ is equal to $V_{\sigma'}A_\beta$, where $\sigma' = \sigma(d/x)$
- $V_\sigma(A_\alpha \equiv B_\alpha) = \{s \mid V_\sigma(A_\alpha) = V_\sigma(B_\alpha)\}$

A *P-HYPE* model is a structure $\mathfrak{M}_1 = \langle S, \mathfrak{D}, \kappa, P, E, V, \pi, \circ, \perp\rangle$, such that:

(i) $\mathfrak{D} = \bigcup_{\alpha\in\mathscr{D}} D_\alpha$

(ii) $\langle S, \mathfrak{D}, V, \circ, \perp\rangle$ is a HYPE model.

(iii) $\kappa : D_e \times S \mapsto P$, is a function that associates a unique perspective index $\kappa(d,s)$ to each individual $d$ in a state $s$.

(iv) $P$ is a set of perspective indices generally of the form $\kappa(d,s)$, for $d \in D_e$, $s \in S$.

(v) $E \in P$ is a distinguished isolated perspective index such that $\neg(\exists x \in D_e, s \in S.(\kappa(x,s) = E))$

(vi) $\pi : P \times S \mapsto \mathscr{P}(S)$ maps every perspective index $\kappa(d,s) \in P$ and state $s \in S$ to a set of states $\pi(\kappa(d,s),s) \subseteq S$, the *perspective set* or *p*-set of $d$ at $s$. When $d_i$ is enlightened then $\pi(\kappa(d,s),s) = \{s\}$. Moreover, to ensure monotonicity of truth under $\leq$, if $s \leq s'$ then $\pi(\kappa(d_i,s),s) \leq \pi(\kappa(d_i,s),s')$, where $S \leq S'$ is defined by the condition $\forall s' \in S'.\exists s \in S.\ s \leq s'$.[14]

(vii) For any $d_i \in D_e$ at state $s$, we can define a HYPE-model $\langle \pi(\kappa(d_i,s),s), D, V, \circ, \perp\rangle$, whose set of states is the *p*-set of $d_i$ at $s$.

(viii) For all $d_1, d_2 \in D_e$ for which $d_1 \neq d_2$, and all $s \in S$, $\pi(\kappa(d_1,s),s) \cap \pi(\kappa(d_2,s),s) = \emptyset$. This condition ensures that perspectives and the denotations of expressions relative to perspectives, are entirely subjective and do not overlap with one another.

Let us consider some aspects of P-HYPE models. $\pi$ allows us to collect together states that contain the private denotations of words for a given agent. These states may in turn be fusions of other states, or they may be atomic. There are various ways we might think of the fusion of states in a perspective. One is to construe them as collections of states such that the propositions true at these states are about some subject matter. Another is to think of them as collections of situations which store information about certain discourse referents, and to think of $\circ$ as a way of transitioning to states with different discourse referents. However, other than the property of monotonicity, which is preserved under fusion extension in the logic HYPE, we leave open for further research what exact role fusion might have in the perspective of an agent. Thus, whilst we left open what the perspective of an agent is, we have come up with a proposal which we will explore further elsewhere. It might even be the case that via P-HYPE we can simulate different theories of hyperintensionality.

[14]Truth is monotonic under with respect to $\leq$ in HYPE.

| WORD | DENOTATION | ABBREVIATION |
|---|---|---|
| Spiderman | $\lambda i. \begin{cases} sm(i) & \text{if } \exists s \in S.\ i = \kappa(h,s) \\ pp(i) & \text{else } \exists s \in S.\ i = E \end{cases} : \Diamond e$ | *spiderman* |
| Inductive | $\lambda x, \lambda j. \begin{cases} I(j)(x(j)) & \text{if } \exists s \in S.\ j = \kappa(h,s) \\ finite(j)(x(j)) & \text{else } \exists s : S.\ j = E \end{cases} : \Diamond e \to \Diamond t$ | *inductive* |
| Love | $\lambda y,x,i..\{\, s \mid \forall s'[s' \in \pi(i,s) \supset s' \in Love(x, y(\kappa(x,s')))]\} : \Diamond e \to e \to \Diamond t$ | *love* |
| Believe | $\lambda p,x,i.\{\, s \in S \mid \forall s'[s \leq s' \wedge s' \in \pi(i,s)] \supset \forall s''$ | |
| | $[DOX(x, s \circ s', s'') \supset s'' \in p\ (\kappa(x,s''))]\,\} : \Diamond t \to e \to \Diamond t$ | $\lambda p,x,i.bel(i,x,p(\kappa(x,s''))$ |
| Prove | $\lambda p,x,i.\{\, s \in S \mid \forall s'[s \leq s' \wedge s' \in \pi(i,s)] \supset \forall s''$ | |
| | $[PROV(x, s \circ s', s'') \supset s'' \in p\ (\kappa(x,s''))]\,\} : \Diamond t \to e \to \Diamond t$ | $\lambda p,x,i.prove(i,x,p(\kappa(x,s''))$ |
| the primes | $\lambda i.\iota x : prime.number(x(i)) : \Diamond e$ | *the.primes* |

**Table 2** Simplified lexical entries

## 3.3. Lexical entries and examples

There are four comments to make about these lexical entries in **Table 2**. Firstly, $h$ denotes 'Harold' (who features in our examples), '$E$' denotes the enlightened perspective and '$u$' denotes the perspective index of the utterer of a sentence. The enlightened perspective index is the perspective index which, if supplied to an expression whose denotation takes a perspective index as an argument, returns the intension of that expression. Secondly, many of these lexical entries are simplified. For example, we are assuming (for expository simplicity) that *prove* is a guarded universal quantifier over worlds–though we haven't specified what sort of universal quantifier it is–and that *Prove* is factive, and so presupposes the truth of its complement.

Thirdly, a crucial aspect of the lexical entries for verbs, is that we are able to formalise the intuition of AG that such complements are always interpreted relative to a perspective which the utterer thinks is the perspective of another person. Consider the denotation of *believe*, which combines with a proposition of type $\Diamond t$ (i.e, a function from perspective indices to states to truth values), an individual and a perspective index. We assume that, in the case of propositional attitude verbs, this perspective index must always be the utterer's perspective index. Where $u$ is the utterer's perspective index at state $s$, *Believe* then universally quantifies over both (i) all the states $s' \geq s$, such that $s' \in \pi(u,s)$, where $s$ is the world in which the sentence is being evaluated and (ii) all the states $s''$ which are doxastically accessible from $s \circ s'$. *x believes p* is then true iff $p$ is true in $s''$ relative to the perspective index associated with $x$ at $s''$.

3.4. 'Prove' and the complements of attitude verbs

Consider (14) and (15):

(14) Harold proves that the primes are not inductive.

(15) Harold proves that the primes are infinite.

Using the lexical entries above, we can derive (16) for a sentence like (14) and (17) for a sentence like (15):

(16) $\{ s \in S \mid \forall s' [s \leq s' \wedge s' \in \pi(u,s)] \supset \forall s''[PROV(h, s \circ s', s'') \supset s'' \in \neg I(\kappa(h,s''))(\iota x.prime.number(x(\kappa(h$

(17) $\{ s \in S \mid \forall s' [s \leq s' \wedge s' \in \pi(u,s)] \supset \forall s''[PROV(h, s \circ s', s'') \supset s'' \in \neg finite(\kappa(h,s''))(\iota x.prime.number(x$

Crucially, (16) and (17) will differ in truth value, if Harold associates distinct denotations with *inductive* and *finite*.

We can also derive the following readings for (14):

(18) $\{ s \in S \mid \forall s' [s \leq s' \wedge s' \in \pi(E,s)] \supset \forall s''[PROV(h, s \circ s', s'') \supset s'' \in \neg I(E)(\iota x.prime.number(x(E)))] \}$

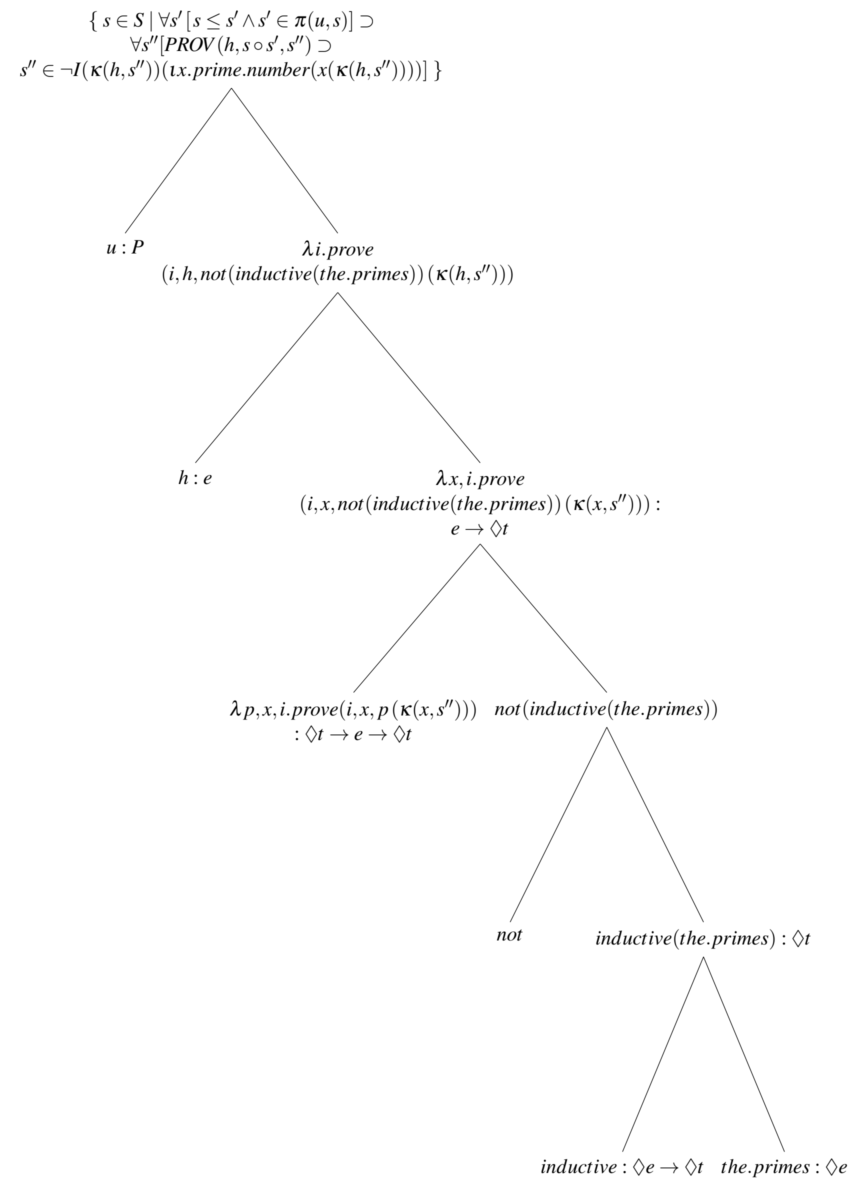
(19) $\{ s \in S \mid \forall s' [s \leq s' \wedge s' \in \pi(u,s)] \supset \forall s''[PROV(h, s \circ s', s'') \supset s'' \in \neg I(u)(\iota x.prime.number(x(u)))] \}$

(20) $\{ s \in S \mid \forall s' [s \leq s' \wedge s' \in \pi(u,s)] \supset \forall s''[PROV(h, s \circ s', s'') \supset s'' \in \neg I(u)(\iota x.prime.number(x(\kappa(h,s''))))$

(21) $\{ s \in S \mid \forall s' [s \leq s' \wedge s' \in \pi(u,s)] \supset \forall s''[PROV(h, s \circ s', s'') \supset s'' \in \neg I(\kappa(h,s''))(\iota x.prime.number(x(u)))$

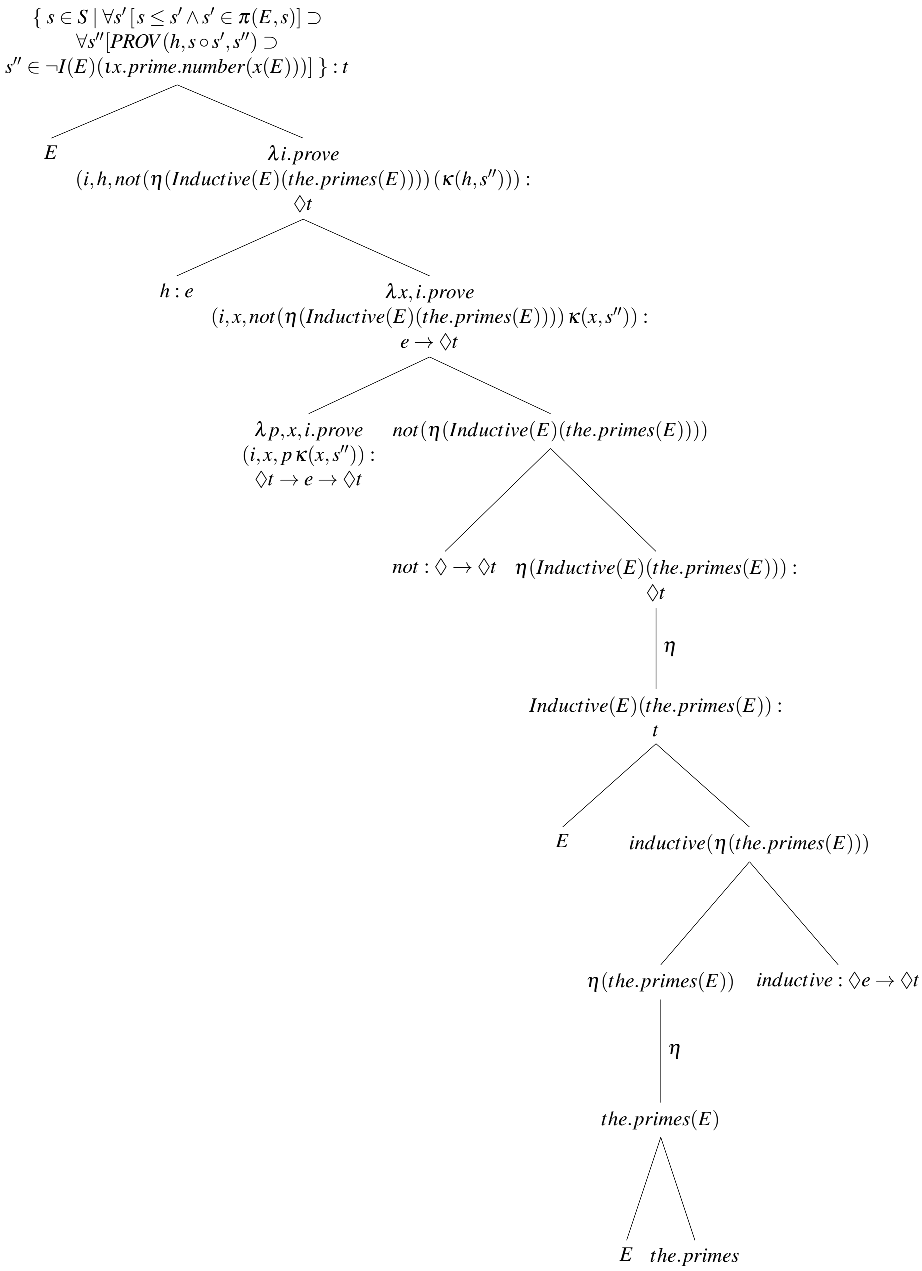
(16) is derived via the tree in (22) below (we make free use of the abbreviations in **table 2**). In (22),'the primes' and 'inductive' remain *in situ* and we feed in the utterer's perspective index $u : P$ at the top of the tree (from now on we call this final perspective index at the top of the tree, the *top-level* perspective index). Consequently, these predicates are caught by the $\kappa$ operator, and so we get the interpretation where the these expressions are interpreted in the way that the utterer thinks that Harold thinks they are interpreted:

(22)



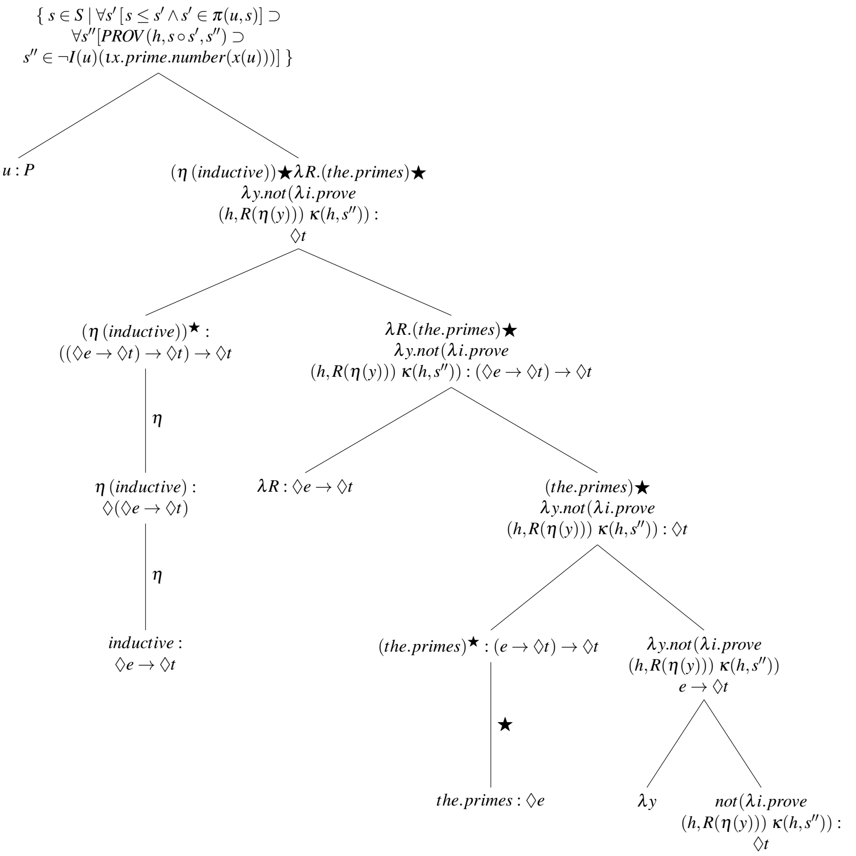

(18) is derived via the tree in (23) below. On this interpretation, (14) receives its usual intension, since we when supply the enlightened perspective index to an expression that expression has its ordinary intension as its semantic value. For this derivation we feed the enlightened perspective index to 'the primes' and 'inductive', using $\eta$ to ensure that these functions can combine, and to ensure that 'the primes are inductive' is fixed to its ordinary intension before combining with 'prove'. Then we feed in the enlightened perspective index as the top-level perspective index.

(23)

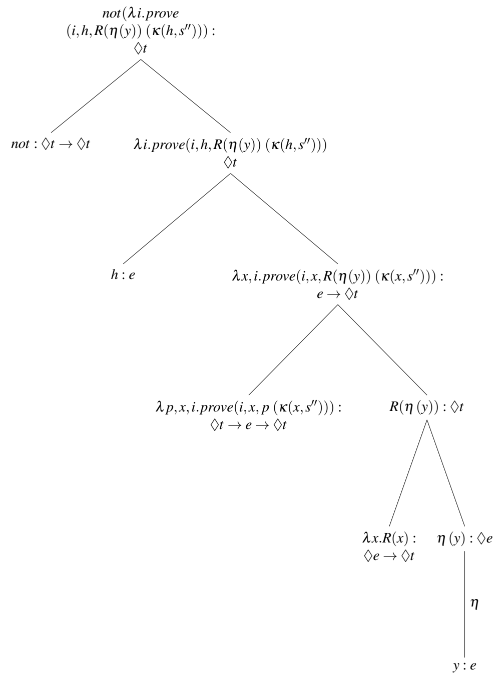



For readabiity, we represent the derivation of (19) by two trees: (24) (for the upper half of the tree) and (25) (for the lower half). (19) is the reading on which both ‘inductive’ and ‘the primes’ are interpreted from the utterer’s perspective. This reading requires we scope both ‘inductive’ and ‘the primes’ above ★, so that they escape $\kappa$:

(24)




(25)




Expressions scoping above ★ are evaluated relative to the utterer's perspective, and expressions scoping below are evaluated according the perspective index associated with the subject, and this allows us to generate 'mixed interpretations' in which some expressions in a sentence are evaluated at the utterer's perspective and others are not. But we don't want some expressions to be evaluated with respect to the enlightened perspective and others not, since it is not clear to us whether such contrasts correspond to valid interpretations of sentences. For this reason, whenever we use ★, the top-level perspective index that we feed to the sentence denotation must be the utterer's perspective index. We only use the enlightened perspective in one case: when all expressions in a sentence are to express their usual intensional values.

In cases where ★ is not used, the top-level perspective index can be either the enlightened perspective index or the utterer.

We use ★ to derive the readings in (19), (20) and (21).[15] But, to derive (19), we could have produced a tree isomorphic to (23) without ★, but in which the utterer's perspective index replaces every instance of the enlightened perspective index. Similarly, (20) and (21) could be derived without ★ with trees in which 'inductive' and 'the primes' remain *in-situ* under the denotation of *prove*, which we omit due to space constraints. So we might think ★ is unnecessary. If, however, we want the priviledged status of the utterer's perspective to be somehow indicated in the trees that we give for various sentences, one option is to require that the utterer's perspective on the interpretation of expressions in a sentence is only available if those expressions scope above ★, and that terms that scope above ★ are always interpreted relative to the utterer's perspective. If we choose this option, then we preserve one feature of the account of Asudeh and Giorgolo (2016), according to which perspectivally-sensitive expressions are somewhat like expressions which take wide-scope when interpreted *de re*. This might be a reason to suggest keeping ★, at least if we think this analogy between perspectivally-sensitive expressions and *de-re*, scoping expressions is significant.

One interesting group of cases which might be thought to pose a challenge to our account involves contrastive focus. Consider an utterance of (26b) in the scenario (26a) ($[\text{ e }]_F$ indicates that the expression 'e' has focus):

(26) a. *Scenario*: Harold and Bill are both enlightened about inductive and finite (i.e, they both know that the prime numbers are finite if, and only, if they are inductive) and it is mutual knowledge that they are both enlightened. Bill utters (26b) to Harold:

b. You proved the prime numbers are $[\text{ not finite}]_F$, not that the prime numbers are $[\text{ not inductive}]_F$.

In the scenario (26a), (26b) is felicitous. However, our semantics predicts that, if the utterer is aware of Harold's being enlightened, (26b) would be infelicitous, given the assumption that a speaker who is enlightened about two co-intensional expressions assigns them the same intension. The felicitous reading of this sentence is arguably (Chris Barker, p.c) a case of metalinguistic focus (Li, 2017), in which Bill is rejecting the use appropriateness of using 'not inductive' as opposed to 'not finite'. Such cases could be captured via the monad for metalinguistic focus (see Li, 2017).[16]

[15] Patrick Elliott (p.c) has asked whether in P-HYPE we need a monad, as opposed to an applicative. In a monad we have $\Diamond\Diamond A \equiv \Diamond A$. In an applicative we have $\eta : \alpha \to \Diamond\alpha$, but we do not have the 'join' $\mu : \Diamond\Diamond\alpha \to \Diamond\alpha$ of the monad, nor ★. We think certain perspective relative phenomena require $\mu$, but don't have the space to discuss these here. The argument of this paragraph would also require ★, if correct.

[16] Perhaps (26b) in (26a) is also felicitous when another perspective, say the perspective of a listener who Harold

3.5. Unembedded mathematical and logical truths

We end by briefly considering how our account might extend to mathematical and logical truths which are not embedded under propositional attitude verbs. There are two sorts of problems we encounter. Firstly, if we allow predicates to be perspective relative across the board, it might seem we predict that sentences like (27) could be true relative to some perspectives, and therefore not plainly false:

(27) There are ten natural numbers.

This objection misunderstands what we mean by saying something is true relative to a perspective. To say this is simply to say that there exists a speaker whose whose mental model verifies the sentence because of the mental representations associated with its parts.[17] This is compatible with (27) being plainly false, and the truth/falsity *simpliciter* of a sentence is determined relative to the enlightened perspective index. We are not denying, unlike MacFarlane (2014), that certain sentences are true *simpliciter*. For us, (27) is strictly speaking false, however it is true (or coherent) relative to some bizarre perspective index, which is unlikely to be associated with all but the strangest of individuals, and unlikely to be salient in any context.[18]

Secondly, consider the seemingly infelicitous (28). (28) receives a coherent reading (30) on the perspective relative semantics (where *u* and *a* represent the perspective of the utterer of the sentence and of the audience), which might usually be expressed by a sentence like (29):

(28) ?The primes are not finite but the primes are finite.

(29) The primes are not finite but the primes are inductive.

(30) $\underbrace{\text{the primes}}_{u}\text{ are }\underbrace{\text{not finite}}_{u}\text{ but }\underbrace{\text{the primes}}_{a}\text{ are }\underbrace{\text{finite}}_{a}$

One response (Chris Barker, p.c) to this problem is that there is in fact a coherent, albeit pragmatically dispreferred/marginal reading of (28), but that in most cases we assess the sentence relative to some perspective on which *not finite* and *finite* are contradictory, perhaps because we usually take predicates in the same sentence to be interpreted from the same perspective. For this reason we would tend to prefer (29) to (28). In fact, Asudeh and Giorgolo (2016), consider a similar case contrasting (31) and (32):

(31) Mary Jane loves Spiderman, but she doesn't love Spiderman.

(32) Mary Jane loves Peter Parker, but she doesn't love Spiderman.

They observe that whilst there is a coherent reading of (31) in their semantics, (32) is preferable on pragmatic grounds to (31), namely, because it is "a clearer way of expressing the relevant

and Bill are aware of, is salient in the context. We can capture this reading by making 'is inductive' and 'is finite' relative to the perspective of an audience member for whom these predicates have a different meaning.

[17]We could even talk of the *coherence* of a statement relative to a perspective, instead of the truth of a statement relative to a perspective, if this is less liable to confuse.

[18]A limitation of this strategy is that we would only distinguish the meaning of one contradiction from another contradiction in a context where a bizzare perspective index is available to be fed to whichever constituent of the contradiction is taken to be perspective sensitive. This contrasts with the intuition that one contradictory sentence might intuitively differ in meaning from another, even to a fully enlightened speaker. We think these cases can be dealt with in similar ways to (26b).

proposition and so is preferred over the version with two instances of the same name." We suspect that if a detailed pragmatic explanation of the contrast between (31) and (32) of this kind can be given, it would also apply to the contrast between (28) and (29).

No doubt this explanation needs developing; in any case, (28) is an example which merely shows the need to have a more detailed theory of when perspective shifting is possible. Apart from the difficult cases we have considered, P-HYPE applies straightforwardly to hyperintensionality outside attitude contexts. For this reason, we hypothesise that our semantic theory constitutes a possible account of unembedded mathematical/logical truths, albeit one which requires development and refinement in various respects.

## 4. Conclusion

P-HYPE provides a possible basis for a compositional hyperintensional phenomena, particularly involving propositional attitude verbs, but also for cases of unembedded mathematical/logical truths. It is able to capture certain aspects of Asudeh and Giorgolo (2016) which are not formalised in their account. Elsewhere, we will develop a more detailed theory of how to constrain and structure perspectives in P-HYPE, in addition to providing an informative comparison between P-HYPE and other hyperintensional semantic theories.

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