Comparatives Revisited: Downward-Entailing Differentials Do Not Threaten Encapsulation Theories

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Abstract

We analyze comparative morphemes (e.g., *-er*, *more*) as intervals of type $\langle dt \rangle$ that serve as differentials in comparatives. We propose that comparatives are about the distance between two intervals on a scale: the differential, which is an interval, is the result of subtracting the interval representing the position of the comparative standard on a scale from the interval representing the position of the comparative subject.

We show that our analysis has at least two advantages. First, it accounts for the semantics of comparatives with downward-entailing or non-monotone differentials in a very natural way, without relying on any strategy that essentially makes quantifiers inside of the *than*-clause take scope over the matrix clause. Second, it opens up new possibilities to give a unified account for various uses of comparative morphemes (e.g., *the more*, comparative correlatives, etc). We mainly focus on the first advantage in this paper.

1 Introduction

A large body of recent literature on comparatives has been focusing on comparatives that contain quantifiers inside the *than*-clause (see [21, 15, 17, 9, 7, 18, 20, 2, 1, 3, 5] among many others). These data raise a crucial question: whether *than*-clause-internal quantifiers take scope over the matrix clause.

As (1) illustrates, there are two ways to analyze the meaning of this sentence: (i) the **endpoint-based analysis** (see (1a)), according to which John's height is compared with the height of the tallest girl, and (ii) the **distribution-based analysis** (see (1b)), according to which John's height is compared with the height of each girl. In terms of truth condition, these two analyses are equivalent here: if John's height exceeds the height of the tallest girl, it follows necessarily that John's height exceeds the height of each girl, and vice versa.

(1) John is taller than <u>every girl</u> is.

a. height(John) > height(the tallest girl) — **The endpoint-based analysis**

b. $\forall x[\text{girl}(x) \rightarrow \text{height}(\text{John}) > \text{height}(x)]$ — **The distribution-based analysis**

Evidently, the endpoint-based analysis does not involve distribution of *than*-clause-internal quantifiers. Consequently, theories that adopt this analysis (called **encapsulation theories** in [6]) do not require *than*-clause-internal quantifiers take scope over the matrix clause. In contrast, the distribution of *than*-clause-internal quantifiers is a necessary ingredient in the distribution-based analysis, and consequently, theories that adopt this analysis (called **entanglement theories** in [6]) necessarily require *than*-clause-internal quantifiers take scope over the matrix clause ([9, 3, 5]) or at least part of the matrix clause ([17]).

*We thank Chris Barker, Anna Szabolcsi and the anonymous reviewers for feedback.

While it is still debatable whether and how *than*-clause-internal quantifiers *can* take scope over the matrix clause in a syntactically plausible way (i.e., how they scope out of a syntactic island),[1] [6] suggests that somehow *than*-clause-internal quantifiers *must* take scope. As (2) and (3) show, [6] argues that only entanglement theories (e.g., [17, 9, 3, 5]), but not encapsulation theories (e.g., [2, 1]), can account for the semantics of the *than*-clause in a unified way, no matter whether there are non-monotone (see (2b) and (3b)) or downward-entailing (DE) differentials (see (2c) and (3c)). Thus, [6] concludes that only entanglement theories, i.e., theories that essentially require *than*-clause-internal quantifiers take scope, are empirically adequate.

(2) Entanglement theories: $[\![\text{than every girl is (tall)}]\!] \approx \forall x[\text{girl}(x) \rightarrow \text{height}(x)...]$
  a. John is taller than every girl is.
  b. John is **exactly 4 inches** taller than every girl is
  c. John is **less than 4 inches** taller than every girl is.

(3) Encapsulation theories: how to interpret $[\![\text{than every girl is (tall)}]\!]$ in a unified way?
  a. John is taller than every girl is. — MAX reading
     height(John) > height(the tallest girl)
  b. John is **exactly 4 inches** taller than every girl is. — MAX=MIN reading
     height(John) > height(the tallest/shortest girl) $\rightsquigarrow$ Girls are of the same height.
  c. John is **less than 4 inches** taller than every girl is. — MAX-&-MIN reading
     height(the shortest girl) $+4'' >$ height(John) > height(the tallest girl)

In this paper, we show that DE or non-monotone differentials do not necessarily threaten encapsulation theories, and thus *than*-clause-internal quantifiers do not *have to* take scope.

Following [17, 14], we cast our endpoint-based analysis of comparatives not in terms of degrees, but in terms of intervals (i.e., convex sets of degrees), with the differential and the comparative standard analyzed as two intervals. In a nutshell, we claim that:

(4) a. In comparatives, *more/-er* refer to intervals that play the role of differentials.
  b. The differential (i.e., result of interval subtraction) is the distance between intervals.

Based on these new claims, we provide a simple and unified mechanism showing how to compositionally derive the truth conditions in (3) and explaining why the interpretation of *than*-clauses seems to vary with differentials and give rise to MAX/MAX=MIN/MAX-&-MIN readings.

§2 presents empirical motivation for our claims. §3 introduces the definition of interval subtraction. §4 shows how basic data of comparatives are analyzed with our proposal, and based on this, §5 shows how various kinds of differentials contribute to the computation of the semantics of comparatives. §6 compares the current analysis with [17]. §7 further shows that the current analysis of comparative morphemes opens up new possibilities to give a unified account for their various uses. §8 concludes this paper.

2 Empirical Motivation: New Observations

2.1 Comparatives Express the Distance Between Two Positions

Comparatives are a most interesting type of degree constructions. **Degrees**, which are points, are elements of **scales** (i.e., totally ordered sets); convex subsets of scales are often called **intervals**. Here we show that in analyzing comparatives, it is useful to (i) distinguish **interval**

[1] See, e.g., [15, 17], for arguments against the view that usual quantifier raising strategies can work.

scales from **ratio scales**, and (ii) consider the essential meaning of comparatives as a relation among **two intervals on an interval scale** and **one interval on a ratio scale**.

Interval scales and ratio scales are subtly different in whether they contain a meaningful, non-arbitrary and unique zero point: interval scales do not necessarily contain one, while ratio scales necessarily contain one. The distinction between **interval scales** and **ratio scales** as well as the use of both of them in comparatives are clearly shown in (5) and (6). Evidently, the scales of time and ranking are interval scales (e.g., *Rank 0* makes no sense; *8 o'clock* does not mean twice of *4 o'clock*). In contrast, the relevant scales that measure differentials are ratio scales: they have a meaningful, non-arbitrary and unique zero point – zero means no difference.

(5) We arrived 2 hours earlier than the check-in time.

a. On an interval scale: (i) our arrival time; (ii) the check-in time.

b. On a ratio scale: the differential *2 hours*.

(6) FSU ranked 3 spots higher than UNC.

a. On an interval scale: (i) the position of FSU; (ii) the position of UNC.

b. On a ratio scale: the differential *3 spots*.

This distinction between interval scales and ratio scales explains why comparatives cannot express the absolute position of the comparative subject or comparative standard on a scale: the absolute position depends on the choice of the origin (i.e., zero point), and this choice can be arbitrary on an interval scale. Instead, comparatives express the absolute distance between the positions standing for the comparative subject and the comparative standard: once these two positions on an interval scale are settled, the distance between them remains constant, no matter how the zero point is chosen and how the absolute positions are defined accordingly.

Thus, we consider the essential meaning of comparatives as a relation among three things: **two positions on an interval scale** (i.e., the one representing the comparative standard, e.g., the check-in time in (5), and the one representing the comparative subject, e.g., our arrival time in (5)) and **the distance between them**.[2] Based on this, we follow [17, 14] and use intervals (i.e., convex sets of points), instead of degrees (i.e., points), to represent positions. An interval represents a value as **a range of possibilities**, and thus intuitively, they can be seen as larger and more generalized markers of positions on a scale. Intervals not only mark positions, but also have size (consider error bars) and carry endpoint information (e.g., boundedness, closeness).[3]

(7) Interval notation: Type of **degree**: d; type of **interval**: $\langle dt \rangle$

An interval $\lambda\delta_d.\{\delta|D_{\min} \leq \delta \leq D_{\max}\}$ can be written as $[D_{\min}, D_{\max}]$.[4]

[2] One of the anonymous reviewers pointed out that our view is inconsistent with traditional assumptions in studying comparatives (e.g., [4, 21, 8, 18]). According to traditional assumptions, comparatives express relations between (positive) thresholds: thresholds can be ordered in such a way that (i) if something meets or exceeds one of them, it meets or exceeds all **lower thresholds**, and (ii) if the highest threshold A meets exceeds the highest threshold B meets, A meets or exceeds **more thresholds** than B does. Crucially, this threshold-based view implicitly assumes that the highest thresholds A and B meet respectively are not infinitely far away (compared to the distance between them) from a certain reference point, i.e., the highest thresholds A and B meet cannot be situated at a $+\infty$ position. Otherwise, even though the highest threshold A meets exceeds the highest threshold B meets, A does not exceed *more* thresholds than B does (e.g., $+\infty + 5$ is not larger than $+\infty$). This underlying assumption is certainly not guaranteed in the worst cases: in (5), when the temporal scale extends to an infinite future, even though our arrival time exceeds the check-in time in being early, they are equally far away from the zero point, which is in infinite future. Since the scale on which the comparative subject and the comparative standard are situated can have an arbitrary zero point, considering lower thresholds in analyzing comparatives can potentially ruin the analysis and thus makes no sense. After all, only the distance between the positions standing for the comparative subject and the comparative standard matters.

[3] See [17] for additional arguments for using intervals, instead of degrees, as position markers on a scale.

[4] When $D_{\min} = D_{\max}$, it is a singleton set, i.e., it contains a single point.

2.2 Comparative Morphemes Represent Differentials

The semantic contribution of comparative morphemes is a fundamental issue in studying comparatives. Here we show that comparative morphemes play the role of differentials.

The crucial empirical motivation is shown in (8) and (9). The most natural interpretation for the use of *more* in (8) is not that the amount he then drank is (a bit) larger than the amount he had drunk previously, but just an amount (a bit) over zero. In other words, *more* is related to the part that is added onto some *augend* (i.e., thing to be increased). Similarly, in (9), *more* signals a second event (i.e., a bringing-chaos event) being added onto the event already existing in the context (i.e., the bringing-depression event), i.e., *more* corresponds to the differential part between the first event e_1 and the sum of the two events $[e_1 + e_2]$. Thus *more* is reminiscent of additive words (e.g., *other, also, too*) in (i) expressing an additive meaning and (ii) being anaphoric: it is felicitous only when there is already an augend in the context.

(8) He drank till he blacked out. Then he drank (a bit) **more**.

(9) War brings depression; **what's more**, it brings chaos.

Then how to account for the use of *more* in (8) and (9) and its use in comparatives in a unified way? If we start from comparatives and analyze the fundamental contribution of *more* as relating two degree (or interval) expressions (i.e., *more* is of type $\langle d, \langle dt \rangle\rangle$ or $\langle dt, \langle dt, t\rangle\rangle$), the use of *more* in (8) and (9) remains a puzzle.

However, if we start from (8) and (9) and analyze *more* as an addend (or differential), i.e., the difference between a sum and an augend, this analysis can be immediately extended to cover comparative data. In comparatives, the augend, the addend (or differential) and the sum are all in the same sentence: (i) the comparative standard plays the role of augend, (ii) the comparative subject the role of sum, and (iii) comparative morphemes the role of differential.

In §2.1, we have proposed to use intervals of type $\langle dt \rangle$ to represent positions on a scale. When an interval (e.g., the interval marking the position of the comparative subject) minus another interval (e.g., the interval marking the position of the standard), the result, i.e., the distance between two positions, is also an interval. Thus, if *more/-er* are analyzed as differentials, then in comparatives, they should be intervals of type $\langle dt \rangle$. We propose that they are intervals in the domain $\lambda D.[D \subseteq (0, +\infty)]$. When a comparative sentence contains a more specific differential, e.g., *2 hours* in (5) and *a bit* in (8), this specific differential further restricts the value of *more/-er*. (10) shows how intervals of type $\langle dt \rangle$ can be compared to individuals of type e:

(10)	x_e:	*someone*	*other*	*the other*	*another*	*John*	*John, a linguist*
	$D_{\langle dt \rangle}$:	*some*	*more*	*the more*	*one more*	$[3'', +\infty]$	*3 feet more/-er*

3 Interval Subtraction

In §2, we have motivated our analysis of using three intervals (i.e., two representing positions on a scale and one representing the differential/distance between them) to characterize the semantics of comparatives. Here we introduce the definition of interval operations:

(11) Interval operations: $[x_1, x_2]\langle\text{op}\rangle[y_1, y_2] = [\text{MIN}(x_1\langle\text{op}\rangle y_1, x_1\langle\text{op}\rangle y_2, x_2\langle\text{op}\rangle y_1, x_2\langle\text{op}\rangle y_2), \text{MAX}(x_1\langle\text{op}\rangle y_1, x_1\langle\text{op}\rangle y_2, x_2\langle\text{op}\rangle y_1, x_2\langle\text{op}\rangle y_2)]$ (see [16])

(12) Interval subtraction: $[x_1, x_2] - [y_1, y_2] = [x_1 - y_2, x_2 - y_1]$

(13) a. $[5, 8] - [1, 2] = [3, 7]$
b. $[5, 8] - [3, 7] = [-2, 5]$

Since an interval represents a value as a range of possibilities, as (11) shows, interval operations result in the largest possible range. Thus, we can simply write interval subtraction as shown in (12). (13) shows two examples. Notice that interval subtraction is different from subtraction defined in number arithmetic: when X, Y and Z represent numbers, if $X - Y = Z$, it follows necessarily that $X - Z = Y$; however, when they represent intervals, as (13a) and (13b) illustrate, if $X - Y = Z$, generally speaking, it is not the case that $X - Z = Y$.

A consequence is that in interval arithmetic, given $X - Y = Z$ and given the values of Y and Z, to compute the value of X, we cannot perform interval addition on Y and Z (see (14)).

(14) a. If $X - [a, b] = [c, d]$, then generally speaking, it is not the case that $X = [a+c, b+d]$.
b. If $X - [a, b] = [c, d]$, X is undefined when $b + c > a + d$ (i.e., when the lower bound of X is larger than the upper bound of X); when defined, $X = [b + c, a + d]$.

4 Accounting for Basic Data

4.1 The Semantics of Scalar Adjectives

We follow standard treatments of scalar adjectives (see [4, 21, 8, 13, 10, 12, 11] among others): scalar adjectives relate individuals with abstract representations of measurement on a scale. Since we use intervals of type $\langle dt \rangle$ to represent positions on a scale, scalar adjectives are of type $\langle dt, et \rangle$ in our analysis, as shown in (15). (16) shows the semantics of the positive form. In (16), D_c is is definite. It is shorthand for 'the contextually salient interval such that it is from the lower bound to the upper bound of being tall for a relevant comparison class'. Its semantic contribution is somehow similar to that of, e.g., [11]'s *pos* operator. In (17), *exactly 6 feet* is interpreted as an interval, which is a singleton set of degrees, i.e., $[6', 6']$.

(15) $[\![\text{tall}]\!]_{\langle dt,et \rangle} \stackrel{\text{def}}{=} \lambda D_{\langle dt \rangle}.\lambda x_e.[\text{height}_{\langle e,dt \rangle}(x) \subseteq D]$
i.e., the height of the individual x is in the interval D.

(16) $[\![\text{John is D}_c \text{ tall}]\!] \Leftrightarrow \text{height(John)} \subseteq D_c$
i.e., the height of John is in the contextually salient interval of being tall.

(17) $[\![\text{John is exactly 6 feet tall}]\!] \Leftrightarrow \text{height(John)} \subseteq [6', 6']$
i.e., the height of John is at the position '6 feet' on the height scale.

4.2 The Semantics of Comparatives

As we have proposed in §2.2, (18) shows that comparative morphemes denote an interval. As shown in (19), we propose that $[\![\text{than}]\!]$ takes two interval arguments – D_{standard} (i.e., the interval standing for the comparative standard) and $D_{\text{differential}}$ (i.e., the differential) – and returns the unique interval that is $D_{\text{differential}}$ away from D_{standard}.[5]

(18) $[\![\text{more/-er}]\!]_{\langle dt \rangle} \stackrel{\text{def}}{=} D$ such that $D \subseteq (0, +\infty)$
(Presupposition requirement: there is an augend in the context.)

(19) $[\![\text{than}]\!]_{\langle dt,\langle dt,dt \rangle\rangle} \stackrel{\text{def}}{=} \lambda D_{\text{standard}}.\lambda D_{\text{differential}}.\iota D[D - D_{\text{standard}} = D_{\text{differential}}]$

Evidently, based on the definition of interval subtraction (12), the operation of $[\![\text{than}]\!]$ is well defined if and only if the sum of the lower bound of $D_{\text{differential}}$ and the upper pound of D_{standard} is not larger than the sum of the upper bound of $D_{\text{differential}}$ and the lower bound of D_{standard}.

[5]The semantic operation we propose in (19) might be carried out by a silent item. We stay ignorant on this.

Based on (18) and (19), we show in (20) details of a compositional derivation for the truth condition of a comparative sentence containing a specific differential.

(20) Computing the truth condition of ⟦John is 5 inches taller than Mary is (tall)⟧:

a. ⟦Mary is D (tall)⟧ $\Leftrightarrow$ height(Mary) $\subseteq D$ i.e., Mary is D tall.

b. Following, e.g., [14, 2, 1, 3], we assume that there is a lambda abstraction. We also assume a silent operator ⟦THE⟧ here (defined as $\lambda P_{\langle \alpha t\rangle}.\iota x[P(x)]$) (see [9]), which turns [λD.[height(Mary) $\subseteq D$]] into a contextually unique interval (that allows some vagueness), i.e., the definite interval standing for Mary's height. ⟦THE⟧[λD.[height(Mary) $\subseteq D$]] can be written as $[D_{\text{Lower-Mary}}, D_{\text{Upper-Mary}}]$, i.e., the interval from the lower bound to the upper bound of Mary's height.

c. ⟦5 inches ... -er⟧ $\Leftrightarrow [5'', 5''] \cap (0, +\infty) \Leftrightarrow [5'', 5'']$

d. ⟦5 inches ... -er than Mary is⟧ $\Leftrightarrow$ ⟦than⟧$([D_{\text{Lower-Mary}}, D_{\text{Upper-Mary}}])([5'', 5''])$ $\Leftrightarrow \iota D.[D - [D_{\text{Lower-Mary}}, D_{\text{Upper-Mary}}] = [5'', 5'']]$

e. ⟦John is 5 inches taller than Mary is (tall)⟧ $\Leftrightarrow$ ⟦tall⟧⟦5 inches ... -er than Mary is⟧(John) $\Leftrightarrow$ height(John) $\subseteq \iota D.[D - [D_{\text{Lower-Mary}}, D_{\text{Upper-Mary}}] = [5'', 5'']]$ i.e., on the height scale, John's height is at such a position that it is $[5'', 5'']$ away from the interval $[D_{\text{Lower-Mary}}, D_{\text{Upper-Mary}}]$.

f. After simplification: height(John) $\subseteq [D_{\text{Upper-Mary}} + 5'', D_{\text{Lower-Mary}} + 5'']$. i.e., on the height scale, John's height is at the position represented by the interval $[D_{\text{Upper-Mary}} + 5'', D_{\text{Lower-Mary}} + 5'']$. This interval is defined only when $D_{\text{Upper-Mary}} + 5'' \leq D_{\text{Lower-Mary}} + 5''$, i.e., $D_{\text{Upper-Mary}} = D_{\text{Lower-Mary}}$. In other words, the position that stands for Mary's height has to be a single point, and John's height is a point $5''$ farther away from the point representing Mary's height on the scale.

5 The Interplay between Differentials and the Interpretation of *Than*-Clause-Internal Quantifiers

Here we analyze comparatives containing various kinds of differentials. To begin with, we first show in (21) the interpretation of a comparative standard that contains a universal quantifier.

(21) a. ⟦every girl is D (tall)⟧ $\Leftrightarrow \forall x.[\text{girl}(x) \rightarrow \text{height(x)} \subseteq D]$ i.e., for each girl x, x's height is situated in the interval D on the height scale.

b. After a lambda abstraction and the application of a silent ⟦THE⟧, it becomes ⟦THE⟧[λD.[$\forall x$.[girl(x) $\rightarrow$ height(x) $\subseteq D$]]] i.e., the contextually unique interval in which every girl's height is situated. In the following, we write this as $[D_{\text{Lower-Girls}}, D_{\text{Upper-Girls}}]$, i.e., the interval from the lower bound to the upper bound of girls' height.

(22) shows the derivation of the so-called MAX reading. In fact, when the differential is upward-entailing, the upper bound of $D_{\text{differential}}$ is unbounded, i.e., $+\infty$, and the sum of the lower bound of the comparative standard and $+\infty$ is still $+\infty$, which is necessarily larger than the sum of the lower bound of the differential and the upper bound of the comparative standard. This has two consequences: (i) there is no extra requirement to make the interval representing John's height well defined; (ii) only the upper bound (but not the lower bound) of the comparative standard shows up in the truth condition after simplification (see (22c)).

(22) John is taller than every girl is.

a. $D_{\text{differential}} = [\![\text{-er}]\!] = (0, +\infty)$

b. [[John is taller than every girl is (tall)]]

$\Leftrightarrow$ [[tall]][[... -er than every girl is (tall)]](John)

$\Leftrightarrow \text{height(John)} \subseteq \iota D[D - [D_{\text{Lower-Girls}}, D_{\text{Upper-Girls}}] = (0, +\infty)]$

c. After simplification: $\text{height(John)} \subseteq (D_{\text{Upper-Girls}}, +\infty)$

(23) and (24) show the derivation of the MAX-&-MIN reading for two sentences containing DE differentials. It is evident that the MAX-&-MIN reading is due to the fact that when the overt differential is downward-entailing, $D_{\text{differential}}$ is bounded at both endpoints. Moreover, to make the interval representing John's height well defined here, it has to be the case that $D_{\text{Upper-Girls}} < D_{\text{Lower-Girls}} + 4''$, i.e., the length of the interval containing girls' height is less than 4 inches. Also notice that *less than X* and *at most X* differ in that the upper bound of *less than X* is open while the upper bound of *at most X* is close: the openness of the upper bound of the DE differential also determines whether the interval representing the position of the comparative subject has an open or close upper bound.

(23) John is **less than 4 inches** taller than every girl is.

a. $D_{\text{differential}} = [\![\text{less than 4 inches ... -er}]\!] = (0, +\infty) \cap (-\infty, 4'') = (0, 4'')$

b. [[John is less than 4 inches taller than every girl is (tall)]]

$\Leftrightarrow \text{height(John)} \subseteq \iota D[D - [D_{\text{Lower-Girls}}, D_{\text{Upper-Girls}}] = (0, 4'')]$

c. After simplification: $\text{height(John)} \subseteq (D_{\text{Upper-Girls}}, D_{\text{Lower-Girls}} + 4'')$

(24) John is **at most 4 inches** taller than every girl is.

a. $D_{\text{differential}} = [\![\text{at most 4 inches ... -er}]\!] = (0, +\infty) \cap (-\infty, 4''] = (0, 4'']$

b. [[John is at most 4 inches taller than every girl is (tall)]]

$\Leftrightarrow \text{height(John)} \subseteq \iota D[D - [D_{\text{Lower-Girls}}, D_{\text{Upper-Girls}}] = (0, 4'']]$

c. After simplification: $[\text{height(John)} \subseteq (D_{\text{Upper-Girls}}, D_{\text{Lower-Girls}} + 4'']$

Finally, (25) and (26) illustrate the meaning derivation of comparatives containing non-monotone differentials. To make the interval representing John's height well defined, in (25), it has to be the case that all the girls have the same height (i.e., $D_{\text{Upper-Girls}} = D_{\text{Lower-Girls}}$), and thus the sentence has the so-called MAX-&-MIN reading. Similarly, in (26), the length of the interval containing girls' height cannot be larger than 2 inches.

(25) John is **exactly 2 inches** taller than every girl is.

a. $D_{\text{differential}} = [\![\text{exactly 2 inches ... -er}]\!] = (0, +\infty) \cap [2'', 2''] = [2'', 2'']$.

b. [[John is exactly 2 inches taller than every girl is (tall)]]

$\Leftrightarrow \text{height(John)} \subseteq \iota D[D - [D_{\text{Lower-Girls}}, D_{\text{Upper-Girls}}] = [2'', 2'']]$

c. After simplification: $\text{height(John)} \subseteq [D_{\text{Upper-Girls}} + 2'', D_{\text{Lower-Girls}} + 2'']$

(26) John is **between 2 and 4 inches** taller than every girl is.

a. $D_{\text{differential}} = [\![\text{between 2 and 4 inches ... -er}]\!] = (0, +\infty) \cap [2'', 4''] = [2'', 4'']$.

b. [[John is between 2 and 4 inches taller than every girl is (tall)]]

$\Leftrightarrow \text{height(John)} \subseteq \iota D[D - [D_{\text{Lower-Girls}}, D_{\text{Upper-Girls}}] = [2'', 4'']]$

c. After simplification: $\text{height(John)} \subseteq [D_{\text{Upper-Girls}} + 2'', D_{\text{Lower-Girls}} + 4'']$

In sum, we have shown how to compositionally derive the correct truth condition of comparatives containing various kinds of differentials in an effortless and precise way: no distributive operation is needed, and no *ad hoc* tweak is employed. In our account, since we do not need to have access to each individual's height, it follows naturally that we do not need to make *than*-clause-internal quantifiers take scope over the matrix clause. The whole mechanism only requires that we have access to the lower and upper bounds of the girls' height. We assume a silent $[\![\text{THE}]\!]$ to achieve this in our account, i.e., we interpret the part following *than* as a definite interval (see also [9]). Other existing encapsulation theories (e.g., [2, 1]) have proposed their own mechanisms to derive the semantics of the endpoints of the comparative standard. A detailed comparison among these mechanisms is left for future work.

5.1 Extension: Accounting for *Fewer Than*

Here we extend our account to comparative data using *fewer/less than*. We propose that the semantics of *less/fewer* includes two parts: (i) the comparative morpheme $[\![\text{-er}]\!]$ and (ii) an operator that changes the direction of comparison (see (27)). How to connect the analysis in (27) with other syntactic/semantic behaviors of *few/less* is left for future research.

(27) $[\![\text{few- than}]\!]_{\langle dt,\langle dt,dt\rangle\rangle} \overset{\text{def}}{=} \lambda D_{\text{standard}}.\lambda D_{\text{differential}}.\iota D[D_{\text{standard}} - D = D_{\text{differential}}]$

(28) If $[a, b] - X = [c, d]$, X is undefined when $b + c > a + d$; when defined, $X = [b - d, a - c]$.

(29) John is **more than 4 inches** less tall than every girl is.

a. $D_{\text{differential}} = [\![\text{more than 4 inches ... -er}]\!] = (0, +\infty) \cap (4'', +\infty) = (4'', +\infty)$.

b. $[\![\text{John is more than 4 inches less tall than every girl is (tall)}]\!]$
$\Leftrightarrow \text{height(John)} \subseteq \iota D[[D_{\text{Lower-Girls}}, D_{\text{Upper-Girls}}] - D = (4'', +\infty)]$

c. After simplification: $\text{height(John)} \subseteq (-\infty, D_{\text{Lower-Girls}} - 4'')$

(30) John is **at most 4 inches** less tall than every girl is.

a. $D_{\text{differential}} = [\![\text{at most 4 inches ... -er}]\!] = (0, +\infty) \cap (-\infty, 4''] = (0, 4'']$.

b. $[\![\text{John is at most 4 inches taller than every girl is (tall)}]\!]$
$\Leftrightarrow \text{height(John)} \subseteq \iota D[[D_{\text{Lower-Girls}}, D_{\text{Upper-Girls}}] - D = (0, 4'']]$

c. After simplification: $\text{height(John)} \subseteq [D_{\text{Upper-Girls}} - 4'', D_{\text{Lower-Girls}})$

6 Comparison with [17]

[17] also uses intervals, instead of degrees, to implement the semantics of comparatives. A crucial difference between our analysis and [17]'s consists in the definition of interval subtraction, and along with it, the definition of differential. Our account follows the standard definition developed in interval arithmetic (see [16]). (31) shows [17]'s definition of subtraction. (32) illustrates how the definitions (31) and (12) differ: the contrast between (32a) and (32b) clearly shows that the analysis of [17] is problematic.

(31) Assuming I is above K, we want $[I - K]$ to pick out the part of the scale that is below I and above K. The differential is considered as the length of $[I - K]$.
For intervals I, K:
If $K \prec I$, then: $\forall J : (J \prec I \;\&\; K \prec J) \leftrightarrow J \sqsubseteq [I - K])$
Otherwise: $[I - K] = 0$ (56) in [17]

(32) Suppose the height of each boy is somewhere between $5'8''$ and $5'11''$, and suppose the height of each girl is somewhere between $5'3''$ and $5'7''$.

a. According to (12), the result of $[5'8'', 5'11''] - [5'3'', 5'7'']$ is $[1'', 8'']$, and in our account this result is the differential. To describe the situation, we would say:
Every boy is <u>between 1″ to 8″</u> taller than every girl. **True** in the scenario

b. According to (31), the result of $[5'8'', 5'11''] - [5'3'', 5'7'']$ is $(5'7'', 5'8'')$, and in [17], the differential in comparatives is understood as the length of this subtraction result: in this case, it is less than $1''$. To describe the situation, we would say: *Every boy is <u>less than 1″</u> taller than every girl.* **False** in the scenario

7 Discussion: other uses of *more*

As we have shown in §2.2, *more* essentially refers to a differential. We have also suggested in (10) that *more* should behave quite similarly to indefinites in many cases. The crucial difference between *more* and usual indefinite expressions is that *more* brings a presuppositional requirement: there has to be an augend in the context. Our analysis of *more* in comparatives opens up new possibilities to relate various data of *more*, comparatives and superlatives.

The more. [19] questions how *more* is related to *the more*, and points out that while *more* can take a *than*-clause, *the more* cannot. Under our analysis, the meaning of *taller than Bill is* (in (33a)) is totally parallel to the meaning of D_c *tall* (in (34a)). Thus, it is unsurprising that if *the* cannot compose with *tall* to form a grammatical construction (in (34b)), *the* cannot compose with *taller than Bill is* to form a grammatical construction either (in (33b)). Then it should be due to the same reason that (35b) is ungrammatical.

Now when we look back at our lexical entry for *than* in (19), evidently, the result of performing $[\![\text{than}]\!]$ on D_{standard} and $D_{\text{differential}}$ is already a definite interval. Thus, our analysis explains why *than*-clause is no longer compatible with an overt *the*.

(33) a. John is taller than Bill is. $[\![(33a)]\!] \Leftrightarrow [\![\text{tall}]\!][\![\text{ ... -er than Bill is}]\!](\text{John})$
b. *John is the taller than Bill is.

(34) a. John is D_c tall. $[\![(34a)]\!] \Leftrightarrow [\![\text{tall}]\!][\![D_c]\!](\text{John})$
b. *John is the tall.

(35) a. John earns more money than Bill does.
b. *John earns the more money than Bill does.

Comparative correlatives. Interestingly, *the more* seems to be a cross-linguistically very prevailing pattern in expressing correlations. By analyzing *more/-er* as differential, a unified account for comparatives and comparative correlatives should be readily available.

One intriguing question here is in comparative correlatives, as illustrated in (36), whether the correlation is between two sums, or just between two differentials. Given the previous discussion, it seems that in comparative correlatives, if the correlation is established between two sums, there cannot be an overt *the* in the part *the more*. Thus, *the more* should refer to only the differential part (consider also *the other* (see (10))), and the correlation should be established between two differentials. A further question is whether there is a binding relation between the two uses of *the more* in comparative correlatives. This is left for future research.

(36) The more I read, the more I understand.

8 Conclusion

In this paper, we provide a new implementation of the endpoint-based analysis to account for comparative data. Our new implementation is based on two claims: (i) comparatives express the relation among three intervals, among which one represents the differential between the other two, and (ii) comparative morphemes should be analyzed as differentials. Technically, our implementation is based on interval arithmetic. With this new implementation, we account for comparative data containing various kinds of differentials in an easy and unified way. More particularly, for comparatives containing *than*-clause-internal quantifiers, no scope taking is needed in our account. Hopefully, our analysis will shed light on more issues concerning comparatives, *more* and uses of intervals in natural language.

References

[1] Peter Alrenga and Christopher Kennedy. *No more* shall we part: Quantifiers in English comparatives. *Natural Language Semantics*, 22(1):1–53, 2014.

[2] Sigrid Beck. Quantifiers in *than*-clauses. *Semantics and Pragmatics*, 3(1):1–72, 2010.

[3] Sigrid Beck. Plural predication and quantified 'than'-clauses. In *The Art and Craft of Semantics: A Festschrift for Irene Heim*, volume 1, pages 91–115, 2014.

[4] Max Cresswell. The semantics of degree. In Barbara Partee, editor, *Montague grammar*, pages 261–92. New york: Academic Press, 1976.

[5] Jakub Dotlačil and Rick Nouwen. The comparative and degree pluralities, 2015. Manuscript.

[6] Nicholas Fleisher. Comparing theories of quantifiers in *than* clauses: Lessons from downward-entailing differentials. *Semantics and Pragmatics*, 2015 (to appear).

[7] Jon Gajewski. More on quantifiers in comparative clauses. In *Semantics and Linguistic Theory (SALT) 18*, pages 340–357, 2008.

[8] Irene Heim. Notes on comparatives and related matters, 1985. Manuscript, University of Texas, Austin.

[9] Irene Heim. Remarks on comparative clauses as generalized quantifiers, 2006. Manuscript, MIT.

[10] Christopher Kennedy. *Projecting the adjective: The syntax and semantics of gradability and comparison*. Routledge, 1999.

[11] Christopher Kennedy. Vagueness and grammar: The semantics of relative and absolute gradable adjectives. *Linguistics and Philosophy*, 30(1):1–45, 2007.

[12] Christopher Kennedy and Louise McNally. Scale structure, degree modification, and the semantics of gradable predicates. *Language*, 81(2):345–381, 2005.

[13] Ewan Klein. Comparatives. In A. von Stechow and D. Wunderlich, editors, *Semantik: Ein internationales Handbuch der zeitgenössischen Forschung*, pages 673–691, 1991.

[14] Fred Landman. Internal and interval semantics for CP-comparatives. In *Logic, Language and Meaning: Proceedings of the 17th Amsterdam Colloquium*, pages 133–142. Springer, 2010.

[15] Richard Larson. Scope and comparatives. *Linguistics and Philosophy*, 11(1):1–26, 1988.

[16] Ramon E. Moore. *Methods and Applications of Interval Analysis*. SIAM, 1979.

[17] Roger Schwarzchild and Karina Wilkinson. Quantifiers in comparatives: A semantics of degree based on intervals. *Natural Language Semantics*, 10(1):1–41, 2002.

[18] Roger Schwarzschild. The semantics of comparatives and other degree constructions. *Language and Linguistics Compass*, 2(2):308–331, 2008.

[19] Anna Szabolcsi. Compositionality without word boundaries: *(the) more* and *(the) most*. In *Semantics and Linguistic Theory (22)*, pages 1–25, 2013.

[20] Robert van Rooij. Comparatives and quantifiers. *Empirical issues in Syntax and Semantics*, 7:1–22, 2008.

[21] Arnim von Stechow. Comparing semantic theories of comparison. *Journal of Semantics*, 3(1):1–77, 1984.