Coordination as a tuple: A unified analysis of 'respective', symmetrical and summative predicates

Yusuke Kubota
Ohio State University
kubota.7@osu.edu

Robert Levine
Ohio State University
levine@ling.ohio-state.edu

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Abstract

This paper proposes a unified analysis of the 'respective' readings of plural and conjoined expressions, the internal readings of symmetrical predicates such as *same* and *different*, and the summative readings of expressions such as *a total of $10,000*. These expressions pose significant challenges to compositional semantics, and have been studied extensively in the literature. However, almost all previous studies focus exclusively on one of these phenomena, and the close parallels and interactions that they exhibit have been mostly overlooked to date. We point out two key properties common to these phenomena: (i) they target all types of coordination, including right-node raising and dependent cluster coordination, two representative types of non-canonical coordination; (ii) the three phenomena all exhibit multiple dependency, both by themselves and with respect to each other. These two parallels between the three phenomena suggest that one and the same mechanism is at the core of their semantics. Building on this intuition, we propose a unified analysis of these phenomena, in which the meanings of expressions involving coordination are formally modelled as tuples. The analysis is couched in Hybrid Type-Logical Categorial Grammar. The flexible syntax-semantics interface of this framework enables an analysis that yields a simple and principled solution for both the interactions with non-canonical coordination and the multiple dependency noted above.

Keywords: 'respective' reading, symmetrical predicate, parasitic scope, hypothetical reasoning, coordination, Hybrid Type-Logical Categorial Grammar

1 The meanings of 'respective', symmetrical and summative predicates

The so-called 'respective' readings of plural and conjoined expressions (cf., e.g., Kay 1989; McCawley 1998; Gawron and Kehler 2004; Winter 1995; Bekki 2006; Chaves 2012) and the semantics of symmetrical predicates such as *same* and *different* (cf., e.g., Carlson 1987; Moltmann 1992; Beck 2000; Barker 2007; Brasoveanu 2011) have been known in the literature to poses significant challenges to theories of compositional semantics. Each of these two constructions alone present a set of quite complex problems of its own, and previous authors have thus mostly focused on studying the properties of one or the other. However, as we discuss below, the problems that the two phenomena exhibit are remarkably similar. A less frequently discussed type of sentences but one which raises essentially the same problem for compositionality comes from the interpretation of expressions such as *a total of X* and *X in total*. We call these expressions 'summative' predicates. Summative predicates have been discussed in the literature mostly in the context of Right-node Raising (RNR) (Abbott 1976; Jackendoff 1977). Some representative examples of each construction are given in (1).

(1) a. John and Bill sang and danced, respectively.
(= 'John sang and Bill danced')
b. {The same performer/Different performers} sang and danced.
(≈ 'The performer who sang and the performer who danced are the same/different')
c. John spent, and Bill lost, a total of $10,000 last year.
(= 'The amount that John spent last year and the amount that Bill lost last year add up to $10,000')

The difficulty that these phenomena pose can be illustrated by the following examples involving 'respective' readings:

(2) a. John and Bill bought the book and the CD, respectively. (NP coordination)
b. John and Bill ran and danced, respectively. (VP coordination)
c. John read, and Bill listened to, the book and the CD, respectively. (RNR)
d. John gave the book and the CD to Sue on Wednesday and to Mary on Thursday, respectively. (dependent-cluster coordination (DCC))

These examples exhibit readings that can be paraphrased by the sentences in (3).[1]

(3) a. John bought the book and Bill bought the CD.
b. John ran and Bill danced.
c. John read the book and Bill listened to the CD.
d. John gave the book to Sue on Wednesday and gave the CD to Mary on Thursday.

To assign the right interpretation, we need to have access to the denotations of parts of a phrase (for example, *John* in *John and Bill* and *John read* in *John read, and Bill listened to*, where in the latter case neither the whole nor the part of the coordinate structure is even a

[1]One might be inclined to think that the adjective *respective* in examples like the following should be given a parallel treatment:

(i) John and Bill talked to their respective supervisors.

However, as convincingly argued by Okada (1999) and Gawron and Kehler (2002), the properties of the adjective *respective* is significantly different from those of the *respectively* sentences in (2). In particular, contrasts such as the following suggest that the adjective *respective* takes scope strictly within the NP in which it occurs:

(ii) a. Intel and Microsoft combined their respective assets.
b#Intel and Microsoft combined their assets respectively.

We thus set aside the adjective *respective* in the rest of this paper. See Gawron and Kehler (2002) for an analysis *respective* that captures its strictly local scope correctly.

constituent in the standard sense). But this of course infringes the principle of compositionality, at least in its strictest formulation. Once the meaning of the whole is constructed, the grammar should no longer have direct access to the meanings of its parts.[2]

One might object to such a characterization (see, e.g., Chaves (2012); we address his approach in detail in Sect. 3.3): at least cases like (2a) can be dealt with by an independently needed mechanism for yielding the so-called cumulative readings of plurals (Scha 1981):

(4) 700 Dutch companies have used 10,000 American computers.

In the cumulative reading of (4), a set of 700 Dutch companies is related to a set of 10,000 American computers in the 'x-used-y' relation. The sentence does not specify which particular company used which particular computer, but it only says that the total number of companies involved is 700 and the total number of computers involved is 10,000.

The 'respective' reading in (2a) could then be thought of as a special case of this cumulative reading. Unlike the more general cumulative reading, the 'respective' reading is sensitive to an established order among elements in each of the conjoined or plural terms (that is, (2a) is false in a situation in which John bought the CD and Bill bought the book), but one could maintain that the core compositional mechanism is the same.

However, an attempt to reduce the 'respective' reading to the cumulative reading fails, at least if we adhere to the conventional assumption about the cumulative reading that it is induced by a lexical operator that directly applies to the meanings of verbs. As should be clear from the examples in (2b-d), it is not just co-arguments of a single verb that can enter into the 'respective' relation. Thus, a lexical operator-based approach is not general enough.

But there is a grain of truth in this attempt to relate cumulative and 'respective' readings. The 'violation' of compositionality under discussion exhibited by 'respective', symmetrical and summative readings arises only in connection with coordinated or plural expressions. (Examples involving symmetrical and summative predicates are introduced below.) Thus, instead of claiming that these constructions pose serious challenges to the tenet of compositionality (as some authors indeed have; cf. Kay 1989), it seems more plausible to find ways to relax compositionality just in the context of coordination and plural expressions, in such a way that these apparent violations of compositionality (but nothing more) is allowed.

Building on related ideas explored by previous authors, in particular, Gawron and Kehler (2004) and Barker (2007), we argue precisely for such an approach in this paper. In fact, in the case of conjoined NPs, which are standardly taken to denote sums (at least since Link (1983)), the issue of compositionality is already moot since the denotation itself ($\mathbf{j} \oplus \mathbf{b}$) retains the internal structure of the conjunction that can be accessed by other operators such as the distributivity operator commonly assumed in the semantics literature (compare this situation to the generalized conjunction of the lifted versions of the individual terms $\lambda P.P(\mathbf{j}) \wedge P(\mathbf{b})$, for which the individual parts are no longer directly accessible). As shown

[2] The notion of compositionality and its relevance for linguistic theory is a controversial issue. See Dowty (2007) for a good overview. There are of course various approaches involving underspecified semantics to relax this strictest notion of compositionality. See, for example, Egg et al. 1998, Richter and Sailer 2004 and Copestake et al. 2005. But such approaches are mostly only good for handling scope ambiguity of quantifiers, and do not seem to have any obvious advantage when applied to the class of phenomena under consideration. Iordăchioaia and Richter (to appear) speculate on a possibility of extending their underspecified framework to the analysis of symmetrical predicates, but since there currently does not exist a fully fleshed out proposal, we refrain from comparing this approach to ours in this paper.

in detail for ‘respective’ readings by Gawron and Kehler (2004), the complex semantics that these expressions exhibit can be uniformly handled by modelling the meanings of expressions involving plural expressions or conjunction by a structured object—either (generalized) sums (Gawron and Kehler 2002) or tuples (Winter 1995; Bekki 2006). The present paper extends this approach to the other two empirical phenomena (symmetrical and summative predicates), as well as embedding the analysis in a flexible syntax-semantics interface. In particular, we formulate our analysis in a version of Type-Logical Categorial Grammar (TLCG) called Hybrid TLCG. This has the advantage that the interactions with nonconstituent coordination (NCC) exhibited by data such as (2c) and (2d) (and their counterparts involving symmetrical and summative predicates introduced below) becomes straightforward. This is especially important since interactions between NCC and each of these phenomena have been known to pose significant problems for analyses of coordination in the literature (see, e.g., Abbott (1976), Jackendoff (1977) and Beavers and Sag (2004) for some discussion).

Having laid out the main goals of the paper, we now turn to the specifics of each of the three phenomena. For the ‘respective’ reading, note first that if we remove the adverb *respectively*, the sentences still have the ‘respective’ reading as one of their possible interpretations.

(5) John and Bill bought the book and the CD.

But in this case, the sentence is multiply ambiguous. For example, in (5), both the subject and object NPs could be construed as group-denoting expressions: the two people bought the two things together. The sentence also allows for readings in which only the subject or the object NP exhibits a distributive reading (e.g. ‘John bought the book and the CD and Bill also bought the book and the CD’). The presence of the adverb *respectively* disambiguates the interpretation to the ‘respective’ reading.

In the ‘respective’ readings with overt conjunction (as in the examples in (2)), the n-th conjunct in one term is matched up with the n-th conjunct in the other term. As noted by many (cf., McCawley (1998); Kay (1989) among others), if the order of elements is clear from the context, not just conjoined NPs but plural NPs can also enter into ‘respective’ predication, as in the following examples:

(6) a. [Caption under a picture showing five men standing next to each other:]
These five men are Polish, Irish, Armenian, Italian and Chinese, respectively.
b. The three best students received the three best scores, respectively.

McCawley (1998) also notes that, when there are more than two plural or conjoined terms in the sentence, multiple ‘respective’ relations can be established among them. Disambiguation with *respectively* works in the same way as in the simpler examples, with the consequence that (7b) with a single *respectively* is ambiguity whereas (7a) with two occurrences of *respectively* is unambiguous:

(7) a. George and Martha sent a bomb and a nasty letter respectively to the president and the governor respectively.
b. George and Martha sent a bomb and a nasty letter to the president and the governor respectively.

As we discuss below, the availability of this multiple ‘respective’ reading turns out to be particularly important in formulating a compositional semantic analysis of ‘respective’ readings.

Turning now to symmetrical predicates, note first that symmetrical predicates such as *same*, *different*, *similar* and *identical* exhibit an ambiguity between the so-called 'internal' and 'external' readings (Carlson 1987).

(8) a. The same waiter served Robin and poured the wine for Leslie.
b. Different waiters served Robin and poured the wine for Leslie.

When uttered in a context in which some waiter is already salient (for example, when (8a) is preceded by *I had a very entertaining waiter when I went to that restaurant last week, and yesterday evening...*), *the same waiter* in (8a) anaphorically refers to that individual already introduced in the discourse. This is called the EXTERNAL READING. But this sentence can be uttered in an 'out of the blue' context too, and in this case, it simply asserts that the individual who acted as Robin's server and the one who poured Leslie's wine were identical, and that that individual was indeed a waiter—the so-called INTERNAL READING. The external reading is just an anaphoric use of these expressions and does not pose a particularly challenging problem for compositional semantics. For this reason, we set it aside and focus on the internal reading in the rest of the paper.[3]

The distribution of the internal reading of symmetrical predicates is remarkably similar to that of 'respective' readings. First, like 'respective' readings, the internal reading is available in all types of coordination:

(9) a. John and Bill read {the same book/different books}. (NP coordination)
b. {The same waiter/Different waiters} served Robin and poured the wine for Leslie. (VP coordination)
c. John read, and Bill reviewed, {the same book/different books}. (RNR)
d. I gave {the same book/different books} to John on Wednesday and to Bill on Thursday. (DCC)

Examples like (9c) and (9d) are especially problematic since they show that deletion-based analyses of nonconstituent coordination (which derives, for example, the RNR example (9c) from an underlying source *John read the same book and Bill reviewed the same book*) do not work (Abbott 1976; Jackendoff 1977; Carlson 1987).

Second, both plural and conjoined expressions induce the internal reading. Thus, by replacing *John and Bill* in (9a) by *the men*, we still get both the external and internal readings:

(10) The men read {the same book/different books}.

Third, just like multiple 'respective' readings, multiple internal readings are possible:

(11) a. John and Bill bought the same book at the same store.
b. John and Bill bought the same book at different stores.
c. John and Bill bought the same book at the same store on the same day for the same price.

[3]But see Sect. 2.2.2, where we briefly discuss a possibility in which the internal and external readings may be related in our setup. Whether a unified analysis of internal and external readings is desirable seems still controversial. See Brasoveanu (2011) and Bumford and Barker (2013) for some recent discussion.

Note moreover that the two phenomena interact with one another:

(12) a. John and Bill showed the same book to Sue and Mary (respectively).
b. John gave, and Bill lent, different books to Sue and Mary on the same day (respectively).

These similarities, especially the fact that the two phenomena interact with one another completely systematically as in (12), suggest that one and the same mechanism is at the core of the compositional semantics of these constructions.

The parallel distributional pattern in fact extends to the interpretations of summative predicates such as *a total of N* and *N in total* as well. The problem that summative predicates pose for the syntax-semantics interface is best known in the context of RNR, in examples such as the following (Abbott 1976):

(13) John spent, and Bill lost, a total of \$10,000 last year.

Just like the internal reading for symmetrical predicates (cf. (9c)), (13) has a reading that is not equivalent to its 'paraphrase' with clausal coordination:

(14) John spent a total of \$10,000 last year and Bill lost a total of \$10,000 last year.

But the summative reading exhibited by (13) (where \$10,000 corresponds to the sum of amounts that respectively satisfy the two predications) is not limited to RNR. The same reading is found in the full range of coordination constructions:

(15) a. The two men spent a total of \$10,000. (NP coordination)
b. A total of \$10,000 was spent and lost. (VP coordination)
c. John donated a total of \$10,000 to Red Cross on Thursday and to Salvation Army on Friday. (DCC)

Note also that here too, both plural NPs (as in (15a)) and conjoined expressions (as in (15b,c)) can enter into summative predication.

Moreover, just as with 'respective' readings and internal readings, iterated summative readings are also possible, and these phenomena interact with one another:

(16) a. A total of three boys bought a total of ten books.
b. John gave, and Bill lent, a total of \$10,000 to Mary and Sue, respectively.
c. John gave, and Bill lent, a total of \$10,000 to $\left\{\begin{array}{l}\text{the same student}\\ \text{different students}\end{array}\right\}$.

In the next section, we propose a unified analysis of 'respective', symmetrical and summative predicates that accounts for the parallels and interactions among these phenomena straightforwardly. The key idea that we exploit is that all these expressions denote tuples, that is, ordered lists of items, and that the same 'respective' predication operator mediates the complex (yet systematic) interactions they exhibit that pose apparent challenges to compositionality. While the semantics of each of these phenomena have been studied extensively in the previous literature by several authors, to our knowledge, a unified and fully detailed compositional analysis of these constructions (especially one that extends straightforwardly to cases involving interactions with NCC) has not yet been achieved. (But see Chaves (2012) for a recent attempt, some of whose key ideas are similar to ours; see Sect. 3.3 for a comparison). We believe that the unified analysis we offer below clarifies the compositional mechanism underlying these phenomena, especially the way it interacts with the general syntax and semantics of coordination that covers not just ordinary constituent coordination (of noun phrases (denoting individuals), verb phrases (denoting predicates), etc.) but also non-canonical coordination including RNR and DCC.

2 The compositional semantics of 'respective' predication

In this section, we present a unified analysis of 'respective', symmetrical and summative predicates in a variant of categorial grammar (CG) called Hybrid Type-Logical Categorial Grammar (Hybrid TLCG; Kubota 2010, 2014, to appear; Kubota and Levine 2013b). Key to our proposal is the idea that the same underlying mechanism of pairwise predication between terms that denote tuples as their meanings is involved in the semantics of these phenomena. This analytic idea itself is theory-neutral, but we show that formulating the analysis in Hybrid TLCG enables us to capture the complex yet systematic properties of this class of phenomena particularly transparently. More specifically, the order-insensitive mode of inference involving the 'vertical slash' (|), the key novel feature of Hybrid TLCG (explained below), enables a unified analysis of the two essential properties of these phenomena identified in Sect. 1: (i) interactions with NCC and (ii) multiple dependency that these predicates exhibit including the interactions of the three phenomena with one another.

2.1 The syntax-semantics interface of Hybrid TLCG

Hybrid TLCG is essentially an extension of the Lambek calculus with a novel kind of slash called VERTICAL SLASH (|), whose most basic application is to model 'covert' movement (but the analytic possibilities in Hybrid TLCG go beyond those available in the standard derivational architecture; see the final paragraph in this subsection and also Kubota (to appear); Kubota and Levine (2014a)). We start with a brief introduction of the framework.[4]

[4]Hybrid TLCG resembles certain approaches in contemporary CG. A full comparison with related approaches is beyond the scope of the present paper, but some comments are in order. Among various contemporary variants of TLCG, our approach resembles most closely Morrill et al.'s (2011) Displacement Calculus. The main difference between the two is how they treat discontinuous constituency: while our approach employs a fully general λ-calculus for modelling the prosodic component of linguistic expressions, Morrill et al.'s system treats discontinuity in terms of a purpose-specific calculus. We believe that there are several empirical phenomena that argue for the more general approach that we adopt Kubota and Levine (2012, 2013c), but since the phenomena we discuss in this paper can all be treated by discontinuous strings with string-type 'gaps', the results of the present work can be directly translated to their setup. Another closely related approach, perhaps more familiar to linguists, is Barker and Shan's (to appear) continuation-based grammar. In particular, λ-abstraction in the prosodic component tied to the vertical slash in Hybrid TLCG has a direct analog in Barker and Shan's NL_λ system: the 'λ' structural postulate. As we see it, there are two key differences between the two approaches, one conceptual and the other empirical. The conceptual difference is that Barker and Shan's 'λ' postulate does not have any formal connection with their logical inference system, and its precise formal and conceptual underpinnings are somewhat obscure. Our vertical slash (|), on the other hand, is much more deeply tied to the fundamental property of the underlying logic: λ-abstraction is inherent in the rules for introduction of implication connectives, as a direct expression of the Curry-Howard isomorphism. The empirical difference is that the directional slashes / and \ in NL_λ are *non-associative* whereas they are

We have tried to make the presentation below as self-contained as possible, but it is relatively compact due to space limitations. For a more complete exposition, see Kubota (2010) and Kubota (to appear). Readers who are interested in the formal properties of the framework should consult Kubota and Levine (2014b), which spells out some formal details that were only implicit in previous work, and especially Moot (2014), which proves that Hybrid TLCG can be embedded in first-order linear logic. This latter work is a major breakthrough in the investigations of the mathematical properties of Hybrid TLCG, as its precise formal underpinnings have been unknown in previous work.

Following Morrill (1994), we start by recasting the Lambek calculus in the 'labelled deduction' format (see also Oehrle (1994)), writing linguistic expressions as tuples $\langle \phi, \sigma, \kappa \rangle$ of phonological form ϕ, semantic translation σ, and syntactic type κ (or category—following the convention in TLCG, we use the terms 'syntactic type' and 'syntactic category' interchangeably), as in the following sample lexicon:

(17) a. john; $\mathbf{j}$; NP
b. mary; $\mathbf{m}$; NP
c. walks; $\mathbf{walk}$; NP\S
d. loves; $\mathbf{love}$; (NP\S)/NP

Complex categories are built from atomic categories (including S, NP and N) recursively with the connectives / and \ (to which | will be later added). We adopt the Lambek-style notation of slashes, where what appears under the slash (A in $A \backslash B$) is always the argument. Parentheses for a sequence of the same type of slash is omitted, as in S/NP/NP, NP\NP\S and S|NP|NP, which are abbreviations of (S/NP)/NP, NP\(NP\S) and (S|NP)|NP, respectively.

The following proof in (18) illustrates how larger linguistic expressions are built from smaller ones using the rules of grammar, of which we first introduce the Elimination rules for / and \ in (19) (these rules should be thought of as directional variants of modus ponens $B \to A, B \vdash A$). Here, a transitive verb, of category (NP\S)/NP, is combined with its two arguments on the right (object) and left (subject).

(18)
$$\cfrac{\mathsf{john};\ \mathbf{j};\ \mathrm{NP} \qquad \cfrac{\mathsf{loves};\ \mathbf{love};\ (\mathrm{NP}\backslash\mathrm{S})/\mathrm{NP} \qquad \mathsf{mary};\ \mathbf{m};\ \mathrm{NP}}{\mathsf{loves} \circ \mathsf{mary};\ \mathbf{love}(\mathbf{m});\ \mathrm{NP}\backslash\mathrm{S}}\ /\mathrm{E}}{\mathsf{john} \circ \mathsf{loves} \circ \mathsf{mary};\ \mathbf{love}(\mathbf{m})(\mathbf{j});\ \mathrm{S}}\ \backslash\mathrm{E}$$

(19) a. Forward Slash Elimination
$$\cfrac{a;\ \boldsymbol{\mathcal{F}};\ A/B \qquad b;\ \boldsymbol{\mathcal{G}};\ B}{a \circ b;\ \boldsymbol{\mathcal{F}}(\boldsymbol{\mathcal{G}});\ A}\ /\mathrm{E}$$

b. Backward Slash Elimination
$$\cfrac{b;\ \boldsymbol{\mathcal{G}};\ B \qquad a;\ \boldsymbol{\mathcal{F}};\ B\backslash A}{b \circ a;\ \boldsymbol{\mathcal{F}}(\boldsymbol{\mathcal{G}});\ A}\ \backslash\mathrm{E}$$

The Elimination rules can roughly be thought of as subcategorization cancellation rules. Note that, by applying the rules in (19), the right surface word order is obtained in (18), paired with the right meaning. The prosodic effect of these rules is string concatenation. / (\) combines the argument to the right (left) of the functor. The semantic effect is function application in both cases.

associative in Hybrid TLCG. This means that the interactions between NCC and 'respective', symmetrical and summative predicates in our analysis cannot be directly imported to their system. Extending Barker and Shan's system with associativity is certainly a conceivable move, but then, it is not clear whether the formal properties that Barker and Shan (to appear) prove for their NL_λ (such as soundness and completeness) carry over to the extended system.

Type-Logical CG (TLCG) takes the analogy between language and logic quite literally. Thus, in addition to the Elimination rules there are also Introduction rules for the two connectives / and \. These rules should be thought of as directional variants of implication introduction (or HYPOTHETICAL REASONING; drawing the conclusion $A \to B$ given a proof of B by hypothetically assuming A).

(20) a. FORWARD SLASH INTRODUCTION

$$\dfrac{\dfrac{\vdots \quad \vdots \quad \underline{[\varphi;\, x;\, A]^n} \quad \vdots \quad \vdots}{\vdots \quad \vdots \qquad \vdots \quad \vdots \qquad \vdots \quad \vdots} }{\dfrac{\mathsf{b} \circ \varphi;\ \boldsymbol{\mathcal{F}};\ B}{\mathsf{b};\ \lambda x.\boldsymbol{\mathcal{F}};\ B/A}\,/\mathrm{I}^n}$$

b. BACKWARD SLASH INTRODUCTION

$$\dfrac{\dfrac{\vdots \quad \vdots \quad \underline{[\varphi;\, x;\, A]^n} \quad \vdots \quad \vdots}{\vdots \quad \vdots \qquad \vdots \quad \vdots \qquad \vdots \quad \vdots} }{\dfrac{\varphi \circ \mathsf{b};\ \boldsymbol{\mathcal{F}};\ B}{\mathsf{b};\ \lambda x.\boldsymbol{\mathcal{F}};\ A\backslash B}\,\backslash\mathrm{I}^n}$$

We first illustrate the workings of these rules with an analysis of RNR, and then come back to the relevant formal details. The significance of the /I and \I rules is that they permit the reanalysis of any substring of a sentence as a (derived) constituent, which gives (TL)CG a distinct advantage over other syntactic theories in analyzing phenomena such as coordination. This is illustrated in (22), which analyzes the string *John loves* in (21) as a full-fledged constituent of type S/NP.

(21) John loves, but Bill hates, Mary.

(22)

$$①\!\to \dfrac{\mathsf{john};\ \mathbf{j};\ \mathrm{NP} \qquad \dfrac{[\varphi; x; \mathrm{NP}]^1 \quad \mathsf{loves};\ \mathbf{love};\ (\mathrm{NP}\backslash\mathrm{S})/\mathrm{NP}}{\mathsf{loves} \circ \varphi;\ \mathbf{love}(x);\ \mathrm{NP}\backslash\mathrm{S}}\,/\mathrm{E}}{②\!\to \dfrac{\mathsf{john} \circ \mathsf{loves} \circ \varphi;\ \mathbf{love}(x)(\mathbf{j});\ \mathrm{S}}{\mathsf{john} \circ \mathsf{loves};\ \lambda x.\mathbf{love}(x)(\mathbf{j});\ \mathrm{S/NP}}\,/\mathrm{I}^1}\,\backslash\mathrm{E}$$

By hypothesizing a direct object NP, we first prove an S (①). (The brackets around premises identify hypotheses, and the index is for keeping track of where the hypothesis is withdrawn in the proof.) At this point, since the phonology of the hypothesized NP φ appears on the right periphery, we can apply /I (②) to withdraw the hypothesis. Intuitively, what is going on here can be paraphrased as follows: since we've proven that there is a complete S by assuming that there is an NP on the right periphery (①), we know that, without this hypothetical NP, what we have is something that becomes an S if there is an NP to its right (②). Note that the lambda abstraction on the corresponding variable in semantics assigns the right meaning (of type $e \to t$) to the derived S/NP.

In the CG analysis of RNR (see, e.g., Morrill (1994); the original analytic insight goes back to Steedman (1985)) such nonconstituents are directly coordinated as constituents and then combined with the RNR'ed expression as in (23) (we use calligraphic letters ($\mathcal{U}, \mathcal{V}, \mathcal{W}, \ldots$) for polymorphic variables; copperplate letters ($\mathscr{P}, \mathscr{Q}, \ldots$) are reserved for higher-order variables (with fixed types)):

(23)

$$\dfrac{\dfrac{\begin{array}{c}\vdots\quad\vdots\\ \textsf{john}\circ\textsf{loves};\\ \lambda x.\mathbf{love}(x)(\mathbf{j}); \mathrm{S/NP}\end{array} \quad \dfrac{\begin{array}{c}\textsf{and};\\ \lambda\mathcal{W}\lambda\mathcal{V}.\mathcal{V}\sqcap\mathcal{W};\\ (X\backslash X)/X\end{array} \quad \begin{array}{c}\vdots\quad\vdots\\ \textsf{bill}\circ\textsf{hates};\\ \lambda x.\mathbf{hate}(x)(\mathbf{b}); \mathrm{S/NP}\end{array}}{\begin{array}{c}\textsf{and}\circ\textsf{bill}\circ\textsf{hates};\\ \lambda\mathcal{V}.\mathcal{V}\sqcap\lambda x.\mathbf{hate}(x)(\mathbf{b}); (\mathrm{S/NP})\backslash(\mathrm{S/NP})\end{array}}/\mathrm{E}}{\textsf{john}\circ\textsf{loves}\circ\textsf{and}\circ\textsf{bill}\circ\textsf{hates};\ \lambda x.\mathbf{love}(x)(\mathbf{j})\sqcap\lambda x.\mathbf{hate}(x)(\mathbf{b});\ \mathrm{S/NP}}\backslash\mathrm{E} \quad \begin{array}{c}\textsf{mary};\\ \mathbf{m}; \mathrm{NP}\end{array}}{\textsf{john}\circ\textsf{loves}\circ\textsf{and}\circ\textsf{bill}\circ\textsf{hates}\circ\textsf{mary};\ \mathbf{love}(\mathbf{m})(\mathbf{j})\wedge\mathbf{hate}(\mathbf{m})(\mathbf{b});\ \mathrm{S}}/\mathrm{E}$$

Note in particular that this analysis assigns the right meaning to the whole sentence compositionally ($\sqcap$ designates generalized conjunction (Partee and Rooth 1983)). As discussed in detail in Kubota and Levine (2013a), this analysis (and the parallel analysis of DCC discussed below) receives independent motivation from the scopal properties of several semantic operators including generalized quantifiers and focus-sensitive particles. The present paper demonstrates that the 'respective' readings, symmetrical predicate, and summative predicates constitute yet another class of phenomena whose semantics argues for this analysis of NCC.

We now return to some formal details. Since $\circ$ is string concatenation, the fragment up to this point is equivalent to the (associative) Lambek calculus **L** (Lambek 1958). This means that a hypothesis can be withdrawn as long as its phonology appears either on the left or right periphery of the phonological representation of the input expression.[5] Note also that in this formulation, the phonological term labelling, rather than the left-to-right order of the premises in the proof tree (as is more commonly done in the literature of mathematical linguistics), is relevant for the applicability conditions of the /I and \I rules (so far as we are aware, Morrill (1994) was the first to recast the Lambek calculus in this format). This point should be clear from the proof in (22), where we have deliberately placed the hypothetical object NP to the *left* of the verb in the proof tree. This also means that the order of the two premises in the Elimination rules does not play any role. In practice, we often write premises in an order reflecting the actual word order, for the sake of readability of derivations.

Though the Lambek calculus is notable for its smooth handling of phenomena exhibiting flexibility of surface constituency (such as nonconstituent coordination including RNR just illustrated), it runs into problems in dealing with phenomena that exhibit 'covert' movement. Thus, following Oehrle (1994), we introduce a new, order-insensitive mode of implication | called VERTICAL SLASH, the Introduction and Elimination rules for which are formulated as follows:

(24) a. VERTICAL SLASH INTRODUCTION

$$\dfrac{\dfrac{\begin{array}{c}\vdots\quad\vdots\quad \dfrac{[\varphi; x; A]^n}{\vdots\quad\vdots}\quad \vdots\quad\vdots\\ \vdots\quad\vdots\qquad\qquad\vdots\quad\vdots\end{array}}{b; \mathcal{F}; B}}{\lambda\varphi.b; \lambda x.\mathcal{F}; B|A}|\mathrm{I}^n$$

b. VERTICAL SLASH ELIMINATION

$$\dfrac{a; \mathcal{F}; A|B \qquad b; \mathcal{G}; B}{a(b); \mathcal{F}(\mathcal{G}); A}|\mathrm{E}$$

Unlike the /I and \I rules, in the |I rule, the missing position of A within $B|A$ (as with /, we write the argument to the right) is explicitly kept track of by means of λ-binding in phonology. This means that we admit functional expressions in the phonological component.

[5]For a more elaborate setup incorporating the notion of 'multi-modality', see, e.g., Moortgat and Oehrle (1994); Dowty (1996); Muskens (2007); Kubota (2014).

Such functional phonologies are applied to their arguments via the |E rule, whose phonological effect is function application. Note the close parallel between the semantic and phonological operations in these rules.

As was first noted by Oehrle (1994), hypothetical reasoning with | enables a formal modelling of Montague's (1973) quantifying-in, or what (roughly) corresponds to covert movement in derivational frameworks. This is illustrated in (25) for the $\forall > \exists$ reading for the sentence *Someone talked to everyone yesterday*:

(25)

$$
\dfrac{
\begin{array}{l}\lambda\sigma.\sigma(\mathsf{everyone});\\ \mathbf{\forall}_{\mathbf{person}};\\ \mathrm{S}|(\mathrm{S}|\mathrm{NP})\end{array}
\quad
\dfrac{
\dfrac{
\begin{array}{l}\lambda\sigma.\sigma(\mathsf{someone});\\ \mathbf{\exists}_{\mathbf{person}};\\ \mathrm{S}|(\mathrm{S}|\mathrm{NP})\end{array}
\quad
\textcircled{1}\!\rightarrow
\dfrac{
\dfrac{
\dfrac{
\left[\begin{array}{l}\varphi_2;\\ y;\\ \mathrm{NP}\end{array}\right]^2
\quad
\dfrac{
\begin{array}{l}\mathsf{talked}\circ\mathsf{to};\\ \textbf{talked-to};\\ (\mathrm{NP}\backslash\mathrm{S})/\mathrm{NP}\end{array}
\quad
\left[\begin{array}{l}\varphi_1;\\ x;\\ \mathrm{NP}\end{array}\right]^1
}{
\begin{array}{l}\mathsf{talked}\circ\mathsf{to}\circ\varphi_1;\\ \textbf{talked-to}(x);\mathrm{NP}\backslash\mathrm{S}\end{array}
}\,/\mathrm{E}
}{
\varphi_2\circ\mathsf{talked}\circ\mathsf{to}\circ\varphi_1;\ \textbf{talked-to}(x)(y);\ \mathrm{S}
}\,\backslash\mathrm{E}
\quad
\begin{array}{l}\mathsf{yesterday};\\ \textbf{yest};\mathrm{S}\backslash\mathrm{S}\end{array}
}{
\begin{array}{c}\varphi_2\circ\mathsf{talked}\circ\mathsf{to}\circ\varphi_1\circ\mathsf{yesterday};\\ \textbf{yest}(\textbf{talked-to}(x)(y));\mathrm{S}\end{array}
}\,\backslash\mathrm{E}
}{
\begin{array}{c}\lambda\varphi_2.\varphi_2\circ\mathsf{talked}\circ\mathsf{to}\circ\varphi_1\circ\mathsf{yesterday};\\ \lambda y.\textbf{yest}(\textbf{talked-to}(x)(y));\mathrm{S}|\mathrm{NP}\end{array}
}\,|\mathrm{I}^2
}{
\textcircled{2}\!\rightarrow\ \begin{array}{c}\mathsf{someone}\circ\mathsf{talked}\circ\mathsf{to}\circ\varphi_1\circ\mathsf{yesterday};\\ \mathbf{\exists}_{\mathbf{person}}(\lambda y.\textbf{yest}(\textbf{talked-to}(x)(y)));\mathrm{S}\end{array}
}\,|\mathrm{E}
}{
\begin{array}{l}\lambda\varphi_1.\mathsf{someone}\circ\mathsf{talked}\circ\mathsf{to}\circ\varphi_1\circ\mathsf{yesterday};\\ \lambda x.\mathbf{\exists}_{\mathbf{person}}(\lambda y.\textbf{yest}(\textbf{talked-to}(x)(y)));\mathrm{S}|\mathrm{NP}\end{array}
}\,|\mathrm{I}^1
}{
\begin{array}{l}\mathsf{someone}\circ\mathsf{talked}\circ\mathsf{to}\circ\mathsf{everyone}\circ\mathsf{yesterday};\\ \mathbf{\forall}_{\mathbf{person}}(\lambda x.\mathbf{\exists}_{\mathbf{person}}(\lambda y.\textbf{yest}(\textbf{talked-to}(x)(y))));\mathrm{S}\end{array}
}\,|\mathrm{E}
$$

Here, $\mathbf{\exists}_{\mathbf{person}}$ abbreviates the term $\lambda P.\exists x[\mathbf{person}(x) \wedge P(x)]$ (similarly for the universal quantifier). Thus, a quantifier has the ordinary GQ meaning, but its phonology is a function of type $(\mathsf{st} \to \mathsf{st}) \to \mathsf{st}$ (with st the type of strings). In (25), by abstracting over the position in which the quantifier 'lowers into' in an S, we first form an expression of type S|NP (①), a sentence missing an NP inside (phonologically of type $\mathsf{st} \to \mathsf{st}$) where the 'gap' position is explicitly kept track of via λ-binding in the prosodic component. By giving this expression as an argument to the quantifier (②), the subject quantifier *someone* semantically scopes over the sentence and lowers its string phonology to the 'gap' position. The scopal relation between multiple quantifiers depends on the order of application of this hypothetical reasoning. We get the inverse scope reading ($\forall > \exists$) in this derivation since the subject quantifier is combined with the sentence first.

Not only is this formalization of quantifying-in theoretically illuminating, capturing the tight correlation between the semantic and phonological effects of quantification transparently, it has an empirical advantage over quantifying-in (and its analogs) as well, in that it extends straightforwardly to more complex types of scope-taking, such as PARASITIC SCOPE in symmetrical predicates (Barker 2007; Pollard and Smith 2012) and SPLIT SCOPE of negative quantifiers (Kubota and Levine 2014a) and number determiners (**?**). Our own analysis of 'respective', symmetrical and summative predicates can be thought of as a further refinement of the parasitic scope analysis of symmetrical predicates by Barker (2007), and it exploits the flexible syntax-semantics interface enabled by the use of prosodic variable binding in this approach. Further empirical motivation for the non-directional mode of implication comes from the analysis of ‘overt’ movement phenomena (Muskens 2003; Mihaliček and Pollard 2012), and Gapping (Kubota and Levine 2014a). In particular, Gapping is especially interesting in this connection in that it exhibits the properties of both ‘overt’ and ‘covert’ movement simultaneously, thus constituting a case that goes beyond the analytic possibilities of the standard derivational architecture.

2.2 Analysis of ‘respective’, symmetrical and summative predicates

2.2.1 Hypothetical reasoning and ‘respective’ predication

We start with the analysis of ‘respective’ readings since our analysis of the other two phenomena builds on the core semantic operator that we introduce for this construction. The underlying intuition of most formal analyses of ‘respective’ readings (cf. Gawron and Kehler 2004; Winter 1995; Bekki 2006) (which we also adopt) is that sentences like (26) involve pairwise predication between two (or more) sets of entities where the ‘corresponding’ elements of the two sets are related by some predicate in the sentence.

(26) Mary and Sue married John and Bill (respectively).

In the case of (26), this ‘predicate’ is simply the lexical meaning of the verb, but in more complex cases that we discuss below, the predicate that relates the elements of the two sets (as well as the elements in the two sets themselves) can be of a more complex type.

Among the previous approaches, Gawron and Kehler (2004) (G&K) work out the relevant compositional mechanism in most detail (see Sect. 3.1 for more on their approach). G&K model the meanings of expressions to be related in a ‘respective’ manner in terms of the notion of sums. While there may be some conceptual appeal to relying on the notion of sums if one restricts one’s attention to cases involving NPs, the advantage of a sum-based approach is not so obvious for more complex cases; note that the analysis needs to be extended to cases involving ‘sums’ of non-entity-type objects, in particular, ‘sums’ of higher-order predicates, since—as we have already seen in the previous section—it is not just NPs that can be related in a ‘respective’ manner. Note in particular that an example like the following shows that making recourse to the notion of (sums of) events to treat such cases is not general enough:

(27) The numbers 9 and 6 are odd and even, respectively.

Moreover, since elements of a sum are inherently unordered, G&K invoke the notion of ‘sequencing functions’—a device external to the sums themselves—to keep track of the relevant ordering among elements of a sum so that ‘corresponding’ elements of different sums can be matched up against each other. We take it that the relevant ordering is a more fundamental property of the ‘sums’ themselves, and that making this information directly retrievable from the data structures of the relevant semantic objects is preferable. Thus, following Winter (1995) and Bekki (2006), we first recast the relevant aspects of G&K’s analysis in terms of the notion of tuple, whose semantic type is a product of the semantic type of its members. Mathematically, tuples are just a straightforward generalization of the notion of ordered pairs, where, unlike ordered pairs, the number of elements is not restricted to two. Formally, an n-tuple can be defined via ordered pairs as follows:

(28) a. For any a and b, the ordered pair $\langle a, b\rangle$ is a two-tuple and is written as (a, b).

b. If A $(= (a_1, a_2, \ldots a_n))$ is an n-tuple, then for any b, the ordered pair $\langle A, b\rangle$ is an $n+1$-tuple and is written as $(a_1, a_2, \ldots a_n, b)$

Since we do not need to distinguish between the ordered pair notation (with nested brackets) and the tuple notation, and since parentheses are used for other purposes in the logical translation language, in what follows we abuse the notation slightly and write tuples with angle brackets. For example, (a, b, c) will be written as $\langle a, b, c\rangle$. Importantly, unlike sums (or sets), tuples are inherently ordered. Thus, $a \oplus b \oplus c = a \oplus c \oplus b$, but $\langle a, b, c\rangle \neq \langle a, c, b\rangle$.

We assume that the conjunction word *and* denotes the following tuple-forming operator (this needs to be generalized to cases involving more than two conjuncts, but we omit this detail since it is not directly relevant for the ensuing discussion):

(29) and; $\lambda \mathcal{W} \lambda \mathcal{V}.\langle \mathcal{V}, \mathcal{W}\rangle$; $(X\backslash X)/X$

This enables us to assign tuples of individuals like $\langle \mathbf{mary}, \mathbf{sue}\rangle$ and $\langle \mathbf{mary}, \mathbf{sue}, \mathbf{ann}\rangle$ as the meanings of expressions like *Mary and Sue* and *Mary, Sue and Ann*.[6] Then, to assign the right meaning to (26), the two tuples $\langle \mathbf{mary}, \mathbf{sue}\rangle$ and $\langle \mathbf{john}, \mathbf{bill}\rangle$, each denoted by the subject and object NPs, need to be related to each other in a 'respective' manner via the relation **married**: Mary married John and Sue married Bill. Establishing this 'respective' relation is mediated by the **resp** operator in (30), which takes a relation and two tuple-denoting terms as arguments, and returns a tuple consisting of propositions obtained by relating each member of the two tuples in a pairwise manner with respect to the relation in question:

(30) $\mathbf{resp} = \lambda \mathcal{R} \lambda \mathcal{T}_{\times_n} \lambda \mathcal{U}_{\times_n}. \prod_i^n \mathcal{R}(\pi_i(\mathcal{T}_{\times_n}))(\pi_i(\mathcal{U}_{\times_n}))$

We assume, following Gawron and Kehler (2004), that the adverb *respectively* denotes this **resp** operator and that the 'respective' readings of sentences without an explicit occurrence of *respectively* are derived via an empty operator having the same denotation as (30). The variables $\mathcal{T}_{\times_n}$ and $\mathcal{U}_{\times_n}$ range over n-tuples. We omit the subscript n if its value is contextually obvious. Thus, in (30), the cardinality of the input tuples need to match. π_i is the projection function which returns the i-th member of the tuple. $\prod_i^n$ is a tuple constructor defined as follows:

(31) $\prod_i^n a_i = \langle a_1, a_2, \ldots, a_n\rangle$

From this, it should be clear that the cardinality of the output tuple matches that of the input tuples.

By giving the relation denoted by the verb and the two tuples denoted by the subject and object NPs as arguments to the **resp** operator, we obtain the following result:

(32) $\mathbf{resp}(\mathbf{married})(\langle \mathbf{m}, \mathbf{s}\rangle)(\langle \mathbf{j}, \mathbf{b}\rangle) = \prod_i \mathbf{married}(\pi_i(\langle \mathbf{m}, \mathbf{s}\rangle))(\pi_i(\langle \mathbf{j}, \mathbf{b}\rangle))$
$= \langle \mathbf{married}(\mathbf{j})(\mathbf{m}), \mathbf{married}(\mathbf{b})(\mathbf{s})\rangle$

[6] We assume that plural NPs denote sums lexically, and that they are converted to tuples via an empty operator when some contextually salient ordering is available. Admitting both sums and tuples may seem to lead to an unnecessary enrichment of the ontology, but it seems unnatural to assume that the denotations of ordinary plural NPs such as *the students at the department* in *The students at the department are all doing well* come with inherent orderings of their members in the lexicon.

This tuple of two propositions is then mapped to a boolean conjunction via a phonologically empty operator with the following meaning:

(33) $\lambda p_{\times}.\bigwedge\limits_{i}\pi_i(p_{\times})$

By applying (33) to (32), we obtain the proposition $\mathbf{married}(\mathbf{j})(\mathbf{m}) \wedge \mathbf{married}(\mathbf{b})(\mathbf{s})$. As will become clear below, keeping the two components separate in the form of a tuple after the application of the **resp** operator is crucial for dealing with multiple 'respective' (or symmetrical/summative) readings in examples like those in (7), (11) and (16).

The next question is how to get this semantic analysis hooked up with a compositional analysis of the sentence. Things may seem simple and straightforward in examples like (26), where the two terms to be related to each other in a pairwise manner are co-arguments of the same predicate. However, as noted already, this is not always the case. For treating more complex cases, G&K propose to treat 'respective' predication in terms of a combination of recursive applications of both the 'respective' operator and the distributive operator, but their approach quickly becomes unwieldy. Since we need to deal with complex examples involving interactions with nonconstituent coordination, we simply note here that the compositional mechanism assumed by G&K is not fully general and turn to an alternative approach (see Sect. 3.1 for a more complete critique of their approach, and also Sect. 4 for some more general remarks about the relationship between the present proposal and G&K's approach).

It turns out that a more general (and simpler) approach which serves our purpose here is straightforwardly available in Hybrid TLCG, by (two instances of) hypothetical reasoning via the vertical slash, as we show momentarily. Crucially, the interdependence between the two product-type terms is mediated by double abstraction via | in the syntax, whose output (together with the two product-type terms themselves) is immediately given to the **resp** operator, which then relates them in a pairwise manner with respect to some relation $\mathcal{R}$. This is essentially an implementation of Barker's (2007) 'parasitic scope'. (For a comparison between the present proposal and Barker's analysis of *same*, see Sect. 3.2.)

In Hybrid TLCG, we can abstract over any arbitrary positions in a sentence to create a relation that obtains between objects belonging to the semantic types of the variables that are abstracted over. This is illustrated in the following partial derivation for (26). By abstracting over the subject and object positions of the sentence, we obtain an expression of type S|NP|NP, where the 'gaps' in the subject and object positions are kept track of via explicit λ-binding in the phonology, just in the same way as in the analysis of quantifier scope above (here and elsewhere, VP abbreviates NP\S).

(34)
$$\dfrac{\dfrac{\dfrac{[\varphi_2; y; \mathrm{NP}]^2 \quad \dfrac{\mathsf{married}; \mathbf{married}; \mathrm{VP/NP} \quad [\varphi_1; x; \mathrm{NP}]^1}{\mathsf{married} \circ \varphi_1; \mathbf{married}(x); \mathrm{VP}}\,/\mathrm{E}}{\varphi_2 \circ \mathsf{married} \circ \varphi_1; \mathbf{married}(x)(y); \mathrm{S}}\,/\mathrm{E}}{\lambda\varphi_2.\varphi_2 \circ \mathsf{married} \circ \varphi_1; \lambda y.\mathbf{married}(x)(y); \mathrm{S|NP}}\,|\mathrm{I}^2}{\lambda\varphi_1\lambda\varphi_2.\varphi_2 \circ \mathsf{married} \circ \varphi_1; \lambda x\lambda y.\mathbf{married}(x)(y); \mathrm{S|NP|NP}}\,|\mathrm{I}^1$$

The 'respective' operator, defined as in (35), then takes such a doubly-abstracted proposition as an argument to produce another type S|NP|NP expression. Phonologically, it is just an identify function, and its semantic contribution is precisely the **resp** operator defined above.

(35) $\lambda\sigma\lambda\varphi_1\lambda\varphi_2.\sigma(\varphi_1)(\varphi_2)$; **resp**; $(Z|X|Y)|(Z|X|Y)$

The derivation completes by giving the two product-type arguments denoted by *John and Bill* and *Mary and Sue* to this 'respectivized' type S|NP|NP predicate, and converting the pair of propositions to a boolean conjunction by the boolean reduction operator in (33). (Dotted lines in derivations correspond to reductions of semantic translations to enhance readability of derivations, and should not be confused with the application of logical rules designated by solid lines. Unlike the latter, purely from a formal perspective, these reduction steps are completely redundant.)

(36)

$$\dfrac{\begin{array}{l}\lambda\sigma\lambda\varphi_1\lambda\varphi_2.\sigma(\varphi_1)(\varphi_2);\\ \mathbf{resp};\\ (Z|X|Y)|(Z|X|Y)\end{array} \qquad \begin{array}{l}\vdots \quad \vdots\\ \lambda\varphi_3\lambda\varphi_4.\\ \varphi_4 \circ \mathsf{married} \circ \varphi_3;\\ \mathbf{married}; (\mathrm{S}|\mathrm{NP})|\mathrm{NP}\end{array}}{\begin{array}{l}\lambda\varphi_1\lambda\varphi_2.\varphi_2 \circ \mathsf{married} \circ \varphi_1;\\ \mathbf{resp}(\mathbf{married}); \mathrm{S}|\mathrm{NP}|\mathrm{NP}\end{array}}\,|\mathrm{E}$$

$$\dfrac{\begin{array}{l}\vdots \quad \vdots\\ \mathsf{john} \circ \mathsf{and} \circ\\ \mathsf{bill};\\ \langle \mathbf{j}, \mathbf{b}\rangle; \mathrm{NP}\end{array} \qquad \begin{array}{l}\lambda\varphi_1\lambda\varphi_2.\varphi_2 \circ \mathsf{married} \circ \varphi_1;\\ \mathbf{resp}(\mathbf{married}); \mathrm{S}|\mathrm{NP}|\mathrm{NP}\end{array}}{\begin{array}{l}\lambda\varphi_2.\varphi_2 \circ \mathsf{married} \circ \mathsf{john} \circ \mathsf{and} \circ \mathsf{bill};\\ \mathbf{resp}(\mathbf{married})(\langle \mathbf{j}, \mathbf{b}\rangle); \mathrm{S}|\mathrm{NP}\end{array}}\,|\mathrm{E}$$

$$\dfrac{\begin{array}{l}\vdots \quad \vdots\\ \mathsf{mary} \circ \mathsf{and} \circ\\ \mathsf{sue};\\ \langle \mathbf{m}, \mathbf{s}\rangle; \mathrm{NP}\end{array} \qquad \begin{array}{l}\lambda\varphi_2.\varphi_2 \circ \mathsf{married} \circ \mathsf{john} \circ \mathsf{and} \circ \mathsf{bill};\\ \mathbf{resp}(\mathbf{married})(\langle \mathbf{j}, \mathbf{b}\rangle); \mathrm{S}|\mathrm{NP}\end{array}}{\begin{array}{l}\mathsf{mary} \circ \mathsf{and} \circ \mathsf{sue} \circ \mathsf{married} \circ \mathsf{john} \circ \mathsf{and} \circ \mathsf{bill};\\ \mathbf{resp}(\mathbf{married})(\langle \mathbf{j}, \mathbf{b}\rangle)(\langle \mathbf{m}, \mathbf{s}\rangle); \mathrm{S}\\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\ \mathsf{mary} \circ \mathsf{and} \circ \mathsf{sue} \circ \mathsf{married} \circ \mathsf{john} \circ \mathsf{and} \circ \mathsf{bill};\\ \langle \mathbf{married}(\mathbf{j})(\mathbf{m}), \mathbf{married}(\mathbf{b})(\mathbf{s})\rangle; \mathrm{S}\end{array}}\,|\mathrm{E}$$

$$\dfrac{\begin{array}{l}\lambda\varphi_1.\varphi_1;\\ \lambda p_\times.\bigwedge_i \pi_i(p_\times);\\ \mathrm{S}|\mathrm{S}\end{array} \qquad \begin{array}{l}\mathsf{mary} \circ \mathsf{and} \circ \mathsf{sue} \circ \mathsf{married} \circ \mathsf{john} \circ \mathsf{and} \circ \mathsf{bill};\\ \langle \mathbf{married}(\mathbf{j})(\mathbf{m}), \mathbf{married}(\mathbf{b})(\mathbf{s})\rangle; \mathrm{S}\end{array}}{\begin{array}{l}\mathsf{mary} \circ \mathsf{and} \circ \mathsf{sue} \circ \mathsf{married} \circ \mathsf{john} \circ \mathsf{and} \circ \mathsf{bill};\\ \mathbf{married}(\mathbf{j})(\mathbf{m}) \wedge \mathbf{married}(\mathbf{b})(\mathbf{s}); \mathrm{S}\end{array}}\,|\mathrm{E}$$

Note in particular that prosodic λ-binding with | enables 'lowering' the phonologies of the two product-type terms in their respective positions in the sentence, thus mediating the syntax-semantics mismatch between their surface positions and semantic scope (of the **resp** operator that they are arguments of) in essentially the same way as with quantifiers.

It should be clear that the analysis extends straightforwardly to cases where one of the product-type terms appears in a sentence-internal position, such as the following:

(37) John and Bill sent the bomb and the letter to the president yesterday, respectively.

For this sentence, we first obtain the following doubly abstracted proposition in the same way as the simpler example above:

(38) $\lambda\varphi_1\lambda\varphi_2.\varphi_1 \circ \mathsf{sent} \circ \varphi_2 \circ \mathsf{to} \circ \mathsf{the} \circ \mathsf{president} \circ \mathsf{yesterday}; \lambda x\lambda y.\mathbf{yest}(\mathbf{sent}(y)(\mathbf{the\text{-}pres}))(x);$ S|NP|NP

The **resp** operator then takes this and the two product-type terms as arguments to produce a sign with the surface string in (37) paired with the following semantic interpretation (after the application of the boolean reduction operator):

(39) $\mathbf{yest}(\mathbf{sent}(\mathbf{the\text{-}bomb})(\mathbf{pres}))(\mathbf{j}) \wedge \mathbf{yest}(\mathbf{sent}(\mathbf{the\text{-}letter})(\mathbf{the\text{-}pres}))(\mathbf{b})$

We now turn to an interaction with NCC, taking the following example as an illustration:

(40) I lent *Syntactic Structures* and *Barriers* to Robin on Thursday and to Mary on Friday, respectively.

The analysis is in fact straightforward.[7] In TLCG, dependent cluster coordination is analyzed by treating the apparent nonconstituents that are coordinated in examples like (40) to be (higher-order) derived constituents, via hypothetical reasoning (with the directional slashes / and \) in essentially the same way as in the analysis of RNR illustrated in Sect. 2.1.

Specifically, via hypothetical reasoning, the string *to Robin on Thursday* can be analyzed as a constituent of type NP\(VP/PP/NP)\VP, that is, an expression that combines with a prepositional ditransitive verb (of type VP/PP/NP) and an NP to its left to become a VP (see (109) in Appendix A for a complete proof):

(41) $\mathsf{to} \circ \mathsf{robin} \circ \mathsf{on} \circ \mathsf{thursday}$; $\lambda x \lambda P.\mathbf{onTh}(P(x)(\mathbf{r}))$; NP\(VP/PP/NP)\VP

We then derive a sentence containing gap positions corresponding to this derived constituent and the object NP ((110) in Appendix A):

(42) $\lambda \varphi_1 \lambda \varphi_2.\mathsf{I} \circ \mathsf{lent} \circ \varphi_1 \circ \varphi_2$; $\lambda x \lambda \mathscr{F}.\mathscr{F}(x)(\mathbf{lent})(\mathbf{I})$; S|(NP\(VP/PP/NP)\VP)|NP

The rest of the derivation involves giving this relation and the two product-type arguments of types NP and NP\(VP/PP/NP)\VP as arguments to the **resp** operator, which yields the following translation for the whole sentence:

[7]Though space considerations preclude a detailed discussion, it is worth noting here that the present analysis extends straightforwardly to interactions between 'respective' readings and another type of non-canonical coordination, namely, unlike-category coordination (UCC), exemplified by sentences such as (i).

(i) Robin and Leslie are a Republican and tactful, respectively.

We assume the basic analysis of UCC given in Morrill (1994), Bayer and Johnson (1995) and Bayer (1996), utilizing a category logic with the meet and join connectives in addition to implication. In terms of this logic, *a Republican and tactful* is assigned the syntactic category NP$\vee$AP. We continue to assume (as in the main text) that the semantics of conjunction is simply the tuple consisting of the meanings of the conjuncts, i.e., $\langle \mathbf{republican}, \mathbf{tactful} \rangle$ for (i). The syntactic category NP$\vee$AP satisfies the subcategorization requirement of the copula (which we take to be VP/(NP$\vee$AP$\vee$PP$\vee$VP$_{ing}$)), and via the **resp** operator the conjoined property is distributed to the conjoined subject in the 'respective' manner to yield the final translation $\mathbf{republican}(\mathbf{r}) \wedge \mathbf{tactful}(\mathbf{l})$ (after boolean reduction).

In phrase structure-based approaches, UCC has long been known to pose a significant problem theoretically (see, e.g., Sag et al. (1985); Bayer (1996); Daniels (2002); Levy and Pollard (2002); Sag (2003) for some discussion), and its adequate analysis is still highly controversial. Data such as (i) above suggest that an ellipsis-based analysis of UCC suggested, for example, by Beavers and Sag (2004) is not on the right track since ordinary like-category coordination recovering the verb in the second conjunct (from which (i) is supposedly derived in such an analysis) eliminates the 'respective' reading available in (i):

(ii) Robin and Leslie are a Republican and are tactful (#respectively).

Indeed, both Chaves (2012) and Yatabe (2012) note that the analysis of UCC via ellipsis is strongly contraindicated by an abundance of data. But the alternative approach, which invokes a highly intricate elaboration of syntactic categories (proposed in various forms by authors such as Daniels (2002) and Levy and Pollard (2002)) mimicking the logical properties of the $\vee$ connective posited in the Morrill/Bayer/Johnson-type analysis of UCC outlined above, is not particularly attractive: as Bayer (1996, 599) notes, it is a consequence of the logical properties of the meet and join connectives in the assumed category logic that a category $X/(Y \vee Z)$ will correspond to just those functors which are compatible with both Y and Z as their arguments. In other words, both $X/(Y \vee Z) \vdash X/Y$ and $X/(Y \vee Z) \vdash X/Z$ are theorems in this logic. No such deduction is possible in the analysis proposed by Daniels (2002) or Levy and Pollard (2002). Thus, to the extent that the logical properties of the $\vee$ connective correspond to the empirical generalizations found in the domain of UCC, the TLCG analysis is distinctly preferable over the phrase structure adaptation of the mechanism in question.

(43) $\mathbf{onTh}(\mathbf{lent}(\mathbf{s})(\mathbf{r}))(\mathbf{I}) \wedge \mathbf{onFr}(\mathbf{lent}(\mathbf{b})(\mathbf{l}))(\mathbf{I})$

Finally, multiple 'respective' readings, exemplified by (44), is straightforward.

(44) Tolstoy and Dostoevsky sent *Anna Karenina* and *The Idiot* to Dickens and Thackeray (respectively).

As in G&K's analysis, the right meaning can be compositionally assigned to the sentence via recursive application of the **resp** operator, without any additional mechanism. The key point of the derivation is that we first derive a three-place predicate of type S|NP|NP|NP, instead of a two-place predicate of type S|NP|NP (as in the simpler case in (36) above), to be given as an argument to the first **resp** operator:

(45) $\lambda\varphi_1\lambda\varphi_2\lambda\varphi_3.\varphi_3 \circ \mathsf{sent} \circ \varphi_1 \circ \mathsf{to} \circ \varphi_2$; $\mathbf{sent}$; S|NP|NP|NP

After two of the tuple-denoting terms are related to each other with respect to the predicate denoted by the verb, the resultant S|NP expression denotes a tuple of two properties (see (111) in Appendix A for a complete proof):

(46) $\lambda\varphi_3.\varphi_3 \circ \mathsf{sent} \circ \mathsf{AK} \circ \mathsf{and} \circ \mathsf{Id} \circ \mathsf{to} \circ \mathsf{Di} \circ \mathsf{and} \circ \mathsf{Th}$; $\langle \mathbf{sent}(\mathbf{ak})(\mathbf{di}), \mathbf{sent}(\mathbf{id})(\mathbf{th})\rangle$; S|NP

And the remaining conjoined term $\langle \mathbf{to}, \mathbf{do}\rangle$ is related to this product-type property by a derived two-place respective operator in the following way:

(47)

$$\begin{array}{c}
\dfrac{\begin{array}{l}\mathsf{To} \circ \mathsf{and} \circ \mathsf{Do};\\ \langle \mathbf{to}, \mathbf{do}\rangle; \mathrm{NP}\end{array} \qquad \dfrac{\begin{array}{l}\qquad\qquad\vdots\quad\vdots\\ \lambda\sigma_2\lambda\varphi_2.\sigma_2(\varphi_2);\\ \lambda P_\times \lambda X_\times.\prod_i \pi_i(P_\times)(\pi_i(X_\times)); (\mathrm{S}|\mathrm{NP})|(\mathrm{S}|\mathrm{NP})\end{array} \quad \begin{array}{l}\qquad\qquad\vdots\quad\vdots\\ \lambda\varphi_3.\varphi_3 \circ \mathsf{sent} \circ \mathsf{AK} \circ \mathsf{and} \circ \mathsf{Id} \circ\\ \mathsf{to} \circ \mathsf{Di} \circ \mathsf{and} \circ \mathsf{Th};\\ \langle \mathbf{sent}(\mathbf{ak})(\mathbf{di}), \mathbf{sent}(\mathbf{id})(\mathbf{th})\rangle; \mathrm{S}|\mathrm{NP}\end{array}}{\begin{array}{l}\lambda\varphi_2.\varphi_2 \circ \mathsf{sent} \circ \mathsf{AK} \circ \mathsf{and} \circ \mathsf{Id} \circ \mathsf{to} \circ \mathsf{Di} \circ \mathsf{and} \circ \mathsf{Th};\\ \lambda X_\times.\prod_i \pi_i(\langle \mathbf{sent}(\mathbf{ak})(\mathbf{di}), \mathbf{sent}(\mathbf{id})(\mathbf{th})\rangle)(\pi_i(X_\times)); \mathrm{S}|\mathrm{NP}\end{array}}\,|\mathrm{E}}{\begin{array}{l}\mathsf{To} \circ \mathsf{and} \circ \mathsf{Do} \circ \mathsf{sent} \circ \mathsf{AK} \circ \mathsf{and} \circ \mathsf{Id} \circ \mathsf{to} \circ \mathsf{Di} \circ \mathsf{and} \circ \mathsf{Th};\\ \prod_i \pi_i(\langle \mathbf{sent}(\mathbf{ak})(\mathbf{di}), \mathbf{sent}(\mathbf{i})(\mathbf{th})\rangle)(\pi_i(\langle \mathbf{to}, \mathbf{do}\rangle)); \mathrm{S}\end{array}}\,|\mathrm{E} \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\begin{array}{l}\mathsf{To} \circ \mathsf{and} \circ \mathsf{Do} \circ \mathsf{sent} \circ \mathsf{AK} \circ \mathsf{and} \circ \mathsf{Id} \circ \mathsf{to} \circ \mathsf{Di} \circ \mathsf{and} \circ \mathsf{Th};\\ \langle \mathbf{sent}(\mathbf{ak})(\mathbf{di})(\mathbf{to}), \mathbf{sent}(\mathbf{id})(\mathbf{th})(\mathbf{do})\rangle; \mathrm{S}\end{array}
\end{array}$$

The two place **resp** operator, which directly relates the product-type property (of type S|NP) with the product-type NP occupying the subject position via pairwise function application of the corresponding elements, can be derived from the lexically specified three-place **resp** operator via hypothetical reasoning. The proof is given in (112) in Appendix A.

2.2.2 Extending the analysis to symmetrical and summative predicates

We exploit the **resp** operator introduced above in the analysis of symmetrical and summative predicates as well. The intuition behind this approach is that NPs containing *same*, *different*, etc. (we call such NPs 'symmetrical terms' below) in examples like (48) denote tuples (linked to the other tuple denoted by the plural *John and Bill* in the same way as in the 'respective' readings above) but that they impose special conditions on each member of the tuple.

(48) John and Bill read the same book.

In (48), John and Bill need to be each paired with an identical book, and in the case of *different*, they need to be paired with distinct books. To capture this additional constraint on the tuples denoted by symmetrical terms, we assign to them GQ-type meanings of type S|(S|NP), where the abstracted NP in their arguments are product-type expressions semantically. More specifically, we posit the following lexical entries for *the same* and *different*:[8]

(49) a. $\lambda\varphi_0\lambda\sigma_0.\sigma_0(\mathsf{the}\circ\mathsf{same}\circ\varphi_0);$
$\lambda P\lambda Q.\exists X_\times\forall i\, P(\pi_i(X_\times)) \wedge \forall i\forall j[\pi_i(X_\times) = \pi_j(X_\times)] \wedge Q(X_\times); \mathrm{S}|(\mathrm{S}|\mathrm{NP})|\mathrm{N}$

b. $\lambda\varphi_0\lambda\sigma_0.\sigma_0(\mathsf{different}\circ\varphi_0);$
$\lambda P\lambda Q.\exists X_\times\forall i\, P(\pi_i(X_\times)) \wedge \forall i\forall j[i \neq j \rightarrow \pi_i(X_\times) \neq \pi_j(X_\times)] \wedge Q(X_\times); \mathrm{S}|(\mathrm{S}|\mathrm{NP})|\mathrm{N}$

In both cases, the relevant tuple (which enters into the 'respective' relation with another tuple via the **resp** operator) consists of objects that satisfy the description provided by the noun. The difference is that in the case of *same*, the elements of the tuple are all constrained to be identical, whereas in the case of *different*, they are constrained to differ from one another.

We now outline the analysis for (48) (the full derivation is given in (113) in Appendix A). The key point is that we first posit a variable that semantically denotes a tuple and relate it with the other tuple-denoting expression (*John and Bill* in this case) via the **resp** operator. This part of the analysis follows proof steps completely parallel to the analysis of 'respective' readings shown in the previous section (see (113) in Appendix A for a complete derivation). Specifically, by hypothetically assuming an NP with phonology φ and semantics $X_\times$, we can derive the expression in (50).

(50) $\mathsf{john}\circ\mathsf{and}\circ\mathsf{bill}\circ\mathsf{read}\circ\varphi;\ \bigwedge_i \pi_i(\mathbf{resp}(\mathbf{read})(X_\times)(\langle\mathbf{j},\mathbf{b}\rangle));\ \mathrm{S}$

At this point (where boolean reduction has already taken place), we withdraw the hypothesis to obtain an expression of type S|NP. This is then given as an argument to the symmetrical term *the same book*, which, as noted above, has the GQ type category of type S|(S|NP). The symmetrical term lowers its phonology to the gap position and semantically imposes the identity condition on the members of the relevant tuple. These last steps are illustrated in the following derivation:

(51)

$$
\dfrac{\begin{array}{c}\vdots\quad\vdots\\ \lambda\sigma_0.\sigma_0(\mathsf{the}\circ\mathsf{same}\circ\mathsf{book});\\ \mathbf{same}(\mathbf{book}); \mathrm{S}|(\mathrm{S}|\mathrm{NP})\end{array}
\qquad
\dfrac{\dfrac{\begin{array}{c}\vdots\quad\vdots\quad \dfrac{[\varphi; X_\times; \mathrm{NP}]^1}{\vdots\quad\vdots}\quad\vdots\quad\vdots\\ \vdots\quad\vdots\qquad\qquad\vdots\quad\vdots\end{array}}{\begin{array}{c}\mathsf{john}\circ\mathsf{and}\circ\mathsf{bill}\circ\mathsf{read}\circ\varphi;\\ \bigwedge_i \pi_i(\mathbf{resp}(\mathbf{read})(X_\times)(\langle\mathbf{j},\mathbf{b}\rangle)); \mathrm{S}\end{array}}}{\begin{array}{c}\lambda\varphi.\mathsf{john}\circ\mathsf{and}\circ\mathsf{bill}\circ\mathsf{read}\circ\varphi;\\ \lambda X_\times.\bigwedge_i \pi_i(\mathbf{resp}(\mathbf{read})(X_\times)(\langle\mathbf{j},\mathbf{b}\rangle)); \mathrm{S}|\mathrm{NP}\end{array}}{|\mathrm{I}^1}}{\begin{array}{c}\mathsf{john}\circ\mathsf{and}\circ\mathsf{bill}\circ\mathsf{read}\circ\mathsf{the}\circ\mathsf{same}\circ\mathsf{book};\\ \mathbf{same}(\mathbf{book})(\lambda X_\times.\bigwedge_i \pi_i(\mathbf{resp}(\mathbf{read})(X_\times)(\langle\mathbf{j},\mathbf{b}\rangle))); \mathrm{S}\end{array}}{|\mathrm{E}}
$$

[8]There is a close connection between the lexical entries for the internal readings posited in (49) and those for the external readings. The lexical entries in (49) essentially establish (non-)identity among each element of a tuple, and in this sense, it can be thought of as involving a reflexive anaphoric reference. By replacing this reflexive anaphoric reference with an anaphoric reference to some external object and stating the (non-)identity conditions to pertain to the object identified by the symmetrical term and the anaphorically invoked external object, we obtain a suitable lexical meaning for the external readings for *same* and *different*. Thus, while it may not be possible to unify the lexical entries for the two readings completely, we believe that our approach provides a basis for understanding the close relationship between the two readings.

One way to see what's going on in this derivation is that the hypothetical reasoning with the tuple-denoting variable $X_\times$ and 'respective' predication involving it is needed since a symmetrical term denotes a quantifier over tuple-denoting expressions and hence cannot directly fill in an argument slot of a lexical verb (which is looking for a non-tuple-denoting expressions as its arguments).

The final translation is unpacked in (52):

(52) $\mathbf{same}(\mathbf{book})(\lambda X_\times . \bigwedge_i \pi_i(\mathbf{resp}(\mathbf{read})(X_\times)(\langle \mathbf{j}, \mathbf{b} \rangle)))$
$= \exists X_\times \forall i\, \mathbf{book}(\pi_i(X_\times)) \wedge \forall i \forall j [\pi_i(X_\times) = \pi_j(X_\times)] \wedge \bigwedge_i \pi_i(\mathbf{resp}(\mathbf{read})(X_\times)(\langle \mathbf{j}, \mathbf{b} \rangle))$
$= \exists X_\times \forall i\, \mathbf{book}(\pi_i(X_\times)) \wedge \forall i \forall j [\pi_i(X_\times) = \pi_j(X_\times)] \wedge \mathbf{read}(\pi_1(X_\times))(\mathbf{j}) \wedge \mathbf{read}(\pi_2(X_\times))(\mathbf{b})$

Since, by definition, $\pi_1(X_\times) = \pi_2(X_\times)$, this correctly ensures that the book that John read and the one that Bill read are identical.

Before we move on to more complex cases, a comment is in order on the semantic analysis of *same* and *different* given in (49). So far as we can tell, the lexical meanings given in (49) capture the truth conditions for the internal readings of *same* and *different* correctly. One might think that (49a) is too weak as the meaning of *same* since according to this definition, (48) can be true in a situation in which the sets of books that John and Bill read are different, as long as one can identify some common book read by both individuals. Thus, one might think that some kind of maximality condition should be imposed on the set of books identified by *same*. We do not agree with this judgment. We believe that (48) is true and felicitous as long as one can identify (at least) one book commonly read by John and Bill. They may have read other books in addition, but that doesn't make (48) false or infelicitous. Such an implication, if felt to be present, is presumably a conversational implicature since it is clearly cancellable:

(53) John and Bill read the same book, although they both read several different books in addition.

Similarly, (49b), as it stands, does not exclude a possibility in which there is some set of books commonly read by John and Bill. We again take this to be the correct result. The following example shows that the implication excluding the existence of common books read by the two (if present at all) is not part of the entailment of the sentence:

(54) John and Bill read different books, although they read the same books too.

We now move on to multiple dependency cases. In fact, the present analysis already assigns the right meanings to these sentences. Specifically, since the same **resp** operator is at the core of the analysis as in the case of 'respective' readings, we immediately predict that symmetrical predicates can enter into multiple dependencies both among themselves and with respect to 'respective' predication, as exemplified by examples like the following:

(55) a. John and Bill gave the same book to Mary and Sue (respectively).
b. John and Bill gave the same book to the same man.

Since the relevant derivations can be reconstructed by taking the derivation for multiple 'respective' readings presented in the previous section as a model, we omit the details and reproduce here only the derived meanings for (55a) and (55b) in (56) and (57), respectively (see (114) in Appendix A for a complete derivation for (55b)).

(56) $\mathbf{same}(\mathbf{book})(\lambda X_\times.\mathbf{gave}(\mathbf{m})(\pi_1(X_\times))(\mathbf{j}) \wedge \mathbf{gave}(\mathbf{s})(\pi_2(X_\times))(\mathbf{b}))$
$= \exists X_\times \forall i\, \mathbf{book}(\pi_i(X_\times)) \wedge \forall i \forall j[\pi_i(X_\times) = \pi_j(X_\times)] \wedge \mathbf{gave}(\mathbf{m})(\pi_1(X_\times))(\mathbf{j}) \wedge \mathbf{gave}(\mathbf{s})(\pi_2(X_\times))(\mathbf{b})$

(57) $\mathbf{same}(\mathbf{book})(\lambda X_\times.\mathbf{same}(\mathbf{man})(\lambda Y_\times.\mathbf{gave}(\pi_1(Y_\times))(\pi_1(X_\times))(\mathbf{j}) \wedge \mathbf{gave}(\pi_2(Y_\times))(\pi_2(X_\times))(\mathbf{b})))$
$= \exists X_\times \forall i\, \mathbf{book}(\pi_i(X_\times)) \wedge \forall i \forall j[\pi_i(X_\times) = \pi_j(X_\times)] \wedge \exists Y_\times \forall i\, \mathbf{man}(\pi_i(Y_\times)) \wedge$
$\forall i \forall j[\pi_i(Y_\times) = \pi_j(Y_\times)] \wedge \mathbf{gave}(\pi_1(Y_\times))(\pi_1(X_\times))(\mathbf{j}) \wedge \mathbf{gave}(\pi_2(Y_\times))(\pi_2(X_\times))(\mathbf{b})$

The derivation for the multiple *same* sentence (55b) involves first positing two product-type variables $X_\times$ and $Y_\times$, which are linked to the plural term *John and Bill* via the recursive application of the **resp** operator and then bound by the two GQs over product-type terms *the same man* and *the same book*.

In the present analysis, the interaction between multiple 'respective' predication with NCC, exemplified by sentences like the following, is similarly straightforward:

(58) Terry gave the same gift to Bill and Sue as a Christmas present on Thursday and as a New Year's gift on Saturday (respectively).

The full derivation, which combines the proof steps for the NCC/'respective' interaction and multiple 'respective' readings already outlined, is given in (115) in Appendix A. We reproduce here the final translation and unpack it:

(59) $\mathbf{same}(\mathbf{gift})(\lambda X_\times.\mathbf{onTh}(\mathbf{asChP}(\mathbf{gave}(\pi_1(X_\times)(\mathbf{b}))))(t) \wedge \mathbf{onS}(\mathbf{asNYG}(\mathbf{gave}(\pi_2(X_\times)(\mathbf{s}))))(t))$
$= \exists X_\times.\forall i\, \mathbf{gift}(\pi_i(X_\times)) \wedge \forall i \forall j[\pi_i(X_\times) = \pi_j(X_\times)] \wedge ..$
$\mathbf{onTh}(\mathbf{asChP}(\mathbf{gave}(\pi_1(X_\times))(\mathbf{b})))(t) \wedge \mathbf{onS}(\mathbf{asNYG}(\mathbf{gave}(\pi_2(X_\times))(\mathbf{s})))(t)$

The present analysis also assigns intuitively correct truth conditions for sentences such as the following, where two symmetrical terms exhibit interdependency with each other without being mediated by a separate plural term (unlike (55b) above):

(60) a. Different students bought different books.
b. The same student bought different books.

As in the above examples, the derivation proceeds by abstracting over the subject and object positions and 'respectivizing' the relation thus obtained, and then 'quantifying-in' the symmetrical terms in the subject and object positions one by one. This yields the following translation for (60a):

(61) $\exists X_\times.\forall i\, \mathbf{student}(\pi_i(X_\times)) \wedge \forall ij[i \neq j \rightarrow \pi_i(X_\times) \neq \pi_j(X_\times)] \wedge \exists Y_\times.\forall i\, \mathbf{book}(\pi_i(Y_\times)) \wedge$
$\forall ij[i \neq j \rightarrow \pi_i(Y_\times) \neq \pi_j(Y_\times)] \wedge \mathbf{resp}(\mathbf{bought})(X_\times)(Y_\times)$

This asserts of an existence of a set of students and a set of books such that the buying relation is a bijection between these two sets. Thus, no two boys bought the same book and no two books were bought by the same boy. This corresponds to one of the intuitively available readings of the sentence. In Appendix B, we discuss a more complex type of reading for the same sentence according to which the sets of books that each boy bought are different from one another, where, for any given pair of boys b_1 and b_2, there could be a partial (but not total) overlap between the sets of books that b_1 and b_2 respectively bought.

We now turn to an analysis of summative predicates such as *a total of $10,000*. The approach to symmetrical predicates displayed above uses the **resp** operator to create pairings between corresponding elements of two tuples, and then imposes a further condition on one of the two tuples involved. The (in)equality relation incorporated in this analysis is only one possible condition that could be so imposed, however; theoretically there are an unlimited number of other possible conditions, and we could expect a certain variety in the way natural language grammars exploit such possibilities. It turns out that we indeed see evidence of exactly this type of variety. In particular, in examples like (62) involving summative predicates, the tuple elements are required to (together) satisfy a quantity condition:[9] taking $n^{\$}$ and $m^{\$}$ to be the tuple elements denoting amounts of money, then, roughly speaking, (62) asserts that $\mathbf{spent}(\mathbf{r}, n^{\$})$ and $\mathbf{lose}(\mathbf{l}, m^{\$})$ and that $n^{\$} + m^{\$} = \$10{,}000$.

(62) Robin spent and Leslie lost a total of $10,000.

In other words, the condition 'adds up to $10,000' is imposed on the members of the tuple, instead of the (in)equality or similarity relations.

To capture this idea, we once again treat the relevant expressions as GQs over product-type terms of type S|(S|NP), assigning to *a total of* the following meaning:

(63) $\lambda\varphi\lambda\sigma.\sigma(\mathsf{a} \circ \mathsf{total} \circ \mathsf{of} \circ \varphi);\ \lambda S\lambda P.\exists X_{\times_n} \sum_{1\leq i\leq n} \pi_i(X_{\times_n}) = S \wedge P(X_{\times_n});\ \mathrm{S}|(\mathrm{S}|\mathrm{NP})$

This operator takes a sum S and a predicate P (over product-type terms) as arguments and asserts the existence of some tuple $X_{\times_n}$ where the sum of all of the elements of $X_{\times_n}$ equals S and $X_{\times_n}$ itself satisfies the predicate P. Since P is a predicate of product-type terms, this effectively means that $X_{\times_n}$ enters into a 'respective' relation with some other product-type term in the sentence. The tuple $X_{\times_n}$ can be thought of as a possible partitioning of the sum S into subportions that can respectively be related to the other tuple(s), which, in the case of (62), is contributed by the plural NP.

The analysis of (62) then proceeds in the same way as the case involving a symmetrical predicate outlined above. We first derive a sentence in which a hypothetically assumed tuple-denoting expression (φ_1; $X_\times$; NP) enters into a 'respective' predication with an overt conjoined term (which, in this case, is a 'nonconstituent' *John spent and Bill lost*; for details, see the full derivation in (116)):

(64) $\mathsf{john} \circ \mathsf{spent} \circ \mathsf{and} \circ \mathsf{bill} \circ \mathsf{lost} \circ \varphi_1;\ \mathbf{spent}(\mathbf{j}, \pi_1(X_\times)) \wedge \mathbf{lost}(\mathbf{b}, \pi_2(X_\times));\ \mathrm{S}$

By abstracting over $X_\times$, we obtain an expression of type S|NP. This is then given as an argument to the GQ-type S|(S|NP) expression denoted by *a total of $10,000*, and we obtain the final translation in (65), which captures the intuitively correct meaning of the sentence:

(65) $\mathbf{total}(\mathbf{\$10k})(\lambda X_\times.\mathbf{spent}(\mathbf{j}, \pi_1(X_\times)) \wedge \mathbf{lost}(\mathbf{b}, \pi_2(X_\times)))$
$= \exists X_{\times_2}.\pi_1(X_{\times_2}) \oplus \pi_2(X_{\times_2}) = \mathbf{\$10k} \wedge \mathbf{spent}(\mathbf{j}, \pi_1(X_{\times_2})) \wedge \mathbf{lost}(\mathbf{b}, \pi_2(X_{\times_2}))$

At this point, there are some residual issues that need to be addressed. The first question is whether the present account of summative predicates can be extended to cumulative readings. Addressing this question fully is beyond the scope of the present paper, but in Appendix B we suggest one possible line in which such an extension may be implemented. Second, assuming that such an extension is plausible, one might ask further whether the present approach

[9] *An average of X* is yet another such expression, as discussed by Kennedy and Stanley (2008).

predicts the right results in cases in which summative terms interact with other aspects of plural semantics. For example, can it account for the fact that summative terms can hold a distributive relation with one plural term but at the same time enter into a cumulative relation with another plural term, exemplified by the following example, a type of sentence originally discussed by Schein (1993)?:

(66) A total of three ATMs gave a total of 1000 customers two new passwords.

On the relevant reading, *three ATMs* is related to *1000 customers* in the cumulative manner, and *two new passwords* distributes over each ATM-customer pair (thus involving 2000 distinct passwords issued). In fact, deriving the right interpretation for examples like (66) is relatively straightforward in the present approach. To derive the relevant reading for (66), we merely need to let the plural term *two new passwords* denoting an ordinary cardinal quantifier scope below the 'respective' operator that establishes the cumulative relation between the other two plural terms. We provide a full derivation in Appendix B.

While the extension to plural semantics that we outline in Appendix B is admittedly sketchy, and several aspects of it need to be elaborated further for it to count as a full-fledged analysis, we take it to be promising that a relatively minor extension of the present proposal provides a simple and general mechanism that adequately accounts for a rather recalcitrant case such as the interaction of cumulative and distributive readings exemplified by (66).

3 Comparison with related approaches

In the previous section, we have proposed a unified analysis of 'respective', symmetrical and summative predicates. The key components of our analysis are the treatment of expressions involving conjunction as denoting tuples of meanings and the flexible syntax-semantics interface of Hybrid TLCG in which hypothetical reasoning for relating the relevant tuples in the 'respective' manner that underlies the semantics of these expressions interacts fully systematically with hypothetical reasoning for forming syntactic constituents that enter into that 'respective' predication. While our proposal builds on several key insights from previous proposals on these phenomena, we are unaware of any other proposal, at any level of formal explicitness, which accounts for the same range of data for which we have provided an explicit account. In this section, we discuss three previous approaches closely related to our own and discuss their key insights as well as limitations.

3.1 Gawron and Kehler (2004)

3.1.1 Overview

Gawron and Kehler (2004) (G&K) present what is, to our knowledge, the first fully compositional formal treatment of the grammar of 'respective' readings, building on earlier observations in McCawley (1968, 1998). Central to their approach is the premise that the semantics of *respectively* establishes a bijection between two sum-denoting expressions, yielding a sum as a result—an idea which clearly allows for the possibility of iteration, since this resulting sum is itself of the right form to serve as an argument to the operation effected by *respectively*. In this way, they can in effect chain together a series of local applications of *respectively* to create one-to-one mappings over arbitrary numbers of pair (or triples, etc.) of objects. Moreover, they extend the notion of sum to higher-order objects, so as to cover all of the following possibilities:

(67) a. Robin and Leslie mailed a letter and a postcard (respectively).
b. Robin and Leslie mailed a letter and a postcard yesterday and today (respectively).
c. Robin and Leslie mailed and hand-delivered a letter and a postcard to Terry and Chris (respectively).

Sums themselves are unordered, whereas ‘respective’ readings impose orders on the pairings which, in the simplest case, reflect the order of mention: what is correlated in (67c) is Robin, mailing, a letter, and Terry on the one hand, and Leslie, hand-delivering, a postcard, and Chris on the other. To impose an ordered structure on the sums, G&K posit a ‘sequencing function’ f which, for each element in a sum, bijectively assigns an integer. The normal arithmetic order on integers thus allows us to interpret the sum effectively as a tuple, with any given sum element’s position in the tuple determined by the value that f assigns to it.

3.1.2 Details and limitations

G&K define a sequencing function f which maps the atoms in sums to the sequence of integers, thus defining an ordering on these atoms, with $f \in (\mathscr{U}^{\mathbb{Z}})^{\mathscr{G}}$, where $\mathscr{G}$ is ‘the set of sums of all types’ (p. 173), $\mathscr{U}$ ‘the set of (possibly atomic) entities of all types’, and $\mathbb{Z}$, as per standard mathematical notation, the set of integers. In particular, the function f takes a given sum g_0 as input and assigns an integer to each successive member of the sum which imposes a sequence structure on g_0; in other words, f effectively maps sums to tuples of the atoms those sums comprise.[10] Given this sequencing function, it is straightforward to specify a ‘respective’ operator ($\mathbf{Resp}_f$) whose ‘effect’ is identical to (30) but which is technically defined for sums rather than tuples:

(68) $$\mathbf{Resp}_f = \lambda P \lambda g. \bigsqcup_{1 \leq i \leq |f|} f(P)(i)(f(g)(i))$$

where $\bigsqcup$ is a generalized sum-forming operator (where ‘sums’ include not just entity sums but sums of higher-order semantic objects) and $|f|$ denotes the ‘cardinality’ of f, i.e., the number of components of the sum that f takes as its first argument. Since the same sequencing function appears in both the property and argument sum in (68), only sums of the same cardinality can be related by the **Resp** operator.

In addition, G&K assume the following **Dist** operator, which is identical to the standard distributive operator in plural semantics (except that it returns a sum instead of a boolean conjunction):

(69) $$\mathbf{Dist} = \lambda P \lambda g. \bigsqcup_{x\,\mathrm{atom}-\sqsubseteq g} P(x)$$

[10]G&K take f to be a variable over such functors, with any particular instantiation of f contextually determined. Alternatively, and equivalently, one can take tuples rather than sums to be basic, as in our analysis in Sect. 2, with the particular order imposed by pragmatic factors bearing on the discourse.

$\mathbf{Resp}_f$ and **Dist** work together straightforwardly to provide 'respective' readings. Thus, in the case of *Robin and Leslie walk*, we have (70) (here, following G&K, we write $\sqcup$ (instead of $\oplus$) for the sum operator):

(70) $\mathbf{Dist}(\mathbf{walk})(\mathbf{r} \sqcup \mathbf{l}) = \mathbf{walk}(\mathbf{r}) \sqcup \mathbf{walk}(\mathbf{l})$

which is a sum of propositions $\mathbf{walk}(\mathbf{r})$ and $\mathbf{walk}(\mathbf{l})$, subsequently converted to a conjunction of the sum elements by the boolean conjunction operator. **Dist** will distribute the objects of transitive predicates in exactly the same way: replacing *walk* with *married*, as in *married Robin and Leslie (respectively)* yields the following:

(71) $\mathbf{Dist}(\mathbf{married})(\mathbf{r} \sqcup \mathbf{l}) = \mathbf{married}(\mathbf{r}) \sqcup \mathbf{married}(\mathbf{l})$

If an appropriate plural subject (e.g., *Chris and Terry*) combines with this VP, we get the following interpretation:

(72) $\mathbf{Resp}_f(\mathbf{married}(\mathbf{r}) \sqcup \mathbf{married}(\mathbf{l}))(\mathbf{c} \sqcup \mathbf{t}) = \mathbf{married}(\mathbf{r})(\mathbf{c}) \sqcup \mathbf{married}(\mathbf{l})(\mathbf{t})$

In examples involving multiple 'respective' relations such as (73), iterated application of $\mathbf{Resp}_f$ following a single application of **Dist** to **sent** and $\iota(\mathbf{bomb}) \sqcup \iota(\mathbf{letter})$ will yield the correct readings.

(73) John and Bill sent the bomb and the letter to the president and the provost, respectively.

Here, **sent** is distributed over $\iota(\mathbf{bomb}) \sqcup \iota(\mathbf{letter})$, yielding a sum over which $\iota(\mathbf{president}) \sqcup \iota(\mathbf{provost})$ is then distributed by $\mathbf{Resp}_f$, with a resulting sum over which $\mathbf{j} \sqcup \mathbf{b}$ is finally distributed, yielding $\mathbf{sent}(\iota(\mathbf{bomb}))(\iota(\mathbf{president}))(\mathbf{j}) \sqcup \mathbf{sent}(\iota(\mathbf{provost}))(\iota(\mathbf{letter}))(\mathbf{b})$. These successive applications of $\mathbf{Resp}_f$ are made possible by the fact that each application of this operator yields a sum over which the subsequent application maps another sum of the same cardinality.

But this picture, while apparently both tidy and general, turns out to be rather unwieldy in the case of examples such as the following ((74b) is the same example as (37) above):

(74) a. John and Bill read and reviewed the book, respectively.
b. John and Bill sent the bomb and the letter to the president yesterday, respectively.

For example, in (74b), $\mathbf{Dist}(\mathbf{sent})$ applies to $\iota(\mathbf{bomb}) \sqcup \iota(\mathbf{letter})$ to yield a property sum, but this sum cannot combine with its next argument $\iota(\mathbf{president})$ via $\mathbf{Resp}_f$ since the latter is not a sum, nor can **Dist** reapply to distribute $\iota(\mathbf{president})$ over the property sum $\mathbf{sent}(\iota(\mathbf{bomb})) \sqcup \mathbf{sent}(\iota(\mathbf{letter}))$ since **Dist** can only distribute a functor over the components of a sum of argument objects, not vice versa. Such data are not addressed in Gawron and Kehler (2004), and on the face of it do not appear amenable to their analysis. Nor is it obvious in the phrase-structural approach they assume how to treat *sent __ to the president* as in effect a transitive verb denoting a relation which could then be distributed over the denotation of *the bomb and the letter*.

One can imagine a range of responses to this problem. For example, we could posit a separate operator, call it $\mathbf{Dist}'$, which would distribute an argument over a sum of two or more predicates—a solution which however seems to involve a high degree of coincidentality, with **Dist** and **Dist′** doing essentially the same thing, but with no logical relationship between them.[11] Another possibility would be to take *to the president* in (74b) to be a type-lifted functor, where **Dist** then distributes $\mathbf{sent}(\iota(\mathbf{bomb})) \sqcup \mathbf{sent}(\iota(\mathbf{letter}))$ over this functor. One might suppose that type-lifting could be added to the semantics as a kind of primitive operation, in a fashion similar to CCG. As a single rule on its own, type-lifting does not appear to be particularly costly; but there are two objections to such a move, one conceptual and one empirical. Conceptually, such a rule would be just an *ad hoc* add-on, falling out of nothing in the native properties of the phrase-structure based architecture that G&K assume. Empirically, type lifting in the semantics alone will be of no use in examples like (40) involving NCC from the previous section. Examples like this, where one of the terms that the **resp** operator takes as an argument is not a syntactic constituent in phrase structure-based theories, suggest that type-lifting needs to be recognized as a fully general rule at the syntax-semantics interface.

Our own proposal, although clearly heavily indebted to G&K's insight that the semantics of 'respective' readings operate on pairs of sums/tuples, is not subject to the difficulties noted above, based as it is not on a phrase structure model of grammar but rather on the flexible syntax-semantics interface of Hybrid TLCG. The crucial advantage that our approach offers is the retention of a unitary approach to 'respective' readings via a single operator which is seemingly unavailable in a phrase structure-based setup such as G&K's.

3.2 Barker (2007, 2012)

3.2.1 Barker's analysis of symmetrical predicates

Barker (2007) offers an account of symmetrical predicates which (according to him) is the first analysis that is both semantic and compositional. The key idea behind his proposal is that *same* maps elements of a sum to a single individual in the denotation of some property. For example, in (75), *same* maps each member of the set $\{\mathbf{j}, \mathbf{b}\}$ to a particular book $b_i \in \mathbf{book}$. Barker refers to the semantic action of *same* and similar predicates as 'parasitic scope'.[12]

(75) John and Bill read the same book.

The specific semantic operator Barker posits for *same* is given in (76):

(76) $\lambda\mathscr{F}\lambda X\exists\mathfrak{f}\forall x <_a X.\mathscr{F}(\mathfrak{f})(x)$

where $\mathscr{F} \in D_{(\tau(\mathrm{Adj})\to\tau(\mathrm{Pred}))\to\tau(\mathrm{Pred})}$ with $\tau(\mathrm{Adj}) = et \to et$ and $\tau(\mathrm{Pred}) = et$. Here $\mathfrak{f}$ denotes a variable over CHOICE FUNCTIONS, each of which takes a set as argument and returns as

[11] G&K speculate on a possibility of unifying their **Dist** and **Resp** operators toward the end of their paper; if this unification is successfully done, both the argument-distributing **Dist** operator in (69) above and the functor-distributing **Dist′** operator under discussion here might be thought of as special cases of a single unified 'predication' operator. But this part of their proposal remains somewhat obscure and not worked out in full detail.

[12] This phrasing originates in the Heim and Kratzer (1998) formulation of the syntax/semantics interface (relying on QR to create quantificational scope), which Barker uses to illustrate his analysis in the first half of his paper. In the TLCG analysis Barker himself offers, the analysis extensively relies on hypothetical reasoning, independent of any subsequent introduction of a quantifier into the proof which can take the derived property as an argument.

output a singleton set consisting of a single member of that set. The effect of this operator is illustrated in (77), where we have translated Barker's proof format into our own (here and below, Adj abbreviates N/N):

(77)

$$\dfrac{\begin{array}{c}\vdots \quad \vdots \quad \left[\begin{array}{l}\varphi_1;\\ \xi;\mathrm{Adj}\end{array}\right]^1 \quad \vdots \quad \vdots\\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots\end{array}}{\lambda\sigma_1.\sigma_1(\mathsf{the}\circ\varphi_1\circ\mathsf{book});\ \iota(\xi(\mathbf{book}));\mathrm{NP}}$$

$$\dfrac{\begin{array}{l}\mathsf{read};\\ \mathbf{read};\\ (\mathrm{NP}\backslash\mathrm{S})/\mathrm{NP}\end{array} \qquad \begin{array}{l}\lambda\sigma_1.\sigma_1(\mathsf{the}\circ\varphi_1\circ\mathsf{book});\\ \iota(\xi(\mathbf{book}));\mathrm{NP}\end{array}}{\begin{array}{l}\mathsf{read}\circ\mathsf{the}\circ\varphi_1\circ\mathsf{book};\\ \mathbf{read}(\iota(\xi(\mathbf{book})));\mathrm{NP}\backslash\mathrm{S}\end{array}}$$

$$\dfrac{\begin{array}{l}\mathsf{read}\circ\mathsf{the}\circ\varphi_1\circ\mathsf{book};\\ \mathbf{read}(\iota(\xi(\mathbf{book})));\mathrm{NP}\backslash\mathrm{S}\end{array} \qquad \left[\begin{array}{l}\varphi_3;\\ y;\\ \mathrm{NP}\end{array}\right]^2}{\begin{array}{c}\varphi_3\circ\mathsf{read}\circ\mathsf{the}\circ\varphi_1\circ\mathsf{book};\\ \mathbf{read}(\iota(\xi(\mathbf{book}))(y);\mathrm{S}\end{array}}$$

$$① \rightarrow \dfrac{\begin{array}{c}\varphi_3\circ\mathsf{read}\circ\mathsf{the}\circ\varphi_1\circ\mathsf{book};\\ \mathbf{read}(\iota(\xi(\mathbf{book}))(y);\mathrm{S}\end{array}}{\begin{array}{c}\lambda\varphi_3.\varphi_3\circ\mathsf{read}\circ\mathsf{the}\circ\varphi_1\circ\mathsf{book};\\ \lambda y.\mathbf{read}(\iota(\xi(\mathbf{book}))(y);\mathrm{S|NP}\end{array}}\ |\mathrm{I}^2$$

$$② \rightarrow \dfrac{\begin{array}{c}\lambda\varphi_3.\varphi_3\circ\mathsf{read}\circ\mathsf{the}\circ\varphi_1\circ\mathsf{book};\\ \lambda y.\mathbf{read}(\iota(\xi(\mathbf{book}))(y);\mathrm{S|NP}\end{array}}{\begin{array}{c}\lambda\varphi_1\lambda\varphi_3.\varphi_3\circ\mathsf{read}\circ\mathsf{the}\circ\varphi_1\circ\mathsf{book};\\ \lambda\xi\lambda y.\mathbf{read}(\iota(\xi(\mathbf{book}))(y);(\mathrm{S|NP})|\mathrm{Adj}\end{array}}\ |\mathrm{I}^1$$

$$③ \rightarrow \dfrac{\begin{array}{l}\lambda\sigma_1.\sigma_1(\mathsf{same});\\ \lambda\mathscr{F}\lambda X\exists\mathfrak{f}\forall x <_a X\,\mathscr{F}(\mathfrak{f})(x);\\ (\mathrm{S|NP})|((\mathrm{S|NP})|\mathrm{Adj}))\end{array} \qquad \begin{array}{c}\lambda\varphi_1\lambda\varphi_3.\varphi_3\circ\mathsf{read}\circ\mathsf{the}\circ\varphi_1\circ\mathsf{book};\\ \lambda\xi\lambda y.\mathbf{read}(\iota(\xi(\mathbf{book}))(y);(\mathrm{S|NP})|\mathrm{Adj}\end{array}}{\lambda\varphi_3.\varphi_3\circ\mathsf{read}\circ\mathsf{the}\circ\mathsf{same}\circ\mathsf{book};\ \lambda X\exists\mathfrak{f}\forall x <_a X.\,\mathbf{read}(\iota(\mathfrak{f}(\mathbf{book})))(y);\ \mathrm{S|NP}}$$

$$\dfrac{\begin{array}{l}\mathsf{john}\circ\\ \mathsf{and}\circ\mathsf{bill};\\ \mathbf{j}\oplus\mathbf{b};\\ \mathrm{NP}\end{array} \qquad \lambda\varphi_3.\varphi_3\circ\mathsf{read}\circ\mathsf{the}\circ\mathsf{same}\circ\mathsf{book};\ \lambda X\exists\mathfrak{f}\forall x <_a X.\,\mathbf{read}(\iota(\mathfrak{f}(\mathbf{book})))(y);\ \mathrm{S|NP}}{\mathsf{john}\circ\mathsf{and}\circ\mathsf{bill}\circ\mathsf{read}\circ\mathsf{the}\circ\mathsf{same}\circ\mathsf{book};\ \exists\mathfrak{f}\forall x <_a \mathbf{j}\oplus\mathbf{b}.\mathbf{read}(\iota(\mathfrak{f}(\mathbf{book})))(x);\ \mathrm{S}}$$

The transitive verb *read* combines with an NP containing a variable of category Adj (where Adj abbreviates N/N); after the complete sentence is built, we first abstract on the subject variable φ_3 (step ①) and then on the adjective variable φ_1 (step ②), creating exactly the type of category that Barker's denotation for *same* looks for as an argument. Then (step ③), the functional prosody specified for *same* 'lowers' the string *same* into the string preceding the prosodic representative of *book*, while on the semantic side, the operator representing the associated semantics applies to the doubly abstracted proposition of type S|NP|Adj. The resultant translation asserts that a reading relation holds between each member of the sum **john**$\oplus$**bill** on the one hand and a specific book on the other. More generally, the abstractions at steps ① and ② in effect yield a relation between a specific individual and a sum, with the role of *same* nothing more than to identify the former with a unique individual and the latter with each of the atomic elements of the sum.

It should be clear from the above exposition that our own analysis in the previous section takes Barker's original work as its basis. In particular, the core of our own proposal is Barker's double abstraction treatment of *same*. Comparison between (77) and our analysis in section 2.2.2 makes the point: our **resp** operator, applied to **read**, yields a functor which maps each component of one aggregate object (the individuals in the conjunction *John and Bill*) with uniquely corresponding elements of some other composite object. The key difference between our proposal and Barker's is that the former decomposes two components of Barker's semantics of *same* and attributes one of them to the more general **resp** operator. Specifically, in our analysis, the semantics of symmetrical and summative predicates in effect consists of (i) an operation, attributed to the general **resp** operator, which can apply iteratively to composite objects to yield new objects of the same type (specifically, tuples), and (ii) a component, specific to the lexical meanings of *same*/*different*, etc., which imposes particular conditions on one of the tuples targeted by this operation. By contrast, in Barker's analysis, the single operation corresponding to *same*/*different*, etc., applies to composite objects (via the abstraction on sums in (76)) but yields a proposition in which reference to that composite object is buried, as it were, within the quantification over parts of the abstracted sum. This aspect of Barker's analysis makes a unified analysis of symmetrical, 'respective' and summative predicates difficult. Moreover, the lack of a recursive mechanism that keeps track of the structure of a sum/tuple-type object entails severe empirical difficulties when multiple instances of symmetrical predicates are present, as we discuss in detail in Sect. 3.2.2.

Barker (2012) does offer a revision of his earlier analysis to cover the special case of *same/same* interaction, but, as we argue directly, this modification fails when extended to other kinds of 'mixed' predication, such as *same/different* or *different/different* interactions. The difficulty with the definition of *same* in Barker (2007) (i.e. (76)) emerges in the following outline of what a derivation would be like for (78) following Barker (2007).

(78) John and Bill put the same object in the same box.

The proof begins by deriving the term in (79) (in what follows, $\mathfrak{f}, \mathfrak{g}, \mathfrak{h}$ are variables over choice functions):

(79) $\varphi_1 \circ \mathsf{put} \circ \mathsf{the} \circ \varphi_2 \circ \mathsf{object} \circ \mathsf{in} \circ \mathsf{the} \circ \varphi_3 \circ \mathsf{box}; \mathbf{put}(\iota(\xi(\mathbf{object})))(\iota(\zeta(\mathbf{box})))(u); \mathrm{S}$

Abstraction first on φ_1 and then on φ_2 yields a doubly abstracted proposition, with syntactic category S|NP|Adj, which is of the right type to be given as an argument to the denotation of *same*. By applying (76) to this term, we obtain the following expression:

(80) $\lambda\varphi_1.\varphi_1 \circ \mathsf{put} \circ \mathsf{the} \circ \mathsf{same} \circ \mathsf{object} \circ \mathsf{in} \circ \mathsf{the} \circ \varphi_3 \circ \mathsf{boxes};$
$\lambda X.\forall x <_a X \exists \mathfrak{f}.\mathbf{put}(\iota(\xi(\mathbf{object})))(\epsilon(\zeta(\mathbf{box})))(x); \mathrm{S|NP}$

Then, abstracting on φ_3 once again yields an expression of type S|NP|Adj, which is of the right type to be given as an argument to the *same* operator displayed in (76). Reapplying the *same* operator to this expression yields the semantics in (81):

(81) $\lambda Y \exists \mathfrak{g} \forall y <_a Y.\lambda X[\exists \mathfrak{f}.\forall x <_a X.\mathbf{put}(\iota(\mathfrak{f}(\mathbf{object})))(\iota(\mathfrak{g}(\mathbf{box})))(x)](y)$

The result in (81) is a clear mismatch between the sum-type variable X and the variable y (which is supposed to be substituted to X), identified by the use of $<_a$ as an *atom* of another sum Y, i.e., an entity that itself has no parts. (81) is therefore semantically incoherent, an outcome which seems to have been fixed in the revised definition of *same* in Barker (2012) in (82).

(82) $\lambda \mathscr{F} \lambda x \exists \mathfrak{f}.\mathscr{F}(\mathfrak{f})(x)$

Here, 'x can ranger over individual[s], or else over sets (sums)' (Barker 2012, 11). This revision effectively corresponds to taking X in (81) to be a variable over either atoms or sums, and suppressing the quantification over the atoms of X in the latter case. The abstraction on X thus becomes an abstraction on an individual variable underspecified for subtype. In order for this alternative approach to give rise to the required outcome, however, another modification is required, whereby every verb has a lexical entry incorporating a distributive operator and a sum-type subject (in fact, the same assumption needs to be made on all other argument positions, since an NP containing *same* can appear in any argument position); thus, for example, the verb *put* has the following lexical entry:[13]

(83) put; $\lambda x \lambda w \lambda Y.\forall y <_a Y.\mathbf{put}(x)(w)(y)$; (NP\S)/PP/NP

for the subject-distributive reading. The double application of the *same* operator in (82) to (83) will yield a functor which, when applied to *John and Bill*, yields the following:

(84) $\exists \mathfrak{g} \exists \mathfrak{f}.\forall y <_a \mathbf{j} \oplus \mathbf{b}.\mathbf{put}(\iota(\mathfrak{f}(\mathbf{object})))(\iota(\mathfrak{g}(\mathbf{box})))(y)$

This is the correct denotation, since the wide-scoping existentially-bound choice functions guarantee that there is at least one object and one box such that both John and Bill put that object in that box.

Unfortunately, the only such mixed predications for which this proposal yields the correct interpretation are *same*/*same* sentences. Further examination of a range of mixed symmetrical predication constructions, where we assume (82) as the translation for *same* throughout, reveals a serious lack of generality in this proposal. This outcome suggests that 'parasitic scope' alone isn't enough to provide a fully general compositional analysis of symmetrical predicates, especially when mixed predication cases are taken into account.

3.2.2 Empirical issues with mixed predication

We can illustrate the problem with multiple symmetrical predicates with example (85):

(85) John and Bill put the same object in different boxes.

Barker (2007) gives the semantics of *different* as in (86):

(86) $\lambda \mathscr{F} \lambda X \forall \mathfrak{g}\, \forall z, v <_a X.\ [\mathscr{F}(\mathfrak{g})(z) \wedge \mathscr{F}(\mathfrak{g})(v)] \rightarrow [z = v]$

with which the translation in (87b) is assigned to (87a).[14]

(87) a. John and Bill read a different book.
b. $\forall \mathfrak{f} \forall z, v <_a \mathbf{j} \oplus \mathbf{b}[\mathbf{read}(\iota(\mathfrak{f}(\mathbf{book})))(z) \wedge\ \mathbf{read}(\epsilon(\mathfrak{f}(\mathbf{book})))(v)] \rightarrow [z = v]$

To paraphrase, (87b) says that whatever choice function one chooses, the only way in which two (potentially distinct) people out of the set $\{\mathbf{j}, \mathbf{b}\}$ read the book that the choice function returns is when the two people are the same ones. In other words, there's no single common

[13] Barker (2012) is actually not explicit on this point, but we believe that this (rather than a separate distributive operator) is his assumption given that his translation (for *The men put the same object in the same box*):

(i) $\mathbf{the\text{-}men}(\lambda x.\exists \mathfrak{g} \exists \mathfrak{f}.\mathbf{put}(\iota(\mathfrak{f}(\mathbf{object})))(\iota(\mathfrak{g}(\mathbf{box})))(x))$

will lead to an incorrect scope relation between the existential quantifiers for the choice functions and the (covert) universal quantifier associated with the distributive operator (making the sentence true just in case each of John and Bill put *some* object in *some* box) if the (implicit) distributive operator were to take scope over the whole property ($\lambda x.\exists \mathfrak{g} \exists \mathfrak{f} \ldots$) that is the denotation of the VP.

[14] ϵ here is the meaning of the indefinite article *a*. Since the choice function returns a singleton set, the choice of the article (between *the* for *same* and *a* for *different*) is immaterial in Barker's analysis.

book that John and Bill both read. Barker (2007, 442) states that '[a]s near as I can tell, this denotation gives reasonable truth conditions for NP-internal uses' of *different*.[15]

The translation in (86) might work for (87a), but in the case of (85), the analysis becomes much more difficult. The following translation is predicted for (85) in Barker's approach, by modelling the analysis on the *same*/*same* example illustrated in the previous section:[16]

(88) $\forall \mathfrak{g} \forall y, u <_a \mathbf{j} \oplus \mathbf{b}[\exists \mathfrak{f}.\ \mathbf{put}(\iota(\mathfrak{f}(\mathbf{object})))(\epsilon(\mathfrak{g}(\mathbf{box})))(y)] \wedge$
$[\exists \mathfrak{h}.\ \mathbf{put}(\iota(\mathfrak{h}(\mathbf{object})))(\epsilon(\mathfrak{g}(\mathbf{box})))(u)] \rightarrow [y = u]$

But since the variables bound by $\exists$ ($\mathfrak{f}$ and $\mathfrak{h}$) in these two conjuncts are distinct, there is no guarantee that the choice function in the first conjunct picks out the same object as the choice function in the second conjunct. Thus, (88) means that there is no box in which both John and Bill put some (possibly distinct) object in. This means that if John put an apple in Box A and Bill put an orange in Box A, that alone makes (88) false. However, the original sentence (85) can still be true in such a situation: suppose John put Bill's orange in Box B in addition. (88) is obviously wrong as the truth conditions for (85).

An equally incorrect result follows when the subject is singular, as in (89):

(89) John put the same object in different boxes.

For in this case, we obtain:

(90) $\forall \mathfrak{g} \forall y, u <_a \mathbf{j}[\exists \mathfrak{f}.\ \mathbf{put}(\iota(\mathfrak{f}(\mathbf{object})))(\epsilon(\mathfrak{g}(\mathbf{box})))(y)] \wedge$
$[\exists \mathfrak{h}.\ \mathbf{put}(\iota(\mathfrak{h}(\mathbf{object})))(\epsilon(\mathfrak{g}(\mathbf{box})))(u)] \rightarrow [y = u]$

Here, the same type mismatch problem between sum-type variables and atomic individual-type variables arises as in the case of (81). To resolve this type mismatch, let us assume a more complex interpretation of $x <_a X$, where $<_a$ designates an (atomic-)part-whole relation when X is a sum but reduces to equality when X is an atom. This, after all, is essentially the same approach reflected in (82) with underspecified x.

On the interpretation of $x <_a X$ as $x = X$ when X is atomic, we have $y = u = \mathbf{j}$, so that the consequent in (90) is just $\mathbf{j} = \mathbf{j}$, a tautology entailing that (89) is necessarily true regardless of the circumstances.

In the foregoing discussion, we assumed that *different* scopes over *same*. There is another derivation for (85) with the opposite scoping relations, and indeed, except for abstraction on the PP argument preceding the abstraction on the direct object, the proof proceeds just as already discussed, mutatis mutandis, resulting in (91).

(91) $\lambda X \exists \mathfrak{f}. \forall \mathfrak{g}. \forall y, u <_a X[\mathbf{put}(\iota(\mathfrak{f}(\mathbf{object})))(\epsilon(\mathfrak{g}(\mathbf{box})))(y) \wedge$
$\mathbf{put}(\iota(\mathfrak{f}(\mathbf{object})))(\epsilon(\mathfrak{g}(\mathbf{box})))(u)] \rightarrow [y = u]$

[15] But note that (87b) can be true when neither John nor Bill read any book, which seems to be too weak as the truth conditions for (87a). One might think that this would follow if we assumed that (87a) presupposes that both John and Bill read at least one book. We refrain from speculating on this point further, since Barker himself does not address this issue explicitly. In any event, this point is orthogonal to the failure of multiple *same*/*different* sentences in Barker's (2007, 2012) approach.

[16] For simplicity, we suppress the covert distributive operator in the lexical meanings of verbs, assuming that **put** in what follows (if appropriate) is an abbreviation of (83) (or some other appropriate lexical meaning). Also, we keep the translation for *different* in Barker (2007), since Barker (2012) does not offer a revision for the translation for *different* (nor is such a revision obvious given the revised translation for *same* given in (82)).

This is the right result for (85).[17] But it does not work for (89), for if the subject is singular, we again wind up with a tautological interpretation for exactly the same reasons as in (90):

(92) $\exists \mathfrak{f}.\forall \mathfrak{g}.[\mathbf{put}(\iota(\mathfrak{f}(\mathbf{object})))(\epsilon(\mathfrak{g}(\mathbf{box})))(\mathbf{j}) \wedge \mathbf{put}(\iota(\mathfrak{f}(\mathbf{object})))(\epsilon(\mathfrak{g}(\mathbf{box})))(\mathbf{j})] \rightarrow [\mathbf{j} = \mathbf{j}]$

Thus, regardless of what the scopal relation between *same* and *different* is, we obtain an interpretation that has nothing to do with the actual truth conditions on such sentences.

The difficulties raised so far turn out to be only the tip of a very deep iceberg. The case of *different*/*different* interactions leads to even stranger results. For example, (93a) winds up with the semantics for the VP as in (93b).

(93) a. John and Bill put different objects in different boxes.
b. $\lambda U \forall w, y <_a U\, \forall \mathfrak{f}\, \forall \mathfrak{g} \lambda W[\forall z, v <_a W[\mathbf{gave}(\iota(\mathfrak{f}(\mathbf{thing})))(\epsilon(\mathfrak{g}(\mathbf{person})))(z) \wedge$
$\mathbf{gave}(\iota(\mathfrak{f}(\mathbf{thing})))(\epsilon(\mathfrak{g}(\mathbf{person})))(v)] \rightarrow [z = v]](w)$
$\wedge \lambda W[\forall z, v <_a W[\mathbf{gave}(\iota(\mathfrak{f}(\mathbf{thing})))(\epsilon(\mathfrak{g}(\mathbf{person})))(z) \wedge$
$\mathbf{gave}(\iota(\mathfrak{f}(\mathbf{thing})))(\epsilon(\mathfrak{g}(\mathbf{person})))(v)] \rightarrow [z = v]](y) \rightarrow [y = w]$

The subtyping mismatch problem is again evident, with atoms supplied as arguments to abstracts on sums, again requiring an interpretation of $<_a$ which reduces to equality in the case where the second relatum is an atom. But the assumption that $x <_a u$ entails $u = x$ has, as a corollary, the consequence that $x, y <_a u$ entails that $u = x = y$. The result is that (93b) reduces to (94):

(94) $\lambda U \forall w, y <_a U\, \forall \mathfrak{f}\, \forall \mathfrak{g}\, [[\mathbf{gave}(\iota(\mathfrak{f}(\mathbf{thing})))(\epsilon(\mathfrak{g}(\mathbf{person})))(w)] \rightarrow [w = w] \wedge$
$[\mathbf{gave}(\iota(\mathfrak{f}(\mathbf{thing})))(\epsilon(\mathfrak{g}(\mathbf{person})))(y)] \rightarrow [y = y]] \rightarrow [y = w]$

But this result makes no sense. What we have in (94) is an implication, whose antecedent is a tautology (itself composed of a conjunction of tautologies of the same general form $\psi \rightarrow (\alpha = \alpha)$), which means that the whole conditional statement is equivalent to its consequent $y = w$. The variables w and y range over the atoms of whatever sum the property in (94) denotes. Thus, it is predicted that (93a) means that John and Bill are the same person.[18]

Finally, in view of what we have argued in Sect. 2 to be the parallel semantic action of symmetrical predicates on the one hand and 'respective' readings on the other, one might wonder whether Barker's analysis outlined above extends to 'respective' readings. Although Barker himself does not attempt to establish a connection between these two phenomena, Kubota (to appear) shows how to extend Barker's (2007) analysis of *same* to 'respective' readings. Unsurprisingly, this approach turns out not to carry over to multiple 'respective' readings. The same kind of problem outlined above in connection to multiple symmetrical predicate sentences—a type mismatch which blocks iterated applications of operators—arises in cases of multiple 'respective' readings. In short, after the first application of the 'respective'

[17]This result would mean that, in examples like (85), we somehow need to ensure that *same* always outscopes *different*, whenever the two cooccur in a sentence and induce internal readings 'parasitic' on the same plural or quantificational expression, whatever their syntactic relation is. Such a constraint is unheard of in the semantics literature.

[18]Note that none of the difficulties displayed above with respect to (85), (89) and (93a) are ameliorated in any sense by the presupposition discussed above in connection with (87), since in none of these cases, adding the (alleged) presupposition to the predicted meanings yields interpretations anywhere close to the correct meanings of the sentences.

operator, the result is a boolean conjunction whose parts are no longer accessible as sum components, making iterative application in principle impossible.

Given the above discussion, there seems no alternative to adopting a different kind of analysis, such as one along the lines of G&K and our own, incorporating a different formal interpretation of the phenomena, in order to get iterated symmetrical predicates and 'respective' readings to work.

3.3 Chaves (2012)

3.3.1 Chaves' analysis of 'respective' readings

The proposal offered in Chaves (2012) differs from the approach taken in Gawron and Kehler (2004) and our own analysis in that it takes 'respective' readings in cases such as (95), where a one-to-one invitation relation holds between the conjuncts of two conjoined NPs, to be only a special case of the cumulative readings.

(95) Bill and Tom invited Sue and Anne to the party (respectively).

The cumulative reading is typically exhibited by examples like (96). On this reading, (96) is true as long as the using relation holds between members of a set of Dutch companies (whose cardinality is 700) and members of a set of American computers (whose cardinality is 10,000) but the sentence does not specify which particular company is related to which particular computer.

(96) 700 Dutch companies used 10,000 American computers.

Chaves maintains that the semantics of individual-to-individual 'respectively' readings as in (95) (but *not* cases such as (1), repeated below as (97)) is nothing more than such a cumulative reading with an additional assumption that the mapping in question is a bijection defined, typically, by the order in which the pluralities participating in the mapping are presented, forced by an overt token of *respectively*.

But this approach will not suffice to capture the truth conditions on (97) (= (1)):

(97) John and Bill sang and danced, respectively. (= 'John sang and Bill danced')

The problem is that in this case, atomic individuals are mapped not to other atomic individuals, as in the case of (95), but to predicates of which they themselves are the arguments. This cannot be handled by any of the mechanisms previously suggested for cumulative readings. Chaves therefore proposes the translation in (98) for the coordination marker *and*, which, according to him, has the effect of providing two possible interpretations for (97) (and mutatis mutandis all other cases involving bijective mappings from individuals to relations): a 'weak identity' reading, in which the conjoined individuals are mapped bijectively to the separate properties in the sum denoted by *sang and danced*, and a 'strong identity' reading, whereby both individuals sang and both individuals danced.[19]

[19] In the translation of *and* that Chaves (2012) provides, the variable e is already existentially bound and hence cannot be identified with the argument of his 'respective' operator, leading to incorrect truth conditions. (98) is a modification of Chaves' translation which corrects this problem. We follow Chaves in leaving the variables x_k, y_k unbound.

(98) $\lambda P \lambda Q \lambda z_0 \ldots \lambda z_n \lambda e.[e = (e_1 \oplus e_2) \wedge Q(x_0)\ldots(x_n)(e_1) \wedge P(y_0)\ldots(y_n)(e_2) \wedge z_0 = (x_0 \oplus y_0) \wedge \ldots \wedge z_n = (x_n \oplus y_n)]$

This operator conjoins two relations, and distributes their conjoined shared dependents so that each of the relations takes one of the conjuncts in each syntactic argument position as its own semantic argument, leaving the other conjunct as an argument of the other relation. We can illustrate how this operator works with a simple derivation of (97) (we take the liberty of reformulating Chaves' analysis in our proof format, but it should be borne in mind that nothing substantial changes by this reformulation):

(99)

$$\dfrac{\dfrac{\dfrac{\begin{array}{l}\textsf{sang};\\ \lambda x \lambda e' \mathbf{sang}(x)(e');\\ \mathrm{VP}\end{array} \quad \dfrac{\begin{array}{l}\textsf{and};\\ \lambda P \lambda Q \lambda z_0 \lambda e.(e = (e_1 \oplus e_2) \wedge Q(x_0)(e_1)\\ \wedge P(y_0)(e_2) \wedge z_0 = (x_0 \oplus y_0); (\mathrm{VP}\backslash\mathrm{VP})/\mathrm{VP}\end{array} \quad \begin{array}{l}\textsf{danced};\\ \lambda x \lambda e' \mathbf{danced}(x)(e'));\\ \mathrm{VP}\end{array}}{\begin{array}{l}\textsf{and} \circ \textsf{danced};\\ \lambda Q \lambda z_0 \lambda e.(e = (e_1 \oplus e_2) \wedge Q(x_0)(e_1)\\ \wedge \mathbf{danced}(y_0)(e_2) \wedge z_0 = (x_0 \oplus y_0)); \mathrm{VP}\backslash\mathrm{VP}\end{array}}}{\begin{array}{l}\textsf{sang} \circ \textsf{and} \circ \textsf{danced};\\ \lambda z_0 \lambda e.(e = (e_1 \oplus e_2) \wedge \mathbf{sang}(x_0)(e_1) \wedge \mathbf{danced}(y_0)(e_2) \wedge z_0 = (x_0 \oplus y_0)); \mathrm{VP}\end{array}} \quad \begin{array}{l}\vdots \quad \vdots\\ \textsf{john} \circ\\ \textsf{and} \circ\\ \textsf{bill};\\ \mathbf{j} \oplus \mathbf{b};\\ \mathrm{NP}\end{array}}{\begin{array}{l}\textsf{john} \circ \textsf{and} \circ \textsf{bill} \circ \textsf{sang} \circ \textsf{and} \circ \textsf{danced};\\ \lambda e.(e = (e_1 \oplus e_2) \wedge \mathbf{sang}(x_0)(e_1) \wedge \mathbf{danced}(y_0)(e_2) \wedge (\mathbf{j} \oplus \mathbf{b}) = (x_0 \oplus y_0)); \mathrm{S}\end{array}}}{\begin{array}{l}\textsf{john} \circ \textsf{and} \circ \textsf{bill} \circ \textsf{sang} \circ \textsf{and} \circ \textsf{danced};\\ \exists e''.\, e'' = (e_1 \oplus e_2) \wedge \mathbf{sang}(x_0)(e_1) \wedge \mathbf{danced}(y_0)(e_2) \wedge (\mathbf{j} \oplus \mathbf{b}) = (x_0 \oplus y_0); \mathrm{S}\end{array}}$$

We assume that the event variable e is, by convention, existentially bound at the last step of the derivation (i.e., at the 'topmost' S node). If $x_0 = y_0$ in (98), we obtain the 'strong identity' interpretation; if the two variables are distinct, we get the 'weak identity' interpretation. The *respectively* operator is supposed to impose a further constraint on the interpretation obtained above, to force the pairings $\mathbf{j} = x_0, \mathbf{b} = y_0$, thereby eliminating the 'strong identity' interpretation possibility.

3.3.2 Difficulties with Chaves' proposal

One advantage that Chaves claims for his approach, relative to the solution offered by G&K, is that his analysis is semantically monotonic; that is, semantic representations that have been derived at one stage of composition are never subsequently altered in the course of further derivation. In particular, his analysis leaves the sum $\mathbf{j} \oplus \mathbf{b}$ undivided, and the bijective interpretation is imposed on (97) by virtue of the requirement that the pairing between event variables and argument variables must, by the *and* operator, be one-to-one. In contrast, G&K (and according to Chaves all other previous approaches as well) take the semantics of coordination in 'respectively' constructions to be a sum, but then provide a semantic analysis which breaks up ('destroys', in Chaves' phrasing) the sums by distributing their atoms over relations and properties, as described in Sect. 3.3.1. Our own analysis presented in Sect. 2 is clearly non-monotonic in this specific sense.

But to the extent that monotonicity is a virtue, it is one that Chaves' account lacks to at least the same (or even a greater) degree as G&K's. To begin with, consider the consequences of (100) for his analysis.

(100) Different newspapers are running conflicting reports. The Guardian and the Telegraph reported that Michael Phelps won the silver medal and the gold medal respectively. (Chaves 2012, 316)

There is a major difficulty for Chaves' approach here, one which neither of the analytic strategies that he appeals to can resolve. The 'bijection on cumulative analysis' approach is inapplicable here, since analyses of cumulative readings require the sums related to each other cumulatively to be co-arguments of the same predicate—clearly not the case with *The Guardian and the Telegraph* (subject of *reported*) and *the silver medal and the gold medal* (object of *won*). This leaves only the individual/relation mapping analysis represented by (98) as an option—but this too is unusable, since the individuals in this mapping must be the arguments of the corresponding relations, again a strictly local relationship. Both approaches, dependent as they are on the local relationship between the terms involved in the 'respective' mapping, therefore appear untenable.

Chaves' counter to the challenge posed by the nonlocality of (100) is to claim that it is only apparent, arising as an instance of prosodic deletion, as per the linearization-based ellipsis (LBE) treatment of nonconstituent coordination in the recent HPSG literature (see, e.g., Beavers and Sag (2004), Chaves (2008) and Hofmeister (2010) for some representative proposals, and Levine (2011) and Kubota and Levine (2013b) for extensive critiques of this approach). For (100), the overt prosodic form supposedly corresponds to an underlying structure in (101):

(101) The Guardian and the Telegraph reported [that Michael Phelps won the silver medal] and [~~that Michael Phelps won~~ the gold medal] respectively.

We now have two conjoined arguments of a single predicate (*reported*), which can be treated in a fashion strictly parallel to (95), via whatever cumulative interpretation mechanism is assumed along with the 'respective' operator whose semantic action was sketched earlier. But by accepting the Beavers and Sag (2004) LBE analysis, Chaves winds up allowing a solution which not only (i) violates his own superiority-of-monotonicity doctrine, but also (ii) falsifies his own account of 'respective' readings based on (98) and his 'respective' operator.

So far as the first point is concerned, consider the following example:

(102) John told some joke to Bill on Tuesday and to Sue on Wednesday.

Since neither *to Bill on Tuesday and to Sue on Wednesday* nor *some joke to Bill on Tuesday and to Sue on Wednesday* is a constituent, the coordination in (102) must involve ellipsis:

(103) John told some joke to Bill on Tuesday and ~~told some joke~~ to Sue on Wednesday.

The problem posed by having a full VP coordination in the underlying structure corresponding to the apparent nonconstituent coordination in (102) was recognized in Beavers and Sag (2004). Since in the underlying structure the coordination outscopes *some joke*, whereas the prosodic yield of the ellipsis operation allows one to construe *some joke* as having scope over the conjunction, Beavers and Sag add the following special condition to the interpretation of coordinate structures:

(104) Quantifier Merger (Beavers and Sag 2004, 62)
For any elided phrase denoting a generalized quantifier in the domain of either conjunct, the semantics of that phrase may optionally be identified with the semantics of its non-elided counterpart.

This, of course, constitutes a complete break with semantic monotonicity (see Levine (2011) and Kubota and Levine (2013a) for a detailed critique). 'Representations previously built' are indeed changed and deleted, and there seems to be no other possibilities for achieving the necessary readings under LBE assumptions.

But the challenge posed by the need to introduce ellipsis into the interpretation process goes well beyond issues of doctrinal consistency. Consider, for example, the NCC/'respectively' example (40) discussed above, repeated here as (105a):[20]

(105) a. I lent *Syntactic Structures* and *Barriers* to Robin on Thursday and to Mary on Friday (respectively).

b. I [lent *Syntactic Structures* and *Barriers* to Robin on Thursday] and ~~(I) lent *Syntactic Structures* and *Barriers*~~ to Mary on Friday (respectively).

In order for Chaves' setup to work as required, it is crucial that the arguments x_k, y_k which are going to enter into the bijective mapping imposed by *and* in (98) each be associated with unsaturated corresponding argument positions. But it is far from obvious how this condition can be fulfilled in the elliptical analysis depicted in (105b), for that analysis appears to entail that the predicate *lent* is already saturated for its direct and dative object arguments in each of the two conjuncts. Moreover, if (as in Chaves (2009) and following the standard assumptions in HPSG) we take adjuncts to be both syntactic and semantic functors on VPs, then P, Q in (98) must correspond to the two temporal adjuncts, which are themselves not conjoined in (105b). Thus, the terms which must be put in pairwise correspondence—*Syntactic Structures*/Robin/Thursday and *Barriers*/Mary/Friday—cannot be associated in the right way by any conceivable application of (98) (nor does the cumulative reading strategy work). Whatever operation is required to obtain the correct semantics on the basis of (105b) would have to be severely nonmonotonic. While in principle such an extremely specific and 'purpose-built' operation can probably be formulated, any such solution almost certainly fails to capture the relevant empirical generalization.

In a sense, Chaves' proposal can be thought of as an attempt to lexicalize the effects of the 'respective' operator of the sort posited in G&K's and our approach. While a strictly lexical approach may be attractive if it can handle all the relevant data uniformly, the discussion above suggests that such an approach faces severe empirical challenges.

[20]The empirical failure of an ellipsis-based solution for non-local 'respective' readings can be corroborated by considering cross-linguistic evidence. First, as discussed in Kubota (to appear), examples completely parallel to (105) can be constructed in Japanese. Second, in Japanese, nominal coordination involves a different conjunction morpheme from verbal coordination. Thus, a reduction to verbal coordination in examples parallel to (101) does not work. The Japanese counterpart of (101) in (i) does have the 'respective' reading, and is thus problematic for Chaves's analysis.

(i)

Asahi-shinbun-to	Mainichi-Shinbun-wa	sorezore	toosho	[Phelps-ga	kin-medaru-to
Asahi-and	Mainichi-TOP	respectively	at.first	Phelps-NOM	gold-medal-and

gin-medaru-o	tot-ta]-to	gohoo-si-ta.
silver-medal	win-PAST-COMP	falsely.report-PAST

'Asahi and Mainichi at first falsely reported that Phelps won the gold medal and the silver medal respectively.'

4 Conclusion

As should be clear from the discussion above, our analysis incorporates ideas from both G&K's analysis of 'respective' readings and Barker's analysis of symmetrical predicates. While the strictly local approach in G&K's original formulation and the nonlocal approach by Barker via 'parasitic scope' may initially look quite different, the effects of the two types of operations (or series of operations) that they respectively invoke are rather similar: they both establish some correspondence between the internal structures of two terms that do not necessarily appear adjacent to each other in the surface form of the sentence. The main difference is *how* this correspondence is established: G&K opt for a series of local composition operations (somewhat reminiscent of the way long-distance dependencies are handled in lexicalist frameworks such as CCG and G/HPSG), whereas Barker does it by a single step of nonlocal mechanism (in a way analogous to a movement-based analysis of long-distance dependencies).

A question that arises at this point is whether we have gained any deeper understanding of the relationship between the respective solutions proposed by these authors, by recasting them within the general syntax-semantics interface of Hybrid TLCG. We do think that we have. Note first that, by recasting these proposals in our setup, we have a clearer picture of the core mechanism underlying both of these phenomena. This in turn enabled us to overcome the major empirical limitations of both G&K's analysis (with respect to non-canonical coordination) and Barker's analysis (with respect to iterated symmetrical predicates).

But we can go even further. As discussed in detail in Kubota and Levine (2014c), the local and nonlocal modelling of 'respective' predication from the two previous works can be shown to have a very close relationship formally, since in Hybrid TLCG, the local composition rules for 'percolating up' the tuple structure from coordination that are crucially involved in G&K's setup can be derived as theorems in a grammar that essentially implements Barker's approach of nonlocal 'respective' predication via hypothetical reasoning. More specifically, by introducing some auxiliary assumptions, the following two rules (where Rule 1 corresponds to G&K's **Dist** operator and Rule 2 corresponds to the **Dist′** operator needed but apparently missing from G&K's setup) are both derivable as theorems in the system we presented in Sect. 2 (for proofs, see Kubota and Levine (2014c)):

(106) a. RULE 1

$$\frac{\mathsf{a};\ \boldsymbol{\mathcal{F}};\ A/B \qquad \mathsf{b};\ \langle \boldsymbol{a_1} \ldots \boldsymbol{a_l} \rangle;\ B}{\mathsf{a} \circ \mathsf{b};\ \langle \boldsymbol{\mathcal{F}}(\boldsymbol{a_1}) \ldots \boldsymbol{\mathcal{F}}(\boldsymbol{a_l}) \rangle;\ A}$$

b. RULE 2

$$\frac{\mathsf{a};\ \langle \boldsymbol{\mathcal{F}_1} \ldots \boldsymbol{\mathcal{F}_n} \rangle;\ A/B \qquad \mathsf{b};\ \boldsymbol{a};\ B}{\mathsf{a} \circ \mathsf{b};\ \langle \boldsymbol{\mathcal{F}_1}(\boldsymbol{a}) \ldots \boldsymbol{\mathcal{F}_n}(\boldsymbol{a}) \rangle;\ A}$$

It can moreover be formally proven that the local and nonlocal modellings of 'respective' predication make exactly the same predictions as to the range of available 'respective' readings and internal readings for symmetrical predicates: in both approaches, it is possible to relate two (or more) terms embedded arbitrarily deeply in different parts of the sentence in the 'respective' manner. This is an interesting result, since one might *a priori* be inclined to think that the local modelling would be inherently less powerful than the nonlocal modelling. It is of course conceivable to entertain a constrained version of local modelling in which percolation of a tuple structure is blocked in certain syntactic environments (such as islands and complements of certain types of predicates). But similar effects can probably be achieved in the nonlocal modelling as well, by constraining the steps of hypothetical reasoning involved in 'respective' predication in some way or other (in relation to this, see Pogodalla and Pompigne

(2012) for an implementation of scope islands in Abstract Categorial Grammar, a framework of CG closely related to Hybrid TLCG).

One may then ask the following question: can we make a case for either the local modelling or the nonlocal modelling by comparing how naturally one can implement the relevant locality constraints? This is an interesting and important question, but unfortunately, we have to leave a detailed investigation of this issue to a future study. The main challenge that this task faces is that the empirical question of what exactly is the nature of the relevant locality constraints (if there are any) for 'respective' and symmetrical predicates is heavily underinvestigated, in contrast to other domains such as long-distance dependencies. In fact, so far as we were able to identify, this issue has been explicitly discussed only with respect to symmetrical predicates, and in a fairly cursory manner in the literature. The major disputes were all from the late 80s and early 90s and since then there does not seem to have been any extensive discussion: Carlson (1987) claimed that the internal readings of *same/different* obeys the same syntactic island constraints as filler-gap dependencies, but this observation was disputed by Dowty (1985) and Moltmann (1992). We concur with these latter authors. Note in particular the following cases, all of which readily induce the internal reading for the symmetrical predicates:[21]

(107)
a. Robin and Leslie believe that the acquisition of $\left\{\begin{array}{c}\text{the same}\\ \text{different}\end{array}\right\}$ skill sets is crucial to success in the business world. (Subject island violation on the *de re* reading)

b. [Robin and Leslie usually agree about who's a bad singer at a karaoke party, and they both get immediately mad when such a person starts singing.]
But today, something weird happened: Robin and Leslie got mad when different people started singing. (Adjunct island violation)

c. The Smiths$_1$ and the Jones's$_2$ go to the same$_1$ psychiatrist and to different$_2$ psychiatrists respectively. (Coordinate Structure Constraint violation; Dowty (1985, 6))

Similar facts seem to hold for the 'respective' reading:

(108)
a. Robin and Leslie thought that studying category theory and intuitionistic logic respectively would be all that was needed for success. (Subject island violation)

b. Robin and Leslie got home before the train and the bus stopped running respectively. (Adjunct island violation)

c. Robin and Leslie named someone who was innocent and guilty respectively. (Complex NP constraint violation)

[21] Though the nature of 'covert movement' is still considerably unclear and controversial (see, for example, Ruys and Winter (2010) for a recent review), this at least means that a movement-based analysis of these expressions (via QR) cannot be motivated in terms of island-sensitivity. On the other hand, if one takes island effects to be by-products of functional factors such as constraints on real-time processing and felicity conditions on discourse (Deane 1991; Kluender 1992, 1998; Kehler 2002; Hofmeister and Sag 2010), what the above data suggests is that the processing and discourse factors which come into play in filler-gap dependencies are not the same ones which govern the interpretation of 'respective' and symmetrical predicates. In the former case, island violation *does* result in reduced acceptability, but it can be ameliorated by manipulating various non-structural factors; in the latter case, island violation simply does not seem to arise to begin with, as evidenced by the near-perfect acceptability of the examples in (107) and (108).

Whatever turns out to be the nature of locality constraints on 'respective' and symmetrical predicates, we believe that the kind of general setup we have offered in this paper should be useful for comparing different hypotheses about them, as it enables one to formulate both the local and nonlocal modelling of 'respective' predication within a single platform. At the very least, the unifying perspective we have offered on these two approaches is interesting in that it relativizes the debate between 'derivational' and 'nonderivational' theories (where the difference between the two architectures may at times have been over-emphasized by proponents of each). So far as the semantics of 'respective' predicates is concerned, our analysis shows that the extra machinery one needs to invoke in each setup (unless otherwise constrained) are largely equivalent, and that the difference in the two types of strategies representative in the two theories is more superficial than real.

A Ancillary derivations

(109)

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\varphi_1; P; \mathrm{VP/PP/NP}]^1 \quad [\varphi_2; x; \mathrm{NP}]^2}{\varphi_1 \circ \varphi_2;\ P(x);\ \mathrm{VP/PP}}/\mathrm{E} \quad \begin{array}{l}\mathsf{to} \circ \mathsf{robin};\\ \mathbf{r}; \mathrm{PP}\end{array}}{\varphi_1 \circ \varphi_2 \circ \mathsf{to} \circ \mathsf{robin};\ P(x)(\mathbf{r});\ \mathrm{VP}}/\mathrm{E} \quad \begin{array}{l}\mathsf{on} \circ \mathsf{thursday};\\ \mathbf{onTh}; \mathrm{VP\backslash VP}\end{array}}{\varphi_1 \circ \varphi_2 \circ \mathsf{to} \circ \mathsf{robin} \circ \mathsf{on} \circ \mathsf{thursday};\ \mathbf{onTh}(P(x)(\mathbf{r}));\ \mathrm{VP}}\backslash\mathrm{E}}{\varphi_2 \circ \mathsf{to} \circ \mathsf{robin} \circ \mathsf{on} \circ \mathsf{thursday};\ \lambda P.\mathbf{onTh}(P(x)(\mathbf{r}));\ \mathrm{(VP/PP/NP)\backslash VP}}\backslash\mathrm{I}^1}{\mathsf{to} \circ \mathsf{robin} \circ \mathsf{on} \circ \mathsf{thursday};\ \lambda x \lambda P.\mathbf{onTh}(P(x)(\mathbf{r}));\ \mathrm{NP\backslash(VP/PP/NP)\backslash VP}}\backslash\mathrm{I}^2
$$

(110)

$$
\dfrac{\dfrac{\dfrac{\begin{array}{l}\mathsf{I};\\ \mathbf{I};\\ \mathrm{NP}\end{array} \quad \dfrac{\begin{array}{l}\mathsf{lent};\\ \mathbf{lent};\\ \mathrm{VP/PP/NP}\end{array} \quad \dfrac{\begin{bmatrix}\varphi_1;\\ x; \mathrm{NP}\end{bmatrix}^1 \quad \begin{bmatrix}\varphi_2;\\ \mathscr{F}; \mathrm{NP\backslash(VP/PP/NP)\backslash VP}\end{bmatrix}^2}{\varphi_1 \circ \varphi_2;\ \mathscr{F}(x);\ \mathrm{NP\backslash(VP/PP/NP)\backslash VP}}/\mathrm{E}}{\mathsf{lent} \circ \varphi_1 \circ \varphi_2;\ \mathscr{F}(x)(\mathbf{lent});\ \mathrm{VP}}/\mathrm{E}}{\mathsf{I} \circ \mathsf{lent} \circ \varphi_1 \circ \varphi_2;\ \mathscr{F}(x)(\mathbf{lent})(\mathbf{I});\ \mathrm{S}}\backslash\mathrm{E}}{\lambda\varphi_2.\mathsf{I} \circ \mathsf{lent} \circ \varphi_1 \circ \varphi_2;\ \lambda\mathscr{F}.\mathscr{F}(x)(\mathbf{lent})(\mathbf{I});\ \mathrm{S|(NP\backslash(VP/PP/NP)\backslash VP)}}|\mathrm{I}^2}{\lambda\varphi_1\lambda\varphi_2.\mathsf{I} \circ \mathsf{lent} \circ \varphi_1 \circ \varphi_2;\ \lambda x\lambda\mathscr{F}.\mathscr{F}(x)(\mathbf{lent})(\mathbf{I});\ \mathrm{S|(NP\backslash(VP/PP/NP)\backslash VP)|NP}}|\mathrm{I}^1
$$

(111)

$$
\dfrac{\begin{array}{l}\mathsf{Di} \circ \mathsf{and} \circ \mathsf{Th};\\ \langle \mathbf{di}, \mathbf{th} \rangle; \mathrm{NP}\end{array} \quad \dfrac{\begin{array}{l}\mathsf{AK} \circ \mathsf{and} \circ \mathsf{Id};\\ \langle \mathbf{ak}, \mathbf{id} \rangle; \mathrm{NP}\end{array} \quad \dfrac{\begin{array}{l}\lambda\sigma_0\lambda\varphi_1\lambda\varphi_2.\sigma_0(\varphi_1)(\varphi_2);\\ \mathbf{resp};\\ (Z|X|Y)|(Z|X|Y)\end{array} \quad \begin{array}{l}\vdots \quad \vdots\\ \lambda\varphi_1\lambda\varphi_2\lambda\varphi_3.\\ \varphi_3 \circ \mathsf{sent} \circ \varphi_1 \circ \mathsf{to} \circ \varphi_2;\\ \mathbf{sent}; \mathrm{S|NP|NP|NP}\end{array}}{\begin{array}{l}\lambda\varphi_1\lambda\varphi_2\lambda\varphi_3.\varphi_3 \circ \mathsf{sent} \circ \varphi_1 \circ \mathsf{to} \circ \varphi_2;\\ \mathbf{resp}(\mathbf{sent}); \mathrm{S|NP|NP|NP}\end{array}}|\mathrm{E}}{\begin{array}{l}\lambda\varphi_2\lambda\varphi_3.\varphi_3 \circ \mathsf{sent} \circ \mathsf{AK} \circ \mathsf{and} \circ \mathsf{Id} \circ \mathsf{to} \circ \varphi_2;\\ \mathbf{resp}(\mathbf{sent})(\langle \mathbf{ak}, \mathbf{id} \rangle); \mathrm{S|NP|NP}\end{array}}|\mathrm{E}}{\begin{array}{l}\lambda\varphi_3.\varphi_3 \circ \mathsf{sent} \circ \mathsf{AK} \circ \mathsf{and} \circ \mathsf{Id} \circ \mathsf{to} \circ \mathsf{Di} \circ \mathsf{and} \circ \mathsf{Th};\\ \mathbf{resp}(\mathbf{sent})(\langle \mathbf{ak}, \mathbf{id} \rangle)(\langle \mathbf{di}, \mathbf{th} \rangle); \mathrm{S|NP}\\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\ \lambda\varphi_3.\varphi_3 \circ \mathsf{sent} \circ \mathsf{AK} \circ \mathsf{and} \circ \mathsf{Id} \circ \mathsf{to} \circ \mathsf{Di} \circ \mathsf{and} \circ \mathsf{Th};\\ \langle \mathbf{sent}(\mathbf{ak})(\mathbf{di}), \mathbf{sent}(\mathbf{id})(\mathbf{th}) \rangle; \mathrm{S|NP}\end{array}}|\mathrm{E}
$$

(112)

$$
\dfrac{\begin{array}{l}\lambda\rho\lambda\sigma\lambda\varphi.\rho(\sigma)(\varphi);\\ \lambda\mathcal{R}\lambda\mathcal{T}_\times\lambda\mathcal{U}_\times.\prod_i \mathcal{R}(\pi_i(\mathcal{T}_\times))(\pi_i(\mathcal{U}_\times));\\ (Z|X|Y)|(Z|X|Y)\end{array} \quad \dfrac{\dfrac{\dfrac{[\sigma; f; \mathrm{S|NP}]^1 \quad [\varphi; x; \mathrm{NP}]^2}{\sigma(\varphi);\ f(x);\ \mathrm{S}}|\mathrm{E}}{\lambda\varphi.\sigma(\varphi);\ \lambda x.f(x);\ \mathrm{S|NP}}|\mathrm{I}^2}{\lambda\sigma\lambda\varphi.\sigma(\varphi);\ \lambda f\lambda x.f(x);\ \mathrm{(S|NP)|(S|NP)}}|\mathrm{I}^1}{\lambda\sigma_1\lambda\varphi_1.\sigma_1(\varphi_1);\ \lambda P_\times\lambda X_\times.\prod_i \pi_i(P_\times)(\pi_i(X_\times));\ \mathrm{(S|NP)|(S|NP)}}|\mathrm{E}
$$

(113)

$$\dfrac{\lambda\sigma_0\lambda\varphi_1\lambda\varphi_2.\,\sigma_0(\varphi_1)(\varphi_2);\ \mathbf{resp};\ (Z|X|Y)|(Z|X|Y) \qquad \begin{matrix}\vdots\\ \lambda\varphi_3\lambda\varphi_4.\,\varphi_4 \circ \text{read} \circ \varphi_3;\ \mathbf{read};\ \mathrm{S|NP|NP}\end{matrix}}{\lambda\varphi_1\lambda\varphi_2.\varphi_2 \circ \text{read} \circ \varphi_1;\ \mathbf{resp}(\mathbf{read});\ \mathrm{S|NP|NP}}\ |\mathrm{E}$$

$$\dfrac{\left[\varphi;\ X_\times;\ \mathrm{NP}\right]^1 \qquad \lambda\varphi_1\lambda\varphi_2.\varphi_2 \circ \text{read} \circ \varphi_1;\ \mathbf{resp}(\mathbf{read});\ \mathrm{S|NP|NP}}{\lambda\varphi_2.\varphi_2 \circ \text{read} \circ \varphi;\ \mathbf{resp}(\mathbf{read})(X_\times);\ \mathrm{S|NP}}\ |\mathrm{E}$$

$$\dfrac{\text{john} \circ \text{and} \circ \text{bill};\ \langle \mathbf{j}, \mathbf{b}\rangle;\ \mathrm{NP} \qquad \lambda\varphi_2.\varphi_2 \circ \text{read} \circ \varphi;\ \mathbf{resp}(\mathbf{read})(X_\times);\ \mathrm{S|NP}}{\text{john} \circ \text{and} \circ \text{bill} \circ \text{read} \circ \varphi;\ \mathbf{resp}(\mathbf{read})(X_\times)(\langle \mathbf{j}, \mathbf{b}\rangle);\ \mathrm{S}}\ |\mathrm{E}$$

$$\dfrac{\lambda\varphi_1.\varphi_1;\ \lambda p_\times.\bigwedge_i \pi_i(p_\times);\ \mathrm{S|S} \qquad \text{john} \circ \text{and} \circ \text{bill} \circ \text{read} \circ \varphi;\ \mathbf{resp}(\mathbf{read})(X_\times)(\langle \mathbf{j}, \mathbf{b}\rangle);\ \mathrm{S}}{\text{john} \circ \text{and} \circ \text{bill} \circ \text{read} \circ \varphi;\ \bigwedge_i \pi_i(\mathbf{resp}(\mathbf{read})(X_\times)(\langle \mathbf{j}, \mathbf{b}\rangle));\ \mathrm{S}}\ |\mathrm{E}$$

$$\dfrac{\text{john} \circ \text{and} \circ \text{bill} \circ \text{read} \circ \varphi;\ \bigwedge_i \pi_i(\mathbf{resp}(\mathbf{read})(X_\times)(\langle \mathbf{j}, \mathbf{b}\rangle));\ \mathrm{S}}{\lambda\varphi.\text{john} \circ \text{and} \circ \text{bill} \circ \text{read} \circ \varphi;\ \lambda X_\times.\bigwedge_i \pi_i(\mathbf{resp}(\mathbf{read})(X_\times)(\langle \mathbf{j}, \mathbf{b}\rangle));\ \mathrm{S|NP}}\ |\mathrm{I}^1$$

$$\dfrac{\lambda\varphi_0\lambda\sigma_0.\sigma_0(\text{the} \circ \text{same} \circ \varphi_0);\ \mathbf{same};\ \mathrm{S|(S|NP)|N} \qquad \text{book};\ \mathbf{book};\ \mathrm{N}}{\lambda\sigma_0.\sigma_0(\text{the} \circ \text{same} \circ \text{book});\ \mathbf{same}(\mathbf{book});\ \mathrm{S|(S|NP)}}$$

$$\dfrac{\lambda\sigma_0.\sigma_0(\text{the} \circ \text{same} \circ \text{book});\ \mathbf{same}(\mathbf{book});\ \mathrm{S|(S|NP)} \qquad \lambda\varphi.\text{john} \circ \text{and} \circ \text{bill} \circ \text{read} \circ \varphi;\ \lambda X_\times.\bigwedge_i \pi_i(\mathbf{resp}(\mathbf{read})(X_\times)(\langle \mathbf{j}, \mathbf{b}\rangle));\ \mathrm{S|NP}}{\text{john} \circ \text{and} \circ \text{bill} \circ \text{read} \circ \text{the} \circ \text{same} \circ \text{book};\ \mathbf{same}(\mathbf{book})(\lambda X_\times.\bigwedge_i \pi_i(\mathbf{resp}(\mathbf{read})(X_\times)(\langle \mathbf{j}, \mathbf{b}\rangle)));\ \mathrm{S}}\ |\mathrm{E}$$

(114)

$$\dfrac{\begin{matrix}\vdots\\ \lambda\varphi_1\lambda\varphi_2\lambda\varphi_3.\varphi_2 \circ \text{gave} \circ \varphi_1 \circ \text{to} \circ \varphi_3;\ \lambda x\lambda y\lambda w.\,\mathbf{gave}(x)(w)(y);\ \mathrm{S|NP|NP|NP}\end{matrix} \qquad \begin{matrix}\vdots\\ \lambda\sigma_0\lambda\varphi_4\lambda\varphi_5\ \sigma_0(\varphi_4)(\varphi_5);\ \lambda\mathcal{R}\lambda\mathcal{T}_{\times_n}\lambda\mathcal{U}_{\times_n}.\prod_i^n \mathcal{R}(\pi_i(\mathcal{T}_{\times_n}))(\pi_i(\mathcal{U}_{\times_n}));\ \mathrm{(Z|X|Y)|(Z|X|Y)}\end{matrix}}{\lambda\varphi_4\lambda\varphi_5\lambda\varphi_3.\varphi_5 \circ \text{gave} \circ \varphi_4 \circ \text{to} \circ \varphi_3;\ \lambda\mathcal{T}_\times\lambda\mathcal{U}_\times \prod_i^n \lambda w.\mathbf{gave}(\pi_i(\mathcal{T}_\times))(w)(\pi_i(\mathcal{U}_\times));\ \mathrm{S|NP|NP|NP}}$$

$$\dfrac{\lambda\varphi_4\lambda\varphi_5\lambda\varphi_3.\varphi_5 \circ \text{gave} \circ \varphi_4 \circ \text{to} \circ \varphi_3;\ \ldots;\ \mathrm{S|NP|NP|NP} \qquad \varphi_6;\ X_\times;\ \mathrm{NP}}{\lambda\varphi_5\lambda\varphi_3.\varphi_5 \circ \text{gave} \circ \varphi_6 \circ \varphi_3;\ \lambda\mathcal{U}_\times \prod_i^n \lambda w.\mathbf{gave}(\pi_i(X_\times))(w)(\pi_i(\mathcal{U}_\times));\ \mathrm{S|NP|NP}}$$

$$\dfrac{\lambda\varphi_5\lambda\varphi_3.\varphi_5 \circ \text{gave} \circ \varphi_6 \circ \varphi_3;\ \ldots;\ \mathrm{S|NP|NP} \qquad \text{john} \circ \text{and} \circ \text{bill};\ \langle \mathbf{j}, \mathbf{b}\rangle;\ \mathrm{NP}}{\lambda\varphi_3.\text{john} \circ \text{and} \circ \text{bill} \circ \text{gave} \circ \varphi_6 \circ \text{to} \circ \varphi_3;\ \langle \lambda w.\mathbf{gave}(\pi_1(X_\times))(w)(\mathbf{j}), \lambda w.\mathbf{gave}(\pi_2(X_\times))(w)(\mathbf{b})\rangle;\ \mathrm{S|NP}}$$

$$\dfrac{\lambda\varphi_3.\text{john} \circ \text{and} \circ \text{bill} \circ \text{gave} \circ \varphi_6 \circ \text{to} \circ \varphi_3;\ \ldots;\ \mathrm{S|NP} \qquad \lambda\sigma_2\lambda\varphi_7.\sigma_2(\varphi_7);\ \lambda P_\times\lambda W_\times.\prod_i \pi_i(P_\times)(\pi_i(W_\times));\ \mathrm{(S|NP)|(S|NP)}}{\lambda\varphi_3.\text{john} \circ \text{and} \circ \text{bill} \circ \text{gave} \circ \varphi_6 \circ \text{to} \circ \varphi_3;\ \lambda W_\times.\langle \mathbf{gave}(\pi_1(X_\times))(\pi_1(W_\times))(\mathbf{j}), \mathbf{gave}(\pi_2(X_\times))(\pi_2(W_\times))(\mathbf{b})\rangle;\ \mathrm{S|NP}}$$

$$\dfrac{\lambda\varphi_3.\text{john} \circ \text{and} \circ \text{bill} \circ \text{gave} \circ \varphi_6 \circ \text{to} \circ \varphi_3;\ \ldots;\ \mathrm{S|NP} \qquad \varphi_8;\ Y_\times;\ \mathrm{NP}}{\text{john} \circ \text{and} \circ \text{bill} \circ \text{gave} \circ \varphi_6 \circ \text{to} \circ \varphi_8;\ \langle \mathbf{gave}(\pi_1(X_\times))(\pi_1(Y_\times))(\mathbf{j}), \mathbf{gave}(\pi_2(X_\times))(\pi_2(Y_\times))(\mathbf{b})\rangle;\ \mathrm{S}}$$

$$\dfrac{\lambda\varphi.\varphi;\ \lambda p_\times.\bigwedge_i \pi_i(p_\times);\ \mathrm{X|X} \qquad \text{john} \circ \text{and} \circ \text{bill} \circ \text{gave} \circ \varphi_6 \circ \text{to} \circ \varphi_8;\ \ldots;\ \mathrm{S}}{\text{john} \circ \text{and} \circ \text{bill} \circ \text{gave} \circ \varphi_6 \circ \text{to} \circ \varphi_8;\ \mathbf{gave}(\pi_1(X_\times))(\pi_1(Y_\times))(\mathbf{j}) \wedge \mathbf{gave}(\pi_2(X_\times))(\pi_2(Y_\times))(\mathbf{b});\ \mathrm{S}}$$

$$\dfrac{\begin{matrix}\vdots\\ \text{john} \circ \text{and} \circ \text{bill} \circ \text{gave} \circ \varphi_6 \circ \text{to} \circ \varphi_8;\ \mathbf{gave}(\pi_1(X_\times))(\pi_1(Y_\times))(\mathbf{j}) \wedge \mathbf{gave}(\pi_2(X_\times))(\pi_2(Y_\times))(\mathbf{b});\ \mathrm{S}\end{matrix}}{\lambda\varphi_8.\text{john} \circ \text{and} \circ \text{bill} \circ \text{gave} \circ \varphi_6 \circ \text{to} \circ \varphi_8;\ \lambda Y_\times.\mathbf{gave}(\pi_1(X_\times))(\pi_1(Y_\times))(\mathbf{j}) \wedge \mathbf{gave}(\pi_2(X_\times))(\pi_2(Y_\times))(\mathbf{b});\ \mathrm{S|NP}}$$

$$\dfrac{\begin{matrix}\vdots\\ \lambda\sigma_3.\sigma_3(\text{the} \circ \text{same} \circ \text{man});\ \mathbf{same}(\mathbf{man});\ \mathrm{S|(S|NP)}\end{matrix} \qquad \lambda\varphi_8.\text{john} \circ \text{and} \circ \text{bill} \circ \text{gave} \circ \varphi_6 \circ \text{to} \circ \varphi_8;\ \ldots;\ \mathrm{S|NP}}{\text{john} \circ \text{and} \circ \text{bill} \circ \text{gave} \circ \varphi_6 \circ \text{to} \circ \text{the} \circ \text{same} \circ \text{man};\ \mathbf{same}(\mathbf{man})(\lambda Y_\times.\mathbf{gave}(\pi_1(X_\times))(\pi_1(Y_\times))(\mathbf{j}) \wedge \mathbf{gave}(\pi_2(X_\times))(\pi_2(Y_\times))(\mathbf{b}));\ \mathrm{S}}$$

$$\dfrac{\text{john} \circ \text{and} \circ \text{bill} \circ \text{gave} \circ \varphi_6 \circ \text{to} \circ \text{the} \circ \text{same} \circ \text{man};\ \ldots;\ \mathrm{S}}{\lambda\varphi_6.\text{john} \circ \text{and} \circ \text{bill} \circ \text{gave} \circ \varphi_6 \circ \text{to} \circ \text{the} \circ \text{same} \circ \text{man};\ \lambda X_\times.\mathbf{same}(\mathbf{man})(\lambda Y_\times.\mathbf{gave}(\pi_1(X_\times))(\pi_1(Y_\times))(\mathbf{j}) \wedge \mathbf{gave}(\pi_2(X_\times))(\pi_2(Y_\times))(\mathbf{b}));\ \mathrm{S|NP}}$$

$$\dfrac{\begin{matrix}\vdots\\ \lambda\sigma_4.\sigma_4(\text{the} \circ \text{same} \circ \text{book});\ \mathbf{same}(\mathbf{book});\ \mathrm{S|(S|NP)}\end{matrix} \qquad \lambda\varphi_6.\text{john} \circ \text{and} \circ \text{bill} \circ \text{gave} \circ \varphi_6 \circ \text{to} \circ \text{the} \circ \text{same} \circ \text{man};\ \ldots;\ \mathrm{S|NP}}{\text{john} \circ \text{and} \circ \text{bill} \circ \text{gave} \circ \text{the} \circ \text{same} \circ \text{book} \circ \text{to} \circ \text{the} \circ \text{same} \circ \text{man};\ \mathbf{same}(\mathbf{book})(\lambda X_\times.\mathbf{same}(\mathbf{man})(\lambda Y_\times.\mathbf{gave}(\pi_1(X_\times))(\pi_1(Y_\times))(\mathbf{j}) \wedge \mathbf{gave}(\pi_2(X_\times))(\pi_2(Y_\times))(\mathbf{b})));\ \mathrm{S}}$$

(115)

$$\dfrac{\mathsf{gave};\ \mathbf{gave};\ \mathrm{VP/PP/NP} \qquad \begin{bmatrix}\varphi_1;\\ x;\\ \mathrm{NP}\end{bmatrix}^1}{\mathsf{gave} \circ \varphi_1;\ \mathbf{gave}(x);\ \mathrm{VP/PP}}\ /\mathrm{E}$$

$$\dfrac{\mathsf{gave} \circ \varphi_1;\ \mathbf{gave}(x);\ \mathrm{VP/PP} \qquad \begin{bmatrix}\varphi_2;\\ y;\\ \mathrm{PP}\end{bmatrix}^2}{\mathsf{gave} \circ \varphi_1 \circ \varphi_2;\ \mathbf{gave}(x)(y);\ \mathrm{VP}}\ /\mathrm{E}$$

$$\dfrac{\mathsf{gave} \circ \varphi_1 \circ \varphi_2;\ \mathbf{gave}(x)(y);\ \mathrm{VP} \qquad \begin{bmatrix}\varphi_3;\\ \mathscr{F};\\ \mathrm{VP\backslash VP}\end{bmatrix}^3}{\mathsf{gave} \circ \varphi_1 \circ \varphi_2 \circ \varphi_3;\ \mathscr{F}(\mathbf{gave}(x)(y));\ \mathrm{VP}}\ /\mathrm{E}$$

$$\dfrac{\mathsf{terry};\ \mathbf{t};\ \mathrm{NP} \qquad \mathsf{gave} \circ \varphi_1 \circ \varphi_2 \circ \varphi_3;\ \mathscr{F}(\mathbf{gave}(x)(y));\ \mathrm{VP}}{\mathsf{terry} \circ \mathsf{gave} \circ \varphi_1 \circ \varphi_2 \circ \varphi_3;\ \mathscr{F}(\mathbf{gave}(x)(y))(\mathbf{t});\ \mathrm{S}}\ \backslash\mathrm{E}$$

$$\dfrac{\mathsf{terry} \circ \mathsf{gave} \circ \varphi_1 \circ \varphi_2 \circ \varphi_3;\ \mathscr{F}(\mathbf{gave}(x)(y))(\mathbf{t});\ \mathrm{S}}{\lambda\varphi_3.\mathsf{terry} \circ \mathsf{gave} \circ \varphi_1 \circ \varphi_2 \circ \varphi_3;\ \lambda\mathscr{F}.\mathscr{F}(\mathbf{gave}(x)(y))(\mathbf{t});\ \mathrm{S|(VP\backslash VP)}}\ |\mathrm{I}^3$$

$$\dfrac{\lambda\varphi_3.\mathsf{terry} \circ \mathsf{gave} \circ \varphi_1 \circ \varphi_2 \circ \varphi_3;\ \lambda\mathscr{F}.\mathscr{F}(\mathbf{gave}(x)(y))(\mathbf{t});\ \mathrm{S|(VP\backslash VP)}}{\lambda\varphi_2\lambda\varphi_3.\mathsf{terry} \circ \mathsf{gave} \circ \varphi_1 \circ \varphi_2 \circ \varphi_3;\ \lambda y\lambda\mathscr{F}.\mathscr{F}(\mathbf{gave}(x)(y))(\mathbf{t});\ \mathrm{S|(VP\backslash VP)|PP}}\ |\mathrm{I}^2$$

$$\dfrac{\lambda\varphi_2\lambda\varphi_3.\mathsf{terry} \circ \mathsf{gave} \circ \varphi_1 \circ \varphi_2 \circ \varphi_3;\ \lambda y\lambda\mathscr{F}.\mathscr{F}(\mathbf{gave}(x)(y))(\mathbf{t});\ \mathrm{S|(VP\backslash VP)|PP}}{\lambda\varphi_1\lambda\varphi_2\lambda\varphi_3.\mathsf{terry} \circ \mathsf{gave} \circ \varphi_1 \circ \varphi_2 \circ \varphi_3;\ \lambda x\lambda y\lambda\mathscr{F}.\mathscr{F}(\mathbf{gave}(x)(y))(\mathbf{t});\ \mathrm{S|(VP\backslash VP)|PP|NP}}\ |\mathrm{I}^1$$

$$\dfrac{\begin{array}{l}\lambda\sigma_0\lambda\varphi_1\lambda\varphi_2.\\ \quad \sigma_0(\varphi_1)(\varphi_2);\\ \mathbf{resp};\\ (Z|X|Y)|(Z|X|Y)\end{array} \qquad \begin{array}{l}\lambda\varphi_1\lambda\varphi_2\lambda\varphi_3.\mathsf{terry} \circ \mathsf{gave} \circ \varphi_1 \circ \varphi_2 \circ \varphi_3;\\ \lambda x\lambda y\lambda\mathscr{F}.\mathscr{F}(\mathbf{gave}(x)(y))(\mathbf{t});\mathrm{S|(VP\backslash VP)|PP|NP}\end{array}}{\begin{array}{l}\lambda\varphi_1\lambda\varphi_2\lambda\varphi_3.\mathsf{terry} \circ \mathsf{gave} \circ \varphi_1 \circ \varphi_2 \circ \varphi_3;\\ \mathbf{resp}(\lambda x\lambda y\lambda\mathscr{F}.\mathscr{F}(\mathbf{gave}(x)(y))(\mathbf{t}));\mathrm{S|(VP\backslash VP)|PP|NP}\end{array}}\ |\mathrm{E}$$

$$\dfrac{\begin{array}{l}\lambda\varphi_1\lambda\varphi_2\lambda\varphi_3.\mathsf{terry} \circ \mathsf{gave} \circ \varphi_1 \circ \varphi_2 \circ \varphi_3;\\ \mathbf{resp}(\lambda x\lambda y\lambda\mathscr{F}.\mathscr{F}(\mathbf{gave}(x)(y))(\mathbf{t}));\mathrm{S|(VP\backslash VP)|PP|NP}\end{array} \qquad \begin{bmatrix}\varphi_4;\\ X_\times;\\ \mathrm{NP}\end{bmatrix}^4}{\lambda\varphi_2\lambda\varphi_3.\mathsf{terry} \circ \mathsf{gave} \circ \varphi_4 \circ \varphi_2 \circ \varphi_3;\ \mathbf{resp}(\lambda x\lambda y\lambda\mathscr{F}.\mathscr{F}(\mathbf{gave}(x)(y))(\mathbf{t}))(X_\times);\ \mathrm{S|(VP\backslash VP)|PP}}\ |\mathrm{E}$$

$$\dfrac{\lambda\varphi_2\lambda\varphi_3.\mathsf{terry} \circ \mathsf{gave} \circ \varphi_4 \circ \varphi_2 \circ \varphi_3;\ \mathbf{resp}(\lambda x\lambda y\lambda\mathscr{F}.\mathscr{F}(\mathbf{gave}(x)(y))(\mathbf{t}))(X_\times);\ \mathrm{S|(VP\backslash VP)|PP} \qquad \begin{array}{l}\mathsf{to} \circ\\ \quad \mathsf{bill} \circ\\ \quad \mathsf{and} \circ\\ \quad \mathsf{sue};\\ \langle \mathbf{b}, \mathbf{s}\rangle;\\ \mathrm{PP}\end{array}}{\begin{array}{l}\lambda\varphi_3.\mathsf{terry} \circ \mathsf{gave} \circ \varphi_4 \circ \mathsf{to} \circ \mathsf{bill} \circ \mathsf{and} \circ \mathsf{sue} \circ \varphi_3;\\ \mathbf{resp}(\lambda x\lambda y\lambda\mathscr{F}.\mathscr{F}(\mathbf{gave}(x)(y))(\mathbf{t}))(X_\times)(\langle \mathbf{b}, \mathbf{s}\rangle);\mathrm{S|(VP\backslash VP)}\end{array}}\ |\mathrm{E}$$

$$\begin{array}{l}\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\ \lambda\varphi_3.\mathsf{terry} \circ \mathsf{gave} \circ \varphi_4 \circ \mathsf{to} \circ \mathsf{bill} \circ \mathsf{and} \circ \mathsf{sue} \circ \varphi_3;\\ \langle \lambda\mathscr{F}.\mathscr{F}(\mathbf{gave}(\pi_1(X_\times))(\mathbf{b}))(\mathbf{t}), \lambda\mathscr{F}.\mathscr{F}(\mathbf{gave}(\pi_2(X_\times))(\mathbf{s}))(\mathbf{t})\rangle;\mathrm{S|(VP\backslash VP)}\end{array}$$

$$\dfrac{\begin{array}{l}\vdots \quad \vdots\\ \lambda\sigma_1\lambda\varphi_1.\\ \quad \sigma_1(\varphi_1);\\ \lambda Y_\times\lambda P_\times.\\ \quad \prod_i \pi_i(P_\times)\\ \quad (\pi_i(Y_\times));\\ (X|Y)|(X|Y)\end{array} \qquad \begin{array}{l}\vdots \quad \vdots\\ \lambda\varphi_3.\mathsf{terry} \circ \mathsf{gave} \circ \varphi_4 \circ\\ \quad \mathsf{to} \circ \mathsf{bill} \circ \mathsf{and} \circ \mathsf{sue} \circ \varphi_3;\\ \langle \lambda\mathscr{F}.\mathscr{F}(\mathbf{gave}(\pi_1(X_\times))(\mathbf{b}))(\mathbf{t}),\\ \quad \lambda\mathscr{F}.\mathscr{F}(\mathbf{gave}(\pi_2(X_\times))(\mathbf{s}))(\mathbf{t})\rangle;\\ \mathrm{S|(VP\backslash VP)}\end{array}}{\begin{array}{l}\lambda\varphi_3.\mathsf{terry} \circ \mathsf{gave} \circ \varphi_4 \circ \mathsf{to} \circ \mathsf{bill} \circ \mathsf{and} \circ \mathsf{sue} \circ \varphi_3;\\ \lambda Y_\times.\prod_i \pi_i(\langle \lambda\mathscr{F}.\mathscr{F}(\mathbf{gave}(\pi_1(X_\times))(\mathbf{b}))(\mathbf{t}),\\ \quad \lambda\mathscr{F}.\mathscr{F}(\mathbf{gave}(\pi_2(X_\times))(\mathbf{s}))(\mathbf{t})\rangle)(\pi_i(Y_\times));\\ \mathrm{S|(VP\backslash VP)}\end{array}}\ |\mathrm{E}$$

$$\dfrac{\begin{array}{l}\mathsf{as} \circ \mathsf{a} \circ \mathsf{christmas} \circ \mathsf{present} \circ\\ \quad \mathsf{on} \circ \mathsf{thursday} \circ \mathsf{and} \circ \mathsf{as} \circ\\ \quad \mathsf{a} \circ \mathsf{new} \circ \mathsf{year's} \circ \mathsf{gift} \circ\\ \quad \mathsf{on} \circ \mathsf{saturday};\\ \langle \lambda P.\mathbf{onTh}(\mathbf{asChP}(P)),\\ \quad \lambda P.\mathbf{onS}(\mathbf{asNYG}(P))\rangle;\\ \mathrm{VP\backslash VP}\end{array} \qquad \begin{array}{l}\lambda\varphi_3.\mathsf{terry} \circ \mathsf{gave} \circ \varphi_4 \circ \mathsf{to} \circ \mathsf{bill} \circ \mathsf{and} \circ \mathsf{sue} \circ \varphi_3;\\ \lambda Y_\times.\prod_i \pi_i(\langle \lambda\mathscr{F}.\mathscr{F}(\mathbf{gave}(\pi_1(X_\times))(\mathbf{b}))(\mathbf{t}),\\ \quad \lambda\mathscr{F}.\mathscr{F}(\mathbf{gave}(\pi_2(X_\times))(\mathbf{s}))(\mathbf{t})\rangle)(\pi_i(Y_\times));\\ \mathrm{S|(VP\backslash VP)}\end{array}}{\begin{array}{l}\mathsf{terry} \circ \mathsf{gave} \circ \varphi_4 \circ \mathsf{to} \circ \mathsf{bill} \circ \mathsf{and} \circ \mathsf{sue} \circ \mathsf{as} \circ \mathsf{a} \circ \mathsf{christmas} \circ \mathsf{present} \circ\\ \quad \mathsf{on} \circ \mathsf{thursday} \circ \mathsf{and} \circ \mathsf{as} \circ \mathsf{a} \circ \mathsf{new} \circ \mathsf{year's} \circ \mathsf{gift} \circ \mathsf{on} \circ \mathsf{saturday};\\ \langle \mathbf{onTh}(\mathbf{asChP}(\mathbf{gave}(\pi_1(X_\times)(\mathbf{b}))))(\mathbf{t}), \mathbf{onS}(\mathbf{asNYG}(\mathbf{gave}(\pi_2(X_\times)(\mathbf{s}))))(\mathbf{t})\rangle;\mathrm{S}\end{array}}\ |\mathrm{E}$$

$$\dfrac{\begin{array}{l}\lambda\varphi_1.\varphi_1;\\ \lambda p_\times.\bigwedge_i\\ \quad \pi_i(p_\times);\\ \mathrm{S|S}\end{array} \qquad \begin{array}{l}\mathsf{terry} \circ \mathsf{gave} \circ \varphi_4 \circ \mathsf{to} \circ \mathsf{bill} \circ \mathsf{and} \circ \mathsf{sue} \circ \mathsf{as} \circ \mathsf{a} \circ \mathsf{christmas} \circ \mathsf{present} \circ\\ \quad \mathsf{on} \circ \mathsf{thursday} \circ \mathsf{and} \circ \mathsf{as} \circ \mathsf{a} \circ \mathsf{new} \circ \mathsf{year's} \circ \mathsf{gift} \circ \mathsf{on} \circ \mathsf{saturday};\\ \langle \mathbf{onTh}(\mathbf{asChP}(\mathbf{gave}(\pi_1(X_\times)(\mathbf{b}))))(\mathbf{t}), \mathbf{onS}(\mathbf{asNYG}(\mathbf{gave}(\pi_2(X_\times)(\mathbf{s}))))(\mathbf{t})\rangle;\mathrm{S}\end{array}}{\begin{array}{l}\mathsf{terry} \circ \mathsf{gave} \circ \varphi_4 \circ \mathsf{to} \circ \mathsf{bill} \circ \mathsf{and} \circ \mathsf{sue} \circ \mathsf{as} \circ \mathsf{a} \circ \mathsf{christmas} \circ \mathsf{present} \circ\\ \quad \mathsf{on} \circ \mathsf{thursday} \circ \mathsf{and} \circ \mathsf{as} \circ \mathsf{a} \circ \mathsf{new} \circ \mathsf{year's} \circ \mathsf{gift} \circ \mathsf{on} \circ \mathsf{saturday};\\ \mathbf{onTh}(\mathbf{asChP}(\mathbf{gave}(\pi_1(X_\times)(\mathbf{b}))))(\mathbf{t}) \wedge \mathbf{onS}(\mathbf{asNYG}(\mathbf{gave}(\pi_2(X_\times)(\mathbf{s}))))(\mathbf{t});\mathrm{S}\end{array}}\ |\mathrm{E}$$

$$\dfrac{\begin{array}{l}\mathsf{terry} \circ \mathsf{gave} \circ \varphi_4 \circ \mathsf{to} \circ \mathsf{bill} \circ \mathsf{and} \circ \mathsf{sue} \circ \mathsf{as} \circ \mathsf{a} \circ \mathsf{christmas} \circ \mathsf{present} \circ\\ \quad \mathsf{on} \circ \mathsf{thursday} \circ \mathsf{and} \circ \mathsf{as} \circ \mathsf{a} \circ \mathsf{new} \circ \mathsf{year's} \circ \mathsf{gift} \circ \mathsf{on} \circ \mathsf{saturday};\\ \mathbf{onTh}(\mathbf{asChP}(\mathbf{gave}(\pi_1(X_\times)(\mathbf{b}))))(\mathbf{t}) \wedge \mathbf{onS}(\mathbf{asNYG}(\mathbf{gave}(\pi_2(X_\times)(\mathbf{s}))))(\mathbf{t});\mathrm{S}\end{array}}{\begin{array}{l}\lambda\varphi_4.\mathsf{terry} \circ \mathsf{gave} \circ \varphi_4 \circ \mathsf{to} \circ \mathsf{bill} \circ \mathsf{and} \circ \mathsf{sue} \circ \mathsf{as} \circ \mathsf{a} \circ \mathsf{christmas} \circ \mathsf{present} \circ\\ \quad \mathsf{on} \circ \mathsf{thursday} \circ \mathsf{and} \circ \mathsf{as} \circ \mathsf{a} \circ \mathsf{new} \circ \mathsf{year's} \circ \mathsf{gift} \circ \mathsf{on} \circ \mathsf{saturday};\\ \lambda X_\times.\mathbf{onTh}(\mathbf{asChP}(\mathbf{gave}(\pi_1(X_\times)(\mathbf{b}))))(\mathbf{t}) \wedge \mathbf{onS}(\mathbf{asNYG}(\mathbf{gave}(\pi_2(X_\times)(\mathbf{s}))))(\mathbf{t});\mathrm{S|NP}\end{array}}\ |\mathrm{I}^4$$

$$\dfrac{\begin{array}{l}\mathsf{the} \circ \mathsf{same} \circ\\ \quad \mathsf{gift};\\ \mathbf{same}(\mathbf{gift});\\ \mathrm{S|(S|NP)}\end{array} \qquad \begin{array}{l}\lambda\varphi_4.\mathsf{terry} \circ \mathsf{gave} \circ \varphi_4 \circ \mathsf{to} \circ \mathsf{bill} \circ \mathsf{and} \circ \mathsf{sue} \circ \mathsf{as} \circ \mathsf{a} \circ \mathsf{christmas} \circ \mathsf{present} \circ\\ \quad \mathsf{on} \circ \mathsf{thursday} \circ \mathsf{and} \circ \mathsf{as} \circ \mathsf{a} \circ \mathsf{new} \circ \mathsf{year's} \circ \mathsf{gift} \circ \mathsf{on} \circ \mathsf{saturday};\\ \lambda X_\times.\mathbf{onTh}(\mathbf{asChP}(\mathbf{gave}(\pi_1(X_\times)(\mathbf{b}))))(\mathbf{t}) \wedge \mathbf{onS}(\mathbf{asNYG}(\mathbf{gave}(\pi_2(X_\times)(\mathbf{s}))))(\mathbf{t});\mathrm{S|NP}\end{array}}{\begin{array}{l}\mathsf{terry} \circ \mathsf{gave} \circ \mathsf{the} \circ \mathsf{same} \circ \mathsf{gift} \circ \mathsf{to} \circ \mathsf{bill} \circ \mathsf{and} \circ \mathsf{sue} \circ \mathsf{as} \circ \mathsf{a} \circ \mathsf{christmas} \circ \mathsf{present} \circ\\ \quad \mathsf{on} \circ \mathsf{thursday} \circ \mathsf{and} \circ \mathsf{as} \circ \mathsf{a} \circ \mathsf{new} \circ \mathsf{year's} \circ \mathsf{gift} \circ \mathsf{on} \circ \mathsf{saturday};\\ \mathbf{same}(\mathbf{gift})(\lambda X_\times.\mathbf{onTh}(\mathbf{asChP}(\mathbf{gave}(\pi_1(X_\times)(\mathbf{b}))))(\mathbf{t}) \wedge \mathbf{onS}(\mathbf{asNYG}(\mathbf{gave}(\pi_2(X_\times)(\mathbf{s}))))(\mathbf{t}));\mathrm{S}\end{array}}\ |\mathrm{E}$$

(116)

$$\dfrac{[\varphi_1; f; \mathrm{S/NP}]^1 \qquad [\varphi_2; x; \mathrm{NP}]^2}{\varphi_1 \circ \varphi_2;\ f(x);\ \mathrm{S}}\ |\mathrm{E}$$

$$\dfrac{\varphi_1 \circ \varphi_2;\ f(x);\ \mathrm{S}}{\lambda\varphi_2.\varphi_1 \circ \varphi_2;\ \lambda x.f(x);\ \mathrm{S|NP}}\ |\mathrm{I}^2$$

$$\dfrac{\lambda\varphi_2.\varphi_1 \circ \varphi_2;\ \lambda x.f(x);\ \mathrm{S|NP}}{\lambda\varphi_1\lambda\varphi_2.\varphi_1 \circ \varphi_2;\ \lambda f\lambda x.f(x);\ \mathrm{S|NP|(S/NP)}}\ |\mathrm{I}^1$$

$$\dfrac{\begin{array}{l}\lambda\sigma\lambda\varphi_1\lambda\varphi_2.\sigma(\varphi_1)(\varphi_2);\\ \mathbf{resp};\\ (Z|X|Y)|(Z|X|Y)\end{array} \qquad \lambda\varphi_1\lambda\varphi_2.\varphi_1 \circ \varphi_2;\ \lambda f\lambda x.f(x);\ \mathrm{S|NP|(S/NP)}}{\lambda\varphi_1\lambda\varphi_2.\varphi_1 \circ \varphi_2;\ \lambda P_\times\lambda X_\times.\prod_i \pi_i(P_\times)(\pi_i(X_\times));\ \mathrm{S|NP|(S/NP)}}\ |\mathrm{E}$$

$$
\dfrac{
\dfrac{\begin{array}{l}\lambda\varphi\lambda\sigma.\sigma(\mathsf{a} \circ \\ \quad \mathsf{total} \circ \\ \quad \mathsf{of} \circ \varphi);\\ \mathbf{total};\\ \mathrm{S|(S|NP)|N}\end{array}
\quad
\begin{array}{l}\mathsf{\$10k};\\ \mathbf{\$10k};\\ \mathrm{N}\end{array}}
{\begin{array}{l}\lambda\varphi\lambda\sigma.\sigma(\mathsf{a} \circ \mathsf{total} \circ \\ \quad \mathsf{of} \circ \mathsf{\$10k});\\ \mathbf{total}(\mathbf{\$10k});\\ \mathrm{S|(S|NP)}\end{array}}
\quad
\dfrac{
\dfrac{
\begin{array}{l}\lambda\varphi_1.\varphi_1;\\ \lambda p_\times.\bigwedge_i\\ \quad \pi_i(p_\times);\\ \mathrm{S|S}\end{array}
\quad
\dfrac{
\dfrac{
\begin{array}{c}\vdots \quad \vdots\\ \lambda\varphi_1\lambda\varphi_2.\varphi_1 \circ \varphi_2;\\ \lambda P_\times\lambda X_\times.\prod_i\\ \quad \pi_i(P_\times)(\pi_i(X_\times));\\ \mathrm{S|NP|(S/NP)}\end{array}
\quad
\begin{array}{l}\vdots \quad \vdots\\ \mathsf{john} \circ \mathsf{spent} \circ \mathsf{and} \circ \mathsf{bill} \circ \mathsf{lost};\\ \langle \lambda x.\mathbf{spent}(\mathbf{j}, x), \lambda x.\mathbf{lost}(\mathbf{b}, x)\rangle;\\ \mathrm{S/NP}\end{array}}
{\begin{array}{l}\lambda\varphi_2.\mathsf{john} \circ \mathsf{spent} \circ \mathsf{and} \circ \mathsf{bill} \circ \mathsf{lost} \circ \varphi_2;\\ \lambda X_\times.\prod_i \pi_i(\langle \lambda x.\mathbf{spent}(\mathbf{j}, x), \lambda x.\mathbf{lost}(\mathbf{b}, x)\rangle)(\pi_i(X_\times));\\ \mathrm{S|NP}\end{array}}{\scriptstyle |\mathrm{E}}
\quad
\begin{bmatrix}\varphi_3;\\ X_\times;\\ \mathrm{NP}\end{bmatrix}^3}
{\begin{array}{l}\mathsf{john} \circ \mathsf{spent} \circ \mathsf{and} \circ \mathsf{bill} \circ \mathsf{lost} \circ \varphi_3;\\ \prod_i \pi_i(\langle \lambda x.\mathbf{spent}(\mathbf{j}, x), \lambda x.\mathbf{lost}(\mathbf{b}, x)\rangle)(\pi_i(X_\times)); \mathrm{S}\end{array}}{\scriptstyle |\mathrm{E}}}
{\begin{array}{l}\mathsf{john} \circ \mathsf{spent} \circ \mathsf{and} \circ \mathsf{bill} \circ \mathsf{lost} \circ \varphi_3;\\ \mathbf{spent}(\mathbf{j}, \pi_1(X_\times)) \wedge \lambda x.\mathbf{lost}(\mathbf{b}, \pi_2(X_\times)); \mathrm{S}\end{array}}{\scriptstyle |\mathrm{E}}}
{\begin{array}{l}\lambda\varphi_3.\mathsf{john} \circ \mathsf{spent} \circ \mathsf{and} \circ \mathsf{bill} \circ \mathsf{lost} \circ \varphi_3;\\ \lambda X_\times.\mathbf{spent}(\mathbf{j}, \pi_1(X_\times)) \wedge \lambda x.\mathbf{lost}(\mathbf{b}, \pi_2(X_\times)); \mathrm{S|NP}\end{array}}{\scriptstyle |\mathrm{I}^3}}
{\begin{array}{l}\mathsf{john} \circ \mathsf{spent} \circ \mathsf{and} \circ \mathsf{bill} \circ \mathsf{lost} \circ \mathsf{a} \circ \mathsf{total} \circ \mathsf{of} \circ \mathsf{\$10k};\\ \mathbf{total}(\mathbf{\$10k})(\lambda X_\times.\mathbf{spent}(\mathbf{j}, \pi_1(X_\times)) \wedge \mathbf{lost}(\mathbf{b}, \pi_2(X_\times))); \mathrm{S}\end{array}}{\scriptstyle |\mathrm{E}}
$$

B Extension to cumulative readings

With a slight extension, the tuple-based analysis of 'respective' predication that we have proposed offers a simple way of characterizing cumulative readings. Specifically, all we need to do is to assume that tuples corresponding to plural terms can contain elements that are themselves sums of individuals rather than just atomic individuals. We sketch here a basic analysis of (117) and then discuss some implications.

(117) A total of 4 students read a total of 5 books.

Assuming that we have four students $\mathbf{s1}, \mathbf{s2}, \mathbf{s3}, \mathbf{s4}$ and five books $\mathbf{b1}, \mathbf{b2}, \mathbf{b3}, \mathbf{b4}, \mathbf{b5}$ and that the reading relation that holds between the two sets is given by the following:

(118) $\{\langle \mathbf{s1}, \mathbf{b1}\rangle, \langle \mathbf{s1}, \mathbf{b2}\rangle, \langle \mathbf{s2}, \mathbf{b1}\rangle, \langle \mathbf{s2}, \mathbf{b2}\rangle, \langle \mathbf{s2}, \mathbf{b3}\rangle, \langle \mathbf{s3}, \mathbf{b2}\rangle, \langle \mathbf{s3}, \mathbf{b4}\rangle, \langle \mathbf{s4}, \mathbf{b5}\rangle\}$

then, the situation can be modelled by a 'respective' predication between two products $X_\times = \langle \mathbf{s1}, \mathbf{s2}, \mathbf{s3}, \mathbf{s4}\rangle$ and $Y_\times = \langle \mathbf{b1} \oplus \mathbf{b2}, \mathbf{b1} \oplus \mathbf{b2} \oplus \mathbf{b3}, \mathbf{b2} \oplus \mathbf{b4}, \mathbf{b5}\rangle$. Thus, the following translation that is assigned to the sentence compositionally by the present analysis suffices to capture the cumulative reading of (117):

(119) $\exists S.|S| = 4 \wedge \mathbf{student}(S) \wedge \exists X_\times \sum_i^n \pi_i(X_\times) = S \wedge \exists S'.|S'| = 5 \wedge \mathbf{book}(S') \wedge \exists Y_\times. \sum_i^n \pi_i(Y_\times) = S' \wedge \mathbf{resp}(X_\times)(Y_\times)(\mathbf{read})$

This extension offers a promising approach to characterizing the more complex reading for sentences involving two occurrences of *different* such as the following that we have mentioned in passing in Sect. 2.2.2.

(120) Different students read different books.

The relevant reading links students to sets of books that (s)he read and asserts that for no pair of two students, the sets of books that they respectively read are completely identical. Thus, the sentence is true on this reading in a situation (call it situation 1) described above with the reading relation in (118) but false in a situation where $\mathbf{s1}$ read $\mathbf{b3}$ in addition (call it situation 2), since in situation 2, the sets of books that $\mathbf{s1}$ and $\mathbf{s2}$ read are exactly identical. By assuming that the tuple of students consists of atomic students but that the tuple of books can consist of sums of books, we can model this reading with our **resp** operator and the semantics for *different* already introduced above. The sentence comes out true in situation 1 but false in situation 2, since, according to the semantics of *different*, each element of the book tuple needs to be distinct from each other, a condition satisfied in situation 1 but not in situation 2.

With the analysis of cumulative readings above, the cumulative/distributive reading for (66), repeated here as (121), is straightforward.

(121) A total of three ATMs gave a total of 1000 customers two new passwords.

The crucial assumption here is that, in (the relevant reading of) this sentence, the plural term *two new passwords* denotes an ordinary cardinal quantifier that scopes below the 'respective' operator that establishes the cumulative relation between the other two plural terms. The analysis goes as in (122) (where **bool** is the denotation of the boolean conjunction operator in (33)).

(122)

$$\dfrac{\mathsf{gave};\ \mathbf{gave};\ \mathrm{VP/NP/NP} \qquad \begin{bmatrix}\varphi_2;\\ y;\\ \mathrm{NP}\end{bmatrix}^2}{\mathsf{gave} \circ \varphi_2;\ \mathbf{gave}(y);\ \mathrm{VP/NP}}$$

$$\dfrac{\mathsf{gave} \circ \varphi_2;\ \mathbf{gave}(y);\ \mathrm{VP/NP} \qquad \begin{bmatrix}\varphi_3;\\ z;\\ \mathrm{NP}\end{bmatrix}^3}{\mathsf{gave} \circ \varphi_2 \circ \varphi_3;\ \mathbf{gave}(y)(z);\ \mathrm{VP}}$$

$$\dfrac{\begin{bmatrix}\varphi_1;\\ x;\\ \mathrm{NP}\end{bmatrix}^1 \qquad \mathsf{gave} \circ \varphi_2 \circ \varphi_3;\ \mathbf{gave}(y)(z);\ \mathrm{VP}}{\dfrac{\varphi_1 \circ \mathsf{gave} \circ \varphi_2 \circ \varphi_3;\ \mathbf{gave}(y)(z)(x);\ \mathrm{S}}{\lambda\varphi_3.\varphi_1 \circ \mathsf{gave} \circ \varphi_2 \circ \varphi_3;\ \lambda z.\mathbf{gave}(y)(z)(x); \mathrm{S|NP}}}$$

$$\dfrac{\mathsf{two} \circ \mathsf{passwords};\ \mathbf{two\text{-}pw};\ \mathrm{S|(S|NP)} \qquad \lambda\varphi_3.\varphi_1 \circ \mathsf{gave} \circ \varphi_2 \circ \varphi_3;\ \lambda z.\mathbf{gave}(y)(z)(x); \mathrm{S|NP}}{\dfrac{\varphi_1 \circ \mathsf{gave} \circ \varphi_2 \circ \mathsf{two} \circ \mathsf{passwords};\ \mathbf{two\text{-}pw}(\lambda z.\mathbf{gave}(y)(z)(x)); \mathrm{S}}{\lambda\varphi_2.\varphi_1 \circ \mathsf{gave} \circ \varphi_2 \circ \mathsf{two} \circ \mathsf{passwords};\ \lambda y.\mathbf{two\text{-}pw}(\lambda z.\mathbf{gave}(y)(z)(x)); \mathrm{S|NP}}}$$

$$\dfrac{\lambda\varphi.\varphi;\ \mathbf{dist};\ \mathrm{(S|NP)|(S|NP)} \qquad \lambda\varphi_2.\varphi_1 \circ \mathsf{gave} \circ \varphi_2 \circ \mathsf{two} \circ \mathsf{passwords};\ \lambda y.\mathbf{two\text{-}pw}(\lambda z.\mathbf{gave}(y)(z)(x)); \mathrm{S|NP}}{\dfrac{\lambda\varphi_2.\varphi_1 \circ \mathsf{gave} \circ \varphi_2 \circ \mathsf{two} \circ \mathsf{passwords};\ \mathbf{dist}(\lambda y.\mathbf{two\text{-}pw}(\lambda z.\mathbf{gave}(y)(z)(x)));\ \mathrm{S|NP}}{\lambda\varphi_1\lambda\varphi_2.\varphi_1 \circ \mathsf{gave} \circ \varphi_2 \circ \mathsf{two} \circ \mathsf{passwords};\ \lambda x.\mathbf{dist}(\lambda y.\mathbf{two\text{-}pw}(\lambda z.\mathbf{gave}(y)(z)(x)));\ \mathrm{S|NP|NP}}}$$

$$\vdots \quad \vdots$$

$$\dfrac{\lambda\sigma_0\lambda\varphi_1\lambda\varphi_2.\ \sigma_0(\varphi_1)(\varphi_2);\ \mathbf{resp};\ (Z|X|Y)|(Z|X|Y) \qquad \lambda\varphi_1\lambda\varphi_2.\varphi_1 \circ \mathsf{gave} \circ \varphi_2 \circ \mathsf{two} \circ \mathsf{passwords};\ \lambda x.\mathbf{dist}(\lambda y.\mathbf{two\text{-}pw}(\lambda z.\mathbf{gave}(y)(z)(x)));\ \mathrm{S|NP|NP}}{\lambda\varphi_1\lambda\varphi_2.\varphi_1 \circ \mathsf{gave} \circ \varphi_2 \circ \mathsf{two} \circ \mathsf{passwords};\ \mathbf{resp}(\lambda x.\mathbf{dist}(\lambda y.\mathbf{two\text{-}pw}(\lambda z.\mathbf{gave}(y)(z)(x))));\ \mathrm{S|NP|NP}}$$

$$\dfrac{\begin{bmatrix}\varphi_1;\\ X_\times;\\ \mathrm{NP}\end{bmatrix}^4 \qquad \lambda\varphi_1\lambda\varphi_2.\varphi_1 \circ \mathsf{gave} \circ \varphi_2 \circ \mathsf{two} \circ \mathsf{passwords};\ \mathbf{resp}(\lambda x.\mathbf{dist}(\lambda y.\mathbf{two\text{-}pw}(\lambda z.\mathbf{gave}(y)(z)(x))));\ \mathrm{S|NP|NP}}{\lambda\varphi_2.\varphi_1 \circ \mathsf{gave} \circ \varphi_2 \circ \mathsf{two} \circ \mathsf{passwords};\ \mathbf{resp}(\lambda x.\mathbf{dist}(\lambda y.\mathbf{two\text{-}pw}(\lambda z.\mathbf{gave}(y)(z)(x))))(X_\times);\ \mathrm{S|NP}}$$

$$\dfrac{\begin{bmatrix}\varphi_2;\\ W_\times;\\ \mathrm{NP}\end{bmatrix}^5 \qquad \lambda\varphi_2.\varphi_1 \circ \mathsf{gave} \circ \varphi_2 \circ \mathsf{two} \circ \mathsf{passwords};\ \mathbf{resp}(\lambda x.\mathbf{dist}(\lambda y.\mathbf{two\text{-}pw}(\lambda z.\mathbf{gave}(y)(z)(x))))(X_\times);\ \mathrm{S|NP}}{\varphi_1 \circ \mathsf{gave} \circ \varphi_2 \circ \mathsf{two} \circ \mathsf{passwords};\ \mathbf{resp}(\lambda x.\mathbf{dist}(\lambda y.\mathbf{two\text{-}pw}(\lambda z.\mathbf{gave}(y)(z)(x))))(X_\times)(W_\times); \mathrm{S}}$$

$$\dfrac{\lambda\varphi.\varphi;\ \mathbf{bool};\ \mathrm{S|S} \qquad \varphi_1 \circ \mathsf{gave} \circ \varphi_2 \circ \mathsf{two} \circ \mathsf{passwords};\ \mathbf{resp}(\lambda x.\mathbf{dist}(\lambda y.\mathbf{two\text{-}pw}(\lambda z.\mathbf{gave}(y)(z)(x))))(X_\times)(W_\times); \mathrm{S}}{\dfrac{\varphi_1 \circ \mathsf{gave} \circ \varphi_2 \circ \mathsf{two} \circ \mathsf{passwords};\ \mathbf{bool}(\mathbf{resp}(\lambda x.\mathbf{dist}(\lambda y.\mathbf{two\text{-}pw}(\lambda z.\mathbf{gave}(y)(z)(x))))(X_\times)(W_\times)); \mathrm{S}}{\lambda\varphi_2.\varphi_1 \circ \mathsf{gave} \circ \varphi_2 \circ \mathsf{two} \circ \mathsf{passwords};\ \lambda W_\times.\mathbf{bool}(\mathbf{resp}(\lambda x.\mathbf{dist}(\lambda y.\mathbf{two\text{-}pw}(\lambda z.\mathbf{gave}(y)(z)(x))))(X_\times)(W_\times)); \mathrm{S|NP}}}$$

$$\dfrac{\lambda\varphi\lambda\sigma.\sigma(\mathsf{a} \circ \mathsf{total} \circ \mathsf{of} \circ \mathsf{1000} \circ \mathsf{customers});\ \mathbf{total}(\mathbf{1k\text{-}cus});\ \mathrm{S|(S|NP)} \qquad \lambda\varphi_2.\varphi_1 \circ \mathsf{gave} \circ \varphi_2 \circ \mathsf{two} \circ \mathsf{passwords};\ \lambda W_\times.\mathbf{bool}(\mathbf{resp}(\lambda x.\mathbf{dist}(\lambda y.\mathbf{two\text{-}pw}(\lambda z.\mathbf{gave}(y)(z)(x))))(X_\times)(W_\times)); \mathrm{S|NP}}{\dfrac{\varphi_1 \circ \mathsf{gave} \circ \mathsf{a} \circ \mathsf{total} \circ \mathsf{of} \circ \mathsf{1000} \circ \mathsf{customers} \circ \mathsf{two} \circ \mathsf{passwords};\ \mathbf{total}(\mathbf{1k\text{-}cus})(\lambda W_\times.\mathbf{bool}(\mathbf{resp}(\lambda x.\mathbf{dist}(\lambda y.\mathbf{two\text{-}pw}(\lambda z.\mathbf{gave}(y)(z)(x))))(X_\times)(W_\times))); \mathrm{S}}{\lambda\varphi_1.\varphi_1 \circ \mathsf{gave} \circ \mathsf{a} \circ \mathsf{total} \circ \mathsf{of} \circ \mathsf{1000} \circ \mathsf{customers} \circ \mathsf{two} \circ \mathsf{passwords};\ \lambda X_\times.\mathbf{total}(\mathbf{1k\text{-}cus})(\lambda W_\times.\mathbf{bool}(\mathbf{resp}(\lambda x.\mathbf{dist}(\lambda y.\mathbf{two\text{-}pw}(\lambda z.\mathbf{gave}(y)(z)(x))))(X_\times)(W_\times))); \mathrm{S|NP}}}$$

$$\dfrac{\lambda\varphi\lambda\sigma.\sigma(\mathsf{a} \circ \mathsf{total} \circ \mathsf{of} \circ \mathsf{3} \circ \mathsf{atms});\ \mathbf{total}(\mathbf{3\text{-}atms});\ \mathrm{S|(S|NP)} \qquad \lambda\varphi_1.\varphi_1 \circ \mathsf{gave} \circ \mathsf{a} \circ \mathsf{total} \circ \mathsf{of} \circ \mathsf{1000} \circ \mathsf{customers} \circ \mathsf{two} \circ \mathsf{passwords};\ \lambda X_\times.\mathbf{total}(\mathbf{1k\text{-}cus})(\ldots); \mathrm{S|NP}}{\mathsf{a} \circ \mathsf{total} \circ \mathsf{of} \circ \mathsf{3} \circ \mathsf{atms} \circ \mathsf{gave} \circ \mathsf{a} \circ \mathsf{total} \circ \mathsf{of} \circ \mathsf{1000} \circ \mathsf{customers} \circ \mathsf{two} \circ \mathsf{passwords};\ \mathbf{total}(\mathbf{3\text{-}atms})(\lambda X_\times.\mathbf{total}(\mathbf{1k\text{-}cus})(\lambda W_\times.\mathbf{bool}(\mathbf{resp}(\lambda x.\mathbf{dist}(\lambda y.\mathbf{two\text{-}pw}(\lambda z.\mathbf{gave}(y)(z)(x))))(X_\times)(W_\times)))); \mathrm{S}}$$

Assuming that the sum of ATMs consists of three ATMs **atm1**, **atm2**, and **atm3**, and that the product corresponding to this sum consists of atomic ATMs as the tuple members in this order, that is, $X_{\times} = \langle \mathbf{atm1}, \mathbf{atm2}, \mathbf{atm3} \rangle$, the final translation reduces to the following:

(123) $$\begin{aligned} &\exists W_{\times_3}. \sum_{1 \leq i \leq 3} \pi_i(W_{\times_3}) = \mathbf{1k\text{-}cus} \wedge \\ &\mathbf{dist}(\lambda y.\mathbf{two\text{-}pw}(\lambda z.\mathbf{gave}(y)(z)(x)))(\mathbf{atm1})(\pi_1(W_{\times_3})) \wedge \\ &\mathbf{dist}(\lambda y.\mathbf{two\text{-}pw}(\lambda z.\mathbf{gave}(y)(z)(x)))(\mathbf{atm2})(\pi_2(W_{\times_3})) \wedge \\ &\mathbf{dist}(\lambda y.\mathbf{two\text{-}pw}(\lambda z.\mathbf{gave}(y)(z)(x)))(\mathbf{atm3})(\pi_3(W_{\times_3})) \end{aligned}$$

This means that the total of 1000 customers can be partitioned into three groups such that for each of these groups, one of the three ATMs gave two distinct passwords to each individual in that group. This corresponds to the relevant reading of the sentence.

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