Wh-questions in dynamic inquisitive semantics

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Abstract

This paper presents a type-theoretic dynamic inquisitive semantics framework, extending the first-order system presented in Dotlačil and Roelofsen (2019). Within this framework, we develop a compositional treatment of wh-questions whose basic premise is that a wh-phrase introduces a discourse referent u, just like non-interrogative indefinites do, and requires the presence of an operator in the left periphery of the clause which requests a witness for u, i.e., it raises an issue whose resolution requires identifying at least one individual that has all the properties ascribed to u. In previous work, it has been argued that a dynamic semantic analysis of questions is needed to account for anaphora with wh-antecedents and for certain kinds of intervention effects (Haida, 2007). Here, we further develop the general approach and argue that it has several additional benefits. Namely, it allows for a uniform treatment of single-wh and multiple-wh questions, it derives mention-some and mention-all readings in a principled way, as well as an often neglected partial mention-some reading in multiple-wh questions, it can capture certain constraints on the availability of mention-some readings, and the effects of number marking on *which*-phrases across single-wh and multiple-wh questions.

Contents

1 Introduction

1.1 A dynamic perspective on meaning

Formal semantic theories often construe the semantic value of a declarative sentence as a formal object representing the *truth conditions* of the sentence. While many semantic phenomena can be fruitfully analyzed within such a truth-conditional approach, it is well-known that certain phenomena are beyond its reach. A familiar example is the licensing of anaphoric pronouns. Consider (1) and (2), adapted from Heim (1982) who in turn attributes such examples to Partee.

(1)   a.   Normally there are ten marbles in the box, but today a^u marble is missing.
        b.   It$_u$ may be under the sofa.

(2)   a.   Normally there are ten marbles in the box, but today there are only nine.
        b. #It may be under the sofa.

Truth-conditionally, (1a) and (2a) are equivalent. Yet, only (1a) licenses anaphoric reference to the missing marble using the anaphoric pronoun *it* in the next sentence. This observation demonstrates that there is more to the meaning of a declarative sentence than its truth conditions, and has led to the development of *dynamic semantics* (Kamp, 1981; Heim, 1982; Groenendijk and Stokhof, 1991, a.m.o.), a framework in which the semantic value of a sentence is a formal object representing its *context change potential*, i.e., the way in which it changes the context in which it is uttered, rather than its truth-conditions. In this framework, the indefinite *a marble* in (1a) changes the context by introducing a discourse referent, which serves as a suitable antecedent for the anaphoric pronoun in (1b). By contrast, (2a) does not introduce such a discourse referent, and the anaphoric pronoun in (2b) therefore lacks a suitable antecedent. This, in dynamic semantics, explains the contrast between (1) and (2).

1.2 A dynamic perspective on questions

While dynamic semantics has led to important new insights into anaphora and several other phenomena (e.g., presuppositions, modals, conditionals, and tense), dynamic semantic theories have typically restricted their attention to declarative sentences. Semantic analyses of *questions* have, with very few exceptions (see below), usually been developed within a static framework. In such static analyses the semantic value of a question is typically a formal object representing its *answerhood conditions* (Hamblin, 1973; Karttunen, 1977) or its *resolution conditions* (Ciardelli *et al.*, 2018). Yet, it is straightforward to see that, just like the meaning of a declarative sentence goes beyond its truth conditions, the meaning of a question goes beyond its answerhood/resolution conditions. Consider, for instance, the following examples modeled after Partee's marble cases.

(3)   a.   Which^u one of her three sons will inherit the house?
        b.   And is he$_u$ going to live there?

(4)   a.   Which^u two of her three sons will not inherit the house?
        b. #And is he$_u$ going to live there?

These examples show that even if two questions are equivalent in terms of resolution/answerhood conditions, they may still differ in their potential to license pronominal anaphora.[1]

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[1]See Ciardelli *et al.* (2018), Roelofsen (2019b) and Theiler (2019) for discussion of other phenomena showing that the meaning of a question goes beyond its answerhood/resolution conditions.

The present paper contributes to the development of a dynamic framework for the analysis of questions, bringing together and extending insights from dynamic theories of declaratives and static theories of questions, in particular those developed within the inquisitive semantics framework (Ciardelli *et al.*, 2018). The basic idea that underlies the approach can be conveyed on the basis of two simple examples. First consider the declarative in (5):

(5)   Someone fainted.

In dynamic semantics, this statement is taken to give rise to three consecutive updates of the context in which it is uttered. First, a *discourse referent*, let's call it u, is introduced; then the context is updated with the information that u is a person; and finally, the context is further updated with the information that u fainted. This sequence of updates can be represented as follows (the details of this notation will be reviewed later, but are not needed to understand the general idea we want to convey at this point):

(6) $[u]; person\{u\}; fainted\{u\}$

        $\underset{\text{someone}}{\nwarrow\nearrow} \quad \underset{\text{fainted}}{\uparrow}$
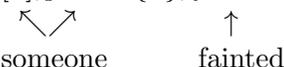
Now consider the question in (7).

(7)   Who fainted?

What does this question have in common with the declarative in (5), and what is it that makes the two different? In dynamic semantics it is possible to give an answer to this question which we find particularly attractive (for reasons to be discussed shortly) and which cannot be formulated in a static semantic framework. Namely, we will analyze the question in (7) as follows. First, it introduces a discourse referent u and updates the context with the information that u is a person and the information that u fainted. This part is shared with the declarative in (5). But then a further update occurs, contributed by an operator which we assume to generally be present in the left periphery of interrogative clauses and which for now we will denote as C^{int}.[2] This update raises an issue whose resolution requires identifying one or more individuals that have the properties ascribed to u, i.e., one or more persons who fainted. In short, this additional update requires identifying a *witness set* for u. We therefore refer to it as a *witness request update* and denote it as $?u$.

The sequence of updates that (7) gives rise to can thus be represented as follows:

(8) $[u]; person\{u\}; fainted\{u\}; ?u$

        $\underset{\text{who}}{\nwarrow\nearrow} \quad \underset{\text{fainted}}{\uparrow} \quad \underset{C^{\text{int}}}{\uparrow}$
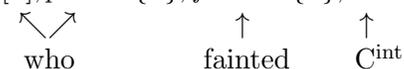
Note that, on this proposal, the semantic contribution of the wh-word *who* is exactly the same as that of the indefinite *someone*. Both introduce a discourse referent u and update the context with the information that u is a person. This is in line with the well-known observation that in many languages wh-words are similar, or even identical in form to non-interrogative indefinites (Ultan, 1978; Haspelmath, 1997; Bhat, 2000; Hengeveld *et al.*, 2021, among others). For instance, *somewhere* and *somehow* are based on *where* and *how*, respectively, and the Dutch word *wat* can mean either 'something' or 'what'.

On our proposal, the difference between wh-elements and non-interrogative indefinites is that the

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[2]In Section 4 we will make more precise assumptions about the left periphery of interrogative and declarative clauses, and we will also revisit the notation adopted here.

former stand in a particular relationship with C^{int}. Specifically, we assume a semantic dependency between wh-elements and C^{int} that is akin to dynamic binding: the $?u$ operator contributed by C^{int} anaphorically accesses the discourse referent u introduced by the wh-element and requests a witness set for it.

The main precedent of our proposal is the work of Haida (2007). There is earlier work on questions in dynamics semantics as well (e.g., Groenendijk, 1998; van Rooij, 1998; Aloni and van Rooij, 2002). However, this earlier work does not adopt the specific approach pursued here. In particular, it does not separate the introduction of a discourse referent, which we take to be the contribution of wh-elements, from the witness request update, which we take to be the contribution of an operator in the interrogative left periphery. Nor does it assume a semantic dependency between wh-elements and the operator in the left periphery.

Empirical motivation for Haida's work as well as the earlier work cited above mainly came from anaphora with wh-antecedents, illustrated in (3) above. Haida in addition argued that a dynamic theory of questions which assumes a semantic dependency between wh-elements and a witness request operator in the left periphery (which his theory indeed does assume but earlier work on questions in dynamic semantics did not) provides a principled account of certain *intervention effects* in questions. Illustrating such effects and how a dynamic theory can account for them would take us too far afield here, so we refer to Haida (2007) for details. For current purposes it suffices to note that this body of work already provided some compelling empirical arguments for a dynamic approach to questions.

A limitation of Haida's proposal, however, is that it is based on the *partition* approach to questions (Groenendijk and Stokhof, 1984). The partition theory, which was initially developed within a static semantic framework, holds that the semantic value of a question is a set of propositions which (i) are mutually incompatible and (ii) together cover the set of all possible worlds. In short, these propositions form a partition of the set of all possible worlds. Haida (2007) develops a dynamic version of the partition theory, and thereby inherits its shortcomings. The main problem is that there are many questions whose basic answers/resolutions are not mutually incompatible and therefore do not form a partition of the set of all possible worlds. Consider the example in (9):

(9)   Who has a bike that I could borrow for 15 minutes?

This question can be resolved by establishing that, say, Amy has a bike that the speaker can borrow, or by establishing that Bob has one, etcetera. These propositions are not mutually exclusive: it is very well possible that both Amy and Bob have a bike that the speaker could borrow. Therefore, such questions pose a problem for the partition approach to questions, a problem that Belnap (1982) referred to as the *unique answer fallacy*.

The recently developed inquisitive semantics framework (Ciardelli *et al.*, 2018) overcomes this problem. In inquisitive semantics, the semantic value of a question is still a set of propositions, just like in partition semantics as well as approaches to questions based on the work of Hamblin (1973) and Karttunen (1977). However, in inquisitive semantics these propositions need not be mutually incompatible. Rather, the constraint that inquisitive semantics places on the set of propositions that make up the semantic value of a question is that it be *downward closed*. This means that if the set contains a certain proposition p, then it should contain every stronger proposition $p' \subset p$ as well. The rationale behind this constraint is simple: the propositions that make up the semantic value of a question are intended to be all and only those propositions that *resolve* the issue that the question expresses. If a proposition p resolves the issue expressed, then any stronger proposition $p' \subset p$ will resolve the issue as well.

Inquisitive semantics avoids the unique answer fallacy and has been argued to have several

further advantages over previous frameworks as well (Roelofsen, 2013; Ciardelli *et al.*, 2017; Ciardelli and Roelofsen, 2017; Ciardelli, 2017; Ciardelli *et al.*, 2018). However, the basic inquisitive semantics framework is static in nature. As such, it cannot capture anaphora with wh-antecedents, nor the intervention effects that Haida's dynamic theory accounts for. This motivates the development of a dynamic inquisitive semantics framework.

A first step in this direction was taken in Dotlačil and Roelofsen (2019). There, an elementary dynamic inquisitive semantics for a first-order logical language was presented, and its utility in analyzing the interaction between anaphora and questions was demonstrated. This framework, however, is still limited in a number of ways. Most importantly, it only provides a semantic interpretation of expressions in a first-order logical language, which means that it does not yet facilitate a *compositional* analysis of semantic interpretation in natural language, i.e., an analysis that makes precise what the semantic contribution is of every meaningful element of a sentence, and how these basic semantic building blocks are combined to form the semantic value of the sentence as a whole. To make this possible, a dynamic inquisitive semantics must be formulated for a suitable type-theoretic logical language, rather than just a first-order language.

1.3   Contributions of the present paper

The present paper aims to make two contributions. The first concerns *framework building*. It presents a full-fledged dynamic inquisitive semantics for a suitable type-theoretic logical language, incorporating and extending insights from previous type-theoretic dynamic frameworks which were restricted to declarative sentences (Muskens, 1996; van den Berg, 1996; Brasoveanu, 2008, among others), and demonstrates how this framework can be used to formulate a compositional account of the semantic interpretation of declarative and interrogative sentences in English.

The second aim of the paper is to offer *additional empirical support* for a dynamic approach to questions. As discussed above, the empirical focus of previous dynamic theories of questions has been on anaphora and intervention effects. We will argue that there are other empirical phenomena which lend support to a dynamic approach as well. Specifically, we claim that a dynamic analysis of wh-questions in English, unlike existing static theories, can simultaneously satisfy all of the following desiderata:

A) **Non-stipulative derivation of mention-some and mention-all readings**
Wh-questions generally can have *mention-some* and *mention-all* readings. For instance, the question in (10) on its most prominent reading requests a specification of *all* people who were invited to the workshop, while (11) can be resolved by specifying just *one* good speaker, even if there are in principle multiple good candidates.

(10)   Who has been invited to the workshop?

(11)   Who would be a good speaker to invite for the workshop?

This flexibility in the interpretation of questions should be derived in a non-stipulative way, ideally relying on mechanisms that are *independently needed* elsewhere in the grammar.

B) **Uniform treatment of single-wh and multiple-wh questions**
In many languages, including English, questions may involve multiple wh-expressions, as exemplified in (12).

(12)   Who ordered what?

The interpretation of single-wh and multiple-wh questions should be derived *uniformly*. That is, unless there is strong independent evidence to the contrary, a theory of questions should assume that single-wh and multiple-wh questions involve essentially the same basic semantic operations.

C) **Partial mention-some readings in multiple-wh questions**
While the distinction between mention-some and mention-all readings in single-wh questions has received quite some attention, related distinctions in the interpretation of *multiple*-wh questions have not been discussed much at all. Usually, it is assumed that such questions only allow for a mention-all interpretation. We argue, however, building on an earlier observation by Xiang (2016), that in some cases multiple-wh questions also allow for a 'partial mention-some reading', which requires an answer that is mention-some w.r.t. one of the wh-phrases but mention-all w.r.t. the others. Suppose, for instance, that Mary has a large herb garden. The herbs that Mary has in her garden each grow in several places. Susan has a list of five herbs that she needs to collect for a certain recipe and asks Mary the following question:

(13)   Which of these herbs grows where?

On the most natural reading of this question, Mary can resolve it by mentioning just one location for each herb. She does not have to exhaustively specify all locations for every herb. A theory of questions should be able to derive such partial mention-some readings in multiple-wh questions.

D) **Constraints on the availability of mention-some readings**
Questions with plural *which*-phrases, e.g., *which participants*, typically lack a mention-some reading. For instance, the question in (14) does not have a mention-some interpretation, unlike its variant in (15) which involves a number-neutral wh-phrase.

(14)   *Participant of workshop in Amsterdam to the workshop organiser:*
        Which local participants have a bike that I could borrow for 15 minutes?

(15)   Who has a bike that I could borrow for 15 minutes?

This constraint can be obviated, however, in the presence of an existential modal operator, as illustrated by the following example from Dayal (2016):

(16)   *Context: A researcher needs a few people with AB blood type to test a new drug. The study requires her to test multiple patients but not necessarily all the patients in the hospital with AB blood type. The researcher has a list of all patients in the hospital, but not their blood types. The administrator does have information mapping patients to blood types. The researcher asks the administator:*
        Which patients can I approach for this test?

A theory of questions should account for the fact that plural *which*-phrases generally block mention-some readings though not in the presence of existential modals.

E) **Uniqueness presuppositions due to singular number marking in *which*-phrases**
Questions with singular *which*-phrases, e.g. *which girl*, typically invoke a uniqueness presupposition. For instance, (17) presupposes that there is a unique girl who danced with Robin.

(17)   Which girl danced with Robin?   ⤳ just one girl danced with Robin

In multiple-wh questions, however, this uniqueness presupposition gets a twist. For instance, (18) does not presuppose that there is a unique girl and a unique boy who danced. It does presuppose that every boy who danced did so with a unique girl, but all in all, there could have been many boys and girls dancing.

(18)   Which girl danced with which boy?   ⤳̸ just one girl and just one boy danced

Another observation that needs to be captured is that the uniqueness requirement normally induced by a singular *which*-phrase can be obviated in questions involving an existential modal operator (Hirsch and Schwarz, 2019) or disjunction (Socolof *et al.*, 2020). For instance, it is perfectly natural to ask (19) without assuming that there is a unique letter that can be inserted in *fo_m* to make a word.

(19)   Which letter can be inserted in *fo_m* to make a word?

Similarly, (20) can be felicitously asked without assuming that there is a unique town in which Shakespeare was born or Bach died.

(20)   In which town was Shakespeare born or did Bach die?

A theory of questions should therefore not just derive the uniqueness presupposition in questions like (17), but also the absence of such a presupposition in cases like (18), (125), and (20).

To our knowledge, no existing theory of questions meets all of these desiderata at once. For instance,[3] Groenendijk and Stokhof (1984) provide a uniform account of single-wh and multiple-wh questions, but their proposal does not successfully capture **mention-some** readings and the effects of **number marking** on wh-phrases.

Dayal (1996) pays close attention to the effects of number marking, but her account does not successfully capture **mention-some** readings and does not provide a **uniform** treatment of single-wh and multiple-wh questions, in the sense that it takes such questions to involve different operators in the left periphery. Moreover, as pointed out by Xiang (2016, 2022a) and discussed in more detail below, the so-called **domain exhaustivity** presupposition that Dayal's account predicts for multiple-wh questions is problematic.

Fox (2012) provides a uniform account of single-wh and multiple-wh questions which captures the fact that singular *which*-phrases give rise to uniqueness presuppositions in single-wh but not in multiple-wh questions. However, Fox's account wrongly predicts that uniqueness presuppositions should also be present in cases like (125) and (20). Moreover, as discussed by Xiang (2016, 2022a), Fox's account, just like Dayal's, derives the problematic **domain exhaustivity** presupposition for multiple-wh questions. In addition, while Fox (2013) derives mention-some readings for single-wh questions, it is not clear yet whether his approach could also capture partial mention-some readings in multiple-wh questions.

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[3]The following paragraphs are not meant as an exhaustive survey of the rich literature on the topic, but rather as a brief overview of some of the most prominent lines of work, and how they fare w.r.t. the above desiderata. In Appendix A we compare our approach to some other recent proposals in some more detail.

Xiang (2016, 2022a) captures mention-some readings and effects of number marking, but her account of single-wh and multiple-wh questions is not **uniform**, in the sense that it assumes special LF ingredients for multiple-wh questions which are not present in single-wh questions (this point is discussed in more depth in Appendix A.3).

We do not claim that it is impossible to further extend and refine currently available static theories to meet the above desiderata. Rather, our aim here is only to show that a dynamic theory can achieve this quite straightforwardly, and therefore has empirical advantages over existing static theories. To keep the paper of manageable length, we will restrict ourselves here to the development of a compositional dynamic inquisitive framework that does *not* include modal operators. Within this framework it will not be possible yet to provide a formally explicit account of the interaction between wh-elements and modal operators, such as exemplified in (16) and (125) above. While we believe that our framework, when extended with modal operators, could provide a natural account of such cases, we leave this for future work.

The paper is organised as follows. First, in Section 2 we provide a concise, informal presentation of the main ideas underlying our account. Then, in Sections 3–5 we present the account in greater detail. In Section 3, we develop a type-theoretic dynamic inquisitive semantics framework, $\mathsf{Inq_D}$. In Section 4, we spell out some basic assumptions about the syntax of declarative and interrogative sentences in English, and provide a compositional semantic analysis of simple declaratives and interrogatives in $\mathsf{Inq_D}$. Finally, in Section 5, we turn to more complex cases and show that the given dynamic account of questions satisfies the desiderata formulated above. Section 6 concludes and present some avenues for future work. The appendix compares our proposal in some depth to three other recent approaches.

2   Informal proposal

In this section we present the proposal informally. Many details are suppressed here, so as to provide direct access to the main ideas. The account rests on five key ideas, which are presented in Sections 2.1-2.5, respectively. A formal implementation will be provided in Sections 3–5.

2.1   Wh-phrases and the witness request operator

As already outlined in Section 1, at the core of our proposal is the idea that, first, the semantic contribution of a wh-phrase, just like that of indefinites in dynamic semantics, is to introduce a discourse referent u and to constrain the range of possible values of u in a certain way (e.g., to people, things, times, or places), and second, that wh-questions involve an update that raises an issue whose resolution requires identifying a set of individuals that satisfy the properties attributed to u, in short a *witness set* for u. This update is denoted as $?u$ and is contributed by an element in the left periphery of a wh-clause. We refer to $?u$ as the *witness request operator*.

2.2   Mention-all readings from maximization of witness sets

As is well-known, when indefinites serve as antecedents for so-called donkey anaphora, two readings can arise, **strong** and **weak**. In (21) the strong reading is most salient: the sentence conveys that whenever Bill eats one or more apples, he peels *all* of the apples he eats.

(21)   If Bill eats an^u apple he peels it$_u$ first.

In (22) on the other hand, the weak reading is most prominent: whenever Bill parks his car and has one or more dimes in his pocket, he throws *one* of the dimes into the parking meter, not necessarily all of them.

(22)   If Bill parks his car and has a^u dime in his pocket, he throws it$_u$ into the parking meter.

This strong/weak ambiguity of donkey anaphora has been a central topic in the dynamic semantics literature (see, e.g., Heim, 1990; Kanazawa, 1994; Brasoveanu, 2008; Champollion *et al.*, 2019). We will follow Brasoveanu (2008) here, who argues that the strong reading is derived by a witness maximization operator, **max**$^*\{u\}$, in the interpretation of the indefinite. Intuitively, in (22a) **max**$^*\{u\}$ ensures that all possible values of u (all the apples that Bill eats) are considered in the consequent. When **max**$^*\{u\}$ is absent, only some values might be considered, which captures the weak reading of (22).

If Brasoveanu (2008) is right that the weak/strong reading of donkey anaphora resides in an ambiguity of indefinites, and if the core idea behind our dynamic approach to questions—namely,that wh-phrases behave semantically just like indefinites—is correct as well, then we should expect that wh-phrases, just like indefinites, can appear with or without the **max*** operator as well. Recall the simple example in (7) above, repeated in (23), and the basic dynamic analysis of this question in (24).

(23)   Who fainted?

(24) $[u]; person\{u\}; fainted\{u\}; ?u$

If wh-phrases are treated as optionally contributing a **max*** operator, then we predict that (23) cannot only be interpreted as in (24) but also as in (25).

(25) $[u]; person\{u\}; fainted\{u\}; \mathbf{max}^*\{u\}; ?u$

On this analysis, resolving the question requires identifying the *maximal witness set* for u. This corresponds to the **mention-all** reading of the question. On the other hand, the analysis in (24), without the **max**$^*\{u\}$ operator, captures the **mention-some** reading. So, our dynamic semantics of wh-questions makes it possible to connect the mention-some/mention-all ambiguity in wh-questions to the weak/strong ambiguity in donkey anaphora. In other words, the ambiguity between mention-some and mention-all readings can be derived in a principled way, relying on a mechanism that is independently needed elsewhere in the grammar.[4]

2.3   Scaling up to multiple-wh questions

The account sketched so far is about single-wh questions but can be generalized in a natural way to deal with multiple-wh questions as well. We propose that in a multiple-wh question, each wh-phrase introduces a discourse referent (dref for short), and that the witness request operator asks for a **functional witness** in this case, i.e., a function mapping the possible values for one dref to the possible values of another. For instance, the multiple-wh question in (26a) is analyzed as in (26b), where $?u_1u_2$ is a functional witness request operator.

(26)   a.   Who ate what?
        b. $[u_1]; [u_2]; ate\{u_1, u_2\}; ?u_1u_2$

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[4]Champollion *et al.* (2019) have recently proposed an alternative strategy to derive strong and weak readings in donkey anaphora, which does not make use of maximization. We leave open here whether this strategy may also be applied in the domain of questions to derive mention-some and mention-all readings.

Informally, $?u_1u_2$ raises an issue whose resolution requires determining, for every possible witness set of u_1, the corresponding witness set of u_2. Or, in other words, a suitable function from the witness sets of u_1 to those for u_2. We will see in Section 3 that both $?u$ and $?u_1u_2$ can be seen as special instances of a general n-ary witness request operator, where $n = 1$ or $n = 2$, respectively. This means that the account is uniform, satisfying desideratum A formulated on page 7.

2.4   Partial mention-some readings in multiple-wh questions

It is typically assumed in the literature that multiple-wh questions only have a *mention-all* pair-list reading.[5] For instance, to resolve the question in (27), one must specify, for each party guest, *everything* that they ate (rather than just *something* that they ate).

(27)   Who ate what at the party last night?

We have observed, however, that in some cases multiple-wh questions also allow for a partial mention-some reading. This was exemplified in (13), repeated in (28).

(28)   Which of these herbs grows where?

This question can, in a suitable context, be resolved by mentioning just one location for each herb. Our account can straightforwardly derive this reading. Just like in single-wh questions, mention-all readings arise through maximization, and mention-some readings arise in the absence of maximization.

Note however that, while (28) is mention-some w.r.t. locations on the relevant reading, it is still mention-all w.r.t. herbs. That is, the question requests information about *all* herbs on the list: it does not suffice to specify just for one herb where it grows.

One might think that the mention-all reading with respect to herbs is a consequence of the fact that the wh-phrase *which of these herbs* is a *which*-phrase whose domain of quantification is clearly given in the context (it is so-called 'D-linked'). Such wh-phrases are known to favour a mention-all reading in single-wh questions as well. However, the same effect arises with non-D-linked wh-phrases which favor a mention-some reading in single-wh questions. For instance, while (29a) has a very salient mention-some reading, (29b) only has a reading that is mention-some w.r.t. *one* of the wh-phrases, not w.r.t. both.

(29)   a.   What is a typical French dish?
        b.   What is a typical dish where?

In English, this appears to be the case in general: multiple-wh questions always require what we will call **domain maximality**—their resolution must map as many potential witnesses of the 'domain dref' as possible to corresponding witnesses of the 'target dref'.

It is important to note that domain maximality as we understand it is different from **domain exhaustivity** in the sense of Dayal (1996). Domain exhaustivity, adapted to our setting, would require that *every* potential witness of the 'domain dref' be matched with a corresponding witnesses of the 'target dref'. Xiang (2019) shows that this requirement is too strong, based on the following

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[5]It is sometimes argued that multiple-wh questions can also have a so-called 'single-pair reading' which is separate from their 'pair-list' reading. In our view, cases in which multiple-wh questions have been reported to have a 'single-pair' reading are just cases where the (most likely) context is one in which only one pair of objects stands in the relation that the question inquires about (see also Dayal, 2016, p.95). If this is correct, an account of multiple-wh questions does not need to derive single-pair readings separately from pair-list readings. In any case, we will concentrate here on pair-list readings. If it turns out that there *is* a need to derive 'single-pair' readings separately, our account will have to be refined.

scenario. Suppose that 100 candidates have applied for 3 jobs. Mary knows the outcome of the search procedure, but Bill doesn't. Then it is natural for Bill to ask:

(30)   Which candidate got which job?

Xiang observes that resolving this question does not require specifying for *every* candidate which job they got, something that would be impossible to do because there were fewer jobs than candidates. However, an additional observation to make about this example is that a resolution of the question must still specify for *as many candidates as possible* (in this case three) which jobs they got. This requirement is what we call domain maximality.

   In our formal account, domain maximality will be captured by an operator that is always present in the left periphery of an interrogative clause. In particular, it will be derived no matter whether a $\mathbf{max}^*$ operator applies to the domain dref or not. Our account will derive two readings for multiple-wh questions: one that is mention-all with respect to all wh-phrases, and one that is mention-all with respect to the 'domain wh-phrase(s)' but mention-some with respect to the 'range wh-phrase'. For this to be possible, it is crucial that on our account mention-some is not a property connected to a question as a whole but rather to individual wh-phrases. In this respect our account differs from many others (e.g., Beck and Rullmann 1999, Chapter 2 of George 2011, Theiler 2014, Champollion *et al.* 2015), which take mention-some to be a property connected to questions as a whole. On such accounts, it is not possible to derive partial mention-some readings of multiple-wh questions (at least not without substantial modifications). To our knowledge, the only previous account that derives such readings is that of Xiang (2016, §5.4.3).

2.5   Effects of number marking

Several effects of number marking need to be captured but the main puzzle is that singular *which*-phrases induce a uniqueness presupposition in single-wh questions but allow for multiple witness-pairs in multiple-wh questions (e.g., Dayal, 1996).

(31)   Which girl danced with Robin? $\rightsquigarrow$ just one girl danced with Robin

(32)   Which girl danced with which boy? $\not\rightsquigarrow$ just one girl and just one boy danced

We will assume, following Brasoveanu (2008) and much other work on number marking in dynamic semantics, that singular number marking, e.g., on *which girl*, invokes an **atomicity requirement**. This ensures that all the possible values assigned to the dref introduced by *which girl* are atomic individuals. Furthermore we will assume, following Roelofsen (2015), that every interrogative clause involves a **presuppositional closure operator**, †, which requires that it is already known in the input context that the actual world lies in one of the alternatives that the question introduces (see also Abusch, 2010). This presupposition is trivially satisfied whenever the alternatives cover the entire logical space, but not when the alternatives cover only part of the logical space.

   A question like (31), repeated in (33a), is then analyzed as in (33b):

(33)   a.   Which^u girl danced?

   b. $\dagger \begin{pmatrix} [u]; \mathbf{atom}\{u\}; girl\{u\}; \\ danced\{u\}; \\ \mathbf{max}^*\{u\}; \\ ?u \end{pmatrix}$

As we will see in detail in Section 5, a uniqueness presupposition is derived in this case from the interplay between $\mathbf{atom}\{u\}$, $\mathbf{max}^*\{u\}$, $?u$, and †. A particularly important role is played by $?u$.

This operator removes from the context any possibilities where more than one girl dances.

For a multiple-wh question we get:

(34)   a.   Which^{u_1} girl danced with which^{u_2} boy?

$$
\text{b.} \quad \dagger \left(
\begin{array}{l}
[u_1]; \mathbf{atom}\{u_1\}; girl\{u_1\}; \\
[u_2]; \mathbf{atom}\{u_2\}; boy\{u_2\}; \\
danced\_with\{u_1, u_2\}; \\
\mathbf{max}^*\{u_1\}; \mathbf{max}^*\{u_2\}; \\
?u_1 u_2
\end{array}
\right)
$$

A global uniqueness requirement is not derived here because, instead of $?u$, the two-place witness request operator $?u_1 u_2$ is at play in this case. This does not remove possibilities from the input context where more than one girl or more than one boy danced, but only ones where there is no function mapping each girl who danced with a boy to the unique boy she danced with. As a result, the account derives a 'relativized' uniqueness presupposition to the effect that no girl danced with more than one boy.

This concludes the informal presentation of the main ideas behind our approach. We will now start to work out these ideas more rigorously.

3   Dynamic inquisitive semantics

3.1   Contexts

In dynamic semantics the meaning of a sentence is viewed as its context change potential. One common way to formalize this idea is to take the semantic value of a sentence to be a function that maps any input context (the context in which the sentence is uttered) to a corresponding output context (the new context after the utterance). A central question, then, is how such contexts are to be modelled.

3.1.1   Contexts in basic inquisitive semantics

We start by considering the notion of context in $\mathsf{Inq_B}$, the basic, static inquisitive semantics framework. In $\mathsf{Inq_B}$, contexts are intended to represent information and issues. For instance, in a certain context it may be known that Bill is in Paris, and it may be an open issue whether Sue is in Paris too. To capture both the information that has been established and the issues that have been raised, contexts are formally modelled in $\mathsf{Inq_B}$ as *sets of information states*, where each information state in turn is a set of possible worlds. The information states that make up a context c are precisely those that (i) contain enough information to resolve the issues that have been raised in the conversation so far and that (ii) do not contain any possible worlds that are already ruled out by the information established in the conversation so far (Ciardelli *et al.*, 2018). For instance, the context informally described above consists of (a) information states s such that all worlds in s are ones in which Bill and Sue are both in Paris, as well as (b) information states s' such that all worlds in s' are ones in which Bill is in Paris but Sue isn't. All these information states support the information that Bill is in Paris and resolve the issue whether Sue in Paris.

The union of all the elements of a context c, $\cup c$, is the set of all worlds that are compatible with the information available in c.[6] This union is denoted as INFO(c). In our example, INFO(c) is the

---

[6]To see this, note in particular that if a world w is compatible with the information established so far in c, then the singleton information state $\{w\}$ will always satisfy conditions (i) and (ii) above.

set of all worlds in which Bill is in Paris.

Not just any set of information states constitutes a proper context representation in $\mathsf{Inq_B}$. Rather, this only holds for sets of information states that are *downward closed*: if they contain a certain information state s, they must also contain any stronger information state $s' \subset s$. This requirement follows from how contexts are construed. To see this, suppose that a context c contains an information state s. This means (i) that s contains enough information to resolve the contextual issues and (ii) that s does not contain any possible worlds that are ruled out by the information available in c. But then, the same goes for any stronger information state $s' \subset s$, which in turn means that these stronger information states must also be in c.

Furthermore, it is assumed in $\mathsf{Inq_B}$ that the inconsistent information state, $\emptyset$, trivially resolves any issue and is therefore contained in any context representation. Given the requirement that contexts are downward closed, the additional requirement that any context contains the inconsistent information state is equivalent to the requirement that any context be non-empty.

In sum, contexts are modeled in $\mathsf{Inq_B}$ as non-empty, downward closed sets of information states, which in turn are sets of possible worlds. This allows us to represent both the information that has been established and the issues that have been raised.

However, a limitation of the context representations in $\mathsf{Inq_B}$ is that they only capture information and issues *about the world*. They do not capture *discourse information*. In particular, they do not keep track of which discourse referents have been introduced, let alone of the information that has been established and the issues that have been raised about these discourse referents.

How should the context representations of $\mathsf{Inq_B}$ be enriched in order to capture such discourse information? Our proposal is to keep the general $\mathsf{Inq_B}$ notion of contexts as downward closed sets of information states intact, but to refine the notion of an *information state*. In $\mathsf{Inq_B}$, information states are simply sets of possible worlds. In dynamic semantics, however, more fine-grained models of information states have been developed, with the intention to capture both world information and discourse information. We will first discuss, in Section 3.1.2, a very simple dynamic model of information states, based on Groenendijk *et al.* (1996) and other work in dynamic semantics. Incorporating this model of information states into inquisitive semantics leads to a basic dynamic inquisitive framework, $\mathsf{Inq_B^D}$, which we have presented in Dotlačil and Roelofsen (2019). In the present article, however, we will work with a more general dynamic model of information states, based on van den Berg (1996); Nouwen (2003) and Brasoveanu (2007, 2008). This model will be presented in Section 3.1.3, and the inquisitive notion of contexts that we build on top of it in Section 3.1.4.

3.1.2 A simple dynamic model of information states

It is quite common in dynamic semantics to model an information state as a set of pairs $\langle w, g \rangle$, where w is a possible world and g an assignment function mapping all discourse referents introduced so far to some concrete individual (Groenendijk *et al.*, 1996, among others). Such pairs are called *possibilities* in Groenendijk *et al.* (1996). A context c formally represented as a set of possibilities captures both information about the world, as well as information about discourse referents. Finally, it also captures possible *dependencies* between these two types of information. To give an example of a dependency, it may be known that the first discourse referent denotes either Alice or Kim, and that the actual world is one in which the individual denoted by this discourse referent, whoever it is, is the tallest woman on earth. Further, it may be known that the individual denoted by the second discourse referent is a friend of the woman denoted by the first discourse referent (Sue for Alice and Mary for Kim). In this context, the value of the second discourse referent depends on that of the first, and the value of the first discourse referent depends on what the world is like.

Let us briefly illustrate how these dynamic information states are updated as a discourse proceeds. Consider the following mini-discourse:

(35) A^{u_1} woman came in. She$_{u_1}$ sat down.

Suppose that the domain of discourse consists of two individuals, a and b, and that when the discourse starts our information state consists of three possibilities $\langle w_1, g_1 \rangle$, $\langle w_2, g_2 \rangle$, and $\langle w_3, g_3 \rangle$. Since no discourse referents have been introduced yet, we have that $g_1 = g_2 = g_3 = \emptyset$. Suppose that w_1, w_2 and w_3 are as follows:

- w_1: a and b are both women, they both came in, only a sat down.

- w_2: a and b are both women, they both came in, only b sat down.

- w_3: a and b are both women, neither of them came in or sat down.

The interpretation of the first sentence, A^{u_1} *woman came in*, involves two steps. The first step is the introduction of a new discourse referent u_1. This means that each possibility $\langle w_i, \emptyset \rangle$ in our initial information state is replaced by two new possibilities $\langle w_i, u_1 \mapsto a \rangle$ and $\langle w_i, u_1 \mapsto b \rangle$, whose world component is the same as before, but whose assignment function maps the new discourse referent u_1 to a or b, respectively. This first step in the interpretation of the sentence thus leads to the following information state:

(36) $\left\{ \begin{array}{l} \langle w_1, u_1 \mapsto a \rangle \\ \langle w_1, u_1 \mapsto b \rangle \\ \langle w_2, u_1 \mapsto a \rangle \\ \langle w_2, u_1 \mapsto b \rangle \\ \langle w_3, u_1 \mapsto a \rangle \\ \langle w_3, u_1 \mapsto b \rangle \end{array} \right\}$

The second step in the interpretation of A^{u_1} *woman came in* is to eliminate all possibilities $\langle w, g \rangle$ from our information state which are such that the individual assigned to u_1 by g is not a woman who came in according to w. More concretely, the possibilities $\langle w_3, u_1 \mapsto a \rangle$ and $\langle w_3, u_1 \mapsto b \rangle$ are eliminated because according to w_3, neither a nor b came in. This leaves us with the following information state:

(37) $\left\{ \begin{array}{l} \langle w_1, u_1 \mapsto a \rangle \\ \langle w_1, u_1 \mapsto b \rangle \\ \langle w_2, u_1 \mapsto a \rangle \\ \langle w_2, u_1 \mapsto b \rangle \end{array} \right\}$

The second sentence, *She$_{u_1}$ sat down*, places further constraints on the possibilities in our information state. Namely, it eliminates possibilities $\langle w, g \rangle$ which are such that the individual assigned to u_1 by g did not sit down according to w. This leads to the following information state:

(38) $\left\{ \begin{array}{l} \langle w_1, u_1 \mapsto a \rangle \\ \langle w_2, u_1 \mapsto b \rangle \end{array} \right\}$

This information state captures that we have obtained the information that the actual world is either w_1 or w_2, and also that the denotation of the discourse referent u_1 depends on what the actual world is. If we are in w_1, then u_1 denotes a; if we are in w_2, then u_1 denotes b. So, again, we

see that information states, modelled as sets of possibilities, capture information about the world, discourse information, and dependencies between them.

The example also highlights another aspect of how information states are updated. Namely, it exemplifies that there are two kinds of updates: *constructive* updates, which introduce new discourse referents and create new possibilities, and *eliminative* updates, which remove possibilities.

As noted above, an inquisitive semantics based on this simple dynamic notion of information states has been developed in Dotlačil and Roelofsen (2019). We now turn to a more general dynamic model of information states which allows us to deal with pluralities, including pluralities in questions. Initial empirical motivation for this more general model came from so-called dependent anaphora. An example of a dependent anaphoric expression is the pronoun it_{u_1} in (39). This pronoun does not refer to a particular paper. Rather, for every student, it refers to the paper written by that student.

(39)   Every student wrote a^{u_1} paper. Most of the students submitted it$_{u_1}$ to a journal.

As has been argued by Krifka (1996) and van den Berg (1996) that dependent anaphora require a non-trivial treatment of the way plural entities are assigned to discourse referents (cf. Nouwen 2003, 2007 for further refinements). It turned out that this analysis of dependent anaphora is also useful in accounting for donkey anaphora (Brasoveanu, 2007), adjectives such as *different* and *same* (Brasoveanu, 2011; Brasoveanu and Dotlačil, 2012), reciprocals (Dotlačil, 2012), and dependent indefinites (Henderson, 2014; Kuhn, 2017). We will argue in the course of this paper that the more sophisticated model also has significant advantages, when incorporated in inquisitive semantics, in the analysis of questions.

3.1.3   A more general dynamic model of information states

In the simple model above, an assignment function g specifies a simultaneous *instantiation* of the active drefs ('simultaneous' in the sense that it specifies an instantiation of each dref). When a new discourse referent u_1 is introduced, new possibilities are created, each involving a randomly picked instantiation of u_1. To determine whether a given possibility $\langle w, g \rangle$ survives a certain eliminative update we have to look at the individuals specified by g as instantiating the active drefs and check whether they satisfy certain properties in w.

The important thing to note is that possibilities are always taken to involve a *single* instantiation of the active drefs. In particular, in constructive updates, newly created possibilities always involve a single instantiation of the newly introduced dref, and in eliminative updates, determining whether a given possibility survives the update only requires us to check a single instantiation of the drefs.

A natural way to generalize this model is to drop the assumption that every possibility involves a *single* simultaneous instantiation of the active drefs. Instead, as proposed in van den Berg (1996) and Brasoveanu (2007, 2012) (see also Väänänen 2007), we may allow for multiple simultaneous instantiations as well. Formally, this means that a possibility is modelled as a pair $\langle w, G \rangle$ where w is a possible world and G a *set* of assignment functions, each specifying a simultaneous instantiation of the active drefs. We will refer to such sets of assignments G as *assignment matrices*, because a set of assignment functions can be displayed as a matrix, where each row lists the values assigned by a specific assignment function to the various drefs, and each column lists the values assigned to a specific dref by the various assignment functions. An example of a dref assignment matrix consisting of three assignment functions, g_1, g_2, g_3, each assigning values to three active drefs, u_1, u_2, u_3, is given in (40):

$$(40) \quad \begin{array}{c|ccc} & u_1 & u_2 & u_3 \\ \hline g_1 & a & b & c \\ g_2 & b & c & a \\ g_3 & c & a & b \end{array}$$

Once we generalize the notion of possibilities in this way, going from pairs $\langle w, g \rangle$ to pairs $\langle w, G \rangle$, a broader range of natural update functions becomes available. Consider constructive updates, which introduce new discourse referents. In the simple model above, it was assumed that each new possibility that such an update creates involves a single instantiation of the newly introduced drefs. Indeed, this is the only natural option we have in that simple model. In the generalized model, however, there are other natural options as well. For instance, while certain constructive updates may create possibilities whose assignment matrices only need to specify *some* possible instantiations of the newly introduced drefs, other constructive updates may only generate possibilities whose assignment matrices specify *all* possible instantiations of the newly introduced drefs. Brasoveanu (2007) makes crucial use of these two types of constructive updates, existential and universal, to account for weak and strong interpretations of donkey anaphora, respectively. We will make use of it as well in accounting for mention-some and mention-all question interpretations.

3.1.4 Contexts in $\mathsf{Inq_D}$

We define contexts in $\mathsf{Inq_D}$ exactly as they are defined in static inquisitive semantics, $\mathsf{Inq_B}$, namely as non-empty downward closed sets of information states. However, the simple static notion of information states as sets of possible worlds is now replaced by the dynamic notion of information states defined above. That is, information states are modelled as sets of possibilities, where each possibility consists of a possible world and an assignment matrix. We think of these information states, just like in $\mathsf{Inq_B}$, as those that (i) contain enough information to resolve the contextual issues, and (ii) do not contain any possibilities that are ruled out by the contextually established information. Only now, the contextual issues and the contextually established information may concern not only what the world is like but may also pertain to the discourse referents that have been introduced. This way, our new notion of context encompasses both information and issues about the world and about discourse referents.

3.1.5 Formal definitions

We now provide explicit formal definitions of the notions discussed above in a type-theoretic logical framework.

Types and frames. First, let us fix the set of types that we assume. There are basic types and complex types. As usual, we assume a basic type e for entities, a basic type s for possible worlds, and a basic type t for truth values. In addition, we also assume a basic type r for discourse referents.[7] Complex types are constructed from these four basic types in two ways. First, for every two types σ and τ, $(\sigma\tau)$ is also a type, namely the type of functions from objects of type σ to objects of type τ. Besides these functional types, which are standardly assumed in type-theoretic semantics, we will also include so-called relational types: for every two types σ and τ, $(\sigma \times \tau)$ is also a type, namely the type of pairs whose first element is of type σ and whose second element is of type τ (Muskens, 1995; Carpenter, 1997).

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[7]As we will see, this means that assignments receive a complex type, namely that of functions from discourse referents to entities. An alternative is to assume a basic type for assignments and a complex type for discourse referents (Muskens, 1996; Brasoveanu, 2007). Nothing essential seems to hinge on this choice.

Definition 1 (Inq$_D$ types)**.**

- Inq$_D$ has four basic types: e for entities, s for possible worlds, t for truth values, and r for discourse referents;

- The set of all Inq$_D$ types Types is the smallest set containing the four basic types, and which is such that for any two types $\sigma \in$ Types and $\tau \in$ Types, we have that $(\sigma\tau) \in$ Types and $(\sigma \times \tau) \in$ Types as well. We will often omit brackets around complex types.

An Inq$_D$ *frame* F, based on a given set of individuals $\mathcal{I}$, specifies for every type $\tau \in$ Types a set D_τ^F, which is called the *domain* of objects of type τ in F. We will often omit the F index on the domains in F, and simply write D_τ. The domain of entities D_e consists of non-empty subsets of $\mathcal{I}$.

Definition 2 (Inq$_D$ frames)**.**
An Inq$_D$ frame based on a set of individuals $\mathcal{I}$ is a set $\{D_\tau \mid \tau \in$ Types$\}$ such that:

- D_e, D_s, D_t, and D_r are pairwise disjoint sets.

- D_e is the set of all non-empty subsets of $\mathcal{I}$. That is, $D_e = \wp(\mathcal{I}) \setminus \emptyset$.

- D_s is a non-empty set of objects that we refer to as possible worlds.

- D_r is a non-empty set of objects that we refer to as discourse referents.

- $D_t = \{0, 1\}$

- For any functional type $\sigma\tau \in$ Types, $D_{\sigma\tau}$ is the set of all functions from D_σ to D_τ.

- For any relational type $\sigma \times \tau \in$ Types, $D_{\sigma\times\tau}$ is the set of all pairs in $D_\sigma \times D_\tau$.

Singleton sets in D_e are called atomic entities; non-singleton sets are called plural entities. The sum of two (atomic or plural) entities $a, b \in D_e$ is defined as their union and is written as $a \oplus b$. Similarly, the sum of a set of (atomic or plural) entities $E \subseteq D_e$ is defined as their union and written as $\oplus E$. The part-of relation, $\leq$, over elements of D_e is the partial order induced by set inclusion, and the proper part-of relation, $<$, holds between non-identical entities that satisfy the part-of relation.

We refer to elements of D_r as discourse referents, and to functions from discourse referents to entities in D_e as *dref assignment functions*. A dref assignment function can be a partial function. That is, it does not necessarily map all discourse referents to an entity. The set of discourse referents $\delta \subseteq D_r$ that it does map to an entity is called its *domain*. We refer to a set of dref assignment functions that have the same domain δ as a *dref assignment matrix* with domain δ. We will abbreviate the type $(re)t$ of dref assignment matrices as m. All abbreviations used for types are summarized in Table 1.

Definition 3 (Dref assignment functions and matrices)**.**
Let F be an Inq$_D$ frame and $\delta \subseteq D_r$ a set of discourse referents in F. Then:

- A function in D_{re} with domain δ is called a *dref assignment function* with domain δ;

- A non-empty set of dref assignment functions with domain δ is called a *dref assignment matrix* with domain δ.

Note that dref assignment matrices are by definition non-empty. In the initial context of a conversation, when no drefs have been introduced yet, we assume that the dref assignment matrix is $\{\emptyset\}$, i.e., a singleton set containing only an empty assignment function, rather than a set containing no assignment functions at all. This will make some notions introduced below easier to define in full generality.

Object	Type	Abbreviation
dref assignment function	re	-
dref matrix	$(re)t$	m
possibility	$s \times m$	-
information state	$(s \times m)t$	i
context	it	k

Table 1: Some commonly used types and their abbreviations.

Definition 4 (Collecting the values of a discourse referent).
If G is a dref assignment matrix with domain δ and u a discourse referent in δ, then:

- $G(u) \coloneqq \{g(u) \mid g \in G\}$

- $\oplus G(u) \coloneqq \oplus \{g(u) \mid g \in G\}$

Possibilities, information states, and contexts. Given these preliminary notions we can now formally define the notions that played a central role in the informal discussion above—possibilities, information states, and contexts. From now on, we assume a given $\mathsf{Inq_D}$ frame F. This means, for instance, that when we say 'let δ be a set of discourse referents' we mean a set of discourse referents in F.

Definition 5 (Possibilities).
For any set of discourse referents δ, a *possibility* with domain δ is a pair $\langle w, G \rangle$, where w is a possible world and G a dref assignment matrix with domain δ. Possibilities are thus of type $s \times m$.

Definition 6 (Information states).

- An *information state* is a set of possibilities.

- Information states are thus of type $(s \times m)t$. We abbreviate this type as i.

In order to define contexts we need one auxiliary notion, that of downward closed sets of information states.

Definition 7 (Downward closed).
A set S of information states is *downward closed* just in case for every information state $s \in S$, every more informed information state $s' \subset s$ is also in S.

Definition 8 (Contexts).

- A *context* is a non-empty, downward closed set of information states.

- Contexts are thus of type (it). We abbreviate this type as k.

Definition 9 (The domain of an information state and a context).

- The domain of an information state s is the union of the domains of the possibilities in s.

- The domain of a context c is the union of the domains of the information states in c.

3.1.6 Informed and inquisitive contexts

As in $\mathsf{Inq_B}$, the union of all the information states in a context c, $\cup c$, is the set of possibilities that are compatible with the information available in c. We will continue to write $\text{INFO}(c)$ for $\cup c$.

Definition 10 (The information available in a context)**.**
For any context c, $\text{INFO}(c) := \cup c$.

A context with domain δ contains non-trivial information if there is a possibility $\langle w, G \rangle$ with domain δ which is *not* in $\text{INFO}(c)$. In this case, we think of the possibility $\langle w, G \rangle$ as having been excluded by the information that has been established in c. If no possibilities have been excluded yet, then the context does not contain any non-trivial information (yet).

Definition 11 (Informed and uninformed contexts)**.**

- A context c with domain δ is informed just in case there is a possibility $\langle w, G \rangle$ with domain δ such that $\langle w, G \rangle \notin \text{INFO}(c)$.

- Otherwise, c is called uninformed.

Resolving the open issues in a context c (if there are any) requires extending the contextually available information, represented by $\text{INFO}(c)$, in such a way as to reach one of the information states in c. In case $\text{INFO}(c)$ is itself an element of c, all the contextual issues are already resolved, i.e., there are no open issues in c. In this case, we say that c is *non-inquisitive*. On the other hand, if $\text{INFO}(c) \notin c$ we say that c is *inquisitive*.

Definition 12 (Inquisitive and non-inquisitive contexts)**.**

- A context c is inquisitive just in case $\text{INFO}(c) \notin c$.

- Otherwise, c is non-inquisitive.

Given a context c, we can always construct a context $!c$ which contains exactly the same information as c, both about the world and about the discourse referents introduced so far, but is not inquisitive. This is achieved by letting $!c$ consist of the information state $\text{INFO}(c)$ plus all subsets thereof. We will refer to this context, again using terminology familiar from $\mathsf{Inq_B}$, as the *non-inquisitive closure* of c. In the definition below we use a downarrow to represent closure under subsets, i.e., for any set of information states S, $S^{\downarrow} := \{s' \mid s' \subseteq s \text{ for some } s \in S\}$.

Definition 13 (Non-inquisitive closure)**.**
For any context c, $\quad !c := \{\text{INFO}(c)\}^{\downarrow}$

Since contexts are always downward closed they can often be represented by means of their *maximal elements*.[8] These elements are information states which contain *just enough* information to resolve the contextual issues. Non-maximal elements also contain enough information to resolve these issues, but they contain more information than is strictly needed to do so. The maximal elements of a context representation are referred to in inquisitive semantics as the *alternatives* that are under consideration in that context. To resolve the contextual issues, one of these alternatives must be established.

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[8]In particular, a context is always fully determined by its maximal elements in case the set of all possibilities is *finite*, which can be assumed in all our examples below.

Definition 14 (Alternatives).
An alternative in a context c is an information state s such that $s \in c$ and there is no strictly weaker information state $t \supset s$ such that $t \in c$.

It generally holds that if a context c contains more than one alternative, it is inquisitive. Vice versa, if a context is non-inquisitive, then it contains only one alternative.[9]
    Finally, we define trivial contexts, the initial context and the inconsistent context as follows.

Definition 15 (Trivial contexts).
A context is trivial just in case it is neither informed nor inquisitive.

Definition 16 (The initial context $c_\top$).
The initial context $c_\top$ is the trivial context whose domain is empty.

Definition 17 (The inconsistent context $c_\perp$).
The inconsistent context $c_\perp := \{\emptyset\}$ only contains the inconsistent information state.

3.1.7 Context extension and subsistence

When exactly should one context be seen as an extension of another? In $\mathsf{Inq_B}$, where information states are plain sets of worlds, the answer to this question is straightforward, namely, c' is an extension of c if and only if $c' \subseteq c$. This guarantees not only that c' contains at least as much information as c but also that the open issues in c' subsume those in c, in the sense that any piece of information that resolves the open issues in c' also resolves those in c.
    In $\mathsf{Inq_D}$, however, where information states are sets of possibilities rather than sets of possible worlds, set inclusion is not the right notion of context extension. To see this, consider the following two contexts:

- $c = \{\{\langle w, \{g\}\rangle\}\}^\downarrow$

- $c' = \{\{\langle w, \{g'\}\rangle\}\}^\downarrow$

where the domain of g is $\{u\}$, that of g' is $\{u, u'\}$, and g' agrees with g on the value of u, i.e., $g'(u) = g(u)$. Both c and c' carry the same information about the world. The only difference is that c pertains to a situation in which there is a single discourse referent, u, whereas c' pertains to a situation in which there is another discourse referent, u', as well. In this case we would like to say that c' is an extension of c. After all, all the information available in c is also available in c', together with additional information about the discourse referent u'. Yet, $c' \not\subseteq c$. This shows that set inclusion is not the right notion of context extension in $\mathsf{Inq_D}$. Below we provide a notion of context extension that does suit our purposes, building on Groenendijk *et al.* (1996). In order to define this notion, we first specify what it means to extend a dref assignment function and dref assignment matrix, a possibility, and an information state.

Definition 18 (Extending dref assignment functions and matrices).

- A dref assignment function g' is an extension of another dref assignment function g if and only if $g' \supseteq g$. Note that for this to hold it is necessary (but not sufficient) that the domain of g' contains the domain of g.

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[9]If the set of all possibilities is finite, then a context always contains at least one alternative. But in general, it is also possible that a context does not contain any alternatives. In this case, by Definition 12, it is inquisitive.

- A dref assignment matrix G' is an extension of another dref assignment matrix G if and only if every $g' \in G'$ is an extension of some $g \in G$ and every $g \in G$ is extended by some $g' \in G'$. In this case we write $G' \geq G$.

Definition 19 (Extending possibilities).
A possibility $\langle w', G' \rangle$ is an extension of another possibility $\langle w, G \rangle$ if and only if $w' = w$ and $G' \geq G$. In this case we write $\langle w', G' \rangle \geq \langle w, G \rangle$.

Definition 20 (Extending information states).
An information state s' is an *extension* of another information state s if and only if every possibility in s' is an extension of some possibility in s. In this case we write $s' \geq s$.

Definition 21 (Extending contexts).
A context c' is an *extension* of another context c if and only if every information state in c' is an extension of some information state in c. In this case we write $c' \geq c$.

It is useful to also define a specific kind of context extension, one which only involves the addition of discourse information. In Groenendijk *et al.* 1996 this is called *subsistence* and we will use the same term here.

Definition 22 (Subsistence of one information state in another).
Let s, s' be information states such that $s' \geq s$. Then we say that s *subsists* in s' if and only if every possibility in s has an extension in s'.

Definition 23 (Subsistence of an information state in a context).
Let s be an information state and c a context. We say that s subsists in c if and only if there is one or more $s' \in C$ such that s subsists in s'. We call all s' in c that satisfy this condition *descendants* of s in c.

Definition 24 (Subsistence of one context in another).
Let c, c' be two contexts such that $c' \geq c$. Then we say that c *subsists* in c' if and only if every state in c subsists in c'.

3.2   A basic formal language and its interpretation

The central formal notion introduced so far is that of $\mathsf{Inq_D}$ *frames*, type-theoretic frames in which, besides the usual basic types e, s, and t, we also have a basic type r for discourse referents. All other notions (possibilities, information states, contexts, update functions, etc.) were defined in terms of such frames.

In principle, we could now turn to natural language and develop a semantic theory which maps expressions of, say, English to objects in our type-theoretic frames. However, following Montague (1973) and many others, we will take a more indirect approach. That is, we will first introduce a *logical* language, and a semantics that maps expressions in this language to objects in our type-theoretic frames. Once this logical language will be in place, we will specify how expressions in a fragment of English, given a certain syntactic analysis, can be translated into our logical language.

We will introduce the logical language and its semantic interpretation in two steps. First, in the remainder of the present subsection we will specify a basic, mostly familiar, type-theoretic logical language and its interpretation in terms of $\mathsf{Inq_D}$ frames. The main novelty at this point will be that there are individual constants of type r, denoting discourse referents, and that there are dref assignment functions which assign values to constants of type r. In Section 3.3 we will add several pieces of new vocabulary to the language. This new vocabulary will all be definable in terms of our basic vocabulary, so it does not extend the expressive power of the language, but it will allow for a more transparent mapping from English expressions to expressions in our logical language.

	Type	Examples of constants	Examples of variables
Entities	e	a, b	x, y
Intransitive predicate	$s(et)$	play, leave	
Transitive predicate	$s(e(et))$	trust, buy	
Discourse referent	r	$u, u_1, u_2, \ldots$	
Context update	$\mathbf{T}$		$\mathcal{U}, \mathcal{U}'$

Table 2: Examples of constants and variables of some commonly used types.

The basic logical language We define a type-logical language $\mathcal{L}$, in which every expression is of a certain type $\tau \in$ Types. The set of all expressions of type τ is denoted as Expr_τ. Expressions are built out of constants and variables, which also each have a type. The set of all constants of a given type τ is denoted as Con_τ, and the set of all variables of type τ is denoted as Var_τ.

Definition 25 (Constants and variables in $\mathcal{L}$).
For any type $\tau \in$ Types, $\mathcal{L}$ contains a set of constants of type τ, Con_τ, and a set of variables of type τ, Var_τ. Some constants and variables in $\mathcal{L}$ are specified in Table 2.

Definition 26 (Expressions in $\mathcal{L}$).
$\mathcal{L}$ is the smallest set such that for any $\sigma, \tau \in$ Types:

1. $\mathsf{Con}_\tau \cup \mathsf{Var}_\tau \subseteq \mathsf{Expr}_\tau$

2. If $\alpha \in \mathsf{Expr}_{\sigma\tau}$ and $\beta \in \mathsf{Expr}_\sigma$, then $\alpha(\beta) \in \mathsf{Expr}_\tau$

3. If $v \in \mathsf{Var}_\sigma$ and $\alpha \in \mathsf{Expr}_\tau$, then $\lambda v.\alpha \in \mathsf{Expr}_{\sigma\tau}$

4. If $\alpha, \beta \in \mathsf{Expr}_\sigma$, then $(\alpha = \beta) \in \mathsf{Expr}_t$

5. If $\alpha, \beta \in \mathsf{Expr}_t$ and $x \in \mathsf{Var}_\sigma$, then $\neg\alpha, (\alpha \wedge \beta), (\alpha \vee \beta), (\alpha \rightarrow \beta), (\exists x \alpha), (\forall x \alpha) \in \mathsf{Expr}_t$

6. If $\alpha \in \mathsf{Expr}_\sigma$ and $\beta \in \mathsf{Expr}_\tau$, then $\langle \alpha, \beta \rangle \in \mathsf{Expr}_{\sigma \times \tau}$

7. If $\alpha \in \mathsf{Expr}_{\sigma \times \tau}$, then $\pi_1(\alpha) \in \mathsf{Expr}_\sigma$ and $\pi_2(\alpha) \in \mathsf{Expr}_\tau$

8. If $R \in \mathsf{Expr}_{s(e^n t)}$, then $*R \in \mathsf{Expr}_{s(e^n t)}$ as well[10]

9. If $G \in \mathsf{Expr}_m$ and $u \in \mathsf{Expr}_r$, then $G(u) \in \mathsf{Expr}_{et}$

10. If $\alpha, \beta \in \mathsf{Expr}_e$, then $\alpha \oplus \beta \in \mathsf{Expr}_e$ and $\alpha < \beta \in \mathsf{Expr}_t$

11. If $\alpha \in \mathsf{Expr}_{et}$, then $\oplus\alpha \in \mathsf{Expr}_e$

12. If $g \in \mathsf{Expr}_{re}$, then $\mathbf{dom}(g) \in \mathsf{Expr}_{rt}$

13. If $g, g' \in \mathsf{Expr}_{re}$ and $u \in \mathsf{Expr}_r$, then $g[u]g' \in \mathsf{Expr}_t$

14. If $G, G' \in \mathsf{Expr}_m$ and $u \in \mathsf{Expr}_r$, then $G[u]G' \in \mathsf{Expr}_t$

15. If $p, p' \in \mathsf{Expr}_{s \times m}$ and $u \in \mathsf{Expr}_r$, then $p[u]p' \in \mathsf{Expr}_t$

---

[10]We use $e^n t$ here, for any $n \geq 1$, to abbreviate the type of functions that take n individuals as input and yield a truth value as output. That is, when $n = 1$, $e^n t \coloneqq et$, and when $n > 1$, $e^n t \coloneqq e(e^{n-1}t)$.

Thus, our basic language $\mathcal{L}$ is built on a standard type-theoretic language with functional and relational types (generated by the first seven rules). $\mathcal{L}$ also includes several additional elements that are less commonly used (rules 8 to 15) and whose interpretation will be explained in the following paragraphs.

Semantic interpretation We now turn to the semantic interpretation of the expressions in $\mathcal{L}$. First, we define interpretation functions and variable assignment functions in the usual way, i.e., as determining the denotation of constants and variables, respectively.

Definition 27 (Interpretation functions).
Given an $\mathsf{Inq_D}$ frame F, an interpretation function into F is a function which, for any type $\tau \in \mathsf{Types}$, maps every constant $\alpha \in \mathsf{Con}_\tau$ to some object in D_τ^F.

Definition 28 (Variable assignment functions).
Given an $\mathsf{Inq_D}$ frame F, a variable assignment function into F is a function which, for any type $\tau \in \mathsf{Types}$, maps every variable $v \in \mathsf{Var}_\tau$ to some object in D_τ^F.

If θ is a variable assignment function into a frame F, $v \in \mathsf{Var}_\tau$ a variable of some type $\tau \in \mathsf{Types}$, and $d \in D_\tau^F$ an object of type τ in F, then we write $\theta^{v/d}$ for the assignment function which is identical to θ except that it maps v to d.

We write $\llbracket \alpha \rrbracket^{F,I,\theta}$ for the semantic value of an expression $\alpha \in \mathcal{L}$ relative to a frame F, an interpretation function I into F, and a variable assignment function θ into F. $\llbracket \alpha \rrbracket^{F,I,\theta}$ is recursively defined as follows.

Definition 29 (The semantic value of an expression).
Let F be an $\mathsf{Inq_D}$ frame, I an interpretation function into F, θ a variable assignment function into F, and $\sigma, \tau \in \mathsf{Types}$. Then:

1. $\llbracket \alpha_\tau \rrbracket^{F,I,\theta} = I(\alpha)$ if $\alpha \in \mathsf{Con}_\tau$

2. $\llbracket \alpha_\tau \rrbracket^{F,I,\theta} = \theta(\alpha)$ if $\alpha \in \mathsf{Var}_\tau$

3. $\llbracket \alpha_{\sigma\tau}(\beta_\sigma) \rrbracket^{F,I,\theta} = \llbracket \alpha \rrbracket^{F,I,\theta}(\llbracket \beta \rrbracket^{F,I,\theta})$

4. $\llbracket \lambda v_\sigma.\alpha_\tau \rrbracket^{F,I,\theta} =$ the function $f \in D_{\sigma\tau}$ which maps any object $d \in D_\sigma^F$ to $\llbracket \alpha \rrbracket^{F,I,\theta^{v/d}}$

5. $\llbracket \alpha_\tau = \beta_\tau \rrbracket^{F,I,\theta} = \begin{cases} 1 & \text{if } \llbracket \alpha \rrbracket^{F,I,\theta} = \llbracket \beta \rrbracket^{F,I,\theta} \\ 0 & \text{if } \llbracket \alpha \rrbracket^{F,I,\theta} \neq \llbracket \beta \rrbracket^{F,I,\theta} \end{cases}$

6. $\llbracket \neg\alpha_t \rrbracket^{F,I,\theta} = 1$ iff $\llbracket \alpha \rrbracket^{F,I,\theta} = 0$

7. $\llbracket \alpha_t \wedge \beta_t \rrbracket^{F,I,\theta} = 1$ iff $\llbracket \alpha \rrbracket^{F,I,\theta} = 1$ and $\llbracket \beta \rrbracket^{F,I,\theta} = 1$

8. $\llbracket \alpha_t \vee \beta_t \rrbracket^{F,I,\theta} = 1$ iff $\llbracket \alpha \rrbracket^{F,I,\theta} = 1$ or $\llbracket \beta \rrbracket^{F,I,\theta} = 1$

9. $\llbracket \alpha_t \rightarrow \beta_t \rrbracket^{F,I,\theta} = 1$ iff $\llbracket \alpha \rrbracket^{F,I,\theta} = 0$ or $\llbracket \beta \rrbracket^{F,I,\theta} = 1$

10. $\llbracket \exists x_\sigma \alpha_t \rrbracket^{F,I,\theta} = 1$ iff $\llbracket \alpha \rrbracket^{F,I,\theta^{x/d}} = 1$ for some $d \in D_\sigma^F$

11. $\llbracket \forall x_\sigma \alpha_t \rrbracket^{F,I,\theta} = 1$ iff $\llbracket \alpha \rrbracket^{F,I,\theta^{x/d}} = 1$ for all $d \in D_\sigma^F$

12. $\llbracket \langle \alpha_\sigma, \beta_\tau \rangle \rrbracket^{F,I,\theta} = \langle \llbracket \alpha_\sigma \rrbracket^{F,I,\theta}, \llbracket \beta_\tau \rrbracket^{F,I,\theta} \rangle$
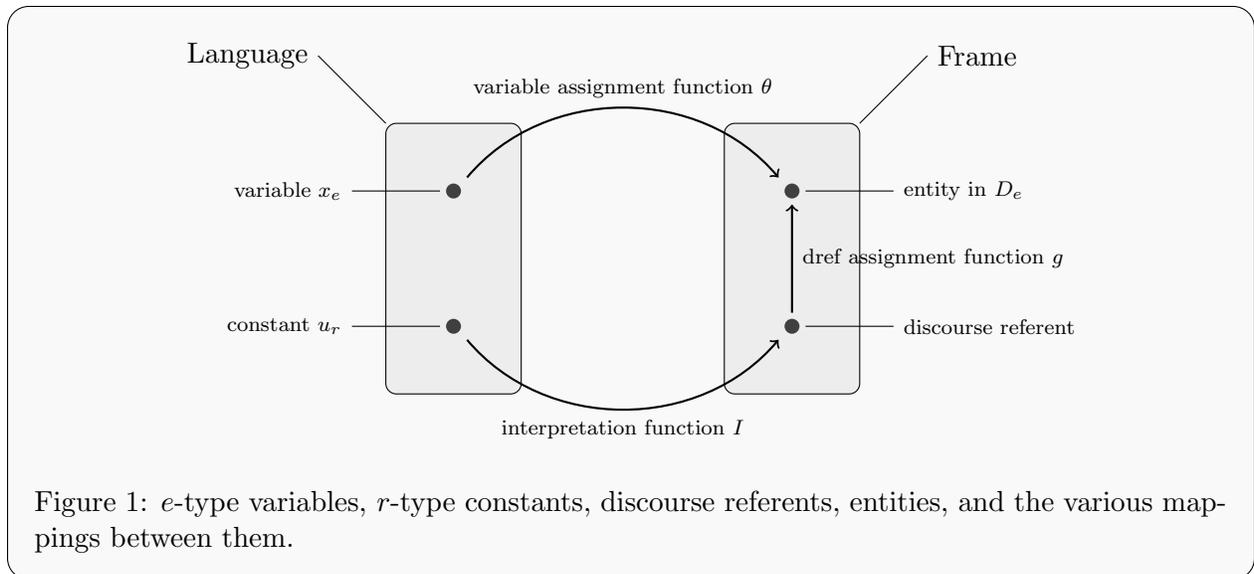
13. $[\![\pi_1(\alpha_{\sigma\times\tau})]\!]^{F,I,\theta}$ = the first element of the pair $[\![\alpha]\!]^{F,I,\theta}$

14. $[\![\pi_2(\alpha_{\sigma\times\tau})]\!]^{F,I,\theta}$ = the second element of the pair $[\![\alpha]\!]^{F,I,\theta}$

15. $[\![^*R_{(s(e^nt))}]\!]^{F,I,\theta}$ is the smallest set such that:

- For all $d_1, \ldots d_n, d'_1, \ldots d'_n \in D_e^F$ and for all $w \in D_s^F$:
- If $[\![R]\!]^{F,I,\theta}(w)(d_1)\ldots(d_n) = 1$, then $[\![^*R]\!]^{F,I,\theta}(w)(d_1)\ldots(d_n) = 1$
- If $[\![^*R]\!]^{F,I,\theta}(w)(d_1)\ldots(d_n) = 1$ and $[\![^*R]\!]^{F,I,\theta}(w)(d'_1)\ldots(d'_n) = 1$,
  then $[\![^*R]\!]^{F,I,\theta}(w)(d_1 \oplus d'_1)\ldots(d_n \oplus d'_n) = 1$

16. $[\![G(u)]\!]^{F,I,\theta} = \{[\![g(u)]\!]^{F,I,\theta} \mid g \in G\}$

17. $[\![\oplus\alpha_{et}]\!]^{F,I,\theta} = \bigcup[\![\alpha]\!]^{F,I,\theta}$

18. $[\![\alpha_e \oplus \beta_e]\!]^{F,I,\theta} = [\![\alpha]\!]^{F,I,\theta} \cup [\![\alpha]\!]^{F,I,\theta}$

19. $[\![\alpha_e < \beta_e]\!]^{F,I,\theta} = \begin{cases} 1 & \text{if } [\![\alpha]\!]^{F,I,\theta} \subset [\![\beta]\!]^{F,I,\theta} \\ 0 & \text{otherwise} \end{cases}$

20. $[\![\mathbf{dom}(g)]\!]^{F,I,\theta} = \{r \in D_r^F \mid \langle r,d \rangle \in [\![g]\!]^{F,I,\theta} \text{ for some } d \in D_e^F\}$

21. $[\![g[u]g']\!]^{F,I,\theta} = 1$ iff

- $[\![\mathbf{dom}(g')]\!]^{F,I,\theta} = [\![\mathbf{dom}(g)]\!]^{F,I,\theta} \cup \{[\![u]\!]^{F,I,\theta}\}$ and
- $[\![\forall v_r((v \neq u \wedge v \in \mathbf{dom}(g)) \to g(v) = g'(v))]\!]^{F,I,\theta} = 1$

22. $[\![G[u]G']\!]^{F,I,\theta} = 1$ iff

- $[\![\forall g \in G.\ \exists g' \in G'.g[u]g']\!]^{F,I,\theta} = 1$ and
- $[\![\forall g' \in G'.\exists g \in G.\ g[u]g']\!]^{F,I,\theta} = 1$

23. $[\![p[u]p']\!]^{F,I,\theta} = 1$ iff

- $[\![\pi_1(p) = \pi_1(p')]\!]^{F,I,\theta} = 1$ and
- $[\![\pi_2(p)[u]\pi_2(p')]\!]^{F,I,\theta} = 1$

The semantic rules in 1–11 are standard. Let us comment on those in 12–23. Rule 12 says that the semantic value of a pair $\langle \alpha, \beta \rangle$ (e.g., a possibility) is, as expected, a pair whose first element is the semantic value of α and whose second element is the semantic value of β. Rules 13 and 14 specify that if α is a pair, $\pi_1(\alpha)$ denotes the first element of the pair and $\pi_2(\alpha)$ its second element.

Rule 15 says that the denotation of $^*R$ is obtained by closing the denotation of R under sums of entities, which are constructed using $\oplus$. That is, whenever two entities a and b are in the extension of R, then a, b, as well as $a \oplus b$ are in the extension of $^*R$. Closure of relations under sum formation is common when the domain consists of both plural and atomic entities, as is the case here (see Krifka, 1989; Landman, 2000; Kratzer, 2008). The * operator is usually called the *pluralization operator*.

Rule 16 says that the semantic value of $G(u)$ is the collection of all those entities that are assigned to u by some element of G. For instance, if $G = \{g_1, g_2\}$, g_1 maps u to a, and g_2 maps u to b, then the semantic value of $G(u)$ is the set $\{a, b\}$.

Rules 17-19 specify the interpretation of $\oplus$ and $<$ in the logical language. These are the same as in the meta-language: $\oplus$ is used to 'merge' two entities, or an arbitrary set of entities, into a

Figure 1: e-type variables, r-type constants, discourse referents, entities, and the various mappings between them.

single, plural entity. If α and β are two entity-denoting expressions then $\alpha < \beta$ is true just in case all the individuals that α consists of are also part of β.

Rule 20 defines the domain of a dref assignment function. Rules 21–23 will be used to define update functions that introduce new discourse referents. Rule 21 specifies the interpretation of $g[u]g'$, which says that two dref assignments, g and g', differ from each other at most with respect to the value assigned to a, possibly newly introduced, dref u. Rules 22 and 23 extend Rule 21 to dref assignment matrices and possibilities, respectively. The definition in rule 22 follows Brasoveanu (2007) and subsequent dynamic systems that operate with dref assignment matrices. It is assumed that G' differs from G at most w.r.t. u if and only if for all g in G there is a corresponding g' in G' that differs from g at most w.r.t. u, and vice versa, for all g' in G' there is a corresponding g in G such that g' differs from g at most w.r.t. u. Finally, rule 23 says that the possibility p' differs from the possibility p at most w.r.t. u if and only if they have the same world (the first projection) and the dref assignment matrix of p' differs from that of p at most w.r.t. u.

It may be useful at this point to highlight the difference between *variables* and *discourse referents* in our system. Variables are expressions in our logical language, while discourse referents are objects in a frame. Discourse referents are mapped to entities in D_e by dref assignment functions, which are themselves objects in D_{re}. Variables of type e are mapped to entities in D_e by variable assignment functions. These functions are not part of a frame, but specify a mapping from elements of the logical language to elements of the frame. Finally, note that our logical language contains expressions $u, u', \ldots$ whose semantic values are discourse referents. However, these expressions are not discourse referents themselves. Neither are they variables. Rather, they are constants. Their semantic value is not determined by a variable assignment function, but by an interpretation function. These notions are all depicted in Fig. 1.

3.3 Extending the basic formal language with update expressions

We now extend the basic logical language introduced above with expressions that will make it easier to translate English sentences into the logical language. These additional expressions in the logical language will all be of type **T**, which means that they all denote context update functions. We will therefore refer to them as update expressions.

3.3.1 Predicational and relational update expressions

We start with expressions specifying that a certain dref has a particular property, e.g., $smile\{u\}$, or that two or more drefs stand in a particular relation to each other, e.g., $see\{u_1, u_2\}$.[11] Semantically, $smile\{u\}$ denotes a context update function. This function takes an input context c and returns a new context, which is the set of information states s in c such that for every possibility p in s and every assignment g in G_p,[12] it holds that $g(u)$ is in the extension of *$smile$ in w_p. We adopt the common assumption that lexical predicates like $smile$ are closed under sums, i.e., they are cumulative (Krifka, 1989; Landman, 2000; Kratzer, 2008). In (41), this is captured by the * preceding the predicate. The semantic interpretation of *R was given in Section 3.2.[13]

(41) $smile\{u\} \coloneqq \lambda c_k \lambda s_i.\ s \in c\ \wedge\ \forall p \in s.\ \forall g \in G_p.\ \textit{*smile}(w)(g(u))$

The predicational update function $smile\{u\}$ can be generalized to the n-ary relational update function $R\{u_1, \ldots u_n\}$ in (42):

(42) $R\{u_1, \ldots u_n\} \coloneqq \lambda c_k \lambda s_i.\ s \in c \wedge \forall p \in s.\ \forall g \in G_p.\ \textit{*R}(w)(g(u_1)) \ldots (g(u_n))$

To illustrate the effects of update functions, we will use diagrams like the one in Figure 2 for $smile\{u\}$. In this diagram, the context on the left is the input context and the one on the right is the output context. Recall that a context is a downward closed set of information states. In diagrams, we always depict only the maximal information states in the context, i.e., the alternatives. In Figure 2, both the input context and the output context contain just one alternative.

The grey arrow in the middle of Figure 2 represents the transition from input to output context due to the update function expressed by $smile\{u\}$. The update eliminates all states in the input context containing at least one possibility according to which one or more entities assigned to u do not smile. Note that the update does not eliminate possibilities $\langle w_{a,b}, G \rangle$ which are such that all assignments in G map u to $a \oplus b$ (see the second column, fourth line in the output context). This is a consequence of closure under sums, which lets the plurality $a \oplus b$ be in the extension of *$smile$ if a and b are in the extension of $smile$.

3.3.2 Conjunction

Update functions can be combined by means of conjunction. As usual in dynamic semantics, we take conjunction to express function composition: the input context is first updated by the first conjunct and then by the second.

(43) $\mathcal{U}_{\mathbf{T}}; \mathcal{U}'_{\mathbf{T}} \coloneqq \lambda c_k.\ \mathcal{U}'(\mathcal{U}(c))$

---

[11]We use curly brackets in these expressions rather than round ones to flag that predicates like $smile$ and see do not directly apply to drefs ($smile$ is not of type rt but rather of type $s(et)$). Context update expressions like $smile\{u\}$ and $see\{u_1, u_2\}$ are abbreviations of complex expressions in the basic logical language, as defined below.

[12]Here and throughout we use w_p as an abbreviation of $\pi_1(p)$ and G_p as an abbreviation of $\pi_2(p)$.

[13]When u has not been introduced yet in a context c, then the result of applying $R\{u\}$ to c is undefined. This is a case of unresolved anaphora. We do not provide an explicit analysis of how such undefinedness projects, i.e., of how it affects the semantic value of complex expressions containing $R\{u\}$, because this is orthogonal to our main concerns here and adding an explicit account of undefinedness projection would significantly complicate the presentation of the framework. As far as we know it is possible to combine $\mathsf{Inq_D}$ with dynamic systems that deal with undefinedness, like partial CDRT (Haug, 2013) and dynamic semantics with stacks (Vermeulen, 1993; Nouwen, 2007; van Eijck and Unger, 2010).

Figure 2: The update effect of $smile\{u\}$. In this and the following diagrams, subscripts on worlds specify which individuals smile in that world.
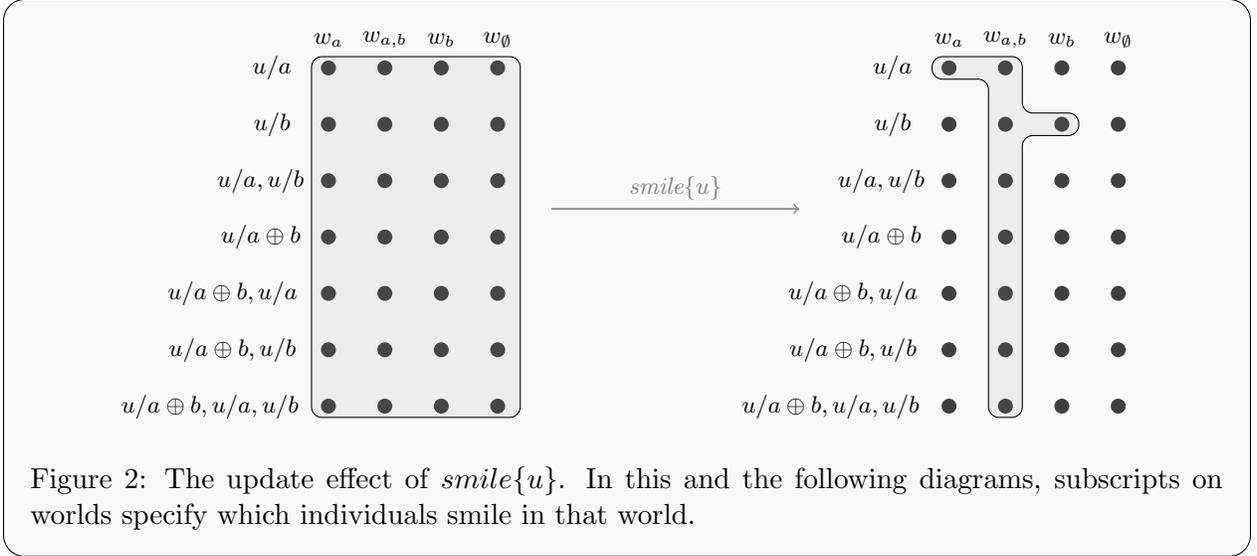
3.3.3   Introducing new discourse referents

We write $[u]$ for the update function that introduces a new discourse referent u. Loosely speaking, $[u]$ updates a state in the input context by 'randomly assigning' any possible values to u across the dref assignment matrices of the possibilities in the state. The formal definition, given in (44), makes use of the logical vocabulary $p[u]p'$, which was defined in Sect. 3.2: it expresses that the dref assignment matrix in the possibility p' differs from that in p at most w.r.t. u.

(44) $[u] := \lambda c_k \lambda s_i.\ \exists s' \in c. \left(\begin{array}{l} \forall p \in s.\ \ \exists p' \in s'.(p'[u]p)\ \wedge \\ \forall p' \in s'.\ \exists p \in s.\ (p'[u]p) \end{array} \right)$

Procedurally speaking, we create a state s in the output context of $[u]$ by taking a state s' from the input context c and requiring that every possibility p in s differs from some possibility p' in s' only w.r.t. u, and that for every possibility p' in s' there is some possibility p in s which only differs from p' w.r.t. u.[14]

The effect of the update function $[u]$ is illustrated in Figure 3. In the initial context, no dref is present yet. The dref assignment matrix in each possibility is a singleton set containing an empty dref assignment function. After the update, every possibility in every state assigns some entity to u.[15] If the context is further updated with $smile\{u\}$, it is restricted to states consisting of possibilities with dref assignment matrices assigning only entities to u that smile.

3.3.4   Disjunction

Disjunction is defined in a similar way as in basic (static) inquisitive semantics. We assume that the result of applying a disjunction of two update functions to a context c is obtained by applying the two update functions separately to c and then forming the union of both output contexts:

(45) $\mathcal{U}_{\mathbf{T}} \sqcup \mathcal{U}'_{\mathbf{T}} := \lambda c_k.\mathcal{U}(c) \cup \mathcal{U}'(c)$

---

[14]Readers familiar with inquisitive semantics might notice that the introduction of discourse referents does not introduce inquisitiveness in $\mathsf{Inq}_{\mathsf{D}}$, in contrast to existential quantification in $\mathsf{Inq}_{\mathsf{B}}$.

[15]Recall that we assume that there are only two atomic entities a and b in the domain of discourse.

Figure 3: An illustration of a sequence of updates starting with the introduction of a dref.

Figure 4: The update effect of $([u_1]; sing\{u_1\}) \sqcup ([u_1]; smile\{u_1\})$. The input context assigns no value to any dref. It is assumed here that D_e contains only one entity, a. w_{sing} is a world in which a sings, w_{smile} a world in which a smiles, $w_{sing,smile}$ a world in which a does both, and $w_\emptyset$ a world in which a does neither.

Disjunctions can turn a non-inquisitive context into an inquisitive one. This is illustrated in Figure 4, where the input context contains a single alternative, but the output context contains two alternatives, each corresponding to one of the disjuncts.

In dynamic semantics, it is often assumed that disjunction is 'internally dynamic' and 'externally static' (see, e.g., Groenendijk and Stokhof, 1991; Kamp and Reyle, 1993; Groenendijk *et al.*, 1996). This means that discourse referents introduced by one of the disjuncts cannot be 'picked up' by anaphoric expressions outside of the disjunction. The treatment of disjunction in $\mathsf{Inq_D}$ is more subtle. Following Stone (1992), we predict that when both disjuncts introduce the same new discourse referent, it is accessible for anaphoric expressions outside of the disjunction. This can be seen in Figure 4: every possibility in the output context assigns a value to the discourse referent introduced inside the disjunction, u_2. This treatment of disjunction accounts for the following contrast observed by Stone (1992). See Fig. 5 for a visual illustration.

(46)   a.   Bill either rented a^u station wagon or a^u van. He took it$_u$ out for three days.
          b.   Bill either rented a^u car or decided to hitchhike. *He took it$_u$ out for three days.
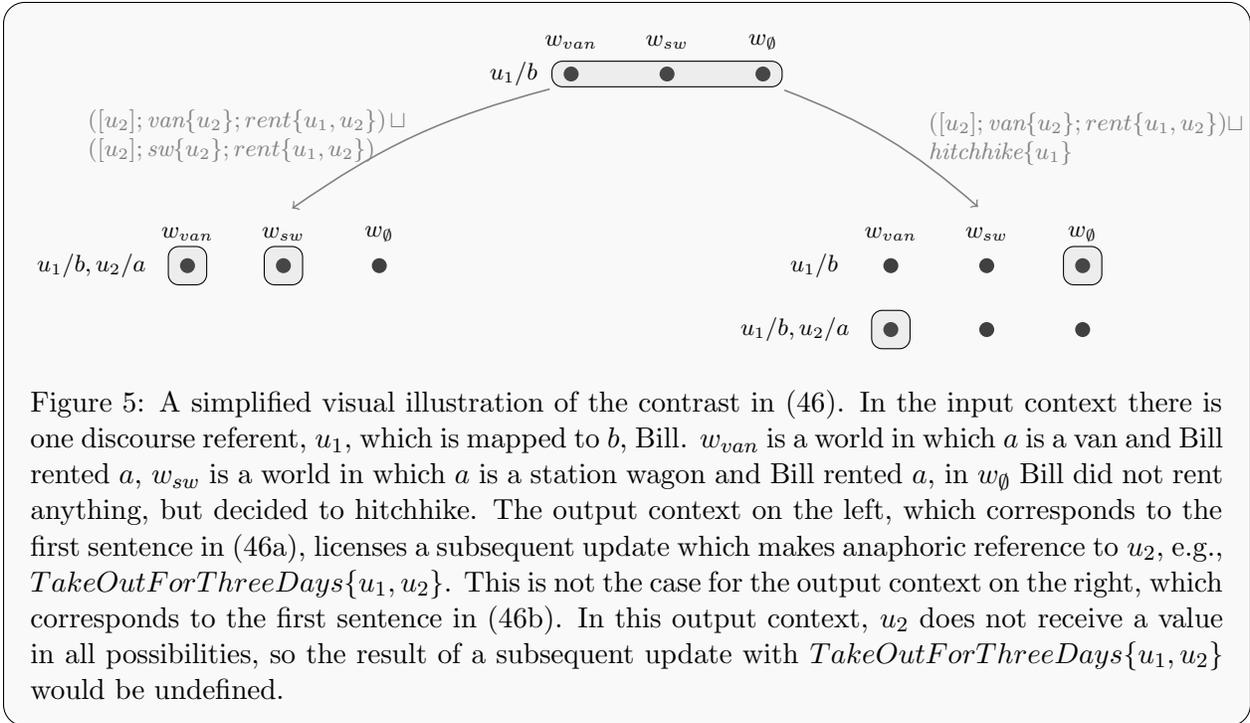
Figure 5: A simplified visual illustration of the contrast in (46). In the input context there is one discourse referent, u_1, which is mapped to b, Bill. w_{van} is a world in which a is a van and Bill rented a, w_{sw} is a world in which a is a station wagon and Bill rented a, in $w_\emptyset$ Bill did not rent anything, but decided to hitchhike. The output context on the left, which corresponds to the first sentence in (46a), licenses a subsequent update which makes anaphoric reference to u_2, e.g., $TakeOutForThreeDays\{u_1, u_2\}$. This is not the case for the output context on the right, which corresponds to the first sentence in (46b). In this output context, u_2 does not receive a value in all possibilities, so the result of a subsequent update with $TakeOutForThreeDays\{u_1, u_2\}$ would be undefined.

Our analysis of disjunction also predicts that if subsequent updates resolve the issue raised by the disjunction by confirming the disjunct that introduced a new dref, then that dref becomes accessible. This captures the felicity of the following two mini-dialogues:[16]

(47) **A:** Bill either rented a^u car or decided to hitchhike.
      B: The former, of course. He took it$_u$ out for three days.

(48) **A:** Did Bill rent a^u car$^\uparrow$ or did he decide to hitchhike$^\downarrow$?
      B: The former, of course. He took it$_u$ out for three days.

3.3.5 Negation

Our treatment of negation, given in (49), is close in spirit to the treatment of negation in static inquisitive semantics. Recall that, informally speaking, a state t subsists in an output context c if, modulo additional discourse referents, t is still present in c. The formal definition of subsistence

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[16]Famously, anaphoric expressions in the second disjunct can also pick up discourse referents introduced in the scope of a negation operator within the first disjunct, as in the following example due to Partee:

(i)   Either this house doesn't have a bathroom or it's in a funny place.

Our treatment of disjunction currently leaves such cases unaccounted for. An exploration of how existing accounts of such cases (e.g., Krahmer and Muskens, 1995; Rothschild, 2017) may be incorporated in $\mathsf{Inq_D}$ is left for future work. Such an extension of $\mathsf{Inq_D}$ would be needed to account for questions like (ii), which, to our knowledge, no existing theory can deal with.

(ii)   Does this house not have a bathroom or is it in a funny place?

was given in Section 3.1.7.[17]

(49) $\neg\mathcal{U}_{\mathbf{T}} \coloneqq \lambda c_k \lambda s_i.\ s \in c\ \wedge\ \forall t \subseteq s.(t \neq \emptyset \to t \text{ does not subsist in } \mathcal{U}(c))$

The update function $\neg\mathcal{U}$, when applied to an input context c, yields an output context consisting of those states in c whose non-empty substates do not subsist in $\mathcal{U}(c)$. Note in particular that states in the output context are always ones that were already present in the input context as well. This means that any discourse referents introduced within the scope of a negation operator (by $\mathcal{U}$) are inaccessible outside of that scope. In other words, negation is 'externally static', as is standardly assumed in dynamic semantics (see, e.g., Groenendijk and Stokhof, 1991; Kamp and Reyle, 1993).

Another property of negation as defined in (49), inherited from static inquisitive semantics, is that a update function expression by $\neg\mathcal{U}$ never raises any issues, even if $\mathcal{U}$ itself does. These two features of negation—that it is externally static and discharges issues raised inside its scope—are illustrated in Figure 6.

Figure 6: Negation applied to an inquisitive update expression, $\neg(smile\{u_1\} \sqcup smile\{u_2\})$, and to an update expression introducing a new discourse referent, $[u_3]; smile\{u_3\}$. The output context is the same in both cases. Note that this output context is not inquisitive, and that u_3 is not present in the output context.
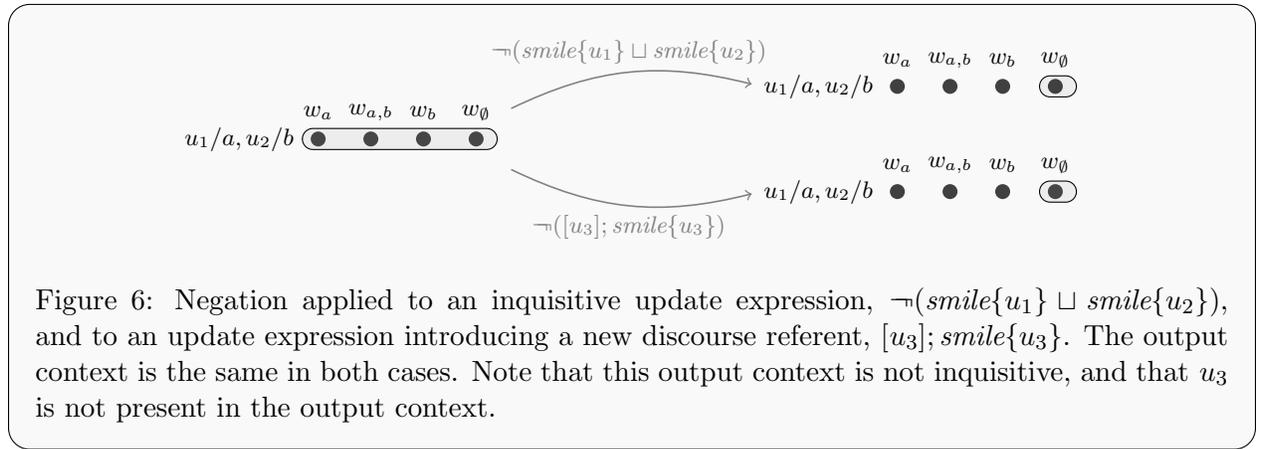
3.3.6 Double negation and removing inquisitiveness

A doubly negated update expression $\neg\neg\mathcal{U}$ always conveys exactly the same information about the world as $\mathcal{U}$ itself, but it never raises any issues and it always renders the drefs introduced by $\mathcal{U}$ inaccessible, due to the two features of negation highlighted above.

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[17]In Sect. 3.1.7 we defined extension and subsistence in the meta-language. These notions can also be defined in our logical language, as follows:

(i)   Dref matrix extension: $G' \geq G \coloneqq \begin{cases} \forall g \in G.\ \exists g' \in G'.g \subseteq g'\ \wedge \\ \forall g' \in G'.\exists g \in G.\ g \subseteq g' \end{cases}$

(ii)   Possibility extension: $p' \geq p \coloneqq\ (w_{p'} = w_p) \wedge (G_{p'} \geq G_p)$

(iii)   State extension: $s' \geq s \coloneqq\ \forall p' \in s'.\exists p \in s.\ p' \geq p$

(iv)   Context extension: $c' \geq c \coloneqq\ \forall s' \in c'.\exists s \in c.\ s' \geq s$

(v)   Subsistence of a state in another state: s subsists in $s' \coloneqq\ (s' \geq s) \wedge \forall p \in s.\exists p' \in s'.\ p' \geq p$

(vi)   Subsistence of a state in a context: s subsists in $c \coloneqq\ \exists s' \in c.(s \text{ subsists in } s')$

(vii)   Subsistence of a context in a context: c subsists in $c' \coloneqq\ \forall s \in c.(s \text{ subsists in } c')$

As we will see, it will be useful in the analysis of English to have another operator that only discharges issues in its scope without affecting information about the world and discourse information. Following work on static inquisitive semantics, we will refer to this operator as the issue-cancelling operator and denote it as '!'. This operator will be used, for instance, in our analysis of declarative sentences, which do not raise issues but are externally dynamic.

We define the update effect of $!\mathcal{U}$ as follows:[18]

(50) $!\mathcal{U}_{\mathbf{T}} := \lambda c_k \lambda s_i . s \in !(\mathcal{U}(c)) \wedge \exists s' \in c. s \geq s'$

The first conjunct says that states in the output context $!\mathcal{U}$ must be in the non-inquisitive closure of $\mathcal{U}(c)$. This condition, however, is not sufficient since if we simply took $!\mathcal{U}$ to be the output context, any issues that were present in the input context c would be discharged. What $!\mathcal{U}$ is meant to do is to discharge any issues that are raised by $\mathcal{U}$, but not to discharge issues that are already present in the input context. The second conjunct in (50) therefore requires that all states in the output context must be extensions of states in the input context. This means that all states in the output context still resolve all issues that were present in the input context.

Fig. 7 highlights the difference between ! and double negation when applied to the inquisitive update $smile\{u_1\} \sqcup smile\{u_2\}$ and the dref-introducing update $[u_3]; smile\{u_3\}$. Both ! and double negation eliminate the issue raised by the disjunction (left column). By contrast, when ! applies to an update that introduces a discourse referent such as $[u_3]; smile\{u_3\}$ it does not make the discourse referent inaccessible for subsequent anaphoric expressions, unlike double negation.

3.3.7   Introducing inquisitiveness

While the ! operator removes inquisitiveness, static inquisitive semantics also comes with the ? operator, which *introduces* inquisitiveness by taking the disjunction of a sentence and its own negation. In our dynamic setting, this operator can be defined as follows:

(51) $?\mathcal{U}_{\mathbf{T}} := \mathcal{U} \sqcup \neg \mathcal{U}$

Applying ? to a given update $\mathcal{U}$ always results in an inquisitive update, except when $\mathcal{U}$ is tautological or contradictory. However, it is not always the most *conservative* way of turning $\mathcal{U}$ into an inquisitive update. In particular, if $\mathcal{U}$ is already inquisitive itself, forming the disjunction with $\neg \mathcal{U}$ is of course not needed to obtain an inquisitive update; we can simply leave $\mathcal{U}$ untouched. Following work on static inquisitive semantics (e.g., Roelofsen 2015, Roelofsen and Farkas 2015 and Ciardelli *et al.* 2018, p.103) we therefore also introduce the following variant of the ? operator:

(52) $\langle ? \rangle \mathcal{U} := \begin{cases} ?\mathcal{U} & \text{if } \mathcal{U} \text{ is not inquisitive yet} \\ \mathcal{U} & \text{if } \mathcal{U} \text{ is already inquisitive} \end{cases}$

---

[18]Recall that for any context c, $!c$ is the non-inquisitive closure of c as defined in the meta-language in Sect. 3.1.6. The definition in the logical language is given here in two steps. First we provide auxiliary definitions for $c^{\downarrow}$ and $\text{INFO}(c)$:

(i) $c^{\downarrow} := \lambda s. \exists s' \in c. s \subseteq s'$

(ii) $\text{INFO}(c) := \lambda p. \exists s \in c. p \in s$

Next, we define $!c$:

(iii) $!c := (\lambda s. s = \text{INFO}(c))^{\downarrow}$

Recall that $s \geq s'$ means that s is an extension of s', see Sect. 3.1.7 and fn. 17.

Input context

$$w_a \quad w_{a,b} \quad w_b \quad w_\emptyset$$

$u_1/a, u_2/b$ [● ● ● ●]

$smile\{u_1\} \sqcup smile\{u_2\}$

$$w_a \quad w_{a,b} \quad w_b \quad w_\emptyset$$
$u_1/a, u_2/b$ [● ● ●] ●

$[u_3]; smile\{u_3\}$

$$w_a \quad w_{a,b} \quad w_b \quad w_\emptyset$$
$u_1/a, u_2/b, u_3/a$ [● ●] ● ●
$u_1/a, u_2/b, u_3/b$ ● [● ●] ●

$!(smile\{u_1\} \sqcup smile\{u_2\})$

$$w_a \quad w_{a,b} \quad w_b \quad w_\emptyset$$
$u_1/a, u_2/b$ [● ● ●] ●

$!([u_3]; smile\{u_3\})$

$$w_a \quad w_{a,b} \quad w_b \quad w_\emptyset$$
$u_1/a, u_2/b, u_3/a$ [● ●] ● ●
$u_1/a, u_2/b, u_3/b$ ● [● ●] ●

$\neg\neg(smile\{u_1\} \sqcup smile\{u_2\})$

$$w_a \quad w_{a,b} \quad w_b \quad w_\emptyset$$
$u_1/a, u_2/b$ [● ● ●] ●

$\neg\neg([u_3]; smile\{u_3\})$

$$w_a \quad w_{a,b} \quad w_b \quad w_\emptyset$$
$u_1/a, u_2/b$ [● ● ●] ●

Figure 7: Comparison of ! and double negation. Each subfigure shows the result of updating the input context that is given at the top of the figure with the update function presented in the box. For simplicity, we assume in the first and second figure on the right that u_3 can only be mapped to a or b (no plural individuals are considered here).

In the work cited above it has been proposed that ensuring inquisitiveness in this more conservative way is one of the semantic contributions of interrogative clause type markers in languages like English. We will incorporate this proposal, which means that the $\langle ? \rangle$ operator will play an important role in our formal analysis of interrogatives.

3.3.8 Presupposing non-informativeness

Another operator which will play an important role in our treatment of interrogatives is the presuppositional closure operator, † (Roelofsen, 2015; Champollion *et al.*, 2017). This operator, when applied to an update $\mathcal{U}$, contributes the presupposition that any information that $\mathcal{U}$ provides about the world must already be present in the input context c. If this presupposition is not met, i.e., if $\mathcal{U}$ provides new information about the world that is not available yet in c, then $\mathcal{U}(c)$ is undefined.

(53) $\quad \dagger\mathcal{U}_\mathbf{T} \coloneqq \lambda c_k. \begin{cases} \mathcal{U}(c) & \text{if } \bigcup c \text{ subsists in } \bigcup\mathcal{U}(c) \\ \text{undefined} & \text{otherwise} \end{cases}$

This operator is relevant for the semantics of questions because we will assume, following e.g. Roelofsen (2015) and Champollion *et al.* (2017), that questions never provide any at-issue information about the world. If they have non-trivial informative content, then this information is always

presupposed to be already available in the input context.

Figure 8 displays three contexts to illustrate the workings of $\dagger$. Suppose that $\mathcal{U}$ is an update function such that $\mathcal{U}(c_1) = c_3$ and $\mathcal{U}(c_2) = c_3$ as well. Then $\dagger\mathcal{U}(c_1)$ is defined but $\dagger\mathcal{U}(c_2)$ is undefined, because $w_{a,b}$ is considered possible in c_2 but not anymore in $\mathcal{U}(c_2)$. So $\mathcal{U}$ provides information about the world that is not available yet in c_2. This means that $\bigcup c_2$ does not subsist in $\bigcup\mathcal{U}(c_2)$, and therefore $\dagger\mathcal{U}(c_2)$ is undefined.

Figure 8: Two input contexts and one output context to illustrate the workings of $\dagger$.
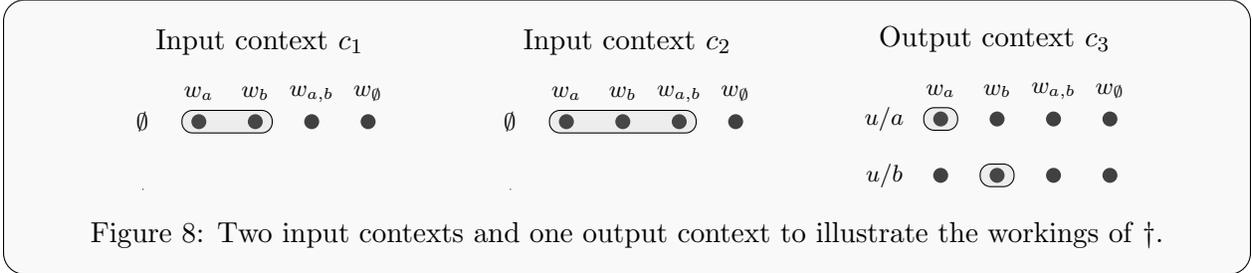
3.3.9 Atomicity

We now define the update function $\mathbf{atom}\{u\}$, which provides the information that the value of u is atomic. The effect of this update function is to eliminate all states from the context containing at least one possibility whose dref assignment matrix includes one or more assignments assigning a non-atomic entity to u. As in earlier work on dynamic semantics (e.g., Brasoveanu, 2008) we will use this update function to capture the semantic contribution of singular number morphology.

(54) $\quad \mathbf{atom}\{u\} \coloneqq \lambda c_k \lambda s_i.\ s \in c\ \wedge\ \forall p \in s.\ \forall g \in G_p.\ \neg\exists y.\ y < g(u)$

The effect of $\mathbf{atom}$ is illustrated in Figure 9. Note that $\mathbf{atom}\{u\}$ only removes states containing possibilities in which some specific dref assignment function assigns a plural entity to u. It does not remove states with possibilities in which multiple values are assigned to u by different dref assignment functions in the dref assignment matrix, as long as all these values are atomic. For instance, in Figure 9, the row $u/a, u/b$ is still present in the output context.

Figure 9: Illustration of the workings of $\mathbf{atom}\{u\}$.
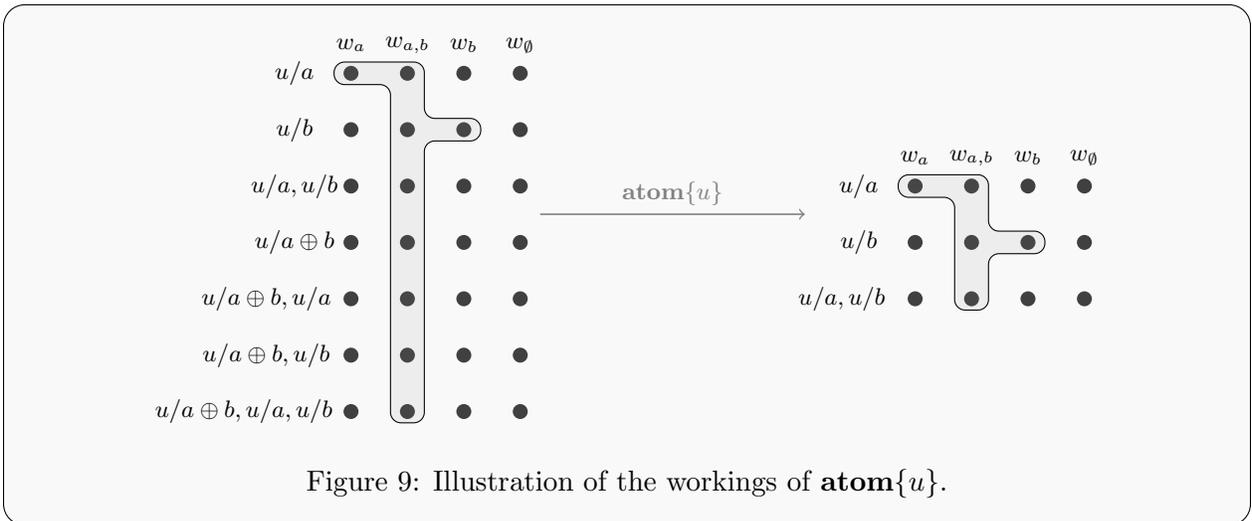
3.3.10 Maximality

Next, we define the update function $\mathbf{max}\{u\}$, which requires that the cumulative value of u in a given possibility p is maximal, compared to other possibilities in the input context with the same world parameter as p.[19] As in previous work on dynamic semantics (e.g., Brasoveanu, 2008) we will use this update function, among other things, to derive strong readings of donkey anaphora.

$$(55) \qquad \mathbf{max}\{u\} := \lambda c_k \lambda s_i . s \in c \; \wedge \left(\begin{array}{l} \forall p \in s. \; \forall p' \in \bigcup c. \\ [w_p = w_{p'} \rightarrow \oplus G_{p'}(u) \subseteq \oplus G_p(u)] \end{array} \right)$$

A graphical illustration of this update function is given in Figure 10. There are two things to notice. First, concentrating on the second column (possibilities with the world parameter $w_{a,b}$), note that only those states that include either of the possibilities in the top two rows are removed. Why are no other states removed from the middle column? This is because $\mathbf{max}\{u\}$ checks whether the *cumulative* value of u is maximal, and the cumulative value $\oplus G(u)$ for any possibility in the third row and lower is the same, namely, $a \oplus b$.

Second, note that states containing the possibility in the leftmost column (the possibility with the world parameter w_a) are not removed by $\mathbf{max}\{u\}$ even though the cumulative value of u in that possibility is just a. The reason that states with this possibility are not removed is that $\mathbf{max}\{u\}$ is sensitive to the world parameter: in checking whether the cumulative value of u is maximal in a given possibility, we do not compare against possibilities with a different world parameter.
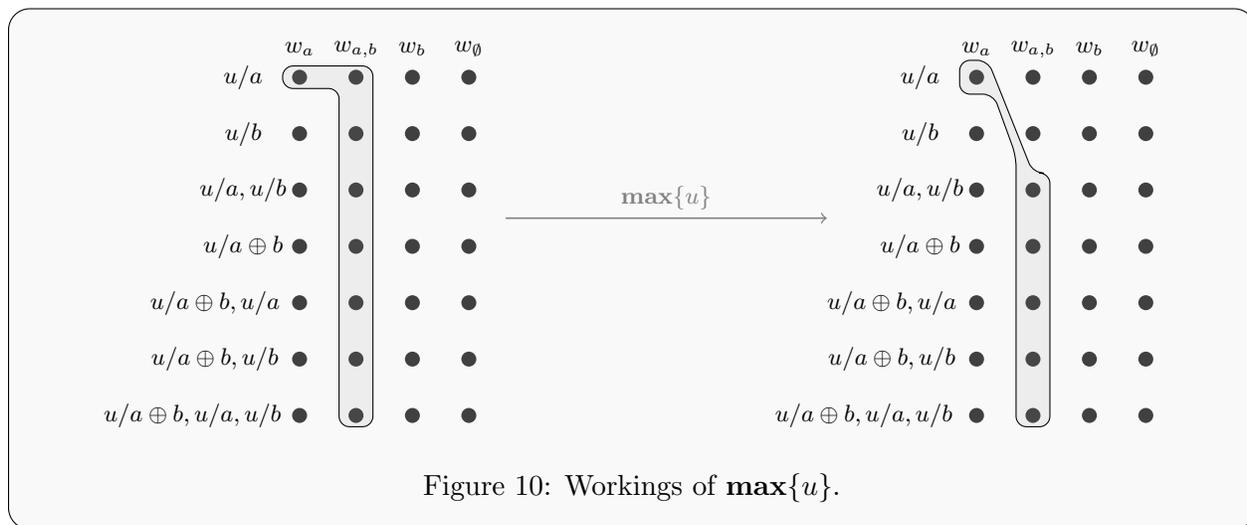
Figure 10: Workings of $\mathbf{max}\{u\}$.

Now, the $\mathbf{max}\{u\}$ update function as defined in (55) suffices to derive strong readings of donkey anaphora involving a single anaphoric expression and a single indefinite antecedent. However, the operator is not sufficient to derive strong readings of donkey anaphora involving multiple anaphoric expressions and indefinite antecedents, and therefore needs to be refined. Consider the following example:

(56)   If a student sees a professor, he greets her.

We did not provide a compositional procedure yet to translate English sentences into $\mathsf{Inq_D}$ expressions, but it should not be hard to see that when we just focus on the antecedent, the clause should be translated as follows:

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[19]The cumulative value of a discourse referent u in a possibility p, $\oplus G_p(u)$, was defined in Section 3.2.

(57) $[u_1]; student\{u_1\}; \textbf{atom}\{u_1\};$
$[u_2]; professor\{u_2\}; \textbf{atom}\{u_2\};$
$see\{u_1, u_2\};$
$\textbf{max}\{u_1\}; \textbf{max}\{u_2\}$

Let us first consider a scenario in which $\textbf{max}\{u\}$ correctly filters out states containing possibilities that assign non-maximal cumulative values to $\{u\}$. Suppose that there are two students, a and b, student a sees professors c and d, and student b sees professors e and f. Then we correctly derive that any state containing a possibility in which the cumulative value of u_1 is strictly contained in $a \oplus b$ or the cumulative value of u_2 is strictly contained in $c \oplus d \oplus e \oplus f$ is removed since in these possibilities the cumulative value of either u_1 or u_2 is non-maximal.

But now let us now consider a second, problematic, scenario. We again assume that there are two students, a and b. Furthermore, we assume that a sees professors c and d, and b sees professors c and e. In this case, $\textbf{max}$ would yield an output context that is intuitively incorrect. In particular, $\textbf{max}$ would not be able to distinguish possibilities with either of the two dref assignment matrices in (58), since for both matrices the cumulative value of u_1 is $a \oplus b$ and the cumulative value of u_2 is $c \oplus d \oplus e$, which are maximal. However, we do want to distinguish these two cases and keep only those possibilites in which the dref assignment matrix is maximal for u_2 with respect to *any value of* u_1. In (58), this only holds for the dref assignment matrix on the right.

(58) Two dref matrices indistinguishable for $\textbf{max}$:

	u_1	u_2
g_1	a	c
g_2	a	d
g_3	b	e

	u_1	u_2
g_1	a	c
g_2	a	d
g_3	b	c
g_4	b	e

This example shows that we need a refined version of the $\textbf{max}$ operator. This refined version, which we will denote as $\textbf{max}^*$, restricts a context c to states consisting of possibilities whose dref assignment functions cumulatively assign a maximal set of individuals to u with respect to any value of other drefs. A formal definition is given in (59). In this definition, $G^{\vec{u} \setminus u_n \mapsto \vec{x}}(u_n)$ is the set of values assigned to u_n by assignments in G which pointwise assign the values $\vec{x}$ to $\vec{u} \setminus u_n$, all drefs except u_n. If u_n is the only dref, then $\textbf{max}^*\{u_n\}$ just amounts to $\textbf{max}\{u_n\}$. But if there are other drefs around, the two operators may come apart.

(59) $\textbf{max}^*\{u_n\} := \lambda c \lambda s. \, s \in c \ \wedge \left(\begin{array}{l} \forall p \in s. \forall p' \in \cup c. \\ [w_p = w_{p'} \to \forall \vec{x}. \oplus G_{p'}^{\vec{u} \setminus u_n \mapsto \vec{x}}(u_n) \subseteq \oplus G_p^{\vec{u} \setminus u_n \mapsto \vec{x}}(u_n)] \end{array} \right)$

The effect of $\textbf{max}^*$ is illustrated in Figure 11. There are two discourse referents in this example and $\textbf{max}^*$ applies to u_2. The output context contains only those possibilities in which u_2 is maximal for any value of u_1. Concretely, states containing the top possibility in the input context are eliminated because u_2 is not maximal there for any value of u_1 compared to the two other possibilities.

3.3.11 Requesting a witness

We now turn to the final update function that we will define, $?u$. This update function will play a crucial role in our analysis of wh-questions. Let us first illustrate this informally on the basis of (60).

(60) Who smiled?
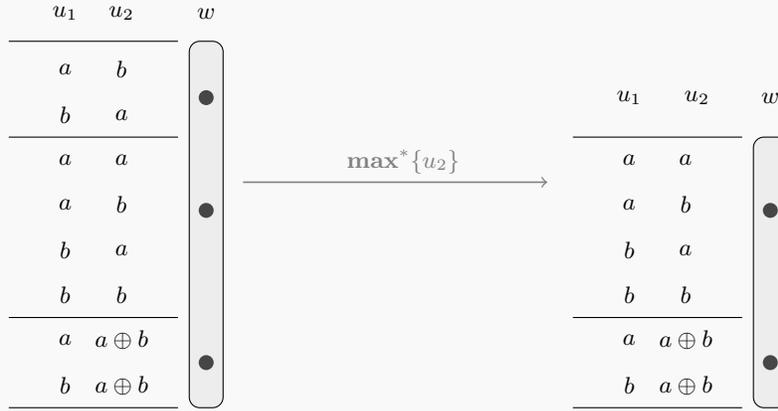
Figure 11: Workings of **max***. There are two discourse referents in this example, u_1 and u_2. Sets of assignment functions G belonging to the same possibility are separated from other assignment functions by a horizontal line. For example, the top possibility in the left diagram has a set of assignments G such that some assignments assign a to u_1 and b to u_2 and the remaining assignments assign b to u_1 and a to u_2.

This question first introduces a discourse referent, u, and updates the context with the information that u is a person and u smiled. These updates are shared with the declarative statement *someone smiled*. In contrast to the declarative, however, we assume that a wh-question induces an additional update, $?u$. The $?u$ update raises an issue whose resolution requires identifying one or more individuals that have the properties ascribed to u, i.e., it requires identifying one or more 'witnesses' for u. For this reason, we call $?u$ the *witness request operator*.

A simple formal definition of the witness request operator is given in (61). As we will see, this definition will have to be further refined, but it is still instructive to consider in some detail what it amounts to.

(61) $?u \coloneqq \lambda c \lambda s.\ s \in c\ \wedge\ \exists x_e.\ \forall p \in s.\ \forall g \in G_p.\ g(u) = x$ (to be modified)

According to (61), a state s in the input context survives an update with $?u$ just in case all the dref assignment functions in all the possibilities in s assign the same entity x to u. In such a state, we know that x satisfies all the properties that have been ascribed to u, so x is a witness for u. A graphical illustration of the update effect of $?u$ as defined in (61) is given in Figure 12.

To see why this simple definition of $?u$ needs to be further refined, suppose that the output context in Figure 12 obtains after an utterance of the question in (60), *Who smiled?*. Intuitively, this question can be resolved by specifying a person who smiled, for example:

(62)   Amy smiled.

However, such an answer does not resolve the issue present in the output context in Figure 12. For the sake of illustration, let us assume that (62) induces the following sequence of updates:

(63) $[u_2]; Amy\{u_2\}; smile\{u_2\}$

Further suppose that in all worlds, the extension of *Amy* is $\{a\}$. Then this sequence of updates eliminates states containing possibilities that have w_b as their world parameter, i.e., possibilities

$$\begin{array}{c}

\end{array}$$

	w_a	$w_{a,b}$	w_b	$w_\emptyset$	$?u_1$		w_a	$w_{a,b}$	w_b	$w_\emptyset$
u_1/a	•	•	•	•	$\longrightarrow$	u_1/a	•	•	•	•
u_1/b	•	•	•	•		u_1/b	•	•	•	
$u_1/a, u_1/b$	•	•	•	•		$u_1/a \oplus b$	•	•	•	•
$u_1/a \oplus b$	•	•	•	•						
$u_1/a \oplus b, u_1/a$	•	•	•	•						
$u_1/a \oplus b, u_1/b$	•	•	•	•						
$u_1/a \oplus b, u_1/a, u_1/b$	•	•	•	•						

Figure 12: The update effect of $?u$ as defined in (61), to be refined below.
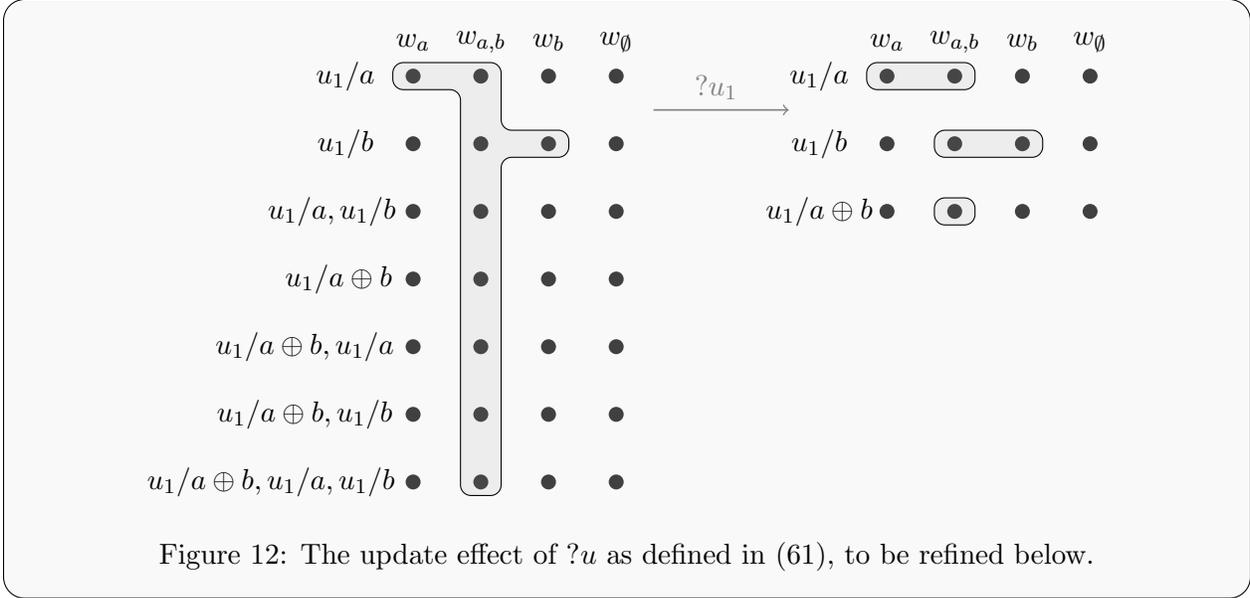
according to which b was the only individual who smiled. Under the simple definition of $?u$ in (61), this update is wrongly predicted *not* to resolve the given question. If we remove all states containing w_b-possibilities from the output context in Figure 12, we still have a context with an unresolved issue, because, even though we have identified a person who smiled, we have *not* determined yet what the value of u is.

The problem is that $?u$, as defined in (61), raises an issue that can only be resolved by establishing two things. First, one must establish certain information about the world, i.e., sufficient information to identify an individual who has the properties ascribed to u. But this alone is not enough according to the definition in (61). Rather, one must also establish the specific value of u. This second requirement is too demanding, because in fact, resolving a question only ever requires establishing information about the world, not about the specific value of a certain discourse referent.

This insight leads us to the refined definition of $?u$ in (64). According to this refined definition, $?u$ selects states s in the input context c for which we can find a witness x such that the world parameter of each possibility in s is compatible with assigning x to u. That is, for every possibility p in s, there is a possibility p' in some state in c with the same world parameter as p whose dref assignment functions all assign x to u. Whether the dref assignment functions in p itself assign x to u doesn't matter. What matters is whether the world parameter of p is compatible with assigning x to u.

(64) $?u := \lambda c_k \lambda s_i.\ s \in c\ \wedge \exists x_e.\ \forall p \in s.\ \exists p' \in \bigcup c.\ (w_p = w_{p'} \wedge \forall g \in G_{p'}.\ g(u) = x)$ (final)

A graphical illustration of the workings of $?u$ as defined in (64) is given in Fig. 13. Note that the output context in this figure contains the same information as the input context, but is inquisitive: two alternatives have been introduced. That is, $?u$ raises an issue, which could be paraphrased as 'What is an individual that has the properties ascribed to u? Is a such an individual (the alternative extending to the left), or is b such an individual (the alternative extending to the right)?' This issue would be resolved by (63), which would yield a context with only the former alternative, by eliminating states containing the w_b-possibility. So the problematic prediction arising from the initial definition of $?u$ no longer obtains.

Our analysis of single-wh questions in English will rely heavily on the $?u$ operator. However,
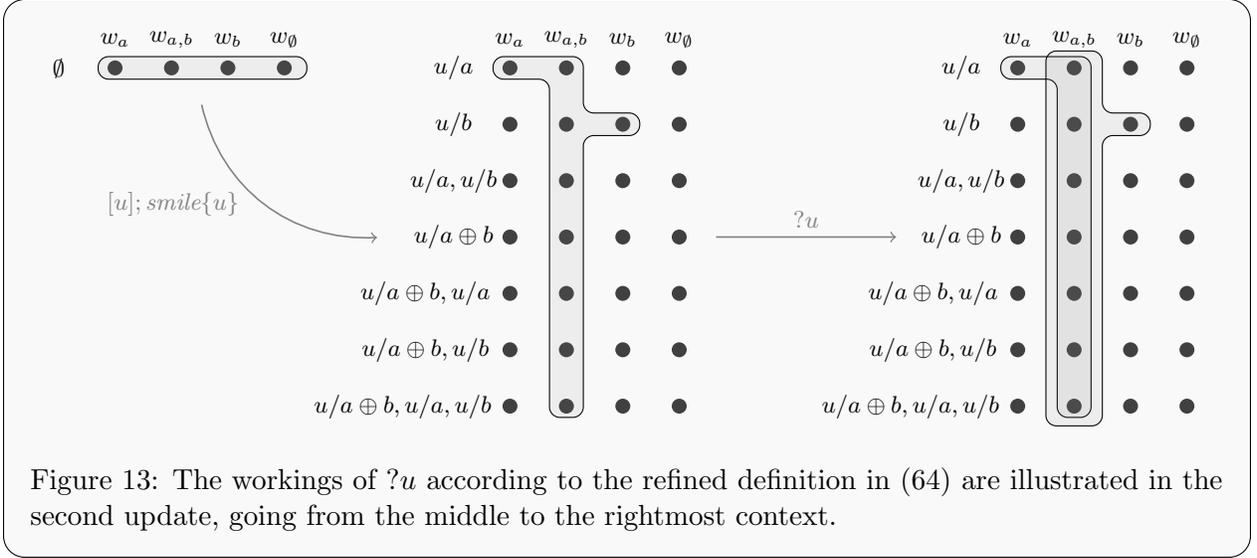
Figure 13: The workings of $?u$ according to the refined definition in (64) are illustrated in the second update, going from the middle to the rightmost context.

for *multiple-wh* questions, we need a generalized witness request operator, which does not make reference to just one discourse referent u but rather to two or more discourse referents, $u_1 \ldots u_n$. The update effect of this operator, which we denote as $?u_1 \ldots u_n$, is defined in (65). The update selects states in the input context which contain enough information about the world to guarantee the existence of a *witness function*, which maps any tuple of entities satisfying the properties ascribed to $u_1 \ldots u_{n-1}$ to a corresponding entity satisfying the properties ascribed to u_n.

(65) $\qquad ?u_1 \ldots u_{n-1}u_n := \lambda c_k \lambda s_i.\ s \in c\ \wedge\ \exists f_{(e^{n-1},e)}.\ \forall p \in s.\ \exists p' \in \bigcup c.$
$$(w_p = w_{p'} \wedge \forall g \in G_{p'}.\ g(u_n) = f(g(u_1), \ldots, g(u_{n-1})))$$

Crucially, when $n = 1$, $?u_1 \ldots u_n$ just boils down to $?u$. In other words, $?u$ is just a specific instance of $?u_1 \ldots u_n$.[20] Because of this, single-wh and multiple-wh questions can be treated in a uniform way in our account, as we will demonstrate in Section 5.

3.3.12 Summary of update expressions

Table 3 gives an overview of all the update expressions introduced in Section 3.3. Predicational update expressions, as well as conjunctions, disjunctions, and negations are familiar both from inquisitive semantics and from dynamic semantics, and our treatment of these expressions was based on earlier treatments in inquisitive and dynamic semantics. The operators !, ?, $\langle ? \rangle$, and †, which regulate the interplay between informative and inquisitive content, come from inquisitive semantics. On the other hand, the operators $[u]$, $\mathbf{atom}\{u\}$, and $\mathbf{max}^*\{u\}$, which regulate discourse referents, come from dynamics semantics. The witness request operator, $?u_1 \ldots u_n$, is the only operator that does *not* have a precedent either in inquisitive semantics or in dynamic semantics. This is because the update it expresses involves *both* discourse referents, which are present in dynamic semantics but not in static inquisitive semantics, and the raising of an issue, which is modeled in static inquisitive semantics but not in basic (non-inquisitive) dynamic semantics. It is exactly this new dynamic-inquisitive witness request operator that will play a crucial role in our

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[20]This is because any 0-place function into the domain D_e can simply be identified with an entity in D_e, namely the entity to which it maps the empty input tuple.

Update expressions	Corresponding operator in inquisitive semantics	Corresponding operator in dynamic semantics
Predicational updates	+	+
Conjunction, disjunction, negation	+	+
!, ?, $\langle ? \rangle$, †	+	−
$[u]$, $\mathbf{atom}\{u\}$, $\mathbf{max}^*\{u\}$	−	+
$?u_1 \ldots u_n$	−	−

Table 3: Update expressions.

analysis of wh-questions.

4   Syntax-semantics interface for a fragment of English

We now specify some basic assumptions about the syntax-semantics interface for a fragment of English, in order to provide a compositional semantic analysis of declaratives, polar questions and wh-questions in $\mathsf{Inq_D}$. Of course, what we present here is not the only way to analyse these sentence types using $\mathsf{Inq_D}$. The assumptions we make about the syntax of English and the syntax-semantics mapping are independent of the semantic framework, and other researchers are free to use $\mathsf{Inq_D}$ in conjunction with different assumptions about the syntax of English and a different mapping between syntax and semantics.

The fragment of English that we consider here can be divided into two parts. On the one hand, we consider certain lexical and morphological elements that are part of the TP, and on the other hand, we consider functional heads that are assumed to be part of the left periphery, external to the TP. Within the TP we first consider verbs, nouns, and number morphology (Section 4.1), and then turn to indefinites and wh-words (Section 4.2). As for the left periphery, we concentrate on two functional heads, one that indicates the clause type, declarative or interrogative, and one that we assume to be always present at the clause boundary, no matter whether the clause is declarative or interrogative (Section 4.3). Specifying the semantic contribution of these elements will be sufficient to obtain a rather general account of declarative and various kinds of interrogative (polar, single-wh, multiple-wh) clauses in English, demonstrating what we consider to be some of the main virtues of the $\mathsf{Inq_D}$ framework.

4.1   Verbs, nouns, and number morphology

In (66), we provide an $\mathsf{Inq_D}$-translation for three verbs: the intransitive verb *sleep*, the transitive verb *see*, and the ditransitive verb *give*. We restrict our attention here to distributive predicates. A detailed analysis of collective predicates in $\mathsf{Inq_D}$ is left for future work.

(66)   a. $[\![\text{sleep}]\!] = \lambda v_r.sleep\{v\}$
       b. $[\![\text{see}]\!] = \lambda Q_{r\mathbf{TT}}\lambda v_r.Q(\lambda v'.see\{v,v'\})$
       c. $[\![\text{give}]\!] = \lambda Q_{r\mathbf{TT}}\lambda Q'_{r\mathbf{TT}}\lambda v_r.Q(\lambda v'.Q'(\lambda v''.give\{v,v',v''\}))$

In (67), we provide $\mathsf{Inq_D}$-translations for the singular noun *boy* and the plural noun *boys*. We assume here that plural number morphology is semantically vacuous, while singular number morphology contributes an atomicity requirement, following Zweig (2008, 2009) and Ivlieva (2013), among others.

(67)   a. $[\![\text{boy}]\!] = \lambda v_r.\mathbf{atom}\{v\}; boy\{v\}$
       b. $[\![\text{boys}]\!] = \lambda v_r.boy\{v\}$

This does not mean that the use of a plural noun does not communicate any cardinality information. Plurals can (and typically do) give rise to a more-than-one inference. We assume, however, that this inference is not semantically encoded, but rather results from pragmatic reasoning (Sauerland 2003; Sauerland *et al.* 2005; Spector 2007; Zweig 2008; Ivlieva 2013; Mayr 2015, among others). Roughly, when a speaker uses the plural *boys*, the fact that she did not use the singular *boy*, which would have been semantically more specific, can give rise to the inference that she is referring to multiple boys. We will return to this point in more depth in Section 5.

4.2 Indefinites and wh-words

The Inq_D-translation of the indefinite article a is given in (68). We treat the indefinite article a, as well as the determiner *some* as generalized determiners that introduce a discourse referent. This treatment of indefinites is standard in compositional dynamic semantics (see, e.g., Muskens 1996). Following Brasoveanu (2007), we furthermore assume that indefinites are ambiguous. They are either weak, as in (68a), or strong, as in (68b). The only difference is that under a strong reading they place a maximality requirement on the values that are assigned to the discourse referent that is introduced. Under the weak reading, this maximality requirement is not present.

(68)   a. $[\![a^u_{weak}]\!] := \lambda P_{r\mathbf{T}}\lambda P'_{r\mathbf{T}}.\ [u]; P(u); P'(u)$

        b. $[\![a^u_{strong}]\!] := \lambda P_{r\mathbf{T}}\lambda P'_{r\mathbf{T}}.\ [u]; P(u); P'(u); \mathbf{max}^*\{u\}$

Brasoveanu (2007) argues for this weak/strong ambiguity in the semantics of indefinites based on constructions involving donkey anaphora. Such constructions are well-known to allow for weak and strong readings, as illustrated in (69). Under its most prominent reading, (69a) implies that whenever Joe sees one or more donkeys he beats all of them. This is called a *strong* reading of donkey anaphora. By contrast, (69b) is most naturally interpreted as saying that whenever Joe has one or more dimes in his pocket, he throws one of them in the parking meter. This is a *weak* reading.

(69)   a.   If Joe^{u_1} sees a^{u_2} donkey, he$_{u_1}$ beats it$_{u_2}$.
        b.   If Joe^{u_1} has a^{u_2} dime in his pocket, he$_{u_1}$ throws it$_{u_2}$ in the parking meter.

We refer to Brasoveanu (2007, 2008) for a detailed analysis of this phenomenon arguing that the ambiguity of such constructions is due to a weak/strong ambiguity in the lexical semantics of indefinite determiners, as we assume here.

     We now turn to wh-words. We assume that they are interpreted just like indefinites: their main function is to introduce a discourse referent. This means in particular that they are not inquisitive; they do not raise an issue. We further assume that simplex wh-phrases (*who, where, what* etc.) have both a weak and a strong reading, just like indefinites.

(70)   a. $[\![\text{who}^u_{weak}]\!] = \lambda P_{r\mathbf{T}}.\ [u]; person\{u\}; P(u)$
        b. $[\![\text{who}^u_{strong}]\!] = \lambda P_{r\mathbf{T}}.\ [u]; person\{u\}; P(u); \mathbf{max}^*\{u\}$

On the other hand, we assume that the wh-determiner *which* only has a strong reading in English:

(71) $[\![which^u]\!] := \lambda P_{r\mathbf{T}}\lambda P'_{r\mathbf{T}}.\ [u]; P(u); P'(u); \mathbf{max}^*\{u\}$

This distinction between simplex wh-phrases and *which*-phrases is motivated by the fact that the former can generate both mention-some and mention-all readings while the latter forces a mention-all reading at least when there is no modal or disjunction in the question. When a modal or a

disjunction is present, *which*-questions may trigger a mention-some reading. This, however, will be derived on our approach without postulating ambiguity in the lexical entry of *which*.

4.3   Left periphery

The left periphery of a clause contains functional heads which syntactically c-command the TP and semantically operate on the semantic content of the TP. Early work in linguistics often assumed that the left periphery involves a single functional head C, referred to as the *complementizer* of the clause. However, Rizzi (1997) and much subsequent work has argued that the left periphery actually involves multiple functional heads, each playing a distinct syntactic role and making a distinct semantic contribution (Rizzi, 2001; Aboh, 2004; Aboh and Pfau, 2011, among many others). There is no general agreement in the literature yet on what the complete inventory of relevant functional heads is, but there seems to be a broad consensus that:

1. All clauses involve a functional head which encodes the *clause type*. The two clause types relevant for us here are declarative and interrogative. We will denote the declarative clause type marker as Dec and the interrogative clause type marker as Int.[21]

2. All declarative and interrogative clauses involve a functional head whose syntactic role is to license *wh-elements* and *focused elements* (if the clause contains any such elements). This head is generally denoted as Foc (short for Focus).

3. Dec and Int syntactically c-command Foc and therefore take higher semantic scope.

These syntactic assumptions about the left periphery, summarised in Fig. 14, are sufficient for our purposes here.
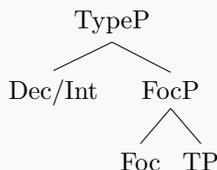
Figure 14: Assumed functional heads in the left periphery.

Below we propose a semantic treatment of Dec, Int, and Foc in $\mathsf{Inq_D}$. The semantic contribution of Foc is discussed in Section 4.3.1, that of Int in Section 4.3.2, and finally that of Dec in Section 4.3.3.

4.3.1   Semantic contribution of Foc

We assume, following Rizzi (1997) and subsequent work, that every argumental wh-phrase must be associated with a Foc head in the left periphery of a clause in which it is contained. In some cases this syntactic association manifests itself as obligatory movement of the wh-phrase to the specifier of the associated Foc head. In other cases, wh-phrases (may) remain in-situ. Even in such cases, however, it is assumed that every wh-phrase is associated with a Foc head in the left periphery

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[21]The interrogative clause type marker is sometimes also referred to as Inter (Aboh, 2004; Aboh and Pfau, 2011).

of a clause in which it is contained.[22] To illustrate what we have in mind, consider the following example from Baker (1970):

(72)   Who remembers where we bought which book?

Both the matrix clause and the embedded clause are assumed to involve a Foc head. The matrix subject wh-phrase *who* associates with the matrix Foc head and has moved to its specifier position. In the embedded clause there are two wh-phrases, *where* and *which book*. The former associates with the embedded Foc head and has moved to its specifier position. On the other hand, *which book* appears in-situ. The assumption is that, even though it has not undergone overt movement, it associates either with the matrix Foc head or with the embedded Foc head. These two possible associations result in two distinct readings, which Baker (1970) already observed. Namely, if *which book* associates with the embedded Foc head, (72) raises an issue whose resolution requires an answer like (73):

(73)   John and Martha remember where we bought which book.

On the other hand, if *which book* associates with the matrix Foc head, (72) raises an issue whose resolution requires an answer like (74):

(74)   John remembers where we bought the physics book and Martha remembers where we bought the biology book.

This example also illustrates that, while every wh-phrase must be associated with one, and only one, Foc head, the converse does not hold. A Foc head may be associated with one wh-phrase, but it may also be associated with multiple wh-phrases or with no wh-phrases at all. We will write Foc_u if Foc is associated with a single wh-phrase introducing the discourse referent u. Similarly, we will write $\mathrm{Foc}_{u_1\ldots u_n}$ if Foc is associated with multiple wh-phrases introducing discourse referents $u_1\ldots u_n$, respectively. And finally, we simply write Foc if there is no associated wh-phrase.

For reasons that will become clear shortly, if Foc is associated with one or more wh-phrases introducing discourse referents $u_1\ldots u_n$, respectively, we will refer to $u_1\ldots u_{n-1}$ as its *domain* discourse referents and to u_n as its *target* discourse referent.

We propose that $\mathrm{Foc}_{u_1\ldots u_n}$ generally makes three semantic contributions, even though, as we will see right below, if $n = 1$ one of these contributions is always vacuous, and if $n = 0$ two of these contributions are vacuous. First, $\mathrm{Foc}_{u_1\ldots u_n}$ discharges the inquisitiveness of the TP that it combines with, by applying the ! operator to the semantic value of the TP. Second, $\mathrm{Foc}_{u_1\ldots u_n}$ places a maximality requirement on all its domain discourse referents. And finally, $\mathrm{Foc}_{u_1\ldots u_n}$ requests a (functional) witness for its target discourse referent by applying the witness request operator $?u_1\ldots u_n$. The proposal is summarized in the following lexical entry:

(75) $[\![\mathrm{Foc}_{u_1\ldots u_n}]\!] := \lambda \mathcal{U}_{\mathbf{T}}.\ !\mathcal{U}\,;\mathbf{max}^*\{u_1\};\ldots;\mathbf{max}^*\{u_{n-1}\};?u_1\ldots u_n$

    where $u_1,\ldots u_n$ are the discourse referents introduced by the associated wh-phrases

As anticipated above, this general entry reduces to something simpler when $n \leq 1$, i.e., if Foc is associated with just one wh-phrase or with no wh-phrases at all. If Foc is associated with a single wh-phrase, i.e., in the case of a simple wh-question, its semantic contribution amounts to (76), and if it is not associated with any wh-phrase whatsoever, which is the case in a polar question, an

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[22]We intentionally remain neutral here as to what the syntactic relationship between a wh-phrase and the associated Foc head amounts to exactly. This can be made concrete in several ways, depending on which syntactic framework one assumes. Our proposal is intended to be compatible with any syntactic framework.

alternative question and a declarative, its semantic contribution reduces to (77).[23]

(76) $\llbracket \text{Foc}_u \rrbracket = \lambda \mathcal{U}_\mathbf{T}. \; !\mathcal{U}; \; ?u$

(77) $\llbracket \text{Foc} \rrbracket \; = \lambda \mathcal{U}_\mathbf{T}. \; !\mathcal{U}$

4.3.2   Semantic contribution of Int

We now turn to the interrogative clause type marker, Int. Following Roelofsen (2015), Roelofsen and Farkas (2015), and Ciardelli *et al.* (2018), we assume that interrogative clause typing has two semantic effects. First, Int ensures that the clause it heads is *inquisitive* (provided that the clause is non-tautological). This is done by applying the $\langle ? \rangle$ operator. Second, Int ensures that the clause is non-informative w.r.t. the input context. This is done by applying the † operator, which requires that all information conveyed by the TP about the world, if any, is already established in the input context. Thus, we assume the following treatment of Int:

(78) $\llbracket \text{Int} \rrbracket = \lambda \mathcal{U}_\mathbf{T}. \; \dagger \, \langle ? \rangle \mathcal{U}$

We will go through four examples to illustrate the interaction between Int and Foc in the interpretation of questions: a simple wh-question, a polar question, an alternative question and a wh-question with a sub-clausal disjunction.

Example 1: simple wh-question   First, consider the simple wh-question in (79) with a weakly interpreted wh-phrase, which should yield a mention-some reading.

(79) Who^u_{weak} has a pen that I could borrow?

Under the assumptions laid out above, this question is translated into Inq_D as follows.

(80)   a. $\llbracket \text{TP} \rrbracket \quad = [u]; person\{u\}; hasPen\{u\}$

      b. $\llbracket \text{Foc}_u \; \text{TP} \rrbracket = !([u]; person\{u\}; hasPen\{u\}); ?u$

                $\equiv [u]; person\{u\}; hasPen\{u\}; ?u$

      c. $\llbracket \text{Int} \; \text{FocP} \rrbracket = \dagger \langle ? \rangle([u]; person\{u\}; hasPen\{u\}; ?u)$

                $\equiv \dagger([u]; person\{u\}; hasPen\{u\}; ?u)$

As seen in (80a), the TP introduces a discourse referent, u, and further updates the context with the information that u is a person who has a pen to borrow. Foc_u enters the stage in (80b), applying the ! operator to discharge any inquisitive content that the TP might carry, and the $?u$ operator to request a witness for u. Note that ! is vacuous in this case because the TP is not inquisitive. Finally, Int comes into play in (80c), applying $\langle ? \rangle$ and † to ensure that the clause is inquisitive and non-informative w.r.t. the input context. $\langle ? \rangle$ is vacuous in this case, since FocP is already

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[23]While we assume here that Foc generally has three semantic tasks—applying !, applying $\mathbf{max}^*$, and applying $?u_1 \ldots u_n$—another way to articulate the account would be to assume that Foc is only responsible for placing a maximality requirement on $u_1, \ldots, u_{n-1}$ and requesting a witness for u_n, i.e., only those tasks that directly concern the discourse referents introduced by the wh-phrases associated with Foc. The third task, applying ! to the semantic content of the TP, would then have to fall under the responsibility of another functional head in the left periphery, scoping below Foc (such a functional head is assumed, for instance, in Roelofsen 2015). While we feel that such a division of labor is conceptually natural, we currently do not have specific empirical arguments for or against it. We therefore choose not to stipulate functional heads in the left periphery beyond necessity.

inquisitive. Figure 15 graphically represents the update effect of $[u]; person\{u\}; hasPen\{u\}; ?u$ on an initial context in which no information and no discourse referents are present yet. Note that the output context has no states containing possibilities that have $w_\emptyset$ as their world parameter, i.e., in which nobody has a pen to borrow. † requires that such possibilities are already ruled out in the input context as well. Overall, then, the question is predicted to raise an issue whose resolution requires identifying at least one individual who has a pen to borrow, and to presuppose that it is already established in the input context that such an individual exists.

Figure 15: The update effect of (79).
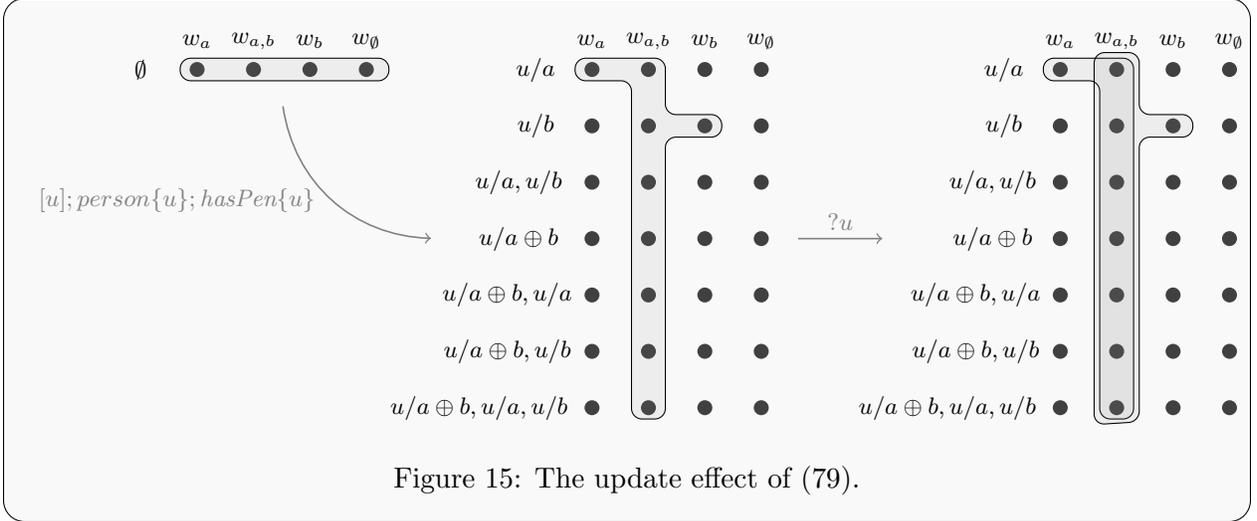
Example 2: polar question   We now turn to a second example, the polar question in (81). This case serves to illustrate in particular why $\langle ? \rangle$, which was vacuous in (79), is a necessary component of the semantic contribution of Int.

(81)   Is it raining?

Under the assumptions we have made, this question is translated into Inq_D as follows:

(82)   a. $[\![\text{TP}]\!]$ $= raining$

   b. $[\![\text{Foc TP}]\!]$ $= {!}\, raining$

   c. $[\![\text{Int FocP}]\!] = {\dagger}\langle ? \rangle {!}\, raining$
   $\equiv ?raining$

We treat $rain$ as a 0-place predicate here. Since neither the TP nor the FocP are inquisitive, the $\langle ? \rangle$ operator contributed by Int in (82c) is crucial here to derive the correct interpretation. It ensures that the sentence is interpreted as a polar question, whose resolution requires establishing whether it is raining or not. The † operator is vacuous in this case, because when $\langle ? \rangle$ applies non-trivially, it always ensures that the question is non-informative w.r.t. the input context.

Example 3: alternative question   Next, we consider the alternative question in (83). This question is to be read with rising intonation on the first disjunct and falling intonation on the second disjunct, as indicated by the $^\uparrow$ and $^\downarrow$ arrows. Other intonation patterns give rise to different

interpretations (see Bartels 1999, Pruitt and Roelofsen 2013, Roelofsen and Farkas 2015, Westera 2017, Meertens 2021, among others).

(83)   Is it raining$^\uparrow$ or is it snowing$^\downarrow$?

We assume that alternative questions involve clausal disjunction (see Han and Romero 2004, Haida 2010, Pruitt and Roelofsen 2011, Uegaki 2014, Roelofsen 2015, Gračanin-Yuksek 2016, Hoeks and Roelofsen 2019). More specifically, we assume that in alternative questions the disjunction operates on two FocPs. This yields the interpretation in (84).[24]

(84)   a. $\llbracket \text{FocP}_1 \text{ or FocP}_2 \rrbracket \quad = \; !\,raining \sqcup !\,snowing$
              $\equiv raining \sqcup snowing$

        b. $\llbracket \text{Int FocP}_1 \text{ or FocP}_2 \rrbracket = \dagger\langle ? \rangle\,(raining \sqcup snowing)$
              $\equiv \dagger\,(raining \sqcup snowing)$

The ! operators contributed by the two Foc heads are vacuous in this case, as in the previous examples. The $\langle ? \rangle$ operator contributed by Int is also vacuous again because the disjunction in its scope already ensures inquisitiveness. The $\dagger$ operator contributed by Int is not vacuous in this case—it requires that it is already established in the input context that it is either raining of snowing. Overall, then, the question is predicted to presuppose that it is either raining or snowing and to request new information that establishes which of the two is the case.

Example 4: wh-question with TP-internal disjunction   Finally, we consider the wh-question in (85) which involves a TP-internal disjunction. This example serves to illustrate in particular why we assume that Foc always discharges inquisitive content generated within the TP by applying !.

(85)   Who$^u_{weak}$ has a pen or or a pencil that I could borrow?

Given the assumptions we have made, this question is translated into $\mathsf{Inq_D}$ as in (86).

(86)   a. $\llbracket \text{TP} \rrbracket \quad = \; [u]; person\{u\}; (hasPen\{u\} \sqcup hasPencil\{u\})$

        b. $\llbracket \text{Foc TP} \rrbracket \; = \; !([u]; person\{u\}; (hasPen\{u\} \sqcup hasPencil\{u\})); ?u$

        c. $\llbracket \text{Int FocP} \rrbracket = \dagger\langle ? \rangle\,(\,!([u]; person\{u\}; (hasPen\{u\} \sqcup hasPencil\{u\})); ?u\,)$
              $\equiv \dagger\,(\,!([u]; person\{u\}; (hasPen\{u\} \sqcup hasPencil\{u\})); ?u\,)$

As seen in (86a), the TP is inquisitive in this case because of the disjunction: it introduces an issue as to whether u has a pen or whether u has a pencil. In (86b), we see that Foc discharges this issue by applying the ! operator. In addition, it raises a different issue by applying the $?u$ operator, requesting a witness for u. Finally, as seen in (86c), Int ensures inquisitiveness by applying the $\langle ? \rangle$ operator, which is vacuous in this case because the FocP is already inquisitive, and ensures non-informativity w.r.t. the input context by applying the $\dagger$ operator. Overall, then, the question is predicted to presuppose that someone has a pen or a pencil to borrow, and to raise an issues whose resolution requires identifying at least one such a person. Importantly, it is predicted that

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[24]If we take a disjunction of two TypePs rather than two FocPs we derive an interpretation that is not correct for (83) and which is in fact not expressible in English without resorting to rather artificial queries such as 'Please answer at least one of the following two questions either positively or negatively: (a) Is it raining? (b) Is it snowing?'. We refer to Hoeks and Roelofsen (2019) and references therein for discussion of syntactic, semantic, and pragmatic mechanisms that may block TypeP disjunctions. Here, we simply take it as a given that disjunction applies at the FocP level in alternative questions.

the issue does not require establishing whether the person in question has a pen or whether she has a pencil. This is a result of Foc discharging the issue introduced by the disjunction. This example thus demonstrates why the ! operator needs to be invoked in questions and why the operator must take scope over the entire TP (to discharge issues raised by disjunctions inside the TP) but below the witness request operator contributed by Foc (otherwise the issue raised by this operator would also be discharged). As already mentioned in footnote 23 above, while we assume here that ! is part of the semantic contribution of Foc itself, one may also assume an additional functional head in the left periphery, below Foc, whose semantic role is specifically to discharge inquisitiveness introduced within the TP by applying !. The role of Foc would then be restricted to requesting a (functional) witness for its target discourse referent, and placing a maximality requirement on its domain discourse referents.[25]

4.3.3 Semantic contribution of Dec

We now turn to the declarative clause type marker, Dec. We assume that Dec simply discharges any inquisitiveness that may have been introduced within its scope, by applying the ! operator.[26]

(87) $[\![\text{Dec}]\!] = \lambda \mathcal{U}_{\mathbf{T}}.\,!\mathcal{U}$

We will consider two examples to illustrate the effect of Dec. The first is a disjunctive declarative sentence:

(88)   It is raining or snowing.

The disjunction may be operating at different levels. Suppose first that it operates TP-internally. Then we obtain the following derivation:

(89) a. $[\![\text{TP}]\!]$ $= raining \sqcup snowing$

    b. $[\![\text{Foc TP}]\!]$ $= !(raining \sqcup snowing)$

    c. $[\![\text{Dec FocP}]\!] = !!(raining \sqcup snowing)$
              $\equiv !(raining \sqcup snowing)$

The correct interpretation is derived, but the semantic contribution of Dec is vacuous here, because Foc already discharges the inquisitiveness introduced by the disjunction. The same is true if the

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[25]The proposed analysis also makes correct predictions for cases in which two wh-questions are disjoined, as in (i), and cases in which a wh-question is disjoined with a polar question, as in (ii), assuming that the disjunction operates at the FocP level in such cases. This is especially noteworthy since these cases are problematic for many other theories of questions (see Ciardelli *et al.*, 2018; Hirsch, 2018; Hoeks and Roelofsen, 2019, for discussion).

(i)   Where can we rent a car or who has one that we could borrow?

(ii)   Where can we rent a car or do you have one that we could borrow?

In particular, our analysis correctly predicts that 'I don't have a car that you could borrow' *fails* to resolve the issue raised by (ii), even though it would succeed in resolving the issue raised by the polar question in the second disjunct when it would appear on its own (not as part of a disjunction). For reasons of space, we do not go through the derivations for these examples in detail—they are analogous to the ones discussed in the main text.

[26]We restrict ourselves here to declarative sentences with *falling* intonation, which, in English, are never inquisitive. Declarative sentences with rising intonation *can* be inquisitive. To account for such cases, our analysis would have to be extended. For instance, we could assume an additional functional head in the left periphery encoding whether the clause is OPEN or CLOSED in the sense of Roelofsen and Farkas (2015) and Ciardelli *et al.* (2018). Another option would be to derive the inquisitive nature of rising declaratives at a pragmatic level (Westera, 2017; Rudin, 2022). For reasons of space, we do not explicitly pursue such extensions here.

disjunction operates at the TP level, i.e., if both disjuncts are TPs, because in this case, disjunction still scopes below Foc.

This is not the case, however, if disjunction operates at the FocP level, i.e., if both disjuncts are FocPs. We have seen above that FocP disjunction must be allowed in the grammar in order to derive the interpretation of alternative questions—see example (83). If we assume that disjunction operates at the FocP level in (88), we obtain the following derivation.

(90)   a. $[\![\text{FocP or FocP}]\!]$ $= \, !raining \sqcup \, !snowing$

                                       $\equiv raining \sqcup snowing$

   b. $[\![\text{Dec (FocP or FocP)}]\!] = \, !(raining \sqcup snowing)$

The end result is the same as when disjunction is taken to operate TP-internally, but now the semantic contribution of Dec is not vacuous because the FocP disjunction is inquisitive and there is no operator other than Dec which discharges this inquisitiveness.

Now, what do we derive if we assume that the disjunction applies at the TypeP level, i.e., that it operates on two full TypePs? In that case we derive the interpretation $!!raining \sqcup !!snowing$, which is equivalent to $raining \sqcup snowing$. This interpretation is in fact not available for (88). That is, (88) cannot be interpreted as a question asking whether it is raining or snowing. Thus, we have to assume that disjunction cannot take scope over the declarative clause type marker, just as we assumed above that it cannot take scope over the interrogative clause type marker. This assumption has also been made in previous work on inquisitive semantics (e.g., Roelofsen and Farkas, 2015; Ciardelli *et al.*, 2018). However, we should note that, while there is work suggesting that the inability of disjunction to scope over Int has a deeper explanation (see Hoeks and Roelofsen, 2019, and references therein), there is no work as of yet providing a deeper explanation for the inability of disjunction to scope over Dec. This assumption, then, must be stipulated at this point.

Now consider a second example, the declarative sentence in (91), which involves an indefinite.

(91)   Someone$_{weak}^u$ called.

This sentence is translated into $\mathsf{Inq_D}$ as follows.

(92)   a. $[\![\text{TP}]\!]$ $= [u]; person\{u\}; called\{u\}$

   b. $[\![\text{Foc TP}]\!]$ $= \, !([u]; person\{u\}; called\{u\})$

   c. $[\![\text{Dec FocP}]\!] = \, !!([u]; person\{u\}; called\{u\})$

                         $\equiv [u]; person\{u\}; called\{u\}$

The example underlines the fact that indefinites are treated on our account as introducing a discourse referent, as in previous work on dynamic semantics, but *not* as introducing inquisitiveness, unlike in several analyses based on static inquisitive semantics (e.g., AnderBois, 2014; Onea, 2016; Ciardelli *et al.*, 2017). Therefore, in a declarative sentence like (91) the ! operators contributed by Foc and Dec are both vacuous, since the TP is already non-inquisitive.

5   Detailed predictions for wh-questions

In Section 4.3.2 we already discussed the predictions of our account for a simple mention-some wh-question, *Who has a pen that I could borrow?*, and a wh-question with TP-internal disjunction, *Who has a pen or a pencil that I could borrow?*. These examples were meant as a first illustration of the interaction between Int, Foc, wh-phrases, and disjunction. We now turn to a more detailed

discussion of the predictions of our account for wh-questions. To what extent does the account meet the desiderata described in Section 1.3? Besides mention-some readings, can it also derive mention-all readings in a non-stipulative way? Does it provide a uniform treatment of single-wh and multiple-wh questions? Does it capture the effects of singular and plural number marking in *which*-questions? In particular, does it derive uniqueness presuppositions for singular *which*-questions without additional stipulations? And does it predict that such uniqueness presuppositions can be obviated by disjunction?

We will demonstrate that, indeed, all these desiderata are met. The assumptions we have made about Int, Foc, wh-phrases, indefinites, disjunction, and number morphology do not only derive correct results in the simple cases we have seen already, but also in many more complex cases. We will show this by going through a number of representative examples, presented in order of increasing complexity. The first example illustrates the derivation of mention-all readings.

5.1 Deriving mention-all readings through maximization

We assumed in Section 4.2 that simplex wh-phrases (*who, where, what* etc.) have both a weak and a strong reading, and that the difference between these two readings is that, on a strong reading, a maximality requirement is placed on the values of the discourse referent that these wh-phrases introduce. The entry for *who* is repeated in (93) for convenience.

(93) a. $[\![\text{who}^u_{weak}]\!] \ \ = \lambda P_{r\mathbf{T}}. \ [u]; person\{u\}; P(u)$
      b. $[\![\text{who}^u_{strong}]\!] = \lambda P_{r\mathbf{T}}. \ [u]; person\{u\}; P(u); \mathbf{max}^*\{u\}$

Importantly, the assumed ambiguity in the interpretation of simplex wh-phrases parallels an ambiguity in the semantics of indefinites, argued for by Brasoveanu (2008) on the basis of donkey anaphora. That is, the assumed ambiguity is not stipulated ad-hoc, but has been motivated independently, based on an empirical phenomenon that lies outside the domain of questions.

We have already seen in Section 4.3.2 that, when *who* is given a weak interpretation, we derive a mention-some reading. We will now show that when *who* is given a strong interpretation, placing a maximality requirement on the discourse referent that it introduces, a mention-all reading is derived. No additional assumptions are needed.

To see this, consider (94) and its $\mathsf{Inq_D}$ translation in (95).

(94) Who^u_{strong} called?

(95) $\dagger([u]; person\{u\}; called\{u\}; \mathbf{max}^*\{u\}; ?u)$

The predicted update effect of (94) is represented graphically in Figure 16 (modulo †). It is instructive to compare this figure with Figure 15. The only difference is that in Figure 16, the $\mathbf{max}^*\{u\}$ operator applies before $?u$ does. This removes all states containing possibilities whose assignments cumulatively assign a non-maximal plural individual to u, compared to other possibilities with the same world-parameter (i.e., other possibilities in the same column of the diagram). Concretely, the operator excludes all states containing either or both of the top two possibilities in the second column.

The output context in Figure 16 contains three mutually exclusive alternatives, while the output context in Figure 15 contained two overlapping alternatives. The alternatives Figure 16 correspond to the three possible exhaustive answers: (i) only a called, (ii) only b called, and (iii) both a and b called. The two overlapping alternatives in Figure 15 on the other hand, correspond to the two non-exhaustive answers: (i) a called (and perhaps b did as well), and (ii) b called (and perhaps a did as well). Thus, the assumed ambiguity in the semantics of simplex wh-phrases indeed derives

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$$\emptyset \qquad \begin{array}{cccc} w_a & w_{a,b} & w_b & w_\emptyset \\ \bullet & \bullet & \bullet & \bullet \end{array}$$

$[u]; person\{u\}; called\{u\}$

$?u$

$\mathbf{max}^*\{u\}$

Figure 16: The update effect of Who^u_{strong} *called?*
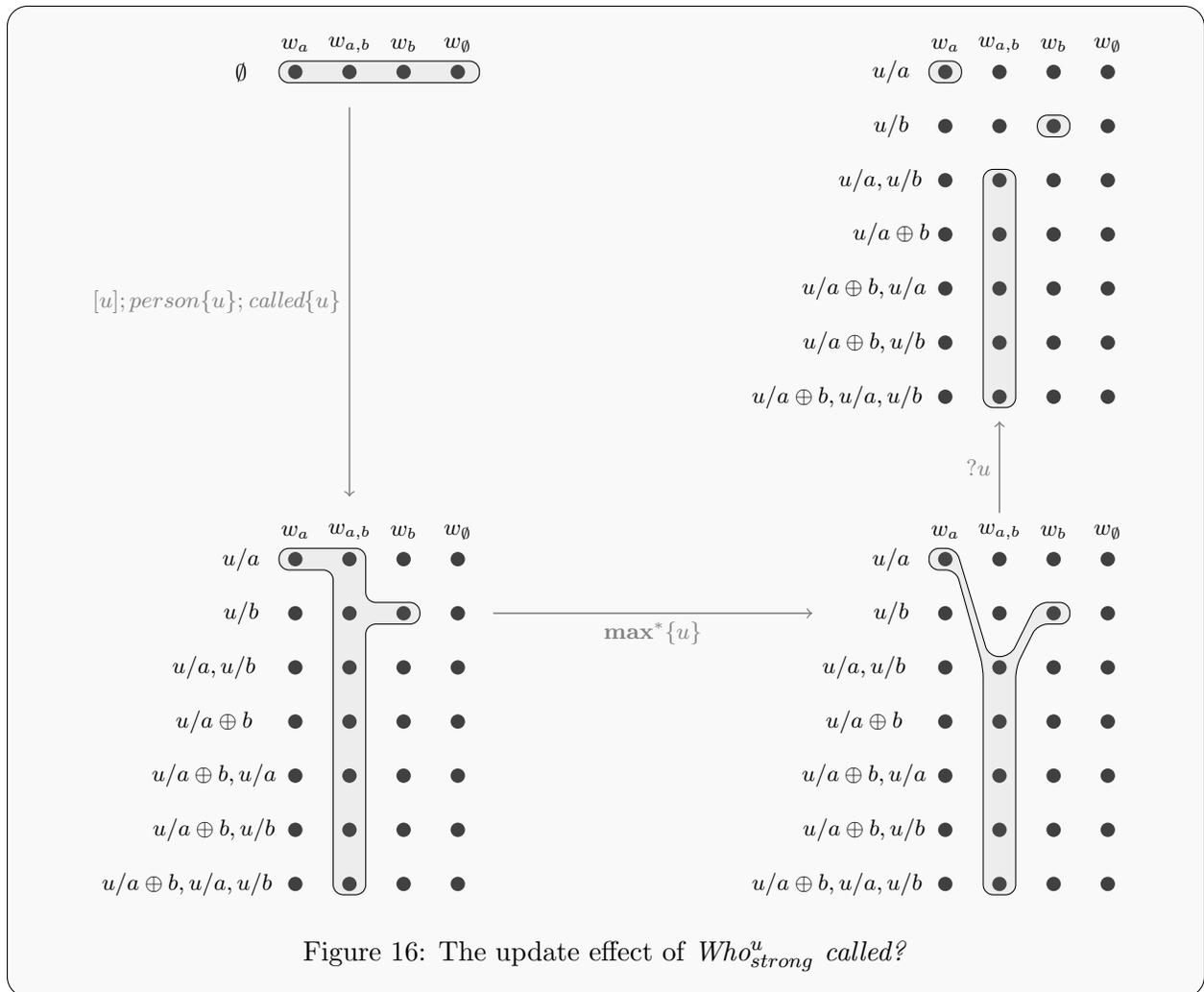
mention-some and mention-all readings.

5.2   *Which*-questions

Comorovski (1996), Dayal (1996) and others have observed that, unlike simplex wh-phrases like *who* and *what*, plural *which*-phrases force mention-all readings and singular *which*-phrases invoke a uniqueness requirement, at least in basic cases (some more complex constructions in which these effects are obviated will be discussed in Section 5.3). This is illustrated in (96).

(96)   a.   Who has a pen that the chair of the committee can borrow to sign the report?
        b.   Which committee members have a pen that the chair can borrow to sign the report?
        c.   Which committee member has a pen that the chair can borrow to sign the report?

While (96a) has a salient mention-some reading, (96b) only permits a mention-all reading. Finally, in (96c), it is presupposed that there is *exactly one* committee member who has a pen that the chair can borrow.

These basic empirical characteristics of plural and singular *which*-questions are derived on our account, under the assumption that the determiner *which*, unlike *who* and *what*, only has a strong

interpretation. That is, it always places a maximality requirement on the values of the discourse referent that it introduces (see the lexical entry for *which* in (71)). Let us demonstrate this.

Plural *which*-questions The case of plural *which*-questions is straightforward. Consider the example in (97).

(97)   Which^u people called? $\rightsquigarrow$ $\dagger\,([u]; person\{u\}; called\{u\}; \mathbf{max}^*\{u\}; ?u)$

The $\mathsf{Inq_D}$ translation of (97) is the same as that of (94) above, so the context update that (97) induces is the one graphically represented in Figure 16. As we discussed above, the issue in the output context captures the mention-all reading of the question.

Singular *which*-questions For singular *which*-questions our assumption that singular number marking invokes an atomicity requirement is crucial. To see the effect of this assumption, consider (98). The atomicity requirement invoked by singular number marking is underlined.

(98)   Which^u person called? $\rightsquigarrow$ $\dagger\,([u]; person\{u\}; \underline{\mathbf{atom}\{u\}}; called\{u\}; \mathbf{max}^*\{u\}; ?u)$

The update effect of (98) is graphically represented in Figure 17 (again modulo $\dagger$). The update is broken down into three steps. In the first step, $\mathbf{atom}\{u\}$ removes from the context all states containing possibilities in which u is assigned a non-atomic value by some assignment. In the second step, $\mathbf{max}^*\{u\}$ eliminates states containing possibilities that cumulatively assign non-maximal plural individuals to u compared to other possibilities with the same world-parameter. Finally, in the third step, the witness request operator $?u$ raises an issue whose resolution requires identifying an individual which has all the properties assigned to u. Crucially, $?u$ removes states containing the possibility $\langle w_{a,b}, \{\{u \mapsto a\}, \{u \mapsto b\}\}\rangle$, because there is no possibility in the input context with the same world parameter whose assignments all agree on the value of u. Thus, after the $?u$ update, all states that remain in the output context consist exclusively of possibilities in which *exactly one* person called, either a or b.

Since the entire sequence of updates is in the scope of a $\dagger$ operator, we correctly derive that the question triggers a uniqueness presupposition. That is, for the update to be well-defined, it should already be established in the input context that there is exactly one person who called. On our account, this uniqueness presupposition is not triggered by a single operator but rather results from the interaction between $\mathbf{atom}$, $\mathbf{max}^*$, $?u$ and $\dagger$. If any of these operators were left out of the derivation, the uniqueness presupposition would not be predicted. In this regard, our account differs from other accounts of uniqueness presuppositions in singular *which*-questions (e.g., Dayal, 1996; Hirsch and Schwarz, 2019), a difference we will return to in Section 5.3.

Pragmatic derivation of more-than-one inferences A plural *which*-question like (97) can be taken to suggest that *more than one* person called. We assume that this inference can arise, in certain contexts, through pragmatic reasoning.[27] The pragmatically strengthened interpretation is given in (99), where the inferred update is underlined.

(99)   Which^u people called? $\rightsquigarrow$ $\dagger\,([u]; person\{u\}; \underline{\neg\mathbf{atom}\{u\}}; called\{u\}; \mathbf{max}^*\{u\}; ?u)$

---

[27]The idea that plural number marking can give rise to pragmatic more-than-one inferences is commonplace in the literature (see for instance Sauerland 2003; Sauerland *et al.* 2005; Spector 2007; Zweig 2008; Ivlieva 2013; Mayr 2015; Sudo 2019). However, to our knowledge, the idea has so far only been proposed for non-wh-phrases, and has not been applied yet to plural wh-phrases.

Figure 17: The update effect of *Which^u person called?*
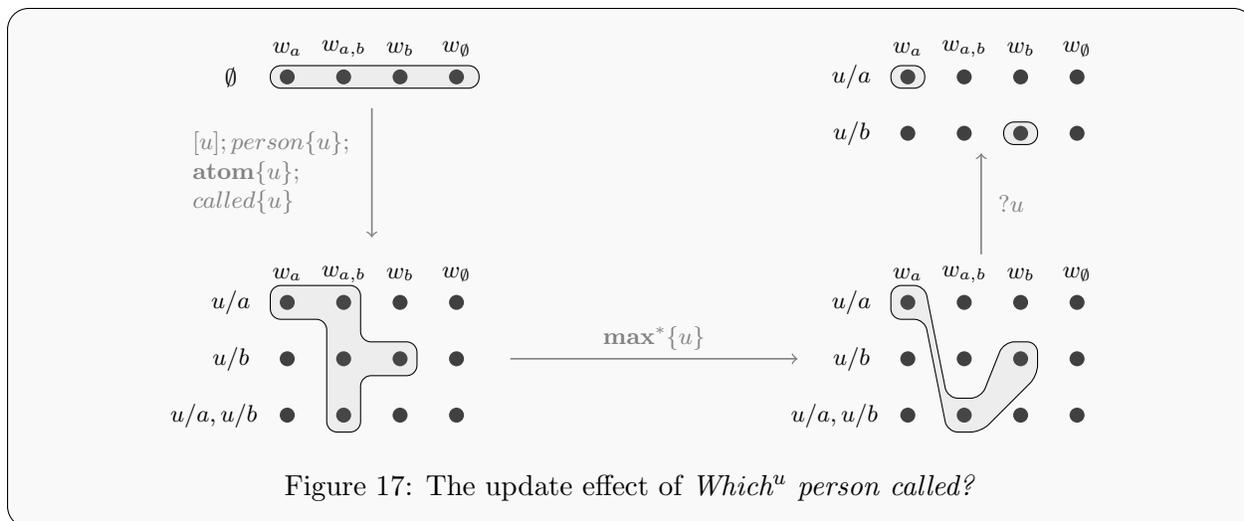
Support for such a pragmatic view on more-than-one inferences in plural *which*-questions comes from a parallel between the behavior of plural *which*-phrases and that of plural non-wh-indefinites when occurring in questions. While plural non-wh-indefinites typically give rise to a more-than-one inference in declarative sentences, they typically don't in questions. For instance, the question posed by A in (100) must be answered affirmatively when the addressee has just one child.

(100)   A: Do you have children?
         B: Yes, I have one.
         B: #No, I have one.

This suggests that plural non-wh-indefinites can range over both atomic and non-atomic entities, and we are assuming that the same holds for plural *which*-phrases.

However, there are cases of non-wh-indefinites in questions that do favour a strengthened interpretation, ranging only over non-atomic entities. This is illustrated in (101) from Sauerland *et al.* (2005). It is odd to respond to (101) with 'yes' if one knows that a dog has only one tail. Indeed, Sauerland *et al.* (2005) show experimentally that adults predominantly respond to questions like (101) with 'no'. This indicates that *tails* in (101) is interpreted as ranging only over non-atomic entities.

(101)   Does a dog have tails?

Sauerland *et al.* (2005) suggest that the crucial difference between questions like (100) and ones like (101) is that the former are naturally taken to be genuine information seeking questions, whose answer is not yet known to the speaker, while the latter are more naturally taken to be 'exam questions', whose answer is already known to the speaker. In the latter case the more-than-one inference is more likely to arise, because the pragmatic reasoning that it is based on only goes through if the speaker can be assumed to have sufficient knowledge to make an informed choice between singular and plural number marking.

We submit that a similar contrast can be observed with plural *which*-phrases. For instance, suppose that Professor Baker always brings a cake if one or more of his students get a paper accepted at a conference. When his colleague sees him come into work with a cake she asks him:

(102)   Which students of yours got a paper accepted?

In this case, a more-than-one inference seems unlikely to arise. This would be explained by the fact that (102) is a genuine information seeking question. Professor Baker's colleague cannot be assumed to have sufficient knowledge to use singular rather than plural number marking.

Now suppose that the following question is posed as part of a history exam:

(103)   In which centuries did the eighty years war take place?

In this case, a more-than-one inference is very likely to arise. The explanation for this, we suggest, is that the person who formulated the question can be assumed to know whether the eighty years war took place in just one century or across two centuries. If she had known that it took place in just one century, she would have used singular number morphology in the *which*-phrase. Since she didn't, we can infer that the war spanned more than one century.

5.3   Obviation of uniqueness effects

Recall from Section 1 that the uniqueness requirement normally induced by a singular *which*-phrase can be obviated in questions involving an existential modal operator (Hirsch and Schwarz, 2019) or disjunction (Socolof *et al.*, 2020). For instance, it is perfectly natural to ask (104) without assuming that there is a unique letter that can be inserted in *fo_m* to make a word.

(104)   Which letter can be inserted in *fo_m* to make a word?

Similarly, (105) can be felicitously asked without assuming that there is a unique town in which Shakespeare was born or Bach died.

(105)   In which town was Shakespeare born or did Bach die?

As Hirsch and Schwarz (2019) and Socolof *et al.* (2020) point out, however, it would be mistaken to conclude that singular *which*-phrases in questions with modals or disjunctions do not contribute a uniqueness requirement at all. To see this, consider (106).

(106)   #Which letter can be inserted in *f_ _m* to make a word?

Based on the observation that this question is infelicitous, Hirsch and Schwarz (2019) argue that, while the question does not presuppose that there is exactly one letter that can be inserted in *f_ _m* to make a word, it does presuppose that it is possible to make a word by inserting exactly one letter in *f_ _m*. We will refer to this presupposition as a 'local uniqueness requirement'.[28] The question is infelicitous, because one can obviously not fill two slots by inserting exactly one letter.

The local uniqueness requirement also arises in singular *which*-questions with disjunction (Socolof *et al.*, 2020). To see this, consider (107):

(107)   a.   Which letter is missing in *pill_w* or was replaced with a dollar sign in bre$d?
         b. #Which letter is missing in *p_ll_w* or was replaced with a dollar sign in bre$d?
         c. #Which letter is missing in *pill_w* or was replaced with a dollar sign in br$$d?

The fact that (107b) and (107c) are infelicitous shows that a local uniqueness requirement pertains to both disjuncts.

As Hirsch and Schwarz (2019) argue, the fact that non-local uniqueness requirements can be obviated by modals and disjunction is problematic for what they call *globalist* accounts, which derive

---

[28]Hirsch and Schwarz (2019) use the term 'low uniqueness requirement'. We opt for 'local' rather than 'low' because we feel that it fits the case of disjunction, discussed below, better.

a uniqueness presuppostion in singular *which*-questions through an 'answer operator' ANS that is located at the highest position in the logical form of the question (Dayal, 1996, 2016; Fox, 2013, 2018; Xiang, 2016). The problem for globalist accounts is that they always predict a uniqueness presupposition at the highest level. They do not derive a local uniqueness presupposition, within the scope of a modal operator or disjunction, because ANS always takes scope above everything else in the question. Hirsch and Schwarz (2019) contrast globalist accounts with what they call *localist* accounts, in which the uniqueness requirement is triggered by the *which*-phrase rather than an answer operator (Champollion *et al.*, 2017; Uegaki, 2018, 2020; Hirsch and Schwarz, 2019). Such accounts can deal with the obviation of a global uniqueness requirement in examples like (104) and (105), and can also correctly derive a local uniqueness requirement in such cases.

What about our analysis? Is it a globalist or a localist account? Certainly, it is not a globalist account, since it derives uniqueness presuppositions without recourse to an answer operator scoping over the entire question. However, it also differs significantly from the localist account of Hirsch and Schwarz (2019). Namely, while Hirsch and Schwarz assume that the uniqueness presupposition of *Which student danced?* is triggered by a single question-internal element, namely the *which*-phrase, on our account there is no single element that contributes a uniqueness presupposition. Rather, the presupposition results from an interplay between **atom**, **max***, $?u$, and $\dagger$, where **atom** and **max*** are contributed by the *which*-phrase, $?u$ is contributed by Foc_u, and $\dagger$ is contributed by Int. We will therefore refer to our account as a *decompositional* account.

How does the decompositional account fare with respect to the obviation data discussed in this section? As already remarked in Section 1, we will not provide a formal analysis of questions with modals here, since it is a non-trivial task to add modals to the Inq_D framework, one that is beyond the scope of this paper. We will therefore concentrate on singular *which*-questions with disjunctions, such as (105). We will show that, for such cases, the decompositional account correctly predicts obviation of a global uniqueness requirement and instead derives the observed local uniqueness requirements pertaining to each individual disjunct.

The assumed syntactic structure for (105) is given in Figure 18. Notice that, just as Socolof *et al.* (2020), we assume that the disjunction scopes below Foc, i.e., at the TP level. We refer to Socolof *et al.* (2020) for evidence supporting this syntactic assumption.

Figure 18: Assumed syntactic structure of (105).

The Inq_D-translation of the question is given in (108). We assume here that the *which*-phrase is interpreted inside each disjunct via reconstruction after Across-the-Board movement. Any lo-

calist account has to assume such a reconstruction mechanism as well to derive local uniqueness requirements.

(108)   a. $[\![\text{TP or TP}]\!] =$
$$
\begin{array}{c}
([u]; \mathbf{atom}\{u\}; town\{u\}; S\_born\_in\{u\}; \mathbf{max}^*\{u\}) \\
\sqcup \\
([u]; \mathbf{atom}\{u\}; town\{u\}; B\_died\_in\{u\}; \mathbf{max}^*\{u\})
\end{array}
$$

   b. $[\![\text{FocP}]\!] = \; ! \left(\begin{array}{c} ([u]; \mathbf{atom}\{u\}; town\{u\}; S\_born\_in\{u\}; \mathbf{max}^*\{u\}) \\ \sqcup \\ ([u]; \mathbf{atom}\{u\}; town\{u\}; B\_died\_in\{u\}; \mathbf{max}^*\{u\}) \end{array} \right) ; ?u$

   c. $[\![\text{TypeP}]\!] = \; \dagger \; (\; ! \left(\begin{array}{c} ([u]; \mathbf{atom}\{u\}; town\{u\}; S\_born\_in\{u\}; \mathbf{max}^*\{u\}) \\ \sqcup \\ ([u]; \mathbf{atom}\{u\}; town\{u\}; B\_died\_in\{u\}; \mathbf{max}^*\{u\}) \end{array} \right) ; ?u)$

The predicted update effect of the question is graphically represented in Figure 19. For simplicity, the diagrams in this figure involve only three worlds:

- w_{actual}: the actual world, in which Shakespeare was born in Stratford upon Avon and Bach died in Leipzig.

- w_{poss}: a non-actual but possible world in which Shakespeare was born in London and Bach died in Berlin.

- w_{imposs}: a non-actual and impossible world in which Shakespeare was born in Stratford upon Avon *and* in London, and Bach died in Leipzig *and* in Berlin.

The third world is included to show that our account derives a local uniqueness requirement.

The first update displayed in Figure 19 is the one induced by the disjunction. This update results in two alternatives, one consisting of possibilities in which u gets assigned atomic values which are, according to the world parameter of that possibility, the place(s) where Shakespeare was born, and one alternative consisting of possibilities in which u gets assigned atomic values which are, according to the world parameter of that possibility, the place(s) where Bach died. In the second update, induced by !, these two alternatives are joined into one big alternative consisting of all possibilities in which u gets assigned atomic values which are, according to the world parameter of that possibility, either the place(s) where Shakespeare was born or the place(s) where Bach died. Finally, in the third update, $?u$ applies. This again introduces two alternatives, but different ones. Namely, the first alternative consists of all possibilities in which Shakespeare was born in Stratford (and not in London) and Bach died in Leipzig (and not in Berlin), and the second alternative consists of all possibilities in which Shakespeare was born in London (and not in Stratford) and Bach died in Leipzig (and not in Berlin). Note that states containing possibilities in which Shakespeare was born in two places and Bach died in two places do not survive this update.

Crucially, as desired, we do *not* derive a global uniqueness requirement. That is, we do not predict that the question presupposes the existence of a unique town in which Shakespeare was born and Bach died. This is because $\mathbf{atom}\{u\}$ and $\mathbf{max}^*\{u\}$ are interpreted inside the two disjuncts. At the same time, we do derive a local uniqueness requirement, that is, we predict that the question presupposes that there is a unique town in which Shakespeare was born and a unique town in which Bach died.

So, just like the localist account of Hirsch and Schwarz (2019) and Socolof *et al.* (2020), our decompositional account successfully derives a local uniqueness requirement in singular *which*-questions with disjunction. As we will see next, our decompositional account also overcomes what

Figure 19: The update effect of (105).

Hirsch and Schwarz (2019, p.765) identify as the main problem that the localist account faces, namely the fact that a multiple-wh question like *Which student talked to which professor?* does not presuppose that exactly one student talked to exactly one professor. A localist account would, without further stipulations, wrongly predict such a presupposition, since it assumes that each *which*-phrase contributes a uniqueness requirement all by itself. As we will see in Section 5.4, our decompositional account does not run into this problem since the operators contributed by a singular *which*-phrase, $\mathbf{atom}\{u\}$ and $\mathbf{max}^*\{u\}$, do not induce a uniqueness presupposition by themselves, but only through interaction with $?u$ and $\dagger$, which are contributed by functional heads in the left periphery.

5.4 Multiple-wh questions with singular *which*-phrases

We now turn to multiple-wh questions. In this and the next two subsections we will discuss several variants. We start with multiple-wh questions containing two singular *which*-phrases, exemplified in (109).

(109)   Which^{u_1} student read which^{u_2} book?

This question has a so-called *pair-list* interpretation. That is, to resolve the issue that the question raises, one has to determine a complete list of pairs $\langle x, y \rangle$ such that x is a student, y a book, and x read y. For instance, if the relevant set of students is $\{a, b, c, d\}$, the relevant set of books is

$\{a', b', c', d'\}$, a read a', b read b', c read c', and d read d', then the list of pairs that needs to be established is:

$$(110) \quad \begin{bmatrix} \langle a, a' \rangle, \\ \langle b, b' \rangle, \\ \langle c, c' \rangle, \\ \langle d, d' \rangle \end{bmatrix}$$

Such a list of pairs encodes a certain relation between students and books. Dayal (1996, 2016) argues that multiple-wh questions with singular *which*-phrases generally presuppose that this relation has two specific properties. First, Dayal proposes that the relation must be *domain exhaustive*, which means that every student in the relevant domain must be related to at least one book. And second, the relation must be *functional*, which means that every student must be related to at most one book. The pair-list in (110) satisfies both properties. On the other hand, the pair-list in (111) violates domain exhaustivity, and the pair-list in (112) violates functionality.

$$(111) \quad \text{Violating domain exhaustivity:} \quad \begin{bmatrix} \langle a, a' \rangle, \\ \langle b, b' \rangle \end{bmatrix}$$

$$(112) \quad \text{Violating functionality:} \quad \begin{bmatrix} \langle a, a' \rangle, \langle a, b' \rangle, \langle a, c' \rangle, \\ \langle b, b' \rangle, \\ \langle c, c' \rangle, \\ \langle d, d' \rangle \end{bmatrix}$$

While there seems to be consensus in the literature that functionality is indeed a necessary component of the pair-list interpretation of multiple-wh questions, Xiang (2019) argues that Dayal's domain exhaustivity requirement in fact need not be obeyed. Specifically, Xiang observes that when 100 candidates have applied for 3 jobs and Mary knows the outcome of the search procedure but Bill doesn't, then it is natural for Bill to ask Mary:

(113)   Which candidate got which job?

Resolving this question does not require specifying for *every* candidate which job they got, something that would be impossible to do because there were fewer jobs than candidates. Still, the question is felicitous. So, Xiang argues, domain exhaustivity is not a necessary component of the pair-list interpretation of multiple-wh questions, and we endorse this conclusion.

We further observe, however, that in order to resolve the question in (113) one must still specify for *as many candidates as possible* (in this case three) which jobs they got. We refer to this requirement as *domain maximality*. For instance, if $\{c_1, \ldots, c_{100}\}$ are the relevant candidates, $\{j_1, j_2, j_3\}$ the relevant jobs, c_{23} got j_1, c_{45} got j_2, and c_{87} got j_3, then to fully resolve (113) one must establish the pair-list in (114), and not some subset thereof.

$$(114) \quad \begin{bmatrix} \langle c_{23}, j_1 \rangle, \\ \langle c_{45}, j_2 \rangle, \\ \langle c_{87}, j_3 \rangle \end{bmatrix}$$

We will now demonstrate that our account captures these empirical properties of multiple-wh questions with singular *which*-phrases, i.e., functionality and domain maximality, without any further stipulations. Given the assumptions we have made about the syntax of English and how logical forms of English sentences are mapped to $\mathsf{Inq_D}$ expressions, the question in (109) is translated as in (115):

$$† ([u_1]; student\{u_1\}; \mathbf{atom}\{u_1\};$$
$$[u_2]; book\{u_2\}; \mathbf{atom}\{u_2\};$$
$$read\{u_1, u_2\}; \mathbf{max}^*\{u_1\}; \mathbf{max}^*\{u_2\}; ?u_1u_2)$$

TypeP

Int

$$[u_1]; student\{u_1\}; \mathbf{atom}\{u_1\};$$
$$[u_2]; book\{u_2\}; \mathbf{atom}\{u_2\};$$
$$read\{u_1, u_2\}; \mathbf{max}^*\{u_1\}; \mathbf{max}^*\{u_2\}; ?u_1u_2$$

FocP

$$\text{Foc}_{u_1,u_2}$$

$$[u_1]; student\{u_1\}; \mathbf{atom}\{u_1\};$$
$$[u_2]; book\{u_2\}; \mathbf{atom}\{u_2\};$$
$$read\{u_1, u_2\}; \mathbf{max}^*\{u_1\}; \mathbf{max}^*\{u_2\}$$

TP

which^{u_1} student read which^{u_2} book

Figure 20: Deriving the translation of (109).

(115)   Which^{u_1} student read which^{u_2} book?

$$† \left(\begin{array}{l} [u_1]; student\{u_1\}; \mathbf{atom}\{u_1\}; \\ [u_2]; book\{u_2\}; \mathbf{atom}\{u_2\}; \\ read\{u_1, u_2\}; \\ \mathbf{max}^*\{u_1\}; \mathbf{max}^*\{u_2\}; \\ ?u_1u_2 \end{array} \right)$$

The main steps in the derivation of this translation are given in Figure 20. Note that the ! operator contributed by Foc_{u_1,u_2} is vacuous in this case and has therefore been omitted from the translation.

The update effect of the question is graphically represented in Figure 21 and Figure 22. In these figures, dref assignment functions belonging to the same possibility are separated from other dref assignment functions by horizontal lines. Super- and subscripts on a world specify who read what in that world. For intance, $w_{b \rightsquigarrow c}^{a \rightsquigarrow c}$ is a world in which a read c and b read c and no other reading happened. For reasons of space, we assume here that it is already known in the input context that a read c, d, or nothing at all, and that b read c, d, or both of them. We furthermore assume that a and b are students and c and d are books in all worlds.

Let us now zoom in on Figure 21, which graphically represents the update induced by the TP, i.e., before $?u_1u_2$ applies. This initial update yields a context with just one alternative, consisting of all possibilities in which (i) u_1 is a student, u_2 a book, and u_1 read u_2, and (ii) the values cumulatively assigned to u_1 and u_2 are maximal in comparison to other possibilities with the same world parameter. For instance, states containing the possibility $\langle w_{b \rightsquigarrow c}^{a \rightsquigarrow c}, \{\{u_1 \mapsto a, u_2 \mapsto d\}\}\rangle$ are eliminated because this possibility does not satisfy (i), and states containing the possibility

$\langle w^{a \leadsto c}_{b \leadsto c}, \{\{u_1 \mapsto a, u_2 \mapsto c\}\}\rangle$ are eliminated because this possibility does not satisfy (ii).

Figure 22 displays the result of subsequently applying the witness request operator $?u_1 u_2$ contributed by Foc_{u_1,u_2}. This operator raises an issue whose resolution requires identifying a function from individuals satisfying all properties ascribed to u_1 to individuals satisfying all properties ascribed to u_2. There are six alternatives in the output context in Figure 22. The leftmost alternative corresponds to the function $\{a \mapsto c, b \mapsto c\}$, the next corresponds to the function $\{a \mapsto c, b \mapsto d\}$, etcetera. Note that the rightmost alternative corresponds to the function $\{b \mapsto d\}$, which does not satisfy domain exhaustivity because student a is not mapped to any book. Also note that all states containing the possibility $\langle w^{b \leadsto c}_{b \leadsto d}, \{\{u_1 \mapsto b, u_2 \mapsto c\}, \{u_1 \mapsto b, u_2 \mapsto d\}\}\rangle$ (fifth row, last column) have been removed by $?u_1 u_2$. This is a desirable result, since the mapping from students to books that they read is not *functional* in $w^{b \leadsto c}_{b \leadsto d}$. After all, student b read two books, c and d, in this world.

This example illustrates three general features of our account. First, it successfully captures the fact that a pair-list answer to a multiple-wh question with singular *which*-phrases must establish a *functional* mapping between individuals in the domain set (in our example, the students) and individuals in the target set (in our example, the books). Second, in line with Xiang's (2019) empirical observations contra an earlier hypothesis by Dayal (1996, 2016), our account does *not* require that the mapping satisfies *domain exhaustivity* (for instance, in our example it is predicted that the question can be resolved by establishing that student b read book c and student a did not read any book). And third, in line with our own empirical observations, the account does require that the mapping satisfies *domain maximality*—it must map as many individuals in the domain set as possible to suitable individuals in the target set. The account thus captures what we identified, building on the work of Dayal (1996, 2016), Xiang (2019) and others, as the main empirical properties of multiple-wh question with singular *which*-phrases.

Moving now from empirical adequacy to theoretical/explanatory adequacy, two additional features of the account are worth highlighting. First, the semantic relation between wh-phrases and Foc is akin to the semantic relation between anaphoric expressions and their antecedents: wh-phrases introduce discourse referents and the witness request operator contributed by Foc targets these discourse referents. This means that, for semantic purposes, it is not necessary for wh-phrases to move either overtly or covertly to the left periphery to take scope there. In particular, in multiple-wh questions in English, the wh-phrase that remains in-situ at the surface can also be interpreted in situ—covert movement to the left periphery is not necessary for interpretation. The same holds for languages in which wh-phrases more generally remain in-situ at the surface. This is a desirable feature of the account given that in situ wh-words can appear inside islands for movement in English and other languages, as exemplified in Reinhart's (1997) famous example in (116).

(116)   Which linguist will be offended if we invite which philosopher?

We assume that, when wh-phrases do move to the left periphery, this is entirely for syntactic reasons.

A second feature of the account that speaks to its theoretical/explanatory adequacy is the fact that single-wh and multiple-wh questions are treated in a *uniform* way. In both cases, all the relevant elements are treated exactly the same: wh-phrases introduce discourse referents, possibly with an atomicity requirement (contributed by number marking) and/or a maximality requirement. Foc contributes a witness request operator, and Int contributes the † operator. Crucially, while in a single-wh question the witness request operator targets just one discourse referent and in multiple-wh questions it targets multiple discourse referents, we have given a single, general definition of $?u_1 \ldots u_n$ which applies to all cases at once. The example we've discussed above involved two wh-phrases, but cases with three or more wh-phrases, exemplified in (117) and (118), can be dealt with

$w_{b\rightsquigarrow c}^{a\rightsquigarrow c}$ $\quad$ $w_{b\rightsquigarrow d}^{a\rightsquigarrow c}$ $\quad$ $w_{b\rightsquigarrow d}^{a\rightsquigarrow d}$ $\quad$ $w_{b\rightsquigarrow c}^{a\rightsquigarrow d}$ $\quad$ $w^{b\rightsquigarrow c}$ $\quad$ $w^{b\rightsquigarrow d}$ $\quad$ $w_{b\rightsquigarrow d}^{b\rightsquigarrow c}$

$\emptyset$

$[u_1]; student\{u_1\}; \mathbf{atom}\{u_1\};$
$[u_2]; book\{u_2\}; \mathbf{atom}\{u_2\};$
$read\{u_1, u_2\}; \mathbf{max}^*\{u_1\}; \mathbf{max}^*\{u_2\}$

u_1	u_2	$w_{b\rightsquigarrow c}^{a\rightsquigarrow c}$	$w_{b\rightsquigarrow d}^{a\rightsquigarrow c}$	$w_{b\rightsquigarrow d}^{a\rightsquigarrow d}$	$w_{b\rightsquigarrow c}^{a\rightsquigarrow d}$	$w^{b\rightsquigarrow c}$	$w^{b\rightsquigarrow d}$	$w_{b\rightsquigarrow d}^{b\rightsquigarrow c}$
a	c	●	●	●	●	●	●	●
a	d	●	●	●	●	●	●	●
b	c	●	●	●	●	●	●	●
b	d	●	●	●	●	●	●	●
b	c							
b	d	●	●	●	●	●	●	●
a	c							
b	c	●	●	●	●	●	●	●
a	c							
b	d	●	●	●	●	●	●	●
a	d							
b	d	●	●	●	●	●	●	●
a	d							
b	c	●	●	●	●	●	●	●

Figure 21: The update effect of $[_{\text{TP}}$ which^{u_1} student read which^{u_2} book]. For simplicity, only a relevant subset of all the possible dref assignment matrices are displayed.

$$w^{a\leadsto c}_{b\leadsto c} \quad w^{a\leadsto c}_{b\leadsto d} \quad w^{a\leadsto d}_{b\leadsto d} \quad w^{a\leadsto d}_{b\leadsto c} \quad w^{b\leadsto c} \quad w^{b\leadsto d} \quad w^{b\leadsto c}_{b\leadsto d}$$

$\emptyset$ $\bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet \quad \bullet$

$[u_1]; student\{u_1\}; \mathbf{atom}\{u_1\};$
$[u_2]; book\{u_2\}; \mathbf{atom}\{u_2\};$
$read\{u_1, u_2\}; \mathbf{max}^*\{u_1\}; \mathbf{max}^*\{u_2\};$
$?u_1u_2$

u_1	u_2	$w^{a\leadsto c}_{b\leadsto c}$	$w^{a\leadsto c}_{b\leadsto d}$	$w^{a\leadsto d}_{b\leadsto d}$	$w^{a\leadsto d}_{b\leadsto c}$	$w^{b\leadsto c}$	$w^{b\leadsto d}$	$w^{b\leadsto c}_{b\leadsto d}$
a	c	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
a	d	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
b	c	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\boxed{\bullet}$	$\bullet$	$\bullet$
b	d	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\boxed{\bullet}$	$\bullet$
b b	c d	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
a b	c c	$\boxed{\bullet}$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
a b	c d	$\bullet$	$\boxed{\bullet}$	$\bullet$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
a b	d d	$\bullet$	$\bullet$	$\boxed{\bullet}$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
a b	d c	$\bullet$	$\bullet$	$\bullet$	$\boxed{\bullet}$	$\bullet$	$\bullet$	$\bullet$

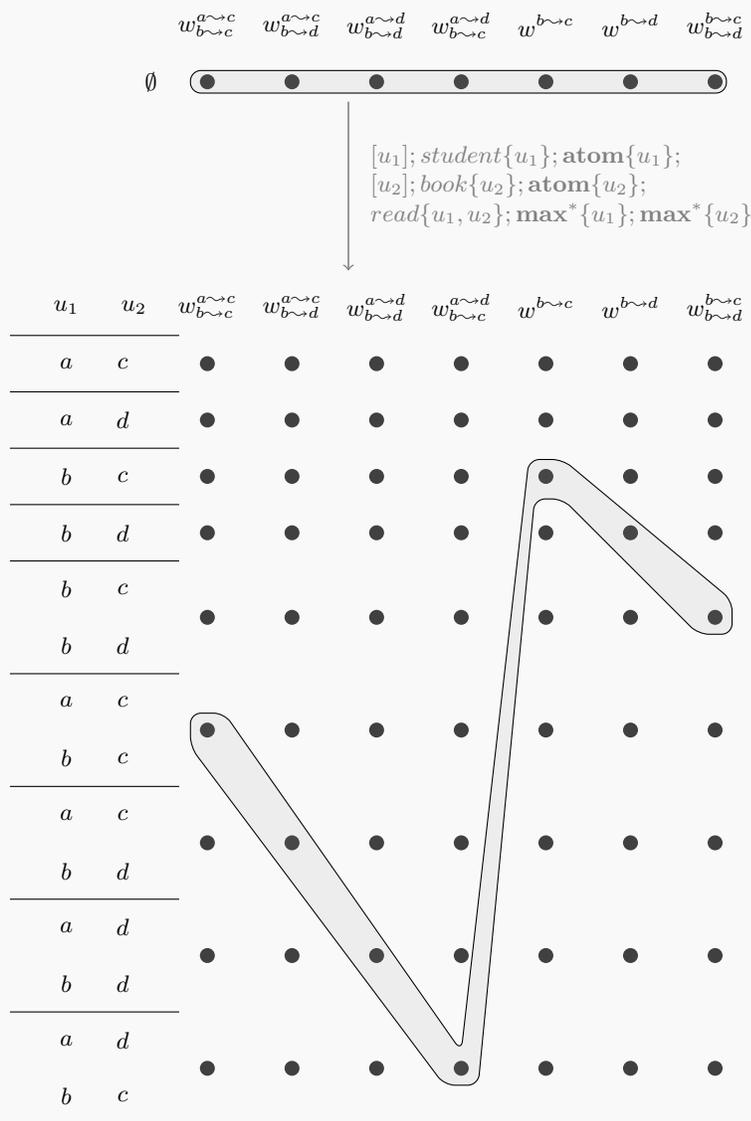Figure 22: The update effect of $[\text{Foc}_{u_1,u_2} [_\text{TP} \text{which}^{u_1} \text{ student read which}^{u_2} \text{ book}]]$. For simplicity, only a relevant subset of all the possible dref assignment matrices are displayed.
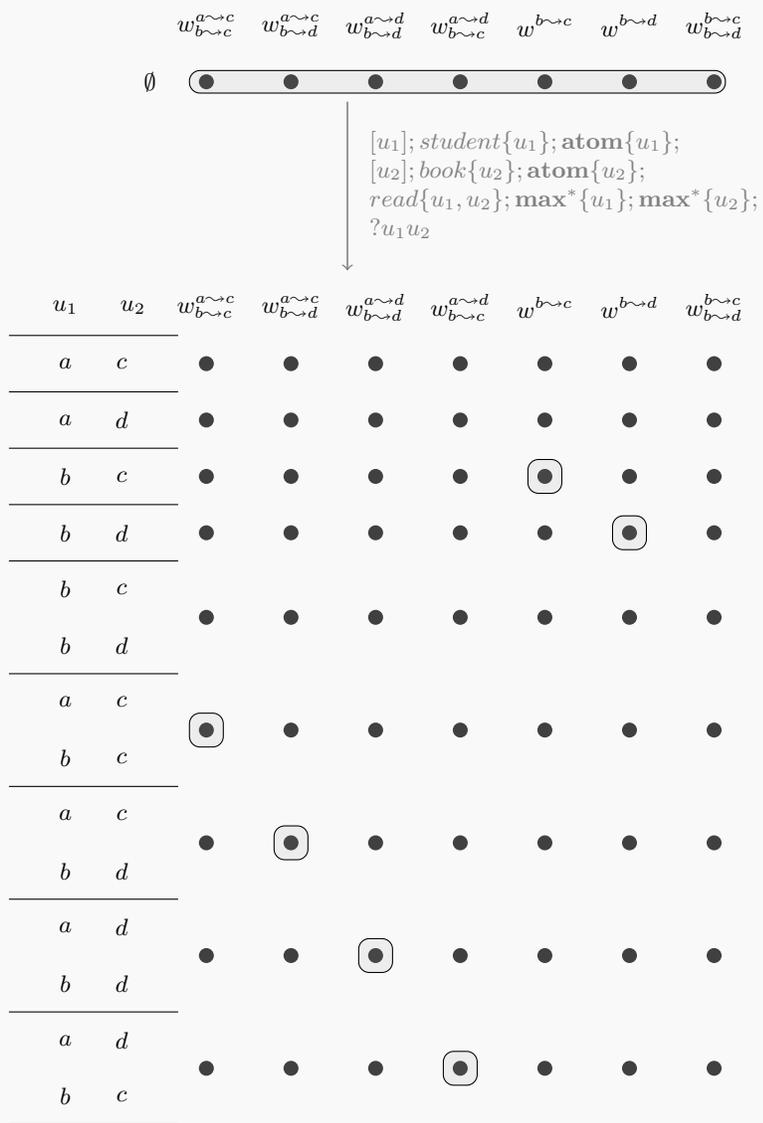
in exactly the same way.[29]

(117)   Which patient will receive which treatment on which day?

(118)   [The library maintains a digital record of. . .]
         which book was taken out by which member on which date for how many days.

Unlike other accounts (e.g., Dayal, 1996, 2016; Xiang, 2019), our account of single-wh questions scales up to multiple-wh questions without the need to stipulate additional semantic mechanisms. This point is discussed in some more detail in Appendix A.3.

5.5   Multiple-wh questions with plural *which*-phrases

We now briefly turn to multiple-wh questions with plural *which*-phrases, exemplified in (119).

(119)   Which^{u_1} students read which^{u_2} books?

$$\dagger \left(\begin{array}{l} [u_1]; student\{u_1\}; \\ [u_2]; book\{u_2\}; \\ read\{u_1, u_2\}; \\ \mathbf{max}^*\{u_1\}; \mathbf{max}^*\{u_2\}; \\ ?u_1u_2 \end{array} \right)$$

The only difference between the $\mathsf{Inq_D}$ translation of (119) and that of (109) is that in (119) no atomicity requirement is placed on the values of u_1 and u_2. As a consequence, we correctly predict that (119), unlike (109), is felicitous in a situation in which it is known that one or more students read multiple books. We also predict that domain maximality still needs to be obeyed. That is, the response needs to pair as many students as possible with the books they read. On the other hand, as in the case above with singular *which*-phrases, domain exhaustivity need not be obeyed. That is, the question can be felicitously asked even if the speaker knows that some students did not read any books.

5.6   Multiple-wh questions with partial mention-some readings

So far, we have focused on multiple-wh questions with *which*-phrases. Such phrases always place a maximality requirement on the values of the discourse referents that they introduce. We now turn to a case in which the target wh-phrase is a simplex wh-phrase rather than a complex *which*-phrase. We have assumed that such simplex wh-phrases, just like indefinites, allow for a strong and a weak interpretation. On their strong interpretation, their behaviour in multiple wh-questions is predicted to be similar to that of plural *which*-phrases, which we have already examined above. However, on their weak interpretation they are predicted to behave differently and give rise to a reading that is impossible to get with *which*-phrases. Consider the example in (120).

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[29]Note in particular that examples like (117) can be asked in a situation in which each patient receives multiple treatments, each on a particular day. For instance, patient p_1 receives treatment t_1 on Monday and treatment t_2 on Wednesday, while patient p_2 receives treatment t_1 on Tuesday and treatment t_2 on Thursday. In such a situation, the type of treatment does not functionally depend on the patient, because there are two treatments per patient. However, the day of the treatment does functionally depend on the patient and the type of treatment. This kind of dependency is permitted by our account. Situations in which both the type of treatment and the day of the treatment functionally depend on the patient are special cases of this.

(120)   Which herb grows where?

$$
\dagger \left(
\begin{array}{l}
[u_1]; herb\{u_1\}; \mathbf{atom}\{u_1\}; \\
[u_2]; place\{u_2\}; \\
grow\_at\{u_1, u_2\}; \\
\mathbf{max}^*\{u_1\}; \\
?u_1u_2
\end{array}
\right)
$$

As we observed in Section 2, this question has a *partial mention-some* reading. On this reading, its resolution requires specifying, for each herb (mention-all), at least one place where it grows (mention-some). Such a partial mention-some reading is impossible to derive for most theories of questions, because the mention-all vs mention-some distinction is generally seen as a distinction that pertains to the interpretation of a question as a whole, rather than to the interpretation of individual wh-phrases (see, e.g., Beck and Rullmann, 1999; George, 2011; Theiler, 2014; Champollion *et al.*, 2015). On our account, the mention-all vs mention-some distinction does pertain to the interpretation of individual wh-phrases. More specifically, whether a given wh-phrase receives a mention-all or a mention-some interpretation depends on whether a maximality requirement is placed on the values of the discourse referent that it introduces. As a result, our account straightforwardly derives partial mention-some readings for questions like (120).

The update induced by (120) is graphically represented in Figure 23. Note that the issue in the output context is mention-all w.r.t. herbs but mention-some w.r.t. places. To resolve the issue, we have to specify at least one place (c, d, e or f) for every herb (a and b). For example, establishing that a grows at c and b grows at e (without excluding that a and b grow in other places as well) is enough to remove all states containing the possibility $\langle w_{b \rightsquigarrow f}^{a \rightsquigarrow d}, \{\{a \mapsto d, b \mapsto f\}\}\rangle$ in the rightmost column, which leads to a new output context with only one alternative, i.e., a context which is no longer inquisitive.

The multiple-wh question in (120) involves two types of wh-phrases: the subject is a complex *which*-phrase, which only has a strong interpretation requiring maximality, while the adverb is a simplex wh-phrase, *where*, which permits both a strong and a weak interpretation, with or without maximality. What if we consider a case with two simplex wh-phrases, neither of which necessarily requires maximality? Is it possible to construct a multiple-wh question in this way which permits a 'pure' mention-some reading, i.e., a reading that is mention-some w.r.t. both wh-phrases?

It seems that, in English, this is impossible. To see this, first consider the single-wh questions in (121).

(121)   a.   What is a typical dish in France?

         b.   *Context: Susan is throwing a party and I want to go. I can't drive, but I my friends, who can drive, also want to go. So I ask my friends:*

               Who is driving there tonight?

Note that each of these questions involves a simplex wh-phrase and has a very salient mention-some reading. However, if we add another simplex wh-phrase, as in (122), we do not obtain questions with pure mention-some readings.

(122)   a.   What is a typical dish where?

         b.   *Context: Sue and Kim are both throwing a party, in different locations. I want to go to one of the parties, I don't care which. In either case I need a lift. So I ask my friends:*

               Who is driving where tonight?

Figure 23: The update effect of *which*^{u_1} *herb grows where*$^{u_2}_{weak}$*?*
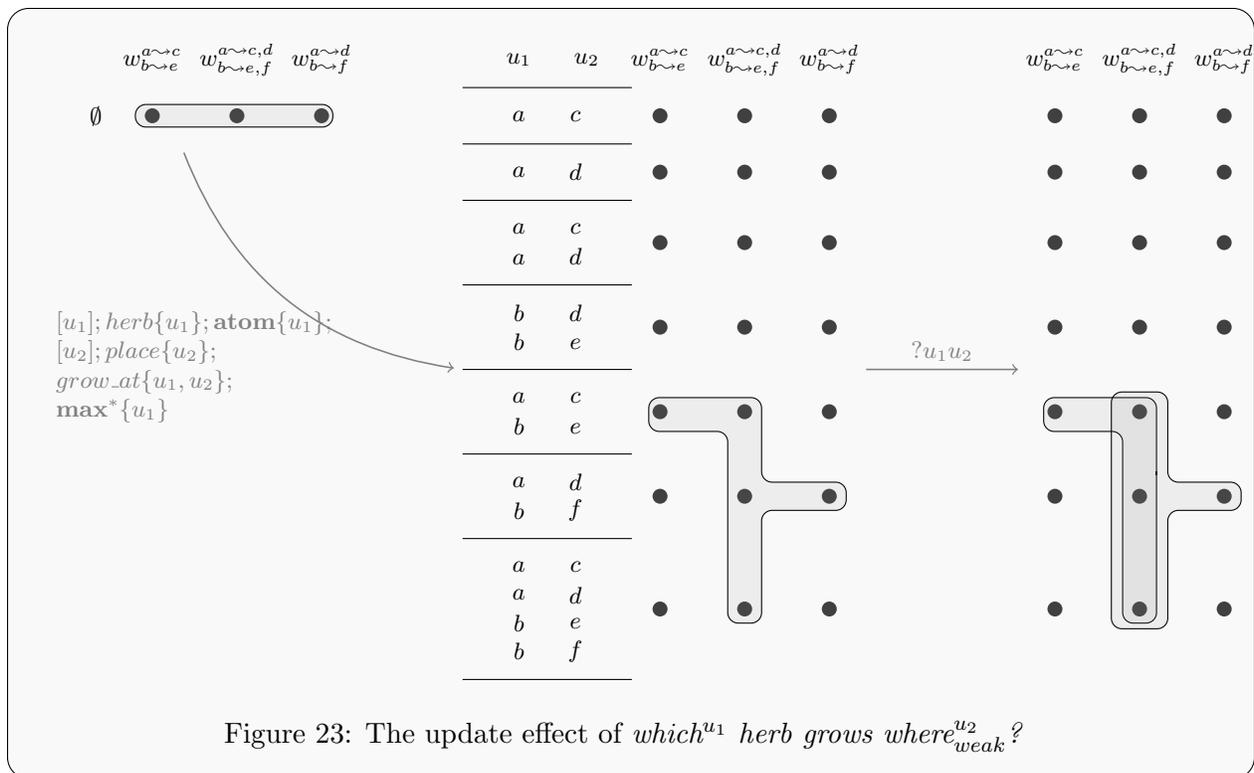
This is predicted by our account, since we assume that Foc applies $\mathbf{max}^*$ to all its domain discourse referents. However, this assumption is otherwise inessential for the account. It is only motivated by the absence of pure mention-some readings in questions like (122a-b).

Interestingly, Lucas Champollion (p.c.) has noted that in German, multiple-wh questions with two simplex wh-phrases do, at least in some cases, permit a pure mention-some reading. An example of such a question is given in (123):

(123)   Wo   kann ich wann Wolfsbarsche fangen?
        Where can   I   when sea bass   catch
        'Where can I catch sea bass? And when?'

This cross-linguistic variation can be captured on our approach if we assume that in German, unlike in English, Foc does not apply $\mathbf{max}^*$ to all its domain discourse referents. This would only affect the predictions of the account concerning questions like (122), all other predictions would remain intact. We leave open here whether this difference between German and English can be tied to other differences between the two languages.

6   Conclusion

We have developed a type-theoretic dynamic inquisitive semantics framework, $\mathsf{Inq_D}$, and have presented a compositional dynamic analysis of wh-questions within this framework. The proposed analysis is based on the assumption that wh-phrases introduce a discourse referent, and an associated operator in the left periphery raises an issue whose resolution requires identifying a witness for this discourse referent, i.e., an individual satisfying all the properties that have been ascribed to it.

We have argued that the analysis improves on previous proposals both in terms of empirical

predictions and in terms of theoretical parsimony. In particular, the analysis derives mention-some and mention-all readings from independently motivated assumptions about wh-phrases, it provides a uniform treatment of single-wh and multiple-wh questions, it derives partial mention-some readings in multiple-wh questions, it captures the effect of singular and plural number marking in *which*-questions, and it predicts a local uniqueness requirement in singular *which*-questions with disjunction. In the Appendix, we compare the analysis in some depth to three other proposals which are, in different ways, related to ours (Haida, 2007; Xiang, 2016, 2021, 2022b; Li, 2019).

There are several natural avenues for further work. One salient open issue concerns the treatment of modal operators in $\mathsf{Inq_D}$. This issue is pertinent because, as we have seen, wh-questions interact with modal operators in interesting ways. A further extension of the present account should explain, for instance, why questions like (124) permit a mention-some reading (Dayal, 2016) and why questions like (125) lack a global uniqueness presupposition (Hirsch and Schwarz, 2019).

(124) *Context: A researcher needs a few people with AB blood type to test a new drug. The study requires her to test multiple patients but not necessarily all the patients in the hospital with AB blood type. The researcher has a list of all patients in the hospital, but not their blood types. The administrator does have information mapping patients to blood types. The researcher asks the administrator:*

Which patients can I approach for this test?

(125) Which letter can be inserted in *fo_m* to make a word?

Other questions that need to be addressed in future work concern the behaviour of various kinds of quantifiers, and relatedly, various types of verbal predicates (collective vs distributive) in wh-questions. For instance, a further extension of the present proposal should ideally leverage insights from earlier work on quantification and predication in dynamic semantics to explain why (126a) permits a reading under which the quantifier takes scope over the question, while (126b) does not (Szabolcsi, 1997, among others), and to derive the intricate resolution conditions of wh-questions with collective predicates like (127) (Xiang, 2021).

(126) a. Which book did every student read?
 b. Which book did fewer than three students read?

(127) Which children formed a team?

Finally, since semantic values in $\mathsf{Inq_D}$ are more fine-grained than in static inquisitive semantics and other static propositional frameworks for question semantics (e.g., Hamblin, 1973; Karttunen, 1977; Groenendijk and Stokhof, 1984), we expect that $\mathsf{Inq_D}$ may be particularly beneficial for the study of phenomena whose analysis has been argued to require sensitivity to sub-propositional semantic structure, including wh-conditionals (Liu 2016; Xiang 2016; Li 2019, see also Appendix A.2), quantificational variability effects (Berman, 1991; Cremers, 2018; Xiang, 2019, 2022a), wh-based free relatives (Xiang, 2021), question embedding under emotive factive verbs (Romero, 2015; Roelofsen, 2019b), so-called 'highlighting-sensitive' discourse particles in questions (Theiler, 2021), and distributivity effects in plural predication and free choice phenomena (Zhao, 2019).

A Related work

In this appendix, we discuss three semantic analyses of questions that are related to ours: the dynamic partition semantics of Haida (2007), the dynamic Hamblin semantics of Li (2019, 2021a,b),

and the categorial approach to questions of Xiang (2016, 2021, 2022a,b). The former two are dynamic theories of questions, just like ours, but they adopt a different static 'baseline' theory: Haida's analysis is based on partition semantics (Groenendijk and Stokhof, 1984), Li's work builds on Hamblin/Karttunen semantics (Hamblin, 1973; Karttunen, 1977). Besides these dynamic approaches, we also discuss Xiang's categorial approach. In this approach, like in the dynamic approaches, semantic values are also more fine-grained than in static propositional theories of questions.

A.1 Haida's dynamic partition semantics

Haida's (2007) dynamic partition semantics, just like the classical static partition semantics of Groenendijk and Stokhof (1982), assumes that a question partitions the logical space into a number of cells, where each cell in the partition corresponds to a possible exhaustive answer to the question. Some questions are indeed naturally thought of as inducing a partition on the logical space. For example, (128a) partitions the logical space into two cells, one corresponding to the proposition that Mary did leave the party and one to the proposition that Mary did not leave the party. Similarly, (128b) partitions the logical space into multiple cells, each corresponding to an exhaustive specification of the set of people who left the party.

(128)   a.   Did Mary leave the party?
         b.   Which people left the party?

A well-known limitation of this approach, however, is that it does not offer a straightforward analysis of wh-questions with a mention-some reading (see, e.g., Belnap, 1982; Ciardelli, 2017). Consider (129).

(129)   Who has a bike that I could borrow?

The issue raised by this question can be resolved by establishing, for instance, that Mary has a bike that I could borrow, or by establishing that Peter does. Crucially, it is not necessary to exclude the possibility that other people might have a bike that I could borrow as well. So the various ways in which this question can be resolved are not mutually exclusive, and therefore do not correspond to the cells of a partition.

Haida's dynamic partition semantics inherits this limitation from static partition semantics. Our proposal on the other hand, since it is based on inquisitive semantics rather than partition semantics, can straightforwardly represent mention-some readings. Furthermore, we have seen that in $\mathsf{Inq_D}$ it is possible to derive mention-some and mention-all readings from independently motivated assumptions. This result would not have been possible in Haida's dynamic partition semantics.

Besides this limitation, which is inherited from the baseline static theory, dynamic partition semantics also faces problems concerning the interaction between questions and anaphora. To illustrate this, we need to briefly sketch how the semantic value of a question is composed on this account. Just like in $\mathsf{Inq_D}$, wh-expressions are interpreted in dynamic partition semantics as introducing discourse referents. The associated interrogative force is contributed by a functional head $C^{[+Q]}$ which Haida assumes to be present in the left periphery of an interrogative clause. The semantic role that Haida attributes to $C^{[+Q]}$ is similar to the semantic role that we ascribe to Foc. Roughly, it requests a witness for the associated discourse referents. A crucial difference, however, is that the semantic operation that $C^{[+Q]}$ expresses on Haida's account is *externally static*. This means that any discourse referent that is introduced within its scope becomes inaccessible for anaphoric reference outside of that scope. This prediction is problematic, since a wh-word can serve as antecedent for a pronoun in the next sentence, as illustrated in (130).

(130)   A: Which man just left?
         B: I don't know his name but I could see that he was angry.

The solution to this problem that Haida proposes relies on the assumption that wh-questions generally carry an existential presupposition. For instance, A's question in (130) presupposes that some man just left, and asks who that man is. On Haida's account it is this existential presupposition that licenses anaphoric reference in subsequent discourse to the man who (is presupposed to have) left. On our account, the relation between wh-expressions and anaphoric pronouns in subsequent discourse is more straightforward. Since Foc and Int are externally dynamic, the discourse referents introduced by wh-expressions remain accessible, and can directly antecede anaphoric pronouns.

Besides being conceptually more complex than necessary, Haida's account of anaphora via the existential presupposition also has an empirical drawback. Namely, it does not extend to polar questions. To see this, consider (131).

(131)   A: Does John have a^{u_1} car?
         B: Yes, but it$_{u_1}$ is broken.

On Haida's account, polar questions involve the same functional head $C^{[+Q]}$ as wh-questions, which makes the discourse referents introduced within its scope inaccessible. Moreover, polar questions clearly do not have an existential presupposition. As a result, it is incorrectly predicted that the pronoun *it* in B's utterance in (131) should not be able to refer to John's car. By contrast, on our account A's question in (131) is taken to introduce two alternatives, one consisting of all possibilities in which John has a car u_1, and one consisting of all possibilities in which John doesn't have a car. B's affirmative response, *yes*, eliminates the second alternative. All possibilities in the remaining alternative assign a value to the discourse referent u_1, so it is unproblematic for this discourse referent to antecede the pronoun *it* in the continuation of B's response.

A.2   Li's dynamic Hamblin semantics

Li (2019, 2021a,b) presents a dynamic Hamblin semantics, and uses this framework to develop an account of Mandarin wh-conditionals (Li, 2019, 2021a) as well complex cases of anaphora with wh-antecedents (Li, 2021b). We illustrate his approach here based on Mandarin wh-conditionals, exemplified in (132).

(132)   Shéi shūle, shéi jiù   qǐngkè.
         who lose   who then pay
         'For every person x, if x loses, then x pays.'

Li treats such constructions as involving two clauses, functioning as the restrictor and the scope, respectively, of a covert universal quantifier. Both clauses contain a wh-element. The semantic role of these wh-elements, according to Li, is to introduce a discourse referent. Further, Li assumes that in the left periphery of both clauses there is an existential disclosure operator (Dekker, 1993), which λ-abstracts over the discourse referents that were introduced by the wh-elements within its scope. Due to this existential disclosure, both the restrictor-clause and the scope-clause denote a property. The covert universal quantifier then says that every individual that satisfies the restrictor-property also satisfies the scope-property.[30]

This approach is similar to ours in two respects. First, in both approaches wh-elements are taken to introduce discourse referents. And second, in both approaches these discourse referents are

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[30]The brief summary of Li's account of wh-conditionals given here glosses over some details that are not relevant for our current purposes. It is only intended as a high-level overview of the account.

targeted by an operator in the left periphery. Li proposes that in Mandarin wh-conditionals, the discourse referents introduced by wh-elements are targeted by an existential disclosure operator. We have proposed that in wh-questions, the discourse referents introduced by wh-elements are targeted by a witness request operator. Thus, the relevant operators are different, but the overall structure is similar.

The most fundamental framework-level difference between Li's approach and ours concerns the way conversational contexts are modelled and the way in which questions are taken to update the context in which they are uttered. On Li's approach, a conversational context is modelled as an *information state*. Such an information state represents information about the world and information about the discourse referents that have been introduced so far. Crucially, however, it does not represent the *issues* that have been raised so far. How, then, is a question taken to update the context in which it is uttered? Li proposes that a question expresses a *non-deterministic* update. Technically, the semantic value of a question is a *set of context updates*. Rather than specifying one specific update, such a set specifies a number of possible updates. By contrast, in $\mathsf{Inq_D}$, contexts are modelled in such a way that they embody both the information that has been established and the issues that have been raised so far, and the semantic value of a question is a deterministic update function, adding a new issue to the context. In short, Li keeps the formal notion of contexts structurally simple (just information, no issues) but has to complicate the formal notion of context updates (allowing both deterministic and non-deterministic updates), while on our approach the formal notion of contexts is structurally more complex (representing both information and issues) but the notion of context updates is simple (only deterministic updates).

One important reason for us to prefer the latter approach is that it allows for a *uniform* semantic treatment of declarative and interrogative clauses. Both types of clauses have the same semantic type—they express a deterministic context update. Such a uniform treatment of declaratives and interrogatives is one of the hallmarks of inquisitive semantics, and various arguments have been made in recent work to highlight its benefits. In particular, a uniform treatment has advantages when one aims to give a semantic account of elements that interact with both types of clauses, such as connectives (Ciardelli *et al.*, 2018), clause-embedding predicates (Theiler *et al.*, 2018; Uegaki, 2019; Roelofsen, 2019a), and certain discourse particles (Theiler, 2021).

There are also differences between the two approaches that do not concern the general architecture of the framework but rather the specific semantic theory of questions that is developed within this framework. One important difference concerns the treatment of wh-expressions as compared to non-wh-indefinites. Even though Li assumes that non-wh-indefinites like *someone* and wh-expressions like *who* both introduce discourse referents, just like we do, he treats these expressions as differing in semantic type. Namely, he takes a non-wh-indefinite to denote a quantifier introducing a discourse referent, while he assumes that a wh-expression denotes a *set* of such quantifiers. This difference in type makes it less evident why wh-expressions are identical in form to non-wh-indefinites in many languages (Ultan, 1978; Haspelmath, 1997; Bhat, 2000; Hengeveld *et al.*, 2021), including Mandarin (see Li 2021a, Section 2, for relevant discussion). Li's treatment of wh-expressions as sets of dynamic quantifiers also has another unwelcome consequence: the restrictor of the wh-expression, unlike that of a non-wh-indefinite, is externally static (see p.444 in Li 2021a for discussion). These issues do not arise in $\mathsf{Inq_D}$, where non-wh-indefinites and wh-expressions are treated on a par, the only difference being that wh-expressions are associated with a witness request operator in the left periphery.

Li's proposal also differs from ours in how it derives maximality and uniqueness effects. Namely, to achieve this, it relies on Dayal's (1996) answerhood operator. As a consequence, it also inherits the challenges that this operator gives rise to: mention-some readings of questions like *Who has a bike that I could borrow?* cannot be derived, and it is not clear how the analysis could handle the

fact that the uniqueness requirement of singular *which*-questions can be obviated by disjunction. As we have seen, mention-some readings and obviation effects under disjunction are straightforwardly derived in $\mathsf{Inq_D}$.

Finally, single-wh questions and multiple-wh questions are not treated uniformly on Li's account. While a single-wh question expresses a set of update functions, a multiple-wh question generates a set of such sets of update functions. In order to reduce such a set of sets to a simple set, Li assumes that multiple wh-questions involve an extra type-shifting operator in the left periphery. Since this type-shifting operator needs to be stipulated only for multiple-wh questions and is not part of the semantic machinery involved in single-wh questions, the approach does not achieve a uniform treatment of single-wh and multiple-wh questions, in contrast to $\mathsf{Inq_D}$.

A.3   Xiang's categorial approach

Xiang (2016, 2021, 2022a,b) pursues a so-called *categorial* approach to questions which, even though it is not dynamic, is still related to our approach in that it uses more fine-grained semantic representations than static proposition-set theories. Namely, building on earlier work by Hausser and Zaefferer (1978), Krifka (2001) and others, Xiang assumes that questions do not denote sets of propositions, but rather functions from individuals to propositions. Moreover, there is considerable overlap in the range of empirical phenomena that Xiang aims to account for and the ones we have been concerned with. Specifically, Xiang's account covers both single-wh and multiple-wh questions, mention-some and mention-all readings, the uniqueness requirement of singular *which*-questions, and the obviation of this requirement in the presence of a modal operator. We cannot provide a comprehensive presentation of the merits and limitations of Xiang's account here. Rather, we will highlight two ways in which it differs from our own approach.

First, even though Xiang's account covers both single-wh and multiple-wh questions, it does not provide a *uniform* analysis of these question types. We will illustrate this on the basis of two concrete examples, (133) and (134), taken from Xiang (2021).

(133)   Which student came?

(134)   Which boy invited which girl?

We will focus on the LF structures that Xiang assumes for these questions. The assumed LF structure for (133) is given in Fig. 24. Note that the wh-phrase is assumed to move out of the TP, leaving a trace x of type e in its base position. This trace x is then abstracted over by a lambda operator which is assumed to be present in the LF structure as a consequence of movement (Heim and Kratzer, 1998).

Xiang further assumes that a type-shifting operator BeDom is adjoined to the wh-phrase that has moved to the left periphery. This BeDom operator takes the wh-phrase *which student* as its argument and returns a function modifier. This modifier turns any function f from individuals to propositions into another function f', whose domain is restricted to students but which is otherwise the same as f. Applying this modifier to the function denoted by the TP (after lambda abstraction) yields the semantic value of the question as a whole. This is a function from individuals to propositions, whose domain is the set of students, and which maps every student x in its domain to the proposition that x came.

The LF structure for the pair-list reading of (134) is given in Fig. 25. For full details and a step-by-step composition, we refer to Xiang (2021). The main point here is that this LF structure includes various elements that are not present in the LF structure assumed for single-wh questions: the $\bigcap$ operator, the ID operator $\lambda q.p = q$ and the lambda-binder of p. Moreover, wh-phrases are
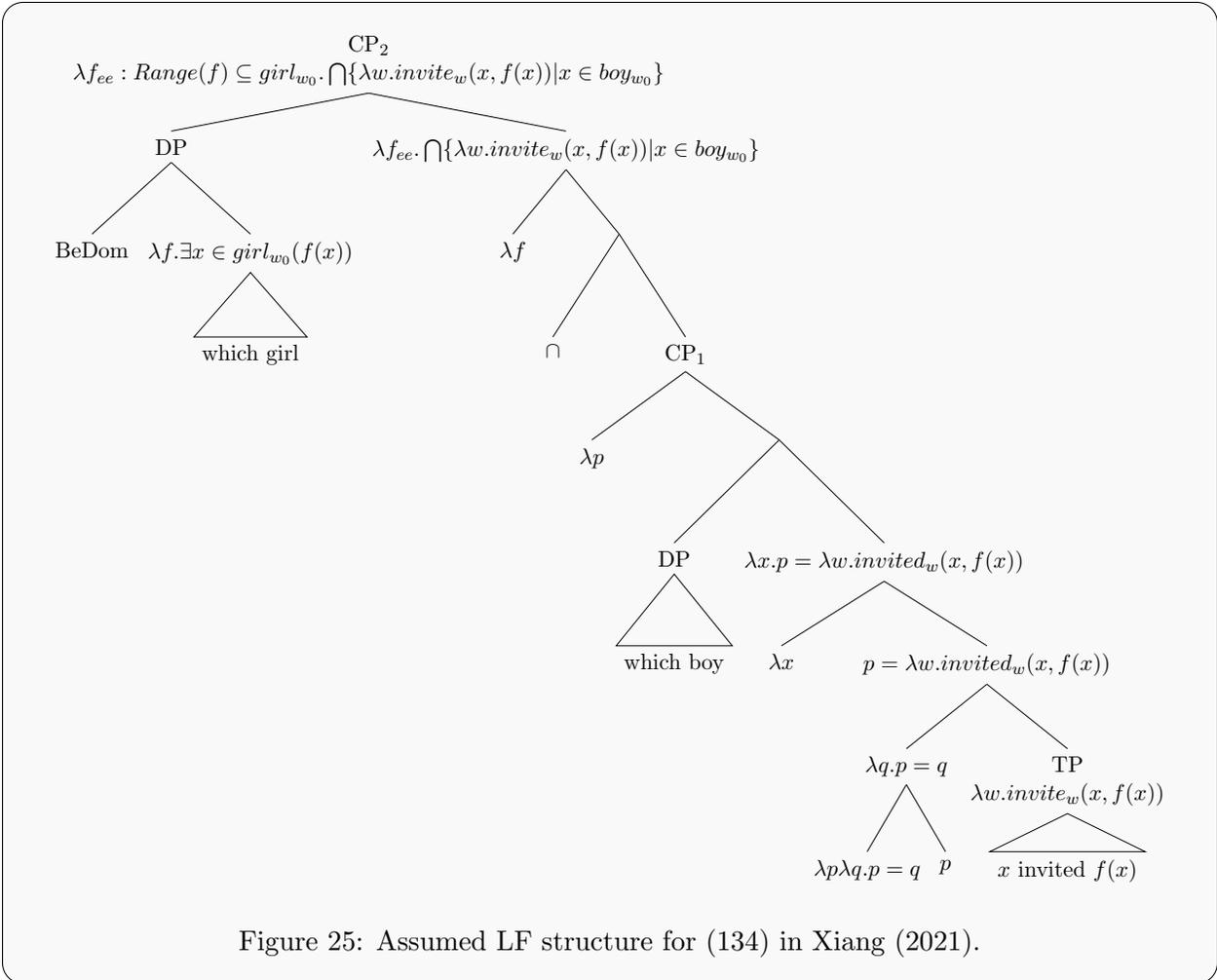
Figure 24: Assumed LF structure for (133) in Xiang (2021).

not interpreted uniformly: the highest wh-phrase in Fig. 25 functions, as a result of the adjoined BeDom operator, as a function modifier, while the lower wh-phrase, which has no adjoined BeDom operator, functions as an existential quantifier. Traces of moved wh-phrases are not interpreted in a uniform way either. In single-wh questions, wh-phrases have as their trace a variable x of type e. The same is true for *which boy* in Fig. 25. However, *which girl* has a complex trace consisting of a function f applied to a variable x, where f is of type (e, e), and x is obligatorily bound by the same lambda operator that binds the variable left by the subject *which boy*. It is not quite clear (to us) how these assumptions could be implemented compositionally.

The question denotations derived for single-wh questions (Fig. 24) and multiple-wh questions (Fig. 25) also differ in semantic type, a result that is not specific to Xiang's account, but is obtained in all categorial approaches. This type mismatch is problematic for the treatment of question coordination, question entailment, and question embedding (see, e.g., Groenendijk and Stokhof, 1984). Xiang (2021) addresses this fundamental issue for categorial approaches but leaves it partially unresolved (see Xiang, 2021, footnote 34).

Finally, note that the derivation of multiple-wh questions (Fig. 25) requires covert movement of the dependent wh-phrase—in our example *which girl*—to the left periphery, even though at the surface this wh-phrase appears in-situ. This is problematic given that in multiple-wh questions, the in-situ wh-phrase can be inside an island, which prevents covert movement to the left periphery (see example (116) above from Reinhart 1997).

In contrast to Xiang's categorial approach, our dynamic account treats all wh-phrases equally, no matter whether they appear in single-wh or in multiple-wh questions. They are always taken to introduce a discourse referent, bound by a witness request operator in the left periphery. The LFs of single-wh and multiple-wh questions involve exactly the same two functional heads left periphery, one contributing a witness request operator and the other a presuppositional closure operator. No additional LF operations absent in single-wh questions are needed to capture pair-list readings of multiple-wh questions. Finally, since wh-phrases are not required to move to the left periphery at LF, the account is compatible with the observation that in-situ wh-phrases can appear inside islands. In fact, the account predicts that in-situ wh-phrases are subject to constrains on anaphora

$$CP_2$$
$$\lambda f_{ee} : Range(f) \subseteq girl_{w_0}. \bigcap\{\lambda w.invite_w(x, f(x))|x \in boy_{w_0}\}$$

DP

$$\lambda f_{ee}. \bigcap\{\lambda w.invite_w(x, f(x))|x \in boy_{w_0}\}$$

BeDom $\quad \lambda f.\exists x \in girl_{w_0}(f(x))$

λf

which girl

$\bigcap$

$$CP_1$$

λp

DP

$$\lambda x.p = \lambda w.invited_w(x, f(x))$$

which boy $\quad \lambda x$

$$p = \lambda w.invited_w(x, f(x))$$

$\lambda q.p = q$

TP
$$\lambda w.invite_w(x, f(x))$$

$\lambda p \lambda q.p = q \quad p$

x invited $f(x)$

Figure 25: Assumed LF structure for (134) in Xiang (2021).

rather than constraints on movement. For instance, if a wh-phrase is in the semantic scope of negation or another externally static operator, the discourse referent that it introduces is predicted to be inaccessible for the witness request operator in the left periphery. For evidence supporting this prediction we refer to Haida (2007) and Haida and Repp (2013).

Returning now to Xiang's categorial approach, besides multiple-wh questions several other empirical phenomena are addressed by this approach as well. In particular, mention-some readings and the uniqueness requirement of singular *which*-questions have received close attention (see in particular Xiang, 2022b). In analysing these phenomena, Xiang builds on Dayal (1996, 2016), who crucially assumes an answerhood/maximality operator in the left periphery. This answerhood/maximality operator, however, is too strong in two respects. First, it only derives mention-all readings, leaving mention-some readings unaccounted for (Fox, 2013). And second, it cannot account for the obviation of uniqueness effects in singular *which*-questions with a modal operator or disjunction (Hirsch and Schwarz, 2019; Socolof *et al.*, 2020). In order to overcome these problems for Dayal's approach, Xiang (2022b) proposes a weaker version of the answerhood/maximality operator, one which evaluates maximality not necessarily w.r.t. the actual world, but possibly w.r.t. modally accessible worlds. This weaker version of Dayal's answerhood/maximality operator overcomes the limitations of the original operator to some extent. Namely, in the presence of a weak modal operator, mention-some

readings can be derived and the obviation of uniqueness effects is predicted. In cases without a modal operator, however, mention-all readings and uniqueness effects are still incorrectly predicted to be obligatory. For instance, the mention-some reading of (135) is not derivable, and the uniqueness obviation in singular *which*-questions with disjunction such as (136) remains unexpected on this account.

(135)   What is a typical French dish?

(136)   In which town was Shakespeare born or did Bach die?

By contrast, as we have seen, our proposal does derive the mention-some reading of (135) and uniqueness obviation in (136). Whether it can be extended to account for uniqueness obviation under modal operators is a question we must leave for future work.

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