

# Comparative Vagueness\*

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## 1 Comparative vagueness

Positive-form predications ('is tall', 'is bald') reign supreme in discussions of linguistic vagueness. Despite the vast literature, little attention has been given to comparatives. It is often assumed in linguistic circles that explicit comparatives—comparatives 'x is ADJ-er/more ADJ than y' with a comparative morpheme—are not vague. For instance (see also [COOPER 1995](#): 246; [KENNEDY 2007](#): 6, [2011](#): 74, 82–83, 93, [2013](#): 271; [MCNALLY 2011](#): 164n.10; [VAN ROOIJ 2011a](#): 65–69):

[T]he comparative form ... is not vague... [A] core semantic difference between the positive and comparative forms ... is that the latter lacks whatever semantic (or pragmatic) features give rise to the vagueness of the former, and simply expresses an asymmetric ordering relation. ([KENNEDY 2013](#): 269–270)

[A]djectives in the comparative are uniformly non-vague. ([BOCHNAK 2013](#): 42)

Such remarks are generally made in passing in view of “prototypical relative adjectives” ([MCNALLY 2011](#): 163) such as 'tall'. Here again is Kennedy:

The comparative predicate *taller than David* ... denotes a property that is true of an object just in case its height exceeds David's height. This is a precise property..., since whether it holds of an object or not is fully determined by facts about that object's height. ([KENNEDY 2013](#): 270)

Vagueness is understood as deriving from fuzziness in standards of application—how many millimeters of height one must have to count as tall, how many cents one

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must have to count as rich, and so on. Hence, “Unsurprisingly, comparatives ... do not give rise to the Sorites paradox, and do not have borderline cases” (McNALLY 2011: 164n.10).

But they do.

Suppose you must decide between saving your dearest friend and saving some number of strangers. Plausibly we have some special obligations to those close to us, so that it is morally better for you to save your dear friend than to save two strangers. But there doesn't seem to be any precise number of strangers that would tip the balance. Consider:

- (1) (P1) Your saving 2 strangers is not morally better than your saving your dear friend.
- (P2) For all  $n$ , if your saving  $n$  strangers is not morally better than your saving your dear friend, then your saving  $n + 1$  strangers is not morally better than your saving your dear friend.
- (C)  $\therefore$  For all  $n > 2$ , your saving  $n$  strangers is not morally better than your saving your dear friend.

No one's friends are that important.

Or suppose you like sugar in your coffee. Yet it's not as if you care exactly how sweet it is. As far as your preferences go, one day's sweetness is as good as any other (okay, at least up to a point, say  $K$ ; there is, perhaps, such a thing as too sweet). Consider (2) — where  $x_s$  is an ordinary cup of coffee, and  $x_1 \dots x_n \dots x_K$  is a series of otherwise identical cups differing only in quantity of sugar, with  $x_n$  being a (pre- $K$ ) cup with  $n$  micrograms of sugar (cf. LUCE 1956).

- (2) (P1)  $x_s$  is more preferable than  $x_1$ .
- (P2') For all  $n < K$ ,  $x_n$  is as preferable as  $x_{n+1}$ .
- (P3) For all  $a, b, c$ , if  $a$  is more preferable than  $b$ , and  $b$  is as preferable as  $c$ , then  $a$  is more preferable than  $c$ . (*PI-transitivity*)
- (C)  $\therefore$  For all  $n < K$ ,  $x_s$  is more preferable than  $x_n$ .

Or in a perhaps more familiar form:

- (3) (P1)  $x_1$  is not more preferable than  $x_s$ .
- (P2) For all  $n$ , if  $x_n$  is not more preferable than  $x_s$ , then  $x_{n+1}$  is not more preferable than  $x_s$ .
- (C)  $\therefore$  For all  $n$ ,  $x_n$  is not more preferable than  $x_s$ .

But not just any cuppa can be the best.

That is: The premises seem true — in (1), given the nature of morality; in (2)/(3), given the nature of one’s preferences. The arguments seem valid. Yet the conclusions are false. There may be something wrong with sugar in one’s coffee, but not that thinking otherwise leads to paradox.<sup>1</sup>

Unlike previous examples of comparative vagueness, (1)–(3) cannot be reduced to cases of indeterminacy (uncertainty, indecision) regarding what dimensions are relevant or unsettledness (imprecision) about measurement procedures.<sup>2</sup> Fix on a particular dimension for moral value or preferability and a basis for determining it, and the force of the comparative sorites remains. Many a monistic indirect consequentialist have countenanced special obligations. Moreover the concern with denying the inductive premises isn’t simply that doing so would be unwarranted or in tension with limited powers of discrimination. As WRIGHT (1987: 239–243) shows, indiscriminability between adjacent items in a sorites series is insufficient to generate the paradox. Only a maximally opinionated coffee maven could deny (P2’) in (2). Discriminable though adjacent cups might be, whether in quantity of sugar or quality of sweetness, one cup is as good as the next given one’s preferences.

Upshot: Linguistic vagueness can be associated not only with how ADJ something needs to be to count as ADJ, but with how ADJ things are.

## 2 Semantics for gradation

It’s common to locate the problem in sorites arguments with positive-form predicates in the inductive premise. (Let  $x_n$  be someone who is  $4' + n$  nanometers tall.)

- (4) (P1) Someone who is  $4'$  isn’t tall (for a pro basketball player).  
(P2) If someone who isn’t tall (for a pro basketball player) grows one nanometer, they still won’t be tall (for a pro basketball player).  
(C)  $\therefore$  No one is tall (for a pro basketball player).
- (5) (P1)  $x_0$  is not tall.

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<sup>1</sup>What if the moral facts or your preferences are such that one stranger or microgram of sugar really does tip the balance? No matter — for purposes of the semantics. Consider Pat, who thinks otherwise and doesn’t care about differences between adjacent cups, and read ‘morally good/better’ as ‘morally good/better according to Pat’ and ‘preferable’ as ‘preferable to Pat’. Any compositional semantics will need to be able to characterize such normative views and states of mind so as to make sense of such explicit relativizations (cf. SWANSON 2011: 696–697).

<sup>2</sup>Contrast WILLIAMSON 1994: 156; ENDICOTT 2000: 43–45, 149–153; KEEFE 2000: 13–14; SASSOON 2013: 76, 172–173. Endicott and Sassoon mention borderline cases arising from multidimensionality; Williamson and Keefe also appeal to issues with measurement procedures. It isn’t said how such cases might give rise to comparative sorites arguments.

- (P2) For all  $n$ , if  $x_n$  is not tall, then  $x_{n+1}$  is not tall.  
 (C)  $\therefore$  For all  $n$ ,  $x_n$  is not tall.

For instance, even if we can't point to any instance of (P2) in (5) that isn't true, perhaps we can know that it isn't true in any context (SOAMES 1999, FARA 2000), or no matter what formally precise language we might be speaking (LEWIS 1970), or no matter how the conversation might evolve (SHAPIRO 2006), or on any competent way of applying 'tall' (KAMP 1981, RAFFMAN 2014). Why not say the same about the generalizations in (1)–(3)?

Comparative sorites arguments such as (1)–(3) raise a distinctive challenge for traditional formal semantics for gradation. Let's start by examining how the arguments would be formalized in two prominent degree-based and non-degree-based frameworks.

## 2.1 Degree-based semantics

A prominent approach is to treat gradable adjectives — adjectives that can figure in comparatives and take degree modifiers — as associating items (individuals, propositions) with degrees on a scale (BARTSCH & VENNEMANN 1973, VON STECHOW 1984, KENNEDY 1999, 2007, HEIM 2001, MORZYCKI 2015). For instance, on a Kennedy-style measure-function implementation, 'tall' denotes a function *tall* from individuals to degrees of height, i.e. the individual's maximal height; 'hot' denotes a function *hot* from individuals to degrees of temperature, i.e. the individual's maximal temperature. Though many theories assume that degrees are isomorphic to rational numbers, a minimal constraint is that the relation  $\geq$  on the set of degrees  $D$  have at least the structure of a partial order, i.e. that  $\geq$  be a reflexive, transitive, antisymmetric relation on  $D$  (KENNEDY 1999, 2007, BARKER 2002, LASSITER 2015). Compositional details aside, the comparative (6) says that the (maximal) degree to which Alice is tall is greater than the (maximal) degree to which Bert is tall ((7)).

- (6) Alice is taller than Bert.  
 (7) (6) is true iff  $tall(Alice) > tall(Bert)$

Recall the comparative sorites argument in (2). The above semantics renders the interpretation of (P3) as in (8),<sup>3</sup> where  $pref$  ( $= \llbracket \text{preferable} \rrbracket$ ) is a function from items to their degree of preferability, a representation of how preferable they are.<sup>4</sup>

$$(8) \quad \forall x \forall y \forall z: ((pref(x) > pref(y)) \wedge (pref(y) = pref(z))) \rightarrow (pref(x) > pref(z))$$

(P3) is an instance of what is sometimes called *PI-transitivity*, which is a weakening of transitivity — i.e., if a relation  $\succeq$  satisfies transitivity, PI-transitivity in (9) is also satisfied, where  $>$  and  $\sim$  are the strict and non-strict parts, respectively, of  $\succeq$ . So (P3) follows from the transitivity of the relation  $\geq$  on the domain of degrees  $D$ .

$$(9) \quad \text{PI-transitivity} \\ (X > Y \wedge Y \sim Z) \rightarrow X > Z$$

Upshot: The premise (P1) is true. (P2') in (2) describes your non-obsessiveness about coffee sweetness; one cup is as good as the next given your preferences. (P3) is entailed by the general structure of scales and thus holds with any adjective denotation. (P2) in (3) encodes these dual properties. Yet the conclusion (C) is false. Hence the paradox.

## 2.2 Delineation semantics

Let's turn to the other main approach to gradation in formal semantics: *delineation semantics* (“partial predicate,” “inherent vagueness” semantics). Delineation theories treat gradable adjectives like non-gradable adjectives (‘digital’) as ordinary predicates. What distinguishes gradable adjectives is that they are sensitive to a contextual comparison class (KLEIN 1980, VON STECHOW 1984, BURNETT 2012). In one context using (10) might express that Alice is tall for a basketball player; in another context it might express that she is tall for an American woman. Gradable adjectives denote partial functions partitioning a comparison class  $CC$  into a positive extension, a negative extension, and an extension gap (the “borderline cases”).<sup>5</sup>

<sup>3</sup>I assume an “equally good” reading of the equative (cf. BHATT & PANCHEVA 2007, RETT 2008).

<sup>4</sup>I use ‘measure function’ broadly not only for adjectives associated with measurement procedures or numerical units of measurement (e.g. height in inches), but for any mapping which would determine an order on objects. What is important about degrees is that they represent assessments of how preferable, tall, etc. things are, and thus that they can be associated with qualitative orderings on the items in adjectives’ domains. Nothing of metaphysical significance is presupposed by talk of things having “degrees” of preferability, value, etc.

<sup>5</sup>Some theories also invoke a parameter  $\delta$  for relevant standards (LEWIS 1970, BARKER 2002), e.g. where the positive extension of ‘tall’ is those individuals in  $CC$  whose height is at least the standard of tallness  $\delta_{tall}$ .

(10) Alice is tall.

(11)  $\llbracket \text{tall} \rrbracket^{CC} = \lambda x: \neg \text{gap}_{CC}(\text{tall})(x) . x$  is tall in  $CC$

A positive-form predication such as (10) is true given a comparison class  $CC$  iff Alice counts as tall in  $CC$ . Following KLEIN 1980, a comparative such as (6) is true (given any comparison class) iff there is some  $CC'$  such that Alice is tall in  $CC'$  and Bert is not tall in  $CC'$ .

To avoid problematic entailments, delineation theories impose qualitative restrictions on comparisons among individuals across comparison classes (FINE 1975, KLEIN 1980, FARA 2000). For instance, if  $x$  counts as tall in some  $CC$  and  $x$  has a greater height than  $y$ , then there is no  $CC'$  in which  $y$  counts as tall and  $x$  doesn't. Delineation theorists have proven that the qualitative restrictions derive a preorder (reflexive, transitive relation)  $\succeq_A$  "at least as ADJ as" on the set of individuals in the domain of 'ADJ', for any adjective 'ADJ' (VAN BENTHEM 1982, KLEIN 1991, VAN ROOIJ 2011a). Degrees and scales may then be derived from these qualitative orderings (CRESSWELL 1977, BALE 2008): the set of degrees  $D$  is the set of equivalence classes under  $\succeq_A$ ; and the relation  $\geq_A$  on  $D$  is defined accordingly such that  $[x]_A \geq_A [y]_A := x \succeq_A y$  (where  $[a]_A$  is an equivalence class  $\{b: b \succeq_A a \wedge a \succeq_A b\}$ ). Upshot: The interpretation of any adjective relies on a preorder  $\succeq_A$  on the set of individuals. The transitivity of  $\succeq_A$  again validates (P3), and the paradox is off and running.

### 2.3 Stock

We noted above that many accounts of vagueness locate the problem with sorites arguments such as (5) in the inductive premise. There is an important difference between the positive-form and comparative-form inductive premises. Consider the degree semantics from §2.1. The positive form is treated as relating a degree to a relevant threshold, or *degree standard*. To a first approximation, (10) is true iff the degree to which Alice is tall,  $\text{tall}(\text{Alice})$ , is at least the degree standard of tallness  $s(\text{tall})$ , i.e. how tall one must be to count as tall.<sup>6</sup> The predicted truth-condition in (12) for (P2) from (5) is false at any point of evaluation; the challenge for theories of vagueness is to explain why speakers nevertheless find it compelling. For instance, perhaps the falsifying instance is never where one is looking (cf. FARA 2000, KENNEDY 2007); or there is uncertainty about which standard is determined

<sup>6</sup>Many degree theories derive the positive form by combining the adjective with a null morpheme *pos* to yield a predicate of individuals (VON STECHOW 1984, KENNEDY 1999). I follow KENNEDY 2007 in treating the variable  $s$  as a function from adjective denotations (measure functions) to degree standards.

by the conversational situation (cf. BARKER 2002); or the speakers are undecided about what standard to accept for purposes of conversation (cf. SILK 2016).

(12) *IND-PRED*

$$\forall n: (\text{tall}(x_n) \not\geq s(\text{tall})) \rightarrow (\text{tall}(x_{n+1}) \not\geq s(\text{tall}))$$

In contrast, the traditional frameworks for gradation in §§2.1–2.2 treat the inductive premise (P3) in (3) as necessarily *true*—in virtue of the basic structure of scales  $\langle D, \geq \rangle$ , per degree-based semantics, or the qualitative ordering  $\succeq_A$  on the set of individuals, per delineation semantics. That leaves (P2'). Yet (P2') would also seem to be true (albeit contingently), given one's preferences. Indeed even a supertaster could accept (13); one simply doesn't care exactly how sweet the coffee is. The non-arbitrariness of morality would seem to straight-up imply (14).

(13) *IND-COMP*

$$\forall n: \text{pref}(x_n) = \text{pref}(x_{n+1})$$

(14)  $\forall n: (\text{morally-good}(\text{save-}n) \not\geq \text{morally-good}(\text{save-friend}))$

$$\rightarrow (\text{morally-good}(\text{save-}n+1) \not\geq \text{morally-good}(\text{save-friend}))$$

It would be surprising if one could rebut fans of special obligations or sugar-taking coffee drinkers with facts about semantic scale structure.

The degree-standard variable  $s$  provides a natural locus for capturing felt vagueness of positive-form predications. Suppose we fix on a comparison class and procedure for measuring height. Still we may not be willing commit to any specific degree of height as how tall one must be to count as tall. What we need is a formal framework that affords an analogous basis for the felt fuzziness of comparative relations such as those in (13)–(14). Tendentiously put, we need a semantics for gradation that avoids validating generalizations such as IND-COMP as a matter of conventional meaning, and allows for a certain kind of intransitivity with gradable expressions.

Some theories of linguistic vagueness offer uniform accounts of vagueness phenomena across syntactic categories ('heap', 'quietly', etc.). (Consider the general frameworks of many-valued logics or supervaluations, or the general metasemantic claims of epistemicists about imperfect knowledge of meanings.) Such apparatus could be applied to comparatives. One might treat the formal semantics as supervaluating over numerical measure functions (SASSOON 2013), where different measure functions provide different counterinstances of the inductive premises. Or one might posit a discourse-level standard of precision/granularity (cf. LEWIS 1979,

SAUERLAND & STATEVA 2007, MORZYCKI 2015), and restrict the evaluation of arguments to “admissible” contexts in which the adjective’s measure function isn’t less “opinionated” than the measures of relevant subvening properties (e.g., quantity of sugar, quality of sweetness). However, a growing body of linguistic work has stressed the importance of distinguishing sources of apparent vagueness phenomena, e.g. by distinguishing kinds of vagueness (SAUERLAND & STATEVA 2007) or by distinguishing vagueness from imprecision or “loose talk” (LASERSOHN 1999, KENNEDY 2007, MORZYCKI 2015). Given the prominence of capturing at least some instances of linguistic vagueness in terms of semantic gradability, in what follows I wish to explore a more local approach that revises the formal semantics of gradation. I leave investigation of apparent vagueness phenomena with non-gradable expressions for elsewhere.

### 3 Degrees and scale structure. Semiorders in a degree semantics

To fix ideas let’s assume a Kennedy-style degree-based framework. §2.1 considered a traditional degree semantics on which scales  $\langle D, \geq \rangle$  impose a relation  $\geq$  with at least the structure of a partial order on the set of degrees  $D$ . I suggest that we reconsider the assumptions about scale structure, and treat the set of degrees as coming with a *semiorder*.<sup>7</sup> Adjective denotations may still associate items with degrees, conceived as points on a scale; yet a scale is now a structure  $\langle D, > \rangle$ , where  $>$  is a semiorder on  $D$ . Semiorders have been used fruitfully in measurement theory and choice theory for modeling intransitive indifferences. The broader research on semiorders provides an independently motivated resource to incorporate into the semantics of gradation.

Formally, a semiorder  $>$  is an interval order that satisfies semitransitivity ((15)). Equivalently,  $>$  is a semiorder iff there is a real-valued function  $f$  such that  $x > y$  iff  $f(x) > f(y) + \epsilon$ , for some fixed positive number  $\epsilon$  (LUCE 1956, SCOTT & SUPPES 1958, FISHBURN 1985, VAN ROOIJ 2011b).

- (15) *Irreflexivity*:  $\forall x: x \not> x$   
*Interval-order*:  $\forall x, y, z, w: (x > y \wedge z > w) \rightarrow (x > w \vee z > y)$   
*Semitransitivity*:  $\forall x, y, z, w: (x > y \wedge y > z) \rightarrow (x > w \vee w > z)$

A relation  $\sim$  can be defined from  $>$ , where  $x \sim y := x \not> y \wedge y \not> x$ . Crucially, unlike the non-strict part of a partial or total order, although  $\sim$  is reflexive and symmetric, it needn’t be transitive.

<sup>7</sup>Alternatively one might revise the representation of degrees, e.g. treating adjectives as associating individuals with sets of points, perhaps intervals (cf. KENNEDY 1999, SOLT 2014).

One way of interpreting the formalism is to understand  $f$  as mapping degrees of ADJ-ness to measures of a property on which ADJ-ness may (possibly trivially) supervene — e.g., mapping degrees of preferability to measures of sweetness. The value  $\epsilon$  can be understood as representing a threshold of distinguishability with respect to the property associated with the adjective (written  $\epsilon_A$ ). Intuitively, the greater the value of  $\epsilon_A$ , the less distinguishing in matters of ADJ-ness. Truth-conditions for the comparative and equative are in (16)–(17), where  $adj$  is the measure function denoted by ‘ADJ’ (n. 3).

- (16) ‘ $a$  is ADJ-er than  $b$ ’ is true  
iff  $adj(a) >_A adj(b)$   
iff  $f_A(adj(a)) > f_A(adj(b)) + \epsilon_A$
- (17) ‘ $a$  is as ADJ as  $b$ ’ is true  
iff  $adj(a) \sim_A adj(b)$   
iff  $|f_A(adj(a)) - f_A(adj(b))| \leq \epsilon_A$

Take ‘preferable’. If quantity of sugar was the sole property determining preferability,  $\epsilon_p$  would represent a level of sugar sufficing to distinguish items in how preferable they are. In such a scenario,  $pref(a) >_p pref(b)$  if the difference in sugar between  $a$  and  $b$  is greater than  $\epsilon_p$ , rendering  $a$  more preferable than  $b$ ; and  $pref(a) \sim_p pref(b)$  if the difference is less than  $\epsilon_p$ , failing to distinguish  $a$  and  $b$  in preferability.

One shouldn’t be misled by the numerical values in the definition of semiorders. A relation  $>$  is a semiorder only if *there is* a function  $f$  and number  $\epsilon$  such that  $x > y$  iff  $f(x) > f(y) + \epsilon$ . As noted in §2.1, degrees needn’t be isomorphic to numbers, and properties of ADJ-ness needn’t be quantifiable. Talk of the numerical relation between  $f_A(adj(a))$  and  $f_A(adj(b))$  is compatible with  $a$  and  $b$  being as ADJ as one another, imperceptibly different in ADJ-ness, or even incomparable.

The threshold  $\epsilon_A$  is used in representing items that do/don’t count as relevantly distinguishable in the context. The notion of (in)distinguishability here is specific to matters of ADJ-ness. Being discriminable in some respect doesn’t imply being “distinguishable,” in the sense of being related by  $>_A$ . Conversely, the fact that  $adj(a) \sim_A adj(b)$  — that  $a$ ’s and  $b$ ’s difference in ADJ-ness doesn’t exceed the threshold of distinguishability  $\epsilon_A$  — doesn’t imply that  $a$  and  $b$  are indiscriminable, either in general or in properties relevant to determining how ADJ they are. Per §2.3, a supertaster might be able to discriminate between adjacent coffee cups in their quantity of sugar or quality of sweetness. Saying that the items’ degrees are related by  $\sim_p$  is to say that any such difference fails to constitute a relevant distinction in

preferability. The act of saving 2 strangers at the expense of your dear friend might be discriminable in (say) utility from the act of saving 3 strangers at the expense of your dear friend. Such a difference may or may not constitute a discriminable difference in moral value.

Let's apply the semantics to the comparative sorites arguments from §1. The truth-conditions for (P2') and (P3) in (2) with 'preferable' are as follows:

- (18) (P2') (in (2)) is true  
iff  $\forall n: \text{pref}(x_n) \sim_P \text{pref}(x_{n+1})$   
iff  $\forall n: |f_P(\text{pref}(x_n)) - f_P(\text{pref}(x_{n+1}))| \leq \epsilon_P$
- (19) (P3) (in (2)) is true  
iff  $\forall a \forall b \forall c: ((\text{pref}(a) >_P \text{pref}(b)) \wedge (\text{pref}(b) \sim_P \text{pref}(c)))$   
 $\rightarrow \text{pref}(a) >_P \text{pref}(c)$

The semantics in (18) captures the truth of (P2'). Every pair of adjacent cups is related by  $\sim_P$ ; discriminable though they might be, one cup is as good as the next given your preferences. However, the PI-transitivity condition in (19) is violated. The counterinstance occurs at the cup  $x_i$  such that  $f_P(\text{pref}(x_s)) - f_P(\text{pref}(x_{i+1})) = \epsilon_P$  (where  $x_s$  is again an ordinary sweetened cup of coffee).  $x_s$  is more preferable than  $x_i$ , since  $f_P(\text{pref}(x_s)) > f_P(\text{pref}(x_i)) + \epsilon_P$ ; yet it's not the case that  $x_s$  is more preferable than  $x_{i+1}$ . The difference between  $x_s$  and  $x_{i+1}$  isn't sufficient to render  $x_s$  more preferable. So, we can accept (i) that  $\text{pref}(x_s) >_P \text{pref}(x_1)$ , i.e. that  $x_s$  is more preferable than  $x_1$ ; (ii) that  $\text{pref}(x_1) \sim_P \text{pref}(x_2) \wedge \dots \wedge \text{pref}(x_{s-1}) \sim_P \text{pref}(x_s)$ , i.e. that adjacent cups aren't distinguished in preferability; and yet, due to the intransitivity of  $\sim_P$ , (iii) that  $\exists x_j: \text{pref}(x_s) \not>_P \text{pref}(x_j)$ , i.e. that there is a cup which  $x_s$  isn't preferable to.

The semantics also avoids validating premises of the form of (P2) in (1)/(3). The counterinstance for (20) is at the act *save-i* such that  $f_A(\text{morally-good}(\text{save-friend})) - f_A(\text{morally-good}(\text{save-}i+1)) = \epsilon_M$ .

- (20) (P2) (in (1)) is true  
iff  $\forall n: \text{morally-good}(\text{save-}n) \not> \text{morally-good}(\text{save-friend})$   
 $\rightarrow \text{morally-good}(\text{save-}n+1) \not> \text{morally-good}(\text{save-friend})$

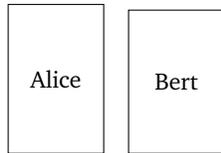
The falsity of the inductive premise is compatible with it being the case that, for any  $n$ ,  $\text{morally-good}(\text{save-}n) \sim_M \text{morally-good}(\text{save-}n+1)$ , i.e. that the (moral) difference between you saving  $n$  strangers over your friend and you saving  $n+1$  strangers over your friend is insufficient to relevantly morally distinguish them in the context.

As FARA (2000) emphasizes in discussing predicative sorites arguments, predicting that the inductive premise is not true doesn't suffice for an overall account of the sorites. If the inductive premise isn't true, why do we find it plausible? What should we say about the seemingly predicted "sharp boundary" between (e.g.) cups that aren't more preferable than  $x_s$  and cups that are? This isn't the place to hazard a general theory of the semantics, epistemology, psychology of vagueness; yet several directions for approaching such questions in the present framework are as follows.

First, the intransitivity of the non-distinguishability relation  $\sim$  provides a locus for some of the sorites' intuitive appeal. Though the formal semantics doesn't verify (P2) in (1)/(3) or PI-transitivity (P3) in (2), it verifies the related claims expressing that adjacent items are relevantly non-distinguishable in preferability, moral value, etc. Second, the distinguishability threshold  $\epsilon_A$  locates a place for importing ideas from broader theories of vagueness (epistemicism, contextualism, supervaluationism). In an epistemicist theory (SORENSEN 1988, WILLIAMSON 1994), facts about competent use across contexts may determine a specific value of  $\epsilon_A$ . Apparent fuzziness in the distinguishability threshold could be diagnosed as uncertainty about what precise language is being spoken. Alternatively, on a broadly contextualist line, the distinguishability threshold may be treated as a contextual parameter, where different contexts determine different levels of distinguishability. For the maximally discriminating and opinionated among us, context may supply a value of  $\epsilon_{A_c} = 0$ ; no difference in properties relevant to determining how ADJ things are goes undetected or uncared-for in matters of ADJ-ness. For the rest of us, context supplies  $\epsilon_{A_c} > 0$  and the comparative sorites is off and running. Even if the compositional semantics takes as given a particular value for  $\epsilon_{A_c}$ , there may be a range of live representations of context and values for  $\epsilon_{A_c}$  compatible with speakers' interests (FARA 2000), psychological states or verbal dispositions (RAFFMAN 1996), or discourse moves (SOAMES 1999, SHAPIRO 2006, SILK 2016, 2019). We may not be able to point to any instance of (P2)/(P2') we reject, or any instance of the sharp boundaries claim we accept.

Previous appeals to semiorders in treatments of vagueness focus on predicative uses and the positive form (LUCE 1956, HALPERN 2008, VAN ROOIJ 2011a,b). To my knowledge, the only precedent in linguistic semantics for invoking semiorders with comparatives is the delineation semantics in VAN ROOIJ 2011a for "implicit" comparatives (KENNEDY 2011) — sentences 'x is ADJ compared to y' in which a comparison is made using the positive form. Unlike with explicit comparatives, which use a comparative morpheme, the truth of implicit comparatives requires that there be a significant difference between the items being compared, as in (21).

(21)



- a. Alice is taller than Bert. (true)  
b. Alice is tall compared to Bert. (false)

VAN ROOIJ (2011a) uses semiorders to capture this “significantly ADJ-er than” relation in the interpretation of implicit comparatives. Explicit comparatives are analyzed via weak orders, as usual (§2.2); van Rooij denies that they are vague. By contrast, the semantics in this section invokes semiorders in the general scale structure, and allows for vagueness phenomena with both positive and comparative forms. Proceeding in this way is compatible with acknowledging contrasts between implicit and explicit comparatives. The semiorder on  $D$  represents a relation of relevant distinguishability in matters of ADJ-ness. The distinguishable difference in height required for the truth of (21a) needn’t be “significant” so as to verify (21b).

Let’s recap. Narrow focus on adjectives such as ‘tall’ has led various theorists to assume that the comparative form cannot be vague (§1). Yet comparative sorites arguments such as (1)–(3) illustrate that linguistic vagueness cannot be wholly traceable to features specific to the positive form. Vagueness can be associated not only with how ADJ something must be to count as ADJ, but with how ADJ things are (SILK 2016, 2019). The latter sort of vagueness cannot be assimilated to indiscriminability or fuzziness in measurement procedures or relevant dimensions.

This section has begun to develop a semiorder-based semantic framework for gradation. The semantics captures relevant intransitivities in comparative sorites cases and allows for vagueness phenomena with both positive and comparative forms. Though the account avoids diagnosing linguistic vagueness in terms of the positive form, there remains a concern that vagueness phenomena are still being addressed piecemeal in terms of gradability. Whether we should prefer a more unified account of apparent vagueness phenomena in natural language remains to be seen (§2.3).

## References

BALE, ALAN. 2008. A universal scale of comparison. *Linguistics and Philosophy* 31, 1–55.

- BARKER, CHRIS. 2002. The dynamics of vagueness. *Linguistics and Philosophy* 25, 1–36.
- BARTSCH, RENATE & THEO VENNEMANN. 1973. *Semantic structures: A study in the relation between syntax and semantics*. Frankfurt: Athäenum.
- VAN BENTHEM, JOHAN. 1982. Later than late: On the logical origin of the temporal order. *Pacific Philosophical Quarterly* 63, 193–203.
- BHATT, RAJESH & ROUMYANA PANCHEVA. 2007. Degree quantifiers, position of merger effects with their restrictors, and conservativity. In CHRIS BARKER & PAULINE JACOBSON (Eds.), *Direct compositionality*, pp. 306–335. New York: Oxford University Press.
- BOCHNAK, MICHAEL RYAN. 2013. *Cross-linguistic variation in the semantics of comparatives*. Ph.D. thesis, University of Chicago.
- BURNETT, HEATHER SUSAN. 2012. *The grammar of tolerance: On vagueness, context-sensitivity, and the origin of scale structure*. Ph.D. thesis, UCLA.
- COOPER, NEIL. 1995. Paradox lost: Understanding vague predicates. *International Journal of Philosophical Studies* 3, 244–269.
- CRESSWELL, M.J. 1977. The semantics of degree. In BARBARA PARTEE (Ed.), *Montague grammar*, pp. 261–292. New York: Academic Press.
- ÉGRÉ, PAUL & NATHAN KLINEDINST (Eds.). 2011. *Vagueness and language use*. New York: Palgrave Macmillan.
- ENDICOTT, TIMOTHY. 2000. *Vagueness in law*. Oxford: Oxford University Press.
- FARA, DELIA GRAFF. 2000. Shifting sands: An interest-relative theory of vagueness. *Philosophical Topics* 28, 45–81.
- FINE, KIT. 1975. Vagueness, truth, and logic. *Synthese* 30, 265–300.
- FISHBURN, PETER C. 1985. *Interval orders and interval graphs: A study of partially ordered sets*. New York: Wiley.
- HALPERN, JOSEPH Y. 2008. Intransitivity and vagueness. *The Review of Symbolic Logic* 1, 530–547.

- HEIM, IRENE. 2001. Degree operators and scope. In CAROLINE FÉRY & WOLFGANG STERNEFELD (Eds.), *Audiatur vox sapientiae: A festschrift for Arnim von Stechow*, pp. 214–239. Berlin: Akademie Verlag.
- KAMP, HANS. 1981. A theory of truth and semantic representation. In JEROEN GROENENDIJK, T. JANSSEN, & M. STOKHOF (Eds.), *Formal methods in the study of language*, pp. 277–322. Amsterdam: Mathematical Centre.
- KEEFE, ROSANNA. 2000. *Theories of vagueness*. Cambridge: Cambridge University Press.
- KENNEDY, CHRISTOPHER. 1999. *Projective the adjective: The syntax and semantics of gradability and comparison*. New York: Garland Publishing.
- KENNEDY, CHRISTOPHER. 2007. Vagueness and grammar: The semantics of relative and absolute gradable adjectives. *Linguistics and Philosophy* 30, 1–45.
- KENNEDY, CHRISTOPHER. 2011. Vagueness and comparison. In ÉGRÉ & KLINEDINST (2011), pp. 73–97.
- KENNEDY, CHRISTOPHER. 2013. Two sources of subjectivity: Qualitative assessment and dimensional uncertainty. *Inquiry* 56, 258–277.
- KLEIN, EWAN. 1980. A semantics for positive and comparative adjectives. *Linguistics and Philosophy* 4, 1–45.
- KLEIN, EWAN. 1991. Comparatives. In ARNIM VON STECHOW & DIETER WUNDERLICH (Eds.), *Semantics: An international handbook of contemporary research*, pp. 673–691. New York: de Gruyter.
- LASERSOHN, PETER. 1999. Pragmatic halos. *Language* 75, 522–551.
- LASSITER, DANIEL. 2015. Adjectival modification and gradation. In SHALOM LAPPIN & CHRIS FOX (Eds.), *The handbook of contemporary semantic theory*, pp. 143–167. Oxford: Wiley-Blackwell, 2nd edn.
- LEWIS, DAVID. 1970. General semantics. *Synthese* 22, 18–67.
- LEWIS, DAVID. 1979. Scorekeeping in a language game. *Journal of Philosophical Logic* 8, 339–359.

- LUCE, R. DUNCAN. 1956. Semiordeers and a theory of utility discrimination. *Econometrica* 24, 178–191.
- MCNALLY, LOUISE. 2011. The relative role of property type and scale structure in explaining the behavior of gradable adjectives. In NOUWEN ET AL. (2011), pp. 151–168.
- MORZYCKI, MARCIN. 2015. *Modification*. Cambridge: Cambridge University Press.
- NOUWEN, RICK, ROBERT VAN ROOIJ, ULI SAUERLAND, & HANS-CHRISTIAN SCHMITZ (Eds.). 2011. *Vagueness in communication*. Berlin: Springer-Verlag.
- RAFFMAN, DIANA. 1996. Vagueness and context relativity. *Philosophical Studies* 81, 175–192.
- RAFFMAN, DIANA. 2014. *Unruly words: A study of vague language*. Oxford: Oxford University Press.
- RETT, JESSICA. 2008. *Degree modification in natural language*. Ph.D. thesis, Rutgers University.
- VAN ROOIJ, ROBERT. 2011a. Implicit versus explicit comparatives. In ÉGRÉ & KLINEDINST (2011), pp. 51–72.
- VAN ROOIJ, ROBERT. 2011b. Vagueness, tolerance and non-transitive entailment. In PETR CINTULA, CHRISTIAN G. FERMÜLLER, GODO LLUÍS, & PETR HÁJEK (Eds.), *Understanding vagueness: Logical, philosophical and linguistic perspectives*, pp. 205–222. London: College Publications.
- SASSOON, GALIT W. 2013. *Vagueness, gradability and typicality: The interpretation of adjectives and nouns*. Leiden: Brill.
- SAUERLAND, ULI & PENKA STATEVA. 2007. Scalar vs. epistemic vagueness: Evidence from approximators. In MASAYUKI GIBSON & TOVA FRIEDMAN (Eds.), *Proceedings of Semantics and Linguistic Theory (SALT) 17*, pp. 228–245. Ithaca, N.Y.: CLC Publications.
- SCOTT, DANA & PATRICK SUPPES. 1958. Foundational aspects of theories of measurement. *Journal of Symbolic Logic* 23, 113–128.
- SHAPIRO, STEWART. 2006. *Vagueness in context*. New York: Oxford University Press.

- SILK, ALEX. 2016. *Discourse Contextualism: A framework for contextualist semantics and pragmatics*. Oxford: Oxford University Press.
- SILK, ALEX. 2019. Evaluational adjectives. *Philosophy and Phenomenological Research* pp. 1–35.
- SOAMES, SCOTT. 1999. *Understanding truth*. Oxford: Oxford University Press.
- SOLT, STEPHANIE. 2014. An alternative theory of imprecision. In TODD SNIDER, SARAH D'ANTONIO, & MIA WEIGAND (Eds.), *Proceedings of SALT 24*, pp. 514–533. Ithaca, N.Y.: CLC Publications.
- SORENSEN, ROY. 1988. *Blindspots*. Oxford: Clarendon.
- VON STECHOW, ARNIM. 1984. Comparing semantic theories of comparison. *Journal of Semantics* 3, 1–77.
- SWANSON, ERIC. 2011. On the treatment of incomparability in ordering semantics and premise semantics. *Journal of Philosophical Logic* 40, 693–713.
- WILLIAMSON, TIMOTHY. 1994. *Vagueness*. London: Routledge.
- WRIGHT, CRISPIN. 1987. Further reflections on the Sorites paradox. *Philosophical Topics* 15, 227–290.