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Dynamic Model Checking of Discourse Representation Structures with Pluralities*

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Abstract

Model checking for First Order Logic is a computationally demanding task. Matters become worse in systems that typically yield fairly large and complex formulas, and that also include the representation of pluralities. This is the case of Discourse Representation Theory (Kamp and Reyle 1993), in which representations encode discourse chunks and deal with various plural phenomena. Not surprisingly, there are virtually no model checkers for DRT. This paper proposes a dynamic model checking strategy that reduces the search space and allows to evaluate non-trivial DRT representations in larger models.

1 Introduction

The *model checking* problem consists in determining if a model/structure $\mathcal{M}$ satisfies a formula ϕ in a logic $\mathcal{L}$. For First Order Logic (FOL) this problem is decidable for finite models, but the combined complexity is PSPACE-complete, and is $O(|\phi|\times|\mathcal{M}|^k)$ for k free variables in every subformula of ϕ (Stockmeyer 1974; Vardi 1982; Libkin 2004). Although there are many model checkers implementations for less expressive logics (modal and temporal logics, typically), there are almost no implementations for FOL (e.g. Blackburn and Bos (2005, 49–50) report finding only one implementation).

These results mean that computational semantics applications run into major problems when using large formulas, even in models of a modest size. For instance, in DRT (Kamp and Reyle 1993) formulas are used to represent not only isolated utterances, but also complex discourse chunks. The problem is made more severe in logical fragments that handle plurals and coordination (e.g. '*some lawyers*', '*twenty lawyers*', '*some men and seventy women*', etc.). Kamp and Reyle (1993) adopt a mereologic model theory

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for pluralities, but even in more conservative accounts like Link (1984), if the model contains 100 base entities then the resulting lattice structure contains 2^{100}–$1 \approx 1,3 \times 10^{30}$ nodes. It is by traversing this structure that plural predicates are evaluated. Other proposals resort to even larger denotation structures, like Hoeksema (1983), Landman (1989), and Ojeda (2001) (e.g. in Landman's system a basic domain with 4 simple atoms results in a structure with $2^{33\,000}$ nodes, due to iterative group formation; see Link (1998) for further discussion). Not only this is linguistically implausible on cognitive grounds, it is also impractical for computational semantics. As far as we know there are no implementations of model checkers for such DRT fragments (including in the DORIS system (Bos 2001), for instance).

There is quite a lot of literature on optimization strategies for modal and temporal logics, driven by industry applications in the domain of system specification and verification. There are two kinds of optimizations which are often discussed: specific operations that are useful in the logic of choice (e.g. computing fixed points is particularly useful to express recursion in temporal logics), and optimizations that are not specific to model checking and consist in search optimizations for the resolution engine (such as literal reordering, tabled resolution, partial order reduction, and clause resolution factoring). In this paper we are mainly interested in optimizations specifically tailored to address the issues raised in DRT representations. In our approach, assignment values are delayed and can be updated in various ways, with the goal of significantly reducing the search space while evaluating DRSs. Our proposal also relies on literal reordering, but it does not require complex formula manipulations nor special resolution strategies (and of course, further performance gains can be obtained if the algorithm is implemented in systems that support tabled resolution, for instance). Section 2 addresses the evaluation of a non-trivial DRS fragment with pluralities, and provides performance results obtained with a Prolog implementation.

2 Delayed Dynamic Assignments

In order to determine if a FOL formula $\forall x \phi$ is true or if a formula $\exists x \phi$ is false, *all* the possible assignments to x in the domain must be tried out. Similarly, in Kamp and Reyle (1993, 425–427) one is in fact quantifying over embedding extensions (an embedding is a set of assignment pairs (v, i), in which v is a discourse referent and i is a domain element), e.g.:

(1) $\mathcal{M} \models_f K_1 \Rightarrow K_2$ *iff* for every extension g of f to U_{K1} such that $\mathcal{M} \models_g K_1$ there is an extension h of g to U_{K2} such that $\mathcal{M} \models_h K_2$

Kamp and van Eijck (1997) propose several alternative semantics for DRSs, including a relational fragment for atomic DRSs, dynamic in the spirit of Groenendijk and Stokhof (1991), of which we reproduce a fragment below.

Definition 1

(i) ${}_s[\![(\{v\},\emptyset)]\!]^{\mathcal{M}}_{s'}$ *iff* $s[v]s'$

(ii) ${}_s[\![(\emptyset,\{P(t_1,...,t_n)\})]\!]^{\mathcal{M}}_{s'}$ *iff* $s = s'$ & $\langle V_{\mathcal{M}s}(t_1),...,V_{\mathcal{M}s}(t_n)\rangle \in I(P)$

(iii) ${}_s[\![(\emptyset,\{v \doteq t\})]\!]^{\mathcal{M}}_{s'}$ *iff* $s = s'$ and $s(v) = V_{\mathcal{M},s}(t)$

(iv) ${}_s[\![\neg K]\!]^{\mathcal{M}}_{s'}$ *iff* $s = s'$ and for no s'' it is the case that ${}_s[\![K]\!]^{\mathcal{M}}_{s''}$

The first case concerns the evaluation of a discourse referent in the universe of an atomic DRS. This is done by taking an input embedding s and outputting an embedding s' which differ at most in the value for x. The remaining cases concern n-ary predicates, equality, and negation.

Spanning the entire domain is a costly operation, specially in the case of plural referents. Below we propose a model checking algorithm inspired in the above dynamic view of satisfaction, where variable assignments are delayed, and may be dynamically updated in various ways. Although model checking for DRSs will remains a computationally complex task, this approach allows real applications to cope with larger models and formulas.

2.1 Basic DRS Fragment

The model checker algorithm ${}_g[\![\cdot]\!]^{\triangleright\mathcal{M}}_{g'}$ that we propose is intended to compute the embedding extensions that verify K in $\mathcal{M}$, under satisfaction conditions that are loyal as possible to the standard denotational definitions.[1] For this purpose, a standard DRS language is adopted, with the standard operators and discourse referents for individuals (x_1, x_2 ...), sets of individuals (X_1, X_2 ...), and events (e_1, e_2 ...). As usual, Greek letters (α_1, α_2 ...) are used to refer to in a neutral fashion, without stating that they are atomic or non-atomic. We will write $g[(x,i)]g'$ to say that g' differs from g only in the value i given to x. So if $(x,i') \in g$ then $g[(x,i)]g'$ yields an updated embedding $g' = (g \setminus \{(x,i')\}) \cup \{(x,i)\}$, and is undefined otherwise.

Our first move is to require that DRS conditions are evaluated by the model checker in a particular fixed order (similarly to the literal ordering optimization (Ullman 1988)). Some DRS components will be able to update delayed variable assignments, while others will not. This means that the former should be evaluated before the latter. The precedence hierarchy holds for conditions containing at least one discourse referent: referential predicates (i.e. nominal predicates) $\prec$ nominal modifiers and '=' conditions $\prec$ verb predicates $\prec$ GQs and complex logical conditions ($\vee$, $\neg$, $\Rightarrow$) $\prec$ cardinality conditions (e.g. $|X| > n$). This order can either be computed on-the-fly or be incorporated into the syntax-semantics interface, and need only hold for sentential DRSs. There is no need of recomputing precedences when a larger

[1] W.r.t. correctedness, there is one kind of DRS in which ${}_g[\![\cdot]\!]^{\triangleright\mathcal{M}}_{g'}$ does not yield the same result as the denotational semantics. See the discussion below in this subsection.

discourse representation is built from merging sentential DRSs. Henceforth we write $\langle\{\alpha_1,\ldots,\alpha_m\},[C_1,\ldots,C_n]\rangle$ to represent a DRS containing a list of ordered conditions $C_1,...,C_n$, instead of a set.

In (i) we define how DRSs are to be checked in $\mathcal{M}$. Each discourse referent in the DRS universe triggers an embedding extension with delayed assignments. These are expressed with the symbol '$\star$', which signals that the bound variable is yet to receive a proper assignment:

Definition 2.1

(i) ${}_g[\![\langle\{\alpha_1,\ldots,\alpha_m\},[C_1,\ldots,C_n]\rangle]\!]^{\triangleright\mathcal{M}}_{g'}$ *iff* $\exists g_1...g_n\, g'$ such that $g_1 = g\cup\{(\alpha_1,\star),\ldots,(\alpha_m,\star)\}$ & ${}_{g1}[\![C_1]\!]^{\triangleright\mathcal{M}}_{g2}$ & ... & ${}_{g_n}[\![C_n]\!]^{\triangleright\mathcal{M}}_{g'}$

(ii) ${}_{g1}[\![p(\alpha_1,\ldots\alpha_m)]\!]^{\triangleright\mathcal{M}}_{g_m}$ *iff* $\langle i_1,\ldots,i_m\rangle \in I(p)$ & $\exists g_2...g_m$ such that $F^{g1}_{g2}(\alpha_1,i_1)$ & ... & $F^{g_{m-1}}_{g_m}(\alpha_m,i_m)$

The satisfaction of relations (n-ary predicates with $n\geq 2$) is defined in (ii). Delayed assignments can be updated with a corresponding element in the extension of the predicate. Discourse referents which already have proper assignments are simply checked against the extension. The function $F^{g}_{g'}$ checks if a given assignment pair (x,i) holds in g, or if the assignment to x is delayed. In the latter case g is updated with the input pair (x,i).

$$F^{g}_{g'}(x,i) = \begin{cases} 1: & (x,i)\in g \;\; \& \;\; g=g' \\ 1: & (x,\star)\in g \;\; \& \;\; g[(x,i)]g' \\ 0: & o.w. \end{cases}$$

For a 1-place predicate $p(\alpha)$, satisfaction is similar: $i\in I(p)$ & $\exists g_2\, F^{g1}_{g2}(\alpha,i)$. The satisfaction conditions for GQs and the connectors '$\vee$', '$\neg$', and '$\Rightarrow$' are very similar to Kamp and Reyle (1993) and Kamp and van Eijck (1997, 200).

(iii) ${}_g[\![\neg K]\!]^{\triangleright\mathcal{M}}_{g'}$ *iff* $g=g'$ & $\neg\exists g''\ {}_g[\![K]\!]^{\triangleright\mathcal{M}}_{g''}$

(iv) ${}_g[\![K_1 \Rightarrow K_2]\!]^{\triangleright\mathcal{M}}_{g'}$ *iff* $g=g'$ & $\forall g_1(\ {}_g[\![K_1]\!]^{\triangleright\mathcal{M}}_{g1} \rightarrow \exists g_2\ \ {}_{g1}[\![K_2]\!]^{\triangleright\mathcal{M}}_{g2}\ )$

(v) ${}_g[\![K_1 \vee K_2]\!]^{\triangleright\mathcal{M}}_{g'}$ *iff* $g=g'$ & $\exists g_1$ (${}_g[\![K_1]\!]^{\triangleright\mathcal{M}}_{g1}$ $\vee$ ${}_g[\![K_2]\!]^{\triangleright\mathcal{M}}_{g1}$)

(vi) ${}_g[\![Qx(K_1,K_2)]\!]^{\triangleright\mathcal{M}}_{g'}$ *iff* $g=g'$ & $Q(A,B)=1$ &
$A{=}\{i : \exists g_1\,({}_g[\![K_1]\!]^{\triangleright\mathcal{M}}_{g_1}$ & $(x,i)\in g_1)\}$
$B{=}\{i : \exists g_1({}_g[\![K_1]\!]^{\triangleright\mathcal{M}}_{g_1}$ & $(x,i)\in g_1$ & $\exists g_2\ {}_{g_1}[\![K_2]\!]^{\triangleright\mathcal{M}}_{g_2}\ )\}$

Finally, we turn to '$\alpha=\beta$' conditions, introduced in discourse continuations. According to the precedence hierarchy, these are evaluated first in their local DRS, which means that β has a delayed assignment:

(vii) ${}_g[\![\alpha \doteq \beta]\!]^{\triangleright\mathcal{M}}_{g'}$ *iff* $\exists i$ such that $\{(\alpha,i),(\beta,\star)\}\subseteq g$ & $g[(\beta,i)]g'$

We can now check if a DRS K is true in a model $\mathcal{M} = (D, I)$. This corresponds to the query ${}_{\{\}}[\![K]\!]_g^{\triangleright\mathcal{M}}$. Consider the toy model given in (2):

(2) $D = \{w_1, ..., w_{10}, m_1, ..., m_{10}, c_1, ..., c_{10}\} \cup \{e_1, ..., e_{10}\}$

$$\begin{array}{lcl} I(woman) & = & \{w_1, ..., w_{10}\} \\ I(man) & = & \{m_1, ..., m_{10}\} \\ I(car) & = & \{c_1, ..., c_{10}\} \\ I(happy) & = & \{m_3, m_5, m_7, w_1, w_2, w_4, w_6, w_9\} \\ I(whistle) & = & \{\langle e_1, m_1\rangle, \langle e_2, m_3\rangle, \langle e_3, w_4\rangle, \langle e_4, w_8\rangle, \langle e_5, m_4\rangle\} \end{array}$$

Let us consider how a DRS like (3) is evaluated in $\mathcal{M}$:

(3)
$$\begin{array}{|c|} \hline \text{x e} \\ \hline \text{man(x)} \\ \text{whistle(e,x)} \\ \hline \end{array}$$

(4) 1. ${}_{\{\}}[\![\langle\{x, e\}, [man(x), whistle(e, x)]\rangle]\!]_g^{\triangleright\mathcal{M}}$ is true *iff* both ${}_{\{(x,\star),(e,\star)\}}[\![man(x)]\!]_{g1}^{\triangleright\mathcal{M}}$ and ${}_{g1}[\![whistle(e, x)]\!]_g^{\triangleright\mathcal{M}}$ hold in $\mathcal{M}$.

So let us compute the first conjunct:

2. ${}_{\{(x,\star),(e,\star)\}}[\![man(x)]\!]_{g1}^{\triangleright\mathcal{M}}$ evaluates as true *iff* some i exists such that $i \in I(man)$ and $F_{g1}^{\{(x,\star),(e,\star)\}}(x, i) = 1$. The query $i \in I(man)$ is solved as $i = m_1$ in two simple membership queries: retrieval of the extension of '*man*' in I, and a membership query to the extension. Both steps succeed, yielding $g_1 = \{(x, m_1), (e, \star)\}$.

Let us now compute the second conjunct:

3. ${}_{\{(x,m_1),(e,\star)\}}[\![whistle(e, x)]\!]_g^{\triangleright\mathcal{M}}$ is true *iff* some $\langle i_1, i_2\rangle \in I(whistle)$ such that $F_{g2}^{\{(x,m_1),(e,\star)\}}(e, i_1) = 1$ and $F_g^{g2}(x, i_2) = 1$. The above membership queries return $i_1 = e_1$ and $i_2 = m_1$ respectively, and $g = \{(x, m_1), (e, e_1)\}$. In the case of x, no update is necessary because $(x, m_1) \in g_2$, and so (3) is true in $\mathcal{M}$ with g.

In this example there are only 3 extension membership queries to $I(p)$, and assignment values to x range only over the denotation of the respective nominal predicate. This minor change in the way assignments are made prevents each existentially quantified discourse referent from yielding at most $n = |D|$ possible assignments. Model checking remains PSPACE-complete, although computations are bound by the size of the denotations, instead of n.

In practice it also becomes simpler to check the truth value of DRSs such as (5). The possible assignments for x in the antecedent no longer ranges over the entire domain. Rather, these range over the individuals in the extension of '*man*'. According to our definition, every possible embedding for the antecedent is required to also satisfy the consequent. In our toy model this entails 10 assignment trials instead of 30 (assuming that event individuals are not considered) for determining the truth value of (5).

(5)

[x | man(x), happy(x)] $\Rightarrow$ [e | whistle(e,x)]

Note that if instead of delaying assignments one were to use sorted individuals for each nominal predicate (e.g. one sort for the individuals in $I(man)$, another sort for the individuals in $I(woman)$, another for individuals in $I(person)$, and so forth) the membership query would still range over the entire domain of individuals, although it is true that invalid assignments would be rejected sooner than in the standard satisfaction definition. Another alternative would be to make the domain much more complex (and redundant) by partitioning D into possibly overlapping sets of individuals $D = \langle Men, Women, People, \ldots \rangle$. Delayed assignments allow us to restrict the membership query to the extension of predicates without making the model more complex (either with a large number of sorts, or with an enumeration of partitions). Furthermore, types and partitions are of little avail when considering plurals. As we shall see in §2.2, the latter can be dealt with by extending our notion of assignment update.

There is one kind of DRS for which our checker does not yield the correct truth conditions. These are cases in which negation intervenes between the discourse referent x and the predicate that secures it's assignment. For example, consider the sentence '*something didn't move*' and a possible translation $\langle \{x\}, \{\neg\langle\{e\}, \{move(e,x)\}\rangle\}\rangle$. This DRS will always come out false by ${}_g[\![\cdot]\!]_{g'}^{\triangleright,\mathcal{M}}$ if at least one thing moves. Still, the problem can be avoided if one assumes that the NP '*something*' receives an explicit semantic representation: $\langle \{x\}, \{thing(x), \neg\langle\{e\}, \{move(e,x)\}\rangle\}\rangle$.[2]

2.2 Plurals and Distributivity

A naive evaluation of plural predications is computationally hopeless, and cannot be implemented even for small toy models. For instance, in Kamp and Reyle (1993, 426) the range of denotations which can be assigned to a non-atomic referent X is given by the semilattice $\mathcal{U}_{\mathcal{M}} = \langle D_{\mathcal{M}}, \subset \rangle$.[3] Because $\mathcal{U}_{\mathcal{M}}$ is obtained from the *entire* domain, this means that the number of sets of individuals that can be assigned to X in our tiny model is already prohibitively large: $2^{30}–1 \approx 1,1\times10^9$. In a sentence like '*Several men saw*

[2]It can be suggested that operators like '*only*' may also require searching over the entire domain. However, this hinges on the semantic representation of choice. For example, consider a sentence like '*Only Tom fell*'. In a Horn-style analysis like $\forall x\,(fall(x) \rightarrow tom'(x)$ our account would only need to search the tuples in the verb extension.

[3]For exposition purposes, we adopt a set notation instead of a sum notation. Non-atomic discourse referents can either be seen as ranging over sums in a mereologic domain $\langle D_{\mathcal{M}}, \subset \rangle$, or equivalently, over sets of individuals in a power set domain $\langle \mathcal{P}(D_{\mathcal{M}}), \subseteq \rangle$.

some women' the search space is 2^{60}–$2 \approx 1,2{\times}10^{18}$. On the other hand, most of these computations are irrelevant in certain cases: the number of sets that can actually satisfy an NP like '*four women*' is a very small subset of $\mathcal{P}(D)$ corresponding to the binomial coefficient $\binom{10}{4} = 210$ where $10 = |I(woman)|$.

For perspicuity, consider the sentence in (6). The mixed predicate '*to rent*' is compatible with both a collective and a distributive interpretation:[4]

(6) Several women rented this car.

a. $\begin{array}{|c|}\hline \text{X y e} \\ \hline \text{woman}^*(\text{X}) \\ \text{car(y)} \\ \text{rent(e,X,y)} \\ \hline \end{array}$

b. $\begin{array}{|c|}\hline \text{X y} \\ \hline \text{woman}^*(\text{X}) \\ \text{car(y)} \\ \begin{array}{|c|}\hline \text{x} \\ \hline \text{x} \in \text{X} \\ \hline \end{array} \Rightarrow \begin{array}{|c|}\hline \text{e} \\ \hline \text{rent(e,x,y)} \\ \hline \end{array} \\ \hline \end{array}$

Note that, as is, our delayed assignment strategy already causes a reduction of the search space assuming a standard evaluation of plurals. Our model checker would search over the power set $\mathcal{P}(I(woman))$, rather than over the much larger $\mathcal{P}(D)$. However, this still yields a combinatorial explosion of at most $2^{|I(woman)|}$ possible assignments: if we have a model with 30 women, then model checking the above DRSs is impractical.

This combinatorial explosion of plural denotations *can* be avoided in several cases, and a fairly practical checking procedure can be devised. We propose to minimize the source of computations by allowing plural denotations to be updated dynamically, each time a condition is evaluated.

We start by stating that $p^*(X)$ always starts by denoting the entire set $I(p)$. Plural assignments will differ from atomic assignments in that the former can be dynamically updated each time a predicate is evaluated. Two types u and r will be used to distinguish between two kinds of assignments made to non-atomic referents: *unrestricted* assignments $(X, A^{\mathsf{u}}) \in g$ allow further updates $g[(X, B^{\mathsf{u}})]g'$ for $B \subset A$, while *restricted* assignments $(X, A^{\mathsf{r}}) \in g$ do not. All assignments made to atomic referents are untyped, and all assignments made to non-atomic referents start out unrestricted:

Definition 2.2 (extending definition 2.1)

(viii) $_{g\supseteq\{(X,\star)\}}[\![p^*(X)]\!]^{\triangleright\mathcal{M}}_{g'}$ *iff* $A = I(p)$ & $\|A\| \geq 2$ & $\exists g'\, g[(X, A^{\mathsf{u}})]g'$

In practice, this condition states that a referent X in a predicate like $woman^*(X)$ yields the unrestricted assignment $(X, \{w_1, ..., w_{10}\}^{\mathsf{u}})$. If X is taken as an argument of a collective predicate like $rent(e, X, y)$ then the assignment (X, A^{u}) is updated to (X, B^{r}) *iff*: $\langle e', B, i\rangle \in I(rent)$ and $B \subseteq A$. For example, if $\langle e_6, \{w_1, w_2, w_3\}, c_2\rangle \in I(rent)$ then $(X, \{w_1, ..., w_{10}\}^{\mathsf{u}})$ is successfully updated into a restricted assignment $(X, \{w_1, w_2, w_3\}^{\mathsf{r}})$. The query $B \subseteq A$ is fairly straightforward because both B and A are known.

[4]We assume in general terms the analysis of distributivity in Kamp and Reyle (1993).

In order to allow (ii) in §2.1 to cope with collective readings we must extend $F^g_{g'}$ so that non-atomic referent assignments are also considered:

$$F^g_{g'}(\alpha,\beta)=\begin{cases} 1: & \alpha = x \;\;\&\;\; \beta = i \;\;\&\;\; (x,i) \in g \;\;\&\;\; g = g' \\ 1: & \alpha = x \;\;\&\;\; \beta = i \;\;\&\;\; (x,\star) \in g \;\;\&\;\; g[(x,i)]g' \\ 1: & \alpha = X \;\;\&\;\; \beta = A \;\;\&\;\; (X,A^r) \in g \;\;\&\;\; g = g' \\ 1: & \alpha = X \;\;\&\;\; \beta = A \;\;\&\;\; (X,B^u) \in g \;\;\&\;\; A \subseteq B \;\;\&\;\; g[(X,A^r)]g' \\ 0: & o.w. \end{cases}$$

We can now evaluate the collective reading represented in (6a), which we have already briefly discussed above. Let us assume that $I(rent)$ contains a tuple $\langle e_6, \{w_2, w_4, w_6\}, c_2\rangle$ in our model, and that $\langle c_2\rangle \in I(car)$. We start by ${}_{\{(X,\star),(y,\star),(e,\star)\}}[\![woman^*(X)]\!]^{\triangleright\mathcal{M}}_{g1}$ obtaining g_1=$\{(X, \{w_1, ..., w_{10}\}^u)$, $(y,\star), (e,\star)\}$. Next, ${}_{g1}[\![car^*(X)]\!]^{\triangleright\mathcal{M}}_{g2}$ succeeds with $g_2 = \{(X, \{w_1, ..., w_{10}\}^u)$, $(y, c_2), (e,\star)\}$. Finally, we have that ${}_{g2}[\![rent(e,X,y)]\!]^{\triangleright\mathcal{M}}_{g3}$ succeeds with the output $g_3 = \{(X, \{w_2, w_4, w_6\}^r), (y, c_2), (e, e_6)\}$. The latter step is the result of case 4 of F verifying that $\{w_2, w_4, w_6\} \subseteq \{w_1, ..., w_{10}\}$. Note that the collective reading also has the effect that the assignment made to X becomes restricted, thus preventing any further updates.[5]

Accordingly, cardinality constraints over referents with restricted assignments like (X, A^r) only need to check the size of the assignment:

(ix) ${}_{g\supseteq\{(X,A^r)\}}[\![|X| = n]\!]^{\triangleright\mathcal{M}}_{g'}$ *iff* $g = g'$ & $\|A\| = n$

Matters are different in regard to distributivity, which can either arise via Link's '$*$ operator, or by a distributive condition as in (6b). We need a different evaluation definition for distributivity. For instance, assume that we wish to evaluate $happy^*(X)$ in an NP like '*happy men*'. We need to determine if $I(man) \cap I(happy)$ contains at least two members, and not to check if $I(happy) \subseteq I(man)$, as $F^g_{g'}$ would have it. Non-delayed distributive predications are thus evaluated with a new function $F_*{}^g_{g'}$ that updates unrestricted assignments by intersecting the respective sets of individuals:

(x) ${}_{g\supseteq\{(X,A)\}}[\![p^*(X)]\!]^{\triangleright\mathcal{M}}_{g'}$ *iff* $B = I(p)$ & $\|B\| \geq 2$ & $\exists g'\, F_*{}^g_{g'}(X,B)$

$$F_*{}^g_{g'}(X,B) = \begin{cases} 1: & (X,A^u) \in g \;\;\&\;\; C{=}A \cap B \;\;\&\;\; \|C\| \geq 2 \;\;\&\;\; g[(X,C^u)]g' \\ 1: & (X,A^r) \in g \;\;\&\;\; A \subseteq B \;\;\&\;\; g = g' \\ 0: & o.w. \end{cases}$$

In restricted assignments, $F_*{}^g_{g'}$ only requires a subset check. This occurs only when a distribution is evaluated *after* some collective predicate has restricted the assignment (e.g. '*Twenty men* [*gathered outside*] *and* [*shouted*]').

[5]The standard semantics (as well as the current formulation) fails to cope with sentences like '*Some men did not lift a stone*': for it to be false, all the $2^{10} - 11$ collections of men must have lifted some stone, which is intuitively wrong. Here we follow Lønning (1989, 59) and others in assuming that negation triggers a distributive 'involvement' reading.

We shall interpret distributive conditions like the one in (6b) indirectly, with a distributive predicate D (where $x \in X$ and K is the scopal DRS):

(xi) $_g[\![D(x,X,K)]\!]^{\triangleright,\mathcal{M}}_{g'}$ *iff* $(Y, B^{\tau}) \in g$ &
$A = \{i : i \in B \;\&\; \exists g_1 \;{}_{g\cup\{(x,i)\}}[\![K]\!]^{\triangleright,\mathcal{M}}_{g_1}\} \;\&\; \|A\| \geq 2 \;\&$
$\exists g'((\tau{=}\mathsf{u} \;\&\; g[(Y, A^{\mathsf{u}})]g') \vee (\tau{=}\mathsf{u} \;\&\; A{=}B \;\&\; g{=}g'))$

The non-empty set A is composed of all the individuals in B that satisfy the DRS in the scope of the distribution. If (X, B^{τ}) is a unrestricted assignment then we update it to a new assignment with the individuals that satisfy the condition (X, A^{u}), but if (X, B^{τ}) is restricted then $A = B$ must hold.

Unfortunately, not all sources of combinatorial explosion can be avoided. Cardinality conditions must be able to non-deterministically update unrestricted assignments with subsets of the input:

(xii) $_{g\supseteq\{(X,A^{\mathsf{u}})\}}[\![|X| = n]\!]^{\triangleright,\mathcal{M}}_{g'}$ *iff* $\exists B\,(\|B\| = n \;\&\; B \subseteq A \;\&\; \exists g' g[(X, B^{\mathsf{r}})]g')$

Still, our strategy pays off because cardinals are evaluated last in a sentential DRS (cf. precedence hierarchy in §2.1), and the search space is reduced by previous evaluations. Moreover, if cardinals are *always* evaluated last in any DRS, then this source of combinatorial explosion is minimized.

We also point out that determiners like '*less than n*' and '*at most n*' pose no problem for our approach. These can receive a GQ (or an abstraction-based) account like in Kamp and Reyle (1993), or even a pluralic analysis along the lines of Link (1998) (e.g. $\lambda P.\lambda Q.\neg\exists X(P(X) \wedge Q(X) \wedge |X| \geq n)$).

2.3 NP Coordination

The semantics of NP Coordination presents many challenges both from a linguistic perspective and from a computational perspective. Here we address a bit of both aspects to show how the model checker scales to conjunction. The complex subject NP in (7a) behaves similarly to a plural noun in the sense that it can have a collective reading in which a man and a woman participate in a unique problem solving situation, and a distributive reading in which each individual solved the problem independently.

(7) a. [A man and a woman]$_Z$ solved the problem.

b. [Several men and four women]$_Z$ solved the problem.

However, it is usually argued (7b) has three readings: a collective reading in which there is one collaborative problem solving situation, an *group distributive reading* in which the men solved the problem collectively and, independently, the women also collectively solve the problem, and a *full distributive reading* in which each individual solved the problem separately.

We will assume that (7b) is represented by the two DRSs given below (and that (7a) is likewise represented by two very similar DRSs):[6]

(8) a. $$\begin{array}{|c|}\hline \text{X Y Z}' \text{ k} \\ \hline \text{man}^*(\text{X}) \quad \text{woman}^*(\text{Y}) \\ \text{problem(k)} \\ \text{X} \in_c \text{Z}' \quad \text{Y} \in_c \text{Z}' \\ \text{solve(e,Z}'\text{,k)} \\ |\text{Y}| = 4 \\ \hline \end{array}$$

b. $$\begin{array}{|c|}\hline \text{X Y Z}' \text{ k} \\ \hline \text{man}^*(\text{X}) \quad \text{woman}^*(\text{Y}) \\ \text{problem(k)} \\ \text{X} \in_c \text{Z}' \quad \text{Y} \in_c \text{Z}' \\ \begin{array}{|c|}\hline \alpha \\ \hline \alpha \in \text{Z}' \\ \hline \end{array} \Rightarrow \begin{array}{|c|}\hline \text{e} \\ \hline \text{solve(e,}\alpha\text{,k)} \\ \hline \end{array} \\ |\text{Y}| = 4 \\ \hline \end{array}$$

In the analysis we assume, the condition '$\in_c$' is introduced by the coordinator '*and*', and is only suitable for Z' referents. Such referents $(Z'_1, Z'_2, ...)$ will be interpreted as *meta-variables*, ranging over a set of discourse referents.[7] Assignment values made to Z' start out as $(Z', \{\})$. We thus reformulate condition (i) in §2.1 to add a function $\triangleright(\alpha)$ which yields a delayed assignment $(\alpha, \{\})$ if α is a conjunction referent Z', and $(\alpha, \star)$ otherwise:

(i') ${}_g[\![\langle\{\alpha_1, \ldots, \alpha_m\}, [C_1, \ldots, C_n]\rangle]\!]^{\triangleright\mathcal{M}}_{g'}$ *iff* $\exists g_1 ... g_n\, g'$ such that
$\exists g_1 = g \cup \{\triangleright(\alpha_1), ..., \triangleright(\alpha_m)\}$ & ${}_{g1}[\![C_1]\!]^{\triangleright\mathcal{M}}_{g2}$ & $\ldots$ & ${}_{g_n}[\![C_n]\!]^{\triangleright\mathcal{M}}_{g'}$

The evaluation of Z' membership assignments amounts to extending the set value with the conjunct's discourse referent:

Definition 2.3 (extending definition 2.2)

(xiii) ${}_g[\![\alpha \in_c Z')]\!]^{\triangleright\mathcal{M}}_{g'}$ *iff* $(Z', A) \in g$ & $\exists g'\, g[(Z', A \cup \{\alpha\})]g'$

We must require that these '$\in_c$' conditions are checked before verb predicates, because the assignment value of Z' is needed to evaluate the predicates in which Z' occurs. Consider an NP like '*a man and several women*' and an embedding $g_1 = \{(x, m_3), (Y, \{w_1 ... w_{10}\}), (Z', \{\})\}$. Evaluating the condition '$x \in_c Z'$' outputs $g_2 = \{(x, m_3), (Y, \{w_1 ... w_{10}\}), (Z', \{x\})\}$, and '$Y \in_c Z'$' outputs $g_3 = \{(x, m_3), (Y, \{w_1 ... w_{10}\}), (Z', \{x, Y\})\}$.

Collective readings of Z' are be obtained by extending $F^g_{g'}$ so that Z' referents are also considered. Basically, each individual that is involved in the collective predication must be mapped into the assignments made to the referents in Z'. For this purpose we resort to a new ancillary function ${F_c}^g_{g'}$:

$F^g_{g'}(Z', A) = 1$ *iff* $\exists B\, (Z', B) \in g$ & ${F_c}^g_{g'}(B, A)$

[6]Or some minor variation thereof, closer to Kamp and Reyle (1993). But see also Chaves (2005) on a HPSG/UDRT interface for the grammar fragment under discussion.

[7]This account is to some extent inspired by the *sub-referent* relation in Krifka (1991), which argues against groups in NP coordination and against some alternative accounts like Schwarzschild (1990). But Krifka (1991) is very different in that these relations (as well a notion of meta-DRS, and of meta-referents) are introduced to handle distributivity.

The recursive function ${F_c}^{g}_{g'}$ ensures that the assignment values of the referents in B exhaust the individuals in A (which comes from the tuple in a collective predicate's extension). Case one and two correspond to the mapping of atomic and non-atomic discourse referents respectively. Case three is the end of recursion, when all the individuals in A have been successfully mapped into the values of the conjoined discourse referents.

$$
{F_c}^{g1}_{g3}(B, A) = \begin{cases} 1: & B = B' \cup \{x\} \ \&\ (x, i) \in g_1 \ \&\ A = A' \cup \{i\} \ \&\ F^{g1}_{g2}(\beta, \alpha) \\ 1: & B = B' \cup \{X\} \ \&\ (X, W_1) \in g_1 \ \& \\ & W_2 = A \cap W_1 \ \&\ \|W_2\| \geq 2 \\ & \&\ F^{g1}_{g2}(X, W_2) \ \&\ (X, W_3) \in g_2 \ \&\ {F_c}^{g2}_{g3}(B', A \setminus W_3) \\ 1: & A = B = \{\} \ \ \&\ \ g_1 = g_3 \\ 0: & o.w. \end{cases}
$$

Consider a collective reading like the one triggered by ${}_{g}[\![solve(e, Z', k)]\!]^{\triangleright\mathcal{M}}_{g'}$. Assume that $B = \{X, Y\}$, $A = \{m_1, m_2, w_3, w_4\}$, and that the input embedding g is $\{(X, \{m_1, ..., m_{10}\}^u), (Y, \{w_1, ..., w_{10}\}^u), (Z', \{X, Y\}), (k, p)\}$. If $\langle e_m, \{m_1, m_2, w_3, w_4\}, p\rangle \in I(solve)$, then the output embedding g' is resolved as $\{(X, \{m_1, m_2\}^r), (Y, \{w_3, w_4\}^r), (Z', \{X, Y\})\}$. Note that the assignments made to the conjunct referents X and Y are updated because the function F is used to compare the initial values of X and Y with their intersection with A.

Distributive readings of Z' referents are captured by defining how the condition $D(\alpha, Z', K)$ is to be checked. One has to ensure that the assignment values of each conjunct in Z' (atomic or not) make K true. The first conditions in (xiv) below state that all the atomic referents $x_0, \ldots, x_m$ in Z' have values that, assigned to α, make K true. The remaining conditions deal with non-atomic referents and are divided in two disjoint cases: one for distributive readings ranging over each of the non-atomic conjuncts, and a second case ranging over the individual atoms in the non-atomic conjuncts. In other words, the first disjunct takes care of standard distributive readings while the second case takes takes care of full distributive readings.

(xiv) ${}_{g}[\![D(\alpha, Z', K)]\!]^{\triangleright\mathcal{M}}_{g'}$ *iff*

$$(Z', \{X_0, ..., X_n, x_0, ..., x_m\}) \in g \ \&$$
$$\{(x_0, i_0), ..., (x_m, i_m)\}) \in g \ \&$$
$$\exists g_t \, ({}_{g\cup\{(\alpha, i_0)\}}[\![K]\!]^{\triangleright\mathcal{M}}_{gt}) \ \&\ ... \ \&\ \exists g_t \, ({}_{g\cup\{(\alpha, i_m)\}}[\![K]\!]^{\triangleright\mathcal{M}}_{gt}) \ \&$$

$$\{(X_0, W_0), ..., (X_n, W_n)\} \subseteq g \ \&$$

$$
\begin{array}{l}
\exists\, g_1 \ldots g_n\, g' \\
\left(\begin{array}{c}
\left(\begin{array}{c}
\exists g^t\, ({}_{g\cup\{(\alpha,W_0)\}}[\![K]\!]^{\rhd\mathcal{M}}_{g^t\supseteq\{(\alpha,W'_0)\}}\ \ \&\ \ F_*{}^{g}_{g1}(X_0,W'_0)) \\
\& \ldots \& \\
\exists g^t\, ({}_{g\cup\{(\alpha,W_n)\}}[\![K]\!]^{\rhd\mathcal{M}}_{g^t\supseteq\{(\alpha,W'_n)\}}\ \ \&\ \ F_*{}^{gn}_{g'}(X_n,W'_n))
\end{array}\right) \\
\vee \\
\left(\begin{array}{c}
A_0 = \{i : i \in W_0\ \&\ \exists g_t\, {}_{g\cup\{(\alpha,i)\}}[\![K]\!]^{\rhd\mathcal{M}}_{gt}\}\ \ \&\ \ F_*{}^{g}_{g1}(X_0,A_0) \\
\& \ldots \& \\
A_n = \{i : i \in W_n\ \&\ \exists g_t\, {}_{g\cup\{(\alpha,i)\}}[\![K]\!]^{\rhd\mathcal{M}}_{gt}\}\ \ \&\ \ F_*{}^{gn}_{g'}(X_n,A_n)
\end{array}\right)
\end{array}\right)
\end{array}
$$

The first disjunct ensures that the assignment value of each non-atomic referent makes K true. Note that these assignments may require updating, which is achieved by the function F_*. Full distributive readings are obtained by finding all the individuals i which are members of the values of $X_0 \ldots X_n$ and that make K true for (α, i). Function F_* ensures that the resulting sets $A_0 \ldots A_n$ are non-empty, and updates the values of $X_0 \ldots X_n$ accordingly.

2.4 Implementation

A Prolog implementation available at `www.clul.ul.pt/clg/ddmc.html` was developed to obtain some preliminary results of the present proposal. In the first section of the table below we show the performance time for computing one possible satisfier embeddings of the DRS below, and in the second section of the table we show the performance result for computing all the possible embeddings. In all of these models, there is only one possible satisfying embedding. Models are of size n, which means that $|I(p)| = n$ for *every* predicate p. In (9) we show the number of logical steps (calls and redos), and CPU time on a 1Gb 3GHz P4 PC running SWI Prolog.

(9)

$$
\boxed{\begin{array}{c}
\text{X Y w Z}'\text{ S e} \\ \hline
\text{man}^*(\text{X})\quad \text{woman}^*(\text{Y})\quad \text{teenager(w)} \\
\text{X}\in_c\text{Z}'\quad \text{Y}\in_c\text{Z}'\quad \text{w}\in_c\text{Z}' \\
\boxed{\begin{array}{c}\alpha \\ \hline \alpha \in \text{Z}'\end{array}} \Rightarrow \boxed{\begin{array}{c}\text{e' c} \\ \hline \text{car(c)} \\ \text{rent(e'},\alpha\text{,c)}\end{array}} \\
\text{student}^*(\text{S})\quad \text{gather(e,S)}\quad |\text{S}| = 6
\end{array}}
$$

$\lvert\mathcal{M}\rvert$	Calls & Redos	Secs.
20	166,984	0.05
40	626,365	0.23
80	2,428,093	0.89
20	238,565	0.09
40	1,823,835	0.67
80	14,413,944	5.17

3 Conclusion

This paper proposes a relational model checker in which variable assignments are delayed and dynamically updated. This allows for a more efficient evaluation of complex DRSs in larger models than in the standard approach, with particularly significant gains for plurals and for complex NP coordination phenomena. A Prolog implementation provided some performance results.

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