PROPORTION QUANTIFIER INTERPRETATIONS OF INDEFINITES AND ENDOCENTRIC RELEVANCE RELATIONS

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Abstract
We show how evidential relevance relations between restrictor predicates N and attribute predicates induce proportion quantifier interpretations of indefinite $DetN$. A paradigm-setting role is played by 'Protagorean' determiners, *many* and *few*, which intuitively designate large and small quantities, but equally express positive and negative relevance. The thesis is explicated and demonstrated within a Decision-Theoretic Semantics (Merin 1999a) and extends Mostowski's (1957) Generalized Quantifier [GQ] framework. Some puzzles about monotonicity of natural language GQs are solved, and several theses about such GQs are advanced and verified.

1. Introduction[1]

In Generalized Quantifier Theory [GQT] as originally presented by Mostowski (1957) and Lindström (1966), quantifiers Q are uniformly and literally specified in terms of quantities, i.e. numbers. They are defined as constraints on the respective or relative numbers of elements of sets of individuals, i.e. on cardinalities $|X_i|$ of n-tuples of sets X_i $(n \geq 1)$ or ratios of them. We develop this approach, taking account of 1980s transfers to natural language by Barwise, Cooper, Keenan and others. Our focus will be Qs whose natural language expression correlates are determiners ('*dets*') in their most common and simple occurrences.[2] Thus, let $Q(A, B)$ be a quantifier proposition denoted by a sentence schema *Det A are B*, for short: $D(A, B)$, e.g. *All artists are beekeepers.*[3] Then the pertinent sets will be elements of the boolean set-algebra, $\mathcal{F}$, generated by A and B.[4,5] Apart from A and B, the sets AB $(= A \cap B)$, $A - B$ $(= A \setminus B)$ will be of particular interest among the elements of $\mathcal{F}$, whose maximal element, Ω, is defined by $\Omega = X \cup \overline{X}$ for any X in $\mathcal{F}$, and whose minimal element, $\emptyset$, is given analogously by $\emptyset = X \cap \overline{X}$.

Repr. in: *Belgian Journal of Linguistics* 19 (2005), 147–186.

For the familiar, 'logical' quantifiers, $Q(A, B)$ can be specified in terms of just *one* of $|A \cap B|$ (write: $|AB|$) or $|A - B|$. Thus, $\exists(A, B)$ [is true] iff $|AB| \neq 0$; $\forall(A, B)$ iff $|A - B| = 0$; $\neg\exists(A, B)$ iff $|AB| = 0$; $\neg\forall(A, B)$ iff $|A - B| \neq 0$ (i.e. iff $|A - B| > 0$). These generalized quantifiers [GQs] are still 'first-order definable', i.e. specifiable without basic appeal to numbers, in terms of boolean, set-algebraic relations: $\exists(A, B)$ iff $AB \neq \emptyset$, $\forall(A, B)$ iff $A\bar{B} = \emptyset$, and so on (see e.g. Westerståhl 1985).

But MOST is not so definable. Here number—indefinite in size—is essential: MOST(A, B) iff $|AB| > |A - B|$. Note that the condition $|AB| > |A - B|$ is equivalent to $|AB|/|A| > 1/2$ when $A \neq \emptyset$. Hence, MOST already introduces the notion of proportion, as will HALF-OF-ALL and TWO-THIRDS-OF.

Proportion, in turn, is explicated in mathematics as a special case of 'measure', namely 'normed measure', also known as 'probability measure' (cf. e.g. Halmos 1950).[6] Cardinality, too, is a special case of measure, namely 'count measure'. Finally, note that any set-algebra is a special case of a boolean algebra.

We shall now treat GQs uniformly as constraints on measure on boolean algebras. In doing so we capture in a uniform way semantics of *Det N*, not only for count N, but also for mass N, as in *much love* and *little money*.[7] But just as importantly, if not more so, we thereby return to the quantitative roots of GQT in Mostowski (1957), which have been somewhat obscured from view and indeed cut back in transport to natural language semantics.[8]

A *measure* $\mu : \mathcal{B} \rightarrow \mathbb{R}_{++}$ is a function from a boolean algebra $\mathcal{B}$, with minimal, 'null' element $\mathbf{0}$ and maximal or 'top' element T, to the non-negative extended real numbers, $\mathbb{R}_{++} = \{r \in \mathbb{R} : r \geq 0\} \cup \{\infty\}$,[9] and which satisfies the axioms

(M1) $\mu(\mathbf{0}) = 0 \neq \mu(T)$;

(M2) $AB = \mathbf{0} \rightarrow \mu(A \vee B) = \mu(A) + \mu(B)$.

Measure is *regular* in the sense of Carnap iff $\mu(X) = 0 \rightarrow X = \mathbf{0}$ [MR]. The cardinality function $|\cdot|$ on sets is a regular measure and is known as *count measure* under the usual embedding of the natural numbers into the reals. A measure μ is *normed* iff $\mu(T) = 1$ [MN]. Its range is therefore $[0, 1]$. Hence, if $C \subseteq A$ and thus $\mu(C)/\mu(A) \in [0, 1]$, the quotient $\mu(C)/\mu(A)$ defines a normed measure, $\mu'(\cdot)$, on subsets of A; the normalizing factor being $1/\mu(A)$. Normed measure is also known as *probability measure* regardless of interpretation, since M1, M2 and MN axiomatize the finitely additive probability calculus. Under the probability interpretation, normed measure $\mu(\cdot)$ is usually written $P(\cdot)$. If A is a proposition,[10] then $P(A)$ denotes the *probability* of A, and $P(T) = 1$ always. The expression $P(B|A)$ denotes the *conditional probability* of B given A, defined by $P(AB)/P(A)$.

Turning now to data, we attend only to those (aspects of) det phrases which are properly quantificational.[11] Such dets must satisfy Permutation Invariance (PERM) or more generally Invariance under boolean Isomorphisms (ISOM). On sets, ISOM says: Let f be any bijection (i.e. a one-to-one total, i.e. invertible mapping) from universes T to T' of entities with $A, B \subseteq T$. Then [ISOM]: $Q^T(A, B)$ [is true] iff $Q^{T'}(f(A), f(B))$ [is]. PERM labels the important special case where $T' = T$ and where f is therefore any permutation π of T.[12] ISOM is part of any definition of logical constant, but it really also defines the concept of a property, which is essentially trans-individual. PERM/ISOM is equivalent to specification of quantifiers in terms of numerical relations (cf. Mostowski 1957, Westerståhl 1998). We call '*q-dets*' those dets which satisfy PERM/ISOM. Deictic and anaphoric dets (or uses of dets) such as *a* and *the* are not q-dets (nor q-det readings of dets).

Outstanding among further properties that GQs may satisfy are 'Extension' [EXT]: $Q^T(A, B)$ iff $Q^{T'}(A, B)$ for $A, B \subseteq T$ and $T \subseteq T'$; and '(Left) Conservativity' [LCONS]: $Q^T(A, B)$ iff $Q^T(A, AB)$. Received opinion is that natural language dets satisfy both, and that *many* and *few* are exceptions which may not even be determiners. We argue that these exceptional q-dets are important for understanding *all* q-dets.

2. 'Weak' and 'Strong' Readings of Determiners

Natural correlates of quantifiers Q come in two broad kinds, 'strong' and 'weak' (Milsark 1977). Weak readings of dets D, said Milsark, are 'cardinal', and go with 'transient' or 'accidental' (i.e. *estar*-type) predicates. Strong readings are 'quantificational', paraphraseable partitively '*DET of the*' and go with 'stable' or 'essential' (i.e. *ser*-type) predicates.[13]

Milsark's syntactic criterion for strong dets was inadmissibility in what might be called 'there-be'-sentences [TBSs]. Weak dets include those traditionally classed as indefinites, notably *a* and *some*. Curiously, the archi-definite det *the* is not securely strong by the TBS test.[14] Familiar formal criteria for the distinction are due to Barwise and Cooper (1981) [B&C] and Keenan and Stavi (1986) [K&S].[15]

Keenan (1987) notes that the B&C criterion for strong dets is a class of sentences without communicative uses, e.g. *All cats are cats.* Worse yet, I feel, would be *Most cats are cats.* This seems to have *cats* designate perceptually apparent cats in the A-restrictor position, and real cats in the B-predicate. However, Keenan's own criterial class, for weak dets, was no less strange phenomenologically. *Some cats in the bar exist* invokes a host of ghostly, non-existent cats drinking alongside their real kinsfolk. And **No cats in the bar exist* would not even be English. Empirical instantiations of the K&S definitional criterion also have a distinctive feature related to this existential strangeness: readings of 'weak' dets in *Det AB exist* retain a proportional reading. Thus, (i) *Some ants in the bar exist* is equivalent, under any natural reading of (i), to (ii) *Some of the ants in the bar exist.* A *proportion* of inexistent ants among ants in the bar is suggested by (i).[16]

We shall now more generally propose conditions for a proportional (i.e. 'strong') reading to emerge for 'weak' determiners. Our theoretical framework will be the doxastic fragment of Decision-Theoretic Semantics [DTS] and Pragmatics [DTP] (Merin 1994b, 1999a).

3. Portion and Proportion

Let us proceed in terms of a theoretical distinction familiar under the label 'cardinal' vs. 'proportional', though in terms which make full use of the concept of 'quantity' at the heart of Mostowski's GQT:

- HYPOTHESIS 1: *Portion quantifiers*, commonly known as 'cardinal' or, more generally, 'intersective' (or loosely: 'weak') quantifiers, are constraints on measure, i.e. on the values of measure functions $\mu(\cdot)$.

- HYPOTHESIS 2: *Proportion quantifiers* commonly known as 'proportional' (or loosely: 'strong') quantifiers are constraints on quotients of measure, i.e. on quotients of values of measure functions $\mu(\cdot)$. The constraints are such as to make them constraints on the values of normed measures.

Portion quantifiers are so called because they measure extensions. They specify that so-and-so-many individuals have a certain property, or that there is so-and-so-much stuff of a particular kind. Proportion quantifiers, by contrast, have for criterial values rational numbers or real numbers approximated by rationals. A rational number $q = a/b$ is a function that takes numerical arguments x to values y with q solving the equation $y = q(x)$ such that $ax = by$. In material terms, a proportion is therefore a function from portions of something to subportions of it.[17]

The warranted assertability- or truth-conditions of *portion* quantifiers instantiate the schema[18]

(1) $D_w(A, B)$ [is true] iff $\mu(AB) \in \mathrm{WCON}_D$

where WCON_D (for 'weak') is a constraint set specific to the particular weak or weakly read determiner $D = D_w$ and where measure is count-measure, $|\cdot|$, for count noun A.[19] The warranted assertability conditions or truth-conditions of *proportion* quantifiers instantiate the schema

(2) $D_s(A, B)$ iff $\mu(AB)/\mu(A) \in \mathrm{SCON}_D$

where SCON_D (for 'strong') is again D_s-specific, $D = D_s$ being a strong or strongly read determiner.

Now, since $AB \subseteq A$, we have $\mu(AB)/\mu(A) \leq 1$. Hence $\mu(AB)/\mu(A)$ is a normed measure: the normed measure of B under the norm induced by $\mu(A)$. When A and B are sets of individuals, it specifies the proportion of Bs in the population A. Analogous relations hold for mass continua, e.g. the volume proportion $\mu_V(E)$ of ethanol, E, in vodka, V. Let us notate $\mu_A(B) =_{df} \mu(B|A) =_{df} \mu(AB)/\mu(A)$. Then two ways of rewriting (2) are

(2′) $D_s(A, B)$ iff $\mu(B|A) \in \mathrm{SCON}_D$.

(2″) $D_s(A, B)$ iff $\mu_A(B) \in \mathrm{SCON}_D$.

Here are two prominent and simple examples:

(3) $All(A, B)$ iff $\mu(B|A) = 1$.

(4) $Most(A, B)$ iff $\mu(B|A) > 0.5$.

Since $\mu(B|A)$ is defined only if $\mu(A) > 0$, A cannot be the $\mathbf{0}$-element—in set-algebras: not $\emptyset$—if proportional $D(A, B)$ is to have warranted assertability or truth conditions. Quantifier readings which demand $A \neq \emptyset$ in order to be assigned a truth value are known as 'presuppositional'. Our definition now *entails* the following

Observation: Proportion quantifiers are presuppositional.

On the Fregean interpretation, *all* is non-presuppositional. When the restrictor, A, is empty, $\forall(A, B)$ will be true by default, as a set-theoretic instance of *ex falso quodlibet*. Accordingly, Mostowski defined $\forall(A, B)$ iff $|A\overline{B}| = 0$. Adaptions to natural language in the wake of Barwise and Cooper (1981) have translated this into the purely set-theoretic or boolean formulation $\forall(A, B)$ iff $A\overline{B} = \mathbf{0}$. Keenan (1987) has labelled the dets having truth conditions specified in terms of $A\overline{B}$ 'co-cardinal' and more generally (with complex dets such as *all but two* included) 'co-intersective'. I will defend the following

Hypothesis: The class of co-cardinal and, more generally, co-intersective determiners in natural languages is empty.

That strong dets tend to presuppositional readings has been asserted in view of the likes of *All survivors of Napoleon's 1812 campaign are regulars at Disneyland.* Explanations have been proposed in terms of processing order (Lappin and Reinhart 1988) and implicature (Abusch and Rooth 2004).[20] However, the assumption that strong quantifiers are literally, i.e. quantitatively, proportional offers a more elegant

> **Explanation**: Division by 0 is undefined in the fields of real and rational numbers.

Previous arguments for proportional readings of *all* and *every* have not, I think, gone as far as actually sustaining the Hypothesis,[21] for it meets putative counterexamples such as

(5) {a. All trespassers/b. Every trespasser} will be prosecuted.

However, the widespread belief that such examples support the Fregean interpretation of *all* and *every* is illusory, and our framework explains why. The key observation is that (5) cannot give us a frequency judgment. No-one can count what has not yet happened, nor therefore establish actual frequencies. The most we can hope for is to be able to entertain *expected frequencies.*[22] Actual frequencies are obtainable in principle for

(6) All trespassers were prosecuted.

By contrast, what (5) says is that the conditional probability for future trespassers being prosecuted is unity. The formal statement of this will require some care in the interpretation of familiar and new symbols. I first give a paraphrase that might fit many determiners: 'For any randomly chosen individual x, the probability that x will be prosecuted, if x is a trespasser, is in the interval $[a, b]$'. In our example $[a, b] = [1, 1]$, i.e. the number 1. Formally, and with care required in interpreting notation,

(7) $\forall x[P(prosecuted(x)|trespasser(x)) = 1]$ for $P \in \mathcal{P}_x^{sym}$.

If the example had been

(5′) Most trespassers will be prosecuted,

we should have written, most liberally,

$$(7') \quad \forall x[P(prosecuted(x)|trespasser(x)) > 0.5] \text{ for } P \in \mathcal{P}_x^{sym}.$$

The intuitive condition on $\forall x$—random choice—is visibly important in this non-extreme case. If we knew for certain of some x, dub it c, that it will *not* be prosecuted if it trespasses, the conditional probability

$$P(prosecuted(c)|trespasser(c))$$

would be zero and hence the bare universal of statement ($7'$), i.e. without the condition $P \in \mathcal{P}_x^{sym}$, would be false.[23] The random choice condition is explicated by the formal condition $P \in \mathcal{P}_x^{sym}$. This says that $P(\cdot)$ is 'symmetric' in staying invariant under permutations of the universe over which x ranges.[24] PERM is satisfied. In other words, the possible instances of x should be 'exchangeable' or 'interchangeable' *salva probabilitate*. An alternative or shorthand way of putting this is that x should be an *arbitrary* individual instantiating the properties in question. 'Arbitrary' means that nothing particular is known about it apart from the information held in the predicates, or rather nothing which would interfere with the probability assignment based on this information.

Exchangeability or symmetry means: only *numbers*, not the order of instances matter in the formation of a reference class for induction to a new instance, x. An intuitive way of putting the condition in ($7/7'$) is indeed that x should be randomly chosen from the pertinent future populations, here of trespassers.[25]

Having made plausible, I hope, the formal representation, I shall now make use of it. Modals such as *will* impose a mandatory interpretation of putative frequencies in terms of epistemic probabilities, i.e. degrees of belief. The prediction of (7) is that (5) is fine only as long as the epistemic probability of someone trespassing is non-zero. If it were zero, (7) would be undefined. To test the prediction, simply utter *There will be no trespassers*, right before uttering (5), and watch the latter's felicity go. The hypothesis that none of the known natural language dets are co-cardinal remains unfalsified.[26]

4. Indefinites and 'Strong' Readings

Determiner phrases *DetN* that have weak readings can have strong readings too, which paraphrase *Det of the N* salva veritate. Exemplars of bona fide weak English determiners with quantifier readings are *a few, some, several, five, many, not a few, few, no* as shown by the Milsark test. All of them can freely instantiate the variable DET in *There are* DET *cats in the garden.* All of them can occur in the collocation DET *of the*, except *no* which must be amended to *none*.[27] Availability of strong readings of plain, prima facie weak dets is most familiar from accounts of *many*. This is held to have upward of three readings, but in return is not classed as a determiner at all by Keenan. Westerståhl (1985: 401–405) offers (modulo order of presentation and numbering)

(8) $Many_{1,k}(A, B)$ iff $|AB| > k$ for contextual k ;

$Many_{2,k'}(A, B)$ iff $|AB| > k' \cdot |A|$ $(0 < k' < 1)$;

$Many_3(A, B)$ iff $|AB| > [|B|/|T|] \cdot |A|$;

$Many_{4,f}(A, B)$ iff $|AB| > f(|T|)\,(0 < f(|T|) < |T|)$;

$Many_{5,k''}(A, B)$ iff $|AB| > k'' \cdot |B|$ $(0 < k'' < 1)$;

$Many_{6*,x,y}(A, B)$ iff $Many_x(A, B)$ and $Many_y(A, B)$
for $(x = 2, k',$ or $3;\ y = 1, k,$ or $4)$.

Indices k, k', etc. will be tacitly understood in what follows.[28] Note for future reference that $Many_3$ is equivalently specified by

$$Many_3(A, B) \text{ iff } |AB|/|A| > |B|/|T|$$

when $|A| \neq 0$, which is always presumed for readings that are essentially proportional. Note also for now that, when $k = k' \cdot |A|$, $Many_2$ becomes a special case of $Many_1$. Similarly, $k' = |B|/|T|$ makes $Many_3$ a special case of $Many_2$.[29] We thus observe:

> Conditions for readings $Many_1$,$Many_2$, and $Many_3$ are simultaneously satisfiable, pairwise and jointly.

Moreover, on dropping Westerståhl's condition $0 < f(|T|) < |T|$, $Many_3$ becomes a special case[30] of $Many_4$, namely $f : x \mapsto |A| \cdot |B| \cdot x^{-1}$. A special case satisfying the condition could be $Many_1$. Note finally, that analogous readings can be had for *few*, identical modulo conversion of '>' to '<'. Example:

(9) $Few_3(A, B)$ iff $|AB| < [|B|/|T|] \cdot |A|$.

In view of their formal unruliness, Keenan has denied *many* and *few* determiner status. He classes them as adjectives (see K&S). Now, *His faults are very* {*few/many*} does attest adjectival occurrence, but I should yet opt for categorial polysemy.[31] Retaining *many* thus, we recall that $Many_2$ is already proportional. It is so intuitively, since its nearest explicit paraphrase, ignoring intimations of anaphoricity, would be *Many of the A are B*. And it would be so formally, give or take definedness for empty A. Alternatively, then:

(10) $\mathit{Many}_{2'}(A, B)$ iff $\mu(B|A) > k'$ $(0 < k' \leq 1)$.

This differs in assertability conditions from $\mathit{Many}_2(A, B)$ only in being undefined when A is empty, whereas $\mathit{Many}_2(A, B)$ will be false.

All of Many_x $(x \neq 1)$ are motivated by the following facts. Westerståhl (1985) remarks a difference in the intuitive meanings and assertability of[32]

(11) Many Scandinavians are Nobel laureates.

(12) Many Nobel laureates are Scandinavians.

You might assent to (12) while rejecting (11). Under a reading Many_1 this would be inexplicable; indeed contradictory, given context constancy. However, Many_2 and a fortiori $\mathit{Many}_{2'}$ afford an explanation. Let N the set of Nobel prize winners in Literature. Let S the set of Scandinavians. Clearly $0 < |NS| < |N| \ll |S|$. At a guess, $|N|$ and $|S|$ differ by a factor of 10^5 to 10^6. Suppose we associate a context of judgment for the pair of sentences with some fixed k'. Then

(11F) $\mathit{Many}_2(S, N)$ iff $|NS| > k' \cdot |S|$ $(0 < k' < 1)$;

(12F) $\mathit{Many}_2(N, S)$ iff $|NS| > k' \cdot |N|$ $(0 < k' < 1)$.

For any k' (12F) holds whenever (11F) does. However, there are k' such that(12F) holds while (11F) does not.[33]

Fernando and Kamp (1996) [F&K] point out that the asymmetry behind Many_2 survives in TBSs:

(13) There are many lawyers who are criminals.

(14) There are many criminals who are lawyers.

In Keenan's "$D(X, Y)$ iff $D(XY, T)$" criterion for weakness, $[\![XY]\!] = [\![YX]\!]$ by definition. And so it would also be in F&K's suggested form, $D(T, Z)$, for TBSs, which contrasts with Keenan's $D(Z, T)$. What $D(T, Z)$ (let us expand $Z = XY$) offers, if the standard ordering is followed—i.e. left: restrictor and possible denominator argument, right: predicate—is the possibility of an abstract proportion reading with truth conditions

(15) $D(T, AB)$ iff $|AB|/|T| \in \Gamma$.

But for finite T (or, given a suitable measure, for infinite T and AB)[34] this yields the frequency of AB, which F&K supplement with a broadly analogous interpretation in terms of probability.[35] The gloss will likewise hold for q-det readings of *some*. To see how smoothly the proportion readings of weak dets fit into a conditional probability 'scale', pragmatize strong det *most*, somewhat in line with Ramsey (1929):

> *Most A are B* signifies: 'If you meet an A, there is an overwhelming probability that it (or he or she) will be a B'.

But now consider analogues where the pair (*most, overwhelming*) is replaced by (*many, very significant*) and (*some, non-negligible*). In the usual context of use, each would be larger than expected. We shall see that in such contexts this means: A is positively relevant to B (cp. Merin 1999a, 2002 on *even*).

5. TBSs and 'Weak' readings of Weak Dets

F&K's empirical discovery that TBSs do not ensure 'cardinal' or 'existential' readings of prima facie weak dets implies that neither of $Q(AB, T)$ or $Q(T, AB)$ can always be the (explanatory) quantifier schema induced by TBSs. Consider

(16) a. There are many Scandinavians who are Nobel laureates.

b. There are many Nobel laureates among Scandinavians.

and analogously with *Nobel laureates* and *Scandinavians* exchanging position. We observe (a) iff (b). Paraphrases (b) confirm that *There are many A who are B* is indeed evaluated in terms of the proportion of As who are Bs, i.e. as a relation involving $\mu(B|A)$.[36] Pure cardinal '$|AB|$' readings for TBS are, I think, guaranteed only by the likes of

(17) There are many individuals who are both criminals and lawyers.

Thus, 'individual who is both an A and a B' is the best English equivalent for the conjunctive property AB. The det *both* ensures commutativity for A *and B*. Indeed, (17) is synonymous with *There are many individuals who are both lawyers and criminals.* Moreover, the same 'cardinal' reading survives in

(18) Many individuals are both lawyers and criminals

which best extends F&K's logical form $D(T, Z)$ for TBSs on expanding $Z = AB$. But of course (18) is a sentence of form $Many(A, BC)$ where A happens to be the set of individuals, presumably of human beings. (Unlike the notion of an all-comprehensive set, this is a coherent notion in familiar working set theories.) Hence, the defining property of A (presumably that of being human) is special at most in terms of relevance relations, to which we come below. One difference between 'individual who is both an A and a B' and 'As who are Bs' is, semi-theoretically, a suspicion that the proportion of ABs to As is engaged in the latter. A more directly apprehended one is a difference in ostensible relevance between A and B.

6. Frequency and Probability: Reflection of Proportion and Projection of Probability

Futurate *all*-sentences such as (5) offered one glimpse of relations between probability and frequency. We had no frequencies, only probabilities. In probability theory there is, of course, a close relation between frequencies and probabilities. Still, it is not so close as to bear a definitional relationship. The two concepts are distinct, but related in inference and indeed shackled together by limit theorems when absolute numbers get large.

We use the proportion information in statistical data to form beliefs by a process of inference, and for the purposes of further inference and action. We use sample size information to establish how resilient to change our beliefs should be in the light of future evidential instance-events (Jeffrey 1965).[37] In the absence of extraneous knowledge or suspicions, we generally let our probabilities become numerically identical to proportions in the sample observed. Proportion is 'reflected' into our probabilities.

'Reflection of Proportion' [ROP], as I propose to call it, is a standard transfer principle from proportion or frequency assessments $\mu(\cdot|\cdot)$ to judgmental probabilities $P(\cdot)$ on widest-scope universally quantified property abstracts. It says: 'assess your epistemic probability for an A being a B as being equal to the proportion of As that are Bs'. In the compact notation introduced for (7), above:

(19) [ROP] $\forall x[P(Bx|Ax)] := \mu(B|A) \quad \text{for } P \in \mathcal{P}_x^{sym}.$

Read schema '$v := z$' as 'Set the value of v to z'. Read '$\forall x$' intuitively in the way suggested for (7), i.e. as specifying 'for arbitrary or randomly chosen x'. An equivalent formulation of ROP is in terms of the single case probability for an arbitrary individual d, arbitrary in the abovementioned epistemic sense:

$$(19') \quad \text{[ROP]} \;\; P(Bd|Ad) := \mu(B|A) \quad \text{for } P \in \mathcal{P}_x^{sym}.$$

Next, note that $\mu(B|A)$ is a proportion of set-sizes, here of AB to A, while the clauses Bx, Ax, and Bd, Ad stand respectively for predicates denoting properties and for sentences denoting propositions.[38] For the special case $A = T$, with T the maximal element in the algebra $\mathcal{F}$ of the underlying measure space, '$\forall x[P(Bx|Tx)] := \mu(B|T)$' rewrites to '$\forall x[P(Bx)] := \mu(B|T)$'. Westerståhl's right-hand side of the condition for $Many_{4,f}$ of (8), as interpreted by him, is in effect of the form $k \cdot \mu(B|T)$.

The next modification to ROP hinges on a problem that would be most acute in a futurate context. Proportions $\mu(B|A)$ interpreted as frequencies are *ceteris paribus* understood as obtaining in our world. But suppose we do not know what the world is like in this respect, or suppose we must consider future worlds or propensities. In such cases, the best we might have to go on would be an expectation of proportions. To 'kill' a variable over 'worlds' or maximally specified knowledge states, we should set our probability to the probability-weighted average of these proportions, frequencies (cf. Halpern 1990, Bacchus 1990, Merin 1996, also 2006 on F&K). This is in essence the procedure legitimized by de Finetti's theorem.

But now consider the possibility of the inverse inferential process, call it 'Projection of Probability' [POP]:

$$(20) \quad \text{[POP]} \;\; \mu(B|A) := \iota y \forall x[P(Bx|Ax) = y] \;\; \text{for } P \in \mathcal{P}_x^{sym}.$$

This says—again modulo the interpretive explanations and qualifications noted for ROP: 'assess the proportion of As that are Bs as being equal to your probability for an A being a B'.

The abstract justification for the transfer in both directions is found in the Representation Theorem of de Finetti (1937) (cf. Jeffrey 1965, 2004). What it says is that we can represent our subjective probabilities, under conditions which amount to permutation invariance (ISOM) as expectations of objective chances, and these in turn can be operationalized least obscurely in terms of frequencies of property instantiations, i.e. proportions.

ROP is standard statistician's fare. POP introduces 'objectification' (Jeffrey 1965) as a possibility and exploits it by way of the inverse rhetorical process:

> **Thesis** (Merin 2001a,b): Putatively intersubjective, rhetorical and thus highly intensional relevance relations serve to induce putatively objective and extensional quantificational relations.

In the case of *many* this would open the possibility that $Many_2(A, B)$ is, at root, a judgment of conditional probability projected onto frequencies. Roughly speaking, it would say that the probability that an A is a B is significantly high. $Many_3(A, B)$ would then be, at root, a judgment that A is positively relevant to B.[39] In the context of verbal expression, the latter is, above all, a rhetorical judgment. In the case of *many* and, dually, *few*, turning the tables on ROP by way of POP seems plausible enough *faute de mieux*.

But there is also a more interesting thesis. This is that *all* proportion quantifiers admit of such a projectivist treatment.[40] I say: 'admit of such a treatment'. I do not have to say: 'this is what they really are'. The de Finetti season ticket does not, formally, discriminate between either Projection or Reflection. Opting for admissibility is sufficient for all the explanatory purposes I consider.

Just as in measure the natural progression of exposition is from portions to proportions to differences or quotients of proportions, in probability a very natural progression is in the opposite direction. This is because the fundamental concept that distinguishes probability theory from general measure theory is the role played therein by a complementary pair of concepts: *dependence* and *independence*. But these are synonyms for relevance and irrelevance, respectively, of which the evidential kind is a prominent special case. And evidential relevance is always equivalent to quotients or differences of conditional probabilities. So the idea would be to *start* with relevance. We then analyse relevance as functions of probabilities. In projecting probabilities onto frequencies, i.e. proportions, we are projecting onto quotients of portions. In certain contexts that give us some portions, we cash out proportions into other portions.

Having explored technicalities underlying this process in the body and footnotes of this section, I proceed as simply as possible, not least in notational polysemy. Expand to taste.

7. Probability and Relevance

Recall that Many_3 as defined predicts $Many_3(A, B)$ iff $|AB| > (|B|/|T|) \cdot |A| = (|B| \cdot |A|)/|T| = (|A|/|T|) \cdot |B| < |AB|$ iff $Many_3(B, A)$. As Westerståhl noted, $Many_3$ will not thus explain meaning and assertability differences between the two forms. Still, $Many_3$, unlike $Many_x$ $(x = 1, 2)$, affords a non-arbitrary criterion. We often do have a reasonable estimate of the size of the relevant universe T alongside those for A and B. The cost, so Westerståhl, is loss of LCONS and EXT. Loss of EXT must worry any right-thinking person. $Many_3(A, B)$ implies that, merely by varying the size of the universe T while keeping that of $A \cup B$ constant, we can change its truth value.[41] Moreover, complicated-looking $Many_3$ has an attraction not hitherto recognized. It has a most natural interpretation by ROP and POP:

> Being an A is *positively relevant* to being a B (and vice versa) in the sense of being evidence for it.

At this point, the interpretation in terms of probability measure has entered our considerations. $Many_1$ had not afforded normed measure. By contrast, ROP performed on the basis of $Many_{2'}$ yields

$Many_2(A, B)$ iff $P(B|A) > k' > 0$.

This says that the probability of something being B conditional on its being A exceeds some positive k'. The comparison of our examples $Many_2(N, S)$ and $Many_2(S, N)$ now also leads to a natural interpretation of the difference in intuitions: The (conditional) probability of a Nobel laureate being Scandinavian is much higher than the (conditional) probability of a Scandinavian being a Nobel laureate, briefly: $P(S|N) \gg P(N|S)$. Next, again utilizing ROP, $Many_3$ yields[42]

$Many_3(A, B)$ iff $P(B|A) > P(B)$.

But, standardly, one says with Keynes, Carnap, Jeffrey and others:

A is *positively relevant* to B iff $P(B|A) > P(B)$.

A is *negatively relevant* to B iff $P(B|A) < P(B)$.

A is *irrelevant* to B in any other case.

These three nominal relations of evidential relevance are symmetric. In obvious shorthand: A xrel $B \rightarrow B$ xrel A, (x = pos, neg, ir, $\emptyset$). A posrel B means roughly: if $P(\cdot)$ represents your current degrees of belief or probabilities, then, on getting to know A (and just A), your degree of belief in B or probability for B should go up. In property terms: on simply learning that something is an A, your probability for it being a B should rise. Analogously for negrel and irrel. Thus, $Many_3$ of (8) expresses the frequency correlate of positive evidential relevance. By ROP, we obtain from frequencies a formulation in evidential terms. By POP we project evidential relations onto frequencies.

Next, we address asymmetry intuitions in terms of relevance.

> **Proposition**: Under quantitative relevance functions that are not symmetric in their arguments, the less probable of two propositions A and B that are positive to one another is more positive to the more probable than conversely:

$$\mathrm{rel}^i_B(A) > 0 \rightarrow \mathrm{rel}^i_B(A) > \mathrm{rel}^i_A(B) \quad \text{iff} \quad P^i(A) < P^i(B).$$

Functions validating this are e.g. $\mathrm{relD}_B(A) =_{df} P(B|A) - P(B)$, $\mathrm{L}_B(A) =_{df} P(A|B)/P(A|\overline{B})$ and $\mathrm{r}_B(A) =_{df} \log[P(A|B)/P(A|\overline{B})]$. Since $\mu(N) \ll \mu(S)$, and therefore $\mu(N)/\mu(T) \ll \mu(S)/\mu(T)$, we have by ROP also $P(N) \ll P(S)$. Being a Nobel laureate (N) is more positively relevant (evidentially, not causally) to being Scandinavian (S) than being S is to being an N.

The relation between this fact and the corresponding one for $Many_2$ is obvious for the relD relevance function. Moreover, whatever naive intuitions support $Many_3$ will also support the yet more complex-looking but equivalent relevance specification by ROP,

$$Many_7(A, B) \text{ iff } |AB|/|A\overline{B}| > |B|/|\overline{B}|.$$

This says that $Many_7(A, B)$ is true or assertable iff the 'Bayes Factor' of A in favour of B is positive, for another way of writing this is $Many_7(A, B)$ iff $\mu(A|B)/\mu(A|\overline{B}) > 1$, yielding by ROP

$$Many_7(A, B) \text{ iff } P(A|B)/P(A|\overline{B}) > 1.$$

Now we can go a step further to

$$Many_{8,k''}(A, B) \text{ iff } \mu(A|B)/\mu(A|\overline{B}) > k'' \ (k'' \geq 1).$$

Here, k'' might be such that $Many_{8,k''}(A, B)$, but not $Many_{8,k''}(B, A)$ its criterion.[43] The Proposition tells us that in this case, A is smaller than B.

The import of these findings is, I think, that a comparative concept of relevance, not simply a nominal concept of it, can reflect in robust intuitions. But if this is so, there is also hope for ranking determiners in terms of evidential relevance relations. And this will raise the question: Relevance to what?

8. Exocentric and Endocentric Relevance Relations

Pure portion quantifier sentences, S, i.e. pure portion readings of sentences S of schematic form $D(A, B)$, offer no scope at all for relevance relations between A and B to be expressed by the sentence so construed. Yet even then $[\![S]\!]$ might be relevant to some other proposition of interest, H. Call such a relevance relation between S or some sub-clause of S and some H lexico-syntactically external to S *exocentric* (to S).[44]

There are also relations between elements of a paradigm of quantifier sentences such as the above, e.g. inductive relations to instances of the sentence schema $All(A, B)$ (Merin 2003b). Instances of it express a deterministic, lawlike relationship or at any rate a universal generalization likely to be explained by such a law. For example, we might compare the relevance of each of $Some(A, B)$ and $Many(A, B)$ to $All(A, B)$ in a context in which the utterer of the first two has not yet sampled all of the underlying universe.

Situations such as these must give rise to a pragmatics of assertion in which quantifier sentences inherit the tacit qualifier periphrased by 'at least' or 'at most'.[45] This need not hold for the first case. However, we might also consider the first case (or a special case of it) as follows: H is chosen such that $All(A, B)$ or its contrary $No(A, B)$ is most positively relevant to it. And then the relevance to H of other expression alternatives in the determiner paradigm is ranked. In many cases this ranking should be identical to the ranking in the condition of inductive support for the extreme item in the expression alternative set.

By contrast, proportion quantifier sentences of form $D(A, B)$ offer scope for relevance relations between properties A and B.[46] Similarly, subclauses of a complex sentence S or propositions canonically associated with phrasal constituents of S might be relevant (or mandatorily irrelevant) to one another.[47] Call such relevance relations *endocentric* (to S).

Endocentric relations can be at the root of certain exocentric relations. For example, suppose *Many cats walk* establishes a significant positive relationship between zoological felinity and a particularly robust form of locomotion. This very relationship may, in turn, be what makes *Many cats walk* positive for *The Baskerville outlet of Kat-Mart will be sited on Bramble Heights*. Below we shall see how endocentric relations between the A and B parts of $D(A, B)$ instances cohere with such relations. But focussing on endocentrics alone we make an

> **Observation:** Endocentric relevance relations between A and B are concomitant with proportion readings of q-dets D occurring in sentences $D(A, B)$.

There is no simple q-det which lacks a proportion reading.[48] All simple q-dets, give or take suffixation of *-one*, admit interpolation of the string *of the* between *Det* and N'. All of them admit of readings paraphraseable in this way even when the string is not interpolated.[49] This knowledge will come in useful for examples (21/22), analogous to Westerståhl's (11/12) for *many*:

(21) Some Scandinavians are Nobel laureates.

(22) Some Nobel laureates are Scandinavians

Recall Westerståhl's observation that (12) sounded more reasonable than (11). The same will, I think, still hold for (22) and (21). Like things might also hold for a variation on F&K's populations for (13/14), introduced after complaints from the legal profession: (i) *Some academics are criminals*, (ii) *Some criminals are academics*. All these examples introduce scope for intuitions primed by set-sizes, folk prejudice, and relative intrinsicity of predicates. So now consider true blandness of relative size and sentiment:

(23) Some acrobats are beekeepers.

(24) Some beekeepers are acrobats.

Here we get as close as we ever get to a cardinal reading by default, short of saying

(25) Some people are both beekeepers and acrobats.

This is presumably because there is no very obvious context of use for these sentences. For all that, even (25) has a proportion reading: the proportion here is that of acrobats-cum-beekeepers in the populace. This reading is not readily intuited, however, because it cannot induce non-zero evidential relevance of peoplehood to AB-hood when relevance is computed—as it ordinarily will be—with respect to the pertinent maximal universe of entities, the universe of people.

9. Determiner 'Scales'

$Many_3^T(A, B)$, when false or inassertable, can usually be massaged into truth by expanding T to a large-enough T'. A true instance can be falsified by contracting the universe. Adopting a dual definition for the corresponding reading of *Few*, we can falsify by expanding, and verify by contracting T.

For *Most* or *All* no such tricks, worthy of Protagoras, the Sophist, will work. In practice, though, we find reasonable bounds on T, and so *Many* and its kin retain their usefulness. For the other non-*No* and non-numeral dets, compatibility with numbers on the ground is enormous, too. Take

(26) DET Scandinavians are Nobel laureates

(27) DET Nobel laureates are Scandinavians.

No matter if we instantiated DET to *Many*, *Some*, *A few*, or *Few*: each instance would be consistent with actual and presumable numbers on the ground.[50] This suggests we try making progress in context-dependent semantics by looking to the tradition of 'structural semantics' (Saussure 1916, Lyons 1963), which contrasts with the 'denotational' tradition in drawing principally on relations within a paradigm of expression alternatives. Ducrot (1973) is indeed very much in this tradition. But, of course, the structural and denotational approach are by no means incompatible, and the issues of present interest will hinge on the choice of spaces of denotata. My proposal is to classify q-dets by ranking them

(a) by ostensible endocentric conditional probability $P(B|A)$ and
(b) by ostensible endocentric relevance e.g. $\mathrm{relD}_B(A) =_{df} P(B|A) - P(B)$.

Each of these will be induced by ROP or will induce frequencies and sometimes portions by POP. For constant $P(B)$, to which constant $\mu(B|T)$ will correspond by ROP or POP, positive relevance is an increasing function of $P(B|A)$. In turn, for constant $P(A)$ (given constant $\mu(T)$) or simply given constant $\mu(A)$, conditional probability $P(B|A)$ will be an increasing function of $P(AB)$ and $\mu(AB)$, respectively. ($\mu(T)$ cancels out here.)

Evidential relevance is the explication I have proposed for Ducrot's notion of *valeur argumentative* (Ducrot 1973, Merin 1994b, 1996, 1999a). And an ordering of quantifying determiners in terms of their ostensible value in argument is a key part of Ducrot's theory. His theory remained informal and by and large and it came with the idea—erroneous it seems—that there was no useful theoretical relation between logical, truth-conditional and argumentative properties of expressions.[51]

Anglo-American 'quantitative' or 'pragmatic' 'scales' were in parts intended to retain this relationship. They took off from quantifier and connective schemata (Fogelin 1967) and inherited the Fregean semantics of *some* and the Aristotelian meaning of *all*. Thus, $Some(A, B)$ was defined by $AB \neq \emptyset$, whose rigorous gloss 'At least one A is B' and quantitative translation $|AB| \geq 1$ is an inequality. Horn (1972) extended this distinctive property of logical *some* to numerals. The lexical meaning of *five* was, not $[\![five]\!] = 5$ but roughly, 'at least 5': $[\![five]\!] = \{x : x \geq 5\}$ or perhaps $[\![five]\!] = \epsilon x[x : x \geq 5]$. The reasons for this were (i) admissibility in collocation with suspender clauses *five if not indeed six/more* and (ii) an effect of negation, observed by Jespersen, namely that *not five* usually meant ‘less than five’. Scalar implicature was to induce final readings equivalent to plain 5 in unnegated contexts.

There is an alternative by way of Act-Based Relevance Orderings (ABROs; Merin 1999a 2003b), which I have offered not least because the ‘Horn scale’ approach turns out to be incoherent. As Sadock (1984) pointed out, elementary reckonings with numbers can no longer be part of English if the approach is right. It also fails to explain why in certain contexts not involving negation the ‘at least’ reading, if any there was, gives way to an ‘at most’ reading. Here are two examples, one imperatival from me, the other indicatival and in essence from Anscombre and Ducrot (1983). Example 1: (Imperative) Compare

(28) (a) Give me $5! vs. (b) All right then: take $5!

Assuming that people like money, we have a distinct intuition for (a) that I shall be happy to get more; and for (b) that I shall be happy to give less. The rationale is found in the ‘demand game’ of bargaining games as explicated by Nash (1953) and one might call it *preference monotonicity*. If you make a Claim, you ipso facto prefer having what you ask for to not having it; and you prefer having more of it. Dually for Concessions. Examples 2: (Indicative)

(29) The meeting was a success! (H)

10 people came (if not more). (S_1)

20 people came (if not more). (S_2)

(30) The meeting was a flop! (H')

10 people came (if that many). (S_1)

If, in response to these examples, yet in line with Horn’s proposal, we plead underspecification of lexical entry, the entry should presumably be $[\![ten]\!] = \epsilon x[x : (x \geq 10) \vee (x \leq 10)]$, which entails, e.g. $[\![ten]\!] = [\![eleven]\!] =$ *any natural number you like.*

I follow Ducrot, modulo explication and extension, in allowing for dual argumentative orientations for independently ordered domains of items such as temperatures or numerical quantities, and in ordering determiners by typical relevance, though I specify what typicality means.[52] I diverge from Ducrot, Horn, and ultimately Fogelin in not having two distinct orderings of ‘positive’ and ‘negative’ items which correspond to or extend the vertical sides of the traditional ‘Square of Oppositions’. Relevance to a proposition H (at issue), unlike entailment, induces a complete ordering of all elements of the proposition algebra which H belongs to. Irrelevance will be the zero point of the ordering and so there is a single scale, roughly as we know it from the integer or real numbers.

The specification of the canonical intermediate variable as indicating a large value or, on the negative side, a small value[53] already determines our choice of prototypical quantifiers. *Many* and *few*, which are the Protagorean quantifiers *par excellence*,[54] will serve as the role models for the scale. My provisional proposal[55] is a modification of $Many_3$ (i.e. Westerståhl's $Many_2$) namely

(31) $Many_{8,k+}(A, B)$ iff $\mu(A|B)/\mu(A|\overline{B}) > k$ $(k = 1 + r)$.

Here r is a small positive real number, or rather a value of a random variable (r.v.) X_r whose probability density peaks about the number r and which assigns zero density to values not exceeding zero. The underlying idea is that A should be positive to B and *very significantly* so. What 'very significant' means, and hence what number the r or X_r actually is, will no doubt depend on extra-epistemic considerations of preference, e.g. losses attending false decisions. These are familiar from statistical decision theory (see Merin 1994b:148–158 for brevity, or any textbook). *Few* will be dual to *many*:

(32) $Few_{8,k-}(A, B)$ iff $\mu(A|B)/\mu(A|\overline{B}) < k$ $(k = 1 - r')$.

These proposals differ from earlier versions[56] in making the relevance relation non-symmetric (B must still be relevant to A but need not be significantly relevant to it). They also differ in the almost non-commital strengthening of 'significant' to 'highly significant'. In fact, the difference would be committal only if a corresponding change were to be introduced in the semantics for its nearest neighbours, *most* and *some*.[57]

Note that A non-negrel B holds iff $\mu(B|A) \geq \mu(B|T)$ assuming $\mu(B|A)$ defined. Thus, an ordering of *all* the main q-dets by endocentric relevance is possible, provided we can assume constraints on the proportion of Bs in the universe. For the extremes we have seen them to be satisfied whenever the sentences so uttered could be informative at all. For the others, I propose that they be considered as default assumptions which hold at least for the mythical 'zero context' of utterances out of the blue, but also whenever the world can be represented credibly as conforming to them.

I prefer to specify meanings in terms that stay as close as possible to measure of physical proportions (where physicality applies) via POP or ROP rather than in terms of probabilities that make appeal in essential ways to counterfactual situations and notions of undirected 'surprise'. Actuaries setting insurance premiums look at proportions and sample sizes on the ground, and this is what perhaps we often pretend to be doing when expressing probabilities in the idiom of quantification.

Positive and negative non-numeral dets should leave a middle ground. The nearest we get to this is *a few*.[58] In the case of *a few* you would expect the distribution to be skewed to the left of that for *some*. The stochastic nature of this is vital. There could be situations where you would be within your rights to say that some people were there when exactly two were there, but you would need three or four to say ‘a few’. In view of such data, I proposed making *a few* relevantially neutral or unspecified.[59] So it would—ceteris paribus—present A as neither significantly positive nor significantly negative to B. It would be like *tepid* on Ducrot’s (1973) temperature scales.[60]

The difference between *some* and *a few* in terms of relevance emerges when we can rule out endocentric relevance to all intents and purposes. *Some people complain* is pretty much of the form $Some(T, B)$, since the only relevant universe T is one of people. (If you counted in robots and corporations, you would have to put emphatic stress on *people*.) This could be an argument towards *We must reduce noise levels*. By contrast, *A few people complain* might recommend tossing a coin whether or not to reduce noise. *Few people complain* is definitely against reducing noise. But again, although the probability distribution over possible proportions will have non-zero support almost everywhere, there will be differences in the relative amounts of support. The distribution might well reflect—by ROP, though perhaps imperfectly so—our experience of frequencies of actual uses.

10. Stochastic Partitions and ABROS: ‘Scales’

So far we have followed the tradition of specifying q-dets in terms of inequalities, though hints arose that this may be as problematic as it is in the special case of Horn’s concept of numeral. Here now is the alternative, and the role model for its base are the purest of quantity expressions, numerals.

The non-negative integer numerals partition the class of sets or count-type properties into equipollent sets corresponding to the natural numbers. Thus *five* means 5, and *a thousand* means 1000. If there is some fuzziness in practice, especially for high numbers, this will be dealt with pragmatically much like the hexagonality of France (see Merin 2003b on ‘exactly’).

The admissibility of suspender-clauses ‘if not more/fewer’ or, equivalently, disjuncts ‘or more/fewer’ is governed by whether or not the speaker adopts a maximizing perspective (appropriate e.g. for Claims of physical quantities) or a minimizing perspective (appropriate e.g. for Concessions of physical quantities). We saw examples from Ducrot in which ‘ten’ will take either slant, depending on rhetorical objective. The suspender will be ‘if not more’ when the party is being argued to be a success. It will be ‘if not less’ or ‘if that many’, when it is being argued to be a flop. The thesis to pursue, now, is an extension of that of Ducrot’s.[61]

Q-dets would not be specified, in terms of their act-independent semantic component, by inequality constraints. Rather, the inequalities would be superimposed on point-valued or fuzzy interval constraints within a semantics that is given by a pragmatics of social acts (Merin 1994a). Thus, simple (Gazdar 1979) use of *Many*, *Most* and *All* or *Every* would mark the utterance as an abstract Claim by default. So would *Some* and *A few* or mass *A little*, with the rider that these would preferentially be counterclaims to a denial. By contrast, *Few* would by default induce Concessions, while *No* would induce the special case of a zero Concession, a Denial.[62]

Endocentric relevance of A to B offers an ordering criterion in terms of direction that is quite independent of extraneous context. Here the proportion reading is all-important. But clearly, from an exocentric perspective, even proportion quantifiers take on a ‘portion’ type quality. For what $All(A, B)$ says is that the largest conceivable portion of As is B. Recall that a proportion is a ‘residuation’ of portions. Given a fixed portion A, the residuation will yield a portion AB, or equivalently a portion of Bs in the universe restricted to A.[63]

Thus, if our evidential context is one in which A is already given, the strong quantifiers *will* deliver portions. For *all*, this will be the maximal portion attainable. For *most* it will be smaller, but still very large and not utterly excluding a welcome windfall of more.

‘Welcome’ is important. The preference structure of the act-pragmatics implies that a Claim for a ranked item x is met if and only if x or some $y > x$ is conceded. This routine pragmatic transformation to “x or better, if there is better” generates a ‘cone’, $\{y : y \geq x\}$, diagnosed by admissibility of suspenders *if not indeed* y or by ‘less than x’ readings under negation. It will include as alternatives items preferred by the utterer under his current evidential or imperatival preference ranking. Thus, we can dissociate meanings into components specified by fuzzy equalities (see below) and by inequalities induced by act type.

Many will yield less than *most*, but still a relatively large portion. The portion yielded by *some* will not be insignificant; the portion yielded by *a few* will be just enough to either counter a claim of zero or to dampen a hopeful claim of *some* or to dampen a claim of *many*. That for *few* cannot be said to yield fewer than *some*, but the direction of exocentric argument is firmly towards minimization of the variable and the ‘cone’ of possibilities which are not excluded does leave possible a windfall of minimal quantity.

Recourse to proportional $Many_2(A, B)$ enabled Westerståhl to explain perceived asymmetries in assertability. A proportional reading $Some_2(A, B)$ could do likewise, albeit for a fainter effect. For suppose our warranted assertability criterion for plain portion *some* was not merely Fregean

($\mu(AB) = |AB| > 0$) or 'Fregean+1' for plural count nouns ($|AB| > 1$). Suppose that 'cardinal' quantificational *some*, instead, signified a minimal *significant* or standard unit portion. There are indeed good Saussurean reasons for supposing so. Consider count-det *a few* and mass-det *a little*. These appear to denote smaller portions than *some*. The test is their comparative politeness in requests, and impoliteness in offers.

I take it that quantifiers such as *some*, *a few* and *many* on their extensional portion readings are given as random variables, i.e. by rough probability distributions over quantities. The rightward (increasing) tail of the *some*-distribution will extend farther than that of *a few*. More importantly, its 'mode'—the portion-size point receiving highest probability mass or density, i.e. the most probable size—is to the right of the mode for *a few*. (Perhaps there exists experimental literature on this question.)

What is a standard or significant portion depends not only on external context—e.g. the perceived consequences of making a mistake—but also, least infrequently, on the size of what it is a portion of. But this is a familiar argument for $Many_2$ that arises well before non-symmetry of its arguments, A and B, is being considered. For *some*, we should have by analogy

(33) $Some_2(A, B)$ iff $|AB| > g' \cdot |A|$ $(0 < g' < 1)$.

Thus, g' stands for an r.v. with a probability distribution whose mode and mean are greater than that of an r.v., g, for *a few*, and smaller than that of an r.v., g'', for *many*.[64]

$Some(A, B)$ pronounced with normal stress will at the very least not intimate that A and B are negative to one another. It might even intimate positivity, though not in any significant sense *ceteris paribus*. Absence of pronounced *ceteris paribus* significance will distinguish its relevance properties from those of *many*. *Ceteris non paribus* this may change.

11. Endocentric Relevance Orderings of Quantifiers

Here we deal with proportion quantifiers (i.e. strong dets and proportion readings of weak dets) only. We have for the strong determiner *all*

(34) $All(A, B)$ iff $\mu(B|A) = 1$.

Conditions on positive relevance by ROP or POP are very modest indeed. A will be positive to B whenever $\mu(B|T) < 1$, i.e. when $\mu(B) < \mu(T)$. In other words, whenever $All(A, B)$ is assertable at all, A will be positive to B provided B is not a wholly uninformative predicate. Indeed, A will be *maximally positive* to B, in that no other property can exceed its positive relevance to B. At the other extreme, we have for the weak determiner *no*

(35) $No(A, B)$ iff $\mu(B|A) = 0$

Here the utterly modest condition $\mu(B|T) > 0$, i.e. $\mu(B) > 0$, which implies for count measure that B is non-empty suffices for negative relevance. Indeed, A will be *maximally negative* to B, in that no other property can exceed its negative relevance to B.

Going back to positives, next in line of decreasing endocentric relevance will be the strong det *most*. Rather than go for the minimal 'more-than-half' definition, it seems reasonable to allow for a parametrized family of quantifiers $Most_{.k}$ which are verified by proportions $0.k > 0.5$.

(36) $Most_{.k}(A, B)$ iff $\mu(B|A) = 0.k$ $(0.5 < 0.k \leq 1)$.

Here a necessary and sufficient condition for A being positive to B is that $\mu(B|T) < 0.k$.[65] Thus, on the minimal requirement for *most*, $\mu(B|A) > 0.5$, a sufficient (but, for this inequality constraint, not a necessary) condition for positive endocentric relevance under ROP or POP is that not more than half of everything is B, i.e. $\mu(B|T) \leq 0.5$. This is still a very modest condition for plausible universes T and for most useful predicates B. Since we specify any member of the family by means of an equality, *Most sharks are harmless* will in general be false or at any rate warrantedly non-assertable if it so happens that all sharks are harmless. The simple inequality specification would rely on 'scalar implicature' to explain why *Most cats are cats* is not seriously assertable. However, there are reasons for proceeding otherwise.

The central idea is that our ordering or 'scale' is not based on a pair of positive and negative rankings by logical or set-inclusion (inclusion chains), but rather on *a single ordered partition or probabilistic fuzzy approximation to a partition.*[66] The partition defines quantitative lexical meanings.

In a second step, 'up-cones' and 'down-cones' will be generated in the act of assertion, conditioned lexically (cp. Ducrot 1973) or else contextually by actual purpose, most notably so for numerals. The principle, recall from Section 10, is that a short-term rational *homo oeconomicus* will not mind getting more (for free!) than demanded, nor having to give away less than conceded. Assertion introduces preferences which might not be prefigured as defaults in the lexicon.

In a third step, upperbounding implicature could undo some or all of the effects of assertoric, act-pragmatic 'cone-formation'.[67]

The following two tables propose not only a ranking in terms of endocentric—i.e. least context-dependent—relevance relations, but also summarize context-relative or context-independent denotations and rough glosses. Where numerical parameters a, b, etc. arise, they stand indifferently for random variables X_a, X_b or their mode value.[68]

The borderline between what are traditionally called 'positive' and 'negative' quantifiers corresponds to change of endocentric relevance sign. The important thing to keep in mind is that the *positive dets* ceteris paribus induce an ostensible *quantity-maximizing preference structure* of the speaker's; and the *negative dets* a dual, *minimizing preference structure*. It is the interaction of these preference structures with ordered fuzzy partitions that yields the inequality readings of intermediate dets which appear to be bounded only by one or the other extreme, and would imply entailments from the extremes towards the mid-point. But such entailments do not hold by general rule, as shown by the absence of any entailment from *all* or *no* to *a few*.

(37)	$All(A,B)$	iff	$\mu(B\|A) = 1.$	[A maxposrel B]
	$Most_a(A,B)$	iff	$\mu(B\|A) = a\ (0.5 < a \leq 1).$	[A v.hiposrel B]
	$Many_b(A,B)$	iff	$\mu(A\|B)/\mu(A\|\bar{B}) = b_r \quad (b_r > 1 + r)$ and $\mu(B\|A) = b\ (b > 0).$	[A sigposrel B]
	$Some_c(A,B)$	iff	$\mu(B\|A) = c\ (c > 0).$	[A [pos/nonneg]rel B]
	$A\ few_d(A,B)$	iff	$\mu(B\|T) + t \geq \mu(B\|A) = d_t \geq \mu(B\|T) - t'.$	[A lowrel B]
	$Few_e(A,B)$	iff	$\mu(A\|B)/\mu(A\|\bar{B}) = e_r \quad (e_r < 1 - r').$ and $\mu(B\|A) = e\ (e < 1).$	[A negrel B]
	$No(A,B)$	iff	$\mu(B\|A) = 0.$	[A maxnegrel B]

$All(A,B)$	$[\mu(B\|A) = 1 \rightarrow A$ posrel B iff $\mu(B) < \mu(T)]$
$Most_a(A,B)$	$[\mu(B\|A) > 0.5 \rightarrow A$ posrel B if $\mu(B) \leq 0.5\mu(T)]$
	$[\mu(B\|A) = 0.k \rightarrow A$ posrel B iff $\mu(B) < 0.k\mu(T)$ for $k > 5]$
$Many_b(A,B)$	A sigposrel B
$Some_c(A,B)$	$[\mu(B\|A) = c\,(0 < c \leq 1) \rightarrow A$ posrel B iff $\mu(B) \leq c \cdot \mu(T)]$
$A\ few_d(A,B)$	$[\mu(B\|A) = d\,(0 < d \leq 1) \rightarrow A$ lowrel B iff $\mu(B) \approx d \cdot \mu(T)]$
$Few_e(A,B)$	A hinegrel B
$No(A,B)$	$[\mu(B\|A) = 0 \rightarrow A$ negrel B iff $\mu(B) > 0]$

The paraphrases are approximate. It is, for example, a moot point whether to class the *ceteris paribus* endocentric relevance of *some* as highly as 'significantly positive' or as modestly as 'non-negative'. What speaks against 'significantly positive' are *Some* {*or/if not indeed*} {*all/*many/*most*} *cats walk.* What speaks for it, is intuition integrating over contexts of use one dimly remembers. Similar uncertainties might affect the glosses attached to other dets. However, in any given context of proportional use, there is a clear ordering of proportions and also of relevances. The random variables X_a, X_b, etc. will have single-peaked probability distributions, the peaks (modes) being a, b, etc. Peaks and centres of gravity of probability mass will yield strict orderings by increasing proportion and *ceteris paribus* relevance:

(38) *no* $\prec$ *few* $\prec$ *a few* $\prec$ *some* $\prec$ *many* $\prec$ *most* $\prec$ *all.*

Proportion, like measure in the above sense, cannot be negative; but relevance can be, very much so. This need not imply a difference in ordering, but will imply a difference of intuitive zero point. For relevance the zero point given by sign change is around or just below *a few*. If you want to be very fancy, you can easily form the complex determiner *an insignificant number of*. A richer ordering would then, naturally, include the q-det *a number of* and the difficult item *several*, which might not be a q-det.[69]

12. Monotonicity Revisited: Complex VPs and NPs

The notion of monotonicity in the GQ literature is 'boolean', being specified in measure-free terms of entailment or set-inclusion. Let 'MON' stand for 'monotone', 'I' for 'increasing' 'D' for 'decreasing', 'L' for 'left', 'R' for 'right', and '$\neg$' for 'not' in exercises. $Q(\cdot,\cdot)$ is right increasing (a.k.a. upward) monotone (RIMON) iff $Q(A,X)$ entails $Q(A,Y)$ for arbitrary $X \subseteq Y$, and is right decreasing (a.k.a. downward) monotone (RDMON) iff $Q(A,Y)$ entails $Q(A,X)$ for $X \subseteq Y$.[70] Similarly, LIMON, LDMON, are specifiable for the restrictor argument. Thus

$$\text{RIMON}: Q(A,B) \models Q(A, B \vee C);\ \text{LIMON}: Q(A,B) \models Q(A \vee C, B);$$

$$\text{RDMON}: Q(A,B) \models Q(A, BD);\ \text{LDMON}: Q(A,B) \models Q(AD, B).$$

Let $X \mid\!\approx Y$ designate one's empirical disposition to judge that, if X holds, then Y holds as well. Now consider *Most*, which on the received account is RIMON. Indeed, $\mid\!\approx$ behaves much like $\models$ in

(39) a. Most Arcadians walk. $\mid\!\approx$

b. Most Arcadians move.

We assume plausibly that the relevant concept of moving is the boolean disjunction of walking (B) and any other forms of locomotion (C). Trouble arrives as soon as we represent $\vee$ by '*or*'.

(40) a. Most Arcadians walk. $??\models\!\approx$

b. Most Arcadians walk or talk.

One explanation might be that talking is *not contiguous in conceptual space* to walking. So the disjunction will not correspond to a *coarsening* of a natural partitition. But this, in turn, can be explicated in terms of relevance to typical activities and attendant decisions.

Yet irrelevant explicit disjunction, as we might call it, let alone the magic of 'implicature' *apud* Grice, is not the only explanation for problems with RIMON judgments. Try first a putative instance of a logically equivalent formulation of RIMON, $Q(A, BC) \models Q(A, B)$:

(41) a. Most upper class Englishmen are gentleman farmers. $?\models\!\approx$

b. Most upper class Englishmen are farmers.

In terms of endocentric relevance relations reflecting stereotypes we might say: being upper class English is very positive to being a gentleman farmer, but not so—indeed negative—to being a farmer. Both relations will feed into presumable exocentric relevance relations. For

(42) a. Most Arcadians are very friendly. $\not\models\!\approx$

b. Most Arcadians are very friendly or very unfriendly.

the same troubles arise even when *most* is replaced by *many* or by *some*, both boolean RIMON, too.

The problematic inferences are problematic because they fail to be relevance-preserving. As in the tables above, let 'X posrel Y' stand for X is positive to Y, and let 'X negrel Y' and 'X irrel Y' stand for the corresponding analogues.

Fact: A posrel $B \not\models A$ posrel $B \vee C$.

Indeed, any of 'A posrel $B \vee C$', 'A irrel $B \vee C$', or 'A negrel $B \vee C$' might hold. In fact, 'A posrel B', 'A posrel C' and 'A negrel $B \vee C$' may simultaneously hold. In (41), $B \subset C$, and hence $B \vee C = C$. Being upper class is very positive to being a gentleman farmer, but not at all positive to being a common garden variety farmer, i.e. someone in $\overline{B}C$. (42) makes vivid the most relevant stronger

> **Theorem** (Merin 1997): In case $P(BC) = 0$, hence in particular when $BC = \emptyset$, $\mathrm{rel}_B(A) \leq \mathrm{rel}_{B \vee C}(A) \leq \mathrm{rel}_C(A)$ or else $\mathrm{rel}_B(A) \geq \mathrm{rel}_{B \vee C}(A) \geq \mathrm{rel}_C(A)$.

Except in degenerate cases, the inequalities will be strict, which implies the following:

> If A posrel B and A negrel C, then $\mathrm{rel}_B(A) > \mathrm{rel}_{B \vee C}(A)$.

Boolean RIMON fails to make the correct predictions, then; and where it does appear to predict correctly we find typical coarsenings to elements $B \vee C$ of 'sufficient partitions' (cf. Merin 1999b) i.e. right *relevance monotonicity*.

Next we appeal to relevance to solve a puzzle about complex DPs which is well known to students of natural language quantifiers.[71] The puzzle is illustrated by a set of robust acceptability judgments:

(43) Most men {but/*and} no women were invited.
(44) Most men {*but/and} some women were invited.
(45) Few men {*but/and} no women were invited.
(46) Few men {but/*and} some women were invited.

B&C observe a generalization: Conjunctions of natural language type <1> quantifiers (i.e. of NPs) must be *and*-conjoined when of equal, and *but*-conjoined when of unequal boolean monotonicity.[72] In Merin (1994b, 1996, 1999a) there is found a theory of the two connectives in terms of relevance relations. A partial statement of a DTS for *but* given there, still sufficiently strong for an explanation, is

> Let A, B denote propositions. Then A *but* B is felicitous in a context i with ostensible belief function P^i only if there is an anaphorically given or accommodable proposition H such that for any numerical relevance function r^i mapping irrelevance to zero, $\mathrm{sgn}[\mathrm{r}^i_H(A)] = -\mathrm{sgn}[\mathrm{r}^i_H(B)] \neq 0$. I.e., A and B have inverse relevance polarity with respect to H.[73]

An important special case of this will be $H = \overline{B}$. This is a case of endocentric relevance with regard to the sentence schema A *but* B. Now consider (43). There may be any number of H to which (43) is relevant overall. Similarly, those parts of it which are obtained by inverting what used to be known as 'conjunction reduction' may each be relevant to it. – We start with a nonstarter. Look for an H such that (43a) *Most men were invited* is positive to it and (43b) *No women were invited* is negative to it. So H might be

(47) The party was as a party should be.

Without further specification of how exactly a party has to be to qualify for being 'as a party should be', (47) will be of little use. For one, (47) would not explain *Most cool cats but* {**∅/also*} *all uncool cats were invited.* The particle *also* is crucial here and not to be neglected (see Merin 1999a). Thus, we should turn to endocentric relevance relations. These require no appeal to extraneous propositions H. Reflective intuition will note that (43) presents being a man as positive to being invited, and being a woman as negative to being invited. In other words, pick an individual at random, an arbitrary individual, and call it x. Then (43a) presents evidence or argues that $Man(x)$ was positive for $Invitee(x)$, while (43b) presents evidence or argues that $Woman(x)$ was negative for $Invitee(x)$.

If quantity is what interests us, then the total number of invitees or indeed their proportion in the universe T of individual will be of interest. So one candidate proposition H might be

(48) A very large number of people were invited.

The corresponding pair H, $\overline{H}$, of propositions is also perfectly good as an intermediate propositional variable $\pm H$. It links (43a) and (43b) to (48) and gives it just the right interpretive twist: Small may be beautiful, but bigger is better.

With (48) as our H, we have lifted the endocentric relevance relations specified in terms of first-order properties to the level of propositions and of a formally exocentric relevance relation. This is because (43a) is positive to (48) precisely to the extent that $Man(x)$ is positive for $Invitee(x)$, both propositions suitably past-tensed. Analogous things hold for (43b). Indeed (48) can be paraphrased thus: For any x, the probability of x having been invited was very high.

The party referred to in (47) would most likely have been presented as a dud. In inviting so few women, the hosts might have discriminated against women, or against men, or whoever. But this kind of reasoning plays no role in explaining (43). Indeed, it should not play a role in explaining constraints which are pretty much syntacticized. Sheer quantity and proportion, by contrast, seem general and simpleminded enough to qualify as a principal part of an explanation. Via (48) considered as an intermediate variable, they could even link up with (47). Both quantity and proportion also underlie the process of simple 'enumerative induction'. This mode of inference is roughly characterized by the following pair of rules, often conflated into one:

> *Enumerative Induction*: (1) The higher the proportion of positive as distinct from negative instances of a property R that you have observed, the more highly you expect the next instance to be positive too. (2) The larger the sample you have considered, the more resilient to counterinstances your expectation will be.[74]

Enumerative induction is one of the fallible mainstays of science, of learning, and of impression-formation.[75] As Bertrand Russell made plain: the chicken that is fed by the familiar hand day after day has a good reason to expect to be fed the next day, though none to anticipate the day when that same hand will wring its neck. A key scientific goal of enumerative induction is support of a universal law. However, both in everyday life and in much of science, support for non-vacuous universal laws ($\forall x[Ax \rightarrow Bx]$, $\forall(A, B)$, $\ldots$) is not always feasible, nor necessary. Thus, our above example scenarios will not in general have *Everybody was invited*, with a fairly literal reading, in place of (47). Otherwise, only sentences such as *Every woman {and/but} DET N VP* would stand a chance of having a non-redundant instance for *DET N*. Yet we have been dealing all along with a larger range of examples. Indeed (48) will engage all of them in support relations and is sufficient thus for most everyday occasions as an issue proposition. It is 'sufficient' (cf. Merin 1999b) for the pragmatic decision problem that is being transacted by means of discourse.[76]

Finally, consider the case $H = \overline{B}$ in our partial semantics for *A but B*. Induction with regard to the reference class (or universe T) of people would make (43a) negatively relevant to (43b). So here we have polarity-consonance of endocentric relevance within each of a pair of de-reduced conjunct propositions. Relevance of these two propositions to one another which is endocentric to the coordinate construction shares that polarity.

The same would hold for the corresponding de-reduced conjuncts of (46). By contrast, the de-reduced conjunct sentences of (44) and (45) would surely be related by non-negative relevance.[77] In Merin (1985, 1994a) I have looked at quantifiers and connectives in terms of a 'maximizer's' perspective and a 'minimizer's' perspective, fully in line with what can be extracted from the work of Ducrot (1972) on French *peu* and *un peu*. It seems to me that this, really, is at the root of monotonicity intuitions. Our above counterexamples to the predictions of boolean monotonicity bear out the primacy of these pragmatic concepts.

13. Conclusion and Outlook

Our aim has been to associate, rather than dissociate, the 'extensional' and 'intensional' aspects of quantification. In brief: there is a progression from portion, to proportion, to relevance, i.e. to difference or indeed proportion of proportions. But there is also a way back, call it 'generalized objectification', and it induces proportion readings where relevance relations are endocentric. In giving denotational semantics in probability spaces that represent socio-mental state spaces, we relied for semantic structure on relations between their elements. In doing so we had little choice but to adopt, in effect, a structuralist approach to meaning in the vein of Saussure, John Lyons, and, indeed, Oswald Ducrot. I think this is what an informatively specified, context-dependent semantics inevitably requires. Quantifiers are concepts of a rather abstract kind. Much if not all of their meaning is given in terms of their conceptual role, explicated in terms of constraints on beliefs and their dynamics. As in the stage world of comedy and drama, abstract roles in the theatre of mind that supervene on physical experience are, in the first place, defined by their relations to one another.

Notes

[1] This article was written with research support from the Thyssen Foundation and the Deutsche Forschungsgemeinschaft under grant SFB 471. It is a shortened and simplified version of Merin (2006) and expands parts of Merin (2001a,b). I owe thanks for comments and queries to convenors and participants at the Jerusalem, Helsinki and, most recently, Brussels conferences where the corresponding presentations were given. I thank most particularly an anonymous referee and the editors of this special issue for very helpful, detailed comments, which I have tried to heed. The object of this paper, in line with their joint advice, is exposition to a wider readership. Technical remarks will, as far as possible, be confined to footnotes.

[2] Q is a Lindström type <1,1> quantifier. Noun phrases, $Det\bar{N}$, denote type <1> quantifiers, in terms of which Mostowski's discussion proceeds.

[3] Johnson-Laird (1983). As usual, context will decide whether A, B denote nouns and verb phrases or whether they denote sets or yet more abstract extralinguistic entities.

[4] This will in general be a sub-algebra of a larger algebra $\mathcal{F}'$ and, in principle, elements of the relative complement $\mathcal{F}' - \mathcal{F}$ could be involved in specifications.

[5] **Notation**: For the sake of simplicity, juxtaposition AB will often denote indifferently conjunction ($A \wedge B$), set-intersection ($A \cap B$), and lattice meet ($A \sqcap B$); $A \vee B$ so denotes disjunction, set-union ($A \cup B$), lattice join ($A \sqcup B$); the bar as in $\bar{A}$ so denotes negation ($\neg$), set-complement, lattice complement. $A - B =_{df} A\bar{B}$.

[6] We shall properly introduce all notions now mentioned.

[7] Higginbotham (1996) and earlier Terry Parsons treated mass quantifiers separately in terms of measure. The present treatment exploits the obvious generalization.

[8] Cp. Barwise and Cooper (1981), Keenan (2002). Linguists adopt wherever possible a non-numerical format. By GQ they usually mean the Fregean (Frege 1884, etc.) higher-predicate, or relational conception of Qs characterized by Richard Montague's typing of NPs as $<<e,t>,t>$ and of dets as $<<e,t>,<<e,t>,t>>$. The deeply syntactic issue of variable-free specification may have drawn attention away from quantity.

[9] In set-algebras, **0** (a.k.a. $\perp$) specializes, thus, to the empty set $\emptyset$, and T (a.k.a. **1** or $\top$) to the universal set Ω. T is my preferred mnemonic for the vacuous property 'thing', as Keenan's (1987) E is for 'exists'. Thus, $Every(T, M)$ might stand for *Everything moves.*

[10] One might, but need not, think of this as a set of 'possible worlds'.

[11] We ignore any det which typically involves speaker reference to particulars.

[12] A permutation is the special case, when T is a set, of an *automorphism* (AUTOM).

[13] Carlson (1978) develops the predicate distinction to 'stage level' and 'individual level'.

[14] Examples: (1°) *There are {five/some/many/few/no/*all/*most} cats in the court.* (2°) *There is {a/#the/*every/*each} cat in the court.* (3°) *There isn't {a/every/any} cat in the court.* The # on *the* in (2°) signifies alleged deviancy, but I find here a perfectly good 'availability' sentence exhibiting the familiarity condition characteristic of *the*. The sentence is, I think, deviant only to the extent that *the* is not a q-det at all.

[15] B&C subdivide Qs into positive Q^+ (e.g. *most, all*) and negative Q^- (e.g. *not all, few*) and define: Q^+ is strong iff $Q^+(A, A)$ holds for all A; Q^- is strong iff $\neg Q^+(A, A)$ holds for all A. All other Q are weak. K&S proceed inversely. They turn into a definition an observation of B&C's: Let T be the pertinent universe of existents, of which A and B are subsets. Then Q is weak iff $Q(A, B) \leftrightarrow Q(AB, T)$, else strong. The English translation of T (K&S write it E) is the verb *exists*. Thus, *Some ants are in the bar* will be true iff *Some ants in the bar exist* is. Equivalence fails on replacing *some* by *most*. *The cat is a cat* will make *the* B&C-strong. K&S (1986: 301) class *the* as non-existential, i.e. strong.

[16] A referee noted 'right conservativity' (i.e. $Q(A, B) \leftrightarrow Q(AB, B)$) employed as a criterion for weakness in Keenan (2003). But now, in place of the existence of inexistent AB intimated by $D(AB, T)$, English $D(AB, B)$ intimates there being ABs that are not Bs. Try: *{Some/*No} cats in the bar are in the bar.* Appeal to 'implicature' does save truth-conditional felicity, but thereby only for a regimented, theory-laden form of English.

[17] Bare plural NPs illustrate the distinction, with proportion lawlike.

[18] Notation: $Q(A, B)$ stands for a proposition, $D(A, B)$ for a sentence. Indexation $D^T(A, B)$ to some universe T is usually understood.

[19] For example, if $D(A, \cdot) =$ *some* N_{pl}, then on the standard construal of natural language dets, $\text{WCON}_D = \{x : x > 1\}$, i.e. $Some(A, B)$ iff $|AB| \in \{x : x > 1\}$. Of course, if $D(A, \cdot) = \exists(A, \cdot)$ (short for $\lambda P \exists x[Ax \wedge Px]$) or if N is in the singular, then $\text{WCON}_D = \{x : x > 0\}$ and we have the standard logical interpretation of 'some': $Some(A, B)$ iff $|AB| \in \{x : x > 0\}$.

[20] Indeed, the present proposal was mooted as a response, at Hans Kamp's 60th Birthday Conference on Presupposition in Stuttgart, October 2–5, 2000, to Abusch and Rooth's presentation.

[21] Milsark's was, I think, essentially for proportionality. Others, to the extent of going farther than acknowledging occasional proportional use, were arguments for presuppositionality, i.e. for mandatory $A \neq \emptyset$, which we saw to be entailed by proportionality. De Jong and Verkuyl (1985) [J&V] see $All(A, B)$ as presuppositional, but do except law-stating sentences such as *All ravens are black* as 'conditional' use based on inherent relations among properties. They hold such use to be 'marked', arguing that our entertaining theories is not a property of natural language. Hegel anticipated an objection to this claim when he observed that people tend more to talk in generalizations, the less sophisticated they are. At any rate, the present framework affords a unified treatment, as will soon become apparent in the main

text. J&V's example, *All unicorns are waiting for the traffic lights*, for a sentence whose truth in a model with no unicorns would be "strange", will contrast not only with the law-stating example *All unicorns have a mane*, which is a real test case for putative $A = \emptyset$ in nomological use, but also with epic *Every unicorn* $\{$*is/was*$\}$ *in love with a maiden.* Hearers will take unicorns to be held existent somewhere, even if only in a world of fiction. There are intelligible semantics for fictional entities (Parsons 1980, Zalta 1988) which explicate this unforced intuition. Lappin and Reinhart (1988) hold that each of *Every unicorn is a unicorn* and *Most American kings are American kings* should be true, granted that there are neither unicorns nor American kings. Again, that unicorn sentences sound all right is surely due to unicorns being accorded existence in a world of fancy which is richly structured enough to count as some form of reality. This would not hold for American kings at present, and indeed ??*Every American king is an American king* is poorly acceptable. So would be the less artefact-prone, pluralistic variant with *nobleman* in place of *king.* (*American* meaning: 'North American', to be safe.) For $Most(A, A)$, recall Section 2.

[22] By the de Finetti (1937) Representation Theorem, expected frequencies are indistinguishable from epistemic probabilities (see Jeffrey 2004). (5b) suggests higher $P(A)$.

[23] A heuristic for reading (7′) comes from quantified modal logic: $\nvdash \forall x \Diamond Qx \rightarrow \Diamond \forall x Qx$. Invalidity of the implication means that wide-scope universal quantification must not be misread for narrow scope here. The problem does not arise in (7) itself. '$P(\cdot) = 1$' behaves like doxastic necessity, '$\Box$', for which $\vdash \forall x \Box Qx \rightarrow \Box \forall x Qx$ holds.

[24] See Bacchus (1990) and Merin (1996: 84–104) for exposition and discussion.

[25] Bacchus (1990) notates $[Qx]_x$, the probability, in frequency terms, that a random element x of our universe ('sample space') has property Q. The conditional version, $[Qx|Rx]_x$, would give the probability or chance relative to the subset $\{x : Rx\}$ of the universe. If R and Q are the respective sets we should have $[Qx|Rx]_x = \mu(Q|R)$. $[Qx]_x$ thus cannot simply be equated with a belief-probability without further ado—least so in the kind of futurate, openly intensional context we have here. See Section 5.

[26] Nor is it falsified by data involving *any.* See the longer version (Merin 2006).

[27] It seems reasonable as an empirical generalization that *There-be* Sentences (TBS) always express availability or unavailability of portions (special case: subsets) of the denotatum of the restrictor common noun. This does not rule out that the PP of a prepositional TBS acts as a tacit norm for proportional measure.

[28] Take for example *Many Scandinavians are Nobel laureates.* The intuition is that $Many_1$ makes $|AB|$ large in some absolute sense; $Many_2$ makes it a large enough proportion of A; and $Many_3$ makes that proportion larger than the proportion of Bs in the set of all individuals considered. Clearly, that last specification takes some intuiting, but we shall soon offer an interpretation of it that makes it most intuitive. $Many_5$ is like $Many_2$, save in having B as the comparison class; $Many_4$ has the universe as a comparison class; $f(\cdot) = k''' \cdot$, i.e. multiplication by a constant ($0 < k''' < 1 = |T|/|T|$) is just one option.

[29] $Many_{5,k''}(A, B) = Many_{2,k'}(B, A)$.

[30] A referee pointed out that, in this case, $Many_3(A, B)$ would not be false when $|AB|/|A| = |B|/|T| = 1$, which suggests replacing "$>$" with "$\geq$" everywhere. I retain formulations with "$>$", since they are more intuitive to read, but clearly, the k, k' could be chosen to make "$\geq$" appropriate.

[31] K&S's policy decision is costly. First, *many* and *few* fit neatly into scalar determiner paradigms, as shown e.g. by $\{$*Many*/**Red*$\}$ *if not most cats walk* and $\{$*Few*/**Abnormal*$\}$ *or no cats walk.* Secondly, asymmetries of intuition about *many* which the literature explains by way of essentially proportional readings (see below) also appear with *some.*

[32] Read 'laureates' short for his 'prize winners in literature'.

[33] Given the enormous difference between $|N|$ and $|S|$, k' might even vary considerably

between the two conditions.

34 As e.g. the set of natural numbers and the set of primes ending in '1' are, which might be addressed in mathematical applications of 'measure quantifiers'.

35 F&K employ such an epistemic probability formulation to model the intuition that *many* means 'larger than expected'; cp. Merin (1994b, 1996, 1999b) on *but*. It differs from the present approach in not linking with frequency and not proposing a relevance interpretation. See Section 6, below; and, for rather more detail, Merin (2006).

36 In the longer version, I argue that standard, default-stressed TBS which carry a location PP have the measure $\mu(PP)$ of their PP inducing a norm yielding a proportion, $\mu(DP)/\mu(PP)$, and thence do not admit DPs other than those which have a portion reading.

37 Three heads after a run of one head and one tail might well sway us to think the coin we tossed is biassed. Three heads after a run of 50 or 51 heads and 49 or 50 tails probably won't.

38 In Bacchus' notation, slightly adapted, this assignment verifies $P(Bd|Ad)\|[Bx|Ax]_x = \alpha) = \alpha$. It is subject to the constraint that $P(([Bx|Ax]_x = \alpha) \wedge K) > 0$, where K is any background knowledge added. It says: conditional on the frequency being α, your probability will be α (if you are the reasonable sort).

39 The present approach differs from F&K's in considering relevance relations, ROP and POP, and the de Finetti route of representing frequency as a degenerate case of epistemic expectations over frequency. Our futurate examples were instances of non-degeneracy.

40 This could be an empirically motivated instance of a projectivist metaphysics of meaning suggested in more general terms by Blackburn (1984) in development of Hume (1739/40).

41 $Many_2$ preserves LCONS and EXT, but only by letting the context provide a parameter, k', for each occurrence of *many* (Westerståhl's observation). We should add: this will preserve EXT only if we can be sure that the parameter is provided in independence of T. That is a very strong assumption.

42 $\mu(B)/\mu(T) = \mu(BT)/\mu(T)$, since $B \subseteq T$ always.

43 $Many_7 = Many_{8,1}$, just as $\exists = Many_{1,0}$.

44 In the case of exocentric relations we might have relations to arbitrary H at issue. Thus *Some cats walk*, *A few cats walk*, perhaps even *Several cats walk*, *Many cats walk*, *Most cats walk*, *All cats walk* and on the other hand *Few cats walk*, *No cats walk* can all be compared in a context by their relevance to some H—even to something as remote as *The price of cheese will rise*. (Think of mice and traps as intermediate causal variables.)

45 Depending on positivity or negativity of the underlying determiner.

46 These will be construable by relevance relations between corresponding propositions obtained upon instantiating them by suitable arbitrarily chosen individuals.

47 This coheres with the Milsark's finding that '*ser*-type' VPs elicit strong readings of weak dets. It might also shed light on de Hoop's (1995) analogous observation specifically about transitive verbs.

48 We have ruled out the articles *a* and *the*, whose overwhelming referential interpretations fail PERM. If they are included, this assertion is obviously false.

49 Fact: $\mu(B|A) > 0$ whenever $\mu(AB) > 0$; and $\mu(AB) > 0$ whenever $\mu(B|A) > 0$ (but note: $\not\vdash\ \mu(B|A) = 0 \leftarrow \mu(AB) = 0$.) Truth conditions of portion and proportion readings of *some* defined by $\exists(A, B)$ coincide, but falsity conditions do not, due to undefinedness of proportion readings. With truth and falsity transposed, the same holds for $\neg\exists(A, B)$ i.e. *no*. The conditional probability induced by ROP from Fregean proportion *some* has a definite lower bound: $P(B|A) \geq 1/|A|$. That induced from proportion *no* has a definite value: $P(B|A) = 0$.

[50] In this particular example DET = *Several* would be infelicitous. Fourteen or so SN is too many. A handful of ABs known by name or by sight, or known metonymically by perception of marks they have left, is my intuition for the assertability condition of *Several*(A, B). The speaker should be able to ***subitize*** the set AB. The knowledge condition might even rule out ***several*** as a q-det. My intuitions for Fr. ***plusieurs***, whose etymology does not hearken back to the notion of being 'distinct', are not secure enough to vouch for a like specification. See Jayez (2005) for a proper account of it.

[51] Ducrot's insight is taken up and his error subjected to constructive critique in Jayez' (1987) formal semantics of ***presque*** ('almost') and ***à peine*** ('hardly'). Their properties are importantly related to ***peu*** ('few/little') and ***un peu*** ('a little'). Jayez' (1987) treatment is in a deterministic framework without appeal made to the notion of evidential relevance.

[52] Orientation is preferred direction of maximization of independently given physical or numeral accounting quantities. The default bias will be for maximization increasing in quantity.

[53] 'Small' intuitively includes zero or has it as a greatest lower bound for physical quantities.

[54] Protagoras said 'Man is the measure of all things', and took payment for teaching people to make the weaker case appear to be the stronger. Recall what $\mu(T)$ means.

[55] Specified in terms of inequalities for simplicity, but due to be respecified in terms of equalities in line with the ABRO principle.

[56] In Merin (2001b), I had proposed $Many_{k+}(A, B)$ iff $\mu(B|A) > \mu(B|T) + r$, and $Few_{k-}(A, B)$ iff $\mu(B|A) < \mu(B|T) - r'$ for $0 < r, r' < 1$. Again, r was specified to stand for an r.v. with a probability density peaked e.g. around 0.1, so that r should represent a noticeable difference and A should be significantly relevant to B.

[57] In (2001b), I had proposed, for the proportion reading, $Some_s(A, B)$ iff $\mu(B|A) \geq s > 0$. The observation attached was that relevance of A to B by ROP or POP was non-negative provided $\mu(B|T) \leq s$, i.e. if the proportion of Bs that also were As was not exceeded by the proportion of Bs in the universe T (under consideration).

Example: A non-negrel B when, of 1 million T, 10.000 are B, 5000 are A and 50 or more are AB. If $|B| = 100.000$ and $|A| < 1000$, then just one [!] AB will suffice to ensure A posrel B, by ROP or POP. It is for this reason that we really want for ***some*** the analogue of the paraphernalia for ***many***: i.e. a minimum size of AB, greater than just one or two.

[58] See Merin (2006) for more argument on ***a few***.

[59] $A\ few_r(A, B)$ iff $\mu(B|T) - r' \leq \mu(B|A) \leq \mu(B|T) + r$.

[60] ***A few*** is incompatible with ***no***. See Merin (2006) for discussion of ***A few spoilsports complain*** when in fact all the spoilsports there exist complain.

[61] Formality and detail apart, it differs doctrinally from his in not treating argumentative relations (e.g. assertions) as constituting a domain ***sui generis***, but as special cases of more general social relations (e.g. claims).

[62] The key principle that links quantifying determiners and the taxonomy of act-types is a naive maxim: Claim as much as possible, concede as little as necessary—***ceteris paribus*** in each case. Label the parameter here involved 'Preference'. The next important taxonomic parameter might be labelled 'Dominance'. In the context of Claims and Concessions it engages the backing (by power, evidence, ...)—or the lack of it—for the pursuit of one's preferences against political or more specifically argumentative opposition. Suppose we make the parameters binary. Let us have one of speaker and addressee preference for a proposition H becoming a constraint on joint speaker addressee commitments. Let one of speaker and addressee ostensibly have the resources to make his or her preference prevail. Then there is already a fourfold taxonomy in principle. Add conventional allocation of initiative in a

transaction, to one of the two, and there is an eightfold taxonomy. Thus, a Claim for some H would instantiate all parameters to Speaker. Concession would instantiate them to Addresee. Denial would instantiate Dominance to Speaker, and the others to Addressee. Thus, act-types stand for parameter configurations. See Merin (1994a) for details and two further loci of binary parametric variation.

63 As J. Lambek noted in the 1950s, a fractional rational number and a material conditional of logic are alike in such respect.

64 *Several* seems below *many* both on its cardinal and relevance reading. It might well be ranked by portion size below *a few* (which does not require AB to be subitizable, as I feel *several* does require), but is more positive in relevance. It seems to imply: 'more than expected', while *a few*, for all its positivity compared to *few*, might yet intimate: 'fewer than expected'. Cp. again Jayez (2005) on *plusieurs*, which is a proper analysis of that difficult item. By contrast, my observations on *several* are sketchy at best. It should, however, become clearer in due course to what extent the two items are synonymous.

65 Make adjustments for the case where $0.k$ stands for the mode of a probability distribution over possible values and $\mu(B|A) = 0.k$ is shorthand for a suitably skewed probability distribution over values of $\mu(B|A)$ whose mode is at $0.k$ and which assigns zero probability to all values up to 0.5.

66 E.g. a series of unimodal probability distributions whose tails overlap considerably. See Merin (1999a, 2003b) on coordinating connectives.

67 I have for long preferred to treat implicature in terms of a bargaining situation. Economists have done so for just as long in terms of signaling, restricted to assertoric claims. Either way, Horn's and perhaps Grice's doctrine based on logical strength—clearly anticipated by Tarski and Schröder—does not generally work (where it appears to work at all) without a substrate of partisan, extra-logical preference rankings, which is common to the bargaining and signaling approaches.

68 Imagine the X_a, X_b, ... as generating skewed bell-shape curves whose peaks are above a, b, ... on the x-axis, height measuring probability of an x-value.

69 For sizeable A, *Several*(A, B) will usually have fewer AB than *Some*(A, B) would, and might even have fewer than *A few*(A, B). Yet it will rank above either in the relevance ordering, if only in virtue of demanding small A. By contrast, *A number of*(A, B), which behaves much like it, has no such restriction; cp. *A number of Europeans have contracted bovine flu.*

70 *Most/Some cats walk* implies *Most/Some cats move*, respectively, so *Most* and *Some* are RIMON. *Few/No cats move* implies *Few/No cats walk*, respectively, so *Few* and *No* are RDMON.

71 Barwise and Cooper (1981: 194–196). It remained, apparently, unsolved, perhaps even unrecognized as unsolved until 2001, when the following explanation was first presented.

72 This is not yet an explanation. To be sure, the conjunction of DPs of distinct RMON types does not yield a monotone quantifier. But this is a claim about truth conditions. The standard theory of coordination represents both connectives truth-conditionally as boolean meet (conjunction, set-intersection). The truth-conditional semantics of the two coordinate NPs will be identical.

There is, as so often, an incidence of syntactization. (I think it is explicable by typical discourse properties being frozen into constraints on collocations.) Thus, B&C will allow that *Most men were invited and no women were invited* is fairly acceptable. Even better would be *Most men were invited and, in the event, no women were invited.* I offer an explanation below, together with that of a referee's challenge.

73 A referee wondered about clausal asymmetry. The fuller statement for prototypical uses

of *A but B* will account for this with the further clause $\mathrm{sgn}[\mathrm{r}^i_H(AB)] = \mathrm{sgn}[\mathrm{r}^i_H(B)]$. This explains the difference between *It's pretty, but expensive* and *It's expensive, but pretty*, inter alia with regard to continuability by *So let's buy it.* See Merin (1996, 1999a).

[74] **Thesis**: Absolute quantity intimations for dets of all kinds (special case: readings) that are non-numerals reflect minimum sample size requirements for reliable induction by ROP. — Recall that, given an estimate of proportion and an estimate of denominator portion A, you have an estimate for AB. This might rationalize, just a little, Milsark's idea that 'weak' interpretations of dets are derivative of 'strong' interpretations.

[75] Its first part also underlies the explanation of a language universal concerning *but* and its translation equivalents. See e.g. Merin (1999a).

[76] A referee challenged *à propos* (43) with the goodness of *Most men and—obviously—no women were invited.* My explanation would be twofold. (I) The syntactic unit is broken up by the parenthetical and so the possible incidence of non-stereotypic use conditions is being indicated. (II) The parenthetical adverb, *obviously*, indicates as already presupposed a specific, known fact which pre-empts the action of the default of blind enumerative induction based on quantity. If we associate properties of minimal syntactic units with stereotypical defaults of usage, these will be frozen into the syntax and thus have a tendency to overgeneralize. Since inference by enumerative induction is the shining example of an epistemic default strategy *faute de mieux*, its syntactization should be expected. When syntactic units are broken up, local contextual evidential and preferential conditions can throw in their weight, and tight acceptability constraints loosen up.

[77] The relationship is therefore like that which characterizes Reichenbach's (1954) Common Cause Theorem: A and B are positive to one another if they have like non-zero relevance sign with regard to some H that may, for instance, be a common reason for them, and are negative to one another if they have different non-zero sign to it. A sufficient condition is that conditional on each of $\pm H$, A and B are probabilistically independent, i.e. are irrelevant to one another.

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Presented at the International Conference on Indefinites and Weak Quantifiers, Royal Flemish Academy of Belgium for Science and the Arts, Brussels, Jan. 6–8, 2005. The present edition is text- and page-identical with the paper appearing in the *Belgian Journal of Linguistics* 19 (2005), 147–186, which was re-formatted from the original LaTeX-produced manuscript. A couple of sense-neutral misprints have been corrected. In-line mathematical typography might be smoother in the present, LaTeX printing at the cost of some uneven linespacing to preserve BJL pagebreaks in the main text.