# Knowing disjunctions with the help of logical grounding

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## 1 Introduction

If Andrea knows that Biden won the last presidential election, then they also know that either Biden won the last presidential election, or Biden is a reptilian. This is the response that epistemic logics based on standard Kripke relational semantics provide [15], which is consistent with the fact that minimally rational agents can perform disjunction introduction. This is not the case in topic-sensitive semantics though. An agent might not grasp the concept of what a reptilian is, and therefore not be able to know any proposition dealing with reptilians. By employing the concept of logical grounding, I propose a semantics that keeps the requirement of topic-grasping but weakens it, re-establishing a more standard treatment of disjunction, in line with the capacities of minimally rational agents. The paper is structured as follows. In Section 2 I introduce topic-sensitive semantics. In Section 3 I introduce the concept of logical grounding – focusing on the grounds of disjunction – and motivate some desiderata. The concept of *minimal topic*, the core of my proposal, is detailed in Section 4 and exploited in Section 5 to propose a new topic-sensitive semantic clause for knowledge. I conclude in Section 6.

## 2 Topic-sensitive semantics

Topic-sensitive epistemic logics [1, 13, 20] aim to achieve non-omniscience by exploiting (i) a theory of topic-sensitive propositional content, inspired by Yablo's theory of aboutness [25] and Fine's truthmakers semantics [7, 8], and (ii) topic-sensitivity of propositional mental states such as knowledge, belief, imagination, etc. According to (i), taking propositional contents to be a set of possible worlds does not tell *how* a sentence is made true. For example, 'Robin is or is not a surgeon' and 'Stone spaces are Hausdorff spaces' are true in all possible worlds but they say different things. To capture this distinction, truth conditions are supplemented with an account of *aboutness*. The content of a sentence becomes a *thick proposition*, namely the pair of its intension – the set of possible worlds that make it true – and its subject matter, *what it is about*. Similarly according to (ii), propositional mental states are topic-sensitive. An agent can grasp facts about Robin and their job without grasping any topological concept. They can therefore know that 'Robin is or is not a surgeon' without knowing that 'Stone spaces are Hausdorff space'. Knowing that $\varphi$ thus requires having information ruling out not-$\varphi$ and grasping $\varphi$'s topic, what it is about. Let's start introducing topic-sensitive epistemic logic, following [13].[1]

**Definition 1** (The language $\mathcal{L}_K$)**.** *Let* $\mathsf{Prop} = \{p_1, p_2, \dots\}$ *be a countable set of propositional variables. The language* $\mathcal{L}_K$ *is defined by the grammar:*

$$\varphi := p_i \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid K\varphi$$

**Definition 2** (Topic model)**.** *A topic model is a tuple* $\mathcal{T} = (T, \oplus, t, \mathfrak{K})$ *where*

- $T$ *is a non-empty set of possible topics;*
- $\oplus$*:* $T \times T \mapsto T$ *is an idempotent, commutative, associative operator;*
- $\mathfrak{K} \in T$ *is a designated topic representing the total topic grasped by the agent;*
- $t : \mathsf{Prop} \mapsto T$ *is a topic function assigning a topic to each element in* $\mathsf{Prop}$*.*

Let $\mathtt{Var}(\varphi)$ denote the set of propositional variables occurring in $\varphi$. The function $t$ extends to the whole language by taking the topic of $\varphi$ to be the fusion of the topics of the propositional atoms occurring in it: $t(\varphi) = \bigoplus\{t(p) : p \in \mathtt{Var}(\varphi)\}$. This entails topic-transparency of operators: $t(\varphi) = t(\neg\varphi) = t(K\varphi)$ and $t(\varphi \wedge \psi) = t(\varphi \vee \psi) = t(\varphi) \oplus t(\psi)$. Topic parthood $\sqsubseteq$ is defined in a standard way: $\forall a, b \in T : a \sqsubseteq b$ iff $a \oplus b = b$. I shall use $\sqsubset$ for strict topic parthood and $\not\sqsubseteq$ for the negation of $\sqsubseteq$.

It is widely accepted that propositional connectives are topic-transparent [9, 25], viz. they have no topic *per se*.[2] The topic of a proposition can therefore be understood as the fusion of the topics of its component. It follows that the topic of a disjunction is the fusion of the topic of each disjunct. 'Biden won the last presidential election' is about Biden winning the presidential election. 'Biden is a reptilian' is about him being a reptilian. The disjunction of these propositions is both about him winning the presidential election and him being a reptilian. Let's now define a (relational) topic-sensitive model and topic-sensitive semantics.

**Definition 3** (Topic-sensitive model)**.** *A* topic-sensitive model *is a tuple* $\mathcal{M} = (W, R, V, \mathcal{T})$ *where* $W$ *is a non-empty set of possible worlds,* $R \subseteq W \times W$ *is*

[1] I simplify the model since [13] also exploits fragmentation (the idea that the mind of an agent is divided into several frames) and has an information update operator. These additional features are not relevant to the current discussion, therefore I get rid of the update operator and consider the mind of the agent to be non-fragmented, i.e. constituted by only one frame. Moreover, no accessibility relation is mentioned in [13], quantifying over all the worlds in a certain frame of mind. This is the same as taking $R$ to be an equivalence relation (reflexive, transitive and symmetric). I take $R$ to be simply reflexive, which is the minimum requirement for epistemic logic, guaranteeing the factivity of knowledge.

[2] The topic transparency of the modal operator $K$ is more controversial. See [20, p. 770] for a possible motivation. Anyway, since in this paper, I will focus on the knowledge of propositional formulas, this point is not crucial. The treatment of the knowledge of modal formulas is left for future work.

*a binary (reflexive) accessibility relation between worlds, $V : \textsf{Prop} \mapsto 2^W$ is a classical valuation function that assigns to each propositional variable in the language $\mathcal{L}_K$ a set of possible worlds, $\mathcal{T}$ is a topic model as defined in Definition 2.*

The truth of propositional formulas is standard, while $K\varphi$ respects the following clause.[3]

$$\mathcal{M}, w \vDash K\varphi \text{ iff (for all } u \in W, Rwu \text{ implies } \mathcal{M}, u \vDash \varphi) \text{ and } t(\varphi) \sqsubseteq \mathfrak{K}.$$

The truth of a modal formula is defined as the conjunction of two requirements. The first is the standard Hintikkian truth in all epistemically accessible possible worlds [15]. This describes an idealized notion of knowledge yielding the problem of logical omniscience: one knows every logical consequence of what one knows [6, 23]. The problem is alleviated by the second requirement: knowledge is restricted to formulas the agent grasps the topic of.[4]

In this framework, to know a disjunction, one needs to grasp the topic of both disjuncts. This is introduced as a positive feature of the framework and respects the following idea expressed by Williamson:

> Although the validity of $\vee$-introduction is closely tied to the meaning of $\vee$, a perfect logician who knows $p$ may lack the empirical concepts to grasp (understand) the other disjunct $q$. Since knowing a proposition involves grasping it and grasping a complex proposition involves grasping its constituents, such a logician is in no position to grasp $p \vee q$, and therefore does not know $p \vee q$ [24, pp. 282–3].

Nonetheless, as Williamson himself underlines, the very meaning of $\vee$ allows performing disjunction introduction. Once we know that Biden won the last presidential election, we only need to be minimally rational agents that master the meaning of $\vee$ to know (or at least to be in a position to know) that either Biden won the last presidential election or he is a reptilian. This is how disjunction is assumed to work given its truth conditions and in fact, "[d]isjunction introduction is an instance of closure that many [...] feel carries particularly acute intuitive weight, so discarding it is a fatal error" [12, p. 2782]. Among these many we can find Dretske [5, p. 1009], Hawthorne [14], Holliday [16, p. 119], Kripke [17, p. 202], and Nozick [19, p. 230]. Once one gets to know $p$, no additional work is needed to get to know $p \vee q$: neither epistemic (inquiry) nor conceptual (topic-grasping) work.[5]

[3] For the sake of simplicity I use a monadic operator like the one that can be found in [13] and [20]. I take my argument and my approach to be prima facie applicable also to the dyadic topic-sensitive operator that can be found, e.g., in [2].

[4] Instead of the Hintikkian clause, other possible world conditions can be and have been used in topic-sensitive semantics. See [22] for some considerations on this point. Anyway, as we shall see, the failure of closure under disjunction introduction does not depend on the possible world semantics of choice, but on the topic-sensitve part of the clause.

[5] I don't consider deductive work since I take agents to be computationally unbounded as standard in the topic-sensitive literature (see e.g. [20, p. 769]).

Nonetheless, I believe that we don't need to throw the baby out with the bathwater. The idea that knowledge is a mental state requiring topic-grasping is an improvement in the field of formal epistemology. I want to keep the intuitive idea that the topic of a proposition (including disjunctions) is defined as the fusion of the topic of its component while proposing a framework in which full topic-grasping is not required for knowledge. To do so, I introduce the concept of *minimal topic*, i.e. the minimal topic required to know a proposition. The concept is defined by the one of logical ground.

## 3 A few words on grounding and some desiderata

Grounding is a non-causal dependence relation that describes the "intuitive notion of one thing holding in virtue of another"[10, p. 37]. Some grounding theorists distinguish between different kinds of grounding: logical, conceptual, and metaphysical [4, p. 21]. I shall focus on the former.

The following is a standard principle about the grounds of disjunction: "whenever it's the case that $\varphi$, the fact that $\varphi$ fully grounds the fact that $\varphi \vee \psi$"[18, p. 567]. Where $A$ is a full ground for $B$ iff nothing apart from $A$ is necessary to ground $B$ [10, p. 50]. The thesis follows directly from the truth conditions for disjunction and the fact that "the classical truth conditions should provide us with a guide to ground"[11, p. 105].

One may suggest then that the necessary and sufficient condition to know a disjunction is to satisfy the standard Hintikkian modal condition and to grasp the topic of at least one of the disjuncts.

$$\mathcal{M}, w \vDash K(\varphi \vee \psi) \text{ iff (for all } u \in W, Rwu \text{ implies } \mathcal{M}, u \vDash \varphi \vee \psi) \text{ and} \\ (t(\varphi) \sqsubseteq \mathfrak{K} \text{ or } t(\varphi) \sqsubseteq \mathfrak{K})$$

This clause validates closure under disjunction introduction but faces the following problems.

(P1) It is an *ad hoc* clause for the knowledge of disjunction, while I want a general clause able to deal with the knowledge of any proposition.
(P2) The clause it too demanding, as detailed in Example A.
(P3) It generates an undesired mismatch between truth and topic-grasping, as detailed in Example B.

**Example A.** Consider the formula $(p \vee q) \vee r$. We are dealing with a disjunction, whose first disjunct is itself a disjunction. Given the proposed semantics, in order to know $(p \vee q) \vee r$ one needs to either grasp the topic of $(p \vee q)$ or the topic of $r$. Nonetheless, given the same semantics, in order to know $p \vee q$ it is sufficient to grasp either the topic of $p$ or the topic of $q$. This seems to be at odds with the endorsement of closure under disjunction introduction. Consider the following scenario. An agent grasps $t(p)$ and moreover knows $p$, but they grasp neither $t(q)$ nor $t(r)$. An example of a model satisfying these conditions is $\mathcal{M}_1 = (W, R, V, \mathcal{T})$ depicted in Figure 1 in which $W = \{w\}$; $R = \{(w, w)\}$;

$T = \{a, b, c\}$; $a = t(p) = \mathfrak{K} \sqsubset b = t(q) = t(r)$; $V(p) = \{w\}$ and $V(q) = \varnothing$. By knowing $p$ and mastering disjunction introduction, the agent is in a position to know $(p \vee q)$, without grasping $t(q)$. By the same reasoning, they are in a position to know $(p \vee q) \vee r$ without grasping $t(r)$. If we accept closure under disjunction introduction, knowing $p$, which requires grasping $t(p)$, is sufficient in order to know $(p \vee q) \vee r$, without the need to grasp any additional subject matter, in particular, there is no need to grasp $t(q)$, in the same way as its grasping is not necessary in order to know $p \vee q$.

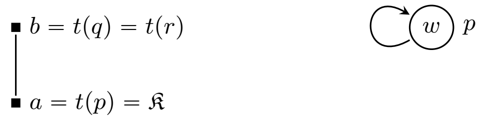


Fig. 1: Topic-sensitive model $\mathcal{M}_1$ for Example A

**Example B.** Consider the formula $p \vee q$. Assume that for all worlds $u$ such that $Rwu : \mathcal{M}, u \vDash p$ but $\mathcal{M}, u \nvDash q$; i.e. $p$ is true and $q$ is false in every world accessible from $w$. Moreover, assume $t(p) \not\sqsubseteq \mathfrak{K}$ but $t(q) \sqsubseteq \mathfrak{K}$, i.e. the topic of $q$ is grasped, while the topic of $p$ is not. An example of a model satisfying these conditions is $\mathcal{M}_2 = (W, R, V, \mathcal{T})$ depicted in Figure 2 in which $W = \{w\}$; $R = \{(w, w)\}$; $T = \{a, b\}$; $a = t(q) \sqsubset b = t(p) = \mathfrak{K}$; $V(p) = \{w\}$ and $V(q) = \varnothing$. It follows $\mathcal{M}_2, w \vDash p \vee q$, but this is undesired since there is no appropriate connection between truth and topic-grasping. The truth of the disjunction in every accessible world from $w$ depends on the truth of $p$ in such worlds, while the agent has grasped the topic of $q$, but not the topic of $p$. The grasped topic is not the relevant one, since the truth of $p \vee q$ depends on the truth of $p$.


Fig. 2: Topic-sensitive model $\mathcal{M}_2$ for Example B

Solving each of these problems constitutes the three desiderata for an appropriate modal clause, in addition to satisfying closure under disjunction introduction. To put forward such a clause, let's first introduce the concept of minimal topic.

## 4 Minimal topic as the topic of logical grounds

To define what a *minimal topic* is, I follow [3] in its definition of logical grounding.

**Definition 4** (Basic rules for grounding)**.** *The following rules are defined on the propositional language* $\mathcal{L}$*.*

$$(\wedge 1)\quad \frac{\varphi \qquad \psi}{\varphi \wedge \psi} \qquad\qquad (\wedge 2)\quad \frac{\neg\varphi}{\neg(\varphi \wedge \psi)} \qquad\qquad (\wedge 3)\quad \frac{\neg\psi}{\neg(\varphi \wedge \psi)}$$

$$(\vee 1)\quad \frac{\neg\varphi \qquad \neg\psi}{\neg(\varphi \vee \psi)} \qquad\qquad (\vee 2)\quad \frac{\varphi}{\varphi \vee \psi} \qquad\qquad (\vee 3)\quad \frac{\psi}{\varphi \vee \psi}$$

$$(\neg)\quad \frac{\varphi}{\neg\neg\varphi}$$

These rules licence only inferences from grounding to grounded statements. Since basic grounding rules are a subset of standard natural deduction rules, their application preserves truth. We can use them to build rooted trees describing the grounding relation.[6] Following standard terminology, I call *literals* propositional atoms and their negations: $\mathsf{Lit} = \mathsf{Prop} \cup \{\neg p : p \in \mathsf{Prop}\}$.

**Definition 5** (TREE)**.** *Let a TREE be a rooted tree whose nodes are occupied by propositional formulas, and whose transitions are given by the basic rules for grounding, in the sense that (i) leaves and only leaves are occupied by literals; (ii) every parent node has either one child or two children, in such a way that the principles depicted in the following table are satisfied.*

| *Node occupied by* | *Number of child(ren)* | *Child(ren) occupied by* |
|---|---|---|
| $\varphi \wedge \psi$ | *2* | $\varphi$ *and* $\psi$ |
| $\varphi \vee \psi$ | *1* | $\varphi$ *or* $\psi$ |
| $\neg(\varphi \wedge \psi)$ | *1* | $\neg\varphi$ *or* $\neg\psi$ |
| $\neg(\varphi \vee \psi)$ | *2* | $\neg\varphi$ *and* $\neg\psi$ |
| $\neg\neg\varphi$ | *1* | $\varphi$ |

Our condition (i) is stronger than Correia's, which only says that no parent node is occupied by a literal.

[6] Rules ($\wedge$2) and ($\wedge$3) are coherent with the fact that by De Morgan laws, $\neg(\varphi \wedge \psi)$ is the same as $\neg\varphi \vee \neg\psi$. What I say about disjunction holds *mutatis mutandis* for formulas of the form $\neg(\varphi \wedge \psi)$. Similarly, ($\vee$1) makes sense since $\neg(\varphi \vee \psi)$ is the same as $\neg\varphi \wedge \neg\psi$. For a discussion about the choice of this set of rules, see [3, pp. 36–8].


The tree on the left is used as an example of a legal tree by Correia [3, p. 34]. The tree on the right is an expansion of it, in which every branch reaches a leaf occupied by a literal. Only the latter is a TREE for us. Following Correia, I define what it means to be a TREE *for* a formula and *from* a set of formulas.

**Definition 6** (TREE for a formula and from a set of formulas)**.** *A TREE is for a formula $\varphi$ and from a set of formulas $\Gamma$ when its root is occupied by $\varphi$ and $\Gamma$ is the set of all the formulas occupying its leaves.*

Since our TREEs only allow literals on their leaves, every TREE for a formula is *from* a set of literals. We can now define the *minimal ground* of a formula.

**Definition 7** (Minimal ground)**.** *A set $\Gamma \subseteq \mathsf{Lit}$ is a minimal ground of $\varphi$, denoted by $\Gamma \rhd_m \varphi$, iff there is a TREE such that its root is occupied by $\varphi$ and $\Gamma$ is the set of all formulas occupying the leaves of the TREE.*

I call this *minimal* ground since one cannot proceed further in the expansion of the TREE and therefore one cannot find a set of less complex formulas grounding $\varphi$. The complexity of a formula is crucial in the definition of logical ground provided by Correia [3] and Poggiolesi [21]. For the scope of our paper, a precise definition of complexity is not important (the two authors provide two slightly different ones). The central feature is that in order for $\Gamma$ to ground a formula $\varphi$, each member of $\Gamma$ must be less complex than $\varphi$. Our definition of minimal ground of course respects this requirement since a minimal ground is always a set of literals. Nonetheless, there is a limit case for which this does not hold: a TREE with only one node, viz. a *degenerated* TREE. Let's consider a degenerated TREE with its root occupied by the literal $p$. The root is also the only leaf, therefore the TREE is *from* $\{p\}$ and *for* $p$. By definition, $\{p\} \rhd_m p$, but $p$ has of course the same complexity of $p$. Correia solves the issue dealing only with non-degenerated TREEs, since grounding is usually taken to be an irreflexive relation: nothing can explain itself. I don't get rid of degenerated TREEs though, otherwise, I wouldn't be able to deal with the knowledge of propositional variables. I don't believe this is a real problem though, since we are dealing with *minimal* ground. One needs to understand minimality as 'minimality wrt a certain environment'. Since we are dealing with logical grounding, our environment is the logical language. Once we reach a leaf of a TREE, we reach the limits of the logical language as far as grounding is concerned. Therefore the best we can do when dealing with literals is to stop, maintaining the same level of complexity, and take the singleton of the literal itself. So, $\{p\}$ is the minimal ground of $p$ just in a negative sense, given the absence of a better candidate inside the language. Another way to put this concerns truth. Grounding is strictly connected to the concept of truth. But truth is assigned directly to propositional variables by a certain valuation. Once we deal with literals we hit rock-bottom (inside the language) as far as truth is concerned, and therefore also as far as grounding is concerned. Analogous reasoning holds for topicality, since subject-matter is assigned directly to propositional variables.

Given the definition of minimal ground, the definition of *minimal topic* is then straightforward.

**Definition 8** (Minimal topic)**.** *Take a set $\Gamma \subseteq \mathsf{Lit}$ and a proposition $\varphi$ such that $\Gamma \rhd_m \varphi$. A minimal topic of $\varphi$ is the topic of $\bigwedge \Gamma$: $t(\bigwedge \Gamma)$.*

Since conjunction is topic-transparent, $t(\bigwedge \Gamma)$ is the fusion of the topics of all the formulas in $\Gamma$: $t(\bigwedge \Gamma) = \bigoplus \{t(\varphi) : \varphi \in \Gamma\}$. Given the definition of minimal topic, one could put forward the following semantic clause for knowledge.

$$\mathcal{M}, w \vDash K\varphi \text{ iff } (\forall u \in W, Rwu \text{ implies } \mathcal{M}, u \vDash \varphi) \text{ and } \exists \Gamma \rhd_m \varphi : t(\bigwedge \Gamma) \sqsubseteq \mathfrak{K}.$$

To know $\varphi$, one needs to satisfy two conditions: having information ruling out not-$\varphi$ and grasping a *minimal* topic of $\varphi$. This clause validates closure under disjunction introduction and solves (P1) and (P2). Nonetheless (P3) is not solved since the same disjunction $\psi \vee \chi$ can have two distinct immediate grounds, i.e. $\psi$ or $\chi$. I elaborate on this point in the following section, before putting forward the appropriate modal clause.

## 5 On the plurality of minimal grounds and the final proposal

Given ($\vee 2$) and ($\vee 3$), for every parental node occupied by $\psi \vee \chi$, we can generate two TREEs, one having one child node occupied by $\psi$ and the other having one child node occupied by $\chi$. Depending on the choice, we obtain different TREEs and therefore different minimal topics for the same formula. A complex formula can therefore possess several distinct minimal grounds and distinct minimal topics. Example B is still problematic since $\{q\} \rhd_m p \vee q$. We need a stricter requirement, connecting the two parts of the semantic clause – the one dealing with truth in a set of possible worlds, and the one dealing with topicality and grounds – in a tighter way.

To further elaborate, we need to consider the fact that grounding is usually taken to be a factive relation in the following sense: "if a statement is grounded in other statements, then both the grounded statement and the grounding statements must be true"[3, p. 35]. A proper notion of grounding must consider a certain truth assignment. We obtain a stronger definition of minimal grounding involving truth.

**Definition 9** (Factive minimal ground)**.** *A set $\Gamma \subseteq \mathsf{Lit}$ is a factive minimal ground of $\varphi$ wrt a world $w$ and a valuation $V$, denoted by $\Gamma \rhd_m^{w,V} \varphi$, iff $\Gamma \rhd_m \varphi$ and $\forall \psi \in \Gamma$: $V$ makes $\psi$ true at $w$.*

We can now put forward an appropriate modal clause.

$$\mathcal{M}, w \vDash K\varphi \text{ iff } \forall u \in W : Rwu \text{ implies } (\exists \Gamma \rhd_m^{V,u} \varphi : t(\bigwedge \Gamma) \sqsubseteq \mathfrak{K})$$

Notice that $\Gamma$ can be different in each accessible possible world and that the truth of every formula in $\Gamma$ assures the truth of $\varphi$ at $u$: $\varphi$ is true in all accessible worlds, and its truth is grounded on the truth of the 'right' sets of literals, i.e., the ones the agent grasps the topic of. In order for $K\varphi$ to be the case at $w$, one needs to *'cover topic-wise'* every accessible world from $w$, as far as the minimal grounds of $\varphi$ are concerned. All desiderata are met. Let's consider the following example in order to better understand the meaning of this clause and why it produces an adequate relation between truth and topic-grasping.

**Example C.** Let $\varphi := (p \vee q) \vee r$ (as in Example A). This proposition has three distinct minimal grounds, i.e. $\{p\}$, $\{q\}$, and $\{r\}$. Knowing either $p$ or $q$ or $r$ is then sufficient in order to know $(p \vee q) \vee r$. Nonetheless, we may know a disjunction without knowing any of its disjunct. Let's consider an example of this kind, by looking at the Kripke model $\mathcal{K} = (W, R, V)$ depicted in Figure 3 such that $W = \{w, u_1, u_2\}$, $R = \{(w, w); (w, u_1); (w, u_2)\}$; $V(p) = \{w, u_1\}$, and $V(q) = \{u_1, u_2\}$, $V(r) = \{u_3\}$.[7]

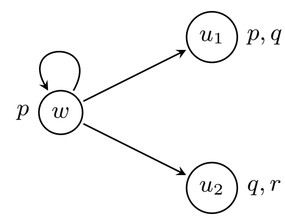


Fig. 3: Kripke model $\mathcal{K}$ for Example C

No propositional variable is true in every world accessible from $w$ and therefore none of them can be known in $w$. Nonetheless, $\varphi$ is true in all such worlds and therefore $\varphi$ is knowable in $w$, if the right amount of topic is grasped. Let us see what the minimal topic required in order to know $\varphi$ is.

In order to do so, let's define a function $\mathscr{G} : W \times \mathcal{L} \mapsto 2^{2^{\mathsf{Lit}}}$ which takes a world $u$ and a propositional formula $\psi$ as input and provides the set of factive minimal grounds of $\psi$ wrt $u$.

$$\mathscr{G}(u, \psi) = \{\Delta : \Delta \rhd_m^{u,V} \psi\}$$

Let's apply the function $\mathscr{G}$ to $\varphi$.

[7]For the sake of the simplicity I omit the fact that, given the reflexivity of $R$, we should also have $(u_1, u_1) \in R$ and $(u_2, u_2) \in R$.

$$\mathscr{G}(w,\varphi) = \{\{p\}\}$$
$$\mathscr{G}(u_1,\varphi) = \{\{p\},\{q\}\}$$
$$\mathscr{G}(u_2,\varphi) = \{\{q\},\{r\}\}$$

One needs to grasp the topic of the element of at least one minimal factive ground for each world accessible from $w$. It follows that, in order to know $(p \vee q) \vee r$ one needs to grasp $t(p) \oplus t(q)$, or $t(p) \oplus t(r)$, or $t(p) \oplus t(q) \oplus t(r)$. Let's comment on why these options are viable. Grasping $t(p) \oplus t(q)$ is sufficient because $p$ is true in both $w$ and $u_1$ and $q$ in both $u_1$ and $u_2$. By grasping $t(p) \oplus t(q)$, one covers topic-wise, every accessible world from $w$, as far as the minimal grounds of $\varphi$ are concerned. The same holds for $t(p) \oplus t(r)$ since $r$ is true in $u_2$. A fortiori, the argument holds for $t(p) \oplus t(q) \oplus t(r)$. Notice that grasping $t(q) \oplus t(r)$ would not be sufficient though, since neither $q$ nor $r$ are true in $w$. Grasping $t(q) \oplus t(r)$ does not cover the totality of accessible possible worlds.

Let's conclude with a comment on logical omniscience. A standard way to describe logical omniscience is by the so-called *rule of monotonicity*: if $\vdash \varphi \rightarrow \psi$, then $\vdash K\varphi \rightarrow K\psi$. While standard epistemic logic validates full monotonicity, standard topic-sensitive semantics validates a topic-restricted version of the rule: if $\vdash \varphi \rightarrow \psi$ and $t(\psi) \sqsubseteq t(\varphi)$, then $\vdash K\varphi \rightarrow K\psi$. My clause validates the following ground-restricted version of the rule.

**Theorem.** *If* $\forall \Gamma \rhd_m \varphi : \exists \Delta \subseteq \Gamma$ *such that* $\Delta \rhd_m \psi$, *then* $\vdash K\varphi \rightarrow K\psi$.

*Proof.* Assume $\mathcal{M}, w \vDash K\varphi$. Then $\forall u \in W, Rwu$ implies $\exists \Theta \rhd_m^{u,V} \varphi : t(\bigwedge \Theta) \sqsubseteq \mathfrak{K}$. Assume $\forall \Gamma \rhd_m \varphi : \exists \Delta \subseteq \Gamma$ such that $\Delta \rhd_m \psi$. Then $\exists \Delta \subseteq \Theta$ such that $t(\bigwedge \Delta) \sqsubseteq t(\bigwedge \Theta) \sqsubseteq \mathfrak{K}$ and $\forall \chi \in \Delta : \mathcal{M}, u \vDash \chi$. Then $\forall u \in W : Rwu$ implies $\exists \Delta \rhd_m^{u,V} \psi : t(\bigwedge \Delta) \sqsubseteq \mathfrak{K}$, i.e. $\mathcal{M}, w \vDash K\psi$.

Knowledge extends from grounding to grounded propositions. This restricted version of monotonicity is well-motivated in virtue of the proposed account and does not collapse into full monotonicity (e.g. $p \vee \neg p$ is not known in case $t(p)$ is not grasped, even if $p \vee \neg p$, being a tautology, is a logical consequence of any proposition), providing a new principled way to alleviate logical omniscience.

## 6 Conclusion

I proposed a semantic clause for knowledge that maintains the good features of topic-sensitive semantics, while improving its treatment of the knowledge of disjunction. I kept the idea that grasping the topic of a proposition is necessary in order to know it but weakened this requirement: only the *minimal* topic of a proposition is required. I exploited the concept of logical grounding to define the minimal parts of a proposition which are relevant truth-wise and topic-wise in order to know such a proposition. The resulting connection between truth and topic-grasping has been shown to be a reasonable one that avoids undesired mismatches. My proposal is particularly relevant for the endorsers of closure under disjunction introduction who wants to keep the core intuitions behind topic-sensitive semantics. Anyway, even if motivated by the treatment of disjunction, the modal clause is a general one, able to deal with the knowledge of any formula. Moreover, the proposed account provides a new principled way to face the problem of logical omniscience. Looking towards future works: the fact that complexity is already built into logical grounding, the proposal seems to be easily extendable in order to face even more directly the problem of logical omniscience, by considering agents that can parse or derive sentences only up to a certain degree of complexity.

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