The logic of Quantifier Raising

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Abstract

Quantifier Raising has long been the standard tool for analyzing scope in natural language. Despite its importance to linguistic theory, its formal properties are almost entirely unknown. For instance, is there an algorithm for deciding whether a given expression has a semantically coherent Quantifier Raising analysis? When there is at least one such analysis, is the number of semantically distinct analyses guaranteed to be finite? Does allowing type lifting (e.g., Partee's LIFT operator) affect the answers to these questions, since it creates new opportunities for scope-taking? This paper addresses these foundational questions by defining NL_{QR}, a formal logic that embodies standard practice by combining Quantifier Raising with an explicit type checking discipline. This leads to three surprising but welcome results. First, I show that Quantifier Raising is equivalent to a simple in-situ theory of scope-taking. This reconciles the age-old conflict between the movement conception of scope and the in-situ perspective. The key observation is that the semantic argument of a scope-taker always corresponds to a connected region of the tree surrounding the scope-taker. I illustrate the equivalence between the movement and the in-situ perspective with the parasitic scope analysis of *same* in *The same bee visited every flower*. Second, I show that Quantifier Raising is directly compositional, despite claims to the contrary. This follows from the interpolation theorem in Barker 2019. Finally, the decidability theorem in Barker 2019 guarantees that NL_{QR} is decidable and has the finite reading property, despite the free availability of type lifting. These results put Quantifier Raising on a reassuringly firm formal footing.

Thanks to Simon Charlow, Sandra Chung, Berit Gehrke, Daniel Lassiter, Richard Moot, and audiences at LENLS 11, ESSLLI 2015, and Stanford. At LENLS 11, Berit Gehrke pressed me on the relation between my logic and Quantifier Raising. I gave her a list of reasons why I thought it was deeply different. "So..." said Berit when I had finished, "it's Quantifier Raising".

1 What's at stake

Scope-taking is one of the most dramatic, distinctive, and pervasive phenomena in natural language, and Quantifier Raising has long been the standard tool for investigating scope. It would be difficult to overstate the importance of Quantifier Raising to modern semantic theory—literally thousands of scholarly discussions depend on

Quantifier Raising, and it is part of the catechism of every well-educated linguist and philosopher of language (chapters 7 and 8 of Heim and Kratzer 1998).

Yet the formal properties of Quantifier Raising have, as far as I know, never been studied. As a result, Quantifier Raising is currently an addictively convenient lingua franca without any guarantee that it behaves formally in a reasonable way. Obviously, this is a less than satisfactory status for such an important tool in the theoretician's toolbox. This paper establishes Quantifier Raising as a well-defined operation with excellent formal properties that can be used with justified confidence.

The key to a deeper understanding of Quantifier Raising is to explicitly regulate the interaction between semantic types and scope-taking. The strategy deployed here will be to make Quantifier Raising a bone fide rule of logical inference in a formal logic, which I will call NL_{QR}. This paper will not include technical details or formal proofs of metatheorems, but will instead describe and interpret the results of Barker 2019. Most notably, it turns out that NL_{QR} is decidable: that is, it is possible to find all semantically distinct scope analyses in an amount of time proportional to the length of the target expression.

It follows that there is no (computational) reason to worry about restricting quantifier raising by, say, prohibiting the raising of non-quantificational DPs, or by prohibiting the re-raising of previously raised expressions, or by restricting the availability of type-lifting (e.g., Partee's LIFT operator), all of which have been proposed in the literature. Instead, we can allow type-lifting and Quantifier Raising to operate freely, secure in the knowledge that there is an algorithm that will deliver all interestingly different analyses.

I'll illustrate how NL_{QR} can help reason about scope by discussing one analysis in some detail that bears on the debate over movement versus in-situ conceptions of scope-taking.

(1) The **same** bee visited **every flower**.

On the analysis of Barker 2007a, the scope of *same* is parasitic on the scope of some other operator, in this case, the quantifier *every flower*. This example motivates and illustrates the Contiguity property defined below, which says that the semantic argument of a scope-taker always corresponds to a contiguous region surrounding the scope-taker. Given contiguity, I show how NL_{QR} can be viewed as characterizing the traditional movement conception of scope-taking, or an in-situ conception, as desired.

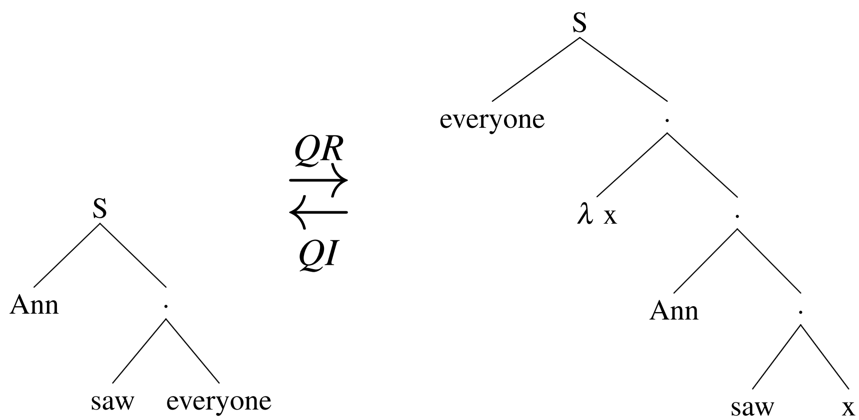
Jacobson (e.g., 1999 and many other works) advocates a particular kind of compositionality she calls direct compositionality, on which every syntactic constituent must be assigned a well-formed semantic denotation. Because the usual Quantifier Raising analysis does not assign a meaning to some phrases containing scope-takers, Barker and Jacobson 2007 hold up Quantifier Raising as their paradigm example of a non-directly compositional system. On the contrary, I show below that Quantifier Raising, as implemented in NL_{QR}, is directly compositional.

As I will discuss below, there must certainly be empirically-motivated constraints on Quantifier Raising, just as there are empirically-motivated constraints on overt syntactic movement. But in view of the results here, these additional scope constraints will be part of the theory of natural language, and not motivated by a need to make Quantifier Raising computationally or conceptually well-behaved.

For those readers who have ever worried about Quantifier Raising, this paper will put your worries to rest. For those who have never worried about Quantifier Raising, this paper will vindicate your confidence.

2 Quantifier Raising as a logical inference

Montague 1974 proposed that scope-takers lower into their surface position via a rule of Quantifying In. May 1978, 1985 developed a similar idea but in reverse: in May's version, the scope-taking expression merges in its surface position, then raises to take scope over its argument via a rule of Quantifier Raising. Glossing over inessential technical details, Quantifying In and Quantifier Raising are inverses of each other:



May's Quantifier Raising maps the tree on the left to the one on the right, and Montague's Quantifying In relates the trees in the opposite direction.

Montague's Quantifying In was part of his syntactic component, and not part of his Intensional Logic. The central innovation explored here is to make Quantifier Raising part of the logic itself—to make it a rule of logical inference:

$$\text{Montague}\downarrow \qquad \frac{\text{everyone}(\lambda x.\text{Ann}\cdot(\text{saw}\cdot x))\vdash S}{\text{Ann}\cdot(\text{saw}\cdot\text{everyone})\vdash S} \qquad \uparrow\text{May}$$

In the top to bottom direction (Quantifying In), this bit of reasoning says that given the judgment expressed by the top line, infer the judgment expressed by the bottom line. That is, given that the logical structure everyone$(\lambda x.\text{Ann}\cdot(\text{saw}\cdot x))$ has syntactic category S, infer that Ann $\cdot$ (saw $\cdot$ everyone) also has category S. In the bottom to top direction (Quantifier Raising), given that Ann $\cdot$ (saw $\cdot$ everyone) is an S, infer that everyone$(\lambda x.\text{Ann}\cdot(\text{saw}\cdot x))$ must be an S. The net result of the bidirectional inference is that the top judgment holds if and only if the bottom one does. In other words, the two structures are equivalent, and therefore syntactically interchangeable.

As I'll explain below, the form of this inference has some unusual elements, including the use of lambdas and variables. Nevertheless, Barker 2019 shows how to make such inferences part of a bone fide logic, opening the path to a treatment of Quantifier Raising that allows us to bring to bear the methods of formal logic for addressing the foundational questions of interest here.

3 NL$_{QR}$, the logic of Quantifier Raising

Readers unfamiliar with Lambek-style grammars may want to skim this section on a first reading, returning later for a deeper understanding of the technical details.

Explaining how the logic works will proceed in two phases: first, I'll present NL$_{QR}$. Then I'll explain how derivations (proofs) licensed by the logic correspond to the traditional Quantifier Raising diagrams familiar from the literature.

What does it mean to use a formal logic to describe the syntax and the semantics of a natural language? Usually, formal logics guarantee preservation of truth: if the premises are true, the conclusion is guaranteed to also be true. Following the work of Lambek, van Benthem, Moortgat, Morrill, and many others (see Moortgat 1997), we can use a logic to describe natural language if we understand that it guarantees instead preservation of grammaticality: if the premises are grammatical, then the conclusion will be equally grammatical.

In this spirit, the formulas of the logic will be interpreted as syntactic categories. For this paper, basic categories include DP, the category of determiner phrases (with semantic type $\mathtt{e}$), and S, the category of ordinary clauses (with semantic type $\mathtt{t}$).

In addition to atomic categories, following Lambek 1958, there are complex categories formed using left implication '$\backslash$' and right implication '$/$'. For instance, an expression in the category DP$\backslash$S has semantic type $\langle \mathtt{e}, \mathtt{t} \rangle$ and can combine with an expression in the category DP to its left in order to form an expression in category S. Likewise, an expression in S$/$(DP$\backslash$S) has the semantic type of a generalized quantifier, namely, $\langle\langle \mathtt{e}, \mathtt{t} \rangle, \mathtt{t} \rangle$, and can combine with an expression in category DP$\backslash$S to its right to form an expression of category S, and so on. I'll use the symbols A, B, and C to stand for arbitrary categories.

Because these categories encode the linear direction of function/argument combination ('$\backslash$' versus '$/$'), they are somewhat more fine-grained than pure semantic types. But it is easy to adapt the logic to a more sophisticated syntax, if desired: just replace each category with its semantic type, and ignore linear order. In this way the logic studied here can easily be adapted to supply scope analyses for any syntactic theory on which there is a well-defined correspondence between syntactic constituents and their semantic types.

Categories can be combined into (binary) tree structures. I'll use Σ, Δ to stand for trees. A *sequent* has the form '$\Sigma \vdash A$', and is interpreted as claiming that the tree Σ is a member of the category A. For instance, the sequent $\mathrm{DP} \cdot \mathrm{DP}\backslash\mathrm{S} \vdash \mathrm{S}$ claims that a tree consisting of an expression in category DP followed by an expression in category DP\S constitutes a (tree-structured) expression in category S.

The rules of the logic describe grammaticality-preserving relations built up out of categories, trees, and sequents.

NL$_{QR}$, the logic of Quantifier Raising:

$$\frac{\Gamma \vdash A \quad \Sigma[B] \vdash C}{\Sigma[\Gamma \cdot A\backslash B] \vdash C}\ \backslash L \qquad \frac{A \cdot \Gamma \vdash B}{\Gamma \vdash A\backslash B}\ \backslash R \qquad \frac{}{A \vdash A}\ \text{Axiom}$$

$$\frac{\Gamma \vdash A \quad \Sigma[B] \vdash C}{\Sigma[B/A \cdot \Gamma] \vdash C}\ /L \qquad \frac{\Gamma \cdot A \vdash B}{\Gamma \vdash B/A}\ /R \qquad \Sigma[\Delta] \equiv_{QR} \Delta \cdot \lambda\alpha\Sigma[\alpha]$$

Here, '$\Sigma[B]$' is a tree containing a specific occurrence of a category B somewhere inside. An example will illustrate:

(2) Ann saw Bill.

$$\cfrac{\cfrac{\mathrm{DP} \vdash \mathrm{DP} \qquad \cfrac{\mathrm{DP} \vdash \mathrm{DP} \quad \mathrm{S} \vdash \mathrm{S}}{\mathrm{DP} \cdot \boxed{\mathrm{DP}\backslash\mathrm{S}} \vdash \mathrm{S}}\ \backslash L}{\mathrm{DP} \cdot (\boxed{\mathrm{DP}\backslash\mathrm{S}}/\mathrm{DP} \cdot \mathrm{DP}) \vdash \mathrm{S}}\ /L}{\mathrm{Ann} \cdot (\mathrm{saw} \cdot \mathrm{Bill}) \vdash \mathrm{S}}\ \textsc{lex} \tag{3}$$

The grayed category DP\S corresponds to the schematic symbol B in the instantiation of the inference rule $/L$. The notation in (3) is standard (Moortgat 1997), except that I will use the term LEX as if it were an inference rule to indicate when I am substituting an English word for the logical formula that characterizes its syntactic category.

We know that the final sequent in this derivation is a theorem of the logic because it follows from axiom instances by application of the inference rules. Therefore this proof says that as long as *Ann* and *Bill* are in category DP, and *saw* is in category $(\mathrm{DP}\backslash\mathrm{S})/\mathrm{DP}$, we have a proof that the tree $\mathrm{Ann} \cdot (\mathrm{saw} \cdot \mathrm{Bill})$ is in category S.

The rule expressing Quantifier Raising in the logic is just a schematic version of the specific inference discussed above in section 2. It says that a structure of the form $\Sigma[\Delta]$

is logically equivalent to a structure of the form $\Delta \cdot \lambda\alpha\Sigma[\alpha]$.[1] The generality of the rule does not mean that anything at all can take scope over anything else—QR inferences will be part of a legitimate proof only if the categories and inferences surrounding it mesh in the appropriate way. Another way of putting it is that NL_{QR} delivers all and only those scope analyses in which the semantic types combine in a coherent way.

To illustrate, we can continue the proof given above in (3) in a way that introduces a scope-taking expression.

(4) Ann saw everyone.

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\mathrm{DP} \vdash \mathrm{DP} \qquad \dfrac{\mathrm{DP} \vdash \mathrm{DP} \quad \mathrm{S} \vdash \mathrm{S}}{\mathrm{DP} \cdot \mathrm{DP}\backslash\mathrm{S} \vdash \mathrm{S}}\,\backslash L}{\mathrm{DP} \cdot ((\mathrm{DP}\backslash\mathrm{S})/\mathrm{DP} \cdot \mathrm{DP}) \vdash \mathrm{S}}\,/L}{\mathrm{DP} \cdot \lambda x(\mathrm{DP} \cdot ((\mathrm{DP}\backslash\mathrm{S})/\mathrm{DP} \cdot x)) \vdash \mathrm{S}}\,\mathrm{QR}}{\lambda x(\mathrm{DP} \cdot ((\mathrm{DP}\backslash\mathrm{S})/\mathrm{DP} \cdot x)) \vdash \mathrm{DP}\backslash\mathrm{S}}\,\backslash R \qquad \mathrm{S} \vdash \mathrm{S}}{\mathrm{S}/(\mathrm{DP}\backslash\mathrm{S}) \cdot \lambda x(\mathrm{DP} \cdot ((\mathrm{DP}\backslash\mathrm{S})/\mathrm{DP} \cdot x)) \vdash \mathrm{S}}\,/L}{\mathrm{DP} \cdot ((\mathrm{DP}\backslash\mathrm{S})/\mathrm{DP} \cdot \mathrm{S}/(\mathrm{DP}\backslash\mathrm{S})) \vdash \mathrm{S}}\,\mathrm{QR}}{\mathrm{Ann} \cdot (\mathrm{saw} \cdot \mathrm{everyone}) \vdash \mathrm{S}}\,\mathrm{LEX}
$$

It is easiest to read this derivation from the bottom upwards. Note that in this derivation, *everyone* has the category $\mathrm{S}/(\mathrm{DP}\backslash\mathrm{S})$, the category of a generalized quantifier. On the third line from the bottom of the proof, this quantifier undergoes Quantifier Raising to the left edge of the structure that contains it, leaving behind a variable.

One interesting feature of this proof is that after two further logical inferences (namely, $/L$ and $\backslash R$, continuing to read from the bottom up), there is a second instance of the QR rule. This double application of the QR rule is characteristic of NL_{QR}. Unlike the lower instance of QR, this upper one operates in the Montagovian Quantifying In direction of travel. The role of the upper QR instance is to make sure that the raised element is replaced with a trace whose type combines smoothly with surrounding material. This type discipline is automatically enforced in NL_{QR}, since without the upper application of QR, the proof would not be complete. So if you ever wondered which of Quantifier Raising or Quantifying In is more natural or essential, the answer given here is that they are both indispensable for a complete understanding of scope taking.

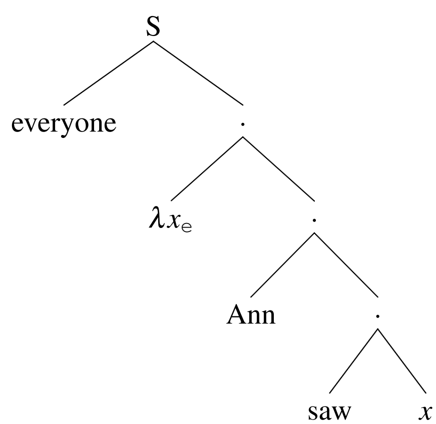
One of the pleasant aspects of Lambek-style grammars, including NL_{QR}, is that the standard Curry-Howard correspondence automatically annotates each NL_{QR} proof

[1] The variable instantiating α must be chosen so that it does not occur anywhere in $\Sigma[\,]$. See Barker 2019 for formal details: `https://rdcu.be/btr3D`.

with a lambda term corresponding to its compositional semantics: *L* inferences correspond to functional application, *R* inferences correspond to lambda abstraction, and structural rules like QR have no effect on the semantic interpretation. For instance, the Curry-Howard labeling for the final S category in (3) is $\mathbf{everyone}(\lambda x.\mathbf{saw}\, x\, \mathbf{ann})$, which is the same interpretation delivered by the traditional interpretation of Logical Form. See Moortgat 1997 for details on the Curry-Howard labeling of Lambek-style grammars.

3.1 Encoding proofs in $\mathbf{NL}_{QR}$ as traditional derivations

The discussion of the proof just given suggests how to map NL_{QR} into traditional Quantifier Raising diagrams. For each reciprocal pair of QR applications, the QR instance that operates in the Quantifier Raising direction (May) corresponds to an application of Quantifier Raising, and the QR instance that operates in the Quantifying In direction (Montague) provides the category whose semantic type characterizes the variable left behind by Quantifier Raising. For instance, the proof above in (3) corresponds to the following diagram:



In (3), the Quantifying In operation moves the category DP into the position formerly occupied by the quantifier, so we label the variable here with e, the semantic type of expressions in category DP.

Another way to characterize the correspondence is to realize that the Curry-Howard labeling for any NL_{QR} proof is essentially a linearalization of the Quantifier Raising analysis. That is, the Curry-Howard labeling of (3) is $\mathbf{everyone}(\lambda x_{\mathsf{e}}.\mathbf{saw}\, x\, \mathbf{ann})$, which is structurally isomorphic to the Quantifier Raising derivation diagram.

There will be additional instances below of proofs in NL_{QR} discussed alongside the corresponding traditional diagrams.

3.2 Unbound traces

As May noticed, unconstrained Quantifier Raising can create unbound traces. This can happen when material containing the trace of a previously raised scope taker raises higher than the lambda that binds the trace. The traditional solution for Quantifier Raising is to simply prohibit unbound traces. The solution for NL_{QR} is essentially the same: we stipulate that QR equivalence is only defined when $\Sigma[\Delta]$ and Δ are both complete structures, where a structure is complete only if it contains no unbound variables. Formal details are given in Barker 2019.

4 Contiguity and movement vs. in-situ

Quantifier Raising is traditionally conceived of as covert movement, similar in essential respects to overt syntactic movement. On this view, Quantifier Raising is part of a procedure for re-configuring syntactic trees into Logical Forms, which then serve as the basis for semantic interpretation.

Of course, there are many other conceptions of scope-taking that reject movement. These 'in-situ' analyses (including Cooper Storage, Flexible types, and so on; see Barker 2015 for one survey) use various strategies for modeling scope that do not involve reordering the pieces of a syntactic tree.

Since NL_{QR} is essentially just Quantifier Raising with a type-checking discipline, it is manifestly compatible with the traditional movement interpretation. Furthermore, to the extent that computational tractability is one of the usual motivations for considering an in-situ approach, the decidability result here weakens one of the main motivations for considering in-situ approaches.

But there is also a view on which NL_{QR} is itself an instance of in-situ scope-taking. Consider the structure $\mathrm{Ann} \cdot (\mathrm{saw} \cdot \mathrm{everyone})$. There is no standard way (though see section 5 below) to combine a transitive verb like *saw*, which has semantic type $\langle \mathtt{e}, \langle \mathtt{e}, \mathtt{t} \rangle \rangle$ with a generalized quantifier like *everyone* with semantic type $\langle \langle \mathtt{e}, \mathtt{t} \rangle, \mathtt{t} \rangle$. Certainly neither one can be a direct argument of the other, and the rule of Predicate Modification doesn't apply. On the conception of scope-taking made popular by chapter 7 of Heim and Kratzer 1998, this is a situation of "type mismatch", and it motivates Quantifier Raising as a strategy for resolving the mismatch.

This is a simple and satisfying narrative. But let's go deeper. From the (limited) perspective of semantic evaluation, the job of the syntax is nothing more (or less) than identifying what the meaningful elements are, and what order they combine in. Given this perspective, let's reframe the question: what combines with what? Certainly, the generalized quantifier does not combine directly with the transitive verb (after all, type mismatch!). But what does it combine with? Quantifier Raising answers this question by creating a new constituent, the nuclear scope of the quantifier. In the case in hand, using the structural vocabulary provided by NL_{QR}, *everyone* combines with the structure $\lambda x(\mathrm{Ann} \cdot (\mathrm{saw} \cdot x))$.

Here is the fact that makes an in-situ view of scope-taking coherent:

> Contiguity: The semantic argument of a scope-taker always corresponds to a connected region of the tree immediately surrounding the scope-taker.

Here, a connected portion of a tree is any subtree from which one or more complete subtrees have been removed (see section 4.1 for a detailed example).

Not all analyses satisfy Contiguity.

(5) I do**n't** think [Bill's very smart].

(6) A [man] entered **who was wearing a raincoat**.

(7) [Ann] seems **nice**.

In each of these cases, it is reasonable to suppose that the bolded expression takes the bracketed expression as its semantic argument, despite the fact that they are not contiguous and adjacent in the intended sense. These constructions all have overt-movement based analyses on which there is a derivational stage at which the bolded and bracketed expressions are adjacent.

Analyses that satisfy Contiguity, however, allow us to take an in-situ perspective, and to assume that the elements combine without syntactic movement. The fact that Quantifier Raising guarantees Contiguity is that an arbitrary Quantifier Raising analysis chops up a syntactic tree into contiguous pieces. As long as we can assign each piece a coherent denotation, and as long as we can represent the order and the manner of the composition of the denotations of the pieces, we have a complete scope analysis without any movement. NL_{QR} satisfies these requirements: we can view a proof in NL_{QR} as a method for finding where to cut in the original tree, how to assign meanings to the parts, and how to combine them into a complete analysis.

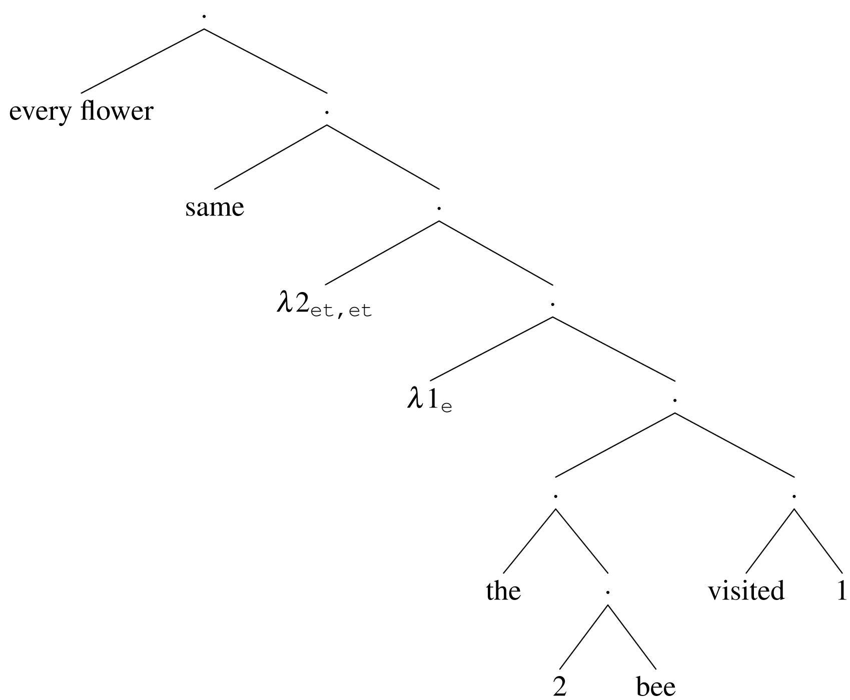
4.1 Case study in contiguity: parasitic scope

I'll illustrate how contiguity leads to an in-situ treatment of scope with a particularly revealing example involving parasitic scope.

(8) The **same** bee visited **every flower**.

On the relevant reading, *same* needs to distribute a property (namely, the property of being visited by a particular bee) over the set of relevant flowers. There is a challenge to compositionality, then, since *same* needs semantic access to a distant constituent that it does not even c-command.

On the analysis of Barker 2007a, the key to a compositional solution is to recognize that any scope-taker creates a new constituent over which some other element can take scope. That is, the scope of the second scope-taker is parasitic on the scope of the first.

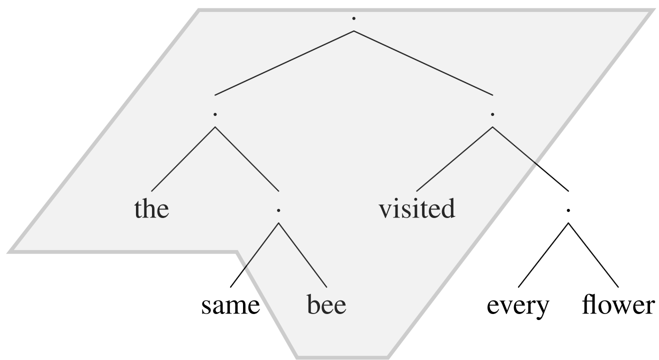


$$\textbf{every-flower}(\textbf{same}(\lambda f \lambda x.\textbf{visited}(x)(\textbf{the}(f(\textbf{bee})))))$$

As the diagram shows, the quantifier *every flower* Quantifier Raises (index 1), then *same* Quantifier Raises (index 2) to a position in between *every flower* and its nuclear scope.

Other phenomena with insightful parasitic scope analyses include superlatives (Szabolcsi 2010), phrasal comparatives (Pollard and Smith 2013), *the average American* (Kennedy and Stanley 2009), and certain types of coordination (Kubota and Levine 2015).

Does parasitic scope satisfy the contiguity claim? In particular, does the semantic argument of *same* correspond to a contiguous region of the surface syntax? In the following diagram, I've marked the portion of the syntactic structure that *same* takes scope over:



This analysis satisfies Contiguity. In particular, the scope-taker *same* is adjacent to a contiguous region that is its semantic argument, namely the grayed region. To see that the grayed region is a connected region of the tree, note that it is the entire tree with two complete subtrees removed (namely, the subtree corresponding to *same*, and the subtree corresponding to *every flower*).

The grayed subproof shows how NL_{QR} recognizes *the _ bee visited _* as a syntactic constituent.

$$
\begin{array}{lll}
\vdots & & \\
(\text{the}\cdot(\textsc{adj}\cdot\text{bee}))\cdot(\text{visited}\cdot\text{DP})\vdash \text{S} & & \\
\hline
\text{DP}\cdot\lambda x((\text{the}\cdot(\textsc{adj}\cdot\text{bee}))\cdot(\text{visited}\cdot x))\vdash \text{S} & & \text{QR}\\
\hline
\lambda x((\text{the}\cdot(\textsc{adj}\cdot\text{bee}))\cdot(\text{visited}\cdot x))\vdash \text{DP}\backslash\text{S} & & \backslash R\\
\hline
\textsc{adj}\cdot\lambda y\lambda x((\text{the}\cdot(y\cdot\text{bee}))\cdot(\text{visited}\cdot x))\vdash \text{DP}\backslash\text{S} & & \text{QR}\\
\hline
\lambda y\lambda x((\text{the}\cdot(y\cdot\text{bee}))\cdot(\text{visited}\cdot x))\vdash \textsc{adj}\backslash(\text{DP}\backslash\text{S}) & \text{DP}\backslash\text{S}\vdash\text{DP}\backslash\text{S} & \backslash R\\
\hline
(\text{DP}\backslash\text{S})/(\textsc{adj}\backslash(\text{DP}\backslash\text{S}))\cdot\lambda y\lambda x((\text{the}\cdot(y\cdot\text{bee}))\cdot(\text{visited}\cdot x))\vdash \text{DP}\backslash\text{S} & & /L\\
\hline
\text{same}\cdot\lambda y\lambda x((\text{the}\cdot(y\cdot\text{bee}))\cdot(\text{visited}\cdot x))\vdash \text{DP}\backslash\text{S} & \text{S}\vdash\text{S} & \text{LEX}\\
\hline
\text{S}/(\text{DP}\backslash\text{S})\cdot(\text{same}\cdot\lambda y\lambda x((\text{the}\cdot(y\cdot\text{bee}))\cdot(\text{visited}\cdot x))\vdash \text{S} & & /L\\
\hline
\text{every-flower}\cdot(\text{same}\cdot\lambda y\lambda x((\text{the}\cdot(y\cdot\text{bee}))\cdot(\text{visited}\cdot x))\vdash \text{S} & & \text{LEX}\\
\hline
\text{every-flower}\cdot\lambda x((\text{the}\cdot(\text{same}\cdot\text{bee}))\cdot(\text{served}\cdot x))\vdash \text{S} & & \text{QR}\\
\hline
(\text{the}\cdot(\text{same}\cdot\text{bee}))\cdot(\text{visited}\cdot\text{every-flower})\vdash \text{S} & & \text{QR}
\end{array}
$$

The Curry-Howard correspondence provides a denotation for the grayed constituent, namely, $\lambda f\lambda x.\mathbf{visited}(x)(\mathbf{the}(f(\mathbf{bee})))$.

Here is the in-situ method suggested by this discussion:

1. Chop a syntactic tree into pieces.

2. Assign each piece a denotation.
3. Combine the denotations in an order that respects contiguity (i.e., elements combine only with adjacent regions)

On this view, Quantifier Raising is exactly the part of the system that implements step (1). Each instance of QR cuts the tree in two places: on the branch leading to the scope-taking element, and on the branch leading to the top of the scope domain. The logical rules of NL_{QR} implements steps (2) and (3).

Incidentally, in the terminology of Barker and Shan 2014, when an element combines with a region that surrounds it, that region counts as one of the *delimited continuations* of that element. So NL_{QR} is well suited to reasoning about delimited continuations.

One reason that it is worthwhile entertaining the in-situ interpretation of Quantifier Raising is that it provides insight into the decidability result discussed below. Here's how: there are a strictly finite number of ways to chop up a given tree into pieces. As long as there is also a strictly finite number of ways of assigning denotations to two pieces that will allow them to combine in a way that respects their semantic types, we can be sure that there are only a finite number of distinct scope analyses for a given syntactic structure. It is not obvious that this is the case, but this follows from the reasoning in Barker 2019. In other words, satisfying Contiguity is essential to the decidability of Quantifier Raising.

In view of Contiguity, then, scope-taking is purely a matter of figuring out what combines with what.

5 Quantifier Raising is directly compositional

According to Barker and Jacobson 2007, Quantifier Raising is not directly compositional. Roughly, a semantic theory is *directly compositional* if every syntactic constituent has a well-defined semantic value. Here is what they say (p. 2) about Quantifier Raising:

> [In the standard analysis using Quantifier Raising,] a verb phrase such as *saw everyone* fails to have a semantic interpretation until it has been embedded within a large enough structure for the quantifier to raise and take scope (e.g., *Someone saw everyone*). On such an analysis, there is no semantic value to assign to the verb phrase *saw everyone* at the point in the derivation in which it is first formed by the syntax (or at any other point in the derivation, for that matter). A directly compositional analysis, by contrast, is forced to provide a semantic value for any expression that is recognized as a constituent in the syntax. Thus if there are good rea-

sons to believe that *saw everyone* is a syntactic constituent, then a directly compositional analysis must provide it with a meaning.

And indeed, if we examine the proof above in (3) of *Ann saw everyone*, there is no stage at which the structure corresponding to *saw everyone* is established as a constituent. In particular, there is no category associated with that particular substructure, and no Curry-Howard labeling that contains the semantic contribution of *saw* and *everyone* and nothing else.

There are many good reasons, of course, to suppose that verb phrases such as *saw everyone* are constituents. For instance, this particular verb phrase can serve as the antecedent of verb phrase ellipsis, as in *Ann saw everyone, and Bill did too*, in which what Bill did was see everyone. In the kind of directly compositional system that Barker and Jacobson have in mind, there will be a semantic value computed for the structure *saw everyone* that will conveniently make salient the semantic value captured by the ellipsis.

So if you think that *saw everyone* is a complete syntactic constituent in $Ann \cdot (\text{saw} \cdot \text{everyone})$, and you think that the only semantic operation allowed in merge structures is function application, then you will have to conclude along with Barker and Jacobson that Quantifier Raising is not directly compositional, at least, not always.

Of course, if we have a more fluid conception of constituency, even the analysis in (3) will count as directly compositional. On the in-situ view described above, the quantifier *everyone* does not form a constituent with *saw* in this particular derivation. Rather, it combines with the contiguous region containing *Ann*, *saw*, and nothing else, that is, with $\lambda x.\text{ann} \cdot (\text{saw} \cdot x)$, as shown in the proof. Once we allow constituents to correspond to any connected region of a structure, we can recognize that every constituent recognized by the derivation does indeed receive a semantic value.

But there is more to be said. Barker 2019 proves an interpolation theorem that says that given any derivation of $\Sigma[\Delta] \vdash A$, there is a category B such that there are derivations of $\Delta \vdash B$ and $\Sigma[B] \vdash A$. That is, in NL_λ, it is possible to find a well-formed category, along with a well-formed meaning, for every structure.

Since NL_{QR} is a fragment of NL_λ, the metalogical theorems in Barker 2019 that characterize NL_λ hold for NL_{QR} as well. [2] Applying the theorem here requires enlarging the fragment of NL_λ covered by NL_{QR} a bit to include conjunction. Then in addition to categories like $A\backslash C$ and C/B, we also have the category $A \times B$. These categories are related by Lambek's residuation laws:

$$A \vdash C/B \quad \text{iff} \quad A \times B \vdash C \quad \text{iff} \quad B \vdash A\backslash C$$

I'll refer to the expanded fragment as $\text{NL}_{QR\times}$. See Barker 2019 for the logical inference rules characterizing $\times$.

[2] NL_λ is discussed in Barker 2007a, Barker and Shan 2014, and Barker 2019. The connectives $\backslash$, $\cdot$, and $/$ in NL_{QR} correspond to $\backslash\!\!\backslash$, $\circ$, and $/\!\!/$ in NL_λ.

Once we have added conjunction to our logic, we can interpolate a category and a semantic value for *saw everyone*. In the case of (3), the category for *saw everyone* (relative to this specific derivation) is $(\mathrm{S}/(\mathrm{DP}\backslash\mathrm{S}))\times(\mathrm{DP}\backslash(\mathrm{DP}\backslash\mathrm{S}))$, whose Curry-Howard labeling (i.e., its semantic value) is $\langle\textbf{everyone}, \textbf{saw}\rangle$, the ordered pair consisting of the generalized quantifier denoted by *everyone* combined with the relation denoted by *saw*.

This sort of derivable direct compositionality is what Barker 2007b calls 'direct compositionality on demand': the formal system allows either a long-distance, raising-style derivation on which we do not pause to provide intermediate structures with categories or denotations; or else an equivalent (but longer) derivation in which these details are supplied.

This is good news for theorists with commitments to direct compositionality: Quantifier Raising, as implemented in $\mathrm{NL}_{QR\times}$, provides direct compositionality on demand. This means that every Quantifier Raising analysis is guaranteed to have an equivalent Quantifier Raising derivation that is directly compositional.

6 Decidability

In addition to interpolation, the decidability result in Barker 2019 also applies to NL_{QR}. Given an arbitrary proof in NL_{QR}, there is an equivalent proof (same conclusion, same logical inferences, same Curry-Howard labeling) whose depth is at most twice the number of $\backslash$'s and $/$'s in the final sequent. Since every derivation is semantically equivalent to a member of a finite set, we have decidability, and furthermore, we know there will be at most a finite number of distinct semantic interpretations for any given sequent.

One reason decidability is not obvious for a grammar with Quantifier Raising is that there can be an unbounded number of analyses for a single sentence. For instance, if it makes sense to Quantifier Raise a generalized quantifier to an enclosing clause, then there is no formal reason why we couldn't allow the quantifier to undergo Quantifier Raising again, and so on, ad infinitum.

Such spurious additional instances of Quantifier Raising are clearly useless semantically. The situation is less clear when type lifting is involved. Type-lifting is freely available in NL_{QR}. For example, Partee's 1987 LIFT operator is a theorem of NL_{QR}:

$$\dfrac{\dfrac{\mathrm{DP}\vdash\mathrm{DP}\qquad\mathrm{S}\vdash\mathrm{S}}{\mathrm{DP}\cdot\mathrm{DP}\backslash\mathrm{S}\vdash\mathrm{S}}\,\backslash L}{\mathrm{DP}\vdash\mathrm{S}/(\mathrm{DP}\backslash\mathrm{S})}\,/R \tag{9}$$

This proof guarantees that any expression in the category DP is also in the category $\mathrm{S}/(\mathrm{DP}\backslash\mathrm{S})$.

Partee 1987 advocates allowing expressions to "live at their lowest type". If proper names like *Ann* live at category DP, then in order to coordinate a proper name with generalized quantifier, the usual strategy is to use LIFT to shift the DP expression into category $S/(DP\backslash S)$. In this spirit, there will be proofs in NL_{QR} that $(\text{Ann} \cdot (\text{and} \cdot \text{everyone})) \cdot \text{left}$ is an S in which the proof in (9) is a subproof.

But freely allowing type-lifting threatens decidability. The reason is that lifting a DP to a generalized quantifier category can force Quantifier Raising. If we allow an already lifted expression to undergo a second instance of lifting, we will need a second instance of Quantifier Raising in order to fully resolve the type mismatch. It follows that if there is no limit on the number of lifting operations, there can be no limit on the number of Quantifier Raising operations.

The reason type-lifting does not break decidability here is that each logical inference that we might hypothesize as part of a derivation involving lifting (or any other derivation, for that matter) consumes a logical connective. That is, inspecting the four logical rules in NL_{QR}, they each have one fewer logical connective in their premises than in their conclusion. You can see this clearly in the proof of LIFT in (9): there are two logical connectives in the final sequent, one logical connective in the middle sequent, and zero logical connectives in the premises on the top line of the proof. This inexorable reduction in the number of logical connectives is the heart of the decidability argument. It follows from the form of the logical inference, and it is why Genzen's sequent presentation is so useful for proving metatheoretical results. Because the inferences involved in the derivation of a lifted category consume two logical connectives (as we just saw for (9)), it follows that the number of lifting inferences that we need to consider while searching for a proof is bounded by the number of logical connectives in the final sequent. In other words, the number of instances of lifting is bounded by the complexity of the semantic types of the lexical items.

A second potential worry comes from Quantifier Raising of non-quantificational expressions. For instance, given a sentence such as *Ann left*, there is no formal reason why *Ann* cannot undergo Quantifier Raising. In particular, in NL_{QR}, we can raise *Ann* using QR, and then immediately QR it back to where it started, with no net difference in semantic interpretation. The diagnosis of Heim and Kratzer 1998:210 is that this kind of excursion is often useless, but semantically harmless, and need not be prohibited. There are empirical arguments that Quantifier Raising of proper names can be not only useful, but essential; see discussions in chapter 9 and 10 of Heim and Krazer 1998, following on insights from Reinhart 1983.

The decidability theorem guarantees a limit on how many times we need to consider allowing Quantifier Raising to target random expressions. The proof shows that when an instance of Quantifier Raising introduces a structural connective that never interacts with a logical rule, that instance of Quantifier Raising is eliminable without affecting the proof. That is, we can safely restrict attention to instances of Quantifier Raising that do non-trivial logical work. Each such instance of Quantifier Raising can be associated with a unique logical connective in the final sequent. Since there are a finite number of logical connectives ('/', '\') in any given sequent, there will be at most a finite number of useful instances of QR.

Note that in NL_{QR}, it is not just quantificational DP's and proper names (category DP) that can undergo QR, it is quite literally any structure at all. The logic will sort out derivations in which such QR'ing is semantically coherent and useful from those in which it is not.

7 Scope constraints

There are additional constraints besides logical coherence that determine which scope interpretations will be available for a given expression (see Szabolcsi 2010). More specifically, it is commonly assumed that just as there are constructions out of which overt movement is prohibited ("island constraints"), there are constructions out of which scope-taking is prohibited ("scope islands"). In a particularly elegant version of the world, islands for overt movement and scope islands would coincide. As Dayal 2013 puts it, "conceiving of Quantifier Raising as a syntactic rule provides a general explanation for some of the restrictions on quantifier scope . . . whatever principles of syntax rule out the formation of overt dependencies in these constructions can be tapped to rule out the creation of problematic covert dependencies at LF."

Indefinites pose a famous challenge to this unified view, since they routinely take scope out of syntactic islands. One well-established strategy for explaining this apparent exception allows indefinites to take scope via some mechanism other than Quantifier Raising. For instance, indefinites might denote choice functions bound by existential closure (Reinhart 1998, Kratzer 1998), or they might contribute sets of alternatives that percolate upwards in the composition via pointwise functional composition (Kratzer and Shimoyama 2002, Alonzo-Ovalle 2006). If indefinites take scope via some other mechanism than Quantifier Raising, that makes it possible to impose limitations on Quantifier Raising that do not affect indefinites. In other words, scope islands might turn out to be constraints on Quantifier Raising, and not on scope taking in general.

It is important to note that although the semantics of indefinites is compatible with the alternative scope-taking mechanisms, it does not require them. At the end of the day, the truth conditions of a sentence involving an indefinite are exactly what they would have been if the indefinite had taken appropriate scope via Quantifier Raising. In fact, Charlow (to appear a) argues that there are good reasons for preferring a Quantifier Raising treatment.

But it's not just indefinites. Other scope-takers are able to take scope out of syntactic islands. To take just one example, relative clauses are one of the stronger islands for overt movement in most languages (**Who did Ann buy the book that was written by?*).

Yet it is possible for universal quantifiers to take scope outside of a relative clause. Here are three naturally-occurring examples:

(10) $^{\gamma}$For the experiment, measure the time that each person took to travel 20 meters.

(11) $^{\gamma}$There is a role that each person is uniquely designed by God to fulfill.

(12) $^{\gamma}$Include the name of the person that each volunteer must report to.

These sentences can describe situations with multiple times, multiple roles, and multiple names. This shows that universals can scope outside of relative clauses.

There are systematic constraints on scope taking, of course; they just don't happen to line up with overt movement islands. For instance, negation and downward monotonic operators create particularly robust scope islands for a wide variety of scope-takers.

(13) No one loves everyone.

This sentence has a logically coherent scope analysis on which it entails that for every person, no one loves that person (i.e., there is no person-loving at all). There may be situations that support special tunes for pronouncing (13) on which the inverse scope reading might become accessible; but if so, the inverse scope reading is dramatically less available than the linear scoping.

To sum up, just like syntactic movement, covert movement must be constrained in various way. It is unlikely that covert movement obeys the same set of constraints as overt movement. Scope islands are strongly sensitive to the identity of the scope-taker, and negation is an important element in any comprehensive picture.

What is the upshot of scope constraints for Quantifier Raising? The theory here provides every logically coherent scope analysis for a given sentence. Only some of those logically possible interpretations will be accessible to native speakers. The view here is that whether constraints on scope-taking turn out to be due to semantic, pragmatic, or processing constraints, or some mixture, they are substantive empirical hypotheses that are independent of the logical notion of scope-taking. It is worth noting that the decidability result discussed here shows that whatever motivates scope islands, it is not the need to render interpretation decidable, though there remains plenty of room for scope constraints to reduce the search space in a way that may make processing easier (White et al. 2017).

8 Three additional issues

8.1 Quantifier Raising is syntactic

The logical equivalence in NL_{QR} that embodies Quantifier Raising is a structural inference rule. As such, it is part of the syntax of the logic. This means that an in-situ quantifier and its Logical Form created by QR are not just semantically related, they are *fully syntactically equivalent*. Put another way, the QR structural rule does not affect semantic labeling at all, since the Curry-Howard correspondence ignores structural inferences. Rather, the QR rule allows a scope-taker to combine directly with material that surrounds it. Thus its role is to help characterize what syntactically combines with what, and in what order. It follows that scope is an essentially syntactic phenomenon.

8.2 Complex traces

There are many analyses that rely on higher-order traces: semantic reconstruction (e.g., Cresti 1995, Sternefeld 1995, Fox 1999, Barker and Shan 2014 inter alia), split-scope analyses (German *kein* (Jacobs 1980), donkey anaphora (Barker and Shan 2014), Haddock sentences (Bumford 2017), and cumulative readings (Charlow to appear b). Higher-order traces are perfectly compatible with the system here. See Charlow to appear b for an especially lucid discussion of the details and the trade-offs of having higher-order traces in a Quantifier Raising analysis.

8.3 The logic of movement?

If computing covert scope analyses is decidable, what about overt movement? NL_{QR} is a proper fragment of NL_λ. As shown in Barker and Shan 2014 and in Barker 2019, NL_λ is able to account not only for in-situ scope-taking, but syntactic movement as well. Because NL_λ is decidable, it provides way to combine syntactic movement and scope-taking in a single unified grammar that is computationally well-behaved. A thorough exploration of the status of overt movement will have to wait for another occasion.

9 Conclusion

Quantifier Raising has long been the standard tool for analyzing scope in natural language. The results in Barker 2019 show that when Quantifier Raising is combined with an explicit method for checking type compatibility, Quantifier Raising is equivalent to a directly compositional, in-situ theory of scope-taking. Furthermore, Quantifier Raising is decidable, and provides a strictly finite number of distinct semantic interpretations for any given expression, even in the presence of type lifting. These results taken together justify full confidence in Quantifier Raising as a coherent and formally well-behaved technique for specifying what combines with what, in which order.

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