Vagueness, Comparatives, and Scale Structure[*]

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Contents

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1 The devil

A dialogue:

> *Devil:* Good news! We're going to play a game. If you win, you go to heaven. But if you should act contrary to morality or offend God, suggesting that good be evil or evil good, or that more be less or less more, or that God is what God is not, or… — okay, you get the point — then, well, you know what happens. Ready? Lovely. To begin: The fates of your best friend and 2 strangers are in my hands. You must choose who shall live and who shall die: either your dearest friend, whom you know and love, or the 2 strangers. Which shall it be?
>
> *You:* What a game! Any chance the initials of one of the strangers is associated with convulsions?
>
> *Devil:* Hah! As if you could be so lucky. Let us suppose the strangers are relevantly similar to your friend.
>
> *You:* In that case: my friend. Final answer.
>
> […*humanity* − 2…]
>
> *Devil:* Good choice! It is indeed morally better for you to save your friend than to save two strangers. You must have been paying attention that day they taught special obligations. Okay, warm-up over. You must now either save your best friend or save 3 strangers. Which shall it be? If saving your friend kept you before, surely it must again now.
>
> *You:* Yes, Devil; you are right, Devil. I will save my best friend.
>
> […*humanity* − 3…]
>
> *Devil:* Lovely! Saving your best friend was morally better than saving 3 strangers. But we're not quite done yet. You must now either save your best friend or save 4 strangers. Don't forget: You mustn't do anything to suggest that God is what God is not, lest you offend the Lord thy God. And the Lord thy God is not an arbitrary god. Surely saving your friend now couldn't be contrary to morality if it wasn't before. You wouldn't accuse God of being arbitrary, would you?
>
> *You:* Surely not!
>
> *Devil:* I wouldn't think so. So you will save your friend?
>
> *You:* Indeed, Devil. You are right again, Devil.
>
> *Devil:* Excellent!

⋮

You: Hey, I see where this is going. You're going to force march me straight into destroying all humanity!

Devil: I never doubted your wits. Worry not, friend, for as it is written, "comparatives… do not give rise to the Sorites paradox."[1] How many grains one must have to have oneself a heap is quite another thing altogether, is it not? Indeed, "adjectives in the comparative are uniformly non-vague."[2] So shall we press on?

You: As you say, Devil.

Devil: Lovely.…

[*…proceeds for some time…*]

Devil: Alas, we appear to be left with only you and this dear friend of yours. Don't fret: I will spare you the burden of choosing. And good news! You have avoided implying that God is arbitrary. Yet I regret to inform you that you have killed off all of humanity — okay, nearly all — and such is in fact grounds for exclusion from, you know, heaven.

You: Hey, wait a minute! You said that there was no Sorites with comparatives.

Devil: I never said that. Quick to accuse the Accuser, eh? Who am I to deny the great guardians of English?

You: Surely something has gone wrong, Devil!

Devil: Indeed — you are on your way to hell! But wrong how?

You: The game was rigged! How could I have avoided acting contrary to morality or suggesting that God is what God is not?

Devil: I never said you could! It is perhaps easier for Bentham to pass through the eye of a needle than for a fan of special obligations to, er — but I digress. Rules are rules. So, as they say, "All the best," or some such.

You just comparative-forced-march yourself into hell.

Less cosmically (or dynamically) put, an argument against special obligations proceeds thus:

[1] McNally 2011: 164n.10

[2] Bochnak 2013: 56

(1) (P1) Your saving 2 strangers is not morally better than your saving your dear friend.

(P2) For all n, if your saving n strangers is not morally better than your saving your dear friend, then your saving $n+1$ strangers is not morally better than your saving your dear friend.

(C) $\therefore$ For all $n > 2$, your saving n strangers is not morally better than your saving your dear friend.

No one's friends are that important.

That is: Suppose for reductio that we have special obligations to those close to us, so that (P1) is true. Yet, though we might have at least some special obligations, so that it is morally better for you to save your friend than to save two strangers, there doesn't seem to be any precise number of strangers that would tip the balance. This, along with the dominant semantic frameworks for gradation in linguistic semantics,* and (P2) is true. (* *See below*.) The argument seems valid; it is validated not only by mathematical induction, but by any transitive consequence relation that validates modus ponens (for the material conditional) and universal instantiation (Hyde 2014). Yet (C) is false. Contradiction. So we don't have special obligations.

(1) is, on the face of it, an instance of the familiar sorites argument form, with 'is not morally better than your saving your dear friend' as the intuitively vague predicate. So why not just take our favorite account of the sorites, apply it to 'is not morally better than your saving your dear friend', and be on our way? Soon enough, friends. It isn't for no reason that comparatives have been said not to be vague.[3] Our opening scene may seem familiar, but, rest assured, the plot will be different.

Whence, then, arguments such as (1)? There are various things one might say in reply. Accepting (C) or rejecting the argument's validity aren't some of them; life is short (and so is this paper), so let us leave such normative and logical revisionaries for elsewhere.

Perhaps it's special obligations that are the problem. Why not bite the bullet and deny (P1)? Bite away, but the relief may be short-lived.

[3] And not merely in the mouth of the devil in §1 (e.g., Cooper 1995: 246; Kennedy 2011: 74, 93; 2013: 267–271, 275; McNally 2011: 164n.10; van Rooij 2011a: 65–69; Bochnak 2013: 41–47). I will focus only on linguistic vagueness associated with gradable adjectives. Unless otherwise noted I use 'comparative' specifically for explicit comparatives, i.e. comparatives using the comparative form ('is ADJ-er than'). I briefly consider implicit comparatives ('is ADJ compared to') in §5. Interadjective comparisons ('x is longer than y is wide'; van Rooij 2011b), metalinguistic comparisons ('more machine than man'; Morzycki 2011), and comparatives with nominal predicates ('more of a bird than'; Sassoon 2013, cf. Sauerland & Stateva 2011) raise special challenges.

You like sugar in your coffee.[4] Yet it's not as if you care exactly how sweet it is. As far as your preferences go, one day's sweetness is as good as any other — okay, at least up to a point (say, K; there is, perhaps, such a thing as too sweet). Now consider (2) — letting x_s be an ordinary cup of coffee, and $x_1, \ldots, x_n, \ldots, x_K$ be a series of otherwise identical cups differing only in quantity of sugar, with x_n being a (pre-K) cup with n micrograms of sugar:

(2) (P1) x_s is more preferable than x_1.
(P2′) For all $n < K$, x_n is as preferable as x_{n+1}.
(P3) For all a, b, c, if a is more preferable than b, and b is as preferable as c, then a is more preferable than c. (*PI-transitivity*)
(C) $\therefore$ For all $n < K$, x_s is more preferable than x_n.

(P1) and (P2′) are true given your preferences. (P3) is validated by standard semantics for gradation.* (* *See below*.) Again the argument seems valid. And yet the conclusion (C) is false. Not every cuppa can be the best. There may be something wrong with sugar in one's coffee, but not that thinking otherwise leads to paradox.[5]

From the objectivity of morality to the subjectivity of taste, there is a reductio in the offing. Moralist or amoralist, the paradox comes for us all.

2 Some details (* *From above*)

The non-vagueness of comparatives has been said to follow from the general structure of standard semantic frameworks for gradation (e.g., McNally 2011, Kennedy 2013). What are these "standard semantic frameworks" that we keep hearing so much about?

[4] Or consider Pat, who does, and read 'preferable' throughout as 'preferable according to Pat'; any compositional semantics will need to be able to characterize Pat's state of mind so as to make sense of such talk. (Likewise for 'morally good' and 'morally good according to God' (or some such).)

[5] Note that the force of (P2′) isn't fundamentally due to limitations in one's powers of discrimination (whether of quantity of sugar or quality of sweetness). As Wright shows in responding to Peacocke, mere indiscriminability between adjacent items is insufficient to generate the paradox (1987: 239–243). Even a supertaster could accept (P2′). One simply doesn't care exactly how sweet the coffee is; one cup is as good as the next, given one's preferences. (P2) with 'morally good' in (1) obviously doesn't turn on matters of indiscriminability. We will return to this. (I now see that certain of the comparative sorites examples which I used in earlier work were problematic in failing to appreciate the above point from Wright (Silk 2016: 198–199, 206). Thanks to Gunnar Björnsson for helpful discussion.)

2.1 Degree-based semantics

First, a prominent approach is to treat gradable adjectives as associating individuals with degrees on a scale (e.g., Bartsch & Vennemann 1973, von Stechow 1984, Kennedy 1999, 2007, Heim 2001). For instance, 'tall' denotes a function *tall* from individuals to (positive) degrees of height, i.e. the individual's maximal height; 'hot' denotes a function *hot* from individuals to (positive) degrees of temperature, i.e. the individual's maximal temperature; and so on.[6] Though many theories assume that degrees are isomorphic to real numbers and that scales are dense linearly ordered sets of degrees (e.g., Bartsch & Vennemann 1973, von Stechow 1984, Klein 1991, Fox & Menendez-Benito 2006, Sassoon 2010, 2013), at minimum it is required that the relation $\geq$ on the set of degrees D have at least the structure of a partial (if not total) order, i.e. that $\geq$ be a (possibly complete) reflexive, transitive, and antisymmetric relation on D (cf. Kennedy 1999, 2007, Barker 2002, Lassiter 2015, Morzycki 2015). Compositional details aside, the comparative (3) says that the (maximal) degree to which Alice is tall, $tall(Alice)$, is greater than the (maximal) degree to which Bert is tall, $tall(Bert)$, as stated in (4).

(3) Alice is taller than Bert.

(4) (3) is true iff $tall(Alice) > tall(Bert)$

Recall the comparative sorites argument in (2). The above semantics renders the interpretation of (P3) as in (5), where *pref* (= ⟦preferable⟧) is a function from items to their degree of preferability, a representation of how preferable they are:[7]

(5) $\forall x \forall y \forall z \big[\big((pref(x) > pref(y)) \wedge (pref(y) = pref(z)) \big) \rightarrow (pref(x) > pref(z)) \big]$

(P3) is an instance of what is sometimes called *PI-transitivity*, which is a weakening of transitivity — i.e., if a relation $\gtrsim$ satisfies transitivity, PI-transitivity in (6) is also

[6] To fix ideas I assume a Kennedy-style measure-function analysis (see also Bartsch & Vennemann 1973), treating gradable expressions as denoting functions from items to degrees (type $\langle e, d \rangle$, or for broadly modal adjectives type $\langle st, d \rangle$). An alternative is to treat gradable expressions as denoting relations between items and degrees (for the non-modal case, type $\langle d, et \rangle$ or $\langle e, dt \rangle$; e.g. Cresswell 1977, von Stechow 1984, Heim 2001). For discussion of different types of degree-based analyses, see e.g. Kennedy 1999, Beck 2011, Demonte 2011, Morzycki 2015. I address non-degree-based frameworks and treatments of the positive form shortly. For simplicity I bracket complications from intensionality. I continue to focus only on vagueness phenomena associated with gradable adjectives.

[7] For simplicity I assume an "equally good" reading of the equative (see Bhatt & Pancheva 2007, Rett 2008).

satisfied, where $>$ and $\sim$ are the strict and non-strict parts, respectively, of $\gtrsim$.

(6) *PI-transitivity*
$(X > Y \wedge Y \sim Z) \rightarrow X > Z$

That is, (P3) is simply a weakening of transitivity. It follows from the transitivity of the relation $\geq$ on the domain of degrees D.

Upshot: (P1) is true (n. 4). (P2′) characterizes your non-obsessiveness about coffee sweetness. (P3) is entailed by the general structure of scales, and thus holds with any adjective denotation (measure function). The argument seems valid. QED.

2.2 Delineation semantics

Degree-based semantics for gradable adjectives aren't the only game in town. Perhaps a non-degree-based approach would fare better? Not so fast. Consider the other main approach in formal semantics: delineation semantics ("supervaluationist," "partial predicate," "inherent vagueness" semantics). Delineation semantics treat gradable and non-gradable expressions alike as ordinary predicates (type $\langle e, t\rangle$, or for broadly modal adjectives type $\langle st, t\rangle$). What distinguishes gradable adjectives is (inter alia) their sensitivity to a contextually supplied comparison class (e.g., Klein 1980, Doetjes et al. 2009, Burnett 2012). In one context 'Alice is tall' might mean that Alice is tall for a basketball player; in another context it might mean that she is tall for an American woman. Gradable adjectives are treated as denoting partial functions partitioning a comparison class *CC* into a positive extension, a negative extension, and an extension gap (the "borderline cases"):[8]

(7) $[\![\text{tall}]\!]^{CC} = \lambda x : \neg gap_{CC}(tall)(x) \,.\, x$ is tall in *CC*

Predicative uses are treated straightforwardly: 'Alice is tall' is true given *CC* iff Alice counts as tall in *CC*. Delineation theories differ on how to analyze the meaning of the comparative given the comparison class. Roughly, according to Klein 1980, a comparative 'x is ADJ-er than y' is true iff there is a comparison class relative to which x counts as ADJ but y does not (cf. van Benthem 1982, von Stechow 1984). For

[8]Some theories also invoke a parameter δ setting relevant standards/thresholds for different adjectives (e.g., Lewis 1970, Barker 2002): 'tall' is treated as denoting those individuals in *CC* whose height is at least as great as the standard of tallness δ_{tall} set (at least in part) by δ. For simplicity I put this potential layer of context-sensitivity aside. In delineation semantics degrees may be invoked in the metalanguage (e.g., Barker 2002), but they aren't included in the type system.

instance, (3) 'Alice is taller than Bert' is true (given any comparison class) iff there is some comparison class CC' such that Alice is tall in CC' and Bert is not tall in CC'.

Degree-based and delineation-based approaches differ on issues regarding the morphology and internal compositional semantics, and the ontological status of degrees. Yet there are well-known logical correspondences between the two frameworks. In order to avoid problematic inconsistencies and entailments, delineation theories impose qualitative restrictions on comparisons among individuals across contexts (comparison classes). For instance, per Klein's (1980) "consistency postulate," if x counts as tall in some CC and x has a greater height than y, then there can be no CC' in which x doesn't count as tall but y does (see also Fine 1975, Kennedy 1999, Fara 2000). Delineation theorists have proven that the relevant qualitative restrictions derive a *preorder* (reflexive, transitive, possibly total relation) $\succsim_A$ "at least as ADJ as" over the set of individuals in the domain of 'ADJ', for any adjective 'ADJ' (Klein 1980, 1991, van Benthem 1982, van Rooij 2011a). The degree-theorist's basic notions of degrees and scales may then be derived from these qualitative orderings $\succsim_A$ (Cresswell 1977, Bale 2008, 2011, Lassiter 2011a, van Rooij 2011a): the set of degrees D is the set of equivalence classes under $\succsim_A$, and the relation $\geq_A$ on D is defined accordingly in terms of $\succsim_A$, i.e. $[x]_A \geq_A [y]_A$ iff$_{def}$ $x \succsim_A y$ (where $[a]_A$ is an equivalence class $\{b : b \succsim_A a \wedge a \succsim_A b\}$). Details of these derivations aside, what is important here are the results. The interpretation of any adjective is treated as relying on a preorder $\succsim_A$ on the set of individuals related by the adjective. The transitivity of $\succsim_A$ again validates the PI-transitivity premise (P3). Whether we go for Klein or for Kennedy, the dialectic from §1 is off and running. The challenge posed by the comparative sorites — more on which shortly — needn't be hostage to debates between degree-based and delineation-based approaches regarding the basic vs. derived status of degrees, the role of degrees in object language and metalanguage, or the morphosyntax of predicative and comparative uses.

3 Idealizations?

The (hopefully not foaming) reader doth protest: "A comparative sorites is a sorites still. Compare (1) to (8)–(9) with 'tall', where x_n is someone $4' + n$ nanometers tall:

(8) (P1) Someone who is $4'$ isn't tall (for a pro basketball player).
(P2) If someone who isn't tall (for a pro basketball player) grows one nanometer, they still won't be tall (for a pro basketball player).
(C) $\therefore$ No one, no matter how tall they are, is tall (for a pro basketball player).

(9) (P1) x_0 is not tall.
(P2) For all n, if x_n is not tall, then x_{n+1} is not tall.
(C) $\therefore$ For all n, x_n is not tall.

A surface-form comparative expression 'is (not) ADJ-er than y' might not have precisely the same syntax/semantics as a simple predicate of individuals (§2) or a positive form use such as 'is (not) tall'. But shall we not still be able to apply our favorite account of the sorites for 'tall' to 'is more preferable than', 'is morally better than', etc.? For instance, a standard move is to locate the problem in the inductive premise: even if we can't point to any instance of the generalization (P2) in (9) that isn't true (cf. Soames 1999, Fara 2000), perhaps we can know it isn't true in any context (Fara 2000), or no matter what formally precise language we might be speaking (Lewis 1970; cf. Silk 2016), or no matter how the conversation might evolve (Shapiro 2006), or on any competent way of applying 'tall' (Kamp 1981, Raffman 2014). Why not say the same about (P2)/(P2′) in (1)/(2)?"

To fix ideas recall our Kennedy-style degree semantics from §2.1. The positive (unmodified) form, in such semantics, is treated as relating a degree to a relevant threshold, or *degree standard*. Roughly put, 'Alice is tall' is true iff the degree to which Alice is tall is at least as great as the relevant degree standard of tallness s_{tall}, i.e. how tall one must be to count as tall:[9]

(10) 'Alice is tall' is true iff $tall(Alice) \geq s_{tall}$

The degree standard s provides a natural place for capturing felt vagueness in predicative uses. Suppose we fix on a particular procedure for measuring height and a relevant comparison class. Still we may not be willing commit to any specific degree of height as constituting how tall one must be to count as tall. Even if the inductive premise (P2) in (9) is false at any point of evaluation according to the formal semantics, as reflected in (11), speakers may find it compelling because (say) the falsifying instance determined by the degree standard is never where one is looking (Fara 2000); or there is indeterminacy or uncertainty about which degree standard is determined in one's concrete conversational situation (cf. Barker 2002);

[9]Details of the internal morphosyntax and compositional semantics which delivers these truth conditions won't be important for our purposes. Many degree-based theories derive the positive form by combining the adjective with a null morpheme, 'pos', to yield a predicate of individuals (e.g., Cresswell 1977, von Stechow 1984, Kennedy 1999, 2007). For simplicity I treat the degree standard simply as a variable (see Kennedy 1999, 2007, Glanzberg 2009 for discussion), and I will abstract away from context-sensitivity from comparison classes.

or the speakers are undecided about what degree standard to accept for purposes of conversation (cf. Silk 2016); and so on.

(11) *IND-PRED*
$\forall n\,[(tall(x_n) \not\geq s_{tall}) \rightarrow (tall(x_{n+1}) \not\geq s_{tall})]$

There is an important difference between the inductive premises in the predicative and comparative sorites arguments. Speakers' assumptions in concrete discourses are typically compatible with a range of degree standards in such a way that makes IND-PRED with 'tall' seem plausible. Yet it is in principle possible to deny IND-PRED and settle on a precise standard of tallness. Treating the positive form 'tall' as semantically interpreted with respect to precise degree standards, or (possibly singleton) sets thereof, allows for this. Drawing the line *here* for what counts as tall may seem unjustified, but, ontological vagueness notwithstanding, it needn't do violence to the worldly facts or manifest a lack of semantic competence.

Not so with premises such as (P2) in (1) or (P2′) in (2). Only the maximally opinionated coffee maven could sincerely deny (12).

(12) *IND-COMP*
$\forall n\,[pref(x_n) = pref(x_{n+1})]$

The normative (and theological) facts in §1 straight-up imply (13).

(13) $\forall n\,[(morally\text{-}good(save\text{-}n) \not> morally\text{-}good(save\text{-}friend))$
$\rightarrow (morally\text{-}good(save\text{-}n{+}1) \not> morally\text{-}good(save\text{-}friend))]$

Idealizations are commonplace in formal semantics. Yet it would distort our representation of speakers' semantic competence — not to mention substantive normative/evaluative facts — to treat measures of preferability, value, etc. as maximally "opinionated" in such a way as to necessarily falsify (12)–(13).

As may be expected, this worry needn't be devastating. "Your contrast between the predicative and comparative cases is spurious. Just as it would be 'physiologically' arbitrary to mark a particular counterinstance of IND-PRED (cutoff) with 'tall', or 'sociologically' arbitrary to mark a particular counterinstance of IND-PRED (cutoff) with 'rich', so would it be (e.g.) 'bouletically' arbitrary to mark a particular counterinstance of (12) with 'preferable', 'normatively' arbitrary to mark a particular counterinstance of (13) with 'morally good', etc. If using sorites-immune formal objects is legitimate in the semantics of the one, it is legitimate in the other." Perhaps. ("One person's modus ponens…" and all that.) Yet the intuitive contrast remains:

Pretheoretically (more-or-less), whereas we might know that at the end of inquiry, no matter how the conversation might evolve, IND-PRED (11) with 'tall' is false, we know that (12)–(13) are *true*, given one's preferences and the normative (*cum*-theological) facts.[10] It is a cost to a theory if it represents us as inevitably semantically incompetent with 'preferable', 'morally good', etc. Semantics is hard, but it shouldn't be that hard.

4 Taking stock

Let's take stock. Positive form predications ('is tall', 'is bald') reign supreme in discussions of linguistic vagueness. Despite the vast literature, little-to-no attention has been given to phenomena with comparatives. In linguistic semantics it is common to assume that the comparative form ('is ADJ-er than') cannot be vague (see n. 3). Vagueness is treated as due to a fuzziness in standards of application — how many millimeters of height one must have to count as tall, how many cents one must have to count as rich, and so on. Hence, "Unsurprisingly, comparatives… do not give rise to the Sorites paradox, and do not have borderline cases" (McNally 2011: 164n.10). But they do.

Vagueness phenomena with comparatives raise a distinctive challenge for traditional formal semantics for gradable expressions. Take (2). The argument seems valid, and we know that the base premise (P1) is true and conclusion (C) is false. The inductive premise (P2′) is true given your lack of concern about precisely how sweet your coffee is. That leaves only (P3), which is validated by the general structure of standard semantics for gradation. This point is perhaps most evident in existing degree-based frameworks, which treat gradable expressions as associating items with degrees on a scale (§2.1); but it generalizes to partial-predicate analyses that don't invoke a notion of degrees (§2.2).

Unlike previous cases (few that there are), the cases of vague comparatives in §1 cannot be reduced to cases of indiscriminability between adjacent items in a sorites series, indeterminacy (uncertainty, indecision) in what dimensions are relevant, or unsettledness (indecision, imprecision) about how items are to be measured (cf. Williamson 1994: 156, Keefe 2000: 12–13, Sassoon 2013). Fix on a particular dimension for preferability or moral value, and procedure for measuring/determining it, and the force of the comparative sorites remains. Many a monis-

[10] Cf. "Each sharpening in the frame represents one way that some indeterminacies can turn out, consistent with the meaning of the predicates, the non-linguistic facts, and the externally fixed contextual factors" (Shapiro 2006: 76; see also, e.g., Raffman 2014). (See nn. 4–5.)

tic indirect consequentialist have countenanced special obligations. Moreover the concern with denying the inductive premises isn't simply that doing so would be unwarranted or in tension with some limited power of discrimination (n. 5). Drawing the line *here* for what counts as tall may seem unwarranted but it isn't incoherent. No such option is available to our poor soul or less-than-maximally opinionated coffee drinker from §1 (cf. §3). It would be surprising if one could rebut fans of special obligations or sugar-taking coffee drinkers with facts about semantic scale structure.

The upshot, as the reader may have guessed a mile (read: 11 pages) back: We need a semantic framework that allows for a certain kind of intransitivity with gradable expressions.

One strategy would be to capture the relevant intransitivities with vague comparatives via some general mechanism. Some theorists tie vagueness phenomena to features of particular types of expressions, often positive form relative gradable adjectives.[11] A frequent worry for such approaches is that seemingly similar phenomena arise also with nominals ('heap'), adverbs ('quietly'), quantifier phrases ('a few dogs'), etc.[12] Consequently, some theories offer uniform accounts of linguistic vagueness, capturing vagueness phenomena across categories of expressions and constructions via general features of the semantics or metasemantics. Witness: the general frameworks of supervaluations or many-valued/fuzzy logics, or the general claims of epistemicists about metasemantic complexity and imperfect knowledge of meanings. Such apparatus could be applied to comparatives. For instance, one might posit systematic semantic association with a discourse-level standard of precision or measure of granularity (cf. Lewis 1979, Krifka 2007, Morzycki 2015), and then restrict the evaluation of arguments to "admissible" contexts in which (inter alia) the adjective's measure function (e.g. *pref*) isn't less "opinionated" than the measures of relevant subvening properties (e.g. quantity of sugar, quality of sweetness). Or one might treat the formal semantics as supervaluating over numerical (real-valued) measure functions (Sassoon 2013) — or treat the formal semantics as interpreted with respect to a particular numerical measure function and treat token uses as adjusting a set of live measure functions (cf. Lewis 1975, Barker 2002, Lassiter 2011b) — where the various measure functions provide different counterinstances of the inductive premises.

There are reasons to be cautious about pursuing "global" strategies such as these. For one thing, there is a growing body of work in linguistic semantics which has

[11] E.g.: "the analysis of vagueness is linked to a particular semantic feature of the positive form (i.e. to a feature of the meaning of pos)" (Kennedy 2011: 83; cf. 74, 2013: 271, Fine 1975: 267).

[12] Cf. Stanley 2003, esp. nn. 6, 8; Keefe 2000: 14; Raffman 2014: 16–18, 22, 131–132.

stressed the importance of distinguishing sources of apparent vagueness phenomena, whether by distinguishing kinds of vagueness (Sauerland & Stateva 2011) or by distinguishing vagueness from imprecision or "loose talk" (Lasersohn 1999, Kennedy 2007, Morzycki 2015). A more specific concern comes from our remarks from §3. Knowing the facts about usage and the extra-linguistic circumstances (think: epistemicism, contextualism), and settling on (meta)normative issues about what determines preferability, moral value, etc. (think: precise contexts), is insufficient to undermine the intuitive force of the comparative sorites. In degree-semantic terms, the problem isn't that our measures are insufficiently precise, or that we are aren't settled on what measure function to associate with the expressions (that we don't know what dimensions are relevant for determining preferability, moral value, etc., or how to measure things with respect to these dimensions). It is that we need the measures to allow for certain intransitivities.

This worry is, again, not decisive. Yet in what follows I would like to put global approaches to the comparative sorites aside, and ask how we might pursue a more local approach that revises the semantics of gradation. Such an approach of course won't speak to apparent vagueness phenomena arising with non-gradable expressions. Given the prominence of capturing at least some apparent vagueness phenomena via features specific to semantic gradability, I ask fans of more global approaches to indulge me in investigating the more specific project. I leave investigation of broader apparent vagueness phenomena for elsewhere.

5 Degrees and scale structure: Semiorders in a degree semantics

To fix ideas let's continue to assume a Kennedy-style degree-based semantic framework. Two natural places for revising the semantics are the representation of *degrees* and the representation of *scales*. On the former: I have been assuming a traditional degree semantics on which degrees are conceived as points on a scale. An alternative would be to treat degrees as *sets* of points, perhaps intervals, and treat adjectives as associating individuals with such (perhaps fuzzy) sets (cf. Kennedy 2001, Schwarzschild & Wilkinson 2002, Solt 2014; though see Silk 2016: 186–187). On the latter: In §2.1 I noted that existing degree-based semantics treat scales $\langle D, \geq \rangle$ as imposing a relation $\geq$ with at least the structure of a partial order on the domain of degrees D. (Analogously in the discussion of delineation theories I noted that the meanings of adjectives are treated as relying on a relation $\gtrsim$ with at least

the structure of a partial preorder on a set of individuals.) In the remainder of the paper I would like to outline one way of revising this assumption about the scale structure. I leave more detailed developments and comparisons with alternatives for future work. (I continue to focus specifically on gradable adjectives.)

Let's continue to treat adjective denotations as associating items with degrees, conceived as points, on a scale. However, I suggest that we treat the domain of degrees as coming with a *semiorder*: I suggest that we treat a scale as a structure $\langle D, \succ \rangle$, where $\succ$ is a semiorder on the set of degrees D. Formally, a semiorder $\succ$ is an interval order that satisfies semitransitivity:

(14) *Irreflexivity:* $\forall x\colon x \not\succ x$
Interval-order: $\forall x, y, z, w\colon (x \succ y \wedge z \succ w) \rightarrow (x \succ w \vee z \succ y)$
Semitransitivity: $\forall x, y, z, w\colon (x \succ y \wedge y \succ z) \rightarrow (x \succ w \vee w \succ z)$

Equivalently, $\succ$ is a semiorder iff there is a real-valued function f such that $x \succ y$ iff $f(x) > f(y) + \epsilon$, for some fixed positive number ϵ (Scott & Suppes 1958, Fishburn 1985). From $\succ$ a relation $\sim$ can be defined, where $x \sim y$ iff $x \not\succ y \wedge y \not\succ x$. Crucially, although $\sim$ is reflexive and symmetric, it needn't be transitive (unlike the non-strict part of a partial or total order).

Semiorders have been used fruitfully in measurement theory and choice theory for modeling intransitive indifferences (Luce 1956, Scott & Suppes 1958, Fishburn 1985). This broader work on semiorders provides an independently motivated piece of apparatus to incorporate into the semantics of gradation. One way of interpreting the above formalism is to understand f as mapping degrees of ADJ-ness to measures of a relevant property on which ADJ-ness may (possibly trivially) supervene — e.g., as mapping degrees of preferability to measures of sweetness. We can think of ϵ as representing a threshold of distinguishability with respect to the property associated with the adjective, now written ϵ_A (hereafter "distinguishability threshold"). Intuitively, the greater the value of ϵ_A, the less distinguishing in matters of ADJ-ness. Truth conditions for comparatives and equatives may be given as follows, where *adj* is the measure function denoted by 'ADJ' (see nn. 7, 9):

(15) 'a is ADJ-er than b' is true
iff $adj(a) \succ_A adj(b)$
iff $f_A(adj(a)) > f_A(adj(b)) + \epsilon_A$

(16) 'a is as ADJ as b' is true
iff $adj(a) \sim_A adj(b)$
iff $|f_A(adj(a)) - f_A(adj(b))| \leq \epsilon_A$

Take 'preferable'. Supposing sweetness is the sole property relevant for preferability, ϵ_P would represent a level of sweetness sufficing to distinguish cups with respect to how preferable they are; anything less than ϵ_P and any differences in sweetness fail to render one more preferable than the other. For degrees of preferability x and y, '$x \succ_P y$' says that x's sweetness is relevantly more than y's sweetness, i.e. greater than ϵ_P; and '$x \sim_P y$' says that the difference in sweetness between x and y is less than ϵ_P.

Several clarificatory remarks on interpreting the formalism: First, it is in general important in semantics for gradation not to read too much into the 'measure' in 'measure function', or into an appeal to "degrees" in the formal semantics. I use 'measure function' broadly, not only for adjectives associated with measurement procedures or numerical units of measurement (e.g. height in inches, for 'tall'), but for any mapping which would determine an ordering on objects (contrast Sassoon 2013, 2010, Crespo 2015, Kamp & Sassoon 2016). Talking of the adjectives' "measure functions" doesn't presuppose that properties of preferability, value, etc. are quantifiable. What is important about "degrees" for our purposes is that they represent assessments of how preferable, tall, rich, etc. things are, and thus that they can be associated with qualitative orderings on the items in the adjectives' domains. Nothing of metaphysical significance is presupposed in our talk of things having "degrees" of preferability, value, etc.

Likewise, one shouldn't be misled by the use of numerical values with f and ϵ in the definition of semiorders. A relation is a semiorder only if *there is* such a function and number that satisfy the stated condition. As noted above, degrees themselves aren't treated as isomorphic to numbers, and properties of ADJ-ness needn't be quantifiable. The numerical threshold ϵ is used in *representing* a relation between items that fail to count as relevantly distinguishable. Talk of the "difference" between $f_A(adj(a))$ and $f_A(adj(b))$ being "less than" or "equal to" ϵ_A is compatible with (inter alia) the items a and b being identical, as ADJ as one another, imperceptibly different in ADJ-ness, or even incomparable.

Finally, the notion of "(non-)distinguishability" is always to be understood as specific to matters of the property described by the adjective in the given context. '$adj(a) \succ_A adj(b)$' says that a and b are relevantly distinguishable *in ADJ-ness* — for instance, that the difference in sweetness between a and b suffices to distinguish them in how preferable they are, and render b more preferable than a. Being discriminable in some respect doesn't imply being "distinguishable," in the present sense of being related by $\succ_A$ in a context. On the flip side, saying '$adj(a) \sim_A adj(b)$', that a's and b's degrees fail to exceed the "threshold of distinguishability" ϵ_A, doesn't imply that a and b are indiscriminable, either in general or in properties relevant to determining how ADJ they are (see n. 5). Indiscriminability is one basis for

being related by $\sim_A$ — for it neither being the case that $x \succ_A y$ nor that $y \succ_A x$ — but it isn't the only one. As noted above, the supertaster among us could perhaps discriminate between adjacent coffee cups x_n, x_{n+1} in their quantity of sugar or even quality of sweetness. Saying that their degrees are related by $\sim_P$ is to say that any such difference fails to constitute a relevant distinction in preferability. The act of leaving your friend to the devil while letting 2 strangers continue on their merry way may be discriminable in, say, utility from leaving your friend to the devil while letting 3 strangers continue on their merry way. Such a difference may even constitute a discriminable difference in moral value.

Let's turn to applying the above semiorder-based account to the comparative sorites. The semantics in (15)–(16) avoids validating PI-transitivity in (2) or satisfying the inductive premise (P2) in (1). Start with (2) with 'preferable'; the predicted truth conditions of (P2′)–(P3) are as follows:

(17) (P2′) is true
iff $\forall n\,[pref(x_n) \sim_P pref(x_{n+1})]$
iff $|f_P(pref(x_n)) - f_P(pref(x_{n+1}))| \leq \epsilon_P$

(18) (P3) is true
iff $\forall a \forall b \forall c[((pref(a) \succ_P pref(b)) \wedge (pref(b) \sim_P pref(c)))$
$\rightarrow (pref(a) \succ_P pref(c))]$

The semantics in (17) captures the truth of (P2′). Every pair of adjacent cups x_n, x_{n+1} are related by $\sim_P$; the difference in how preferable they are falls below the threshold ϵ_P. Discriminable though they might be, one cup is as good as the next, given your preferences. However, PI-transitivity (P3) is violated. The counterinstance occurs at cup x_i such that $f_P(pref(x_s)) - f_P(pref(x_{i+1})) = \epsilon_P$ (where x_s is, again, an ordinary sweetened cup of coffee): x_s is more preferable than x_i, since $f_P(pref(x_s)) > f_P(pref(x_i)) + \epsilon_P$; yet it's not the case that x_s is more preferable than x_{i+1}, since $f_P(pref(x_s)) = f_P(pref(x_{i+1})) + \epsilon_P$. x_{i+1} still fails to differ in sweetness from x_s in such a way as to render it less preferable. So, we can (*a*) accept that $pref(x_s) \succ_P pref(x_1)$, i.e. that x_s is more preferable than x_1; (*b*) allow that $pref(x_1) \sim_P pref(x_2) \wedge \ldots pref(x_{s-1}) \sim_P pref(x_s)$, i.e. that adjacent cups in the series aren't distinguished in preferability; and yet, due to the intransitivity of $\sim_P$, (*c*) maintain that $\exists x_j$: $pref(x_s) \nsucc_P pref(x_j)$, i.e. that there is a cup x_j which x_s isn't preferable to.

The semantics also avoids satisfying the inductive premise (P2) in (1) from our dialogue with the devil:

(19) (P2) is true
iff $\forall n\,[(\textit{morally-good}(\textit{save-}n) \not\succ \textit{morally-good}(\textit{save-friend}))$
$\rightarrow (\textit{morally-good}(\textit{save-}n{+}1) \not\succ \textit{morally-good}(\textit{save-friend}))]$

The counterinstance occurs at act *save-i* such that $f_A(\textit{morally-good}(\textit{save-friend})) - f_A(\textit{morally-good}(\textit{save-}i{+}1)) = \epsilon_M$. This failure of the inductive premise is compatible with accepting $\textit{morally-good}(\textit{save-}n) \sim_M \textit{morally-good}(\textit{save-}n{+}1)$, that the difference — indeed moral difference — between saving n strangers (over your friend) and saving $n + 1$ strangers (over your friend) is insufficient to relevantly morally distinguish them in the context.

In these ways, the present semiorder-based degree semantics avoids validating PI-transitivity or satisfying inductive premises of the classic form in arguments such as (2) (or (9) for the matter); and it does so without needing to represent measures of preferability, value, etc. as maximally discriminating and opinionated (§3). As Fara 2000 emphasizes, however, predicting that the inductive premise is not true doesn't suffice for an account of the sorites (cf. Wright 2003: 87, 97–98, Raffman 2014: 122, a.o.). If the inductive premise isn't true in any context, why do we find it so plausible? What about its classical equivalent, the "sharp boundaries" claim? What should we say about the seemingly predicted sharp boundary between (e.g.) cups that aren't more preferable than x_s and cups that are?

This isn't the place to hazard a general theory of the semantics, epistemology, and psychology of vagueness;[13] however, several directions for approaching the above questions in light of the above semantics are as follows. First, the intransitivity of the defined non-distinguishability relation $\sim$ locates a place for explaining at least some of the sorites' intuitive appeal. Though the semantics doesn't verify (P2) in (1) or PI-transitivity (P3) in (2), it verifies claims expressing how adjacent items in the series are relevantly non-distinguishable in preferability, moral value, etc. So, one idea is that we find the inductive premises compelling because we fail to distinguish them from the true claim that $\textit{adj}(x_1) \sim_A \textit{adj}(x_2) \wedge \cdots \wedge \textit{adj}(x_{n-1}) \sim_A \textit{adj}(x_n)$.

Second, the distinguishability threshold ϵ_A locates a place for importing techniques familiar from broader theories of vagueness (epistemicism, contextualism, supervaluationism, etc.). For instance, epistemicists claim that facts about competent use determine precise extensions for intuitively vague terms (Sorensen 1988, Williamson 1994, Barker 2002). In terms of the present semantics an epistemicist might treat a specific value of ϵ_A as determined by the world of evaluation. Apparent fuzziness in the distinguishability threshold could be diagnosed as uncertainty about

[13] See Silk 2016: chs. 6–7 for one sort of broadly contextualist approach.

what formally precise language (in the sense of Lewis 1975) is being spoken.

Alternatively, on a broadly contextualist line, the distinguishability threshold may be treated as a contextual parameter $\epsilon_c = \langle \epsilon_{A^1_c}, \epsilon_{A^2_c}, \ldots \rangle$, with different contexts determining different levels of distinguishability.[14] For the maximally opinionated among us, context may supply a value of $\epsilon_{A_c} = 0$; no difference in properties relevant to determining how ADJ things are goes undetected or uncared-for (?) in matters of ADJ-ness. For the rest of us, context supplies $\epsilon_{A_c} > 0$ and the comparative sorites is off and running. Further, even if the compositional semantics takes as given a particular value for ϵ_c, there may be indeterminacies in what value is determined in concrete discourses (cf. Silk 2016). In token uses there may be a range of live representations of context and values for ϵ_c compatible with the interlocutors' interests (Fara 2000), psychological states or verbal dispositions (Raffman 1994, 1996), or discourse moves (Kamp 1981, Soames 1999, Shapiro 2006, Silk 2016). We may not be able to point to any instance of the inductive premise (P2)/(P2′) we reject, or any instance of the sharp boundaries claim which we accept.

Before concluding I would like to briefly compare the semantics in this section to the delineation semantics (§2.2) from van Rooij 2011a, which uses semiorders for analyzing so-called *implicit* comparatives (the only precedent, to my knowledge, for invoking semiorders in linguistic semantics). Implicit comparatives are sentences '*x* is ADJ compared to *y*' in which a comparison is made using the positive form (Kennedy 2011, Bochnak 2013), unlike explicit comparatives '*x* is ADJ-er than *y*', as considered in this paper, which use the comparative form. Implicit comparatives differ from ordinary explicit comparatives in intuitively requiring there to be a *significant* difference between the items, as reflected in (20).

[14] 'Broadly contextualist' in the sense of covering different combinations of contextualism/relativism and indexicalism/non-indexicalism in the sense of MacFarlane 2009, i.e. different views on whether the value of the parameter figures in calculations of semantic content or is included in the index/circumstance of evaluation, and whether it is determined by the context of utterance or by a "context of assessment." I take it that many accounts in the vagueness literature classified as "contextualism about vagueness" (see below in the main text) are neutral on such matters of the compositional semantics and postsemantics (cf. Åkerman 2009: 47–53; Kölbel 2010: 324–325; Kennedy 2013: 270n.19; Silk 2016: §§6.1, 6.3.3). For narrowly contextualist and relativist approaches, see e.g. Silk 2016 and Richard 2008; Lassiter 2011b, respectively.

(20)

a. Alice is taller than Bert. (*explicit comparative: true*)
b. Alice is tall compared to Bert. (*implicit comparative: false*)

Though Alice's height is greater than Bert's height, as required by (20a), it isn't *significantly* greater, as intuitively required by (20b). van Rooij uses semiorders to capture this "significantly ADJ-er than" relation in the intuitive interpretation of implicit comparatives. Explicit comparatives are analyzed via weak orders as usual (§2.2), and they are denied to be vague. The semantics in this section, in contrast, invokes semiorders in the scale structure which figures in the interpretation of all uses, and thus allows for vagueness with both positive and comparative forms. Note that this proposal is compatible with contrasts between implicit and explicit comparatives such as in (20). The semiorder on the set of degrees represents a relation of relevant distinguishability in matters of ADJ-ness. The truth of (20a) simply requires that Alice's height be distinguishably greater than Bert's — which it is. The distinguishable difference required by an explicit comparative needn't be "significant" in such a way that verifies a corresponding implicit comparative, such as (20b).

This section has begun to develop a revised semiorder-based semantic framework for gradation. Although semiorders have had wide use in measurement and choice theory, they have received nearly no currency to-date in linguistic semantics. Previous appeals to semiorders in treatments of vagueness have focused specifically on predicative uses and the positive form (Luce 1956, Halpern 2008, van Rooij 2011a,c).[15] The proposal in this section is to invoke semiorders in the basic structure of semantics for gradation. In degree-based terms, the semantics treats the relation on the set of degrees as a semiorder, rather than, say, a partial or total order. This move helps capture vagueness phenomena with gradable expressions in various forms and uses. The proposed semiorder-based semantics allows for vagueness phenomena with both positive and comparative forms, and captures the problematic intransitivities in comparative sorites cases such as from §1.

Many questions remain. The account allows for vagueness with comparatives and avoids diagnosing linguistic vagueness generally in terms of features specific to

[15]Interestingly, Fara's (2000) semantics for the positive form utilizes a relation of "being significantly greater than," which is the interpretation typically given to semiorders in preference theory and measurement theory. Ackerman (1994: 132–135) raises the connection between the sorites and the paradox of intransitive preferences but doesn't develop it further.

positive form relative gradable adjectives (*pace*, e.g., Cooper 1995, Kennedy 2011, 2013, McNally 2011, van Rooij 2011a, Bochnak 2013); yet there remains a potential concern that vagueness phenomena are still being addressed piecemeal, in terms of the semantics of gradation. Whether we should prefer a more unified account of apparent vagueness phenomena in natural language remains to be seen (§4). Extensions to higher-order vagueness and expressions of other categories (e.g. 'definitely'), interactions between positive predications and (implicit and explicit) comparatives, and implications for scalar semantics generally call for careful investigation. For now let us rest content that — the devil and one's special obligations be damned — humanity may yet live to see another day.

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