Few and *fewer*

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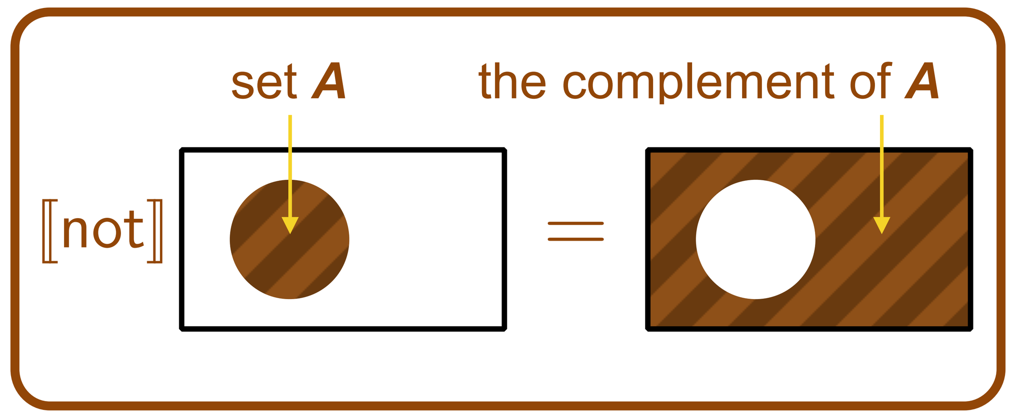
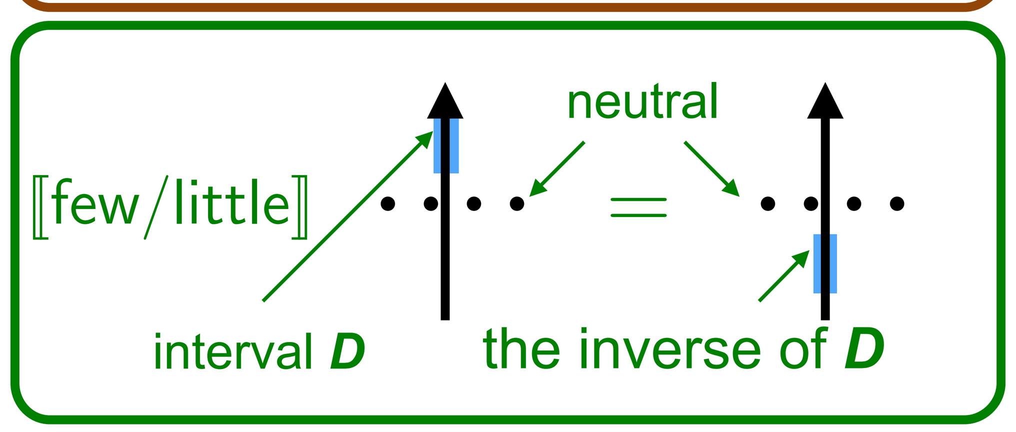
Take-home messages

We claim that **not** and **few/little** represent two different kinds of **negation** operators: While **not** takes a **set** as argument and returns its **complement**, **few/little** takes an **interval** as argument and returns its **inverse** with regard to a certain **neutral**.





Basic facts about *few/little*

- Traditional decompositional analysis:
  $[\![\text{few/little}]\!] \approx$ not many (e.g., Solt 2006)
- Two facts arguing against this analysis:
  - There can be a gap between being few/little and being many/much.

(1) *When Aristotle discusses the question, whether we should have many or few friends, and determines that it is best to have **neither many nor few**, …*

(2) *She is **neither tall nor short**.*

- $[\![\text{few/little} + \text{-er}]\!]$, i.e., the comparative use of *few/little*, remains unexplained.

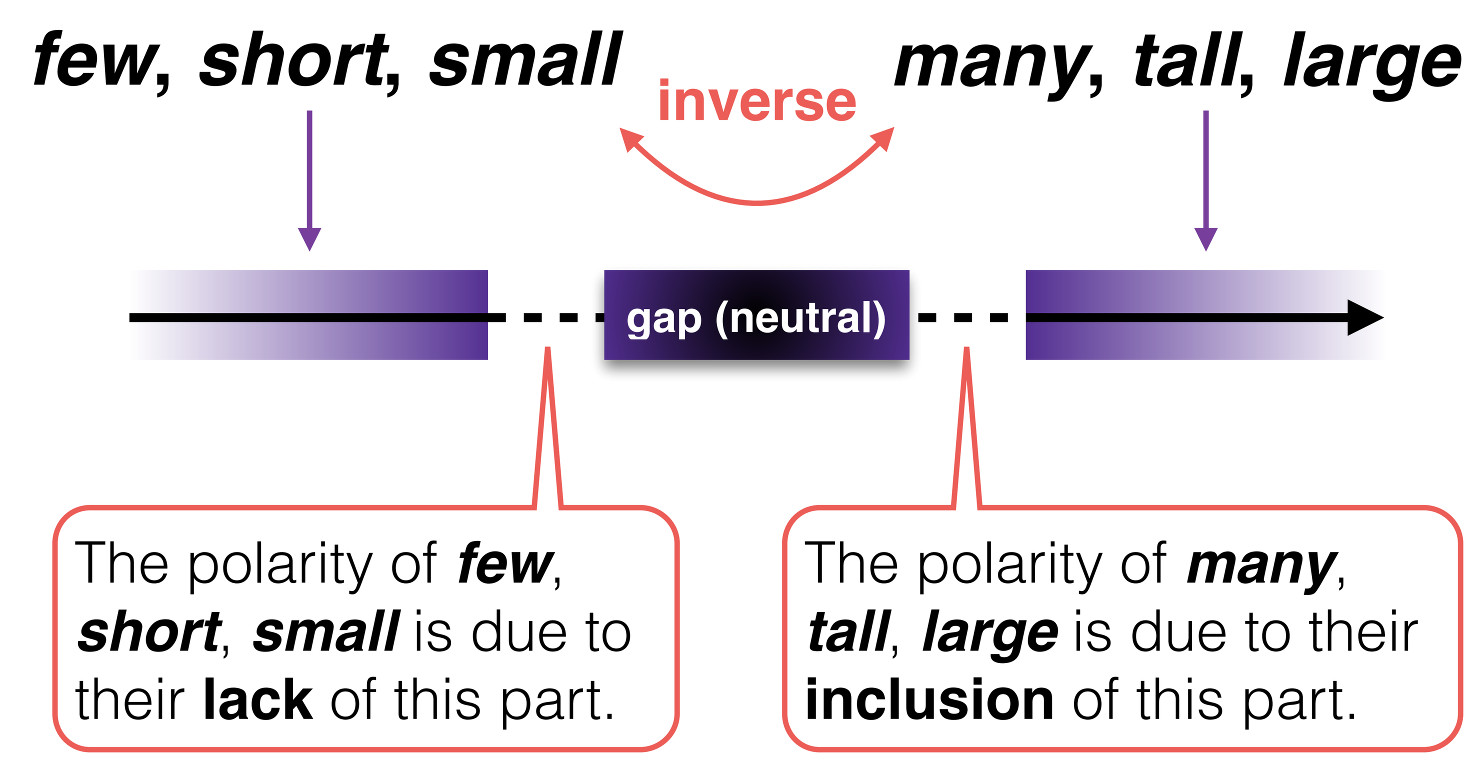
(3) *Lucy has **more** apples than Bill does, but Lucy has **fewer** pears than Bill does.*

(4) *John is **less** tall than Mary is.*

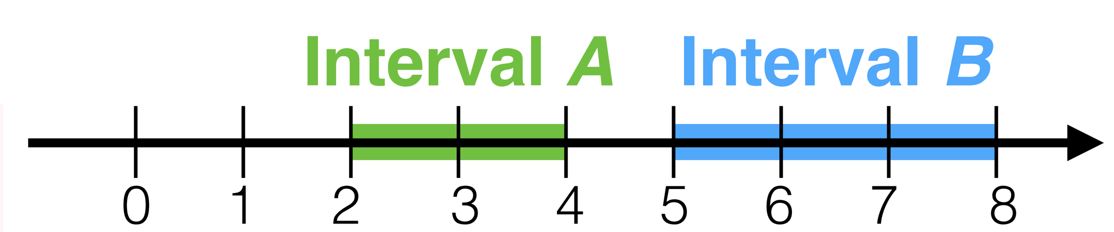
Proposal: few/little$_{\text{op}}$ is an inverse-generating operator

In absolute constructions, scalar antonyms denote intervals that are inverse with regard to gaps.

- Intuitively, as an operator, **few/little$_{\text{op}}$** turns *many* into *few*; it turns *tall* into *short*; it turns *large* into *small* …….
- Kennedy 1997/1999: "tall measures the height an object has; short measures the height an object does not have."



In **comparatives**, differentials denote intervals, and few/little$_{\text{op}}$ turns differentials into inverses.



$B - A = [5, 8] - [2, 4] = [1, 6]$ inverse
$A - B = [2, 4] - [5, 8] = [-6, -1]$

- Comparatives express relations among **3 intervals**: the two intervals representing the **comparative standard** and the **comparative subject**, and the **differential** between them.
- ***more/-er*** : default differentials, i.e., $(0, +\infty)$ (Zhang & Ling 2015).
- **few/little$_{\text{op}}$** turns differentials into their inverses, and here the neutral is [0, 0]—no difference between the comparative standard and the comparative subject.

Formal analysis of few/little$_{\text{op}}$ implemented with interval arithmetic

$[\![\text{more/-er}]\!]_{\langle dt\rangle} \stackrel{\text{def}}{=} D$ such that $D \subseteq (0, +\infty)$

$[\![\text{than}]\!]_{\langle dt,\langle dt,dt\rangle\rangle} \stackrel{\text{def}}{=} \lambda D_{\text{standard}}.\lambda D_{\text{differential}}.\iota D[D - D_{\text{standard}} = D_{\text{differential}}]$

$\text{few/little}_{\text{operator}\langle dt,dt\rangle} \stackrel{\text{def}}{=} \lambda D_{\text{positive}}.\iota D[D_{\text{positive}} - D_{\text{neutral}} = D_{\text{neutral}} - D]$

In absolute constructions, D_{neutral} is a context-dependent gap;
In comparatives, $D_{\text{neutral}} = [0, 0]$.

$[\![\text{many}]\!]_{\langle dt\rangle} \stackrel{\text{def}}{=} \iota D[D - D_{\text{gap}} = (0, +\infty)] = (D_{\text{gap\_upper bound}}, +\infty)$

$[\![\text{few/little}]\!]_{\langle dt\rangle} \stackrel{\text{def}}{=} \text{few/little}[\![\text{many}]\!]$, $\because [\![\text{many}]\!] - D_{\text{gap}} = (0, +\infty)$,

$\therefore [\![\text{few/little}]\!] = \iota D[D_{\text{gap}} - D = (0, +\infty)] = (-\infty, D_{\text{gap\_lower bound}})$

$[\![\text{tall}]\!]_{\langle dt,et\rangle} \stackrel{\text{def}}{=} \lambda D.\lambda x.[\text{height}(x) \subseteq D]$

$[\![\text{John is D}_c \text{ tall}]\!] \Leftrightarrow [\![\text{tall}]\!][\![\text{D}_c]\!][\![\text{John}]\!] \Leftrightarrow \text{height(John)} \subseteq D_c$

(1) John is 3" taller than Mary is.

$D_{\text{standard}} = [\![\text{Mary is (tall)}]\!] = [\![\text{the}]\!]\lambda D.\text{height(Mary)} \subseteq D$

i.e., Mary's height, or $[D_{\text{Mary\_lower}}, D_{\text{Mary\_upper}}]$

$D_{\text{differential}} = [\![3\text{" -er}]\!] = (0, +\infty) \cap [3'', 3''] = [3'', 3'']$

$[\![3\text{" -er than Mary is}]\!] = [\![\text{than}]\!][D_{\text{Mary\_lower}}, D_{\text{Mary\_upper}}][\![3\text{" -er}]\!]$

$= \iota D[D - [D_{\text{Mary\_lower}}, D_{\text{Mary\_upper}}] = [3'', 3'']]$

$\therefore [\![(1)]\!] \Leftrightarrow \text{height(John)} \subseteq \iota D[D - [D_{\text{Mary\_lower}}, D_{\text{Mary\_upper}}] = [3'', 3'']]$

$\Leftrightarrow \text{height(John)} \subseteq [D_{\text{Mary\_upper}} + 3'', D_{\text{Mary\_lower}} + 3'']$. It is defined when $D_{\text{Mary\_upper}} + 3'' \leq D_{\text{Mary\_lower}} + 3''$, i.e., $D_{\text{Mary\_upper}} = D_{\text{Mary\_lower}}$.

(2) John is at most 2" taller than every girl is.

$D_{\text{standard}} = [\![\text{every girl is (tall)}]\!] = [\![\text{the}]\!]\lambda D.[\forall x[\text{girl}(x) \rightarrow \text{height}(x) \subseteq D]]$,

i.e., the contextually most informative interval D such that every girl's height is contained in it, and it can be written as $[D_{\text{girls\_lower}}, D_{\text{girls\_upper}}]$

$D_{\text{differential}} = [\![\text{at most 2" ... -er}]\!] = (0, +\infty) \cap (-\infty, 2''] = (0, 2'']$

$\therefore [\![(2)]\!] \Leftrightarrow \text{height(John)} \subseteq \iota D[D - [D_{\text{girls\_lower}}, D_{\text{girls\_upper}}] = (0, 2'']]$

$\Leftrightarrow \text{height(John)} \subseteq (D_{\text{girls\_upper}}, D_{\text{girls\_lower}} + 2'']$

It is defined when $D_{\text{girls\_upper}} < D_{\text{girls\_lower}} + 2''$.

(3) John is at most 2" less tall than every girl is.

$D_{\text{standard}} = [D_{\text{girls\_lower}}, D_{\text{girls\_upper}}]$

$D_{\text{differential}} = \text{few/little}[\![\text{at most 2" -er}]\!] = \text{few/little}(0, 2'']$

$= \iota D[(0, 2''] - [0, 0] = [0, 0] - D] = [-2'', 0)$

$\therefore [\![(3)]\!] \Leftrightarrow \text{height(John)} \subseteq \iota D[D - [D_{\text{girls\_lower}}, D_{\text{girls\_upper}}] = [-2'', 0)]$

$\Leftrightarrow \text{height(John)} \subseteq [D_{\text{girls\_upper}} - 2'', D_{\text{girls\_lower}})$

It is defined when $D_{\text{girls\_upper}} - 2'' < D_{\text{girls\_lower}}$.

Basics of interval semantics

- **Degrees** are points on a **scale.**
- **Scale**: a totally ordered set of points.
- **Intervals**: convex sets of degrees. An interval represents a value as a range of possibilities.
- **Interval subtraction** (Moore 1979):

$$[a, b] - [c, d] = [a - d, b - c]$$

Selected references:

Kennedy, Christopher. 1997/1999. *Projecting the Adjective: The Syntax and Semantics of Gradability and Comparison.*

Moore, Ramon. 1979. *Methods and Applications of Interval Analysis.*

Solt, Stephanie. 2006. Monotonicity, Closure and the Semantics of *few*. *Proceedings of WCCFL 25.*

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