Appendix for 'On Mates' Puzzle'

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In this Appendix, I define two fragments of English and characterize their corresponding formal semantic computation. The first fragment captures individual *de re* belief reports. The second fragment covers Mates' sentences in their opaque and transparent interpretation. The following aspects of the syntax and semantics are common to both fragments:

Syntax: Each expression is assigned a type among the following:

- u, e, s & t are types
- If a, b are types, $\langle a, b\rangle$ is a type

The language contains infinite variables v_a of each type a. I use the following conventions:

- x, y, z are of type e
- w, v, u are of type s
- G, G' are of type $\langle e, \langle s, e\rangle\rangle$
- φ, φ' are of type $\langle\langle e, \langle s, e\rangle\rangle, \langle s, t\rangle\rangle$
- $\mathcal{H}, \mathcal{H}'$ are of type $\langle u, \langle e, \langle s, t\rangle\rangle\rangle$
- ω, ω' are of type $\langle\langle u, \langle e, \langle s, t\rangle\rangle\rangle, \langle s, t\rangle\rangle$

Semantics: A semantic model is a tuple $M = \langle W, D_e, D_t, D_u, I\rangle$ where:

- $W =$ a set of possible worlds
- $D_e =$ a set of individuals
- $D_t =$ a set of truth values $\{0, 1\}$
- $D_u =$ a set of well-formed expressions, pairs of phonological and syntactic representations
- $I =$ a valuation function mapping constant expressions to (sets of) elements of the various domains

In both fragments, the denotation of a sentence depends on:

- model M
- a context of utterance c
- a variable assignment g
- a possible world w

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Non-logical constants depend on the valuation function I provided by the model; variables depend on the assignment function g. To combine the meanings of expressions, we use Functional Application (FA):

- If α is of type $\langle a, b\rangle$, β is of type a and γ is an expression consisting of α and β, then: $\gamma = \alpha(\beta)$

Functional expressions are represented using lambda-abstraction.

De re about individuals

In the fragment for *de re* reports about individuals, the denotation of a sentence depends on the above parameters and:

- a constant function $\Pi = \lambda x \lambda w.x$

In this fragment, sentences are uniformly functions from objects of type $\langle e, \langle s, e\rangle\rangle$ to propositions (type $\langle s, t\rangle$). To interpret any sentence at an index of evaluation, the index needs to provide more than a possible world. That is the role of Π: it provides an argument for sentences like 'Ortcutt is a spy' whereby, if this clause contains a CG-variable as sister to 'Ortcutt', Π maps 'Ortcutt' to its transparent value, namely Ortcutt. Otherwise Π is idle.

Where α is a sentence, we write $[\![\alpha]\!]^{M,c,g,\Pi,w}$ for 'the denotation of α with respect to M, c, g, Π, w'. However, since M, c, g do not play a role in our derivation, we simply write $[\![\alpha]\!]^{\Pi,w}$.

Constants have the following denotation (assigned by function I):

- Ralph $= r$
- Ortcutt $= o$
- that $= \lambda\varphi.\varphi$
- is-a-spy $= \lambda x\lambda G\lambda w.\mathbf{spy}(x)(w)$ (*de dicto*)
- is-a-spy $= \lambda x\lambda G\lambda w.\mathbf{spy}(G(x)(w))(w)$ (*de re*)
- believes $= \lambda\varphi\lambda x\lambda G\lambda w.\exists G'(G'$ is an $ACG_{x,w}$ & $\forall v \in DOX_{x,w} : \varphi(G')(v))$

The predicate 'is-a-spy' is ambiguous between its *de re* and *de dicto* interpretation. In its *de re* interpretation, it contains a CG-variable that scopes over its first argument.

The sentence (2a) 'Ralph believes that Ortcutt is a spy' has the following LF (in its *de re* interpretation),

(2a) Ralph [believes-w_0 [that [λG$_1$ [λw_1[[[[G$_1$ Ortcutt]w_1] is a spy]w_1]]]]]

which has the following compositional derivation (all transformations are the result of FA):

- believes(that(is-a-spy(Ortcutt)))(Ralph) =
- $\lambda\varphi\lambda x\lambda G\lambda w.\exists G'(G'$ is an $ACG_{x,w}$ & $\forall v \in DOX_{x,w} : \varphi(G')(v))$ $(\lambda\varphi.\varphi(\lambda x\lambda G\lambda w.\mathbf{spy}(G(x)(w))(w)(o)))(r) =$
- $\lambda\varphi\lambda x\lambda G\lambda w.\exists G'(G'$ is an $ACG_{x,w}$ & $\forall v \in DOX_{x,w} : \varphi(G')(v))$ $(\lambda\varphi.\varphi(\lambda G\lambda w.\mathbf{spy}(G(o)(w))(w)))(r) =$
- $\lambda\varphi\lambda x\lambda G\lambda w.\exists G'(G'$ is an $ACG_{x,w}$ & $\forall v \in DOX_{x,w} : \varphi(G')(v))(\lambda G\lambda w.\mathbf{spy}(G(o)(w))(w))(r) =$

- $\lambda x \lambda G \lambda w.\exists G'(G'$ is an $ACG_{x,w}$ & $\forall v \in DOX_{x,w} : \lambda G \lambda w.\mathbf{spy}(G(o)(w))(w)(G')(v))(r) =$
- $\lambda G \lambda w.\exists G'(G'$ is an $ACG_{r,w}$ & $\forall v \in DOX_{r,w} : \lambda G \lambda w.\mathbf{spy}(G(o)(w))(w)(G')(v)) =$
- $\lambda G \lambda w.\exists G'(G'$ is an $ACG_{r,w}$ & $\forall v \in DOX_{r,w} : \mathbf{spy}(G'(o)(v))(v))$

This sentence is interpreted as follows:

- $[\![(2a)]\!]^{\Pi,w} = 1$ iff $\exists G'(G'$ is an $ACG_{r,w}$ & $\forall v \in DOX_{r,w} : \mathbf{spy}(G'(o)(v))(v))$.

In words: there exists an acquaintance-based concept generator G' for Ralph at w such that, across Ralph's doxastic alternatives, G' maps Ortcutt to a spy.

De re **about words**

In this fragment, 'believe' maps *metalinguistic* concept-generators to propositions. Correspondingly, all other expressions have a different semantic type as well. The denotation of a sentence depends on M, c, g, w and:

- a constant disquotational function Δ (see **Definition 6** and the surrounding discussion)

Where α is a sentence, we write $[\![\alpha]\!]^{M,c,g,\Delta,w}$, or $[\![\alpha]\!]^{\Delta,w}$ for short.

Constants have the following denotation (assigned by I):

- Andrea $= a$
- Fiona $= f$
- that $= \lambda \omega.\omega$
- not $= \lambda \omega \lambda \mathcal{H} \lambda w.\mathbf{not}\, \omega(\mathcal{H})(w)$
- is-an-attorney $= \lambda x \lambda \mathcal{H} \lambda w.\mathcal{H}(\textit{attorney})(x)(w)$ (*opaque*)
- is-a-lawyer $= \lambda x \lambda \mathcal{H} \lambda w.\mathcal{H}(\textit{lawyer})(x)(w)$ (*opaque*)
- is-an-attorney = is-a-lawyer $= \lambda x \lambda \mathcal{H} \lambda w.\mathbf{lawyer}(x)(w)$ (*transparent*)
- believes $= \lambda \omega \lambda x \lambda \mathcal{H} \lambda w.\exists \mathcal{H}'(\mathcal{H}'$ is an $MCG_{x,w}$ & $\forall v \in DOX_{x,w} : \omega(\mathcal{H}')(v))$

The predicates 'is-an-attorney' and 'is-a-lawyer' are ambiguous between *transparent* and *opaque* interpretations. In the former case, they have the same denotation; in the latter, they do not: each contains an MCG-variable as sister to a different member of D_u, possibly resulting in different denotations.

The sentence (1a) 'Andrea believes that Fiona is an attorney' has the following LF (in its *opaque* interpretation):

(1a) Andrea [believes-w_0 [that [$\lambda \mathcal{H}_1$ [λw_1 [w_1 [Fiona [$\mathcal{H}_1$ is-an-attorney]]]]]]]

And the following compositional derivation:

- believes(that(is-an-attorney(Fiona)))(Andrea) =
- $\lambda \omega \lambda x \lambda \mathcal{H} \lambda w.\exists \mathcal{H}'(\mathcal{H}'$ is an $MCG_{x,w}$ & $\forall v \in DOX_{x,w} : \omega(\mathcal{H}')(v))$
$(\lambda \omega.\omega(\lambda x \lambda \mathcal{H} \lambda w.\mathcal{H}(\textit{attorney})(x)(w)(f)))(a) =$

- $\lambda\omega\lambda x\lambda\mathcal{H}\lambda w.\exists\mathcal{H}'(\mathcal{H}'$ is an $MCG_{x,w}$ & $\forall v \in DOX_{x,w} : \omega(\mathcal{H}')(v))$
  $(\lambda\omega.\omega(\lambda\mathcal{H}\lambda w.\mathcal{H}(\textit{attorney})(f)(w)))(a) =$
- $\lambda\omega\lambda x\lambda\mathcal{H}\lambda w.\exists\mathcal{H}'(\mathcal{H}'$ is an $MCG_{x,w}$ & $\forall v \in DOX_{x,w} : \omega(\mathcal{H}')(v))$
  $(\lambda\mathcal{H}\lambda w.\mathcal{H}(\textit{attorney})(f)(w))(a) =$
- $\lambda x\lambda\mathcal{H}\lambda w.\exists\mathcal{H}'(\mathcal{H}'$ is an $MCG_{x,w}$ & $\forall v \in DOX_{x,w} : \lambda\mathcal{H}\lambda w.\mathcal{H}(\textit{attorney})(f)(w)(\mathcal{H}')(v))(a) =$
- $\lambda\mathcal{H}\lambda w.\exists\mathcal{H}'(\mathcal{H}'$ is an $MCG_{a,w}$ & $\forall v \in DOX_{a,w} : \lambda\mathcal{H}\lambda w.\mathcal{H}(\textit{attorney})(f)(w)(\mathcal{H}')(v)) =$
- $\lambda\mathcal{H}\lambda w.\exists\mathcal{H}'(\mathcal{H}'$ is an $MCG_{a,w}$ & $\forall v \in DOX_{a,w} : \mathcal{H}'(\textit{attorney})(f)(v))$.

This sentence is interpreted as follows:

- $[\![(1a)]\!]^{\Delta,w} = 1$ iff $\exists\mathcal{H}'(\mathcal{H}'$ is an $MCG_{a,w}$ & $\forall v \in DOX_{a,w} : \mathcal{H}'(\textit{attorney})(f)(v))$

In words: there exists a metalinguistic concept generator for Andrea at w that maps 'attorney' ($\in D_u$) to a property that Fiona has across Andrea's doxastic alternatives. In Context A, the truth of this sentence it witnessed by a metalinguistic concept generator that maps 'attorney' to the property of *being a judge*.

The sentence (1b) ('Andrea believes that Fiona is not a lawyer') has the following LF (in its *opaque* interpretation) and derivation:

(1b) Andrea [believes-w_0 [that [not [$\lambda\mathcal{H}_1$ [λw_1 [w_1 [Fiona [$\mathcal{H}_1$ is-a-lawyer]]]]]]]]

- believes(that(not(is-a-lawyer(Fiona))))(Andrea)=
- $\lambda\omega\lambda x\lambda\mathcal{H}\lambda w.\exists\mathcal{H}'(\mathcal{H}'$ is an $MCG_{x,w}$ & $\forall v \in DOX_{x,w} : \omega(\mathcal{H}')(v))$
  $(\lambda\omega.\omega(\lambda\omega\lambda\mathcal{H}\lambda w.\textbf{not}\ \omega(\mathcal{H})(w)(\lambda x\lambda\mathcal{H}\lambda w.\mathcal{H}(\textit{lawyer})(x)(w)(f))))(a) =$
- $\lambda\omega\lambda x\lambda\mathcal{H}\lambda w.\exists\mathcal{H}'(\mathcal{H}'$ is an $MCG_{x,w}$ & $\forall v \in DOX_{x,w} : \omega(\mathcal{H}')(v))$
  $(\lambda\omega.\omega(\lambda\omega\lambda\mathcal{H}\lambda w.\textbf{not}\ \omega(\mathcal{H})(w)(\lambda\mathcal{H}\lambda w.\mathcal{H}(\textit{lawyer})(f)(w))))(a) =$
- $\lambda\omega\lambda x\lambda\mathcal{H}\lambda w.\exists\mathcal{H}'(\mathcal{H}'$ is an $MCG_{x,w}$ & $\forall v \in DOX_{x,w} : \omega(\mathcal{H}')(v))$
  $(\lambda\omega.\omega(\lambda\mathcal{H}\lambda w.\textbf{not}\ \lambda\mathcal{H}\lambda w.\mathcal{H}(\textit{lawyer})(f)(w))(\mathcal{H})(w))(a) =$
- $\lambda\omega\lambda x\lambda\mathcal{H}\lambda w.\exists\mathcal{H}'(\mathcal{H}'$ is an $MCG_{x,w}$ & $\forall v \in DOX_{x,w} : \omega(\mathcal{H}')(v))$
  $(\lambda\omega.\omega(\lambda\mathcal{H}\lambda w.\textbf{not}\ \mathcal{H}(\textit{lawyer})(f)(w)))(a) =$
- $\lambda\omega\lambda x\lambda\mathcal{H}\lambda w.\exists\mathcal{H}'(\mathcal{H}'$ is an $MCG_{x,w}$ & $\forall v \in DOX_{x,w} : \omega(\mathcal{H}')(v))$
  $(\lambda\mathcal{H}\lambda w.\textbf{not}\ \mathcal{H}(\textit{lawyer})(f)(w))(a) =$
- $\lambda x\lambda\mathcal{H}\lambda w.\exists\mathcal{H}'(\mathcal{H}'$ is an $MCG_{x,w}$ & $\forall v \in DOX_{x,w} : \lambda\mathcal{H}\lambda w.\textbf{not}\ \mathcal{H}(\textit{lawyer})(f)(w)(\mathcal{H}')(v))$
  $(a) =$
- $\lambda\mathcal{H}\lambda w.\exists\mathcal{H}'(\mathcal{H}'$ is an $MCG_{a,w}$ & $\forall v \in DOX_{a,w} : \lambda\mathcal{H}\lambda w.\textbf{not}\ \mathcal{H}(\textit{lawyer})(f)(w)(\mathcal{H}')(v)) =$
- $\lambda\mathcal{H}\lambda w.\exists\mathcal{H}'(\mathcal{H}'$ is an $MCG_{a,w}$ & $\forall v \in DOX_{a,w} : \textbf{not}\ \mathcal{H}'(\textit{lawyer})(f)(v))$

This sentence is interpreted as follows:

- $[\![(1b)]\!]^{\Delta,w} = 1$ iff $\exists\mathcal{H}'(\mathcal{H}'$ is an $MCG_{a,w}$ & $\forall v \in DOX_{a,w} : \textbf{not}\ \mathcal{H}'(\textit{lawyer})(f)(v))$

In words: there exists a metalinguistic concept generator for Andrea at w that maps 'lawyer' ($\in D_u$) to a property that Fiona lacks across Andrea's doxastic alternatives. In Context A, the truth of this sentence it witnessed by a metalinguistic concept generator that maps 'lawyer' to the property of *being a lawyer*, which Andrea knows that Fiona does not have.

Finally, in its *transparent* interpretation, (1a) has the following LF and derivation:

(1a) Andrea [believes-w [that [$\lambda\mathcal{H}_1$ [λw_1 [w_1 [Fiona is an attorney]]]]]]

- believes(that(is-an-attorney(Fiona)))(Andrea)=
- $\lambda\omega\lambda x\lambda\mathcal{H}\lambda w.\exists\mathcal{H}'(\mathcal{H}'$ is an $MCG_{x,w}$ & $\forall v \in DOX_{x,w} : \omega(\mathcal{H}')(v))$
  $(\lambda\omega.\omega(\lambda x\lambda\mathcal{H}\lambda w.$**lawyer**$(x)(w)(f)))(a) =$
- $\lambda\omega\lambda x\lambda\mathcal{H}\lambda w.\exists\mathcal{H}'(\mathcal{H}'$ is an $MCG_{x,w}$ & $\forall v \in DOX_{x,w} : \omega(\mathcal{H}')(v))$
  $(\lambda\omega.\omega(\lambda\mathcal{H}\lambda w.$**lawyer**$(f)(w)))(a) =$
- $\lambda\omega\lambda x\lambda\mathcal{H}\lambda w.\exists\mathcal{H}'(\mathcal{H}'$ is an $MCG_{x,w}$ & $\forall v \in DOX_{x,w} : \omega(\mathcal{H}')(v))$
  $(\lambda\mathcal{H}\lambda w.$**lawyer**$(f)(w))(a) =$
- $\lambda x\lambda\mathcal{H}\lambda w.\exists\mathcal{H}'(\mathcal{H}'$ is an $MCG_{x,w}$ & $\forall v \in DOX_{x,w} : \lambda\mathcal{H}\lambda w.$**lawyer**$(f)(w)(\mathcal{H}')(v))(a) =$
- $\lambda\mathcal{H}\lambda w.\exists\mathcal{H}'(\mathcal{H}'$ is an $MCG_{a,w}$ & $\forall v \in DOX_{a,w} : \lambda\mathcal{H}\lambda w.$**lawyer**$(f)(w)(\mathcal{H}')(v)) =$
- $\lambda\mathcal{H}\lambda w.\exists\mathcal{H}'(\mathcal{H}'$ is an $MCG_{a,w}$ & $\forall v \in DOX_{a,w}$: **lawyer**$(f)(v))$

This sentence is interpreted as follows:

- $[\![(1a)]\!]^{\Delta,w} = 1$ iff $\exists\mathcal{H}'(\mathcal{H}'$ is an $MCG_{a,w}$ & $\forall v \in DOX_{a,w}$: **lawyer**$(f)(v))$

In words: there exists an MCG for Andrea at w and Fiona is a lawyer across Andrea's doxastic alternatives.