

# **Opacity and Paradoxes**

*The Paraconsistent Semantics of Natural Languages*

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(Draft Version 06- $\alpha$ .2. Please, comments, suggestions and corrections are welcome)

## **Abstract**

The purpose of this work is to revisit opaque contexts from the perspective of natural languages. ‘Opacity’ has been understood firstly as failure of the application of Leibniz’ substitution of identicals principle and later as accessibility relations holding between possible worlds. However, opacity in the semantics of natural languages ought to be simply characterised truth-functionally, in which case it results from devices that both avoid paradoxical interpretation of sentences and circumvent the principle of Pseudo-Scotus. Accordingly, what is herein proposed is a solution based on a kind of Paraconsistent Semantics for natural languages.

**Keywords:** Propositional Attitudes, Semantics, Philosophy of Language, Linguistics, Logic, Accessibility Relations, Belief Reports, Consistency, Human Languages Semantics, Intensional Liar, Leibniz’ Law, Moore’s Paradox, Opacity, Paraconsistency.

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## 1. Introduction

### 1.1 The Goals

In this work I shall examine some aspects of the semantic phenomenon called *opacity* from the perspective of human languages in their common usage (rather than artificial languages or usages created by Logicians). I have analysed mainly English data though, the general conclusions of this work supposedly apply to every human language. The two basic questions that will be herein tentatively addressed are:

( $Q_1$ )            How the attitudinal operator contributes to or modifies the truth of the statements?

( $Q_2$ )            Why do human languages need opaque contexts to report epistemic states?

It will be herein argued that the phenomenon called opacity results from semantic mechanisms that avoid the Principle of Pseudo-Scotus, rather than from the assumed *inapplicability* of Leibniz' substitution of identicals principle. Such mechanisms access certain model-situations (in the same or different worlds) from other model-situations indicated by transparent contexts.

Here we shall seek a kind of semantics where a contradiction (or antinomy) does not entail an infinite number of possible consequences, and does not follow from a conjunct of complementary descriptions<sup>1</sup>, and shall show how propositional attitudes contribute to keep contradictions under control. This is in part due to the possibility of separating and later accessing model-situations and controlling entailment relations in a paraconsistent manner.

The whole presentation of the issues aims to be as compact as clear to give a comprehensive but not detailing picture. The initial part is devoted to dispel some myths, and the subsequent ones concentrate on the viable alternatives to the problems discussed.

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<sup>1</sup> See da Costa & Krause (2003) for a discussion in other branches of knowledge.

## 1.2 Background

Opacity has been one of the most intriguing and important matters in Philosophy, Logic and in Linguistics. The *latu sensu* notion of opacity can be initially figuratively described as the phenomenon of a sentential context not allowing the light of a semantic/logic principle to pass through, i.e., a certain context is *opaque* because a certain (mode of) inference is not visibly valid therein. There have been some more specific and/or stronger hypotheses trying to define actual instantiations of this notion occur and/or to predict when and to explain why they occur. Indeed, as far as I know, there have been at least two basic approaches on opacity.

The first is the *classic* or *traditional* approach on opacity, which is due to the so called Fregean tradition, and which has been questioned by the so called Russellian philosophers, assumes the definition of opaque context given in (1) below or variants thereof:

(1) **Opaque context** (classic version)

A sentential context  $\varphi$  containing an occurrence of a term  $t$  is opaque, if the substitution of co-referential terms is an invalid mode of inference with respect to this occurrence. (See Mckinsey 1998, Quine 1956)

Here the mode of inference in question is the application to Leibniz' Law defined in (2):

(2) **Leibniz' Principle of Substitutability of Identicals**

*Eadem sunt quorum unum potest substitui alteri salva veritate.*

For a long time (1) has been the most spread conception and still persists as an option in the recent literature (see, for instance, Aloni 2003).

The second view tries to derive opacity from the notion of *non-symmetry of accessibility relations*. It basically deals with the idea that distinct possible worlds may be somehow interconnected via the (intensional) semantics of natural languages sentences.

Thus, the main views and debates mentioned hereinbefore have gravitated around two axes of discussion respectively:

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- (i) The first axis involves the contrapositions of common and widespread interpretations<sup>2</sup> of the works of four great thinkers, namely Frege (1892), Russell (1905), Strawson (1950) and Quine (1953, 1956);
- (ii) The second axis implies a different paradigm of discussion based on the more recent and equally important works of Kripke (1959, 1986) and Hintikka (1969).

### 1.3 Structure of the paper

In the subsequent Section I shall present a summary of the traditional idea that opacity is defined in relation to Leibniz' Substitutability and shall present five arguments against it. The rejection of such view neither necessarily conducts to some form of Russellianism nor entails that the approach based on world accessibility relations is unproblematic (see Sub-section 3.4).

As it is impossible to make a comprehensive exposition of their ideas and of the large amount of subsequent analyses presented in pursuance of such debates, I shall on purpose skip many details of the discussion and pertinent issues, such as the contrasts between *de re* and *de dicto* contexts. In this work I shall mainly consider basically epistemic reports involving one word that expresses a propositional attitude and embeds a sentence.

In the following section I begin to introduce the issue from the first classic axis of discussion. I shall argue that due to four strong reasons opacity cannot be reduced to some form of restriction on Leibniz' substitutions.

Section 3 will present an alternative view, which treats opacity truth-functionally and makes each propositional attitude correspond to an accessibility relation. Although initially it is tempting to think the accessibility relation holds between possible worlds, I shall argue in favour of the idea of accessing different model-situations.

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<sup>2</sup> The works mentioned above may be read in many different ways, especially from the perspective of more recent linguistic and logic theories. Unfortunately, we have not sufficient space for expanding this issue in this paper. See May (2001) for Frege's identity statements.

Sections 4 and 5 will seek to situate the issue in a form of paraconsistent semantics for human languages.

I shall use the bold small Greek letters  $\gamma$ ,  $\kappa$  and  $\tau$  to designate propositional attitudes; the capital italic  $S$  and  $W$  for situations and worlds; non-bold small Greek letters for expressions, sentences and propositions; Capital Greek letters for domains, sets and contexts; small Roman letters for individuals; non-italic capital Roman letters for predicates;  $\circ$  for consistent,  $\bullet$  for inconsistent;  $\bullet^*$  for trivial;  $@$  for non-consistent but non-trivial;  $=$  for equal;  $\Rightarrow$  for substitution;  $\&$  for commutative conjunction and  $\&_{\Gamma}$  for sequential non-commutative conjunction;  $|=$  for entailment and  $|\neq$  for *it does not necessarily follow from*, as well as the alethic values  $\mathbf{T}$ ,  $f$  and  $\mathbf{U}$  and the usual symbols quantifiers and connectives ( $\forall$ ,  $\leftrightarrow$ ,  $\vee$ ,  $\rightarrow$ , etc.). I shall try to keep the notation as simple as possible and shall often omit the epistemic agent in the formulae.

## 2. Opacity and (Non-) Substitutability

### 2.1 Against the Classic View

Here we are dealing with linguistic phenomenon that was first formalised in Philosophy. Accordingly, Leibniz' substitutability of identicals principle has been one of the first attempts to conjoin the possibility or capacity of one thing to replace another with the observation that anything that is true of the one must also be true of the other. But Leibniz has advanced his principle centuries after Aristotle had already introduced us to a distinction between certain types of contexts, which nowadays are called *opaque* and *transparent*. Aristotle's famous example is that of man called *Coriscus*, who wears a mask, and the situation where not everyone, who refers to the masked man, may knowingly refer to Coriscus or vice-versa<sup>3</sup>. As far as we know, it was Frege (1892) who first

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<sup>3</sup> See **Cohen** (1994) for a more detailed discussion.

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confronted Leibniz' proposals with Aristotle's old philosophical problem from the perspective of Logic.

But it was Church (1956) that brought Leibniz' Law to the realm of linguistic issues, by proposing a more precise enunciation thereof:<sup>4</sup>

(3) **Church' Substitutability of Identicals**  
Things are identical if the name of one can be substituted for that of the other without loss of truth.

The classic view on opacity given in (1), when applied to linguistic issues, defines it in relation to (2) above or to a variant of (3), i.e., in relation to the substitution of linguistic categories and not of other kinds of entities in a world and to the (non-) preservation of truth in natural language sentences and not in utterances pertaining to other kinds of languages. Particularly, for empirical reasons, it has been argued that the sentential context (1) mentions is characterised by the presence of one or more lexical items that express a propositional attitude:

(4) **Opacity of Attitudinal Ascriptions Hypothesis**  
Sentential contexts reporting propositional attitudes are opaque under (1).

Consider the sentences below, which refer to the situation of described in Oedipus' legend:

- (5) a. Oedipus killed the man he met at the crossroads.  
b. Laius/ The man Oedipus met at the crossroads was his biological father.

From (5a&b) above it is possible to infer (5'), by substituting *his biological father* for *the man he met at the crossroads*:

- (5') Oedipus killed his biological father.

The contexts above are deemed transparent to the application of Leibniz' Law. However, contexts involving propositional attitudes seem opaque in relation to the same, as illustrated by (5').

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<sup>4</sup> See Hawthorne (2003) and Krause & Coelho (2003) for a discussion of the consequences of Leibniz' Law in other branches of knowledge.

Notice that it is not possible to infer (11b) from (11a) and (4b) by substituting *his biological father* for *the man he met at the crossroads*, for the result is false:

- (5'')            a. Oedipus intended to kill the man he met at the crossroads.⇒  
                    b. *f* Oedipus intended to kill his biological father.

This has basically been the traditional or standard view on opacity: briefly, both (1) and (3) hold. But this is just a mere observation that opacity obtains where there are words expressing propositional attitudes. The fact that there is no obvious logic connection between (1) from (3) is a more than sufficient motive to do one of the two things below:

- Option (i.)        Either to pursue a way to derive (3) from (1) or  
Option (ii.)      To abandon the traditional thesis on opacity.

I have at least four other stronger arguments, which make the case for the second choice, i.e., counter-arguments to the traditional thesis, which I summarise below:

- (<sup>C</sup>1)            Some substitutions in **transparent** contexts also yield odd results;  
(<sup>C</sup>2)            Substitution tests do not always yield false results in opaque contexts;  
(<sup>C</sup>3)            The data used to illustrate the traditional thesis on opacity have been described in an incomplete manner, comparing things that are not comparable one to another;  
(<sup>C</sup>4)            If one accepts the standard view on opacity, incorrect predictions are made, respecting binding and control phenomena in natural languages.

Let me unfold these arguments.

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### 2.2 Unexpected Substitutions

#### 2.2.1 Failure in Transparent Contexts

The first argument against the strict association between substitutability and opacity ( $C_1$ ) is that even in certain transparent contexts, i.e., where no propositional attitude is reported, the application of the substitutability principle does yield odd results. This evidences that one thing is independent from the other.

Consider the famous Fregean examples: since Venus has is styled *Venus*, *morning star* and *evening star*, the three expressions should be perfectly inter-exchangeable in a transparent context. But, if one takes the sentence (6a) and substitutes *the evening star* for *Venus*, the result (6b) is odd:

- (6)                    a. Venus appears in the morning.  $\Rightarrow$   
                          b. #The evening star appears in the morning.

An interpretation of Frege's work traditionally attributes the oddity of (6b) to the *sense* versus *referent* distinction. Regardless of how to characterise it in theoretic terms of a semantic framework, it is clear that what is in question is not whether (6b) can be inserted into a believe clause, but the very presence of the adjective *evening* that modifies *star* in contradiction to the predicate *appears in the morning*.

Indeed there are numerous analogous examples that show called *impossible syllogisms* in transparent contexts, involving what is herein:

- (7)                    a. Lepidopterans (can) fly.  
                          b. Caterpillars are Lepidopterans.  $\Rightarrow$   
                          c. *f* Caterpillars (can) fly.

In (7) there is no obvious evidence of a (hidden) propositional operator. Rather, what happens to (7) reflects revision of information and the defeasibility property of natural languages (see



Sub-section 4.1). Accordingly (7a) states a default rule for the class of Lepidopterans, the exception being the stage of their lives when they are caterpillars. In other words, (7a) is susceptible of reviewing, as most statements in natural languages. This same reasoning can be applied to sentences containing proper nouns, like (8):

- (8)
- a. Captain Marvel loses his super-strength and his capacity to fly when he says the magic word.
  - b. Billy Batson is Captain Marvel.  $\Rightarrow$
  - c. *f* Billy Batson loses his super-strength and his capacity to fly when he says the magic word.

Sentence (8c) is the wrong depiction of the Comics book mentioned, since Billy Batson actually gains super-strength and the capacity to fly when he says the magic word (and consequently transforms into Captain Marvel).

Of course, in comparison, (6) cannot be explained solely in relation to defeasibility, although one could think of a context where it is uttered as a consequence of someone's astronomical discovery, which forced him to review his former beliefs respecting the stars. The oddity of (6) has to do with issues of consistency, wherefrom one concludes that there is at least another fundamental semantic property involved.

Anyway, the examples above suffice to show that restrictions on substitutions transcend the case of propositional attitude ascriptions.

### *2.2.2 Non-Uniformity of Results in Opaque Contexts*

The second argument is against the idea that attitude ascriptions **always** block Leibniz' substitutability ( $C_2$ ). Indeed, the substitution tests do not uniformly yield false results in all opaque contexts. On the contrary, some substitutions might yield even true results. Let us give one initial example, where the same object is designated by different names (that may imply different manners

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of representing the same object). Yet none of the possible alternative names or expressions to designate the same object changes the truth of the statement:

- (9) Every visitor to the Louvre intends to see the Mona Lisa/ La Gioconda/  
da Vinci's most famous painting.

Of course, there are other cases. Sentences, which initially seem resilient to the application of substitution, may be *fixed* by the use of some *strategies*. Thus, to make unlikely substitutions work it is enough to spell out the conditions or scenarios where they may apply with the preservation of truth. For instance, a true sentence like (10) does not normally yield a true result, if, one substitutes *Spiderman* for *Peter Parker*, as in (10'a). But in a sentential context like (10'b) the substitution preserves the truth:

- (10) Jameson believes that Peter Parker is a mere photographer.  
(10') a. *f* Jameson believes that Spiderman is a mere photographer.  
b. **T** The disguise is so convincing that even Jameson believes that  
Spiderman is a mere photographer. (See Berg 1988, McKinsey 1999)

This argument shows that if (1) and (5) hold, then they do not suffice, for they can only to account for the cases of unsuccessful substitutions, and the successful cases remain unexplained.

### 2.3 One Crucial Detail

The third argument against the traditional view on opacity (<sup>C</sup>3) is that such view is empirically based on the comparison of things that are incomparable. This argument has to do with the incomplete manner examples like (5), (5') and (5'')— often given in the literature— are used to corroborate the standard view on the assumed correlation propositional attitudes and failure of substitutability: crucial details in the description of the situations are omitted to make the comparisons look legitimate.

The presentation of the standard thesis usually comports the following steps in (11) to show that (12) is not derivable. In the following  $x$  is an epistemic agent,  $\tau$  is a propositional attitude,  $P$  is a predicate of  $y$  and  $y$  an individual:

$$(11) \quad \text{a. } x \tau(Py)$$

$$\text{b. } y=z$$

Making the substitution according to (11b) the result is false:

$$(12) \quad \text{f } x \tau(Pz)$$

But the problem is that the pattern in (11) consists of two truths that are not comparable, the second of which being in a transparent context, unlike the first that is an attitudinal ascription. On the other hand, (13c) can be derived from (13a&b), by making the substitution according to (13b):

$$(13) \quad \text{a. } x \tau(Py)$$

$$\text{b. } x \tau(y=z) \Rightarrow$$

$$\text{c. } \text{T } x \tau(Pz)$$

This makes a huge difference for the Semantics of natural languages. To illustrate how this is crucial, let us transpose the Oedipus' examples to a situation in Electra's legend, just changing the characters. First consider (14), which provides the basic information on the legend:

$$(14) \quad \text{a. Electra killed the woman who helped Aegisthus to kill her father.}$$

$$\text{b. The woman who helped Aegisthus to kill Electra's father was}$$

$$\text{Clytemnestra, her mother.}$$

Now, surprisingly we can make successful inferences that were not possible in the Oedipus' case. Among the several possibilities, one can infer (14'b) from (14b) and (14'a) without any difficulty:

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- (14') a. Electra intended to kill the woman who helped Aegisthus to kill her father.⇒
- b. **T** Electra intended to kill her mother/ Clytemnestra.

What is going on? Why is there such difference between Oedipus' and Electra's examples? The contrast between (5), (5') and (5'') and (14) and (14') seems to make the matters even more mysterious. However such mystery is due to the fact that in the case of Oedipus' and Electra's legends used above, (5), (5') and (5'') are incomplete accounts of the situations they respectively refer to. The *(potential) text* of each legend itself has more sentences that provide more details to the *readers*. Indeed, the most important piece of information is missing in (5) above:

- (Key)** Oedipus did not know that the man he met at the crossroads was his biological father.

Unlike Electra, who knew that her mother and Aegisthus killed her father, and who therefore became a **conscious matricide**, Oedipus cannot be attributed the intention of parricide.

However, if (5'') is recomposed in order to add the information contained in the **(Key)** above, then it is possible to infer (15):

- (15) **T** Oedipus intended to kill the man, who, unbeknownst to him, was his biological father.

For the same sort of reason, the substitutions in (16) below will yield false or true results, depending on whether the subjects in question know that Batman is Bruce Wayne or not:

- (16) a. The Joker/ Robin assumes that Batman is always driving the batmobile.
- b. *f* The Joker/ **T** Robin assumes that Bruce Wayne is always driving the batmobile.

## 2.4 Binding and Control Subst

The fourth argument against the strict correlation between Leibniz' Law and opacity (<sup>C</sup>4) has to do with two syntactic-semantic relations: *binding*<sup>5</sup> and *control*.<sup>6</sup> If one accepts the standard view on opacity, then incorrect linguistic predictions about binding and control are made.

In construing sentences, there is a clear connection between Leibniz' Law and the notion of binding, for the latter entails the (possibility of) *mental* or *implicit* substitution operations involving variables (such as anaphors, pronouns, etc). For instance, consider the examples bellow:

(17) Narcissus<sub>1</sub> loves himself<sub>1</sub>.

A claim like (17) is true iff it is true that the person Narcissus loves is Narcissus himself. Accordingly, the binding relation indicates that if *himself* is replaced by *Narcissus*, the formed sentence *Narcissus loves Narcissus* will be true, even if it sounds awkward. This indicates that binding is a form of manifestation of Leibniz' Law, whereby opaque contexts should somehow block binding or control.

Indeed, this prediction was made by Quine (1956), according to whom it is meaningless to quantify into opaque contexts. Suppose that *C*' is any sentential context reporting a propositional attitude. Under (5), *C*' is opaque. If one obtains *C*'' result from *C*' by substituting a variable *v* for the term at the relevant opaque occurrence, then Quine says that binding of this occurrence of *v* by a quantifier outside the scope of *C*'' results in a meaningless expression.

Quine's thesis has been contested in the literature and counter-examples have been given. McKinsey (1998), for instance, points that if a sentence like *David is such that Bruce believes that he is a megalomaniac* is true, so is its existential generalisation: *someone is such that Bruce believes that he is a*

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<sup>5</sup> In Generative syntax, binding is defined binding as an instance of co-indexation between two terms. In broader terms, this concept can be understood in the following manner:

- A lexical item  $\mu$  binds another item  $\varepsilon$ , if  $\varepsilon$  is a variable and  $\mu$  functions as an assignment to  $\varepsilon$ . Cf. Chomsky (1982).

<sup>6</sup> In generative literature, control is a kind of special binding involving an inaudible variable called PRO that functions as the subject of an infinitival clause.

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*megalomaniac*. In the generalised version, the indefinite (weak quantifier) *someone* binds the pronoun *he*. According to Quine's thesis, this existential generalisation sentence should be meaningless. Yet it is a clearly meaningful sentence.

To reinforce this observation, a simple look at the previous examples will reveal that the binding and the control relations *subsist*,<sup>7</sup> in transparent and opaque contexts alike. Below some of the examples are repeated for reasons of convenience with the proper indices represented:

- (5'')
- a. Oedipus<sub>1</sub> killed the man he<sub>1</sub> met at the crossroads.
  - b. The man Oedipus<sub>1</sub> met at the crossroads was his<sub>1</sub> biological father.
- (5''')
- a. Oedipus<sub>1</sub> intended PRO<sub>1</sub><sup>8</sup> to kill the man he<sub>1</sub> met at the crossroads.⇒
  - b. *f* Oedipus<sub>1</sub> intended PRO<sub>1</sub> to kill his<sub>1</sub> biological father.

## 2.5 Partial Conclusions

If the classic view expressed in (1) has to be discarded, then one has consequently to define opacity in terms of more fundamental properties of human languages. Keeping in mind the remarks made hereinbefore, the differences between transparent and opaque context may be tentatively characterised truth-functionally.

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<sup>7</sup> Another connected issue is rigid designation. Under a stronger interpretation of opacity defined in relation to Leibniz's Law would lead us to expect that rigidity should disappear in opaque contexts too. I do not think so though, I have not enough space to discuss it in detail.

<sup>8</sup> In Generative tradition *PRO* is an inaudible variable that functions as the subject of a non-finite sentence.

### 3. Opacity as Accessibility

#### 3.1 General Ideas

Opaque contexts can be compositionally delimited by some lexical item that corresponds to an attitudinal operator like a verb, a noun or an adverb and in several manners, as it is exemplified by the sentences below (by the words in bold):

- (18)
- a. Electra **intended** to kill Clytemnestra.
  - b. It was Electra's **intention** to kill Clytemnestra.
  - c. Electra **intentionally** killed Clytemnestra.

Setting aside the fact that the above categories behave differently in syntax, and considering that the phenomena must be treated in a manner that takes into account scope distinctions, it is possible to formalise epistemic reports in a simple manner by assuming the existence of:

- (19)
- i. An attitudinal operator  $\gamma$  expressed by one lexical item that takes scope over the subordinate clause, which expresses proposition;
  - ii. A proposition  $\pi$  ;
  - iii. A quantifier over propositions  $Q$ .

Given this basic toolbox, there are two hypothetical ways to structure the reports:

- (19')
- a.  $\gamma(Q\pi \ \& \ \pi)$
  - b.  $Q\pi \ \& \ \gamma(\pi)$

For reasons to be shown here, in the case of most attitudinal operators (19a) holds, while (b) is out. There are exceptions to this.

If this tentative formalisation is correct, then three pertinent questions follow:

- ( $Q_1$ )            How the attitudinal operator contributes to or modifies the truth of the statements?

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(Q<sub>2</sub>) Why do human languages need opaque contexts to report epistemic states?

(Q<sub>3</sub>) How can the attitudinal operator be defined in more basic terms?

In answering the three questions above, I shall gradually argue in favour of interpreting the attitudinal operator in part as a model-situation accessing device (rather than a possible world accessibility relation).

### 3.2 Opaque Contexts and Truth

Empirical evidences show that the presence an attitudinal operator crucially changes the truth or the falsity of a statement. For example, a statement, which alone is false, may thereby be embedded within a larger sentence and the resulting epistemic report will be true. Compare the examples below:

- (20)
- a. *f* The earth was flat.
  - b. **T** Folks in the middle age thought that the Earth was flat.

The first sentence of the pair above is and has always been false in the real world, although, it could be the case in another world. On the other hand, the second sentence, which contains the first, is true. This is because persons in the middle age actually thought that Earth was flat, in spite of the fact that, unbeknownst to them, it was not. The contrast with the pair in (21) is startling:

- (21)
- a. **T** The other planets have never revolved around the Earth.
  - b. *f* Ptolemy suggested that the other planets have never revolved around the earth.

The second sentence of the pair above is not true in the real world because Ptolemy was the main theorist of geocentric Astronomy, although in another possible world Ptolemy could have proposed a heliocentric theory. This sort of data shows that an clause  $\phi$  embedded in an opaque context is true if and only if it is true in the epistemic agent's construal of the world, and that the



whole epistemic report is true if and only if it correctly describes the mental state of the epistemic agent<sup>9</sup>. We may extend these observations to many other examples:

- (22)
- a. The sentence *Sandy believes that money grows on trees* is true inasmuch as it is true that Sandy believes that money grows on trees, and regardless of the fact that money does not grow on trees.
  - b. *Linda supposes that there are indigenous elephants in South American Jungles* is true inasmuch as it is true that Linda supposes that there are elephants in South American Jungles, and regardless of the fact that there are not indigenous elephants in South American Jungles.
  - etc.

Or in Bach's (1997) terms: *Sandy believes that money grows on trees* is true iff it is this is one of Sandy's beliefs; *Linda supposes that there are elephants in South American Jungles* is true iff such supposition is among Linda's suppositions, etc.

Accordingly an attitudinal operator seems to separate and at the same time relate two worlds: one that corresponds to the mental state of the epistemic agent and another where the epistemic agent exists. If this is the case, an attitudinal operator may be reinterpreted in terms of accessibility of possible worlds, as already proposed by Kripke (1959), Hintikka (1969) and Lewis (1986). I shall accept this idea and modify it later on, but for the moment let us explore it a little bit.

### 3.3 Possible Worlds

Let us consider the notion of accessing possible worlds:

- (23) **Possible Worlds Accessibility**
- Given two possible worlds  $W'$  and  $W''$  and an accessibility relation  $R$  that corresponds to a propositional attitude:

<sup>9</sup> Cf Jackendoff (1983, 1996) and Lakoff (1987).

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$w'$  is accessible from  $W''$  throughout  $R$  (see Hintikka 1969).

An approach based on the notion above has some advantages and also disadvantages. The first advantage is that it can correctly predict the differences between veridical and non-veridical reports in terms of reflexivity and non-symmetry:

(23')

### i. Non-Symmetric Relation

An accessibility relation  $R$  is non-symmetric if a possible world  $W'''$  is accessible from  $W''$  throughout a relation  $R$ , but  $W''$  is not accessible from  $W'''$  throughout  $R$ .

### ii. Reflexive Accessibility Relation

An accessibility relation  $\mathcal{A}$  is reflexive iff  $\mathcal{A}$  holds between a possible world  $W'$  and  $W'$  itself.

So far we have mainly considered non-veridical epistemic reports with lexical items like *to believe* and *to intend*. These cases are instances of non-symmetric accessibility relations. The veridical reports on the other hand involve lexical items that require a clause that not only corresponds to the epistemic agent's mental state, but that is true in the world wherein such agent exists. Cases of veridical reports include sentences with the verbs *to acknowledge* and *to know*. Let us see some examples:

(24)

a. *f* Spanish is the language of Brazil.

b. *#* Filmmakers know/acknowledge that Spanish is the language of Brazil.

Sentence (24b) is nonsensical because *to acknowledge* and *to know*, unlike *to believe* and *to think*, **seem to** entail that their complement clause must be true in the world the whole epistemic report is true. This sort of fact is supposed to follow from the reflexive character of the relevant accessibility relation.

In spite of the advantages of an analysis of attitudinal operators as accessibility relations, it also carries important problems, which must be considered. Some are minor problems. For instance, to say that some propositional attitudes like *to know* are reflexive relations could imply that the epistemic agent is omniscient. Another problem would be that under the aforesaid presumption, the veridical reports are predicted to be unsuitable to describe fiction.

The first problem is clear. If one says that verbs like *to know* express an accessibility relation between one world and itself, this might mean that the world that corresponds to the knowledge of epistemic agent is the world wherein he is, i.e., the epistemic agent knows all about his world. But inasmuch no epistemic agent is omniscient, it turns out that the possible world that corresponds to his knowledge can at best be a section of the world wherein he lives.

The second problem is also puzzling. Accordingly to the reflexive relation analysis of veridical reports, the pronoun *we* in the sentence below could not refer to persons living in the real world:

(25) We know that Romeo and Juliet died after their secret wedding.

In (25) the accessed world is a fictional world accessed from the real world by epistemic agents expressed by *we*.

These two problems point to the idea that the accessibility relations are always more or less non-symmetric. But these problems can somehow be skipped by a refinement of the involved notions, or just by adding more conceptual instrumentals to the machinery.

It thinks that the real problem with the idea of accessing worlds is that it lacks some of the advantages of using a semantics based on model-situations.

### 3.4 Problems with Paradoxical Readings

The major disadvantage of an analysis of opaque contexts based on the accessibility relations between **possible worlds** is that it does not offer a solution to some important paradoxes. Hence it

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incorrectly predicts that such paradoxes will obtain in the interpretation of natural language sentences, while they usually do not. Here I am referring specially to the following paradoxes exemplified by the respective sentences:

(26) **Fitch's knowability**

We know that there are things we do not know.

(26') **Socrates' Paradox**

One thing I know is that I know nothing.

(27) **Moore's Paradox**

The Earth is round, but I do not believe it.

(28) **The Intensional Liar's (or Lindström's 1998 Epimenides') Paradox**

a. All sentences contemplated by Epimenides are false.

b. The sentence above is contemplated by Epimenides.

(29) **Pollyanna's Paradox**

a. Pollyanna believes everything the Physics teacher says (is true).

b. The Physics teacher says that Pollyanna does not believe anything he says.

These sentences are common in the daily usage of any natural language and are not considered paradoxical in their usual interpretation. But the accessibility relation analysis predicts so. Let us examine those more carefully.

### *3.4.1 Fitch's Knowability and Socrates' Ignorance Paradoxes*

Within the realm of possible worlds approaches, the idea that the concept of knowledge crucially contains the concept of truth perforce brings Fitch's (1963) paradox to the arena of our discussion (Cf Beall 2000, MacFarlane 2004, Restall 2004 and Tennant 2002). The paradox consists of taking a basic notion that there are unknown truths to derive the idea that should the knowability

thesis in (30) hold, then all truths are known, not merely knowable). Thus Fitch's paradox makes incorrect predictions about sentences like (26), although they are common sentences in everyday life.

In standard formalisation of Fitch's paradox it is assumed that there is a predicate *known*  $K$  and the property of being *knowable* is  $\Diamond K$ , so that every truth  $\pi$  is knowable:

$$(30) \quad \textbf{Knowability thesis}$$

$$\forall \pi (\pi \rightarrow \Diamond K \pi)$$

Accordingly, omitting their respective subjects, (26) corresponds to the conjunctions in (30):

$$(31) \quad \text{a. } \exists \pi (\pi \& \neg K \pi)$$

$$\text{b. } K(\exists \pi (\pi \& \neg K \pi))$$

Being the conjunction in (31a) true, it is knowable ( $\Diamond K(\pi \& \neg K \pi)$ ). If one by knowing a conjunction knows the conjuncts, thus  $K(\pi \& \neg K \pi)$  entails  $K \pi$  and  $K(\neg K \pi)$ . But  $K(\neg K \pi)$  means  $\neg K \pi$ , thus  $K(\pi \& \neg K \pi)$  entails the contradiction  $K \pi$  and  $\neg K \pi$ . But (26) does not entail such contradiction in the way it is commonly interpreted in plain English. (Cf. Sub-sections 4.1 and 5.4)

Socrates' paradox reminds Fitch's one outwardly. Anyhow, (29') is in principle contradictory: if the utter of (29') knows nothing, then there should not be one thing he knows. In other words, it should follow that he does not know that he knows nothing, etc. But that is not a conclusion the users of English (or other human languages) commonly draw from a sentence like (29'). The solution to Socrates' paradox is like the ones to Pollyanna's and the Intensional Liar's paradoxes (in Sub-section 4.1).

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### 3.4.2 Moore's Paradox

Moore's paradox<sup>10</sup> involves opacity in sentences that seem contradictory like (27) and the first sentences below:

- (32)            a. I do not believe in witches, but they do exist. (said by Sancho)  
                  b. Witches exist, but Sancho does not believe in them. (said about Sancho)

The utter of (27) firstly asserts that the Earth is round, then claims not to believe in what he just stated, making a seemingly inconsistent sentence. On the other hand, (27) should, as a whole, be true, if the utter does not believe in what the first clause means. The same holds for (32a).

Following the standard understanding of Moore's paradox, while in (32b) the utters merely present the description of the world, in (27) and (32a) they initially seem committed to asserting Earth's roundness and to denying the existence of witches and then break such commitments. In other words, there is a contradiction between the propositional attitudes: the utter at the same time knows what is the truth but does not believe it.

### 3.4.3 Intensional Liar and Pollyanna's Paradoxes

Lindström's Epimenides is just like the Liar's paradox, but it involves the propositional attitude expressed by *to contemplate*.

Pollyanna's paradox is similar, but involves a short dialogue. Assuming that the first sentence in (29) is true and that Pollyanna hears it, one obtains:

- (33)            Pollyanna believes that she does not believe anything the Physics teacher says.

---

<sup>10</sup> Cf. Gillies (2001) and Rosenthal (2002).

But, given (29b), that leads to a chain of contradictory conclusions:

- (33')            i. Pollyanna believes that she believes anything the Physics teacher says.  
                            $\Rightarrow$   
                           ii. Pollyanna believes that she does not believe anything the Physics  
                           teacher says.  $\Rightarrow$  etc.

But, according to the common intuitions of users of any natural language, this is an absurd interpretation of the Pollyanna's sentences.

Let us now consider the advantage of using the notion of accessing situation rather than possible worlds in the attempts to solve paradoxes.

## 4. Re-Discussion

### 4.1 Solutions to the Paradoxes

Among the common approaches to paradoxes in the literature there are those that rely on temporal or spatial segmentation and/or on contextualisation of the statements. These solutions basically seek to assign a non-paradoxical interpretation to sentences that under classic logic should be paradoxical.

Examples of these kinds of solution have been proposed for the knowability paradox. For instance, Restall (2004) argues that the two pieces of the conjunction do not occur at the same time, i.e., not all truths can be known at once, so that the correct formalisation of the conjunction should be something like:

$$(31') \quad \exists \pi (\pi \not\rightarrow_{\Gamma} \neg K\pi)$$

Edgington (1985), on the other hand, argues that two distinct situations  $S'$  and  $S''$  ought to be assumed: the knower knows  $\pi$  in  $S'$  and  $\pi$  is true in  $S''$ , although  $\pi$  is not true in  $S'$  itself. Both of

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the aforesaid approaches can be combined to avoid or solve Fitch's paradox. (See Sub-section 5.4 for more details)

Recent instances of these sorts of solution to the versions of the Liar's paradox are found in Gauker (2001, 2004), Glanzberg (2003) and Lindström (1998). Except for some different details in the formulation of their proposals, Lindström and Glanzberg's solutions go basically in the same direction as Gauker (2004), for whom the relativisation of truth to context may offer the means to avoid paradoxes, if it is possible to deny that a sentence about a context can be true in the very context it is about.

Glanzberg (2003) argues that, as a consequence of high principles, in the Liar's sentence there is a domain restriction phenomenon at work, where a covert quantifier over propositions plays a role in predications of truth. Consider the liar's sentence:

(34) This sentence is not true= $\sigma$

The sentence above entails two distinct conclusions:

- (34') i. That  $\sigma$  is true.  
ii. That  $\sigma$  is not true.

If the conclusions above are taken to refer to two different contexts, relating to two different domains or to two different periods in the history of the same domain, there is no necessary contradiction between them. Glanzberg assumes that this sort of strategy could be used to tackle the strengthened Liar's paradox. If it is possible to assume that if propositions are simply sets of truth conditions, then the correlation between a proposition and the sentence that expresses it reveals a context dependence. As most predications of truth work through the expression relation, they contain a tacit existential quantifier, so that:

(35)  $\exists\pi(\text{Exp}(\varphi, \pi, I) \& \text{Tr}(\pi))$

The proposition  $\pi$  expressed by the sentence  $\varphi$  in context  $I$  is true.



Where, in Glanzberg's system, Exp means *expresses* and Tr *Truth predicate*. The domain of Glanzberg's quantifier  $\exists\pi$  can be contextually restricted, as in the case of all quantifiers in natural language. This implies that any claim like every *proposition is true* means that every proposition in a certain domain is true. The Strengthened Liar reasoning shows that there must be extraordinary context dependence in the Liar's sentence, and the domain of Glanzberg's propositional quantifier  $\exists\pi$  must expand between:

$$(36) \quad \neg\exists\pi(\text{Exp}(\varphi, \pi))$$

and

$$\varphi$$

The first claim may be true, if within the contextual domain of propositions there is no proposition for  $\varphi$  to express. The second claim may express a true proposition, if there is one in the expanded domain of propositions available from its context.

Lindström (1998) tries a relatively equivalent path for the intensional Liar's paradox. He deems it analogous the paradox of the Barber. In the latter, the paradox is avoided by denying the existence of any such person, who shaves all and only those inhabitants of his village that do not shave themselves. Similarly, for Lindström, Epimenides (the intensional Liar) cannot exist, for he would be a person who contemplates (or asserts) one and only one proposition in the domain consisting of available propositions, namely, the proposition that all propositions in the domain he contemplates are false.

Let  $\Sigma$  be the set of all propositions and  $\Omega$  a subset of  $\Sigma$  over which a propositional quantifier ranges. If one re-states the Liar's proposition in the form  $\forall\pi(A\pi \rightarrow \neg\pi)$ , then Lindström's theorem applies:

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(37)

**i. Theorem**

If the domain of quantification  $\Omega$  satisfies the principle of plenitude, then the Liar proposition for  $\Omega$  is not a member of  $\Omega$ .

(See Lindström 1998 for demonstration)

**ii. Corollary**

If the domain of quantification  $\Omega$  satisfies the principle of plenitude, then it cannot be the whole set  $\Sigma$ .

Lindström theorem above refers to Kaplan's plenitude principle:

(38)

**The Plenitude Principle**

There is a property  $A$  of propositions such that: for each proposition  $\pi$ , it is possible that  $\pi$  should have had the property  $A$  and that  $\pi$  should have been the only proposition having that property. (Kaplan, Lindström 1998)

Lindström then proposes an attitudinal version of Kaplan's principle:

(38')

**Attitudinal Plenitude**

For any proposition  $\pi$ , it is possible that the thinker entertains the proposition  $\pi$  at time  $t$  and that  $\pi$  is the only proposition that he entertains at time  $t$ . (see Lindström 1998)

Lindström's interpretation of sentences is relatively to a parameter, namely, the domain  $\Omega$ , whereby one sentence will express different propositions for different values of the parameter  $\Omega$ , which is fixed by context:

(39)

**i.** For every context of use  $\Gamma$ , there is a domain  $\Omega_\Gamma$  of all the propositions that are available in  $\Gamma$ .

**ii.** A proposition unavailable in  $\Gamma$  cannot be quantified over from the

perspective of  $\Gamma$ .

This solution is applicable to Pollyanna's paradox: accordingly, what falls within the scope of Pollyanna's beliefs does not include the proposition expressed by (29b).

Although Lindström tries to show that the intensional Liar paradox can be defused and thus does not pose a threat to possible worlds semantics, his notion of *context of use* seems to refer to a section of a world.

If these solutions are understood correctly, they all point to a situation-model approach. Empirically it is known that the context shift produces either one or both of the following changes:

- The truth-value of a proposition expressed by a sentence changes;
- The proposition a sentence expresses changes.

I take that these two facts together with others show that each context is not a mere section of the possible world, rather it is also a structured model or situation.

Indeed, in principle there is no reason to think that the notion of accessibility cannot be transposed to a situation-model based approach. Accordingly, among the implicit presuppositions underlying a kind of solution like the ones proposed by Glanzberg or Lindström, there is one idealised division of situations into at least three *tiers*. Firstly, there is the **production** situation, i.e., the context where a sentence is uttered, but which is not necessarily the situation it refers to. Secondly, there is the **referential** situation, which the sentence refers to and where the expressed proposition is true or not. In the case of epistemic reports, the referential situation includes the contexts of use where someone entertains a proposition, i.e., has an attitude in relation to it. In a more finely grained manner, the referential situation of an epistemic report is the one that ideally satisfies the attitudinal plenitude principle above. Thirdly, there is the **alternate** situation: they are those situations that coincide with the propositions that are entertained. The referential situation can be accessed from the production situation, and the alternate situation from the former.

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### 4.2 Basic Properties of Human Languages

The introduction of situation accessibility relations into natural language semantics is necessary though, it is not sufficient to capture the phenomena described. These relations have somehow to abide by greater principles, i.e., to comport with certain key properties of human languages.

Schöter (1994), among others, claims that the semantics of natural languages exhibit four basic properties that are already acknowledged and have been investigated in non-classic logic theories: paraconsistency, defeasibility or non-monotonicity, partiality, which contrasts to *totality*, and relevance.

The substitution problems found by Frege and others in opaque contexts have in part to do with non-monotonicity. Here, a non-monotonic inference is the one that does not necessarily obey (40):

(40) **Monotonicity**  
Where it is legitimate to infer a proposition  $\pi$  from a set of propositions  $\Omega$  it is also legitimate to infer  $\pi$  from any other set  $\Omega'$  that contains  $\Omega$ .

This is a clear picture. If one understands the referential situation as the context Lindström's attitudinal plenitude refers to, so the referential situation is defined by a set of propositions (the epistemic reports) that contains the propositions that defines the alternate situation (i.e. the propositions that correspond to an agent's mental state). In simple and direct terms, this should allow monotonic inferences. But the two situations are mediated by a propositional attitude that blocks monotonicity.

Defeasibility and information revision are two sides of the same coin in human languages. In this sense, sentences (27) and (32a) are not normally interpreted as paradoxical, because any of its

componential clause is subject to review by another. Thus (27), under its usual interpretation, expresses the simple constatation that the facts simply contradict what the utter expected to be the case. On the other hand, (32a), besides its intended comic effect, may mean that, in spite of his firm scepticism, the utter admits there are unexplained facts that seem counter-evidences. Another possible interpretation is that, although there are people who claim to be witches, the utter does not accept their claim. In both cases, the utters talk about the very belief revision process in given situations: why it seems necessary and how difficult it may be.

But non-monotonicity and information review are properties present in the interpretation of both opaque and transparent expressions, and cannot make the best answer to the question

- *Why do human languages need opaque contexts to report epistemic states?*

Methinks that the answer rests on the need human languages have to circumvent the principle of Pseudo-Scotus.

### 4.3 Paraconsistent Semantics

The main point of surveying the opacity issue in natural languages is to ask why do human languages need opacity. In other words, one can intuitively think that human languages cannot dispense with propositions and propositional attitudes. But it is not clear why propositional attitudes cause this opaque effect whereby the truth or the falsity of some embedded statements is insulated. This is especially more mysterious if one considers that propositional attitudes are mechanisms to access separate situations. Let us consider a possible answer in a stepwise manner.

Any language has the means to convey negation. In standard classic logic the alethic value false ( $f$ ) and the negation symbol are deemed equivalent:

(41) **Non-contradiction**

$$\top \sigma \leftrightarrow f \neg \sigma.$$

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Given empirical evidences, a very small number of cases can fit into the schema above in natural languages. If, however, from an alternative perspective, one distinguishes falsity from negation, it is possible to admit that an expression and its negation may have the same truth-value. In such cases, a contradiction obtains:

(42) **Contradiction**  
Both  $\sigma$  and  $\neg\sigma$  are true.

Contradictions are nevertheless problematic under the principle of Pseudo-Scotus:

(43) **The Principle of Pseudo-Scotus**  
*Ex contradictio sequitur quodlibet.*  
From a contradiction anything follows.

A semantics that conforms with (43) is called *trivial* or *explosive* in the modern logic jargon<sup>11</sup>. An expression is trivial (or explosive) if it is assigned an interpretation in pursuance of the principle above, which is to say, if it is possible to derive any consequence from it and its negation:

(43')  $(\alpha, \neg\alpha) \models \forall \phi (\alpha \text{ is trivial, i.e., } \bullet^{\circ} \alpha)$

If human languages were explosive in this sense, any sentence expressing a contradiction would result in complete disaster. For instance, the Principle of Pseudo-Scotus predicts that the interpreter of any of the sentences presented before would draw an infinite number of consequences therefrom, if he considers that each sentence somehow contains a conjunction like  $(\sigma \& \neg\sigma)$ . In such case the interpreter would not know what to think or to do, as he would be busy contemplating the consequences forever.

However, that is not the case, for the semantics of any human language can both tolerate the existence of a contradiction and control it, i.e., it restricts the set propositions that are derivable from such contradiction. This property of human languages is defined below:

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<sup>11</sup> Mind that the terms *trivial* and *trivialisation* will not be herein used in another sense than those defined above.

(44) **Paraconsistency**

It is not the case that for any  $\sigma$  and any  $\phi$ ,  $(\sigma, \neg\sigma) \models \phi$ .

According to Gauker (2001), the same anti-paradox mechanisms of his 2004 paper also block contradiction, for the impossibility of such reflexive reference to contexts ensures that there is no sentence of the language such that both it and its negation are assertible in a single context. Here, I assume something slightly similar: the semantics of human languages segregates a proposition and its negation in different situations to avoid both paradoxical and explosive readings.

Under the concept of paraconsistency above, natural language sentences may be divided into consistent and non-consistent statements:

(45) **Consistency**

i.  $(\sigma, \neg\sigma) \models \neg \circ\sigma$  (if  $\sigma$  is contradictory, then it is not consistent);

ii.  $\circ\sigma \models \neg(\sigma, \neg\sigma)$  (if  $\sigma$  is consistent, then it is non-contradictory)<sup>12</sup>.

Propositional reports are consistent statements, since they are in principle non-contradictory.

Let me expand this.

The examples given hereinbefore, such as those illustrating the Moore's paradox, might at first glance seem to assert that both a proposition  $\pi$  and  $\neg\pi$  are true, i.e., a contradiction. But this would be the case only if the piece of the structure that expresses the attitude did not count to contain explosion. The point is that in most of the examples the first proposition ( $\pi$ ) is true in the referential situation  $\mathcal{S}$ , while its negation ( $\neg\pi$ ) is true in the alternate one. Which is to say, they are both true, but not in the same domain. Firstly, consider two potential paraphrases of that (27):

- (46)            a. Earth is round, but I do not believe it is round.  
                   b. Earth is round, but I believe that it is not round.

<sup>12</sup> Here  $\circ\alpha$  means that  $\alpha$  is consistent, and  $\bullet\alpha$  that  $\alpha$  is inconsistent.

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In the first potential paraphrase the propositional attitude falls within the scope of the negation, while in the second the negation is inside of the attitudinal scope. One could re-write (46) in the semi-formal manner below:

- (46')            a.  $\pi$ , but  $\neg\gamma[\pi]$ .  
                    b.  $\pi$ , but  $\gamma[\neg\pi]$ .

In (46a) there is not a clear contradiction. The first conjunct states that *Earth is round*. But the second conjunct does not deny the fact that *Earth is round*. Rather, it denies the belief in that fact, which is to say, the epistemic report does not contain a real negation of the proposition. In (46b) although it is not round is the negation of the Earth is round, it is contained within the alternate situation, so there is not a real contradiction either. This sort of analysis can be extended to the other cases.

### 4.4 More Analyses and Cases

An epistemic report usually contains a super-ordinate clause whose consistency is stronger than that of the subordinate, according to (47):

- (47)            **Consistency Strength**  
  
                    If the consistency of  $\sigma$  implies the consistency of  $\phi$ ,  $\sigma$  is said to have greater consistency strength.

Take the example below:

- (48)            Pollyanna ignores that the Physics teacher is lying.

In (48) the epistemic report excludes the negation of *the Physics teacher is lying*, making the embedded sentence consistent, whereby the consistency of the main clause is stronger.

If one assumes that the fact above may be generalised, one has to deal with apparent counter-examples, such as those (49):



- (49)
- a. Sancho supposes that witches exist and does not suppose it.
  - b. Sancho supposes that witches exist and do not exist.

There are two basic ways to interpret (49): either that both reports are non-consistent or that somehow their consistency is saved. If one maintains that every super-ordinate clause in an epistemic report has stronger consistency, then the second option must be considered.

Considering that *and* corresponds to a temporal connective, like in Restall's (2004) solution to Fitch's paradox, then Lindström's attitudinal plenitude (38') saves the consistency of (49a). On the other hand, (49b) seems to report a genuine non-consistent belief, unless the temporal feature of *and* ( $\mathcal{E}_T$ ) suffices to cancel the contradiction. Accordingly, (49) should be paraphrased as follows:

- (49')
- a. Sancho sometimes supposes that witches exist and at other times does not suppose it.
  - b. Sancho supposes that sometimes witches exist and at other times they do not exist.

But if, according to the interpretation assigned to (49b), the connective is commutative (&) and thus the contradiction is not cancelled, then one has to consider that possibility that epistemic reports may be non-consistent and yet non-trivial (or non-explosive). This possibility will be explored in the next section.

## 5. Further Paraconsistent Issues

### 5.1 Interpretation of Sentences and Depictions of Situations

So far we have seen that opacity must be characterised truth-functionally. It has been pointed out that propositional attitudes contribute to the meaning of expressions by affecting inference. The resulting effect is what is called opacity, although in many cases a better metaphoric name for it would perhaps be *refraction*. Anyhow, as epistemic reports may have more than one possible

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interpretation, it seems sensible to formulate appropriate moods of entailment that work in epistemic reports and yield the different readings. Here we shall seek to obtain moods of entailment compatible with attitudinal operators, by modifying the classical concept of entailment. Let us do it in a stepwise fashion.

The first step is to treat epistemic reports as descriptions of situations. In such case, one has to deal with interpretations assigned to the sentences that describe the situations. In this sense, if a situation  $S$  is describable by a **finite** number of propositions, and if one understands that  $\partial$  entails  $\pi$  iff both  $\partial$  and  $\pi$  are descriptions of the same situation, then entailment is always **naturally non-trivial**. This kind of entailment I shall call descriptive:

$$(50) \quad \partial \models_D \forall \pi \leftrightarrow (\partial \in S) \& (\pi \in S)$$

In the same manner a contradiction belonging to  $s$  can be controlled, for it is not any consequence that is derivable therefrom: only those consequences belonging to  $s$ :

$$(50') \quad (\partial, \neg\partial) \models_D \forall \pi \leftrightarrow ((\partial, \neg\partial) \in S) \& (\pi \in S)$$

However, this reasoning does not work for situations that are describable by a potentially **infinite** number of propositions and excludes the possibility of inferring consequences across the situations that are related via the accessibility relation. Therefore, an extra assumption is necessary.

## 5.2 Complementary Descriptions

The second step is to make a more refined assumption: that one or more epistemic reports by accessing alternate situations from referential situations, function as descriptions in a complementary<sup>13</sup> fashion. Accordingly, a given model situation  $S$  admits (what is herein called) a **complementary interpretation** if the following conditions are satisfied:

$$(51) \quad \text{i. } S \text{ contains at least two descriptions } \partial_1 \text{ and } \partial_2;$$

---

<sup>13</sup> The ideas used in this section are mainly inspired by da Costa & Krause (2001, 2003).

- ii.  $\partial 1$  and  $\partial 2$  refer to the same situation  $S$ ;
- iii. Neither  $\partial 1$  nor  $\partial 2$ , if taken alone, accounts exhaustively for all the aspects of  $s$ ;
- iv.  $\partial 1$  and  $\partial 2$  are mutually exclusive in the sense that their combination into a single description would lead to logical contradictions.

In classical logic a true proposition cannot ‘exclude’ another true proposition. Accordingly, if both  $\alpha$  and  $\beta$  are true propositions in a certain domain  $\Omega$ , then  $\alpha \ \& \ \beta$  is also a true proposition of the same domain. This means that if from  $\alpha$  one deduces  $\gamma$ , and if from  $\beta$  one deduces  $\neg\gamma$ , then  $\gamma \ \& \ \neg\gamma$  is also deducible in the same set of propositions.

However, if one wants to restrict the set of propositions that describe a situation, there must be more about the conjunction  $\alpha \ \& \ \beta$ , so that in  $T \ \alpha \ \models \pi$  and  $\beta \ \models \neg\pi$  obtain, but  $(\pi \ \& \ \neg\pi)$  is not true in the described situation. In other words, there must be a formal way to avoid that  $(\alpha \ \& \ \beta)$  entails a contradiction, since we do not intend to rule out complementary situations.

Let us consider two examples of complementary descriptions in a situation  $s'$  where Sancho knows that Don Quixote is a mad man but accepts to work for him because the latter has promised to make the former the King of an Island. Such situation may be described by two different assertions, each of them having a different consequence:

- (52)            a. Sancho thinks Don Quixote does not deceive people.  
                            $\models$  In such case, he thinks that Don Quixote will fulfil his promise of making him a King.
- b. Sancho thinks Don Quixote is delusional.  
                            $\models$  In such case, he thinks that Don Quixote will not fulfil his promise of making him a King.

It is evident that Sancho’s beliefs do not imply that (53) holds:

- (53)            #Sancho thinks that Don Quixote will fulfil his promise and will not fulfil

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it.

This means that Sancho's beliefs are complementary in the sense above, and that there are ways to block (53) to follow from (52).

Thence, the following step is to envisage two classes of propositional attitudes and different types of entailment that read the types of propositional ascriptions differently. The two classes have been mentioned in the previous Section: those that are consistent ( ${}^{\circ}\tau$ ) and those non-consistent but non-trivial ( $@\tau$ ).

Now consider the common conclusions people intuitively take from the use of an attitudinal term like *to suppose*. From the examples below two different conclusions are drawn, according to how the attitudinal verb is interpreted:

- (54)                    a. Ptolemy supposed that the Earth was the centre of the Universe.  
                              b. Scientists suppose that some very distant planets have water.

In the case of (54a) it is inferred from the use of the verb to suppose that the proposition expressed by subordinate clause is false, which is to say, the attitudinal operator is interpreted as equivalent to negation:

$$(55) \quad \gamma(\pi) \mid =_N \neg\pi$$

In (54b), on the other hand, to suppose implies that the embedded clause expresses an undecided proposition. I take that, under such interpretation, what is inferred is an exclusive disjunction:

$$(56) \quad \gamma(\pi) \mid =_{\epsilon} \pi \vee \neg\pi$$

We can, accordingly, say that there is a negative reading or entailment ( $\mid =_{\epsilon}$ ) and a disjunctive one ( $\mid =_N$ ), which can be extended to conjunctions of descriptions, as shown below:

$$(57) \quad \text{For the propositional attitude } {}^{\circ}\tau \text{ and the complementary descriptions } \alpha \text{ and } \beta, \text{ such that } \alpha \mid =\pi \text{ and } \beta \mid =\neg\pi:$$

- i.  ${}^{\circ}\tau(\alpha\&\beta) \models_{\epsilon} \pi\vee\neg\pi$  (disjunctive interpretation)
  - ii.  ${}^{\circ}\tau(\alpha\&\beta) \models_{\text{N}} \neg(\alpha\&\beta)$  (negative interpretation)
- or  ${}^{\circ}\tau(\alpha\&\beta) \models_{\text{N}} \neg(\pi\&\neg\pi)$

As the combination of devices above conspire to control explosions, they can be extended to non-consistent but non-trivial reports:

- (58) For the propositional attitude  $@\tau$  and the descriptions  $\pi$  and  $\neg\pi$ , such that  $\pi \models \partial$  and  $\neg\pi \models \chi$ :
- i.  $@\tau(\pi\&\neg\pi) \models_{\epsilon} \partial\vee\chi$  (disjunctive interpretation)
  - ii.  $@\tau(\pi\&\neg\pi) \models_{\text{N}} \neg(\partial\&\chi)$  (negative interpretation)

I shall not bother to demonstrate (57ii) and (58ii). It is easy to demonstrate (57i), by assuming that the connectives below are commutative:

- (59) **Demonstration**
- i.  $\tau(\alpha\&\beta) = \tau(\alpha)\&\tau(\beta)$ ;
  - ii.  $\tau(\alpha) \models_{\epsilon} \alpha\vee\neg\alpha$ ,  $\tau(\beta) \models_{\epsilon} \beta\vee\neg\beta$ ;
  - iii.  $\alpha \models \pi \rightarrow (\alpha\vee\neg\alpha) = (\pi\vee\neg\pi)$ ;
  - iv.  $\beta \models \neg\pi \rightarrow (\beta\vee\neg\beta) = (\neg\pi\vee\pi)$ ;
  - v. Given the previous steps,  $\tau(\alpha)\&\tau(\beta) = (\pi\vee\neg\pi)\&(\neg\pi\vee\pi)$ ;
  - vi. By the commutative property,  $(\pi\vee\neg\pi)\&(\neg\pi\vee\pi)$  is simply  $(\pi\vee\neg\pi)$ .

The demonstration of (58i) is something similar.

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### 5.3 Applications

It is possible to apply (57) to examples like (52). Indeed, (57ii) precludes (53) as a valid conclusion. Moreover (57i) predicts correctly that Sancho is taking his chances by working for Don Quixote, i.e., according to Sancho's beliefs, there is a fifty per cent chance that Don Quixote will fulfil his promise, (52) entailing (53'):

(53') Sancho thinks that Don Quixote will either fulfil his promise or will break it.

And that is basically why Sancho does not quit his boss, in spite of the fact that the former deems the latter delusional.

We can analyse (49b) as well, using (58) which provides the non-explosive interpretations for it. Whatever *supposes that witches exist and do not exist* entails it will not make a true conjunction. Assume an interpretation that each part of the coordination in (49b) entails a different proposition:

(49'') a. Sancho supposes that witches exist (...). |= Old chivalry stories are based on real facts.  
b. Sancho supposes that (...) witches do not exist. |= Everything is just the by-product of the imagination of people like Don Quixote.

What is predicted under (58i) is that (60) is not entailed by (49''):

(60) #Old chivalry stories are based on real facts and everything is just the by-product of the imagination of people like Don Quixote.

On the other hand, (58ii) predicts that (60') is the usual and intuitive consequence of (49''):

(60') Either old chivalry stories are based on real facts or everything is just the by-product of the imagination of people like Don Quixote.

#### 5.4 Appendix: the (Non-)Transitivity of veridical reports (Some Residual Points)

If some propositional attitudes are interpretable as negation of propositions, should veridical reports containing verbs like *to know* interpreted as the simple statement of the propositions? This question seems to make sense, since it has been observed that *to know something* entails that that *thing* is true. However, a claim like (61a) might mean that the propositional attitude contributes nothing to the meaning of the whole expression. Moreover, if (61a) is true, then (61b) should be the case:

- (61)            **Hypothesis A**
- a.  $\mathbf{k}(\pi) \models \pi$
- b.  $(\mathbf{k}(\pi) \models \pi) \& (\pi \models \mu) \rightarrow \mathbf{k}(\pi) \models \mu$

Let us call one of the ideas that seems to underlie the intuition formalised in (61a) Hypothesis A' and re-write it as (62):

- (62)            **Hypothesis A'**
- a.  $\mathbf{k}(\pi) = \pi$
- b.  $\therefore$  Adding *to know* to a proposition adds nothing to its meaning.

Hypothesis A' would lead to think that the negation of  $\mathbf{k}(\pi)$  is the negation of  $\pi$  itself:

- (62')             $\neg \mathbf{k}(\pi) = \neg \pi$

Should (62') hold, then statements of the type *not know*  $\pi$  and those of the type *to know that*  $\pi$  *is the not case* would mean the same. But the evident contrast between sentences (63a) and (b) below does not confirm such prediction:

- (63)            a. We know that Giselle is not a singer.  $\neq$
- b. We do not know that Giselle is a singer.

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This contrast suggests that  $\mathbf{K}(\pi)$  is not equal to  $\pi$  but rather that it means one epistemic agent has access to a truth  $\pi$ , while  $\neg\mathbf{K}(\pi)$  does not mean that  $\neg\pi$ , but rather that an agent has not access to a truth  $\pi$ .

Still this finding only excludes hypothesis A', while it would be possible to maintain hypothesis A. Additional evidences, on the contrary, suggest that *to know* is perhaps the most opaque of the attitudinal verbs, in the sense that sentences with *to know* somehow block transitivity. Consider this example:

- (64)            a. People can skate on the lake.  $|=$  Its water has frozen.  
                  b. John knows that people can skate on the lake.  $| \neq$  Its water has frozen.

Being the entailment in (64a) valid, and if (61b) applies, then the addition of *John knows...* should not affect the entailment. But (64b) disconfirms such expectation: the mere fact that John knows that people can skate on the lake does not mean right at the same moment that the water of the lake is covered by a thick layer of ice.

Now consider this other hypothesis:

- (65)            **Hypothesis B**  
 $(\pi | = \mu) \rightarrow (\mathbf{K}(\pi) | = \mathbf{K}(\mu))$

This second hypothesis is not true either, as shown by (66)

- (66)            a. Oedipus killed the man he met at the crossroads.  
                   $|=$  The oracle has been fulfilled.  
                  b. Oedipus knows he killed the man he met at the crossroads.  
                   $| \neq$  Oedipus knows the oracle has been fulfilled.

So evidences point to the contrary conclusion, although an expression of the type *to know*  $\pi$  entails the truth of  $\pi$ , it somehow unmakes  $\pi | = \mu$ :



(67) **Non-transitivity**

$$(\mathbf{k}(\pi) | = \pi) \& (\pi | = \mu) \rightarrow (\mathbf{k}(\pi) | \neq \mu) \& (\mathbf{k}(\pi) | \neq \mathbf{k}(\mu))$$

The non-transitivity of veridical reports requires closer examination and more attention. For the sake of economy, such topic cannot and will not be herein investigated in more detail. Here it will suffice to say that the apparent non-transitivity of veridical reports is also an anti-trivialisation mechanism.

## 6. Epilogue

### 6.1 More Residual Points

I have on purpose chosen to provide a broad comparison between the theoretic points of view explored and some simplified but useful solutions to the related empirical problems, both of which are non-exhaustive. The implicit intention has been to shift the axis of discussion of an old and recurrent theme to the paradigms of a paraconsistent semantics for natural languages. I have also deliberately sought to present a picture that is both accessible and usable by a majority of linguists, and not to restrict the ideas to a small circle of deep connoisseurs of Logic and Philosophy of Language.

Nevertheless, there are a considerable amount of residual issues. Particularly, the proposals advanced in Sections 4 and 5 require the pertinent investigation of more compositional issues.

Moreover the proposed paraconsistent semantics should be expanded to include quantification over situations, which presumes the ordering of situations in the domain of quantification. The transposition of such notions from a possible worlds approach (cf. Kratzer 1981) to a model-situation one is indispensable.

These two points are inter-related and have herein remained untouched. I hope to be able to explore facets and details of such issues with more elaboration in future works.

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## 6.2 Retrospect

Opacity is simply a semantic phenomenon related to the truth conditions of sentential contexts and reflects the paraconsistent character of human languages. Accordingly, a sentential context is opaque in relation to another if the *light* of the truth-conditions of the later is not allowed to pass through the propositional attitude gluing both contexts together, whereby both explosions under the principle of Pseudo-Scotus and paradoxical readings of sentences are avoided or controlled.

The semantics of a language includes contradiction if it derives a sentence or an expression  $\sigma$  and its negation  $\neg\sigma$  for some proposition  $\pi$ , and is deemed *trivial* or *explosive* if any proposition  $\phi$  is derivable therefrom. On the other hand, the semantics of a language is called a paraconsistent if it handles contradiction in a non-trivial way, i.e., if it is not the case that a pair of sentences  $\sigma$  and  $\neg\sigma$  entails any  $\phi$ , and so the explosive character of such semantics is controlled or precluded.

In natural languages, epistemic ascriptions embed statements that are relatively opaque to the truth conditions of the referential situation and not to Leibniz' Principle. Failure of substitution in a given sentential context, on the other hand, may have primarily to do with the defeasibility property of human languages.

If attitudinal operators actually contain or are equal to accessibility relations, then in the case of human languages such relations access situations from situations, whether the situations are part of the same or different possible worlds.

Briefly, propositional attitudes create involucrees to truth-conditions in order to avoid explosion under the Principle of Pseudo-Scotus. Though paraconsistent approaches in Logic have

been developed since the seminal works of Jas'kowsky (1948) and da Costa (1963, 1997)<sup>14</sup>, and though they have important consequences for Linguistics, similar approaches in natural language semantics are only beginning.

We can now answer the questions formulated in Sub-section 3.1:

- (<sup>Q</sup><sub>1</sub>)            How the attitudinal operator contributes to or modifies the truth of the statements?
- (<sup>Q</sup><sub>2</sub>)            Why do human languages need opaque contexts to report epistemic states?
- (<sup>Q</sup><sub>3</sub>)            How can the attitudinal operator be defined in more basic terms?

These questions have been addressed hereinafter, and may be synthesised in three short answers:

- (<sup>R</sup><sub>1</sub>)            An attitudinal operator changes truth conditions by separating the model-situation (of the same or different world) in which the epistemic ascription can be asserted from that where the embedded proposition is true.
- (<sup>R</sup><sub>2</sub>)            Human languages need opaque contexts to avoid trivialisation under Pseud0-Scotus and paradoxical readings.
- (<sup>R</sup><sub>3</sub>)            The attitudinal operator can be decomposed into simpler operators or predicates that provide spatial, temporal or contextual co-ordinates. Alternatively, we may equate an attitudinal operator to an accessibility relation holding between model-situations (of the same or different worlds).

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<sup>14</sup> Current paraconsistent Logic comprehends a varied set of distinct proposals, whose general features and ways of implementation are presented in Priest (2002), de Amo, Carnielli & Marcos (2002), Carnielli (2002), da Costa, Krause & Bueno (2004), Damásio & Pereira (1997), Hunter (1996), Libert (2002), Marcos (2001, 2002) and Tanaka (2003), among others. I shall not comment on the differences between these views.

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