Capturing ignorance inferences and roundness effects of modified numerals

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Some puzzles with unembedded modified numerals

In contrast with bare numerals, numerals modified by ‘at least’ and ‘more than’ tend only to convey a lower-bound and speaker ignorance (Krifka 1999). In the past two decades, we have acquired a much finer-grained understanding of the effect of these modifiers.

[A] Stronger ignorance with superlative than comparative modifiers

(1) I have more than/??at least four children

[B] Comparative, but not superlative, affected by roundness/contextual salience of the numeral

(2) Mary can drink, she’s at least/??more than 27

[C] Question under Discussion (QUD) effects:

- Comparative less frequent and degraded with precise QUD

(3) a. How many people attended the workshop? (I need to inform the administration)
b. At least 15 / ??More than 15.

- Stronger ignorance inferences with precise QUD

(4) a. Exactly how many CDs do you have?
At least 60.
b. Does your CD collection fit in this holder?
No, I have at least 60.

Some previous proposals

- Cummins (2011, 2013) captures roundness effect for comparatives, can capture QUD effect on ignorance, but wrongly predicts superlatives to be more sensitive to roundness. Difference in ignorance is postulated.
- Buccola&Haida (2017) capture most of the data but wrongly predict salience effects with ‘at least’. Enguehard (2018) builds on their proposal to account for comparatives, but does not address superlative modifiers.
- Westera&Brasoveanu (2014) assume that different modifiers trigger different QUDs. This captures the difference in ignorance and sensitivity to roundness, but the assumption is *ad hoc*.
- Many more proposals address the ignorance inference without discussing roundness/salience effects.

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A pragmatic account of roundness and ignorance effects

We propose an Optimality Theory account inspired by Cummins (2011).

Semantic assumptions:

- Exact semantics for bare numerals (to be revised)
- Naive semantics for modified numerals:

Pragmatic assumptions:

Constraints: (φ = expression, s = speaker’s knowledge, Q = QUD)

QUAL: $s \subseteq \varphi$ (Gricean Quality)

QUANT: φ resolves Q as much as s does (Gricean Quantity):
$\{q \in Q \mid s \cap q = \emptyset\} \subseteq \{q \in Q \mid [\![\varphi]\!] \cap q = \emptyset\}$

SIMP: One violation if φ contains a comparative modifier, two violations if it contains a superlative modifier.

NSAL: φ shouldn’t contain numeral that are neither round nor salient.

ISAL: φ should only contain numerals that are *internally salient* to the speaker.

A numeral is internally salient to a speaker if it matches a boundary of their knowledge. ☞ *If you know that between 7 and 12 students smoke, then ‘seven’ and ‘twelve’ are internally salient to you.*

Constraints ranking: QUAL ≫ QUANT ≫ ISAL ≈ NSAL ≫ SIMP
Lower-ranked constraints do not break ties between ISAL and NSAL: more than one expression can be optimal in some contexts (Stochastic OT interpretation).

Application to polar and how-many questions

Q	s	φ	QUAL	QUANT	ISAL	NSAL	SIMP
How many: $\{\{0\},\{1\},\{2\}\ldots\}$	$\{k\}$	n	*!		*		
		☞ k				*	
		more than n		*!	*		*
		more than $k-1$		*!	*	*	*
		at least n		*!	*		**
		at least k		*!		*	**
	$[k,\ldots)$	n	*!		*		
		k	*!			*	
		more than n		*!	*		*
		more than k-1		(*)	*!	*	*
		at least n		*!	*		**
		☞ at least k				*	**
Polar: $\{[0,n),[n,\ldots)\}$	$\{k\}$	n	*!		*		
		☞ k				*	
		☞ more than n			*		*
		more than $k-1$			*!	*!	*
		at least n			*		**!
		at least k				*	**!
	$[k,\ldots)$	n	*!		*		
		k	*!			*	
		☞ more than n			*		*
		more than $k-1$			*!	*!	*
		at least n			*		**!
		☞ at least k				*	**

Summary of the main predictions

‘more than’:

- only ever used with round/salient numerals,
- never used to answer precise “how many?” questions,
- compatible with both precise knowledge and ignorance.

‘at least’:

- only ever used when the speaker is ignorant,
- always used in a “precise” way,
- compatible with any QUD.

Further predictions

PPI behavior of ‘at least’ (Spector 2011)

(5) John doesn’t have more than three/*at least three children

‘At least’ must satisfy ISAL to compete with ‘more than’. Negation flips strict and non-strict comparisons, so “not…at least” always violates ISAL.

Partial orders (Mendia 2016)

(6) #At least two students smoke, but not exactly two.

(7) At least Liz and Mary smoke, but not only them.

$$s = \{\ l\oplus m\oplus a\ ,\ l\oplus m\oplus b\ ,\ l\oplus m\oplus c\ \} \quad \rightarrow \quad \min(s) = \#\ \text{and}\ \inf(s) = l\oplus m$$

Assuming ISAL is sensitive to GLB rather than minimum, we capture Mendia’s observation that partial orders do not require ignorance wrt the exhaustified prejacent. Indeed:

(i) Partial orders allow GLB which are not minima

(ii) Partial orders are incompatible with ‘more than’

(8) *More than Liz and Mary smoke

NB: Condition (i) alone is not sufficient:

(9) How far is the university from here?
#At least three miles, but not exactly three.

Accommodating an ambiguity theory of numerals

Good evidence against exact semantics for bare numerals (Geurts 2006, Spector 2013, a.o.). We can adopt a semantic ambiguity theory for bare numerals (either type-shifting or grammatical exhaustification) provided we add a constraint to deal with ambiguous expressions:

SQUAL: $\langle \varphi, s, Q\rangle$ violates SQUAL if for some parse φ_i of φ: $s \not\subseteq \varphi_i$

Open questions

- Expressions like ‘about’ and ‘exactly’, halo effect of round numerals, and vague speaker knowledge (i.e., cases where the speaker’s knowledge does not have clear boundaries).
- Negative modifiers ‘at most’/‘less than’.
- Interactions between superlative modifiers and modals.
- Cognitive validity of ISAL.