Inquisitive semantics

ESSLLI 2015

Lecture notes

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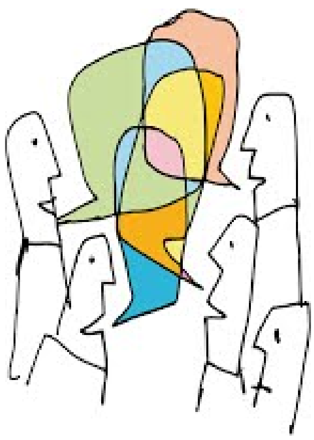

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About this document

These are lecture notes for a course on inquisitive semantics at ESSLLI 2015.

An updated version of these lecture notes is available at:

www.illc.uva.nl/inquisitivesemantics/overview/the-basic-framework

Many of the papers referred to in these lecture notes can be accessed through:

www.illc.uva.nl/inquisitivesemantics/papers

Finally, to become familiar with the framework presented here, one may want to try out the computational tools available at:

www.illc.uva.nl/inquisitivesemantics/resources

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Chapter 1

Introduction

Inquisitive semantics is a new semantic framework mainly intended for the analysis of linguistic information exchange. Information exchange can be seen as a process of raising and resolving issues. Inquisitive semantics provides a new formal notion of issues, which makes it possible to model various concepts that are crucial for the analysis of linguistic information exchange in a more refined and more principled way than has been possible in previous frameworks. In particular:

1. The **meaning** of both declarative and interrogative sentences can be represented in an integrated way, capturing not only the information that such sentences may convey but also the issues that they may raise;

2. Similarly, **conversational contexts** can be modeled as encompassing not just the information that has been established in the conversation so far, but also the issues that have been brought up;

3. And finally, it becomes possible to formally represent a broader range of **propositional attitudes** that are relevant for information exchange: besides the familiar information-directed attitudes like *knowing* and *believing*, issue-directed attitudes like *wondering* can be captured as well.

These lecture notes provide a detailed exposition of the most basic features of inquisitive semantics, and demonstrate some of the advantages that the framework has with respect to previously proposed ways of representing meanings, conversational contexts, and propositional attitudes.

This introductory chapter will proceed as follows. Section 1.1 argues in some detail why a framework like inquisitive semantics is needed for a satisfactory analysis of information exchange, and Section 1.2 provides a global outline of the remaining chapters.

1.1 Motivation

The most basic question that we should address is why a formal notion of issues is needed at all in order to construct a suitable framework for the analysis of linguistic information exchange. This will be done in Section 1.1.1.

A second point that we want to make is that the analysis of linguistic information exchange does not just require a notion of meaning for declaratives and another notion of meaning for interrogatives side by side, but rather a single notion of meaning that is general enough to capture both the information that sentences may convey and the issues that they may raise in an integrated way. Several reasons for this will be discussed in Section 1.1.2.

1.1.1 Why do we need a formal notion of issues?

There are several reasons why a formal notion of issues is needed for the analysis of linguistic information exchange, and each of these is related to one of the three aspects of information exchange listed above: some arise from the need for a suitable notion of meaning, some from the need for a suitable model of conversational contexts, and yet others from the need for a sufficiently refined representation of the mental states of conversational participants. We will discuss each in turn.

Reason 1: To capture the meaning of interrogative sentences

Standardly, the proposition expressed by a sentence is construed as a set of possible worlds, those worlds that are compatible with the information that the sentence conveys (as per the conventions of the language; additional information may be conveyed pragmatically when the sentence is uttered). This works well for declarative sentences like (1) below, whose conversational role is indeed to provide information.

(1) Bill is coming.

But information exchange typically does not just consist in a sequence of declarative statements. An equally important role is played by interrogative sentences, whose main conversational role is to raise issues.

Can the meaning of interrogative sentences be captured in terms of the standard notion of propositions? Consider the example in (2), a polar interrogative:

(2) Is Bill coming?

Frege (1918) famously proposed that the interrogative in (2) and the declarative in (1) can indeed be taken to have the same semantic content:

> "An interrogative sentence and an indicative one contain the same thought; but the indicative contains something else as well, namely, the assertion. The interrogative sentence contains something more too, namely a request. Therefore two things must be distinguished in an indicative sentence: the content, which it has in common with the corresponding sentence-question, and the assertion."
>
> (Frege, 1918, p.294)[1]

So the idea is that declaratives and interrogatives have the same semantic content—a proposition—but come with a different force—either assertion or request. This idea has been quite prominent in the literature, especially in *speech act theory* (Searle, 1969; Vanderveken, 1990).[2] However, as noted by Frege himself, it is limited in scope. It may work for plain polar interrogatives, but not for many other kinds of interrogatives, like (3)-(5):

(3) Is Bill coming, or Sue?

(4) Is Bill coming, or not?

(5) Who is coming?

Moreover, as has been argued extensively in the more recent literature (see especially Groenendijk and Stokhof, 1997), even the idea that a plain polar interrogative has the same content as the corresponding declarative is problematic. In particular, when applied to *embedded* cases it is not compatible with the principle of *compositionality*. To see this, compare the following two examples, which contain embedded variants of the declarative in (1) and the polar interrogative in (2), respectively:

(6) John knows that Bill is coming.

(7) John knows whether Bill is coming.

The two sentences as a whole clearly differ in content. But then, the principle of compositionality dictates that the embedded clauses must differ in content as well.

Thus, the meaning of interrogative sentences cannot be suitably captured in terms of the standard notion of propositional content. Rather, what we need for the semantic analysis of interrogatives is a notion of meaning that directly captures the issues that they raise.[3]

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[1] The page reference is to the translated version, Frege (1956).

[2] See also recent work on questions in dynamic epistemic logic (van Benthem and Minică, 2012).

[3] There is an extensive literature on the semantics of interrogatives (Hamblin, 1973; Karttunen, 1977; Groenendijk and Stokhof, 1984, among many others), and inquisitive semantics strongly builds on the insights that have emerged from this work. A detailed comparison will

Reason 2: To model conversational contexts

It has been argued extensively in the literature that conversational contexts have to be modeled in a way that does not only take account of the information that has been established in the conversation so far, but also of the issues that have been brought up, often referred to as the *quesions under discussion* (Carlson, 1983; Groenendijk and Stokhof, 1984; van Kuppevelt, 1995; Ginzburg, 1996; Roberts, 1996; Büring, 2003; Beaver and Clark, 2008; Tonhauser *et al.*, 2013, among others). We will briefly discuss two reasons why this is important. First, it is needed to develop a formal theory of pragmatic reasoning and the conversational implicatures that result from such reasoning. And second, it is needed for a theory of information structural phenomena like topic and focus marking. Let us first consider pragmatic reasoning.

A key notion in Gricean pragmatic reasoning is the notion of *relevance*. When is a contribution to a conversation relevant for the purposes at hand? One natural answer is that a contribution is relevant just in case it addresses one of the issues under consideration. Even if the issues under consideration only partially characterize what is 'relevant' in a broader sense, this partial characterization is crucial for a formal theory of conversational implicatures. For, the issues under consideration influence which conversational implicatures arise. To see this, consider the following example:

(8) A: What did you do this morning?
B: I read the newspaper. $\rightsquigarrow$ B did not do the laundry

(9) A: What did you read this morning?
B: I read the newspaper. $\not\rightsquigarrow$ B did not do the laundry

B's utterance is exactly the same in both cases, but the issue that it addresses is different. As a result, in (8), where the question under discussion is what B *did* this morning, there is a conversational implicature that B did not do anything besides reading the newspaper, i.e., that he did not do the laundry for instance. On the other hand, in (9), where the question under discussion is what B *read* this morning, there is a weaker conversational implicature, to the effect that B did not read anything besides the newspaper. This does not imply that he did not do other things, such as the laundry. Thus, we see that pragmatic reasoning is sensitive to the issues that are at play in the context of utterance.

Now let us illustrate the importance of contextual issues for information structural phenomena like focus and topic marking. We will concentrate on focus marking. Languages generally have grammatical ways to signal which part of a sentence is in focus and which part is backgrounded. In English,

be provided in Chapter 5.

the focus/background distinction is marked intonationally: focused constituents receive prominent pitch accents, while backgrounded constituents do not. In other languages, focus is sometimes marked by means of special particles or by means of word order.

Which constituents should be marked as being in focus and which should be marked as being backgrounded is determined, at least partly, by the issue that is being addressed. To see this, consider the following examples, where capitalization is used to indicate focus marking by means of prominent pitch accents.

(10) A: Who did Alf rescue?
B: Alf rescued BEA. / #ALF rescued Bea.

(11) A: Who rescued Bea?
B: ALF rescued Bea. / #Alf rescued BEA.

If the question is who Alf rescued, as in (10), then the response that Alf rescued Bea must be pronounced with a prominent pitch accent on *Bea*. Placing a pitch accent on *Alf* instead results in infelicity. On the other hand, if the question is who rescued Bea, as in (11), then the same response, i.e., that Alf rescued Bea, must be pronounced with an prominent pitch accent on *Alf* rather than *Bea*. Thus, we see that focus marking, just like pragmatic reasoning, is sensitive to the issue under discussion.[4]

Reason 3: To model issue-directed propositional attitudes and capture the meaning of verbs that report such attitudes

In order to understand linguistic information exchange, it is important to have a way of representing the information that is available to the agents participating in the exchange, as well as the information that they would like to obtain. In other words, we need to be able to model what the agents *know* or *believe* at any given time, and also what they would like to know, i.e., what they *wonder* about. Knowledge and belief are information-directed propositional attitudes; wondering is an issue-related propositional attitude. The simplest and most common way to model the knowledge and beliefs of an agent is as a set of possible worlds, namely those worlds that are compatible with what the agent knows or believes. Such a set of worlds is thought of as representing the agent's *information state*. Similarly, in order to capture what an agent wonders about, we need a representation of her *inquisitive state*. For such a representation, we again need a formal notion of issues.

[4] Besides pragmatic reasoning and information structural phenomena like topic and focus marking, it has been argued that a model of conversational contexts that comprises the issues that have been raised is also needed for a suitable analysis of discourse particles (see, e.g. Rojas-Esponda, 2013) and presupposition projection (e.g., Tonhauser *et al.*, 2013).

Moreover, turning back to language, just like there are verbs like *know* and *believe* that describe the information state of an agent, see (12) below, there are also verbs like *wonder* and *be curious* that describe the inquisitive state of an agent, as in (13).

(12) John knows that Bill is coming.

(13) John wonders who is coming.

Clearly, in order to analyze the meaning of verbs like *wonder* we do not only need a suitable representation of the meaning of the interrogative clause that the verb takes as its complement (here, *who is coming*), but also a suitable representation of the inquisitive state of the subject of the verb (here, *John*).

1.1.2 Why should we pursue an integrated notion of meaning?

We have seen that the semantic analysis of interrogative sentences requires a notion of meaning that is different from the one that is most commonly assumed when analyzing declarative sentences. A follow-up question that naturally arises, then, is whether declaratives and interrogatives could each be analyzed in isolation, using a different notion of meaning for each type of sentence, or whether a more integrated approach is needed, with a single notion of meaning that is general enough to deal with both declaratives and interrogatives.

We will give several reasons why neither declaratives nor interrogatives can be fully understood in isolation, which makes an integrated approach necessary.

Reason 1: Mutual embedding

Declarative and interrogative sentences can be embedded into one another, as exemplified in (14)-(16).

(14) Bill asked me who won. embedded interrogative

(15) Who told you that Jane won? embedded declarative

(16) Bill asked me who told you that Jane won. two-level embedding

So the meaning of a declarative sentence is sometimes partly determined by the meaning of an embedded interrogative sentence, and vice versa. Clearly, then, a complete semantic account of declaratives cannot be achieved without getting a handle on interrogatives, and the other way around, a complete semantic account of interrogatives is impossible without a treatment of declaratives. Thus, the two have to be analyzed hand in hand; considering them in isolation is bound to lead to incomplete theories.

Reason 2: Interpretational dependencies

As illustrated in (17) and (18), the interpretation of a declarative sentence sometimes partly depends on the issue raised by a preceding interrogative.[5]

(17) A: What did you do this morning?
B: I only read the newspaper. $\rightsquigarrow$ B did not do the laundry

(18) A: What did you read this morning?
B: I only read the newspaper. $\not\rightsquigarrow$ B did not do the laundry

If the question is what you *did* this morning, as in (17), then the declarative statement that you only read the newspaper implies that you did not do other things, like the laundry. On the other hand, if the question is what you *read* this morning, as in (18), then the same declarative statement just implies that you did not read anything else, which leaves open whether you did anything besides reading, such as the laundry. Thus, not just the pragmatic implicatures that a declarative statement may induce, but even its truth-conditional content can depend on the issue that is addressed, which again means that analyzing declaratives in isolation, without taking interrogatives into account as well, is bound to lead to an incomplete theory.

Reason 3: Common building blocks

Declaratives and interrogatives are to a large extent built up from the same lexical, morphological, and intonational elements. Clearly, we would like to have a uniform semantic account of these elements, i.e., an account that captures their semantic contribution in full generality, rather than two separate accounts, one capturing their semantic contribution when they are part of declarative sentences and the other when they are part of interrogative sentences.

To make this concrete, consider the following two examples, a declarative and an interrogative which are built up from exactly the same lexical items and also exhibit the same intonation pattern (we use ↑ and ↓ to indicate rising and falling intonation, respectively).

(19) Luca is from Italy↑ or from Spain↓.

(20) Is Luca from Italy↑ or from Spain↓?

Note that both sentences contain the disjunction word *or*. In declaratives, *or* is generally taken to yield the *union* of the semantic values of the two disjuncts. In (19), each disjuct expresses a property, which is standardly represented as a set of individuals—those individuals that satisfy the property. Thus the semantic

[5] The difference between examples (17)-(18) below and examples (8)-(9) discussed above is that B's response in (17)-(18) contains the particle *only*.

value of the first disjunct is the set of all individuals from Italy, and the semantic value of the second disjunct is the set of all individuals from Spain. The union of these two sets is the set of all individuals that are either from Italy or from Spain. Sentence (19) conveys that Luca is one of the individuals in this set.

This seems a reasonable account of *or* in declaratives. But what is the role of *or* in interrogatives? Ultimately, we would like to have an account of *or* that is general enough to capture its semantic contribution in both declaratives and interrogatives in a uniform way. And similarly for other lexical and morphological elements, as well as intonational features that play a role in both types of sentences.

This can only be achieved if we operate with a notion of meaning, both at the sentential level and at the level of subsentential constituents, that encompasses both informative and inquisitive content. The meaning of a complete sentence should capture both the information that the sentence conveys and the issue that it raises (where of course, one of these, or both, may be trivial), and the meaning of any subsentential constituent should capture the contribution that this constituent makes both to the information conveyed and to the issue raised by the sentence that it is part of.

Reason 4: Entailment

Entailment is normally thought of as a logical relation between declarative sentences. One sentence is taken to entail another if the first conveys at least as much information as the second. This logical relation plays a central role in the standard logical framework for natural language semantics. For one thing, predictions about entailment constitute one of the primary criteria for empirical success of a semantic theory. That is, a theory is assessed by testing its predictions about entailment. But besides this, entailment is important in various other respects as well. For instance, it plays a crucial role in the derivation of quantity implicatures, which involves comparing the sentence that a speaker actually uttered with other sentences that the speaker *could have* uttered instead. This comparison is done in terms of informative strength, which is captured by entailment (Grice, 1975, and much subsequent work). Similarly, entailment is needed to formulate interpretive principles like the Strongest Meaning Hypothesis, which has been argued to play a crucial role in the interpretation of plural predication (Dalrymple *et al.*, 1998; Winter, 2001). Finally, it has been used to characterize the distribution of positive and negative polarity items in terms of upward and downward entailing environments (e.g., Ladusaw, 1980; Kadmon and Landman, 1993).

Clearly, we would like our theories of quantity implicatures, plural predication, polarity items, etcetera, to apply in a uniform way to declarative and interrogative constructions. However, since the standard notion of entailment compares two sentences in terms of their informative, truth-conditional content (and subsentential expressions in terms of their contribution to the informative content of the sentences that they are part of), it does not suitably apply to interrogatives. For this reason, the scope of entailment-based theories such as the ones just mentioned is currently restricted to declaratives.

What we need, then, is a notion of entailment that is general enough to apply to both declaratives and interrogatives in a uniform way. Such a notion must be sensitive to both informative and inquisitive strength. An obvious prerequisite for this is that we operate with a notion of meaning that encompasses both informative and inquisitive content.

Reason 5: Logical operations

Two declarative sentences can be combined by means of conjunction and disjunction.

(21) Peter rented a car and Mary booked a hotel.

(22) Peter rented a car or he borrowed one.

This does not only hold for root declaratives, but also for embedded ones.

(23) Simon believes that Peter rented a car and that Mary booked a hotel.

(24) Simon believes that Peter rented a car or that he borrowed one.

This is also true for interrogatives, both embedded and unembedded ones.[6]

(25) Where can we rent a car, and which hotel should we take?

(26) Where can we rent a car, or who might have one that we could borrow?

(27) Peter is investigating where can we rent a car and which hotel we should take.

(28) Peter is investigating where can we rent a car or who might have one that we could borrow.

These parallels between declaratives and interrogatives do not only exist in English, but in many other languages as well: words that are used to conjoin declaratives are also used to conjoin interrogatives, and words that are used to disjoin declaratives are also used to disjoin interrogatives.

[6]While the possibility of conjoining interrogative sentences is uncontroversial, the possibility of disjoining interrogatives has been disputed by Szabolcsi (1997, 2015a) and Krifka (2001b). In Section 5.2 we will examine Szabolcsi's argument in some detail. On the basis of examples such as (26) and (28), we will argue that disjoining interrogatives is in principle possible, and that the meaning of the resulting disjunction is correctly derived by applying the inquisitive entry for disjunction to the meanings of the two interrogative disjuncts.

What we would like to have, then, is an account of conjunction and disjunction that does not just apply to declaratives, but that is general enough to apply to both declaratives and interrogatives in a uniform way. Such an account again requires an integrated treatment of declaratives and interrogatives, employing a notion of meaning that embodies both informative and inquisitive content.

Besides conjunction and disjunction, another logical operation that can be performed both on declaratives and on interrogatives is *conditionalization*, as exemplified in (29) and (30).

(29) If Bill asks Mary out, she will accept.

(30) If Bill asks Mary out, will she accept?

This calls for a uniform account of conditionals, one that applies uniformly regardless of whether the consequent is a declarative or an interrogative sentence. Again, such an account can only be provided within a semantic framework which encompasses both informative and inquisitive content.

1.2 Outline

The remaining chapters of these lecture notes broadly fall into two parts. The first part, spanning Chapters 2-4, provides a detailed exposition of the basic inquisitive semantics framework. The second part, consisting of Chapters 5-7, discusses several applications of the framework in the anaysis of linguistic information exchange, and compares the approach to previous work.

In a bit more detail: Chapter 2 introduces the new notions of issues, conversational contexts, meanings, and propositions that form the heart of inquisitive semantics; Chapter 3 identifies the basic operations that can be performed on inquisitive propositions; and Chapter 4 presents an inquisitive semantics for the language of first-order logic.

Then, turning to the second part, Chapter 5 discusses the advantages of inquisitive semantics as a framework for the semantic analysis of interrogatives in comparison with previous work on interrogatives; Chapter 6 provides a concrete illustration of the benefits of treating declaratives and interrogatives in a integrated way; and Chapter 7 discusses the representation of information-directed and issue-directed propositional attitudes, as well as the semantics of verbs like *know* and *wonder* which are used to report such attitudes.

Finally, Chapter 8 concludes with a brief summary, Appendix A provides a list of publications, manuscripts, and teaching materials that have served as the main sources for these lecture notes, and Appendix B provides some pointers to work that further extends or applies the framework to be presented here.

1.3 Exercises

Exercise 1.1. In Section 1.1.1 we discussed several reasons why a formal notion of issues is needed for the analysis of linguistic information exchange. One other area where such a notion is arguably needed is in logical frameworks that are designed to reason about *dependencies* (cf., Väänänen, 2007; Yang, 2014; Ciardelli, 2014a). Why are dependencies and issues intrinsically connected? Can you think of other areas where a formal notion of issues is required?

Exercise 1.2. In Section 1.1.2 we argued that declaratives and interrogatives cannot be fully understood in isolation. Rather, they need to be analyzed in an integrated way, operating with a notion of meaning that is general enough to capture both informative and inquisitive content. One domain not yet mentioned above where such an integrated notion of meaning seems needed is that of *discourse particles*. Consider the following examples from Iatridou and Tatevosov (2014) involving the particle *even* in interrogative sentences:

(31) A: Let's go to Oleana's for dinner.
B: Where is that even?

(32) A: I want to study the Penutian language Tunica.
B: Where is that even spoken?

The particle *even* can also be used in declaratives, as in (33).

(33) Even Lev came to the party.

We would of course like to have a uniform account of *even* that applies both to occurrences in declaratives and to occurrences in interrogatives. Can you think of one?

Can you perhaps think of yet other reasons why it is necessary to pursue an integrated account of declaratives and interrogatives?

Chapter 2

Issues, contexts, meanings, and propositions

There are two dominant views on meaning in formal semantics, a *static* one and a *dynamic* one. On the static view, the meaning of a sentence is identified with its *truth conditions*. In order to know the meaning of a sentence one has to know, for any possible world, whether the given sentence is true in that world.

By contrast, on the dynamic view the meaning of a sentence is seen as something that determines the intended effect of an utterance of that sentence in a conversation. That is, when a speaker utters a sentence in a certain context, she intends her utterance to change the context in a particular way, and the meaning of the sentence determines this intended effect. Thus, the meaning of a sentence is conceived of as its *context change potential*, which is modeled formally as a function that maps every discourse context to a new one.

This general dynamic picture of meaning can be made more precise in several ways, depending on what exactly we take a discourse context to be. The simplest and most common option is to think of a discourse context as the body of information that has been established in the discourse so far. This body of information is usually referred to as the *common ground* of the conversation, and the simplest way to model it is as a set of possible worlds—those worlds that are compatible with the information that is publicly shared among the conversational participants.[1]

If the discourse context is identified with the publicly established information, then the context change potential of a sentence boils down to its *information change potential*. Formally, the meaning of a sentence can then be identified with a function that maps information states—sets of possible worlds—to other information states.

[1] Sometimes a distinction is made between the *common ground* of a conversation and the *context set* (Stalnaker, 1978). The common ground is then construed as the set of pieces of information that are publicly shared among the conversational participants, and the context set as the set of possible worlds that are compatible with all these pieces of information. For our purposes, it will not be necessary to keep track of the individual pieces of information that are publicly shared, so we simply construe the common ground as the set of possible worlds that are compatible with all these pieces of information.

Even though under the static view the meaning of a sentence does not inherently determine its conversational effects, a specific connection is usually assumed between the truth-conditions of a sentence and the intended effect of uttering that sentence in a conversation. Namely, it is assumed that the common ground of the conversation is restricted to precisely those worlds that satisfy the truth-conditions of the sentence, i.e., to those worlds in which the sentence is true (Stalnaker, 1978). If for any sentence φ and any context c we let $|\varphi|$ denote the set of all possible worlds where φ is true, and write $c[\varphi]$ for the context that results from uttering φ in c, then the assumed connection between the truth conditions of φ and its conversational effects can be succinctly written as follows:

$$c[\varphi] = c \cap |\varphi|$$

Under these assumptions, the truth-conditions of a sentence completely determine the sentence's context change potential. That is, as long as we restrict ourselves to purely informative sentences, identify discourse contexts with the information that has been established in the discourse so far, and construe the effect of an utterance as restricting the discourse context to those worlds where the uttered sentence is true, the static and the dynamic view yield essentially the same results.

However, under the dynamic view it is much more straightforward to lift one or more of these restrictive assumptions, in order to generalize the overall picture. In particular, identifying a discourse context with the information that has been established so far is only one particular way of spelling out the notion of a discourse context. Of course, the general conception of meaning as context change potential is in principle compatible with richer notions of discourse contexts as well. And in order to analyze many types of discourse, involving more than just purely informative statements, such richer notions are indeed required.

We will focus here on a very basic type of discourse, namely one in which a number of participants *exchange information* by raising and resolving issues. In order to analyze this type of conversation, discourse contexts should not only embody the information that has been established so far, but also the issues that have been raised so far. And similarly, the meaning of a sentence should not only embody its potential to provide new information, but also its potential to raise new issues.

In this chapter we will introduce a formal notion of issues which will allow us to articulate such richer notions of context and meaning. While we will take the dynamic view on meaning as our point of departure, we will eventually also establish a suitably generalized static notion of meaning.

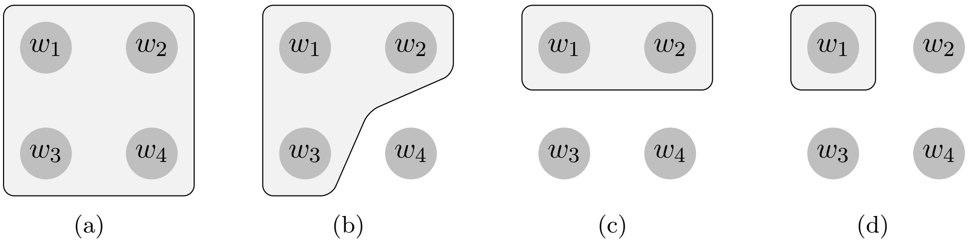


Figure 2.1: Information states.

2.1 Information and issues

We would like to obtain a notion of discourse contexts that embodies both the information that has been established so far and the issues that have been raised so far. We already know how we can model the information established so far, namely as a set of possible worlds, namely those worlds that are compatible with the established information. We will use W to denote the entire logical space, i.e., the set of all possible worlds.

Definition 2.1. [Information states]
An information state s is a set of possible worlds, i.e., $s \subseteq W$.

We will often refer to information states simply as *states*. Figure 2.1 depicts some examples of information states in a logical space consisting of just four possible worlds: w_1, w_2, w_3, w_4. Intuitively, an information state can be thought of as locating the actual world within a certain region of the logical space. For instance, the state in Figure 2.1(d) contains the information that the actual world is located in the upper left corner of the logical space, while the state in Figure 2.1(c) contains the information that the actual world is located in the upper half of the logical space.

If s and t are two information states and $t \subseteq s$, then t contains at least as much information as s; it locates the actual world with at least as much precision. In this case, we call t an *enhancement* of s.

Definition 2.2. [Enhancements]
A state t is called an enhancement of s just in case $t \subseteq s$.

Note that we do not require that t is *strictly* contained in s, i.e., that it contains strictly more information than s. If $t = s$, then we call t a *trivial* enhancement of s. If $t \subset s$, then we say that t is a *proper* enhancement of s.

The four information states depicted in Figure 2.1 are arranged from left to right according to the enhancement order. The state in Figure 2.1(b) is an enhancement of the state in Figure 2.1(a), and so on. The state consisting of all possible worlds, W, depicted in Figure 2.1(a), is the least informed of all information states: any possible world is still taken to be a candidate for the actual world, which means that we have no clue at all what the actual world is like. This state is therefore referred to as the *ignorant state*. Every other state is an enhancement of it.

At the other far end of the enhancement order is the empty state, $\emptyset$. This is an enhancement of any other state. It is a state in which all possible worlds have been discarded as candidates for the actual world, that is, the available information has become inconsistent. It is therefore referred to as the *inconsistent state*.

Now, besides the information that has been established in the conversation so far, we also want to take account of the issues that have been brought up. How should issues be represented formally? Our proposal is to characterise issues in terms of what information it takes to resolve them. That is, an issue is identified with a set of information states: those information states that contain enough information to resolve the issue.

We assume that every issue can be resolved in at least one way, which means that issues are identified with *non-empty* sets of information states. Moreover, a set of information states can only suitably embody an issue if it is *downward closed*. That is, if s is an information state in an issue I, then any $t \subseteq s$ should be in I as well. After all, if $s \in I$, then s contains enough information to resolve I; but then any $t \subseteq s$ clearly also contains enough information to resolve I, and should therefore be included in I as well. Thus, issues are defined as non-empty, downward closed sets of information states.[2]

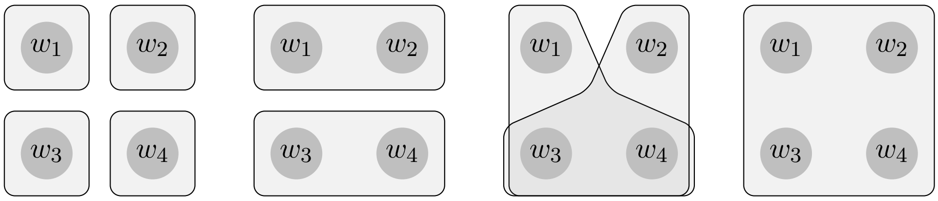
Definition 2.3. [Issues]
An issue is a non-empty, downward closed set of information states.

Definition 2.4. [Resolving an issue]
We say that an information state s resolves an issue I just in case $s \in I$.

If an information state s resolves an issue I, we will also say that I is *settled* in s. Moreover, if the information available in a state s is both sufficient and necessary to ensure that an issue I can be resolved truthfully, then we say that I is an issue *over* s. This holds just in case I forms a *cover* of s, i.e., $s = \bigcup I$.

To see this, first suppose that $s \not\subseteq \bigcup I$, i.e., that there is a world w in s that is not included in any $t \in I$. Then the information available in s does not preclude w from being the actual world. But if w is indeed the actual world,

[2]Notice that this means that the inconsistent information state, $\emptyset$, is an element of every issue. Thus, it is assumed that every issue is resolved in the inconsistent information state. This limit case may be regarded as a generalization of the usual *ex falso quodlibet* principle to issues.



(a) (b) (c) (d)

Figure 2.2: Issues over the state $\{w1, w2, w3, w4\}$.

then it would be impossible to resolve I without discarding the actual world. Thus, in this case s does not contain sufficient information to ensure that I can be resolved truthfully.

Now suppose that $\bigcup I \not\subseteq s$, i.e., that there is a world w in $\bigcup I$ that is not included in s. In this case, the information state $s \cup \{w\}$, which contains strictly less information than s itself, already ensures that I can be resolved truthfully. In other words, in this case s contains more information than necessary to ensure that I can be resolved truthfully.

Definition 2.5. [An issue over a state]
Let I be an issue and s an information state. Then we say that I is an issue over s if and only if $\bigcup I = s$.

Notice that an issue I over a state s may contain s itself. In this case resolving I does not require any information beyond the information that is already available in s. If so, we call I a *trivial* issue over s. Downward closure implies that for any state s there is precisely one trivial issue over s, namely the issue consisting of *all* enhancements of s, i.e., the powerset of s, which we denote as $\wp(s)$. On the other hand, if $s \notin I$, then in order to settle I further information is required, that is, a proper enhancement of s must be established. In this case we call I a *proper* issue over s.

Two issues over a state s can be compared in terms of what it takes for them to be settled: one issue I is at least as inquisitive as another issue J just in case any state that settles I also settles J. Since an issue is identified with the set of states that settle it, this order on issues just amounts to inclusion.

Definition 2.6. [Ordering issues]
Given two issues I, J over a state s, we say that I is at least as inquisitive as J just in case $I \subseteq J$.

Among the issues over a state s there is always a *least* and a *most* inquisitive one. The least inquisitive issue over s is the trivial issue $\wp(s)$ whose resolution, as we saw, requires *no* information beyond the information already available in s. The most inquisitive issue over s is $\{\{w\} \mid w \in s\} \cup \{\emptyset\}$, which can only be settled consistently by providing a complete description of what the actual world is like.

Figure 2.2 shows some issues over the information state $s = \{w_1, w_2, w_3, w_4\}$. In order to keep the figures neat, only the maximal elements of these issues are displayed. The issue depicted in subfigure (a) is the most inquisitive issue over s, which can only be settled consistently by specifying precisely which world is the actual one. The issue depicted in subfigure (b) can be settled either by establishing that the actual world is an element of the set $\{w_1, w_2\}$, or by establishing that it is an element of $\{w_3, w_4\}$. The issue depicted in subfigure (c) can be settled either by establishing that the actual world is an element of $\{w_1, w_3, w_4\}$, or by establishing that it is an element of $\{w_2, w_3, w_4\}$. Finally, subfigure (d) displays the trivial issue over s, which is already settled in s itself. Both (b) and (c) are less inquisitive than (a) and more inquisitive than (d), while neither of them is more inquisitive than the other.

2.2 Contexts

The most straightforward way to proceed would be to define a discourse context c as a pair $\langle \mathsf{info}_c, \mathsf{issues}_c \rangle$, where info_c is an information state and issues_c a set of issues over info_c, representing the information established and the issues raised so far, respectively. The initial discourse context would then be $\langle W, \emptyset \rangle$, consisting of the trivial information state, which does not rule out any world, and the empty set of issues.

This would indeed be a suitable notion of discourse contexts. However, for our current purposes, it will be convenient to simplify this notion somewhat. We will do this in two steps. First, rather than thinking of a discourse context c as a pair $\langle \mathsf{info}_c, \mathsf{issues}_c \rangle$ where issues_c is a *set* of issues over info_c, we will think of a discourse context as a pair $\langle \mathsf{info}_c, \mathsf{issue}_c \rangle$ where issue_c is a *single* issue over info_c. This simplification is justified by the observation that any set of issues Ω over a state s can be merged into a single issue:

$$I_\Omega := \{t \subseteq s \mid t \in J \text{ for every } J \in \Omega\}$$

which is settled precisely by those enhancements $t \subseteq s$ that settle all issues in Ω. Notice that if $\Omega \neq \emptyset$ the issue I_Ω amounts to the intersection $\bigcap \Omega$ of all issues in Ω, whereas if $\Omega = \emptyset$, I_Ω amounts to the trivial issue $\wp(s)$ over s.[3,4]

So we can think of a discourse context c as a pair $\langle \mathsf{info}_c, \mathsf{issue}_c \rangle$, where info_c is an information state, and issue_c a single issue over info_c, and we can take the initial context to be the pair $\langle W, \wp(W) \rangle$, consisting of the trivial information state, which does not rule out any world, and the trivial issue over this state, which is settled even if no information is present yet. But this representation can be simplified even further. After all, since issue_c is an issue over info_c, it must form a *cover* of info_c. So we always have that $\mathsf{info}_c = \bigcup \mathsf{issue}_c$. That is, info_c can always be retrieved from issue_c. But then info_c can just as well be left out of the representation of c. Thus, a discourse context c can simply be represented as an issue over some information state s, i.e., a non-empty, downward closed set of enhancements of s that together form a cover of s. This information state s is then understood to embody the information available in c.

Definition 2.7. [Discourse contexts]

- A discourse context c is a non-empty, downward closed set of states.
- The set of all discourse contexts will be denoted by $\mathcal{C}$.

Definition 2.8. [The information available in a discourse context]

- For any discourse context c: $\mathsf{info}_c := \bigcup c$

We have moved from the commonplace notion of a discourse context as a set of possible worlds—representing the information established so far—to a richer notion of discourse contexts as non-empty, downward closed sets of information states—representing both the information established so far and the issues raised so far. With this enriched notion of discourse contexts, we are in principle ready to specify a notion of *meaning* that embodies both informative and inquisitive content as well. However, before turning to meanings, it will be useful to briefly identify some special properties that discourse contexts may have and some operations that can be performed on them.

First of all, we can make a distinction between *informed* and *ignorant* discourse contexts, ones in which some information has been established and ones in which no information has been established yet, respectively.

[3] Recall from footnote 1 that we are already assuming a similar simplification concerning the informative component of a discourse context: we do not keep track of all the separate pieces of information that have been established in the discourse so far, but rather of the set of worlds that are compatible with all these pieces of information (formally, this is again the *intersection* of all the separately established pieces of information). For certain purposes it may be convenient, or even necessary, to keep track of all the separate pieces of information and/or issues that have been established/raised in a discourse. However, for our current purposes, this would only add unnecessary complexity.

[4] Notice that I_Ω is guaranteed to be an issue in the sense of Definition 2.3. In particular, it is guaranteed to be non-empty, since it always contains the inconsistent information state.

Definition 2.9. [Ignorant and informed discourse contexts]

- A discourse context c is ignorant iff $\mathsf{info}_c = W$.
- A discourse context c is informed iff $\mathsf{info}_c \neq W$.

Similarly, we can make a distinction between *indifferent* and *inquisitive* discourse contexts. A discourse context c is indifferent iff the information that has been established so far settles all the issues that have been raised, i.e., $\mathsf{info}_c \in c$. Otherwise, i.e., if there are unresolved issues, then c is inquisitive.

Definition 2.10. [Indifferent and inquisitive discourse contexts]

- A discourse context c is indifferent iff $\mathsf{info}_c \in c$.
- A discourse context c is inquisitive iff $\mathsf{info}_c \notin c$.

There are two special discourse contexts: the *initial* and the *absurd* discourse context. The initial context, $c_\top$, is the only context that is both ignorant and indifferent. The absurd context, $c_\bot$, is one in which the established information is inconsistent and therefore rules out all possible worlds.

Definition 2.11. [The initial and the absurd discourse context]

- $c_\top := \wp(W)$
- $c_\bot := \{\emptyset\}$

Two discourse contexts can be compared in terms of the information that has been established or in terms of the issues that have been raised. One context c' is at least as informed as another context c if and only if $\mathsf{info}_{c'} \subseteq \mathsf{info}_c$.

Definition 2.12. [Informative order on discourse contexts]
For any $c, c' \in \mathcal{C}$:

- $c' \geq_{\mathsf{info}} c$ iff $\mathsf{info}_{c'} \subseteq \mathsf{info}_c$

Similarly, for any two discourse contexts c and c' that are equally informed, i.e., $\mathsf{info}_c = \mathsf{info}_{c'}$, we can say that c' is at least as inquisitive as c if and only if every state that settles all the issues that have been raised in c' also settles all the issues that have been raised in c, i.e., if and only if $c' \subseteq c$.

Definition 2.13. [Inquisitive order on discourse contexts]
For any $c, c' \in \mathcal{C}$ such that $\mathsf{info}_c = \mathsf{info}_{c'}$:

- $c' \geq_{\mathsf{inq}} c$ iff $c' \subseteq c$

Now, we would like to combine the two orders and say that c' is an *extension* of c just in case c' is both at least as informed and at least as inquisitive as c. But there is a subtlety here. Namely, if c' is strictly more informed than c, then c' and c cannot be compared directly in terms of inquisitiveness. Thus, what we require is that c' be at least as informed as c and moreover, that c' be at least as inquisitive as the *restriction* of c to $\mathsf{info}_{c'}$.

Definition 2.14. [Restriction]
If $s \subseteq \mathsf{info}_c$, the *restriction* of c to s is the context $c \upharpoonright s$ defined by:

$$c \upharpoonright s := \{t \in c \mid t \subseteq s\}$$

Intuitively, $c \upharpoonright s$ is a context that contains the information available in c plus the information embodied by s, and inherits the issues present in c to the extent that they are not resolved by the information embodied by s.

This notion of restricting a context to a certain information state allows us to define the extension relation between discourse contexts. Namely, we say that c' is an extension of c just in case (i) c' is at least as informed as c, i.e., $\mathsf{info}_{c'} \subseteq \mathsf{info}_c$; and (ii) c' is at least as inquisitive as the restriction of c to the information already available in c', i.e., $c' \subseteq c \upharpoonright \mathsf{info}_{c'}$. Now, it can be shown that these two conditions are satisfied if and only if $c' \subseteq c$. Thus, the extension relation between discourse contexts simply amounts to inclusion.

Definition 2.15. [Extending discourse contexts]
For any $c, c' \in \mathcal{C}$:

- c' is an extension of c, $c' \geq c$, iff $c' \subseteq c$

The extension relation forms a *partial order* on $\mathcal{C}$, i.e., it is a reflexive, transitive, and anti-symmetric relation, and $c_\top$ and $c_\bot$ constitute the extremal elements of this partial order: $c_\bot$ is an extension of every discourse context, and every discourse context is in turn an extension of $c_\top$.

Fact 2.16. [Partial order and extrema]

- $\geq$ forms a partial order on $\mathcal{C}$
- For every $c \in \mathcal{C}$: $c_\bot \geq c \geq c_\top$

Finally, the way we have construed discourse contexts allows for a natural definition of the *merge* of two contexts, which we will denote as $c \oplus c'$, and the *difference* between them, which we will denote as $c \oslash c'$. Both of these notions will be useful later on. The merge of two discourse contexts can simply be defined as their intersection.

Definition 2.17. [Merging two discourse contexts]
For any $c, c' \in \mathcal{C}$:

- $c \oplus c' \ := \ c \cap c'$.

It can be shown that $c \oplus c'$ is always a proper discourse context, i.e., a non-empty downward closed set of states.

Fact 2.18. [Merging yields a new discourse context]
For any $c, c' \in \mathcal{C}$, $c \oplus c'$ is also in $\mathcal{C}$.

Moreover, it can be shown that the information available in $c \oplus c'$ is exactly the information available in c plus that available in c', and that a state settles the issues in $c \oplus c'$ just in case it settles both the issues in c and those in c'.

Fact 2.19. [Merging information and issues]
For any $c, c' \in \mathcal{C}$:

1. A possible world is compatible with the information available in $c \oplus c'$ just in case it is compatible with the information available in c and with the information available in c'.

2. A state settles all the issues in $c \oplus c'$ just in case it settles all the issues in c and all the issues in c'.

The *difference* between c and c', $c \oslash c'$, is naturally characterized as the weakest context $c^\dagger$ such that the merge of c' and $c^\dagger$ is an extension of c, i.e., such that $(c' \oplus c^\dagger) \geq c$. Compare this with the difference between two natural numbers, say 7 and 3. This is the smallest number n such that the sum of n with 3 is greater or equal to 7. In this concrete case, n is evidently 4.

At an abstract level, the difference between two discourse contexts is characterized in the same way, but the concrete recipe to compute this difference is of course a bit different, because contexts are more structured than natural numbers. It can be shown that the difference between c and c' is the context consisting of all information states s such that any $t \subseteq s$ that is in c' is also in c.

Definition 2.20. [The difference between two discourse contexts]
For any $c, c' \in \mathcal{C}$:

- $c \oslash c' \ := \ \{s \subseteq W \mid \text{for every } t \subseteq s, \text{ if } t \in c' \text{ then } t \in c \text{ as well}\}$

It can again be shown that $c \oslash c'$ is always a proper discourse context, i.e., a non-empty downward closed set of states.

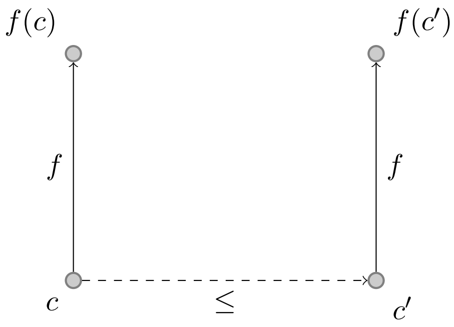


Figure 2.3: A meaning function f applied to two contexts c and c', one an extension of the other. In this case the compatibility condition requires that $f(c') = f(c) \oplus (c' \oslash c)$.

FACT 2.21. [Taking the difference between two contexts yields a new context] For any $c, c' \in \mathcal{C}$, $c \oslash c'$ is also in $\mathcal{C}$.

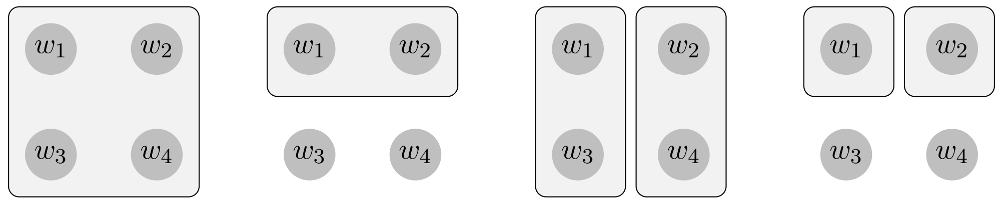
This completes our exposition of the enriched notion of discourse contexts. We are now ready to turn to meanings.

2.3 Meanings

We characterized the meaning of a sentence in general terms as its context change potential, which can be modeled formally as a function that maps every discourse context to a new discourse context. We have now specified what we take discourse contexts to be, so it has also become clearer what we take meanings to be.

However, we will not regard just any function f over discourse contexts as a proper meaning function. First of all, we will assume meanings to be functions that map any context c to a new context that is an *extension* of c. Second, we will assume that meaning functions operate in a *uniform* way across different contexts. The idea is that for any context c and any extension of it c', the difference between $f(c)$ and $f(c')$ should be traceable to the initial difference in information and issues between c and c'. Once the initial gap between c and c' is filled, the difference between $f(c)$ and $f(c')$ should also vanish.

Let us make this intuitive idea more precise. Consider a discourse context c, an extension of it, c', and a meaning function f mapping c and c' to $f(c)$ and $f(c')$, respectively, as depicted in Figure 2.3. What we want is that $f(c')$ can be obtained from $f(c)$ by adding the initial difference between c and c' to it, i.e., that $f(c') = f(c) \oplus (c' \oslash c)$. We call this condition the *compatibility condition.*



(a) c (b) c' (c) $f(c)$ (d) $f(c')$

Figure 2.4: Illustrating the compatibility condition: given that c, c' and $f(c)$ are as depicted in (a-c), $f(c')$ has to be as depicted in (d).

Definition 2.22. [Compatibility condition]
A function f over discourse contexts satisfies the compatibility condition iff for every $c, c' \in \mathcal{C}$ such that $c' \geq c$, we have that $f(c') = f(c) \oplus (c' \oslash c)$.

While the given formulation of the compatibility condition corresponds most directly with the underlying intuitive idea, it can be simplified somewhat, which will make it easier to see what it requires in concrete examples. The simplification is possible because if both c' and $f(c)$ are extensions of c, we have that:

$$\begin{aligned} f(c) \oplus (c' \oslash c) &= (f(c) \oplus c) \oplus (c' \oslash c) \\ &= f(c) \oplus (c \oplus (c' \oslash c)) \\ &= f(c) \oplus c' \end{aligned}$$

Thus, what the compatibility condition requires is just that $f(c') = f(c) \oplus c'$. Recalling that the merge of two contexts simply amounts to their intersection, we get the following, more operational formulation of the compatibility condition.

Fact 2.23. [Compatibility condition simplified]
A function f over discourse contexts satisfies the compatibility condition iff for every $c, c' \in \mathcal{C}$ such that $c' \geq c$, we have that $f(c') = f(c) \cap c'$.

Let us consider a concrete example. Let W be $\{w_1, w_2, w_3, w_4\}$, let c be the context depicted in Figure 2.4(a), c' the context depicted in Figure 2.4(b), and $f(c)$ the context depicted in Figure 2.4(c) (as before we only depict the maximal elements of each context). Then $f(c')$ has to amount to the intersection of $f(c)$ and c', i.e., it has to be the context depicted in Figure 2.4(d).

With the compatibility condition in place, we are now finally ready to specify what we take meanings to be.

Definition 2.24. [Meanings]

- A meaning is a function f which maps every context c to a new context $f(c) \geq c$, in compliance with the compatibility condition.

- The set of all meanings is denoted by $\mathcal{M}$.

The entailment relation between two meanings can be naturally defined in (at least) two ways. First, it may be defined in terms of the extension relation between contexts, i.e., we could say that one meaning f entails another meaning g just in case for every context c, $f(c)$ is an extension of $g(c)$. We will refer to this notion as extension-based entailment, or e-entailment for short, and denote it as $\models_e$.

DEFINITION 2.25. [Extension-based entailment]
For any two meanings f and g:

- $f \models_e g$ iff $f(c) \geq g(c)$ for any $c \in \mathcal{C}$

Another way to characterize entailment, which is very common in dynamic semantics, is in terms of *redundancy*. The idea is that one meaning function f entails another meaning function g just in case for any context c, if we first apply f to c, then applying g after that is redundant, i.e., has no further effect. That is, for any c, $g(f(c))$ is identical to $f(c)$. We will refer to this notion as redundancy-based entailment, or r-entailment for short, and denote it as $\models_r$.

DEFINITION 2.26. [Redundancy-based entailment]
For any two meanings f and g:

- $f \models_r g$ iff $g(f(c)) = f(c)$ for any $c \in \mathcal{C}$

It turns out that in our present setting, due to the compatibility condition, these two notions of entailment actually coincide. To see this, first suppose that $f \models_e g$. Then, $f(c) \geq g(c)$ for any $c \in \mathcal{C}$. Now consider $g(f(c))$ for some arbitrary context c. By the compatibility condition, it has to be the case that $g(f(c)) = g(c) \cap f(c)$. But then, since $f(c) \geq g(c)$, we must have that $g(f(c)) = f(c)$, which means that $f \models_d g$.

Now suppose that $f \models_r g$. Let c be an arbitrary context. Again, by the compatibility condition, it has to be the case that $g(f(c)) = g(c) \cap f(c)$. Furthermore, since $f \models_r g$, we have that $g(f(c)) = f(c)$. But then it follows that $f(c) = g(c) \cap f(c)$, and this can only be the case if $f(c) \geq g(c)$. So $f \models_e g$.

FACT 2.27. [The two notions of entailment coincide]
For any two meanings f and g:

- $f \models_e g$ iff $f \models_r g$

In view of this result, we will henceforth simply speak of entailment between meanings, rather than e-entailment or r-entailment, and denote the relation as $\models$, without any subscript.

2.4 Propositions

Recall that under the static view on meaning, the meaning of a sentence φ is identified with its truth-conditions—a function from worlds to truth-values—or equivalently, with the set of worlds $|\varphi|$ that satisfy these truth-conditions. This set of worlds is usually referred to as the *proposition* expressed by the sentence. Furthermore, as noted above, the proposition expressed by a sentence φ is usually taken to determine the sentence's conversational effect in a particular way, namely, it is assumed that the intended effect of an utterance of φ is to restrict the discourse context—classically modeled as a set of possible worlds—to $|\varphi|$. In other words, the new discourse context is obtained by intersecting the old discourse context with the proposition expressed by φ.

In the present setting, we may also introduce a notion of propositions that fulfils precisely the same role. To get at the right notion, recall from Fact 2.16 that every discourse context c is an extension of $c_\top$. Thus, the compatibility condition ensures that for every meaning f and every discourse context c:

$$f(c) = f(c_\top) \cap c$$

This means that f is completely determined by $f(c_\top)$. If we have a certain discourse context c and we want to know what the new discourse context is that results from applying f to c, we can simply take the intersection of c with $f(c_\top)$. Thus, even though we are no longer restricting ourselves to purely informative discourse, a dynamic meaning function f can in our setting still be identified with a unique static object, $f(c_\top)$, which completely determines its behavior. We will call this static object the *proposition* associated with f. Propositions, then, are the same kind of objects as discourse contexts, just as in the standard, purely information-oriented framework. Only now, discourse contexts and propositions are no longer sets of possible worlds, but rather sets of information states, non-empty and downward closed.

Definition 2.28. [Propositions]

- A proposition is a non-empty, downward closed set of states.
- The set of all propositions is denoted by Π.

Notice that there is a one-to-one correspondence between propositions and meanings: for every meaning f, the associated proposition is $f(c_\top)$, and for any proposition $P \in \Pi$, the associated meaning is the function f_P that maps every discourse context c to $c \cap P$.

Fact 2.29. [Meanings and propositions]
There is a one-to-one correspondence between meanings and propositions.

To get a good grasp of what propositions encode in the present setting, suppose that we are in the initial discourse context $c_\top$, where no information has been established and no issues have been raised yet, and someone utters a sentence expressing a proposition P. In making such an utterance, the speaker proposes to establish a new discourse context, $c_\top \cap P$. Since $c_\top = \wp(W)$, $c_\top \cap P$ just amounts to P. The common ground in this new context is $\bigcup P$, and the issue that has been raised is one that can be settled by further enhancing the common ground to one of the information states in P.

Thus, it is natural to say that the *informative content* of a proposition P, $\mathsf{info}(P)$, is embodied by $\bigcup P$, and that the issue raised by P is one which is resolved in an information state s just in case $s \in P$.

Definition 2.30. [Informative content of a proposition]

- For any proposition P: $\mathsf{info}(P) \coloneqq \bigcup P$

Definition 2.31. [The issue raised by a proposition]

- The issue raised by a proposition P is one that is resolved in an information state s just in case $s \in P$.

We say that a proposition P is *true* in a world w just in case w is compatible with the informative content of P, i.e., $w \in \mathsf{info}(P)$.

Definition 2.32. [Truth]

- A proposition P is true in a world w just in case $w \in \mathsf{info}(P)$.

Finally, we say that an information state s *supports* a proposition P just in case it implies the information content of P and resolves the issue raised by P.

Definition 2.33. [Support]
An information state s supports a proposition P if and only if:

- $s \subseteq \mathsf{info}(P)$, and
- s resolves the issue raised by P.

It is easy to see that s supports P just in case $s \in P$. So support simply amounts to membership.

Fact 2.34. [Support amounts to membership]
An information state s supports a proposition P if and only if $s \in P$.

From the fact that propositions are downward closed it follows that truth and support are closely connected: a proposition P is true in a world w just in case it is supported by the singleton state $\{w\}$.

FACT 2.35. [Truth and support]

- A proposition P is true in a world w if and only if P is supported by $\{w\}$.

The notion of support will become very useful later on. Notice that the relation between propositions and support is exactly the same as that between issues and resolution: a proposition is the set of all states that support it; an issue is the set of all states that resolve it. Moreover, the relation between inquisitive propositions and support is also parallel to the relation between *classical* propositions and truth: a classical proposition is the set of all worlds in which it is true. In the present setting, truth does not relate directly to propositions in this way, but rather to the informative content of a proposition: the informative content of a proposition is the set of all worlds in which the proposition is true. Evidently, the fact that the connection between truth and propositions is more direct in the classical setting is an immediate consequence of the fact that classical propositions exclusively encode informative content.

2.4.1 Informative and inquisitive propositions

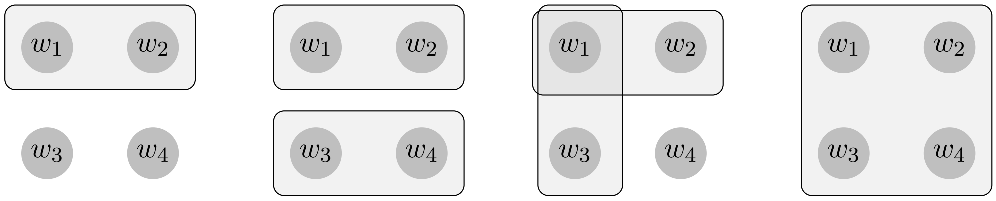
We will say that a proposition P is *informative* just in case its informative content is non-trivial, i.e., $\mathsf{info}(P) \neq W$. On the other hand, we will say that P is *inquisitive* just in case establishing its informative content is not sufficient to settle the issue that it raises, i.e., $\mathsf{info}(P) \notin P$.

DEFINITION 2.36. [Informative and inquisitive propositions]

- A proposition P is informative iff $\mathsf{info}(P) \neq W$.
- A proposition P is inquisitive iff $\mathsf{info}(P) \notin P$.

Suppose that a given information state s settles a proposition P, and there is no weaker information state $t \supseteq s$ that also settles P. Then s contains *just enough* information to settle P, it does not contain any superfluous information. Technically, these states are the *maximal elements* of P, since information states consisting of more worlds contain less information. We will refer to these maximal elements as the *alternatives* in P.[5]

[5]Our use of the term *alternatives* here is very closely related to its use in the framework of *alternative semantics* (cf., Hamblin, 1973; Kratzer and Shimoyama, 2002; Simons, 2005; Alonso-Ovalle, 2006; Aloni, 2007, among others). We will return to the connection between inquisitive semantics and alternative semantics in Section 4.7 and in Section 5.1.



(a) +informative −inquisitive (b) −informative +inquisitive (c) +informative +inquisitive (d) −informative −inquisitive

Figure 2.5: Propositions.

Definition 2.37. [Alternatives]

- The maximal elements of a proposition P are called the alternatives in P.

If a proposition P is non-inquisitive, i.e., if $\mathsf{info}(P) \in P$, then $\mathsf{info}(P)$ is the unique maximal element of P, i.e., the unique alternative in P. On the other hand, if a proposition P contains two or more alternatives then it cannot contain $\mathsf{info}(P)$ and therefore it must be inquisitive. If a proposition contains only finitely many information states, which will be the case in all the examples that we will consider, then the connection between multiple alternatives and inquisitiveness is even stronger. Namely, a proposition with finitely many elements is inquisitive if and only if it contains at least two alternatives.[6]

Fact 2.38. [Inquisitiveness and alternatives]

- A proposition containing finitely many elements is inquisitive if and only if it contains at least two alternatives.

Figure 2.5 depicts a number of propositions. In each case, we only depict the alternatives that the proposition contains. The proposition depicted in Figure 2.5(a) contains just one alternative and is therefore not inquisitive, but it *is* informative, since its informative content does not cover the entire logical space. The proposition depicted in Figure 2.5(b) contains two alternatives and is therefore inquisitive; on the other hand, it is not informative, because its informative content, i.e., the union of the two alternatives, covers the entire logical space. The proposition depicted in Figure 2.5(c) is both informative and inquisitive, since it contains two alternatives and the union of these two alternatives

[6]To see that this does not generally hold for propositions containing infinitely many states, consider a proposition that consisting of an infinite chain of states, $s_1 \subset s_2 \subset s_3 \subset \ldots$, without any maximal element. By Definition 2.36, such a proposition is inquisitive, even though it does not contain any alternatives. So, if a proposition contains at least two alternatives, then it is always inquisitive, but the reverse implication only holds if we restrict ourselves to finite propositions (Ciardelli, 2009).

does not cover the entire logical space. Finally, the proposition depicted in Figure 2.5(d) contains a single alternative, W, and is therefore not inquisitive. It is not informative either because its informative content covers the entire logical space.

The notions of informative and inquisitive propositions are of course closely related to the notions of informed and inquisitive discourse contexts defined in Section 2.2. Namely, a proposition P is informative just in case the associated meaning f_P can turn an ignorant discourse context into an informed discourse context, and similarly, a proposition is inquisitive just in case the associated meaning can turn an indifferent discourse context into an inquisitive discourse context.

Fact 2.39. [Informative and inquisitive propositions and discourse contexts]

- A proposition P is informative iff there is at least one discourse context c such that c is ignorant and $f_P(c)$ is informed.
- A proposition P is inquisitive iff there is at least one discourse context c such that c is indifferent and $f_P(c)$ is inquisitive.

2.4.2 Statements, questions, hybrids, and tautologies

It will be convenient to distinguish several classes of propositions, based on whether they are informative and/or inquisitive. First, we will refer to non-inquisitive propositions as *statements*, and to non-informative propositions as *questions*. The conversational effect of a statement, if any, is just to provide information, while the conversational effect of a question, if any, is just to raise an issue. Second, we will refer to a proposition that is both informative and inquisitive as a *hybrid*, and to a proposition that is neither informative nor inquisitive as a *tautology*. A hybrid provides information and raises an issue; a tautology does neither.

Definition 2.40. We say that a proposition P is:

- a statement iff it is non-inquisitive;
- a question iff it is non-informative;
- a hybrid iff it is both informative and inquisitive;
- a tautology iff it is neither informative nor inquisitive.

Each of these four classes of propositions is instantiated by one of the propositions depicted in Figure 2.5 above. The proposition in Figure 2.5(a) is not

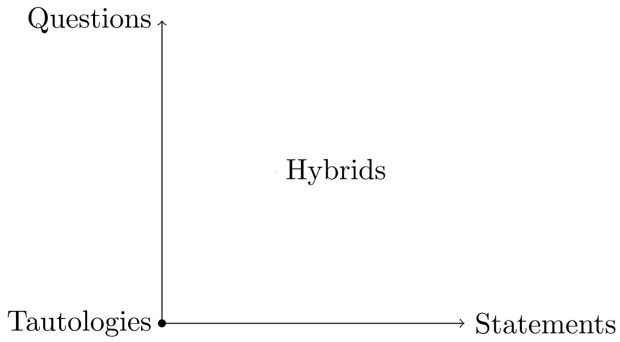


Figure 2.6: Propositions in a two-dimensional space.

inquisitive, so it is a statement; the proposition in Figure 2.5(b) is not informative, so it is a question. the proposition in Figure 2.5(d) is both informative and inquisitive, so it is a hybrid; and finally, the proposition in Figure 2.5(d) is neither informative nor inquisitive, so it is a tautology.

Propositions can be thought of as inhabiting a two-dimensional space, as depicted in Figure 2.6. The horizontal axis is inhabited by statements, which are non-inquisitive. The vertical axis is inhabited by questions, which are non-informative. The 'zero-point' of the space is inhabited by the tautology, which is neither informative nor inquisitive. The rest of the space is inhabited by hybrids, which are both informative and inquisitive.

Spelling out what it means to be non-informative and/or non-inquisitive we obtain the following direct characterization of statements, questions and tautologies.

Fact 2.41. [Direct characterization of statements, questions and tautologies]

- P is a statement $\iff \mathsf{info}(P) \in P$.
- P is a question $\iff \mathsf{info}(P) = W$.
- P is a tautology $\iff W \in P$.

It will be insightful (and useful for later) to consider a number of alternative characterizations of statements as well.

Fact 2.42. [Alternative characterizations of statements]
The following are equivalent for any proposition P:

1. P is a statement;

2. $P = \wp(\mathsf{info}(P))$

3. P is supported by a state s just in case P is true in all worlds in s;

4. P has a greatest element.

The equivalence between 1 and 2 directly follows from the direct characterization of statements in Fact 2.41. After all, if $P = \wp(\mathsf{info}(P))$ then clearly $\mathsf{info}(P) \in P$. Vice versa, if $\mathsf{info}(P) \in P$ then by downward closure every substate of $\mathsf{info}(P)$ must we in P as well, so we have that $\wp(\mathsf{info}(P)) \subseteq P$. But since by definition $\mathsf{info}(P) = \bigcup P$, we also have that $P \subseteq \wp(\mathsf{info}(P))$. Putting the two together, we get that $P = \wp(\mathsf{info}(P))$.

The equivalence between 2 and 3 is also quite immediate. Recall from Fact 2.34 that s supports P just in case $s \in P$. Now, if $P = \wp(\mathsf{info}(P))$ then every state s that supports P, i.e., every $s \in P$, must be contained in $\mathsf{info}(P)$. But this means that for every $w \in s$ we have that $w \in \mathsf{info}(P)$, which is just to say that P is true in w. Vice versa, if 3 holds then $\mathsf{info}(P)$, which is the set of all worlds where P is true, must support P. Thus, $\mathsf{info}(P) \in P$. From this we can derive, as we did above, that $P = \wp(\mathsf{info}(P))$.

Finally, let us establish the equivalence between 4 and the direct characterization of statements in Fact 2.41. Clearly, if P has a greatest element, then this greatest element must be $\bigcup P$, which is $\mathsf{info}(P)$. So we have that $\mathsf{info}(P) \in P$. Vice versa, if $\mathsf{info}(P) \in P$, then $\mathsf{info}(P)$ must be the greatest element of P.

The characterization of statements given in 4 makes it particularly easy to say whether a proposition is a statement given a visualization of it—we just have to check whether it has a greatest element. We already established in Fact 2.38 that a proposition containing finitely many elements is inquisitive if and only if it contains at least two alternatives, and therefore a statement if and only if it contains just one alternative, i.e., one maximal element. The present characterization in terms of *greatest* elements is more general since it applies to infinite propositions as well.

Finally, one particular consequence of the characterization of statements given in 2 is that there is only one proposition that counts as a tautology, namely $\wp(W)$. After all, tautologies are non-informative statements, i.e., ones whose informative content is trivial. So if P is a tautology, then we must have that $\mathsf{info}(P) = W$. But then, according to the characterization in 2, it must be the case that $P = \wp(W)$.

2.4.3 Entailment

Just like discourse contexts, propositions can be ordered either in terms of their informative component or in terms of their inquisitive component. First, a proposition P is at least as informative as another proposition Q if and only if the informative content of P determines with at least as much precision what the actual world is like as the informative content of Q, i.e., $\mathsf{info}(P) \subseteq \mathsf{info}(Q)$.

Definition 2.43. [Informative order on propositions]
Let $P, Q \in \Pi$. Then:

- $P \models_{\mathsf{info}} Q$ iff $\mathsf{info}(P) \subseteq \mathsf{info}(Q)$

Similarly, if P and Q are equally informative, then P is at least as inquisitive as Q just in case any state that settles the issue raised by P also settles the issue raised by Q, i.e., if and only if $P \subseteq Q$.

Definition 2.44. [Inquisitive order on propositions]
Let $P, Q \in \Pi$ and $\mathsf{info}(P) = \mathsf{info}(Q)$. Then:

- $P \models_{\mathsf{inq}} Q$ iff $P \subseteq Q$

Now, we would like to say that a proposition P *entails* a proposition Q just in case P is both at least as informative and at least as inquisitive as Q. But there is a subtlety here, the same that we encountered when defining the extension relation on discourse contexts. Namely, if P is strictly more informative than Q, then P and Q cannot be compared directly in terms of inquisitiveness. Thus, what we require is that P be at least as inquisitive as the *restriction* of Q to $\mathsf{info}(P)$.

Definition 2.45. [Restricting propositions]
Let $P \in \Pi$ and $s \subseteq \mathsf{info}(P)$. Then the *restriction* of P to s is the proposition:

$$P \restriction s := \{t \in P \mid t \subseteq s\}$$

This notion of restricting a proposition to a certain information state allows us to define the entailment relation between propositions. Namely, we say that a proposition P *entails* a proposition Q just in case (i) P is at least as informative as Q, i.e., $\mathsf{info}(P) \subseteq \mathsf{info}(Q)$; and (ii) P is at least as inquisitive as the restriction of Q to the informative content of P, i.e., $P \subseteq Q \restriction \mathsf{info}(P)$. It can be shown that these two conditions are satisfied exactly if $P \subseteq Q$. So the entailment relation between propositions can simply be defined in terms of inclusion, just like the extension relation between discourse contexts.

Definition 2.46. [Entailment]
For any $P, Q \in \Pi$:

- $P \models Q$ iff $P \subseteq Q$

Entailment between two propositions can also be characterized in terms of support, just like classical entailment can be characterized in terms of truth: one proposition entails another just in case and state that supports the former also supports the latter.

FACT 2.47. [Entailment in terms of support]
For any $P, Q \in \Pi$:

- $P \models Q$ iff any state that supports P also supports Q

Entailment forms a partial order on the set of all propositions. The tautology, $\wp(W)$, is entailed by any other proposition, i.e., it is the weakest element of the partial order. We denote it as $\top$. On the other hand, the partial order also has a strongest element, namely $\{\emptyset\}$, which entails all other propositions. We refer to this proposition as the contradictory proposition, and denote it as $\bot$. Note that the meaning associated with $\top$, $f_\top$, maps every discourse context to itself, i.e., it never has any real effect, while the meaning associated with $\bot$, $f_\bot$, maps every discourse context to the absurd context.

DEFINITION 2.48. [Tautology and contradiction]

- $\top := \wp(W)$
- $\bot := \{\emptyset\}$

The entailment order on propositions is closely related to the entailment order on meanings. Namely, P entails Q just in case f_P entails f_Q.

FACT 2.49. [Entailment between propositions and between meanings]
For any $P, Q \in \Pi$:

- $P \models Q$ iff $f_P \models f_Q$

Combining Facts 2.29 and 2.49 we obtain the following result.

FACT 2.50. [Meanings and propositions are isomorphic]
The space of meanings ordered by entailment, $\langle \mathcal{M}, \models \rangle$, and the space of propositions ordered by entailment $\langle \Pi, \models \rangle$, are isomorphic.

In what follows we will mostly focus our attention on propositions rather than on meanings. However, Fact 2.50 ensures that everything we will say about propositions could be said about meanings just as well. For instance, in Chapter 3 we will show that the space of propositions, $\langle \Pi, \models \rangle$, forms a complete Heyting algebra, a result that will be used in Chapter 4 to formulate an inquisitive semantics for a first-order logical language, which is, in a very precise sense, the counterpart of classical first-order logic in the inquisitive setting. Fact 2.50 ensures that the space of meanings, $\langle \mathcal{M}, \models \rangle$, has precisely the same algebraic structure as well.

However, before turning to a more in-depth formal investigation of the notion of propositions that we have arrived at, let us first illustrate how such propositions can be used to capture the informative and inquisitive content of various types of sentences in natural language.

2.5 Illustration

The notion of propositions as sets of information states, non-empty and downward closed, allows us to capture the informative and inquisitive content of a wide range of declarative and interrogative sentences in natural languages in a uniform and transparent way. To illustrate this, consider the following sentences in English, where ↓ and ↑ indicate falling and rising intonation, respectively:

(1) Peter will attend the meeting↓.

(2) Peter will attend the meeting↑?

(3) Will Peter attend the meeting?

(4) Will Peter↑ attend the meeting, or Maria↑?

(5) If Peter attends the meeting, will Maria attend it too?

(6) Who will attend the meeting?

These examples instantiate five different sentence types: (1) a declarative with falling intonation, (2) a declarative with rising intonation, (3) a polar interrogative, (4) an open disjunctive interrogative,[7] (5) a conditional polar interrogative, and (6) a wh-interrogative. Notice that the classical notion of propositions as sets of possible worlds only allows us to capture the informative content of (1), a declarative with falling intonation; it does not allow us to deal appropriately with the rising declarative in (2) and the various types of interrogatives in (3)-(6).

In the framework that we have laid out so far these sentences can all be treated in a uniform way, expressing the propositions depicted in Figure 2.7.

[7] Open disjunctive interrogatives, with rising intonation on all disjuncts, are to be distinguished from *closed* disjunctive interrogatives, often referred to as *alternative questions*, with *falling* intonation on the final disjunct (Roelofsen and van Gool, 2010; Pruitt and Roelofsen, 2013). The latter characteristically carry a non-at-issue implication that exactly one of the disjuncts is supposed to hold (Karttunen and Peters, 1976; Han and Romero, 2004; Beck and Kim, 2006; Pruitt and Roelofsen, 2011; Biezma and Rawlins, 2012; Aloni *et al.*, 2013, among others). This non-at-issue implication cannot be captured in the basic framework presented here, which is only concerned with at-issue content. However, there are natural ways to extend the framework in such a way that non-at-issue content can be captured as well (see, e.g., Ciardelli *et al.*, 2012; AnderBois, 2012).

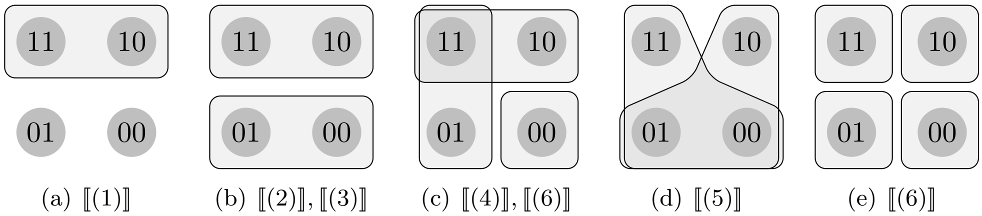


Figure 2.7: Propositions expressed by (1)–(6), exemplifying several types of declarative and interrogative sentences.

We assume here for simplicity that W consists of just four possible worlds: one world where both Peter and Maria will attend the meeting, one where only Peter will attend, one where only Maria will attend, and one where neither Peter nor Maria will attend—in Figure 2.7, these worlds are labeled 11, 10, 01, and 00, respectively. As before, each proposition is visualized by depicting just the alternatives it contains, i.e., its maximal elements.

The falling declarative in (1) can be taken to express the proposition depicted in Figure 2.7(a). This proposition contains a single alternative, which is the set of all worlds where Peter will attend the meeting. Since this single alternative does not cover the entire logical space, the proposition is informative. Moreover, since there is just one alternative, the proposition is not inquisitive. This captures the fact that in uttering (1), a speaker provides the information that Peter will attend the meeting, without requesting any further information.

The rising declarative in (2) and the polar interrogative in (3) can both be taken to express the proposition depicted in Figure 2.7(b). This proposition consists of two alternatives: the set of all worlds where Peter will attend the meeting, and the set of all worlds where Peter will not attend the meeting. Since the two alternatives together cover the entire logical space, the proposition is not informative. On the other hand, since it contains two alternatives, it *is* inquisitive. This captures the fact that in uttering either (2) or (3), a speaker raises the issue whether Peter will attend the meeting or not.

The open disjunctive interrogative in (4) can be taken to express the proposition depicted in Figure 2.7(c). This proposition consists of three alternatives: the set of all worlds where Peter will attend the meeting, the set of all worlds where Maria will attend, and the set of all worlds where neither Peter nor Maria will attend. Since these alternatives together cover the entire logical space, the proposition is not informative. However, since it contains three alternatives, it is inquisitive. It raises an issue which can be resolved by establishing that Peter will attend the meeting, or by establishing that Maria will attend, or by establishing that neither of the two will attend.

The conditional interrogative in (5) can be taken to express the proposition depicted in Figure 2.7(d). This proposition consists of two alternatives, which correspond to the following answers to (5):[8]

(7) a. Yes, if Peter attends the meeting, then Maria will attend as well.
b. No, if Peter attends the meeting, then Maria won't attend.

Since the two alternatives cover the entire logical space, the proposition is not informative. However, since it contains two alternatives, it is inquisitive. This captures the fact that in uttering (5), a speaker requests enough information to determine whether Maria will attend the meeting, under the assumption that Peter will.

Finally, consider the wh-interrogative in (6). In uttering this sentence, a speaker may be taken to request a complete, exhaustive specification of all people who will attend the meeting. Alternatively, she may be taken to request enough information to indentify at least one person who will attend the meeting, if there is such a person, and otherwise to establish that nobody will attend.[9] These two interpretations of wh-interrogatives are generally referred to as a *mention-all* and a*mention-some* interpretation, respectively. Under a mention-all interpretation, (6) expresses the proposition depicted in Figure 2.7(e). Under a mention-some interpretation, it expresses the proposition depicted in Figure 2.7(c). In both cases, the proposition expressed is inquisitive but not informative. But under the mention-all interpretation the proposition expressed is more inquisitive than under the mention-some interpretation.

This completes our brief illustration of how the informative and inquisitive content of various sentence types can be captured using the proposed notion of propositions. Along the way, we also drew attention to several aspects of

[8]Intuitively, there are other natural answers to this conditional interrogative as well:

(i) a. No, it might be that Peter will attend the meeting and that Maria won't.
b. Peter won't attend the meeting.

The first assumes a modal interpretation of (5), the second does not resolve the conditional interrogative as intended but rather denies its antecedent. The basic inquisitive semantics framework that we are presenting here is not fine-grained enough to capture the special nature of these answers. However, the framework may be further extended in such a way that a more refined semantic analysis of conditional interrogatives becomes possible, capturing both modal interpretations and denials of the antecedent (Groenendijk and Roelofsen, 2015).

[9]In uttering a wh-interrogative like (6), a speaker is often taken to *presuppose* that at least one person will attend the meeting. Such a presupposition cannot be captured directly in the basic inquisitive semantics framework that we are presenting here, which is only designed to capture at-issue informative and inquisitive content (see also footnote 7 above). However, if we allow for *partial* meaning functions, i.e., meaning functions that are only defined for certain input contexts, then presuppositions can be captured in a natural way as well (see Ciardelli *et al.*, 2012).

meaning that are beyond the scope of the basic inquisitive semantics framework that we are presenting here. However, this basic framework is set up in such a way that it allows for several natural refinements and extensions. For instance, admitting *partial* meaning functions, which are only defined for certain input contexts, leads to a natural account of presuppositions, as discussed in Ciardelli *et al.* (2012, 2015). Second, we may consider meaning functions that satisfy a weaker version of the compatibility condition. Such meanings may no longer be fully determined by a unique, static proposition; they may become genuinely dynamic. Third, rather than starting out with the commonplace notion of information states as sets of possible worlds, which imposes a very specific (Boolean) algebraic structure on the space of all information states, we may construe information states as primitive objects and assume that the space of all information states has a more generic algebraic structure (Punčochář, 2015a). Fourth, in order to model more than just informative and inquisitive content we may further enrich our notion of discourse contexts and/or propositions, either by adding additional contextual components such as discourse referents (Roelofsen and Farkas, 2015), or by weakening the downward closure constraint that we have placed on contexts and propositions here (Ciardelli *et al.*, 2014; Punčochář, 2015b; Groenendijk and Roelofsen, 2015). Such amendments lead to richer notions of meaning, and further broaden the range of semantic phenomena that can be captured in the framework. However, it largely remains to be investigated whether these refined notions of meaning are as well-behaved as the one we have presented here.

2.6 Exercises

Exercise 2.1. [The extension relation between contexts]
Show that, for any two discourse contexts c and c', the following two conditions are satisfied if and only if $c' \subseteq c$ (see page 25).

1. c' is at least as informed as c, i.e., $\mathsf{info}_{c'} \subseteq \mathsf{info}_c$;
2. c' is at least as inquisitive as the restriction of c to the information already available in c', i.e., $c' \subseteq c \upharpoonright \mathsf{info}_{c'}$.

Exercise 2.2. [The difference between two contexts]
In Definition 2.20, $c \oslash c'$ is defined as follows:

$$c \oslash c' \;:=\; \{s \subseteq W \mid \text{for every } t \subseteq s,\ \text{if } t \in c' \text{ then } t \in c \text{ as well}\}$$

This definition is motivated by the idea that the difference between c and c' is the weakest context $c^\dagger$ such that $(c' \oplus c^\dagger) \geq c$. Show that $c \oslash c'$ indeed satisfies this property.

Chapter 3

Basic operations on propositions

Now that we have introduced a new notion of meaning and propositions, a natural question that arises is what the basic operations are that can be performed on such meanings/propositions. In the classical setting, where propositions are simple sets of worlds, we can form the intersection or the union of two propositions, or the complement of a single proposition. These operations play a central role in logic and in semantic analyses of natural languages: conjunction and disjunction are standardly taken to express intersection and union, respectively, while negation is standardly taken to express complementation. Do these operations have natural counterparts in the inquisitive setting, where propositions are no longer simple sets of worlds?

We will address this question in Section 3.1, adopting an algebraic perspective. We will find that the basic algebraic operations on classical propositions can indeed be applied to inquisitive propositions as well. This result facilitates a very natural way of dealing with connectives and quantifiers, and will allow us to define an inquisitive semantics for the language of first-order logic which is, from an algebraic perspective, the exact counterpart of classical first-order logic in the inquisitive setting.

In Section 3.2, we will consider two additional operations, which trivialize the informative or the inquisitive content, respectively, of any given proposition. For reasons that will become clear below, we refer to such operators as *projection operators*. One projection operator turns any proposition into a statement, trivializing its inquisitive content, while the other turns any proposition into a question, trivializing its informative content. Clearly, these operations do not have a counterpart in the classical setting, where propositions capture only informative content to begin with; but in the inquisitive setting they naturally arise, and we will suggest that they also have an important role to play in the semantic analysis of natural languages.

3.1 Algebraic operations

In this section we will identify the basic algebraic operations that can be applied to inquisitive propositions. To illustrate our approach, we will first briefly review the algebraic perspective on classical logic.

3.1.1 The algebraic perspective on classical logic

In the classical setting a proposition P is simply a set of possible worlds. Let us denote the set of all classical propositions as Π_c. The proposition expressed by a sentence can be thought of as carving out a certain region in the logical space—the set of all possible worlds—and in asserting a sentence, a speaker is taken to provide the information that the actual world is located within this region. One proposition P *entails* another proposition Q, $P \models Q$, just in case $P \subseteq Q$, which means that P carves out a smaller region in the logical space than Q does, thereby encoding more information as to what the actual world is like. Entailment forms a partial order on the set of all classical propositions, i.e., it is a reflexive, transitive, and anti-symmetric relation.

Now, every partially ordered set has a certain algebraic structure and comes with certain basic algebraic operations. The set of classical propositions ordered by entailment, $\langle \Pi_c, \models \rangle$, forms a so-called *Heyting algebra*, which comes with four basic operations: *meet*, *join*, *relative pseudo-complementation* and *absolute pseudo-complementation*.

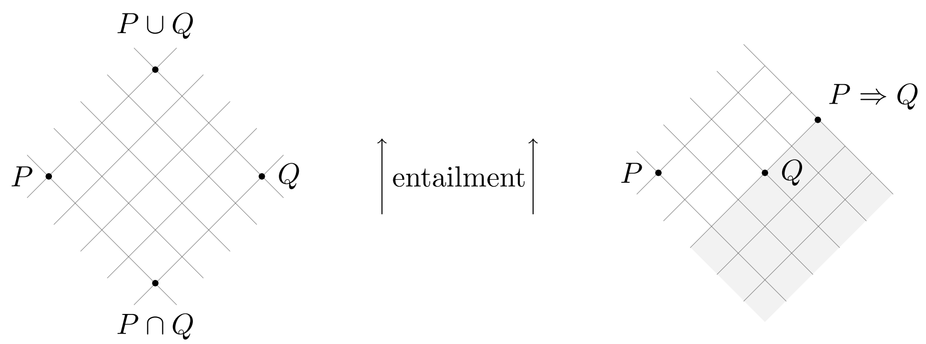
The *meet* of P and Q is the *greatest lower bound* of P and Q with respect to entailment, i.e., the weakest proposition that entails both P and Q. As depicted in Figure 3.1(a), this greatest lower bound amounts to the *intersection* of the two propositions: $P \cap Q$. More generally, the meet of a (possibly infinite) set of propositions Σ amounts to the intersection of all the propositions in that set:

$$\bigcap \Sigma = \{w \mid w \in P \text{ for all } P \in \Sigma\}$$

If Σ is empty, then $\bigcap \Sigma$ is the proposition consisting of all possible worlds, W. This is the weakest of all propositions, since it is entailed by all other propositions. It is denoted as $\top$. On the other hand, if Σ is the set of all propositions, then $\bigcap \Sigma$ is the empty proposition, $\emptyset$. This is the strongest of all propositions, since it entails all other propositions. It is denoted as $\bot$.

The *join* of two propositions P and Q is the *least upper bound* of P and Q with respect to entailment, i.e., the strongest proposition that is entailed by both P and Q. As depicted in Figure 3.1(a), this least upper bound amounts to the *union* of the two propositions: $P \cup Q$. More generally, the join of a (possibly infinite) set of propositions Σ amounts to the union of all the propositions in that set:

$$\bigcup \Sigma = \{w \mid w \in P \text{ for some } P \in \Sigma\}$$



(a) join and meet (b) relative pseudo-complement

Figure 3.1: Join, meet, and relative pseudo-complement.

If Σ is empty, then $\bigcup\Sigma$ is the empty proposition, $\bot$. On the other hand, if if Σ is the set of all propositions, then $\bigcap\Sigma$ is the proposition consisting of all possible worlds, $\top$.

The existence of meets and joins for arbitrary sets of propositions implies that $\langle\Pi_c, \models\rangle$ forms a complete lattice, bounded by $\bot$ and $\top$ as its strongest and weakest elements, respectively.

Now let us turn to relative and absolute pseudo-complementation. The pseudo-complement of a proposition P relative to another proposition Q, which we will denote as $P \Rightarrow Q$, can be thought of intuitively as the *difference* between P and Q: it is the weakest proposition R such that P and R together contain at least as much information as Q.[1] More formally, it is the weakest proposition R such that $P \cap R \models Q$. This is visualized in Figure 3.1(b). The shaded area in the figure is the set of all propositions R which are such that $P \cap R \models Q$. The weakest among these, i.e., the topmost one, is the pseudo-complement of P relative to Q. This proposition consists of all possible worlds which, if contained in P, are also contained in Q:

$$P \Rightarrow Q \;=\; \{w \mid \text{if } w \in P \text{ then } w \in Q \text{ as well}\}$$

Absolute pseudo-complementation is a limit case of its relative counterpart. The absolute pseudo-complement of a proposition P, which we will denote as P^*, is the weakest proposition R such that $P \cap R$ entails *any* other proposition. Since the only proposition that entails any other proposition is $\bot$, P^* can be characterized as the weakest proposition R such that $P \cap R = \bot$. It consists

[1]Indeed, this is precisely the same notion of difference that we introduced for discourse contexts in Section 2.2.

simply of all worlds that are not in P itself:

$$P^* = \{w \mid w \notin P\}$$

In a Heyting algebra it always holds, by definition of P^*, that $P \cap P^* = \bot$. In the specific case of $\langle \Pi_c, \models \rangle$, we also always have that $P \cup P^* = \top$. This means that in this particular setting, P^* is in fact the *Boolean complement* of P, and that $\langle \Pi_c, \models \rangle$ forms a *Boolean algebra*, a special kind of Heyting algebra.

Thus, classical propositions are amenable to certain basic algebraic operations. Classical first-order logic is obtained by associating these operations with the connectives and the quantifiers. Indeed, the usual definition of truth can be reformulated as a recursive definition of the set $|\varphi|$ of models over a domain D in which φ is true. The inductive clauses then run as follows:

- $|\neg\varphi| = |\varphi|^*$
- $|\varphi \wedge \psi| = |\varphi| \cap |\psi|$
- $|\varphi \vee \psi| = |\varphi| \cup |\psi|$
- $|\varphi \to \psi| = |\varphi| \Rightarrow |\psi|$
- $|\forall x.\varphi(x)| = \bigcap_{d \in D} |\varphi(d)|$
- $|\exists x.\varphi(x)| = \bigcup_{d \in D} |\varphi(d)|$

Negation expresses absolute pseudo-complementation, conjunction and disjunction express binary meet and join, respectively, implication expresses relative pseudo-complementation, and quantified formulas, $\forall x.\varphi$ and $\exists x.\varphi$, express the infinitary meet and join, respectively, of $\{|\varphi(d)| \mid d \in D\}$.

Notice that everything started with a notion of propositions and a natural entailment order on these propositions. The entailment order induces certain basic operations on propositions, and classical first-order logic is obtained by associating these basic semantic operations with the connectives and quantifiers.

3.1.2 Algebraic operations on inquisitive propositions

Recall that in inquisitive semantics propositions are not sets of worlds, but rather sets of information states, non-empty and downward closed. In this setting, one proposition P entails another proposition Q just in case P is at least as informative and at least as inquisitive as Q. We have seen that this condition is satisfied just in case $P \subseteq Q$. So technically entailment still amounts to inclusion, just like in classical logic, though now it encompasses both informative and inquisitive strength.

Let us consider the algebraic structure of the space of all inquisitive propositions ordered by entailment, $\langle \Pi, \models \rangle$, in order to determine which operations could be associated with the connectives and quantifiers in an inquisitive semantics for the language of first-order logic. What kind of algebraic operations can be performed on inquisitive propositions? Does every set of propositions still have a unique greatest lower bound (*meet*) and a unique least upper bound (*join*) w.r.t. entailment? Does every proposition still have a pseudo-complement relative to any other proposition?

It turns out that these questions can be answered in the positive: $\langle \Pi, \models \rangle$ forms a complete Heyting algebra, just like $\langle \Pi_c, \models \rangle$. First, any set of propositions $\Sigma \subseteq \Pi$ still has a meet and a join, which can moreover still be characterized in terms of intersection and union.

Fact 3.1. [Meet] Any set of propositions $\Sigma \subseteq \Pi$ has a meet, which amounts to:

$$\bigcap \Sigma = \{s \mid s \in P \text{ for all } P \in \Sigma\}$$

Fact 3.2. [Join] Any set of propositions $\Sigma \subseteq \Pi$ has a join, which amounts to:

$$\bigcup \Sigma = \{s \mid s \in P \text{ for some } P \in \Sigma\}$$

if $\Sigma \neq \emptyset$, and to $\{\emptyset\}$ otherwise.

The existence of meets and joins for arbitrary sets of propositions implies that $\langle \Pi, \subseteq \rangle$ forms a complete lattice. This lattice has a unique strongest element, $\bot := \{\emptyset\}$, and a unique weakest element, $\top := \wp(W)$.

Furthermore, just as in the classical setting, for every two propositions P and Q, there is a unique weakest proposition R such that $P \cap R$ entails Q. Recall that this proposition, the pseudo-complement of P relative to Q, can be thought of intuitively as the difference between P and Q.

Fact 3.3. [Relative pseudo-complement]
For any $P, Q \in \Pi$, the pseudo-complement of P relative to Q amounts to:

$$P \Rightarrow Q \;:=\; \{s \mid \text{for every } t \subseteq s, \text{if } t \in P \text{ then } t \in Q\}$$

The existence of relative pseudo-complements implies that $\langle \Pi, \subseteq \rangle$ forms a Heyting algebra. Finally, recall that the *absolute* pseudo-complement of a proposition P, denoted P^*, is defined as the pseudo-complement of P relative to $\bot$. We saw that in the classical setting, P^* amounts to the set of worlds that are not in P. In the inquisitive setting, P^* amounts to the set of states that are incompatible with any state in P.

FACT 3.4. [Absolute pseudo-complement] For any proposition $P \in \Pi$:

$$P^* = \{s \mid s \cap t = \emptyset \text{ for all } t \in P\}$$

A state s is incompatible with all states in P just in case it is incompatible with $\bigcup P$, which in turn holds just in case s is a subset of $\overline{\bigcup P}$. Thus, P^* can also be characterized as $\wp(\overline{\bigcup P})$, which means in particular that P^* always contains a single alternative, $\overline{\bigcup P}$, and is therefore never inquisitive.

The algebraic operations that we have identified are exactly the ones that are present in the classical setting. One notable difference, however, is that the absolute pseudo-complement of an inquisitive proposition is not always its *Boolean* complement. In fact, most inquisitive propositions do not have a Boolean complement at all. To see this, suppose that P and Q are Boolean complements. This means that:

(i) $P \cap Q = \bot$

(ii) $P \cup Q = \top$

Since $\top = \wp(W)$, condition (ii) can only be fulfilled if either P or Q contains W. Suppose $W \in P$. Then, since P is downward closed, $P = \wp(W) = \top$. But then, in order to satisfy condition (i), we must have that $Q = \{\emptyset\} = \bot$. So the only two elements of our algebra that have a Boolean complement are $\top$ and $\bot$. Hence, the space $\langle \Pi, \models \rangle$ of inquisitive propositions does not form a Boolean algebra, unlike the space $\langle \Pi_c, \models \rangle$ of classical propositions.

This difference has repercussions for the behavior of the logical system that we will specify, in particular for negation, which will be taken to express absolute pseudo-complementation (for instance, the law of double negation will no longer hold). However, the similarity between $\langle \Pi, \models \rangle$ and $\langle \Pi_c, \models \rangle$ that we identified, i.e., the fact that both form a Heyting algebra, is much more important for our current purposes. In particular, the existence of meets, joins, and relative and absolute pseudo-complements in $\langle \Pi, \models \rangle$ will allow us to specify an inquisitive semantics for the language of first-order logic which is, from an algebraic perspective, the exact counterpart of classical first-order logic in the inquisitive setting. We will turn to this in Chapter 4. Before that, however, we will consider two additional operations that are particularly natural to perform on propositions in inquisitive semantics.

3.2 Projection operators

We noted in Section 2.4.2 that propositions in inquisitive semantics can be seen as inhabiting a two dimensional space, with statements living on one axis and questions on the other. Given this picture, it is natural to consider whether it is possible to define general *projection operators* on this space, i.e., operators that turn any given proposition either into a statement or into a question, otherwise preserving as much as possible of the proposition's characteristics. We will refer to such operators as s-projection operators and q-projection operators, respectively.

Let us first consider more precisely what would be required for an operator π to qualify as an s-projection operator. On the one hand, when applied to a proposition P, π should trivialize the inquisitive content of P, so that the resulting proposition is a statement. On the other hand, π should preserve the informative content of P, i.e., πP should have exactly the same informative content as P itself.

Definition 3.5. [Requirements on s-projection]
An operator π qualifies as an s-projection operator just in case for any $P \in \Pi$:

- πP is a statement
- $\mathsf{info}(\pi P) = \mathsf{info}(P)$

Now, in Section 2.4.2 we saw that if P is a statement, then we always have that $P = \wp(\mathsf{info}(P))$. This means that in order to satisfy the above requirements, πP must amount to $\wp(\mathsf{info}(P))$ for any proposition P. Thus, the semantic behavior of π is uniquely determined by the given requirements.

Fact 3.6. [Unique characterization of s-projection]
An operator π qualifies as an s-projection operator just in case for any $P \in \Pi$:

- $\pi P = \wp(\mathsf{info}(P))$

Now let us consider which requirements π should fulfil in order to qualify as a q-projection operator. Obviously, we should require that π trivializes the informative content of the proposition to which it applies, i.e., πP should always be a question. But, given this basic requirement, we cannot further demand that π always preserves the inquisitive content of P. For, if P and πP do not have the same informative content, then their inquisitive content will differ as well.

Fortunately, there is a natural way to overcome this obstacle. Namely, what we *can* require is that π preserve the *decision set* of P, i.e., the set of states that either settle the issue raised by P, or contradict the informative content of P and thereby establish that it is impossible to settle the issue altogether.

Definition 3.7. [Contradicting and deciding on a proposition]
Let s be an information state and P a proposition. Then we say that:

- s *contradicts* P just in case $s \cap \mathsf{info}(P) = \emptyset$;

- s *decides* on P just in case s either supports or contradicts P.

Definition 3.8. [Decision set]
The *decision set* $D(P)$ of a proposition P is the set of states that decide on P.

The decision set of a proposition can be characterized explicitly as follows.

Fact 3.9. [Decision set explicated] For any proposition P:

- $D(P) = P \cup P^*$

Now, what we require of a q-projection operator π is that, besides trivializing the informative content of the proposition it applies to, it preserves the proposition's decision set. This is a requirement that can in principle be met, since P and πP can very well have the same decision set even if they differ in informative content.

Definition 3.10. [Requirements on q-projection]
An operator π qualifies as a q-projection operator just in case for any P:

- πP is a question;
- $D(\pi P) = D(P)$.

Now suppose that π fulfils these requirements. Then for any P, πP is a question, which means that the informative content of πP is W. But then $(\pi P)^* = \{\emptyset\}$, and therefore $D(\pi P) = (\pi P) \cup (\pi P)^* = \pi P$. But since π should preserve the decision set of P, we also have that $D(\pi P) = D(P) = P \cup P^*$. Putting these facts together, we obtain that $\pi P = P \cup P^*$. Thus, the requirements we placed on π again uniquely determine its semantic behavior.

Fact 3.11. [Unique characterization of q-projection]
An operator π qualifies as a q-projection operator just in case for any P:

- $\pi P = P \cup P^*$

Thus, by spelling out the natural requirements on s-projection and q-projection we have arrived at a unique characterization of two projection operators, which we will denote as ! and ?, respectively.

Definition 3.12. [Projection operators] For any proposition P:

- $!P := \wp(\mathsf{info}(P))$
- $?P := P \cup P^*$

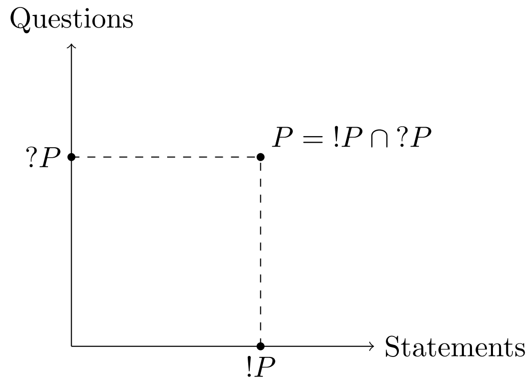


Figure 3.2: Projection operators.

As depicted in Figure 3.2, the projection operators ! and ? turn any proposition P into a statement $!P$ which has the same informative content as P, and a question $?P$ which has the same decision set as P. P itself can always be reconstructed as the meet of these two 'pure components'.

Fact 3.13. [Division] For any proposition P:

- $P = !P \cap ?P$

Finally, let us consider how ? and ! are related to the algebraic operations identified in Section 3.1. Notice that $?P$ is already explicitly characterized in terms of the algebraic operations: it amounts to the join of P and its absolute pseudo-complement P^*. It turns out that $!P$ can also be characterized in terms of pseudo-complementation. Namely, for any proposition P, $!P$ amounts to P^{**}, i.e., to the proposition that results from two successive applications of the absolute pseudo-complementation operator to P.

Fact 3.14. [Projection operators and algebraic operators]
For any proposition P:

- $!P = P^{**}$
- $?P = P \cup P^*$

This concludes our discussion of the basic semantic operations that can be performed on propositions in inquisitive semantics. We end this chapter with a brief remark on the linguistic relevance of these operations, which will be further substantiated in later chapters.

3.3 Linguistic relevance

Even though virtually any introduction to natural language semantics starts out with a discussion of classical logic, the linguistic relevance of the classical treatment of the connectives and quantifiers is not often explicitly argued for. What makes this particular logical system so special? Why is it even called *classical* logic? Is this terminology just a historical coincidence, or does it carry some real substance?

The algebraic perspective that we adopted here makes it possible to answer these questions. Namely, what is special about classical logic is that it takes the connectives and quantifiers to express the most basic algebraic operations on propositions. This also explains why classical logic is of particular interest from a linguistic point of view. After all, since the algebraic operations on propositions that are associated with the connectives and quantifiers in classical logic are so elementary, it is to be expected that natural languages will generally have ways to express them as well; just like one would expect, for instance, that the basic algebraic operations in arithmetic—addition, substraction, multiplication, and division—are generally expressible in natural languages. And indeed, in language after language words have been found that may be taken to fulfill exactly this purpose (see, e.g., Haspelmath, 2007; Gil, 2013; Szabolcsi, 2015b). The algebraic perspective provides a simple explanation for the cross-linguistic ubiquity of such words. This makes the treatment of the connectives and quantifiers in classical logic attractive from a linguistic point of view, which is one important reason for us to try to establish the exact counterpart of classical logic in the inquisitive setting, which is what we will do in Chapter 4.

Similar considerations apply to the projection operators. Again, since these operators are so elementary, it is to be expected that they too are expressible in many natural languages. More specifically, it seems plausible to hypothesize that they are expressed in English and many other languages by declarative and interrogative clause type markers. Such markers may include word order, intonation, as well as specific particles. For instance, on a first approximation, we may hypothesize that declarative word order in English invokes the ! operator, and interrogative word order the ? operator. A more detailed account of clause type marking in English in terms of the projection operators will be presented in Chapter 6.

3.4 Exercises

Exercise 3.1. [Working through some examples]
Consider the four propositions depicted in Figure 2.5 on page 33.

1. Determine the absolute pseudo-complement of each of these propositions.

2. Determine the meet and the join of any pair among these propositions.

3. Determine the outcome of applying the projection operators to each of these propositions.

Exercise 3.2. [Meets and joins]
Show that every set of propositions in inquisitive semantics has a meet (Fact 3.1) and a join (Fact 3.2) with respect to entailment.

Exercise 3.3. [Absolute pseudo-complementation]
Show that for any proposition P in inquisitive semantics, P^* amounts to the set of states that are incompatible with any state in P (Fact 3.4).

Exercise 3.4. [Projection operators]
Suppose we apply both projection operators to a given sentence, one after the other. Does it matter in which order we do this? That is, does the following hold for every proposition P:

$$?!P = !?P$$

Chapter 4

A first-order inquisitive semantics

In this chapter we define an inquisitive semantics for the language of first-order logic, making use of the operations on propositions identified in the previous chapter. We will highlight some of the main features of the system, and illustrate it with a range of examples.

4.1 Logical language and models

We will consider a standard first-order language $\mathcal{L}$, based on a signature that consists of a set of function symbols $\mathcal{F}_{\mathcal{L}}$ and a set of relation symbols $\mathcal{R}_{\mathcal{L}}$, each with an associated arity $n \geq 0$. As usual, 0-place function symbols will be referred to as individual constants. We assume that the language has $\neg$, $\vee$, $\wedge$, $\rightarrow$, $\exists$, and $\forall$ as its basic logical constants.

We will interpret $\mathcal{L}$ with respect to *first-order information models*. Such models consist of a set of possible worlds W, each associated with a standard first-order model. A standard first-order model, in turn, consists of a domain of individuals D and an interpretation function I which maps any function symbol in $\mathcal{F}_{\mathcal{L}}$ to a function over D and every relation symbol in $\mathcal{R}_{\mathcal{L}}$ to a relation over D.

In order to avoid certain issues arising from quantification across different possible worlds, we will restrict our attention to *rigid* first-order information models, in which the domain of individuals as well as the interpretation of function symbols is fixed across worlds. The only thing that may differ from world to world is the interpretation of relation symbols.

Definition 4.1. [Rigid first-order information models]
A rigid first-order information model for $\mathcal{L}$ is a triple $\langle W, D, I \rangle$, where:

- W is a set, whose elements are referred to as possible worlds;
- D is a non-empty set, whose elements are referred to as individuals;

- I is a map that associates every $w \in W$ with a first-order model I_w such that:
  - For every $w \in W$, the domain of I_w is D;
  - For every n-ary function symbol $f \in \mathcal{F}_{\mathcal{L}}$, $I_w(f) : D^n \to D$, with the condition that for every $w, v \in W$, $I_w(f) = I_v(f)$;
  - For every n-ary relation symbol $R \in \mathcal{R}_{\mathcal{L}}$, $I_w(R) \subseteq D^n$.

Unless specified otherwise, we will assume a fixed model throughout our discussion and we will often omit explicit reference to it. So, while in the previous chapters, where we were not yet considering a concrete logical langauge, we simply assumed a set of possible worlds W as our logical space, we now consider a triple $\langle W, D, I \rangle$, where W is still a set of possible worlds, and the other elements specify the interpretation of the function symbols and relation symbols in our language w.r.t. these possible worlds.

In order not to have assignments in the way all the time, we will assume that for any $d \in D$, our language $\mathcal{L}$ contains an individual constant d' such that $I_w(d') = d$ for all $w \in W$: if this is not the case, we simply expand the language by adding new constants, and we expand each I_w accordingly. In this way we can define our semantics just for formulas without free variables, and we can do without assignments altogether. This move is not essential, but it simplifies notation and terminology somewhat.

Finally, it will be convenient to have a notation for the set of worlds in our model in which a given sentence φ is classically true. We will denote this set as $|\varphi|$ and refer to it as the *classical truth-set* of φ.

Definition 4.2. [Classical truth-set]
For any $\varphi \in \mathcal{L}$, the set of worlds where φ is classically true is denoted as $|\varphi|$.

4.2 Semantics

We are now ready to recursively associate each sentence of our first-order language with an inquisitive proposition. The proposition expressed by an atomic sentence $R(t_1, \dots, t_n)$ is defined as the set of all information states that consist exclusively of worlds where $R(t_1, \dots, t_n)$ is true, i.e., as subsets of $|R(t_1, \dots, t_n)|$. The connectives and quantifiers are taken to express the basic algebraic operations that we identified in Section 3.1.

Definition 4.3. [First-order inquisitive semantics]

1. $[R(t_1, \dots, t_n)] \coloneqq \wp(|R(t_1, \dots, t_n)|)$
2. $[\neg\varphi] \coloneqq [\varphi]^*$

3. $[\varphi \wedge \psi] := [\varphi] \cap [\psi]$
4. $[\varphi \vee \psi] := [\varphi] \cup [\psi]$
5. $[\varphi \rightarrow \psi] := [\varphi] \Rightarrow [\psi]$
6. $[\forall x.\varphi(x)] := \bigcap_{d \in D} \ [\varphi(d')]$
7. $[\exists x.\varphi(x)] := \bigcup_{d \in D} \ [\varphi(d')]$

We refer to this first-order system as InqB, where B stands for *basic*. We refer to $[\varphi]$ as the proposition expressed by φ. The clauses of InqB constitute a proper inquisitive semantics in the sense that they indeed associate every sentence $\varphi \in \mathcal{L}$ with a proposition in the sense of Definition 2.28 in Section 2.4.

FACT 4.4. [Suitability of the semantics] For any $\varphi \in \mathcal{L}$, $[\varphi] \in \Pi$.

Many notions that were introduced in Chapter 2 with reference to propositions can now be formulated with reference to the sentences in our logical language. For instance, we define the *informative content* of a sentence φ, $\mathsf{info}(\varphi)$, as the informative content of the proposition it expresses, which amounts to $\bigcup[\varphi]$. Similarly, we say that the issue raised by a sentence φ is the issue raised by $[\varphi]$, which is the issue that is resolved by an information state s just in case $s \in [\varphi]$.

DEFINITION 4.5. [Informative content of a sentence] For any $\varphi \in \mathcal{L}$:

- $\mathsf{info}(\varphi) := \bigcup[\varphi]$

DEFINITION 4.6. [The issue raised by a sentence] For any $\varphi \in \mathcal{L}$:

- The issue raised by φ is one that is resolved by a state s just in case $s \in [\varphi]$.

We say that one sentence φ *entails* another sentence ψ, $\varphi \models \psi$, just in case the proposition expressed by φ entails the proposition expressed by ψ, and we say that φ and ψ are *equivalent*, $\varphi \equiv \psi$, just in case they express exactly the same proposition.

DEFINITION 4.7. [Entailment and equivalence] For any $\varphi, \psi \in \mathcal{L}$:

- $\varphi \models \psi$ just in case $[\varphi] \subseteq [\psi]$
- $\varphi \equiv \psi$ just in case $[\varphi] = [\psi]$

Finally, we say that an information state s *supports* a sentence φ, notation $s \models \varphi$, just in case it supports the proposition expressed by φ, which holds precisely if s implies $\mathsf{info}(\varphi)$ and resolves the issue that φ raises.

Definition 4.8. [Support]
An information state s supports a sentence φ, $s \models \varphi$, if and only if:

- $s \subseteq \mathsf{info}(\varphi)$, and
- s resolves the issue raised by φ.

It is easy to see that these two conditions are satisfied just in case $s \in [\varphi]$. So, just like the proposition expressed by φ in classical logic is the set of worlds where φ is true, the proposition expressed by φ in InqB is the set of states where φ is supported.

Fact 4.9. [Support and propositions]
For any information state s and any sentence φ:

- $s \models \varphi \iff s \in [\varphi]$

As a consequence of this fact, InqB can be characterized by a recursive definition of the support conditions for the sentences in the language. These support conditions are as follows.

Fact 4.10. [Support conditions]

1. $s \models R(t_1, \dots, t_n)$ iff $s \subseteq |R(t_1, \dots, t_n)|$
2. $s \models \neg\varphi$ iff $\forall t \subseteq s$: if $t \neq \emptyset$ then $t \not\models \varphi$
3. $s \models \varphi \wedge \psi$ iff $s \models \varphi$ and $s \models \psi$
4. $s \models \varphi \vee \psi$ iff $s \models \varphi$ or $s \models \psi$
5. $s \models \varphi \to \psi$ iff $\forall t \subseteq s$: if $t \models \varphi$ then $t \models \psi$
6. $s \models \forall x\varphi(x)$ iff $s \models \varphi(d')$ for all $d \in D$
7. $s \models \exists x\varphi(x)$ iff $s \models \varphi(d')$ for some $d \in D$

In much early work on inquisitive semantics (Ciardelli, 2009; Groenendijk and Roelofsen, 2009; Ciardelli and Roelofsen, 2011) as well as in some more recent work (e.g., Ciardelli *et al.*, 2015; Ciardelli, 2015), InqB is in fact characterized directly in terms of support conditions. The proposition expressed by a sentence is then defined in terms of these support conditions, i.e., as the set of all states that support the sentence. An advantage of this approach is that it parallels the usual presentation of classical logic, with truth conditions as the basic notion. Another advantage, at least for certain purposes, is that it allows for a very efficient presentation of the system, bypassing all the more abstract notions that we introduced here before even starting to consider a concrete logical language.

There are two main reasons why we have chosen a less direct route here, following Ciardelli *et al.* (2013a) and Roelofsen (2013a). First, the current presentation of the new inquisitive notion of meaning (Chapter 2) brings out very explicitly how the standard information-centred notion of meaning is enriched, both in its static and in its dynamic flavor, why it is shaped exactly the way it is, and that it naturally allows for various further extensions and refinements (see the references on page 42 as well as Appendix B). Second, the algebraic perspective adopted here (Chapter 3) makes it possible to motivate the treatment of the connectives and quantifiers in InqB in a solid way, relying only on the structure of our new space of meanings. Moreover, it shows that InqB is, in a very precise sense, the exact counterpart of classical logic in the inquisitive setting. Thus, unlike a support-based exposition, this mode of presentation flows directly from the abstract motivations and the philosophical underpinnings of the system to its concrete implementation. On the other hand, for many particular applications of inquisitive semantics, it is more practical to present InqB directly in terms of support conditions. The support based perspective will also be employed in Chapter 7 of the present lecture notes.

4.3 Semantic categories and projection operators

We say that a sentence in informative, inquisitive, a statement, a question, a hybrid, or a tautology just in case the proposition that it expresses is. This amounts to the following.

DEFINITION 4.11. [Informativeness and inquisitiveness] For any $\varphi \in \mathcal{L}$:

- φ is informative just in case $\mathsf{info}(\varphi) \neq W$.
- φ is inquisitive just in case $\mathsf{info}(\varphi) \notin [\varphi]$.

DEFINITION 4.12. [Semantics categories] We say that a sentence $\varphi \in \mathcal{L}$ is:

- a statement iff it is non-inquisitive;
- a question iff it is non-informative;
- a hybrid iff it is both informative and inquisitive;
- a tautology iff it is neither informative nor inquisitive.

FACT 4.13. [Direct characterization of statements, questions and tautologies]

- φ is a statement $\iff$ $\mathsf{info}(\varphi) \in [\varphi]$ $\iff$ $[\varphi]$ has a greatest element.
- φ is a question $\iff$ $\mathsf{info}(\varphi) = W$.

- φ is a tautology $\iff W \in [\varphi]$.

Just like propositions, then, the sentences in our logical language can be thought of as inhabiting a two-dimensional space (see Figure 2.6 on page 35). The horizontal axis is inhabited by statements, which are non-inquisitive. The vertical axis is inhabited by questions, which are non-informative. The 'zero-point' of the space is inhabited by tautologies, which are neither informative nor inquisitive. The rest of the space is inhabited by hybrids, which are both informative and inquisitive.

In Section 3.2 we characterized two projection operators on propositions, which turn any given proposition into a statement or a question. Now that we are considering a concrete logical language, we will introduce two one-place connectives that express these projection operators. We will denote these connectives as ! and ?, just like the operators they express.

Definition 4.14. [Projection operators]
For any $\varphi \in \mathcal{L}$:

- $[!\varphi] := ![\varphi]$
- $[?\varphi] := ?[\varphi]$

Recall from Fact 3.14 on page 51 that the projection operators on propositions can be characterized algebraically:

- $!P = P^{**}$
- $?P = P \cup P^{*}$

Since negation expresses absolute pseudo-complementation and disjunction expresses the join operation, this means that the connectives ! and ? can be characterized in terms of negation and disjunction.

Fact 4.15. [Projection operators in terms of negation and disjunction]
For any $\varphi \in \mathcal{L}$:

- $!\varphi \equiv \neg\neg\varphi$
- $?\varphi \equiv \varphi \vee \neg\varphi$

This means that ! and ? do not have to be added to our logical language as primitive connectives; $!\varphi$ and $?\varphi$ can simply be regarded as abbreviations of $\neg\neg\varphi$ and $\varphi \vee \neg\varphi$, respectively.

Finally, we have that a sentence φ is always equivalent to the conjunction of its two 'pure components' $!\varphi$ and $?\varphi$ (the analogue of Fact 3.13 on page 51).

Fact 4.16. [Division] For any φ:

- $\varphi \equiv !\varphi \wedge ?\varphi$

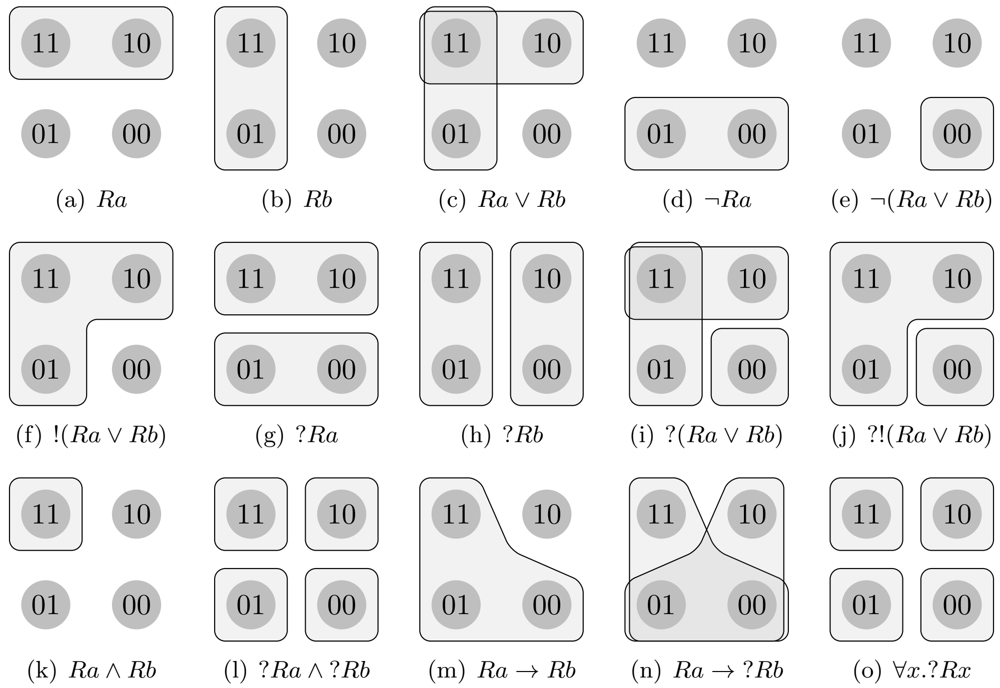


Figure 4.1: The propositions expressed by some simple sentences.

4.4 Examples

Now let us consider some concrete sentences in InqB and the propositions that they express. We will assume that our language contains just one unary predicate symbol, R, and two individual constants, a and b. Accordingly, we will assume that the domain of discourse consists of just two objects, denoted by a and b, respectively. Our logical space, then, consists of four worlds, one in which both Ra and Rb are true, one in which Ra is true but Rb is false, one in which Rb is true but Ra is false, and one in which neither Ra nor Rb is true. These worlds will be labeled 11, 10, 01, and 00, respectively. As usual, in order to keep the pictures orderly we display only the maximal elements of a proposition. For concreteness, we will informally read Ra as 'a is running', and similarly for Rb.

Atomic sentences. Let us first consider the proposition expressed by the atomic sentences Ra and Rb. According to the clause for atomic sentences, $[Ra]$ consists of all states s such that every world in s makes Ra true, i.e., the state $\{11, 10\}$ and all substates thereof. Thus, as depicted in Figure 4.1(a), $[Ra]$ has a unique greatest element, $\{11, 10\}$. Fact 4.13 therefore ensures that Ra is a statement. It provides the information that a is running, and it does not request any further information. So it behaves just as in classical logic. Analogously, Rb is a statement which provides the information that b is running, without requesting any further information. The proposition expressed by Rb is depicted in Figure 4.1(b).

Disjunction. Next, consider the disjunction $Ra \vee Rb$. According to the clause for disjunction, $[Ra \vee Rb]$ consists of those states that are either in $[Ra]$ or in $[Rb]$. These are $\{11, 10\}$, $\{11, 01\}$, and all substates thereof.

Since $\mathsf{info}(Ra \vee Rb) = \bigcup[Ra \vee Rb] = \{11, 10, 01\} \neq W$, the disjunction $Ra \vee Rb$ is informative. It provides the information that at least one of a and b is running. However, unlike in the case of atomic sentences, in this case there is no unique greatest element in $[Ra \vee Rb]$ that includes all the others. Instead, there are *two* maximal elements, $|Ra| = \{11, 10\}$ and $|Rb| = \{11, 01\}$, which together contain all the others. Thus, $Ra \vee Rb$ is not a statement; besides being informative it is also inquisitive. In order to settle the issue that it raises, one has to establish either that a is running, or that b is running.

A note of caution is perhaps in order here: it is important to keep in mind that InqB does not embody an analysis of sentences in natural language, it only provides the tools to formulate such analyses. In particular, a disjunctive sentence in InqB like $Ra \vee Rb$ does not necessarily correspond to a disjunctive declarative in English like (1) below, or to a disjunctive interrogative like (2) for that matter.

(1) Ann is running or Bill is running.

(2) Is Ann running or is Bill running?

In Chapter 6 we will present a concrete analysis of sentences like (1) and (2), using InqB. On that analysis, (1) corresponds to $!(Ra \vee Rb)$ and (2) either to $?(Ra \vee Rb)$ or to $Ra \vee Rb$, depending on intonation.

Negation. Next, we turn to negation. According to the clause for negation, $[\neg Ra]$ consists of all states s such that s does not have any world in common with any state in $[Ra]$. Thus, $[\neg Ra]$ consists of all states that do not contain the worlds 11 and 10, which are $|\neg Ra| = \{01, 00\}$ and all substates thereof, as depicted in Figure 4.1(d). Since this set of states has a greatest element, Fact 4.13 ensures that $\neg Ra$ is a statement. It provides the information that a is not running, and does not request any further information.

Now let us consider the negation of an inquisitive disjunction, $\neg(Ra \vee Rb)$. According to the clause for negation, $[\neg(Ra \vee Rb)]$ consists of all states which do not have a world in common with any state in $[Ra \vee Rb]$. Thus, $[\neg(Ra \vee Rb)]$ consists of all states that do not contain the worlds 11, 10, and 01, which are $\{00\}$ and $\emptyset$, as depicted in Figure 4.1(e). Again, there is a unique maximal element, namely $|\neg(Ra \vee Rb)| = \{00\}$. Thus, $\neg(Ra \vee Rb)$ is a statement, which provides the information that neither a nor b is running, just like in classical logic, and does not request any further information.

These examples of negative sentences exemplify the general observation that we made above concerning pseudo-complementation (just below Fact 3.4): the absolute pseudo-complement of a proposition always contains a unique alternative. This means that a negative sentence $\neg\varphi$ is always a statement, which provides the information that φ is false, and does not request any further information.

Projection operators. Next let us consider $!(Ra \vee Rb)$, which abbreviates $\neg\neg(Ra \vee Rb)$. We have just seen that $\neg(Ra \vee Rb)$ expresses the proposition depicted in Figure 4.1(e). Applying negation again, we arrive at the proposition depicted in Figure 4.1(f), which has $|Ra \vee Rb|$ as its unique alternative. Notice that $!(Ra \vee Rb)$ is not equivalent with $Ra \vee Rb$. The two sentences have the same informative content, but the former is a purely informative statement, while the latter is also inquisitive. This exemplifies the general nature of !: for any sentence φ, $!\varphi$ is a statement with the same informative content as φ. If φ itself is already a statement, then $!\varphi$ and φ are equivalent; if φ is inquisitive, as in the example just considered, the two differ.

Let us now turn to ?. Consider $?Ra$, which is an abbreviation of $Ra \vee \neg Ra$. We have already seen what $[Ra]$ and $[\neg Ra]$ are. According to the clause for disjunction, $[?Ra] = [Ra \vee \neg Ra]$ consists of all states that are either in $[Ra]$ or in $[\neg Ra]$. These states are $|Ra|$, $|Rb|$, and all substates thereof, as depicted in Figure 4.1(g). Since $\mathsf{info}(?Ra) = W$, $?Ra$ is not informative, which means that it is a question. Moreover, since $[?Ra]$ contains two alternatives, it is inquisitive. In order to settle the issue that it raises, one has to establish either that a is running, or that a is not running. That is, one has to establish *whether* a is running. Thus, while $?Ra$ is shorthand for $Ra \vee \neg Ra$, perhaps the most famous classical tautology, it is not a tautology in InqB: instead, it corresponds to the polar question "whether Ra". Analogously, $?Rb$, depicted in Figure 4.1(h), corresponds to the polar question "whether Rb".

If ? applies to the disjunction $Ra \vee Rb$, which is already inquisitive, then it yields the proposition depicted in Figure 4.1(i). $[Ra \vee Rb]$ already contains two alternatives, $|Ra|$ and $|Rb|$; ? adds a third alternative, which is the set of worlds that are neither in $|Ra|$ nor in $|Rb|$. Thus, in order to resolve the issue raised by $?(Ra \vee Rb)$, one either has to establish that a is running, or that b is, or that neither a nor b is.

Finally, let us consider a case where ! and ? both apply, one after the other: $?!(Ra \vee Rb)$. As we already saw above, $[!(Ra \vee Rb)]$ contains a single alternative, consisting of all worlds where at least one of a and b is running. As depicted in Figure 4.1(j), ? adds a second alternative, which is the set of worlds where neither a nor b is running. Notice that the resulting proposition differs from that expressed by $?(Ra \vee Rb)$, which contains three alternatives rather than two. In order to settle the issue expressed by $?!(Ra \vee Rb)$ it is sufficient to establish that at least one of a and b is running. In order to settle the issue expressed by $?(Ra \vee Rb)$ this is not sufficient; rather, it needs to be established *which* of a and b is running (or that neither of them is). We will see in Chapter 6 that the ability to capture such subtle differences is crucial to account for various kinds of disjunctive questions in natural languages.

Conjunction. Next, let us consider conjunction. First, let us look at the conjunction of our two atomic, non-inquisitive sentences, Ra and Rb. According to the clause for conjunction, $[Ra \wedge Rb]$ consists of those states that are both in $[Ra]$ and in $[Rb]$. These are $\{11\}$ and $\emptyset$. Thus, $[Ra \wedge Rb]$ has a unique greatest element, namely $\{11\}$, and accordingly $Ra \wedge Rb$ is a statement which, just like in the classical case, provides the information that both a and b are running.

Now let us look at the conjunction of two inquisitive sentences, $?Ra$ and $?Rb$. As depicted in Figure 4.1(l), the proposition $[?Ra \wedge ?Rb]$ contains four alternatives, $|Ra \wedge Rb|$, $|Ra \wedge \neg Rb|$, $|\neg Ra \wedge Rb|$, and $|\neg Ra \wedge \neg Rb|$. Since these alternatives together cover the entire logical space $?Ra \wedge ?Rb$ is a question. Moreover, since there is more than one alternative, this question is inquisitive. In order to settle the issue that it raises, one has to establish one of $Ra \wedge Rb$, $Ra \wedge \neg Rb$, $\neg Ra \wedge Rb$, $\neg Ra \wedge \neg Rb$. Thus, our conjunction is a question which requests enough information to settle both the issue whether Ra is the case, contributed by $?Ra$, and the issue whether Rb is the case, contributed by $?Rb$.

These two examples of conjunctive sentences exemplify a general fact: if φ and ψ are statements, then the conjunction $\varphi \wedge \psi$ is also a statement, which provides both the information provided by φ and the information provided by ψ; and if φ and ψ are questions, then the conjunction $\varphi \wedge \psi$ is also a question, which requests enough information to settle both the issue raised by φ and the issue raised by ψ.

Implication. Next, let us consider implication. Again, we will first consider a simple case, $Ra \to Rb$, where both the antecedent and the consequent are atomic, and therefore non-inquisitive. According to the clause for implication, $[Ra \to Rb]$ consists of all states s such that every substate $t \subseteq s$ that is in $[Ra]$ is also in $[Rb]$. These are all and only those states that are contained in $|Ra \to Rb| = \{11, 01, 00\}$, as depicted in Figure 4.1(m). So, $[Ra \to Rb]$ has a unique greatest element, $|Ra \to Rb|$, which means that the implication $Ra \to Rb$ is a statement which, just like in the classical setting, provides the information that if a is running, then so is b.

Now let us consider a more complex case, $Ra \to ?Rb$, where the consequent is an inquisitive question. As depicted in Figure 4.1(n), the proposition $[Ra \rightarrow ?Rb]$ contains two alternatives, $|Ra \rightarrow Rb| = \{11, 01, 00\}$, and $|Ra \rightarrow \neg Rb| = \{10, 01, 00\}$. Since these two alternatives together cover the entire logical space, our implication is a question. Moreover, since there is more than one alternative, the implication is inquisitive. In order to settle the issue that it raises, one must either establish $Ra \rightarrow Rb$, or $Ra \rightarrow \neg Rb$. In the former case one establishes that if a is running, then so is b; in the latter case, that if a is running, then b isn't. So $Ra \rightarrow ?Rb$ is a question which requests enough information to establish whether b is running under the assumption that a is.

Again, these two examples of conditional sentences exemplify a general feature of InqB: if ψ is a statement, then $\varphi \rightarrow \psi$ is a statement as well, providing the information that if φ holds, then so does ψ; and if φ a statement and ψ a question, then $\varphi \rightarrow \psi$ is a question as well, requesting enough information to settle the issue raised by ψ assuming the information provided by φ.

Quantification. Finally, let us consider existential and universal quantification. As usual, existential quantification behaves essentially like disjunction and universal quantification behaves essentially like conjunction. In fact, since our current domain of discourse consists of only two objects, denoted by a and b, respectively, $\exists x.Rx$ expresses exactly the same proposition as $Ra \vee Rb$, depicted in Figure 4.1(c), and $\forall x.Rx$ expresses exactly the same proposition as $Ra \wedge Rb$, depicted in Figure 4.1(k). Finally, consider the proposition expressed by $\forall x.?Rx$, depicted in Figure 4.1(o). Notice that this proposition induces a partition on the logical space, where each block of the partition consists of worlds that agree on the extension of P. Thus, $\forall x.?Rx$ is a question that asks for an exhaustive specification of the objects that are running. This concludes our illustration of the behavior or the connectives and quantifiers in InqB.

4.5 Informative content, truth, and support

Recall that $\mathsf{info}(\varphi)$ is defined as $\bigcup[\varphi]$, which is a set of worlds. In classical logic, the informative content of a sentence φ is also embodied by a set of worlds, namely the set of all worlds where φ is true, $|\varphi|$. Thus, the question that naturally arises is how these two notions of informative content relate to each other. It turns out that the two are precisely the same.

FACT 4.17. [Informative content and truth] For any sentence $\varphi \in \mathcal{L}$:

- $\mathsf{info}(\varphi) = |\varphi|$

This shows that InqB fully preserves the classical treatment of informative content. The system only differs from classical logic in that, besides informative content, it takes inquisitive content into consideration as well.

Notice that Facts 2.41 and 4.17 together yield the following characterization of questions and statements in terms of classical truth.

Fact 4.18. [Questions and statements in terms of classical truth]

- φ is a question $\iff |\varphi| = W$
- φ is a statement $\iff |\varphi| \in [\varphi] \iff [\varphi] = \wp(|\varphi|)$

Thus, questions in InqB are precisely those sentences that are classically true at any world. On the other hand, a sentence is a statement in InqB just in case the proposition it expresses is fully determined by its classical truth-set: $[\varphi] = \wp(|\varphi|)$. This means that a statement φ provides the information that φ is true, and does not request any further information. Thus, statements behave exactly as they do in classical logic.

The classical behavior of statements results in a tight connection between their support conditions and their truth conditions. Namely, a statement φ is supported by a state s just in case it is true in every world in s. This holds only for statements; the moment a sentence becomes inquisitive, the connection between support and truth breaks down.

Fact 4.19. [Support and truth]
The following are equivalent for any sentence $\varphi \in \mathcal{L}$:

- φ is a statement
- $s \models \varphi$ just in case φ is true in every world in s

4.6 Syntactic properties of statements and questions

We have defined statements as non-inquisitive sentences, and questions as non-informative sentences. These characterizations are *semantic* in nature. Below we provide some *syntactic* conditions which make it easy to recognize a large class of statements and questions (though not all of them) just based on their form, without inspecting their meaning.

Let us start with statements. The following fact provides some syntactic conditions which guarantee that a sentence is a statement. These conditions generalize some of the more specific observations that were already made in discussing the examples above.

Fact 4.20. [Sufficient conditions for statements]

1. $!\varphi$ is always a statement;
2. Atomic sentences are always statements;

3. $\neg\varphi$ is always a statement;
4. If φ and ψ are statements, then so is $\varphi \wedge \psi$;
5. If ψ is a statement, then so is $\varphi \to \psi$ for any antecedent φ;
6. If $\varphi(d')$ is a statement for all $d \in D$, then so is $\forall x\varphi(x)$.

Now let us turn to questions. Again we provide some syntactic conditions that guarantee that a given sentence is a question, generalizing some of the more specific observations made in discussing the examples above.

FACT 4.21. [Sufficient conditions for questions]

1. $?\varphi$ is always a question;
2. A classical tautology is always a question;
3. If φ and ψ are questions, so is $\varphi \wedge \psi$;
4. If ψ is a question, then so are $\varphi \vee \psi$ and $\varphi \to \psi$, for any φ;
5. If $\varphi(d')$ is a question for all $d \in D$, then so is $\forall x\varphi(x)$;
6. If $\varphi(d')$ is a question for some $d \in D$, then so is $\exists x\varphi(x)$.

4.7 Sources of inquisitiveness

The partial syntactic characterization of statements given in Fact 4.20 implies that disjunction, the existential quantifier, and the ? projection operator are the only sources of inquisitiveness in our logical language.

FACT 4.22. [Sources of inquisitiveness]
Any sentence that does not contain $\vee$, $\exists$, or ? is a statement.

Note that there is a close connection between disjunction, the existential quantifier, and the ? operator in InqB. Namely, they all behave as join operators: $[\varphi \vee \psi]$ is the join of $[\varphi]$ and $[\psi]$, $[\exists x.\varphi(x)]$ is the join of $\{[\varphi(d')] \mid d \in D\}$, and $[?\varphi]$ is the join of $[\varphi]$ and $[\varphi]^*$. In terms of semantic operators, then, the join operator is the essential source of inquisitiveness: without applying this operator, it is impossible to produce inquisitive propositions from non-inquisitive ones.

This fact may provide the basis for an explanation of the well-known observation that in many natural languages, question markers are homophonous with words for disjunction and/or existentials (see Jayaseelan, 2001, 2008; Bhat, 2005; Haida, 2007; Cable, 2010; AnderBois, 2011; Slade, 2011, among others). For instance, Malayalam *-oo* and Japanese *ka* are used for all three purposes:

Malayalam	Japanese	English translation
aar-**oo**	dare-**ka**	someone (existential)
Anna-**oo** Peter-**oo**	Anna-**ka** Peter-**ka**	Anna or Peter (disjunction)
Anna wannu-(w)**oo**	Anna wa kita-**ka**	Did Anna come? (interrogative)

Szabolcsi (2015b) proposes an account of this cross-linguistic phenomenon in inquisitive semantics, suggesting that the inquisitive join operation can indeed be seen as the semantic common core of disjunction, existentials, and question markers in languages like Malayalam and Japanese.

It is also interesting to note that there is a close connection between the treatment of disjunction and existentials in InqB, and their treatment in *alternative semantics* (Kratzer and Shimoyama, 2002; Simons, 2005; Menéndez-Benito, 2005; Alonso-Ovalle, 2006; Aloni, 2007, among others). In both frameworks, disjunction and existentials introduce sets of alternatives. In the case of alternative semantics, this treatment is motivated by a number of empirical linguistic phenomena, including free choice inferences, exclusivity implicatures, and counterfactual conditionals with disjunctive antecedents. The analysis of disjunction and existentials as introducing sets of alternatives has made it possible to develop new accounts of these phenomena which improve considerably on previous accounts. However, work on alternative semantics has not provided motivation for the alternative treatment of disjunction and indefinites *independently* of the linguistic phenomena that it has aimed to capture. Therefore, though successful in terms of empirical coverage, it has limited explanatory power. Moreover, the treatment of disjunction and existentials in alternative semantics has been presented as one that is really *incompatible* with the classical algebraic treatment of disjunction and existentials. For instance, Alonso-Ovalle (2006) writes in the conclusion section of his dissertation:

> "This dissertation has investigated the interpretation of counterfactuals with disjunctive antecedents, unembedded disjunctions, and disjunctions under the scope of modals. We have seen that capturing the natural interpretation of these constructions proves to be challenging if the standard analysis of disjunction, under which *or* is the Boolean join, is assumed."

Similarly, Simons (2005) starts her paper as follows:

> "In this paper, the meanings of sentences containing the word *or* and a modal verb are used to arrive at a novel account of the meaning of *or* coordinations. It has long been known that such sentences [...] pose a problem for the standard treatment of *or* as a Boolean connective equivalent to set union."

One wonders how deep the incompatibility between the classical algebraic treatment of *or* and its treatment in alternative semantics really is. Are we forced to choose one over the other, or is it possible to reconcile the two approaches somehow?

As discussed in detail in Roelofsen (2015b), the algebraic approach we have taken here sheds new light on these two issues. First, it shows that, once we take both informative and inquisitive content into account, general algebraic considerations lead essentially to the treatment of disjunction that was proposed in alternative semantics, thus providing exactly the independent motivation that has so far been missing.[1] Moreover, it shows that the 'alternative' treatment of disjunction is actually a natural generalization of the classical treatment: disjunction can still be taken to behave semantically as a join operator, only now the meanings that this join operator applies to are more fine-grained in order to capture both informative and inquisitive content. Thus, we can have our cake and eat it: we can treat disjunction as a join operator and as introducing sets of alternatives at the same time. In inquisitive semantics, the two go hand in hand.

4.8 Exercises

Exercise 4.1. [De Morgan's laws]
Below are two well-known equivalences from classical logic, known as De Morgan's laws:

$$\neg(\varphi \vee \psi) \equiv \neg\varphi \wedge \neg\psi$$
$$\neg(\varphi \wedge \psi) \equiv \neg\varphi \vee \neg\psi$$

Do these equivalences also hold in inquisitive semantics? If yes, give a proof. If no, provide a counterexample.

Exercise 4.2. [The law of double negation]
Recall that in classical logic, $\neg\neg\varphi \to \varphi$ is a tautology for any φ. Show that in InqB:

1. $\neg\neg\varphi \to \varphi$ is a tautology whenever φ is a statement;
2. It is not the case that $\neg\neg\varphi \to \varphi$ is a tautology for any φ.

Explain why this difference between classical logic and InqB arises, even though $\neg$ and $\to$ express exactly the same algebraic operations in both frameworks (absolute and relative pseudo-complementation, respectively).

[1]It should be noted that, while both in alternative semantics and in inquisitive semantics disjunction generates alternatives, there is also a subtle but important difference. Namely in inquisitive semantics one alternative can never be contained in another. This has certain advantages, which are discussed in Section 5.1.

Chapter 5

A new framework for question semantics

In Chapters 2-4 we laid out the basic architecture of inquisitive semantics. In the present chapter and the ones to follow, we will discuss some of its applications and its relation to other frameworks. As discussed in Chapter 1, one of the main purposes of inquisitive semantics is to serve as a framework for the semantic analysis of questions in natural languages.[1] There is a large body of previous work on the semantics of questions. A number of general frameworks have been proposed, and within these frameworks many specific theories have been formulated. In this chapter we will not yet formulate a specific theory of questions in natural language—this will be done in Chapter 6—but rather compare inquisitive semantics, as a new framework for question semantics, with other general frameworks that have been proposed in previous work. In doing so, we will restrict our attention to those previous proposals that are most closely related to our own. That is, we will consider the *alternative semantics* framework proposed by Hamblin (1973) and Karttunen (1977), the *partition semantics* of Groenendijk and Stokhof (1984), and the *dynamic* framework for question semantics developed in (Jäger, 1996; Hulstijn, 1997; Groenendijk, 1999, 2009; Mascarenhas, 2009).[2] We will argue that inquisitive semantics preserves the essential insights that have emerged from these previous approaches, while overcoming their main shortcomings.

5.1 Alternative semantics

Alternative semantics is probably the most widely used framework for the semantic analysis of questions in natural language. It was first proposed by Hamblin (1973), driven by the following idea:

[1]Throughout this chapter, we use the term 'questions' to refer to a class of sentences in natural languages, not to a class of propositions/sentences in InqB, as we did in Chapter 2-4.

[2]One prominent approach that we will not discuss here is the *functional* approach (sometimes also called the *categorial* approach or the *structured meanings* approach). We refer to Krifka (2001a, 2011) and Groenendijk and Stokhof (1997) for overviews.

> "Questions set up a choice-situation between a set of propositions, namely those propositions that count as answers to it."
> (Hamblin, 1973, p.48)

Thus, Hamblin takes questions to denote sets of classical propositions. These propositions are often referred to as *alternatives*, hence the name of the framework. Karttunen (1977) independently proposed a very similar view on question meanings: he also took questions to denote sets of classical propositions, though he restricted the denotation of a question in a particular world to propositions that correspond to answers that are *true* in that world. In both systems, the *meaning* of a question, i.e., its intension, is a function from worlds to sets of classical propositions. In Hamblin's system, this function maps every possible world to the same set of propositions, corresponding to the set of all possible answers; in Karttunen's system, every world is mapped to a subset of all possible answers, namely those that are true in the given world. As noted by Karttunen (1977, p.10), this difference is inessential. In both cases, the meaning of a question is fully determined by—and could be identified with—the set of all classical propositions that correspond to a possible answer.[3]

This classical view on question meanings faces some fundamental problems. We will discuss these, and show that they no longer arise in inquisitive semantics.

Problem 1: Possible answers

The first problem is that the framework's core notion—that of a *possible answer*—is difficult to pin down. Surely, Hamblin and Karttunen provide a compositional semantics for a fragment of English, and thereby specify what they take to be the possible answers to the questions in that fragment. But in order to assess such a compositional theory, or even to properly understand what its predictions amount to, we first need to have a pre-theoretical notion of possible answers, one that the theoretical predictions can be evaluated against. The problem is that such a pre-theoretical notion is difficult, if not impossible to identify. To illustrate this, consider the question in (1) and the responses in (2):

[3] It should be noted that there are significant differences between Hamblin's and Karttunen's approach concerning the *compositional derivation* of question meanings. In a nutshell, Hamblin proposes a rather radical departure from the standard Montagovian approach to meaning composition, adapting the semantic type of all lexical items and letting the operation that is standardly used to compose the meanings of two constituents, i.e., *function application*, operate in a pointwise fashion. Karttunen on the other hand sticks to the standard Montagovian procedure. While Hamblin's more radical proposal has been adopted more widely, its compositional apparatus faces a number of thorny problems (see, e.g., Shan, 2004; Novel and Romero, 2010; Theiler, 2013; Charlow, 2014; Ciardelli and Roelofsen, 2015a). These problems can be overcome in inquisitive semantics in a principled way; however, a detailed discussion of compositionality is beyond the scope of these lecture notes; we refer to Theiler (2013, 2014) and Ciardelli and Roelofsen (2015a).

(1) Who is coming for dinner tonight?

(2) a. Paul is coming.
b. Paul and Nina are coming.
c. Only Paul and Nina are coming.
d. Some girls from my class are coming.
e. Paul or Nina is coming.
f. Paul is not coming.
g. I don't know.
h. Are we having dinner tonight?

In principle, all the responses in (2) could be seen as possible answers to (1). For Hamblin and Karttunen, only (2a) counts as such. However, it is not clear what the precise criteria are for being considered a possible answer, and on which grounds (2a) is to be distinguished from (2b-h).

In inquisitive semantics, question meanings are also sets of classical propositions, just like in alternative semantics. However, in inquisitive semantics these classical propositions are not thought of as the 'possible answers' to the question. Rather, they are thought of as the information states—or equivalently, the pieces of information—that *resolve* the issue that the question raises. As a consequence, in inquisitive semantics question meanings cannot be defined as arbitrary sets of classical propositions, which is what Hamblin and Karttunen take them to be. Rather, they have to be *downward closed*. After all, if an information state s resolves the issue raised by a given question Q, then any stronger information state $t \subset s$ will also resolve the issue raised by Q.

In our view, pre-theoretical intuitions about which pieces of information resolve a given issue are much more robust than pre-theoretical intuitions about what the 'possible answers' to a given question are. Thus, evaluating theories of questions formulated in inquisitive semantics is more feasible than evaluating theories of questions which, like Hamblin and Karttunen, view question meanings as sets of possible answers.

This said, we should hasten to emphasize that, even though inquisitive semantics does not assume a one-to-one correspondence between the classical propositions that constitute a question meaning and the 'possible answers' to that question, this is not to say that question meanings in inquisitive semantics cannot play a role in characterizing sensible notions of answerhood at all. Quite the contrary.

For instance, it would be natural to characterize the *basic answers* to a question Q as those pieces of information that:

(i) resolve the issue raised by Q, and

(ii) do not provide more information than necessary to do so, i.e., are not strictly stronger than any other piece of information that also resolves Q.

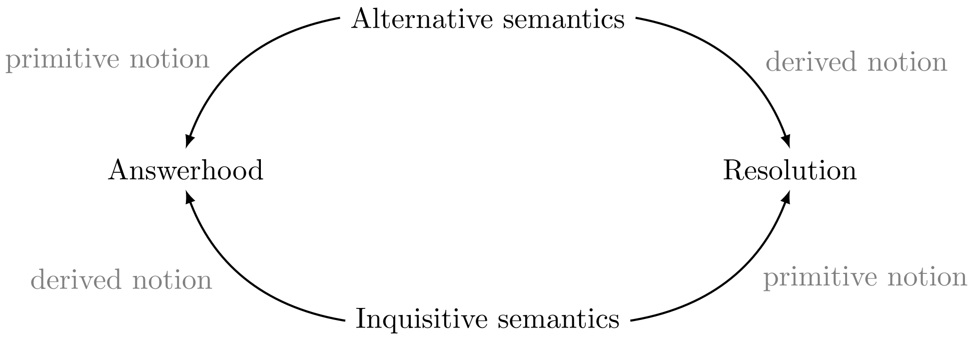


Figure 5.1: Primitive and derived notions in alternative/inquisitive semantics.

Under this definition, the basic answers to Q correpond precisely to what we called the *alternatives* in the proposition expressed by Q.[4,5] Along the same lines, we may also define notions of *partial answerhood* and *subquestionhood* (see, e.g., Groenendijk and Roelofsen, 2009), which are crucial for the analysis of discourse structure and information structure (see, e.g., Roberts, 1996; Büring, 2003).

What is crucial is that in inquisitive semantics question meanings are not characterized *in terms of* basic/possible/complete/partial answers. Rather, as depicted in Figure 5.1, it is the other way around: question meanings, i.e., issues, are defined in terms of what it takes to resolve them, and the basic/possible/complete/partial answers to a question are defined in terms of its meaning. As a consequence, whichever notion of basic/possible/complete/partial answerhood we choose to adopt, there will be no need for such a notion to correspond directly to some pre-theoretical concept. Rather, it will be grounded, in a precisely circumscribed way, in the pre-theoretical notion of what it takes for a given issue to be resolved. For instance, if as suggested above we characterize the basic answers to a question as those pieces of information that resolve the issue that the question raises and do not provide more information than is necessary to do so, then in order to evaluate a theory that associates every question with

[4]Recall from footnote 6 that some propositions in InqB do not contain any alternatives. According to the characterization of basic semantic answers just given, questions expressing such propositions do not have any basic semantic answers. See Ciardelli (2010), Ciardelli *et al.* (2013b), and Roelofsen (2013a) for further discussion of such cases.

[5]Notice that our definition of basic answers as minimal resolving pieces of information implies that the set of basic answers to a given question is always a set $\mathcal{A}$ such that for any $s, t \in \mathcal{A}$, neither $s \subset t$ nor $t \subset s$. After all, if $s \subset t$, then s cannot be a *minimal* resolving piece of information, and vice versa if $t \subset s$. Thus, even if—in the spirit of Hamblin and Karttunen—we were to identify the meaning of a question with the set of its basic answers, not just any set of classical propositions would count as a proper question meaning. We will return to this point below.

a set of basic answers, we can simply rely on judgments concerning resolution, rather than judgments directly concerning 'basic answers'.

Problem 2: Entailment

A second fundamental problem for alternative semantics, which was pointed out and discussed at length in Groenendijk and Stokhof (1984), is that it is difficult to define a suitable notion of *entailment* in this framework that determines when one question is more demanding than another. One consequence of this is that it is hard, if not impossible, to give a principled account of the interaction between questions and logical connectives and quantifiers. For instance, it proves problematic to give a satisfactory treatment of the conjunction of two questions. Without a suitable notion of entailment, conjunction can certainly no longer be treated as a *meet* operator.[6]

This problem does not arise in inquisitive semantics, which has a well-behaved notion of entailment. As discussed in Chapter 3, the space of propositions in inquisitive semantics, ordered by entailment, has a familiar algebraic structure, and a natural treatment of the logical connectives is obtained by associating them with the basic operations in this algebra. Thus, as we have seen, the classical treatment of conjunction as a *meet* operation can be preserved in inquisitive semantics to apply to informative and inquisitive sentences in a uniform way, and the same goes for the other operations.

The two problems that we just discussed for alternative semantics are closely related. After all, if it were possible to ground the notion of 'possible answers' in some pre-theoretical notion, then it would most likely also become clear how to characterize entailment. That is, if there were clear criteria for what it takes to count as a possible answer, we would also know better on which grounds two sets of possible answers should be compared, and under which conditions one set should be seen as entailing another.[7]

Compare the situation with the one we have in classical logic. There, the proposition expressed by a sentence is a set of possible worlds, which are intended to correspond to situations that are compatible with the information that the sentence conveys. In this case, there is a clear pre-theoretical intuition to build

[6]See Roelofsen (2013a) and Ciardelli and Roelofsen (2015a, 2016) for more elaborate discussion of this point, and a critical assessment of some concrete notions of entailment and conjunction that may be considered in alternative semantics.

[7]It is not the formal notion of meaning as such that stands in the way of a suitable notion of entailment, but really the conception of these meanings in terms of possible answers. For instance, if we construe the meaning of a sentence as a set of classical propositions, as in alternative semantics, but think of these propositions as those that the sentence *draws attention to*, rather than as possible answers, then it is quite straightforward to define a suitable notion of entailment, which compares two sentences/meanings in terms of their attentional strength (Roelofsen, 2013b).

on, as to whether a certain situation is or is not compatible with a given piece of information. As a consequence, it is also clear when one sentence should be taken to entail another, namely if it conveys at least as much information, meaning that the proposition it expresses is a subset of the proposition expressed by the other sentence. In alternative semantics, the meaning of a question is a set of classical propositions which are intended to correspond to its possible answers. However, since it is not clear when exactly a proposition should count as a possible answer, it is also difficult to say when one question should entail another.

In inquisitive semantics, the proposition expressed by a question is a set of information states, which are intended to be those information states in which the issue that the question raises is resolved. It is also clear, then, when one question is more demanding than another, namely if every information state that resolves the former also resolves the latter. This immediately delivers the desired notion of entailment, as well as the algebraic operations that are characterized in terms of it.

Problem 3: Overgeneration

A third problem, which is again connected to the other two, is that there are question meanings in alternative semantics which seem impossible to express in natural languages. These are question meanings containing two alternatives α and β such that one is strictly contained in the other, $\alpha \subset \beta$.

One may think that such meanings may be expressed by disjunctive questions, where each disjunct contributes one of the two alternatives. However, in order to get that $\alpha \subset \beta$, we would have to construct the question in such a way that one disjunct classically entails the other. As illustrated in (3) and (4) below, such questions are infelicitous (Ciardelli and Roelofsen, 2016).

(3) #Is John American, or is he Californian?

(4) #Is the value of x different from 6, or is it greater than 6?

It has been well-known since Hurford (1974) that disjunctive declaratives where one disjunct entails the other are generally infelicitous as well.

(5) #John is American or he is Californian.

(6) #The value of x is different from 6 or it is greater than 6.

This phenomenon, known as *Hurford's constraint*, has been given an appealing explanation in terms of *redundancy*. More specifically, Katzir and Singh (2013) propose the following principle (see also Simons, 2001; Meyer, 2014, for closely related proposals):

> **Local redundancy:** a sentence is deviant if its logical form contains a node $O(A, B)$ which is obtained by application of a functor O to two arguments A, B, and the outcome is semantically equivalent to one of the arguments.[8]

Let us briefly consider how this principle predicts Hurford's constraint. In classical semantics, the meaning of a sentence A is a classical proposition $|A|$, the set of worlds where the sentence is true. A entails B just in case $|A| \subseteq |B|$. Moreover, sentential disjunction yields the union of two propositions, that is, $|\mathsf{or}(A, B)| = |A| \cup |B|$.

Now, suppose that the logical form of a sentence contains a node at which disjunction applies to two arguments A and B such that $|A| \subseteq |B|$, as in examples (5) and (6). Then we have that $|\mathsf{or}(A, B)| = |A| \cup |B| = |B|$. So, there is indeed a node that is obtained by application of a binary operator whose outcome is semantically equivalent with one of its inputs. Thus, the given logical form exhibits local redundancy and is therefore predicted to be deviant.

Now, one would of course hope that this explanation of Hurford's constraint in terms of redundancy would apply not only to declaratives like (5) and (6), but also to questions like (3) and (4). But this is not the case in alternative semantics, where the disjuncts express singleton sets, $\{|A|\}$ and $\{|B|\}$, respectively, and disjunction yields the set $\{|A|, |B|\}$. Since the output of the disjunction operator is different from any of its inputs, the local redundancy condition is not violated, and no deviance is therefore predicted.

In inquisitive semantics, the explanation of Hurford's constraint in terms of redundancy does naturally apply to questions like (3) and (4), assuming that each of the disjuncts expresses a proposition containing all states that consist exclusively of worlds where that disjunct is true (just like atomic sentences in InqB), and English or is treated as a join operator, just like disjunction in InqB. We then have that $[A] = \wp(|A|)$, $[B] = \wp(|B|)$, and $[\mathsf{or}(A, B)] = [A] \cup [B] = \wp(|A|) \cup \wp(|B|) = \wp(|B|) = [B]$. Thus, the output of the disjunction operator is identical to one of its inputs, and redundancy is predicted just as for declarative Hurford disjunctions.

Let us try to better understand this contrast between inquisitive semantics and alternative semantics by considering the notion of 'alternatives' that plays a role in the two frameworks. We have seen that both frameworks associate questions with sets of alternatives, but that the status of these alternatives crucially differs from one framework to the other.

In inquisitive semantics, the alternatives in the proposition expressed by a question are characterized as those pieces of information that resolve the issue that the question raises in a minimal way. This implies that sets of alternatives

[8] Katzir and Singh (2013)'s proposal is relativized to a context of utterance c. Since context-dependency plays no role in our discussion, we omit reference to contexts for ease of exposition.

have to be of a particular form: two alternatives are always logically independent, that is, one is never contained in the other.

In alternative semantics on the other hand, there is no such constraint on sets of alternatives: any set will do. This is connected, of course, to the fact that the notion of an alternative is a primitive notion in this framework, not defined in terms of resolution conditions or any other more elementary notion.

Let us refer to sets of classical propositions whose elements are pairwise logically independent as *non-nested sets*. In inquisitive semantics, then, unlike in alternative semantics, only non-nested sets of classical propositions are regarded as proper sets of alternatives. Thus, certain meanings in alternative semantics do not have a counterpart in inquisitive semantics. It is precisely these additional meanings, i.e., nested sets of alternatives, which seem impossible to express in natural languages, at least ones like English. In principle, a Hurford disjunction would be exactly the right kind of construction to express a nested set of alternatives. But we have seen that such disjunctions are infelicitous.[9] This seems to indicate that there is something wrong with nested sets of alternatives as meanings, which is puzzling from the perspective of alternative semantics, since in this framework nested sets of alternatives have exactly the same status as non-nested sets.

In inquisitive semantics, the puzzle does not arise, because nested sets of alternatives simply do not exist. Importantly, such sets are not ruled out by some special purpose constraint: rather, it just follows from the way alternatives are construed that they are never nested. This means that from the perspective of inquisitive semantics, what is special about Hurford disjunctions is not that they express some distinguished class of meanings, but rather that they involve redundant disjuncts, which fail to contribute an alternative to the meaning of the disjunction. As we have seen, this is precisely what makes it possible to explain their infelicity.

5.2 Partition semantics

Groenendijk and Stokhof (1984) propose that a question does not denote a set of

[9]It should be noted that there are apparent counterexamples to Hurford's constraint, which may seem to undermine the argument that we are making here. For instance:

(i) Bill solved two of the homework problems, or he solved all of them.

At first blush, it seems that the second disjunct entails the first, and yet the sentence is felicitous. However, as argued in detail by Chierchia *et al.* (2009), in such cases the weaker disjunct receives a strengthened interpretation—here, that Bill solved *only* two of the homework problems—which in effect makes it logically independent from the other disjunct. For a more detailed exposition of the argument that we are presenting here, taking cases like (i) into account, we refer to Ciardelli and Roelofsen (2016).

classical propositions at each world, but rather a single classical proposition embodying the *true exhaustive* answer to the question in that world. For instance, if w is a world in which Paul and Nina are coming for dinner, and nobody else is coming, then the denotation of (7) in w is the classical proposition expressed by (8).

(7) Who is coming for dinner tonight?

(8) Only Paul and Nina are coming.

The *meaning* of a question, i.e., its intension, then amounts to a function from worlds to classical propositions. In Groenendijk and Stokhof's framework these classical propositions are required to have two special properties: they have to be *mutually exclusive* (since two different exhaustive answers are always incompatible), and together they have to *cover* of the entire logical space (since every world is taken to be compatible with at least one exhaustive answer). So the meaning of a question is a set of classical propositions that together form a *partition* of the logical space.

Problem: Undergeneration

Partitions correspond to a specific kind of propositions in inquisitive semantics. That is, for every partition ρ, there is a corresponding proposition P_ρ in inquisitive semantics, consisting of all states that are contained in one of the blocks of the partition:

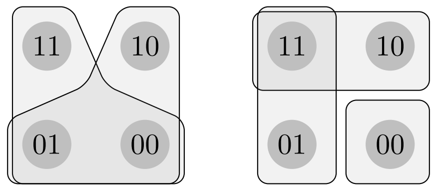
$$P_\rho \coloneqq \{s \subseteq b \mid b \in \rho\}$$

On the other hand, not every proposition in inquisitive semantics corresponds to a partition. This holds in particular for all propositions in inquisitive semantics containing overlapping alternatives, and ones whose elements do not cover the entire logical space.

Thus, while we saw that the range of question meanings in inquisitive semantics is more confined than in alternative semantics, it is broader than in partition semantics. In terms of expressive power, then, there is a strict linear order between the three frameworks:

alternative semantics > inquisitive semantics > partition semantics

We have already discussed some of the benefits of inquisitive semantics w.r.t. alternative semantics, and these benefits are all connected in one way or another to the difference in expressive power between the two frameworks. With respect to partition semantics, the main benefit of inquisitive semantics is that, because of its more general notion of meaning, it allows us to deal with a broader range



(a) [[(9)]] (b) [[(10)]], [[(11)]]

Figure 5.2: Propositions expressed by (9)–(11), problematic for partition semantics.

of questions in natural language. In particular, without further amendments it is impossible in partition semantics to give a satisfactory account of conditional questions, disjunctive questions, and mention-some wh-questions, exemplified in (9)–(11).

(9) If Peter attends the meeting, will Maria attend it too?

(10) Will Peter↑ attend the meeting, or Maria↑?

(11) Who will attend the meeting? (on a mention-some reading)

As we discussed in Section 2.5, these sentences would be treated in inquisitive semantics as expressing the propositions depicted in Figure 5.2. Note that these propositions have *overlapping* alternatives, which means that they do not constitute partitions of the logical space. And yet they correctly capture what is needed to resolve the issues raised by (9)–(11). For instance, in order to resolve the issue raised by the conditional question in (9), it is sufficient to establish that Maria will go if Peter will, or to establish that Maria will not go if Peter goes. These two options correspond precisely to the two overlapping alternatives in Figure 5.2(a). And similarly for (10)-(11). Thus, these types of questions are beyond the scope of partition semantics but can be suitably dealt with in inquisitive semantics.[10]

A concern: disjunctions of questions

We have shown above that inquisitive semantics provides a notion of question meaning that is richer than the one provided by partition semantics, and that this is crucial in order to accommodate several classes of questions which express issues that do not correspond to partitions of the logical space. However, this richer notion of meaning may also raise a certain concern. Namely, Szabolcsi (1997, 2015a) argues that while questions in natural languages can be conjoined,

[10]For a more detailed discussion of the difference in expressive power between inquisitive semantics and partition semantics, we refer to Ciardelli *et al.* (2015, §5).

they can *not* be directly disjoined, and she points out that partition semantics provides an attractive explanation for this contrast.

For instance, Szabolcsi (1997, p.325) notes that the following example, a disjunction of two questions, is decidedly odd.

(12) Who did you marry or where do you live?

This can be explained in partition semantics. Namely, a parition may be identified with an equivalence relation on the space of possible worlds, and while the intersection of two equivalence relations is itself again an equivalence relation, the same is not true of the union of two equivalence relations. If conjunction and disjunction are taken to express intersection and union, respectively, it is to be expected that conjunction, but not disjunction, can apply to two questions to form a new question.[11]

On the other hand, in inquisitive semantics the oddness of (12) is not expected on purely semantic grounds, because if we take disjunction to express the join operator it delivers a perfectly sensible issue, one that can be resolved either by establishing whom the addressee married or by establishing where the addressee lives. This issue does not correspond to a partition, but it is an issue nonetheless in our framework. Thus, while the inquisitive notion of meaning has important advantages w.r.t. partition semantics, it may also be argued to have a certain disadvantage.

However, this argument crucially hinges on Szabolcsi's claim that questions cannot be directly disjoined in natural languages. While we agree with Szabolcsi about the oddness of (12), we are convinced by examples like (13), repeated from Chapter 1, that the general claim is too strong: disjunctions of interrogatives are not always infelicitous.

(13) Where can we rent a car, or who might have one that we could borrow?

We should note that Szabolcsi in fact already remarks that a sentence like (12) may be marginally acceptable if regarded as a case in which the speaker first asks the question *who did you marry*, but then reconsiders and proposes to replace this first question by the second, *where do you live*. In such cases, Szabolcsi suggests, disjunction does not play its usual role but is rather used as a corrective device.

Our example (13), however, can be uttered by someone without any reconsideration halfway, and it can be addressed by an addressee as a single question,

[11] Krifka (2001b) endorses Szabolcsi's empirical claim that questions cannot be disjoined, but offers a different explanation, based on the assumption that questions do not express sets of propositions or partitions, but rather *speech acts*, which Krifka models as operations on commitment states. Speech act disjunction does not exist according to Krifka, because it "would lead to disjunctive sets of commitments, which are difficult to keep track of" (Krifka, 2001b, p.16).

to which both disjuncts contribute. So it seems to be a genuine disjunction of two questions.[12]

We should also note that Szabolcsi (1997) does not base her empirical claim merely on cases like (12) but also on a striking pattern that is found in embedded questions in Hungarian. Hungarian complement clauses, whether declarative or interrogative, are always headed by the subordinating complementizer *hogy*. Szabolcsi argues that this subordinating complementizer expresses a *lifting* operation that needs to be invoked before two interrogative complement clauses can be disjoined (just like proper names have to be lifted into generalized quantifiers when they are conjoined or disjoined with a quantificational noun phrase). Support for this idea comes from examples like (14) and (15) below, which indicate that (i) conjoined interrogative complement clauses can have either two occurences of *hogy*, applying to both individual conjuncts, or a single occurence of *hogy*, applying to the conjunction as a whole, but (ii) disjoined interrogative complement clauses must have two occurences of *hogy*, each applying to one of the individual disjuncts.

(14) János megtudta, **hogy** kit vettél feleségül és (**hogy**) hol
Janos found.out subord whom you.took as.wife and (subord) where
laksz.
you.live
'Janos found out whom you married and where you live.'

(15) János megtudta, **hogy** kit vettél feleségül vagy *(**hogy**) hol
Janos found-out subord whom you.took as.wife or *(subord) where
laksz.
you.live
'Janos found out whom you married or where you live.'

Szabolcsi concludes from this observation that disjunction cannot directly apply to interrogative complement clauses, but always requires intervention of a lifting operation, expressed overtly in Hungarian by *hogy*.

However, there are counterexamples to the generalization. The Hungarian counterpart of our example (13) is a case in point. When embedded, it may come either with one or with two occurences of *hogy*, no matter whether the embedding verb is extensional (e.g., *find out*) or intensional (e.g., *investigate*).[13]

(16) Péter megtudta, hogy hol tudunk autót bérelni vagy **(hogy)** kinek
Peter found.out subord where can.we car rent.inf or (subord) who.to

[12] Haida and Repp (2013) also challenge Szabolcsi's empirical claim, although they maintain a weaker version of it: questions can only be disjoined in downward entailing or non-veridical contexts. Our example (13) presents a challenge for this weaker claim as well.

[13] We are grateful to Donka Farkas, Anikó Liptak, and Anna Szabolcsi for discussion of this datapoint.

van egy, amit kölcsönvehetnénk.
is one which could.borrow.we
'Peter found out where we can rent a car or who has one that we could borrow'

(17) Péter azt vizsgálja, **hogy** hol tudnánk autót bérelni vagy
Peter that.acc investigates subord where could.we car rent.inf or
(hogy) kinek van egy, amit kölcsönvehetnénk.
(subord) who.to is one which could.borrow.we
'Peter is investigating where we could rent a car or who has one we could borrow'

In (17), a single occurence of *hogy* favors a reading on which disjunction takes narrow scope w.r.t. the verb, while two occurences of *hogy* favor a reading on which disjunction take wide scope (Peter is investigating where we can rent a car or he is investigating who has one we could borrow), a pattern that is in line with Szabolcsi's idea that *hogy* expresses a lifting operation.

It thus seems that, at in least some cases, disjunction *can* apply directly to questions, both in English and in Hungarian. A question that naturally arises, then, is whether the general disjunction operation that inquisitive semantics makes available allows us to derive the correct meaning for those disjunctions of questions which *are* felicitous. This indeed seems to be the case. For instance—assuming the natural mention-some interpretation for the two disjuncts—the question in (13) is predicted to express an issue which can be resolved either by identifying a place where the speaker can rent a car, or by identifying a person who might have a car that the speaker can borrow. These are indeed the resolution conditions we expect for (13). Notice that this prediction is obtained simply by applying inquisitive disjunction to the two questions—the same disjunction operation that, as we will discuss in detail in the next chapter, can be taken to be at work in a disjunctive declarative like (18), and also in an alternative question like (19).

(18) John is in London or in Paris.

(19) Is John in London, or in Paris?

Thus, after all, disjunctions of questions seem to provide one more argument in favor of inquisitive semantics over partition semantics, where examples such as (13) can only be handled at the cost of a significant complication of the framework (and one that gives up some of its most attractive features, such as the general account of entailment and coordination among interrogatives; see Groenendijk and Stokhof, 1989).

Of course, an interesting question that remains to be addressed is why our example (13) behaves so differently from Szabolcsi's example (12), both as a standalone question and when embedded. We think that the difference may be explained pragmatically. A disjunction of two questions expresses an issue that may be resolved equally well by providing information resolving the first disjunct, or by providing information resolving the second disjunct. Now, it is difficult to see what kind of motivation (or what kind of decision problem, to follow van Rooij 2003) a speaker could have that would lead her to raise or even consider the issue expressed by (12). This is very different in the case of (13): in this case, it is immediate to reconstruct the sort of motivation that may lead a speaker to consider the relevant issue. We suggest that the different cognitive plausibility of the two issues at stake underlies the difference in the perceived felicity of the associated questions.

5.3 Dynamic semantics

While Hamblin (1973), Karttunen (1977), and Groenendijk and Stokhof (1984) all operate under a static view on meaning, there are also a number of more recent proposals that aim to capture the meaning of questions in a *dynamic* framework. The first such proposals, developed by Jäger (1996), Hulstijn (1997), and Groenendijk (1999), essentially reformulate the partition theory of questions in the format of an update semantics (Veltman, 1996).[14] This means that they explicitly identify meanings with *context change potentials*, i.e., functions over discourse contexts, just as we did in Section 2. Moreover, rather than modeling a discourse context simply as a set of worlds—embodying the information established in the discourse so far—these theories provide a more refined model of the discourse context, one that also embodies the issues that have been raised so far. More specifically, a discourse context is modeled as an *equivalence relation* C over a set of worlds $s \subseteq W$. Such an equivalence relation, which induces a partition on s, can be taken to encode both information and issues. On the one hand, the *domain* of C, i.e., set of worlds s, is taken to encode the information established so far. And on the other hand, C is taken to relate two distinct worlds w and v in s just in case the difference between w and v is not (yet) at-issue, i.e., the discourse participants have not yet expressed an interest in information that would distinguish between w and v. In other words, C is conceived of as a relation encoding *indifference* (Hulstijn, 1997).

Both declaratives and questions can then be taken to have the potential to change the context in which they are uttered. A declarative restricts the domain s to those worlds in which the sentence is true (strictly speaking, it removes all pairs of worlds $\langle w, v\rangle$ from C such that the sentence is false in at least one of the two worlds). Questions *disconnect* worlds, i.e., they remove a pair $\langle w, v\rangle$ from C just in case the true exhaustive answer to the question in w differs from the true exhaustive answer to the question in v.

[14] See the book *Questions in dynamic semantics* (Aloni *et al.*, 2007) for several papers elaborating on these early proposals.

Thus, the dynamic systems of Jäger (1996), Hulstijn (1997), and Groenendijk (1999) provide a notion of context and meaning that embodies both informative and inquisitive content in an integrated way. However, as discussed in detail by Mascarenhas (2009), these proposals still face a number of issues, both empirical and conceptual. Empirically, just as in classical partition semantics, it is still impossible in these dynamic frameworks, at least without further amendments, to satisfactory deal with conditional questions, disjunctive questions, and mention-some *wh*-questions.[15]

Conceptually, if C is primarily thought of as a relation encoding *indifference*, then it is not clear why it should be an *equivalence relation*. In particular, it is not clear why C should be *transitive*. The discourse participants could very well be interested in information that distinguishes w from v, while they are not interested in information that distinguishes either w or v from a third world u. To model such a situation, we would need an indifference relation C such that $\langle w, u\rangle \in C$ and $\langle u, v\rangle \in C$ but $\langle w, v\rangle \notin C$. This is impossible if we require C to be transitive.

These concerns led Groenendijk (2009) and Mascarenhas (2009) to develop a framework in which indifference relations are defined as reflexive and symmetric, but not necessarily transitive relations. Otherwise, the architecture of this framework is still essentially the same as that of Jäger (1996), Hulstijn (1997), and Groenendijk (1999).

Groenendijk (2009) and Mascarenhas (2009) coined the term *inquisitive semantics* to refer to the framework they proposed, and the framework we have presented here can be seen as a generalization of this early incarnation of inquisitive semantics. To distinguish the two, the framework proposed by Groenendijk (2009) and Mascarenhas (2009) has been referred to in later work (e.g., Ciardelli and Roelofsen, 2011; Ciardelli *et al.*, 2015) as *inquisitive pair semantics*, or simply *pair semantics*, since it still defines issues in terms of an indifference relation—technically a set of world-pairs—just like partition semantics (both in its classical static instalment and in its dynamic rendering) and unlike the current inquisitive semantics framework, where issues are defined as sets of information states.

Groenendijk (2009) and Mascarenhas (2009) argued that inquisitive pair semantics, besides avoiding the conceptual problem concerning transitive indifference relations, also overcomes the empirical limitations of partition theory concerning conditional questions, disjunctive questions, and mention-some *wh*-questions. However, whereas conditional questions like (9) and open disjunctive questions with two disjuncts like (10) can be dealt with satisfactorily, disjunctive questions with three or more disjuncts are still problematic, and the same holds

[15] Although see Isaacs and Rawlins (2008) for an analysis of conditional questions in a dynamic partition semantics that allows for hypothetical updates of the context of evaluation.

for mention-some *wh*-questions.[16] Thus, in terms of expressive power, inquisitive pair semantics is situated in between partition semantics and the inquisitive semantics framework presented here:

alternative semantics > inquisitive semantics > pair semantics > partition semantics

Several important aspects of the general philosophy underlying the inquisitive pair semantics persist in the present framework. However, its key ingredient—the notion of issues—has been generalized, and this generality is needed to suitably capture the full range of question types in natural languages. Thus, the inquisitive semantics framework presented here naturally fits within the existing tradition of semantic theories of informative and inquisitive discourse, but it is more general and able to cover more empirical ground than its predecessors.

5.4 Exercises

EXERCISE 5.1. [Inquisitive semantics versus alternative semantics]
Explain in your own words what the difference is between inquisitive semantics and alternative semantics in terms of expressive power, how this difference arises, and how it pertains to the suitability of the two frameworks for the analysis of questions in natural languages.

EXERCISE 5.2. [Inquisitive semantics versus partition semantics]
Explain in your own words in what sense the notion of issues in inquisitive semantics is more general than the notion of question meanings in partition semantics, and why this extended generality is needed for the analysis of questions in natural languages.

EXERCISE 5.3. [Inquisitive semantics versus pair semantics]
Determine the interpretation of a disjunction with three disjuncts, $p \vee q \vee r$, in the pair semantics of Groenendijk (2009) and Mascarenhas (2009). How does this differ from the meaning assigned to $p \vee q \vee r$ in InqB? How does this difference arise? And how does it pertain to the suitability of the two frameworks for the analysis of questions in natural languages?

[16] For a detailed discussion of this issue we refer to Ciardelli and Roelofsen (2011, §7-8) and Ciardelli *et al.* (2015, §5).

Chapter 6

An integrated treatment of declaratives and interrogatives

In the previous chapter we discussed several vantage points of inquisitive semantics as a framework of question semantics. In this section we will formulate a concrete account of certain types of declaratives and interrogatives, highlighting the particular advantage that inquisitive semantics has w.r.t. other frameworks when it comes to formulating a uniform account of lexical and prosodic elements that play a role in both declarative and interrogative constructions.

We will consider sentences consisting of one or more declarative or interrogative clauses separated by disjunction, with different intonation patterns. Some representative examples are given in (1)-(5) below (we use ↑ and ↓ to indicate rising and falling pitch contours, respectively).

(1) Igor speaks English↓.

(2) Igor speaks English↑.

(3) Does Igor speak English↑?

(4) Does Igor speak English↑ or does he speak French↓?

(5) Does Igor speak English↑ or does he speak French↑?

Drawing inspiration from Zimmermann (2000) we will view such declarative and interrogative sentences as *lists*. Lists either consist of a *single clause*, as in (1)-(3), or of *multiple clauses* separated by disjunction, as in (4)-(5). Moreover, we think of lists as being either *open* (signaled by a final rise), as in (2), (3) and (5), or *closed* (signaled by a final fall), as in (1) and (4).

We will present an account of such lists in InqB. While our focus will be on English, we expect that the basic semantic operations that our account associates with the relevant lexical, morphological, and prosodic features play a central role in the interpretation of similar constructions in other languages as well. The division of labor between the various elements is bound to vary from language to language, but we expect that the basic repertoire of semantic operations that our account draws on will be relatively stable across languages.

A general point that we want to make in this section, independently of the details of the specific account that we will present, is that any account which aims to treat disjunction and the relevant prosodic features *uniformly* across declarative and interrogative constructions, has to be couched within a semantic framework which treats informative and inquisitive content in an integrated way. For instance, if we want to give a uniform characterization of the role of disjunction in declaratives and interrogatives, we have to be able to capture how it affects both informative and inquisitive content, independently of the kind of construction that it happens to be part of. And similarly for the relevant prosodic features. Simply put, the fact that declarative and interrogative lists are largely built up from the *same parts* constitutes an important piece of motivation for a semantic framework like inquisitive semantics, which treats informative and inquisitive content in an integrated way, as opposed to approaches in which the standard truth-conditional notion of meaning is maintained for declaratives and a separate notion of meaning is invoked for interrogatives (e.g., Karttunen, 1977; Groenendijk and Stokhof, 1984). Such approaches do not provide a uniform account of disjunction across declaratives and interrogatives—the semantic contribution of disjunction in interrogatives like (4) is taken to be different from its truth-conditional contribution in declaratives.[1]

We will present a simplified version here of the account of declarative and interrogative lists outlined in Roelofsen (2013c) and developed in more detail in Roelofsen (2015a). The simplified account to be presented here has also been presented in Roelofsen and Farkas (2015), where it serves as the basis for a theory of answer particles like *yes* and *no*. The main reason we present only a simplified version of the account in Roelofsen (2015a) here is that the full account does not only aim to capture the informative and inquisitive content of the various types of lists but also their presuppositional content, which requires an extension of the basic InqB system. While such an extension increases the empirical coverage of the account, a simplified non-presuppositional version should suffice to substantiate our general point, i.e., to demonstrate the advantages of inquisitive semantics in formulating a uniform account of lexical and prosodic elements that play a role in both declarative and interrogative constructions.

We will proceed as follows. Section 6.1 provides a more systematic, though still informal characterization of the different types of lists that we will be concerned with, Section 6.2 specifies what we take the logical forms of these different

[1] The uniform treatment of disjunction across declaratives and interrogatives to be presented here is closely related to the treatment of disjunction in alternative semantics (Simons, 2005; Alonso-Ovalle, 2006; Aloni, 2007). See Section 4.7 for some discussion of how the latter treatment of disjunction is related to the inquisitive one. For more elaborate comparison, see Ciardelli and Roelofsen (2015a, 2016); Roelofsen (2015b).

types of lists to be, and Section 6.3 specifies how these logical forms can be interpreted in InqB.

6.1 Declarative and interrogative lists

We take lists to differ along three basic parameters: they can be declarative or interrogative, they can be open or closed, and they can consist of a single clause or of multiple clauses separated by disjunction. We will refer to these clauses as 'list items'. In total, then, we consider $2 \times 2 \times 2 = 8$ basic types of lists, exemplified in (6)-(9).

(6) *Closed declarative lists*

a. *mono-clausal*: Igor speaks English↓.

b. *multi-clausal*: Igor speaks English↑ or he speaks French↓.

(7) *Open declarative lists*

a. *mono-clausal*: Igor speaks English↑.

b. *multi-clausal*: Igor speaks English↑ or he speaks French↑.

(8) *Closed interrogative lists*

a. *mono-clausal*: Does Igor speak English↓?

b. *multi-clausal*: Does Igor speak English↑ or does he speak French↓?

(9) *Open interrogative lists*

a. *mono-clausal*: Does Igor speak English↑?

b. *multi-clausal*: Does Igor speak English↑ or does he speak French↑?

Closed lists characteristically have falling intonation on the final item, while open lists characteristically have rising intonation on the final item. Non-final list items are canonically pronounced with rising intonation, both in open and in closed lists. Moreover, each item is pronounced in a separate intonational phrase, which means that there is an intonational phrase break after each non-final item, before the disjunction word. In fact, two non-final list items may be separated just by an intonational phrase break, i.e., the disjunction word may be omitted if neither of the items is final.

Disjunction can be used to separate list items, but it may also occur *within* a list item. Thus, the lists in (10) below all have a single item, containing disjunction, rather than two items separated by disjunction (we use hyphenation to indicate the absence of an intonational phrase break):

(10) *Lists with a single disjunctive item*

a. *Closed declarative*: Igor speaks English-or-French↓.
b. *Open declarative*: Igor speaks English-or-French↑.
c. *Closed interrogative*: Does Igor speak English-or-French↓?
d. *Open interrogative*: Does Igor speak English-or-French↑?

Note that some types of lists that we consider here are better known under different names. For instance, singleton interrogative lists (either open or closed) are usually referred to as *polar questions* and non-singleton closed interrogative lists are usually referred to as *alternative questions*. We will sometimes use this more familiar terminology alongside our list terminology. The former has the advantage of being easier to recognize; the latter has the advantage of explicating the distinctive features of each type of list and emphasizing that each specific construction is considered here as part of a more general paradigm.

Previous analyses of lists have only been concerned with some types of lists, not with the full range. Zimmermann (2000) focuses on non-singleton declarative lists like (6b) and (7b). On the other hand, Pruitt (2007), Biezma (2009), Biezma and Rawlins (2012), and Aloni *et al.* (2013), who like us also draw inspiration from Zimmermann, focus on singleton open interrogatives (polar questions) like (9a) and (10d), and non-singleton closed interrogatives (alternative questions) like (8b). A uniform analysis of the full range of lists exemplified in (6)-(10) would thus extend the coverage of these previous analyses considerably.[2]

6.2 Logical forms

Schematically, we assume that a list with n items has the following logical form:

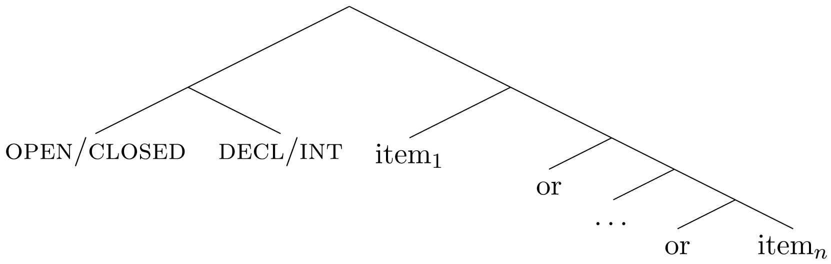
[2]The idea that disjunction can be used to form lists has also been put forth by Simons (2001, p.616), independently of Zimmermann (2000). In Simons' work, however, this idea does not form the basis for a particular semantic treatment of disjunctive sentences and their various prosodic features, but is rather part of a pragmatic explanation for the fact that disjunctive declaratives are typically much more natural in response to a given question than truth-conditionally equivalent non-disjunctive sentences. For instance, if the question is why Jane isn't picking up the phone, then (i) is a much more natural answer than (ii).

(i) Either she isn't home, or she can't hear the phone.

(ii) It's not the case that she is at home and she can hear the phone.

To the extent that Simons' analysis of this phenomenon is successful, it provides independent motivation for our general outlook on disjunctive sentences as lists. A detailed assessment of Simons' analysis, however, is beyond the scope of these lecture notes.

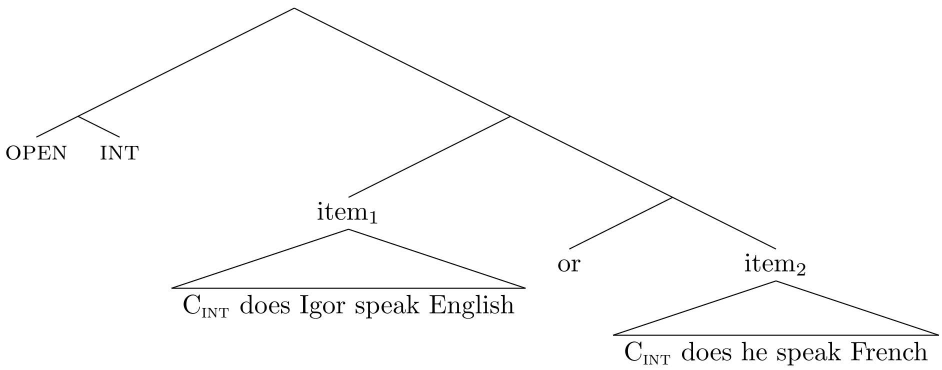
(11)



We will refer to OPEN/CLOSED and DECL/INT as *list classifiers* and to the rest of the structure as the *body* of the list. We assume that each item in the body of the list is a full clause, headed by a declarative or interrogative complementizer, C_{DECL} or C_{INT}, depending on whether the list is classified as DECL or INT, respectively.[3]

To give a concrete example, the open interrogative in (9b), which involves two list items, is taken to have the following structure:

(12)



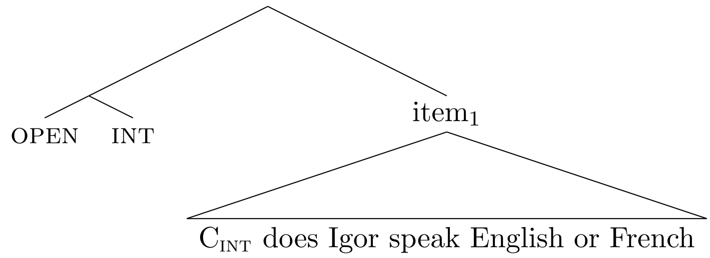
On the other hand, the open interrogative in (10d), which has a single list item containing a disjunctive phrase, is taken to have the following structure:

[3] We are only considering lists here whose items undeniably amount to entire clauses. That is, we do not condider cases like (i), which may be treated either as a single clause containing a disjunctive phrase, or alternatively as a disjunction of two full clauses, the latter partially elided.

(i) Does Igor speak English↑ or French↓?

Such constructions raise a number of syntactic issues which remain largely unresolved in the literature. See, e.g., Larson (1985); Han and Romero (2004); Beck and Kim (2006); Haida (2010); Pruitt and Roelofsen (2011); Uegaki (2014); Roelofsen (2015a) for discussion.

(13)



6.3 Interpreting logical forms

We will specify a semantic interpretation of these logical forms by translating them into the logical language of InqB. Thereby we associate each logical form with a proposition, namely the proposition expressed by the formula that serves as its translation.

We will first consider the body of a list, and after that turn to the list classifiers. Recall that the body of a list consists of one or more list items, separated by disjunction. Every list item, in turn, is a full clause headed by a declarative or interrogative complementizer ($C_{\text{DECL/INT}}$). The rest of the clause is a tense phrase (TP), which may itself contain a disjunction.

The translation procedure is very straightforward. Any disjunction is translated as $\vee$, no matter whether it separates two list items or occurs within one of the list items. Every complementizer, be it declarative or interrogative, is translated as !. The rationale for this is that every list item is seen, intuitively speaking, as one block, i.e., as contributing a single alternative to the proposition expressed by the list as a whole. This is ensured by applying !, which turns any proposition P into a proposition with a single alternative, $\bigcup P$. Otherwise the procedure is completely standard. Thus, the body of a list is translated according to the rule in (14), where $\varphi_1, \ldots, \varphi_n$ are standard translations of TP_1, $\ldots$, TP_n into the language of propositional logic.

(14) *Rule for translating the body of a list*:

$$[[C_{\text{DECL/INT}}\ TP_1] \text{ or} \ldots \text{or } [C_{\text{DECL/INT}}\ TP_n]] \quad \rightsquigarrow \quad !\varphi_1 \vee \ldots \vee !\varphi_n$$

Returning to our concrete examples above, if we translate *Igor speaks English* as p and *Igor speaks French* as q, then we get the following translations for the list bodies of (9b) and (10d), respectively.

(15) [C_{INT} does Igor speak English] or [C_{INT} does he speak French] $\rightsquigarrow$ $!p \vee !q$

(16) [C_{INT} does Igor speak English or French] $\rightsquigarrow$ $!(p \vee q)$

Now let us turn to the list classifiers. To specify their semantic contribution it is convenient to use some notation and terminology from type theory.[4] Recall that in inquisitive semantics, propositions are sets of sets of possible worlds, i.e., objects of type $\langle\langle s,t\rangle,t\rangle$. Let us abbreviate this type as T. Now, we will treat DECL and INT as propositional operators, i.e., as functions that take a proposition as their input, and deliver another proposition as their output. This means that DECL and INT are of type $\langle T,T\rangle$. On the other hand, we will treat OPEN and CLOSED as *modifiers* of propositional operators, i.e., as functions that take a propositional operator as their input, and deliver a modified propositional operator as their output. So OPEN and CLOSED are of type $\langle\langle T,T\rangle,\langle T,T\rangle\rangle$. It will become clear in a moment why OPEN and CLOSED are treated as having this somewhat more complex type, rather than simply $\langle T,T\rangle$, like DECL and INT. First, we need to look at each of the classifiers in somewhat more detail.

First consider DECL. We will treat DECL as making a list purely informative, i.e., as *eliminating inquisitiveness*. This effect can be achieved straightforwardly by treating DECL as a function that takes the proposition P expressed by the body of a list as its input and applies the projection operator ! to it, returning $!P$. Using type-theoretic notation, this can be formulated concisely as follows:

(17) DECL $\rightsquigarrow$ $\lambda P.!P$

Next, consider INT. The proposal in Roelofsen (2013c, 2015a) is to treat interrogativity as having two effects. First, whenever possible, it *ensures inquisitiveness*. This is done by applying a conditional variant of the ? operator, which we will denote here as $\langle ? \rangle$. If the proposition P that $\langle ? \rangle$ takes as its input is not yet inquisitive, then ? is applied to it. On the other hand, if P is already inquisitive, then it is left untouched. The only case in which this procedure does *not* yield an inquisitive output is when P is a tautology or a contradiction. In this case $\langle ? \rangle P$ is a tautology, which is not inquisitive. In all other cases, $\langle ? \rangle P$ is inquisitive.

The second effect of interrogativity proposed in Roelofsen (2013c, 2015a) is that it *ensures non-informativity*, by introducing a presupposition that the actual world must be contained in $\bigcup P$. This second aspect of interrogativity is especially important to account for the presuppositional component of alternative questions (see, e.g., Karttunen and Peters, 1976; Biezma and Rawlins, 2012). However, since presuppositions cannot be captured in the basic InqB system, we simplify the analysis here and restrict ourselves to the first aspect of interrogativity described above, i.e., to ensure inquisitiveness. Thus, we assume the following treatment of INT:

[4] We will only use some type-theoretical notation here in the meta-language to describe functions (as in, e.g., Heim and Kratzer, 1998). A more rigorous approach would be to extend the InqB system to a full-fledged type theoretic framework (as done in Theiler, 2014; Ciardelli and Roelofsen, 2015a). We leave this step implicit here because it would involve quite some technicalities which are to a large extent orthogonal to our present concerns.

(18) INT $\rightsquigarrow$ $\lambda P.\langle ? \rangle P$

Finally, let us consider OPEN and CLOSED. Intuitively speaking, we treat these classifiers as encoding whether the list is left 'open ended' or whether it is 'finished' and ready to be 'sealed off'. The role of CLOSED is to mark the list as being finished, and to allow DECL or INT, whichever is present, to 'seal off' the list. Thus, CLOSED is simply treated as the identity function, leaving the propositional operator π expressed by DECL or INT untouched and letting it apply to the proposition expressed by the body of the list.

(19) CLOSED $\rightsquigarrow$ $\lambda \pi . \pi$

On the other hand, the role of OPEN is to mark the list as being open-ended. It prevents DECL/INT from sealing off the body of the list, and instead applies the ? operator, which adds the set-theoretic complement of $\bigcup P$ as an additional alternative. This captures what we take to be the characteristic semantic property of open lists, which is that they always leave open the possibility that none of the given list items holds. Thus, unlike CLOSED, prevents the operator π expressed by DECL or INT from becoming operative, and instead applies ? to the proposition P expressed by the body of the list.

(20) OPEN $\rightsquigarrow$ $\lambda \pi . \lambda P . ?P$

In total there are four types of lists, each featuring a combination of two classifiers. From the treatment of the individual classifiers given above, it follows that the four types of lists are translated into our logical language as specified in (21) below, where in each case φ stands for the translation of the body of the list, obtained according to the rule in (14) above.

(21) *Rules for translating lists*:

a.	[[CLOSED DECL] body]	$\rightsquigarrow$	$!\varphi$
b.	[[CLOSED INT] body]	$\rightsquigarrow$	$\langle ? \rangle \varphi$
c.	[[OPEN DECL] body]	$\rightsquigarrow$	$?\varphi$
d.	[[OPEN INT] body]	$\rightsquigarrow$	$?\varphi$

The rules in (14) and (21) together give a complete specification of how to translate declarative and interrogative lists in English into our logical language. In Table 6.1 we provide translations for a number of examples that are representative for all the types of lists that we are concerned with. In the translations of these examples, p stands for *Igor speaks English* and q for *Igor speaks French*. In each case we provide the direct translation and also a simpler formula that is semantically equivalent in InqB to the direct translation. The propositions expressed by all these simplified translations are depicted in Figure 6.1.

Note that the mapping from logical forms to propositions is not a one-to-

	Translation:	Simplified:
—Closed delaratives—		
Igor speaks English↓.	$!!p$	p
Igor speaks English-or-French↓.	$!!(p \vee q)$	$!(p \vee q)$
Igor speaks English↑, or he speaks French↓.	$!(!p \vee !q)$	$!(p \vee q)$
—Open declaratives—		
Igor speaks English↑.	$?!p$	$?p$
Igor speaks English-or-French↑.	$?!(p \vee q)$	$?!(p \vee q)$
Igor speaks English↑, or he speaks French↑.	$?(!p \vee !q)$	$?(p \vee q)$
—Open interrogatives—		
Does Igor speak English↑?	$?!p$	$?p$
Does Igor speak English-or-French↑?	$?!(p \vee q)$	$?!(p \vee q)$
Does Igor speak English↑, or does he speak French↑?	$?(!p \vee !q)$	$?(p \vee q)$
—Closed interrogatives—		
Does Igor speak English↓?	$?!p$	$?p$
Does Igor speak English-or-French↓?	$?!(p \vee q)$	$?!(p \vee q)$
Does Igor speak English↑, or does he speak French↓?	$!p \vee !q$	$p \vee q$

Table 6.1: Representative examples of all types of lists considered.

one mapping: various types of lists are given the same translation and are thus associated with the same proposition. For instance, the open polar interrogative in (22a), the closed polar interrogative in (22b), and the open declarative in (22c) are all translated as $?p$, which expresses the proposition in depicted Figure 6.1(c).

(22) a. Does Igor speak English↑? — open interrogative
b. Does Igor speak English↓? — closed interrogative
c. Igor speaks English↑. — open declarative

This is as it should be, because in terms of informative and inquisitive content these sentences are indeed the same: they all raise the issue whether Igor speaks English, and do not provide any information. However, there *is* of course a contrast between (22a) on the one hand and (22b-c) on the other. Namely, while (22a) is generally seen as the *default* way of expressing the issue whether Igor speaks English, (22b-c) are perceived as *marked* ways of doing so, and the range of contexts in which they can be felicitously used is restricted.

The same point applies to the three sentences in (23), which are the disjunc-

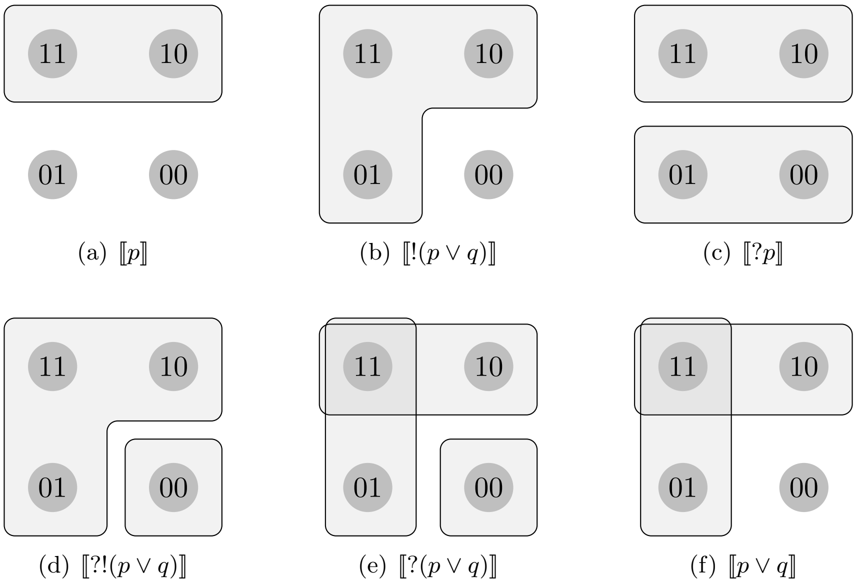


Figure 6.1: Propositions expressed by the examples in Table 6.1.

tive counterparts of those in (22):

(23)	a.	Does Igor speak English-or-French↑?	open interrogative
	b.	Does Igor speak English-or-French↓?	closed interrogative
	c.	Igor speaks English-or-French↑.	open declarative

These sentences are all translated as $?!(p \vee q)$ and thus associated with the proposition depicted in Figure 6.1(d). Again, the open interrogative in (23a) is the default way of expressing this proposition, while the closed interrogative in (23b) and the open declarative in (23c) are marked ways of doing so.

Finally, the same point also applies to the two sentences in (24), which are both translated as $?(p \vee q)$.

(24)	a.	Does Igor speak English↑ or does he speak French↑?	open interrogative
	b.	Igor speaks English↑ or he speaks French↑.	open declarative

The interrogative is again the default way of expressing the given proposition, while the declarative is marked—indeed, in this case it is difficult to think of any context at all in which it could be used felicitously.

Thus, a general distinction can be made between *marked* and *unmarked* sentence types. We will first discuss the unmarked cases in more detail, in Section 6.3.1, and then turn to the marked cases in Section 6.3.2.

6.3.1 Unmarked cases

We start with the simplest unmarked case, namely the non-disjunctive closed declarative in (25):

(25) Igor speaks English↓. closed declarative

This sentence is taken to have the following logical form:

(26) [[CLOSED DECL] [C_{DECL} Igor speaks English]]

The translation of this logical form is $!!p$, which can be simplified to just p. The proposition expressed by this sentence is depicted in Figure 6.1(a). It is correctly predicted that the sentence provides the information that Igor speaks English, without requesting any additional information.

Next, consider the disjunctive closed declaratives in (27) and (28):

(27) Igor speaks English-or-French↓. closed declarative

(28) Igor speaks English↑ or he speaks French↓. closed declarative

These sentences are taken to have the following logical forms, respectively:

(29) [[CLOSED DECL] [C_{DECL} Igor speaks English or French]]

(30) [[CLOSED DECL] [[C_{DECL} Igor speaks English] or [C_{DECL} he speaks French]]]

These logical forms have the same simplified translation, namely $!(p \vee q)$, which expresses the proposition depicted in Figure 6.1(b). Thus, the sentences are correctly predicted to provide the information that Igor speaks English or French, without requesting any additional information.

Now let us turn to interrogatives. The simplest unmarked case here is the mono-clausal open polar interrogative in (31).

(31) Does Igor speak English↑? open interrogative

This sentence is taken to have the following logical form:

(32) [[OPEN INT] [C_{INT} does Igor speak English]]

The simplified translation of this logical form is $?p$, which expresses the proposition depicted in Figure 6.1(c). The sentence is correctly predicted to request information as to whether Igor speaks English, and not to provide any information.

Next, consider the disjunctive open interrogative in (33).

(33) Does Igor speak English-or-French↑? open interrogative

This sentence is taken to have the following logical form:

(34) [[OPEN INT] [C_{INT} does Igor speak English or French]]

The simplified translation of this logical form is $?!(p \vee q)$, which expresses the proposition depicted in Figure 6.1(d). Again, the sentence is predicted to be inquisitive and not informative. In order to resolve the issue that it raises, it either needs to be established that Igor indeed speaks at least one of the two languages, or that he does not speak either.

Next, consider the open interrogative in (35), where the disjunction separates two list items.

(35) Does Igor speak English↑ or does he speak French↑? open interrogative

This sentence is taken to have the following logical form:

(36) [[OPEN INT] [[C_{INT} does Igor speak English] or [C_{INT} does he speak French]]]

The simplified translation of this logical form is $?(p \vee q)$, which expresses the proposition depicted in Figure 6.1(e). As desired, the sentence is predicted to be more inquisitive than (33). Namely, in order to resolve the issue that it raises, it is not sufficient to establish whether or not Igor speaks at least one of the two languages. Rather, it either needs to be established that Igor speaks English, or that he speaks French, or that he speaks neither.

Finally, consider the closed interrogative in (37), again with two list items.

(37) Does Igor speak English↑ or does he speak French↓? closed interrogative

This sentence is taken to have the following logical form:

(38) [[CLOSED INT] [[C_{INT} does Igor speak English] or [C_{INT} does he speak French]]]

The translation of this logical form, on the simplified non-presuppositional account presented here, is $p \vee q$, which expresses the proposition depicted in Figure 6.1(f). Notice that the ? operator is not invoked here because the proposition that INT gets as its input is already inquisitive. Since the role of INT is not to blindly apply ?, but rather just to ensure inquisitiveness, it leaves the input proposition unaltered in this case. The prediction, then, is that the alternative question in (37) provides the information that Igor speaks at least one of the two languages, and raises the issue which of the two he speaks.

This prediction is not entirely satisfactory, because it does not capture the generalization that alternative questions presuppose that exactly one of the disjuncts holds (Karttunen and Peters, 1976; Biezma and Rawlins, 2012). As remarked at the outset, however, it is impossible to properly capture this generalization in InqB, which concentrates exclusively on informative and inquisitive content and leaves presuppositional aspects of meaning out of consideration.

We refer to Ciardelli *et al.* (2012) and Roelofsen (2015a) for a presuppositional extension of InqB, and to the latter work for a more sophisticated version of the account of lists presented here, which does make satisfactory predictions about alternative questions.

Aside from this loose end concerning the presuppositional component of alternative questions, we have seen that the basic semantic properties of unmarked declarative and interrogative lists are accounted for in a straightforward and principled way.

6.3.2 Marked cases

We now turn to the marked cases, listed below:

(39)	Does Igor speak English↓?	closed interrogative
(40)	Igor speaks English↑.	open declarative
(41)	Does Igor speak English-or-French↓?	closed interrogative
(42)	Igor speaks English-or-French↑.	open declarative
(43)	Igor speaks English↑ or he speaks French↑.	open declarative

Our aim here will just be to account for the marked status of these types of sentences—we will *not* try to characterize their special discourse effects or the exact range of contexts in which they could be felicitously used. The general idea that we will pursue, familiar from much work in neo-Gricean pragmatics and optimality theory (see, e.g., Horn, 1984; Blutner, 2000), is that an expression is perceived as marked if there is another expression that has the same meaning and is, other things being equal, better suited to express that meaning. One reason for this may be that the latter expression is easier to produce; another reason may be that it has a greater chance of being interpreted as intended. This second reason will be most relevant for us.

Notice that every sentence in (39)-(43) either involves the classifier combination [CLOSED INT] or the combination [OPEN DECL]. Vice versa, every type of list with one of these two classifier combinations is represented in (39)-(43), except for closed interrogatives with multiple list items, i.e., alternative questions—we will return to this momentarily. Quite generally, then, there is something marked about closed interrogatives and open declaratives. Why would this be? In Farkas and Roelofsen (2014); Roelofsen (2015a) it is proposed that the source of this markedness lies in the fact that these kinds of lists are generally in competition with open interrogative lists, and that the latter are generally preferred because they maximize the chance of being interpreted as intended. This is because, in many configurations, OPEN and INT have precisely the same semantic effect, and even more importantly, in these configurations the same overall interpretation would result if either OPEN or INT were to be *mis*interpreted as CLOSED or DECL, respectively.

Let us look at an example to make this more concrete. Consider the closed interrogative in (39). The simplified translation of this sentence is $?p$, which is also the simplified translation of the open interrogative in (44).

(44) Does Igor speak English↑? open interrogative

Now suppose that someone hears (44) in a conversation and has to determine its meaning. If all goes well, the sentence is recognized as an open interrogative—through the interrogative word order and the final rise. However, even if the sentence is mistakenly parsed as an open *declarative*, or as a *closed* interrogative, the same interpretation would still be derived. Thus, open interrogatives are very robust: if one piece breaks, the whole construction still functions as intended. This is not the case for the closed interrogative in (39). If this sentence is mistakenly parsed as a closed *declarative*, the intended interpretation would not be obtained. This explains the marked nature of this sentence type.

Exactly the same reasoning applies to the open declarative in (40). This sentence also has $?p$ as its simplified translation, so it is also in competition with the open interrogative in (44). And again, it does not have the same robustness as the open interrogative, because if it is mistakenly parsed as a *closed* declarative, the intended interpretation is not obtained.

The other three cases can be explained analogously: (41) and (42) are in competition with the open interrogative in (45), and (43) is in competition with the open interrogative in (46). In all cases the open interrogative is favored because of its supreme robustness.

(45) Does Igor speak English-or-French↑? open interrogative

(46) Does Igor speak English↑ or does he speak French↑? open interrogative

Finally, let us return to the case of alternative questions, i.e., closed interrogatives with multiple items, which are *not* marked, even though closed interrogatives with a single item are, whether they contain a disjunction or not (see examples (39) and (41) above). The reason for this is that closed interrogative lists with multiple items are *not* generally equivalent with the corresponding open interrogative lists. So in this case there is no competition between the two types of lists.

To make this concrete again, consider the closed interrogative in (47).

(47) Does Igor speak English↑ or does he speak French↓? closed interrogative

The simplified translation of this sentence is $p \vee q$. Thus, it does not have the same meaning as the corresponding open interrogative in (46), nor is there any other competing list type. This explains its unmarked nature.

This concludes our analysis of declarative and interrogative lists in InqB. Even though there is much more to say about the linguistic properties of such lists, we hope that the bare bones account that we have presented here has succeeded in substantiating the general point that we set out to make, namely that declaratives and interrogatives are to a large extent 'built up from the same parts', i.e., the same lexical, morphological, and prosodic elements, and that a uniform account of such elements, which applies across declarative and interrogative constructions, requires a framework like inquisitive semantics, which treats informative and inquisitive content in an integrated way.

6.4 Exercises

Exercise 6.1. Determine the logical form of each of the examples below, and derive, step by step, how these logical forms are translated into InqB according to the rules in (14) and (21).

(48)
- a. Martina plays the piano↓.
- b. Martina plays the piano-or-the-cello↓.
- c. Martina plays the piano↑ or she plays the cello↓.
- d. Martina plays the piano↑.
- e. Does Martina play the piano↑?
- f. Does Martina play the piano-or-the-cello↑?
- g. Does Martina play the piano↑ or does she play the cello↑?
- h. Does Martina play the piano↑ or does she play the cello↓?
- i. Does Martina play the piano↓?

Exercise 6.2. Explain how the marked status of (48d) and (48i) is accounted for by the theory presented here.

Chapter 7

A new view on propositional attitudes

In the previous chapters we have seen that inquisitive semantics makes available a new notion of conversational contexts, which does not only capture the information that has been established in the conversation so far but also the issues that have been brought up, as well as a new notion of meaning, which does not just embody informative content but also inquisitive content. In this chapter we will show that the framework also gives rise to a new view on propositional attitudes, especially those that are relevant for information exchange. Namely, besides the familiar information-directed attitudes like *knowing* and *believing* it also allows us to model issue-directed attitudes like *wondering*.

A perspicuous and widely adopted formal treatment of information-directed attitudes is provided by *epistemic logic* (EL), sometimes also called the logic of knowledge and belief, which has its roots in the work of Hintikka (1962) and has been further developed by many authors in subsequent work (see, e.g., Fagin *et al.*, 1995). In this framework, the information state (or knowledge state or belief state) of an agent is modeled as a set of possible worlds, namely those worlds that are compatible with the informative available to the agent (or with what the agent knows or believes, respectively). As we have seen, this notion of information states also plays an important role in inquisitive semantics. However, while the information-directed attitudes of an agent can be suitably captured in terms of her information state, this clearly does not hold for her issue-related attitudes. In order to capture what an agent wonders about, we need a representation of the issues that she entertains, i.e., her *inquisitive state*.

To this end, we will define an *inquisitive epistemic logic* (IEL), which brings together ideas from standard EL and InqB. For simplicity, we will restrict ourselves to a propositional language, leaving the universal and existential quantifier of InqB out of consideration. On the other hand, the modal part of our logical language will be richer than that of basic EL: while the latter only has a modal operator K, which is used to talk about knowledge, the language that we will consider also has a modal operator E, which is used to talk about the issues that the agents entertain.

One purpose of IEL is to serve as a formal framework to describe and reason about information- and issue-directed attitudes as such. However, this is not the only purpose. Of equal importance, it also provides a basic semantic treatment of verbs in natural languages that are used to report such attitudes. In English, such verbs include *know* and *wonder*, and many other languages have verbs that fulfil precisely the same purpose. While the semantics of *know* and its cross-linguistic kin has been considered extensively, its treatment in IEL differs from most previous accounts in that it deals completely uniformly with cases where *know* takes a declarative complement and cases where it takes an interrogative complement, exemplified in (1) and (2), respectively.

(1) John knows that Bill is coming.

(2) John knows whether Bill is coming.

As for *wonder*, IEL does not only capture its interpretation when taking an interrogative complement, as in (3) below, but it also provides a possible semantic explanation of the fact that it can*not* take a declarative complement, illustrated in (4).

(3) John wonders whether Bill is coming.

(4) *John wonders that Bill is coming.

We will proceed as follows. Section 7.1 provides a brief review of standard EL, Section 7.2 presents the IEL framework, and Section 7.3 sketches how the treatment of the modalities in IEL may be generalized to other modal constructions.

7.1 Epistemic logic

Epistemic logic is a particular kind of modal logic, which serves as a formal framework to describe and reason about the knowledge of a set of agents $\mathcal{A}$, both about the world (factual knowledge) and about one another's knowledge (higher-order knowledge).

7.1.1 Logical language and models

The language of epistemic logic is a standard propositional language, based on a set of atomic sentences $\mathcal{P}$, enriched with a modal operator K_a for each agent $a \in \mathcal{A}$. A sentence of the form $K_a\varphi$ is read as 'agent a knows φ.'

Sentences in this logical language are interpreted with respect to *epistemic models*, which consist of a set of possible worlds W, together with (i) a *valuation map* which determines, for each world $w \in W$, which atomic sentences in the language are true in that world, and (ii) a set of *epistemic maps* which specify, for every world $w \in W$, the information state of each agent $a \in \mathcal{A}$ in that world, where information states are, as before, sets of possible worlds.

Definition 7.1. [Epistemic models] An epistemic model for a set of atomic sentences $\mathcal{P}$ and a set of agents $\mathcal{A}$ is a tuple $M = \langle W, V, \sigma_{\mathcal{A}} \rangle$ where:

- W is a set, whose elements are called *possible worlds*.
- $V : W \to \wp(\mathcal{P})$ is a *valuation map* that specifies for every world $w \in W$ which atomic sentences are true in w.
- $\sigma_{\mathcal{A}} = \{\sigma_a \mid a \in \mathcal{A}\}$ is a set of *epistemic maps* from W to $\wp(W)$, each of which assigns to any world $w \in W$ an information state $\sigma_a(w)$.

Note the similarity between epistemic models and the first-order information models we considered in Chapter 4. In both cases, a model consists of a set of possible worlds together with certain elements that describe the state of affairs at each possible world. What these elements are depends on the specific formal language that we consider.

The epistemic maps in an epistemic model are typically required to satisfy certain conditions, depending on the precise kind of knowledge or belief they are intended to capture. For instance, the following conditions are often imposed:

- *Factivity*: for any $w \in W$, $w \in \sigma_a(w)$
- *Introspection*: for any $w, v \in W$, if $v \in \sigma_a(w)$, then $\sigma_a(v) = \sigma_a(w)$

The factivity condition requires that the information available to agents be truthful, so that the information state of an agent is a knowledge state, rather than merely a belief state. The introspection condition requires that agents know what their own knowledge state is, so that if the information state of a in w differs from her state in v, then a can tell the worlds w and v apart. Either of these conditions may be dropped or weakened to model scenarios of false information or not fully introspective agents (see, e.g., Fagin *et al.*, 1995).

The epistemic maps $\sigma_a : W \to \wp(W)$ can be equivalently regarded as binary relations $\sim_a \subseteq W \times W$, where for any w and v: $w \sim_a v$ iff $v \in \sigma_a(w)$. The factivity and introspection conditions on σ_a then translate to the requirement that $\sim_a$ be an equivalence relation. While the presentation of epistemic models that uses equivalence relations rather than functions is more common in the literature, the functional notation has an important advantage for our current purposes: it brings out more clearly that the maps σ_a, together with the valuation V, characterize those aspects of a possible world that are deemed relevant. This suggests that, if we wanted to characterize possible worlds in more detail, taking into account more aspects than just the information available to all the agents involved, we could add further elements to our models to describe these additional aspects. This is indeed the approach we will take in Section 7.2.

Just as we did in Chapter 4, in what follows we will assume a fixed epistemic model M as our logical space and omit reference to it whenever possible.

7.1.2 Semantics

Within the context of an epistemic model M, the language of epistemic logic is interpreted by means of the following truth-conditional clauses.

DEFINITION 7.2. [Semantics of standard epistemic logic]

1. $w \models p \iff p \in V(w)$
2. $w \models \neg\varphi \iff w \not\models \varphi$
3. $w \models \varphi \wedge \psi \iff w \models \varphi$ and $w \models \psi$
4. $w \models \varphi \vee \psi \iff w \models \varphi$ or $w \models \psi$
5. $w \models \varphi \to \psi \iff w \not\models \varphi$ or $w \models \psi$
6. $w \models K_a\varphi \iff$ for all $v \in \sigma_a(w) : v \models \varphi$

The only novelty with respect to classical propositional logic is the interpretation of the modality K_a, which relies on the epistemic map σ_a. Notice that, if we denote by $|\varphi|$ the set of worlds w at which φ is true according to the above clauses, the truth-conditions for a modal formula $K_a\varphi$ may be written as follows:

6.′ $w \models K_a\varphi \iff \sigma_a(w) \subseteq |\varphi|$

Thus, $K_a\varphi$ is true at a world w in case φ follows from the information available to a in w. This reformulation of the clause brings out the fact that the modal statement $K_a\varphi$ makes a claim about the relation between two sets of worlds: the information state $\sigma_a(w)$ of the agent a at w, and the proposition $|\varphi|$ expressed by the argument. This perspective will help us understand how modalities can be generalized to the inquisitive setting, where both the state of an agent and the proposition expressed by a formula are no longer simple sets of worlds, but richer objects encoding both information and issues.

7.1.3 Common knowledge

Besides the agents' individual knowledge, notions of group knowledge also play an important role in the analysis of information exchange. One notion that is of particular importance is that of *common knowledge*, i.e., the information that is publicly shared among the group. One might think that treating this notion would require enriching our models with a map σ_* that specifies, for each world w, an information state $\sigma_*(w)$ embodying the information that is publicly available to all the agents in w. We could then expand our language with a corresponding common knowledge modality K_*, interpreted as follows:

$$w \models K_*\varphi \iff \sigma_*(w) \subseteq |\varphi|$$

However, common knowledge is closely tied to the agents' individual knowledge: in fact, it is *determined* by it. A sentence φ is common knowledge if and only if every agent a knows that φ, and every agent a knows that every agent b knows that φ, and every agent a knows that every agent b knows that every agent c knows that φ, and so on. Thus, the truth-conditions of the formula $K_*\varphi$ should be completely determined by the following condition:

$$w \models K_*\varphi \iff w \models K_{a_1}K_{a_2}\dots K_{a_n}\varphi \quad \text{for any } a_1, \dots, a_n \in \mathcal{A},\ n \geq 1$$

One can show that, in order to guarantee this equivalence for any particular valuation V, the common knowledge map σ_* must be defined precisely as follows:

$$\sigma_*(w) = \{\ v \mid \text{there exist } u_0, \dots, u_{n+1} \in W \text{ and } a_0, \dots, a_n \in \mathcal{A} \text{ such that } u_0 = w,\ u_{n+1} = v, \text{ and for } i \leq n,\ u_{i+1} \in \sigma_{a_i}(u_i)\}$$

This means that the common knowledge map σ_* is uniquely determined by the set of individual epistemic maps $\sigma_{\mathcal{A}}$, and need not be added to our models as an additional component.

7.2 Inquisitive epistemic logic

We now turn to inquisitive epistemic logic, IEL. In this framework it is not only possible to model the information available to a set of agents, but also the issues that they entertain.

7.2.1 Inquisitive epistemic models

While in epistemic logic a possible world w was characterized by (i) a valuation for the atomic sentences in the language, and (ii) an information state for each agent, we now also need to specify (iii) an *inquisitive state* for each agent, encoding the issues that the agent entertains in w. This is where the notion of issues that we introduced in Chapter 2 comes in. Recall that issues were construed as non-empty, downward closed sets of information states, namely precisely those information states that *resolve* the issue. Moreover, recall that it is only possible to *truthfully* resolve an issue I if the actual world is contained in at least one $s \in I$, i.e., if the actual world is contained in $\bigcup I$. We say that

I is an issue *over* the information state $\bigcup I$. Finally, recall that the set of all issues is denoted by $\mathcal{I}$.

In standard epistemic logic, every agent a is assigned an information state $\sigma_a(w)$ in every world w, determining the range of worlds that she considers possible candidates for the actual one. Now, every agent will also be assigned an inquisitive state $\Sigma_a(w)$, which will be modeled as an issue over the information state $\sigma_a(w)$, encoding the ways in which the agent would like to further enhance her current information state, narrowing down the set of possible candidates for the actual world.

Since $\Sigma_a(w)$ will be modeled as an issue over $\sigma_a(w)$, we will always have that $\sigma_a(w) = \bigcup \Sigma_a(w)$. This means that from the inquisitive state $\Sigma_a(w)$ of an agent a in a world w, we can always derive the information state $\sigma_a(w)$ of that agent in that world, simply by taking the union of $\Sigma_a(w)$. Thus, in effect, $\Sigma_a(w)$ encodes both the information available to a and the issues entertained by a in w. This means that the map Σ_a suffices as a specification of the state of the agent at each world, encompassing both information and issues. We do not have to list σ_a explicitly as an independent component in the definition of an inquisitive epistemic model: we can simply derive $\sigma_a(w)$ as $\bigcup \Sigma_a(w)$.

Definition 7.3. [Inquisitive epistemic models]
An inquisitive epistemic model for a set $\mathcal{P}$ of atoms and a set $\mathcal{A}$ of agents is a triple $M = \langle W, V, \Sigma_{\mathcal{A}} \rangle$ where:

- W is a set, whose elements are called possible worlds.
- $V : W \to \wp(\mathcal{P})$ is a *valuation map* that specifies for every world w which atomic sentences are true at w.
- $\Sigma_{\mathcal{A}} = \{\Sigma_a \mid a \in \mathcal{A}\}$ is a set of *state maps* $\Sigma_a : W \to \mathcal{I}$, each of which assigns to any world w an issue $\Sigma_a(w)$.

This general characterization of inquisitive epistemic models may again be supplemented with certain constraints on the agents' information states and inquisitive states. For instance, in analogy with the conditions considered above for standard epistemic models, we may require the following:

- *Factivity*: for any $w \in W$, $w \in \sigma_a(w)$
- *Introspection*: for any $w, v \in W$, if $v \in \sigma_a(w)$, then $\Sigma_a(v) = \Sigma_a(w)$

The factivity condition is just as before, ensuring that the agents' information states are truthful. The introspection condition now concerns both information and issues: agents must be introspective in that they must know not only what information they have, but also what issues they entertain. That is, if the state of a in world w differs from the state of a in v, either in information or in issues, then a must be able to tell w and v apart. These conditions are intended here just as an illustration: the choice of the conditions to be imposed on the state maps Σ_a will depend on the particular intended application of the framework, and in any case, it is orthogonal to the main novelties introduced by IEL.

Clearly, there is much similarity between inquisitive epistemic models and standard epistemic models. Both consist of a set of worlds, each equipped with (i) a valuation for atomic sentences and (ii) a state for each agent. The only difference is that while in standard EL the agents' states describe just their information, in IEL they encompass both their information and their issues.

7.2.2 Logical language and semantics

Let us now turn to the logical language. We will enrich the language of standard EL by adding a new modality E_a for each agent, which we read as 'a entertains φ'.[1] As we will see, this modality allows us to describe the issues that an agent is interested in, and in combination with the modality K_a, it allows us to define a modality W_a which is a reasonable formalization of the attitude of wondering about an issue.

In IEL, every sentence will be associated with an inquisitive proposition rather than a classical proposition. In this respect, the transition from EL to IEL is just like that from classical first-order logic to InqB. This means that the semantics will be characterized not in terms of the relation of truth with respect to a world, but in terms of support with respect to an information state. The proposition expressed by a formula φ, $[\varphi]$, will be defined as the set of information states that support φ. The recursive definition of support for IEL runs as follows.

Definition 7.4. [Semantics of inquisitive epistemic logic]

1. $s \models p \iff$ for all $w \in s,\ p \in V(w)$
2. $s \models \neg\varphi \iff$ for all $t \subseteq s$ such that $t \neq \emptyset :\ t \not\models \varphi$
3. $s \models \varphi \wedge \psi \iff s \models \varphi$ and $s \models \psi$
4. $s \models \varphi \vee \psi \iff s \models \varphi$ or $s \models \psi$
5. $s \models \varphi \to \psi \iff$ for all $t \subseteq s :\ t \models \varphi$ implies $t \models \psi$
6. $s \models K_a\varphi \iff$ for all $w \in s : \sigma_a(w) \models \varphi$
7. $s \models E_a\varphi \iff$ for all $w \in s,$ for all $t \in \Sigma_a(w) : t \models \varphi$

[1] We use 'entertain' here as a technical term, which—unlike the 'wonder' modality that it allows us to define—is not supposed to correspond precisely with its non-technical use.

As far as the non-modal fragment of the language is concerned, the recursive characterization of support runs just as in Chapter 4. That is, the support clauses for the propositional part of the language still associate the connectives with the basic algebraic operations on inquisitive meanings. The novelty introduced by IEL lies in the clauses for the modalities K_a and E_a. To understand these clauses, it is useful to look at the truth-conditions to which they give rise. Recall from Fact 2.35 on page 32 that truth with respect to a world w always amounts to support with respect to $\{w\}$. Thus, by considering what is needed for support with respect to singleton states, we obtain the following truth-conditions for the modalities.

Fact 7.5. [Truth-conditions for modal formulas]

1. $w \models K_a\varphi \iff \sigma_a(w) \models \varphi$
2. $w \models E_a\varphi \iff$ for all $t \in \Sigma_a(w) : t \models \varphi$

Given these truth conditions, it becomes clear that $K_a\varphi$ is supported by a state s just in case it is true at any world in s, and analogously for $E_a\varphi$:

$$s \models \Box\varphi \iff w \models \Box\varphi \text{ for all } w \in s \qquad \text{where } \Box \in \{K_a, E_a\}$$

This means, given Fact 2.42 on page 35, that modal formulas are always statements.

Fact 7.6. For any φ, $K_a\varphi$ and $E_a\varphi$ are statements.

Now, since the semantics of modal formulas is completely determined by their truth-conditions with respect to worlds, we can focus on understanding what it takes for modal formulas of the form $K_a\varphi$ and $E_a\varphi$ to be true at a world.

$K_a\varphi$ is true at a world w just in case the information available to a settles the proposition expressed by φ, i.e., $\sigma_a(w) \in [\varphi]$. As we will discuss in more detail below, this is a natural generalization of the interpretation of $K_a\varphi$ in standard epistemic logic, which does not only deal appropriately with cases where φ is purely informative, but also with cases where φ is inquisitive.

On the other hand, $E_a\varphi$ is true at a world w just in case the *inquisitive state* of a entails $[\varphi]$, i.e., $\Sigma_a(w) \subseteq [\varphi]$. This means that all the states in $\Sigma_a(w)$, i.e., the states that a wants to get to, are ones that settle $[\varphi]$. In other words, the issue expressed by φ is one that a would like to see resolved. This is close to saying that *a wonders* about the issue expressed by φ, with one exception: if a already has enough information to resolve the issue expressed by φ, i.e., if $K_a\varphi$ holds, then according to the above characterization, $E_a\varphi$ holds as well. But in such a scenario, we would not say that *a wonders* about the issue expressed by φ. The situation of an agent *a wondering* about the issue expressed by φ can be characterized as one where the agent does not yet have sufficient information to resolve the issue (so that $\neg K_a\varphi$ holds) but the states she wants to get to are states that do contain such information (so that $E_a\varphi$ holds). So, we can define W_a in terms of K_a and E_a as follows:

$$W_a\varphi := \neg K_a\varphi \wedge E_a\varphi$$

Let us illustrate the workings of our modal operators in some more detail by means of some concrete examples. First consider $K_a\, p$: this is true at a world w in case the information state of a at w supports p. This simply means that p must be true throughout $\sigma_a(w)$. Thus, when applied to an atomic sentence, K_a boils down to the familiar knowledge modality of standard epistemic logic. This holds more generally if K_a applies to statements, atomic or otherwise.

However, now K_a can also be applied to inquisitive sentences. As an example, consider $K_a?p$. This formula is true at a world w in case the information state of a supports $?p$. This means that p must either be true throughout $\sigma_a(w)$ or false throughout $\sigma_a(w)$. That is, a either has to know that p holds or that p does not hold—in other words, a has to know *whether* p holds. Thus, in IEL the K_a modality is generalized in such a way that one and the same semantic clause delivers the expected reading for $K_a\varphi$ both when φ is a statement like p, and when φ is a question like $?p$.

Now let us consider $E_a\, p$. This formula is true in a world w in case any $t \in \Sigma_a(w)$ supports p, that is, in case any $t \in \Sigma_a(w)$ is included in $\mathsf{info}(p)$; clearly, this holds if and only if $\sigma_a(w) = \bigcup \Sigma_a(w) \subseteq \mathsf{info}(p)$. But this is precisely what is required for the truth of $K_a\, p$. In sum, we have $w \models E_a\, p \iff w \models K_a\, p$; since all modal formulas are statements, this implies that $E_a\, p$ and $K_a\, p$ are equivalent. This example illustrates a more general fact: when E_a is applied to a statement, atomic or otherwise, it simply boils down to the K_a modality, and thus coincides with the familiar K_a modality of standard epistemic logic.

Things become more interesting when E_a applies to an inquisitive formula. Consider for example $E_a?p$, which is true at a world w in case $\Sigma_a(w) \subseteq [?p]$. This means that any information state which settles the issues that a entertains at w also resolves the question $?p$. Now, a trivial way in which this may hold is if a's current information state, $\sigma_a(w)$, already resolves $?p$, that is, if we have $w \models K_a?p$. In this case, any $t \in \Sigma_a(w)$, being an enhancement of $\sigma_a(w)$, must also resolve $?p$. On the other hand, it may also be the case that the agent's current information does not settle $?p$, but the states that the agent wants to get to are all ones that do. This holds precisely when we have that $W_a?p$, which we defined as an abbreviation of $\neg K_a?p \wedge E_a?p$. Thus, $W_a?p$ captures the fact that *a wonders whether p*, in the sense that a does not know whether p is true but wants to find out.

Interestingly, the fact that E_a and K_a coincide when applied to statements implies that, whenever φ is a statement, $W_a\varphi$ is contradictory. That is, applying the wondering modality to a statement is bound to give rise to a contradiction. This may offer an explanation for the fact that, in English and many other languages, the verb *wonder* (as well as other inquisitive attitude verbs such as *investigate* and *be curious*) cannot take declarative complements.

Now let us briefly consider the nature of the modal operators in IEL from a more mathematical perspective. Clearly, our K_a and E_a are not standard Kripke modalities; that is, they cannot be regarded as quantifiers asserting the truth of their argument at some/all accessible worlds. Yet, there is a sense in which these operators work in our system precisely the way Kripke modalities work in standard modal logic.

In Section 7.1, we remarked that in EL, the modality K_a can be regarded as expressing a relation between two semantic objects of the same kind: the state $\sigma_a(w)$ associated with the world of evaluation w, and the proposition $|\varphi|$ expressed by the argument. This relation simply amounts to inclusion:

$$w \models K_a\varphi \iff \sigma_a(w) \subseteq |\varphi|$$

All modal operators of standard modal logic can be seen as working in this way: they express a relation between two sets of worlds, a set of worlds associated with the world of evaluation, and the proposition expressed by the sentence that the operator takes as its argument.

Our modal operators K_a and E_a work in a very similar way: they express a relation between two semantic objects of the same kind, the state $\Sigma_a(w)$ associated with the evaluation world, and the proposition $[\varphi]$ expressed by the argument, as the following re-formulation of their truth-conditions shows.

$$\begin{aligned} w \models K_a\varphi &\iff \bigcup \Sigma_a(w) \in [\varphi] \\ w \models E_a\varphi &\iff \Sigma_a(w) \subseteq [\varphi] \end{aligned}$$

The only difference is that, now, both the state $\Sigma_a(w)$ and the proposition $[\varphi]$ are no longer simple sets of worlds: rather, they are downward closed sets of information states, which capture both information and issues. Notice that, since such semantic objects have more structure than simple sets of worlds do, one may naturally consider several other relations, besides the two expressed by K_a and E_a. In this way, the inquisitive perspective suggests a natural generalization of the notion of modal operators. We will briefly come back to this point in Section 7.3 and in the exercises appended to this chapter.

7.2.3 Common knowledge and public issues

Besides the information and issues that are private to each agent, agents also share certain public information and jointly entertain certain issues. In Section 7.1 we saw how the common knowledge construction in epistemic logic allows us to derive a public information map σ_* representing the information that is publicly available to all the agents, starting from the epistemic maps σ_a encoding the information available to each individual agent. The question is whether this construction can be generalized to the present setting. That is, is it possible to derive a public state map Σ_*, encoding public information and issues, from the maps Σ_a describing the information and issues of each individual agent?

One way to go about answering this question is to consider, as we did in the case of common knowledge, the conditions that a public entertain modality E_* associated with the map Σ_* would have to satisfy. This will put constraints on the definition of Σ_*, which may be sufficient to get at a unique characterization. So, let us consider what it would mean for a sentence to be *publicly entertained*. In standard epistemic logic, φ is publicly known in case every agent knows that φ, and every agent knows that every agent knows that φ, and so on. Analogously, it seems natural to say that φ is *publicly entertained* in case every agent entertains φ, and every agent knows that every agent entertains φ, and every agent knows that every agent knows, etcetera. Thus, the behavior of the public entertain modality E_* would have to be subject to the following condition:

$$w \models E_*\varphi \iff w \models K_{a_1} \dots K_{a_{n-1}} E_{a_n}\varphi \quad \text{for all } a_1 \dots a_n \in \mathcal{A},\ n \geq 1$$

If one finds the alternation of the modalities puzzling, there is no need to worry: since K_a and E_a are equivalent when applied to statements, and since any sentence that starts with a modality is a statement, we can simply replace all the K_a's with E_a and obtain the equivalent 'homogeneous' condition:

$$w \models E_*\varphi \iff w \models E_{a_1} \dots E_{a_{n-1}} E_{a_n}\varphi \quad \text{for all } a_1 \dots a_n \in \mathcal{A},\ n \geq 1$$

Does this condition constrain the map Σ_* sufficiently to characterize it uniquely? The answer is *yes*. One can verify that the above condition on E_* holds for any particular valuation V if and only if the map Σ_* is defined as follows:

$$\begin{aligned}\Sigma_*(w) = \{\ s \ \mid \ & \text{there exist } v_0, \dots, v_n \in W \text{ and } a_0, \dots, a_n \in \mathcal{A} \\ & \text{such that } v_0 = w,\ v_{i+1} \in \sigma_{a_i}(v_i) \text{ for all } i < n, \text{ and } s \in \Sigma_{a_n}(v_n)\}\end{aligned}$$

Importantly, the public information map σ_* corresponding to the public state map Σ_*, defined as $\sigma_*(w) := \bigcup \Sigma_*(w)$, coincides exactly with the map we would obtain by performing the common knowledge construction on the individual information maps σ_a. Thus, the standard common knowledge construction from epistemic logic generalizes smoothly to a 'public state' construction which encompasses both information and issues.

Given this construction, we can add modalities K_* and E_* to our logical language, to be interpreted as follows:

$$s \models K_*\varphi \iff \text{for all } w \in s:\ \sigma_*(w) \models \varphi$$

$$s \models E_*\varphi \iff \text{for all } w \in s,\ \text{for all } t \in \Sigma_*(w):\ t \models \varphi$$

If φ is a statement, then $K_*\varphi$ gets its standard meaning, expressing that φ is common knowledge. On the other hand, in our setting K_* also applies to inquisitive sentences: if φ is inquisitive, then $K_*\varphi$ says that the group's common knowledge settles the issue expressed by φ—in short, that φ is *publicly settled*. For instance, the formula $K_*?p$ captures the fact that it is common knowledge among the group whether p.

Moreover, by combining the two public modalities we can define a public version of the wonder modality, W_*: a group of agents jointly wonder about φ if they publicly entertain φ and φ is not yet publicly settled.

$$W_*\varphi := \neg K_*\varphi \wedge E_*\varphi$$

Just like K_*, the modality W_* is very useful in formally describing an information exchange: while K_* lets us describe which issues the conversational participants have publicly settled, W_* lets us describe what the *open issues* are in the exchange, i.e., what the issues are that the group as a whole would like to see resolved and for which no resolution has been publicly established yet.

An interesting feature of the public wondering operator is that $W_*\varphi$ does not entail $W_a\,\varphi$ for a particular agent a. While this may come as a surprise at first, it is just as it should be: if $W_*\varphi$ holds, then φ is publicly entertained but not publicly settled, that is, the common knowledge of the group does not settle φ. It may well be that there is some agent whose *private* knowledge does settle φ. This does not prevent φ from being an open issue for the group, so long as this private information is not made publicly available. In fact, $W_*\varphi$ might even be the case while *every* individual agent can resolve φ, but the information needed to resolve φ has not been made common knowledge: although the issue is settled for each individual agent in this case, it is still open for the group as a whole.

This concludes our brief presentation of IEL. In Ciardelli and Roelofsen (2015b) this basic framework is presented in more detail and extended with a dynamic modal operator which makes it possible to describe how the agents' private and public information and issues *change* when a statement is made or a question is asked, generalizing the 'public announcement operator' of *dynamic epistemic logic* (see, e.g., van Ditmarsch *et al.*, 2007). In Ciardelli (2014b) the logic that IEL gives rise to is investigated and axiomatized. Finally, in Ciardelli and Roelofsen (2014) a more fine-grained system is developed, which does not only deal with 'hard knowledge' but also with beliefs which may be revised or retracted. This inquisitive believe revision framework can not just be used to model linguistic information exchange, but also other information-related processes such as rational inquiry, where the interplay between issues and beliefs has been argued to play a crucial role (see, e.g., Olsson and Westlund, 2006).

7.3 Beyond *know* and *wonder*

We have focused our attention in this chapter on a small set of modal operators, in a particular logical setting. However, the approach that we have taken may well be applicable beyond this restricted setting as well, giving rise to a richer view on the linguistic notion of modality in general. We end this chapter with some programmatic remarks on the potential benefits of such an enriched perspective.

In linguistics, modal expressions are standardly characterized as sentential operators that relate the proposition expressed by their argument (their *prejacent*) to a proposition encoding a set of relevant background assumptions (the *modal base*). Some modal expressions indicate that the prejacent is *consistent* with the modal base (possibility modals), while others indicate that the prejacent is *entailed* by the modal base (necessity modals). The nature of the modal base depends on the particular flavor of the modal expression. For instance, epistemic modals relate their prejacent to a relevant body of information, while deontic modals relate their prejacent to a modal base determined by a relevant set of rules. Finally, modal expressions differ in their grammatical category. Among the most widely investigated kinds of modal expressions are propositional attitude verbs like *know*, *believe*, *want*, and *hope*, and auxiliary verbs like *might*, *may*, *must*, and *should*.

Sophisticated theories have been developed to capture the core mechanisms that underlie the linguistic behavior of all these different types of modal expressions in a unified way (see in particular Kratzer 2012 for a collection of influential articles, and Kaufmann and Kaufmann 2015 for a recent survey). However, while the domain that is covered by these theories is indeed impressively broad, the approach taken in inquisitive epistemic logic suggests a substantial further generalization, both of the linguistic notion of modal expressions as such, and of the theories that deal with them.

Namely, rather than construing modal expressions as relating two classical propositions, we may construe them as relating two inquisitive propositions, just as we did with the K_a and W_a modalities in IEL. This would broaden our linguistic view on modality in three ways. First, as exemplified in a very concrete way in IEL, the class of modal expressions would become richer, now also including operators that take inquisitive constructions as their argument. Thus, it would become possible to pursue a unified account of propositional attitude verbs like *know*, *believe*, *want*, and *hope* on the one hand, and issue-directed attitude verbs like *wonder*, *be curious*, and *investigate* on the other. Second, a more fine-grained notion of modal bases would become available: we could interpret modal expressions not only in the context of a certain body of information, but also in the context of a relevant background *issue*. And third, while on the standard account there are only two salient relations between the prejacent and the modal base, i.e., inclusion (entailment) and overlap (consistency), inquisitive propositions have much more structure than classical propositions, and can therefore be related in many more ways. This would allow for a refinement of the basic dichotomy between possibility and necessity modals.

While these remarks are admittedly very programmatic and clearly stand in need of concrete substantiation, the research programme that they suggest seems an exciting one to pursue. The treatment of *know* and *wonder* developed in IEL just constitutes the first step in this direction.

7.4 Exercises

Exercise 7.1. [Ignorance]
Consider a new modal operator N_a in IEL, where $N_a\,\varphi$ is informally read as 'a is completely ignorant with regard to φ'.

1. Define a suitable semantic interpretation of $N_a\,\varphi$.

2. Check whether $N_a\,\varphi$ is equivalent to $\neg K_a\,\varphi$ whenever φ is a statement.

Exercise 7.2. [Agnosticism]
Consider a new modal operator G_a in IEL, where $G_a\,\varphi$ is informally read as 'a is completely agnostic with regard to φ'.

1. Define a suitable semantic interpretation of $G_a\,\varphi$.

2. Check whether $G_a?\varphi$ is equivalent to $\neg E_a?\varphi$ whenever φ is a statement.

Chapter 8

Conclusion

We will end with a summary of the framework we presented, and will briefly consider to what extent the high-level desiderata discussed in Chapter 1 have been met.

8.1 Summary

Let us start by reviewing the main concepts that play a role in InqB, the basic first-order inquisitive semantics presented in Chapter 4. The diagram in Figure 8.1 provides an overview of these concepts and the dependencies between them. Our starting point was a particular *language* $\mathcal{L}$, in this case the language of first-order logic (the upper leftmost item in the diagram). Given this language, we defined the *models* relative to which the sentences in our language would be interpreted. A model was construed as a set of possible worlds W, associated with a domain of discourse and an interpretation function determining the denotation of the basic elements of our language (function symbols and relation symbols) in each possible world. Thus, a model determines a certain *logical space*, the set of worlds W, as well as a particular connection between the worlds in this space and the basic elements of the language under consideration.

We adopted the standard notion of *information states* as sets of possible worlds, i.e., subsets of W. In terms of information states, we characterized the crucial notion of *issues*, and based on this notion of issues we introduced a notion of *discourse contexts* encompassing both the information established so far and the issues raised so far. Then, as customary under a dynamic view on meaning, we defined *meanings* as context change potentials, i.e., functions over discourse contexts, and we showed that under certain natural assumptions (i.e., that meanings always map a given input context to an extension of that context in a way that satisfies the 'compatibility condition'), every meaning f uniquely corresponds to a static object P_f such that for any context c, $f(c)$ simply amounts to $c \cap P_f$. These static objects, then, play exactly the same

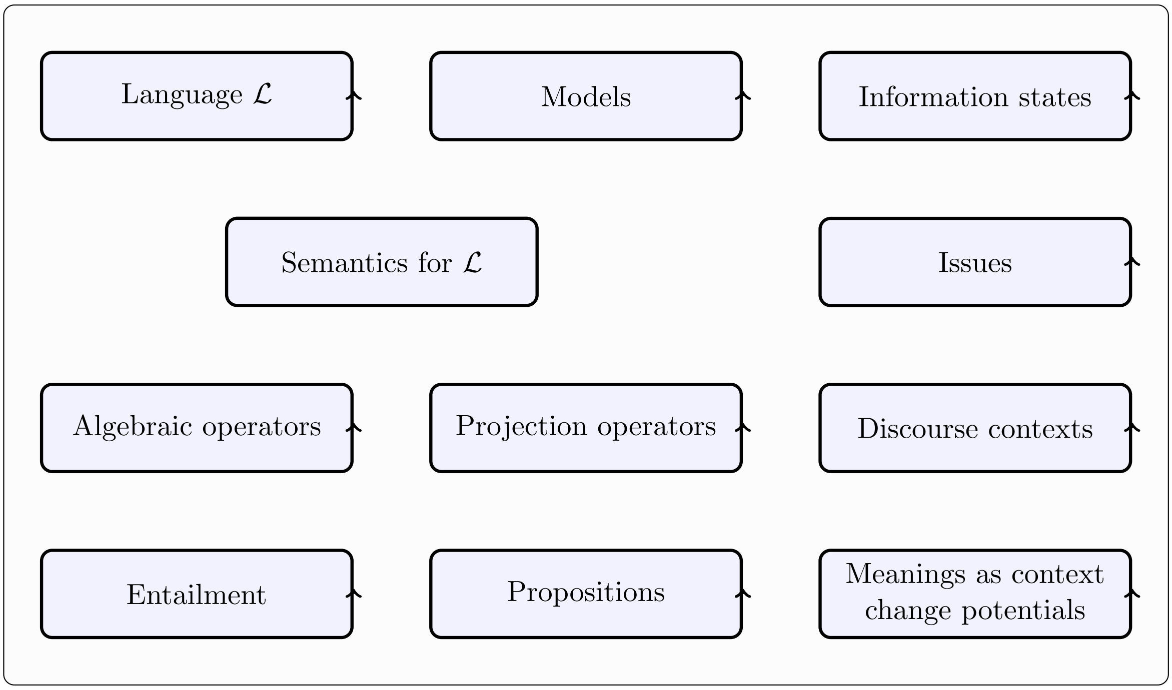


Figure 8.1: Dependency diagram of the main concepts in InqB.

role as *propositions* do in the standard static framework. Only, while in the standard framework both contexts and propositions are simple sets of possible worlds, in inquisitive semantics they are sets of information states, non-empty and downward closed. Thus, we obtained a more general notion of meaning, both at the dynamic level of context change potentials and at the static level of propositions.

We defined a notion of *entailment* between propositions, and in terms of this notion, we characterized a number of basic *algebraic operators* on propositions. On the other hand, we also defined two *projection operators*, which trivialize either the informative or the inquisitive content of any given proposition.

Finally, using the algebraic operators and the projection operators, we defined a semantics for the language $\mathcal{L}$ that we started out with, coming full circle.

Having laid out this schematic overview of InqB, we would like to emphasize two things. First, in many concrete applications of inquisitive semantics, the only new notion that is really needed from all the above is the inquisitive notion of propositions, as opposed to classical propositions. Even though we characterized propositions in terms of context change potentials, which in turn were defined in terms of contexts, which in turn were characterized in terms of issues, it is very well possible to take a shortcut and avoid introducing all these intermediate notions if they are not required for the given application. After all, propositions are just sets of information states, non-empty and downward closed. So after having introduced the (completely standard) notion of information states, one could in principle proceed immediately to characterize the propositions expressed by the sentences in the language under consideration, either directly or through a recursive definition of the support conditions for the sentences in the language (see the discussion in Section 4.2). Of course, for the general purpose of analyzing linguistic information exchange, the intermediate notions are all of interest in themselves. However, if they are not needed for a particular application, they can safely be skipped, which allows for a much more efficient presentation of the framework (see, e.g., Ciardelli and Roelofsen, 2011 for an example of such a compact presentation in a logical setting, and Roelofsen and Farkas, 2015, pp.366-369, for another example in the context of a particular linguistic application).

Something else that we would like to emphasize is that all the notions that play a crucial role in InqB, except for the logical language and the models with respect to which the sentences in the language are interpreted, were already characterized in Chapters 2-3, *without reference to any particular logical or natural language*. This makes these notions highly general and widely applicable.

As we saw in Chapter 4, what becomes necessary when turning to a particular language is a more specific characterization of the assumed logical space. In Chapters 2-3, we just assumed a generic set of possible worlds W as our logical space, without any further specification. The moment we fix a particular logical language, we have to establish a connection between the worlds in our logical space and the basic elements of our language. Thus, in Chapter 4, we supplemented the set of possible worlds W with a domain of individuals D and a function I determining the denotation of the basic elements of our language (in this case, function symbols and relation symbols) w.r.t. each world $w \in W$. Having fixed this connection between 'worlds and words', all the general notions introduced in Chapters 2-3 could be imported straightforwardly.

In Chapter 7 we considered another logical language, namely a propositional language with modal operators to describe the knowledge and issue of a given set of agents. Accordingly, we re-construed our logical space as a set of possible worlds W together with (i) a valuation function, determining the truth value of the atomic sentences in our language at every world $w \in W$, and (ii) a set of *state maps* $\Sigma_{\mathcal{A}}$, determining the information states and inquisitive states of all the agents at every world $w \in W$. Having thus established a suitable connection between the worlds in our logical space and the basic elements of our logical language (in this case, atomic sentence and the modal operators), all the general notions laid out in Chapters 2 could once again be imported straightforwardly.

The fact that the framework is built up in this modular way makes it very flexible. There are many ways in which the basic notions introduced here may be further refined, extended, and applied (see Appendix B for some references).

8.2 Mission accomplished?

Let us now return to the high-level desiderata for a formal framework for the analysis of linguistic information exchange which were discussed in Chapter 1, and indicate how the framework we presented addresses these desiderata.

The first high-level desideratum was a formal notion of issues that allows for a suitable representation of contexts, meanings, and propositional attitudes. In Chapter 2 we introduced such a notion of issues, and in terms of it we defined a notion of discourse contexts and a notion of meaning. In Chapter 5 we argued that this notion of meaning is particularly suitable for the semantic analysis of questions, overcoming the main shortcomings of previous frameworks for question semantics (alternative semantics, partition semantics, and inquisitive pair semantics). Finally, in Chapter 7 we showed that the new notion of issues facilitates a richer view on propositional attitudes as well, encompassing both information-directed attitudes like *know* and *believe*, and issue-directed attitudes like *wonder*.

The second high-level desideratum was a framework that allows for an *integrated* treatment of declarative and interrogative sentences, with a single notion of meaning embodying both informative and inquisitive content, rather than two separate notions of meaning for the two different sentence types. One of the arguments that we made to justify this desideratum was that declarative and interrogative sentences are to a large extent built up from the same lexical, morphological, and intonational elements. A general characterization of the semantic contribution of each of these elements should capture both their contribution to the informative content and to the inquisitive content of the sentence that they are part of. This requires a notion of meaning that encompasses both informative and inquisitive content.

The notion of meaning that we introduced in Chapter 2 satisfies this requirement, and the merits of this feature of the framework were illustrated in Chapter 6 with a concrete analysis of declarative and interrogative sentences involving disjunction and various intonation patterns. In particular, both disjunction and the relevant intonational elements were given a uniform treatment across the two sentence types.

Thus, both desiderata have been met and the ensuing benefits have been concretely substantiated. From a narrow perspective, then, our goals have been achieved. From a broader perspective, however, these result just indicate that our general mission is worthwhile pursuing. We do not see the basic framework presented here as a final product but much rather as a point of departure.

Appendix A

Sources

These lecture notes bring together a number of ideas and results from previous publications, manuscripts, and teaching materials. Below we list the main sources for each chapter, which in many cases contain more in-depth discussion of the ideas presented here.

- Chapter 1: Roelofsen (2014).
- Chapter 2: Ciardelli, Groenendijk, and Roelofsen (2012, §2) and Ciardelli, Groenendijk, and Roelofsen (2013a, §2-5)
- Chapter 3: Roelofsen (2013a, 2015b)
- Chapter 4: Ciardelli (2009); Groenendijk and Roelofsen (2009); Ciardelli and Roelofsen (2011); Roelofsen (2013a)
- Chapter 5: Ciardelli, Groenendijk, and Roelofsen (2013a, §6) and Ciardelli and Roelofsen (2015a, 2016)
- Chapter 6: Roelofsen (2013c, 2015a); Roelofsen and Farkas (2015)
- Chapter 7: Ciardelli and Roelofsen (2015b)

Appendix B

Pointers for further reading

Despite its relatively recent inception, there has already been a lot of work on inquisitive semantics, much more than we have been able to cover in these lecture notes. The basic framework presented here has been further extended, refined, and applied in several ways, the logical properties of the framework have been investigated, and some interesting connections with other logical frameworks have emerged, though in all these areas there are still many open issues to be addressed. Below we provide some pointers for further reading.

Extensions of InqB and IEL.

- A type-theoretical extension of InqB, for full compositionality:
  Theiler (2014); Ciardelli and Roelofsen (2015a)
- A presuppositional extension of InqB:
  Ciardelli *et al.* (2012, 2015); Roelofsen (2015a)
- An extension of InqB with propositional discourse referents:
  Roelofsen and Farkas (2015)
- An extension of IEL with a dynamic 'public announcement' operator:
  Ciardelli and Roelofsen (2015b)
- An extension of IEL with graded beliefs next to hard knowledge:
  Ciardelli and Roelofsen (2014)

Refinements of InqB.

- A refinement of InqB with a weak negation operator, whose treatment requires the existence of propositions that are not downward closed:
  Punčochář (2015b)

- A refinement of InqB that does not model information states as sets of possible worlds but as primitive objects in an algebra that is less specific than the Boolean algebra of information states as sets of worlds:
  Punčochář (2015a)

- A refinement of InqB that does not characterize a proposition just in terms of the states that support it, but also in terms of the states that reject it or 'dismiss a supposition' of it, referred to as InqS:
  Groenendijk and Roelofsen (2015)

- An extension of InqS with operators corresponding to epistemic and deontic modal auxiliaries (*might*, *may*, *must*):
  Aher and Groenendijk (2015)

- A refinement of InqB that is not only concerned with informative and inquisitive content, but also 'attentive content', whose treatment again requires propositions that are not downward closed:[1]
  Ciardelli *et al.* (2014)

Logical investigations.

- Logical investigation of InqB:
  Ciardelli (2009); Ciardelli and Roelofsen (2011)

- Logical investigation of IEL:
  Ciardelli (2014b)

- Logical investigation of various refinements of InqB, listed above:
  Punčochář (2015a,b)

- Logical investigation of inquisitive pair semantics:
  Mascarenhas (2009); Sano (2009, 2011)

- On the general role of questions in logic:
  Ciardelli (2015)

Applications in linguistics.

- Root questions:
  AnderBois (2011, 2012); Roelofsen and Farkas (2015); Roelofsen (2015a)

[1]There is also work which argues that it is in fact impossible to capture all three types of content—informative, inquisitive, and attentive—at once using a single semantic object, and pursues a two-dimensional approach instead (Roelofsen, 2013b).

- Embedded questions:
  Theiler (2014); Roelofsen *et al.* (2014); Herbstritt (2014); Roelofsen *et al.* (2016)

- Potential questions: Onea (2013)

- Answer particles (*yes*/*no*): Roelofsen and Farkas (2015)

- Disjunction: Winans (2012); Roelofsen (2015a,b)

- Modal auxiliaries: Aher (2013); Aher and Groenendijk (2015)

- Conditionals:
  Onea and Steinbach (2012); Starr (2014); Groenendijk and Roelofsen (2015)

- Quantifier particles: Szabolcsi (2015b)

- Ellipsis: AnderBois (2014)

- Exhaustivity implicatures: Westera (2012, 2013a,b)

- Imperatives: Aloni and Ciardelli (2013)

- Scalar modifiers: Coppock and Brochhagen (2013)

- Directional numeral modifiers: Blok (2015)

- Attentive *might*: Roelofsen (2013b); Ciardelli *et al.* (2014)

Applications in cognitive science.

- Reasoning fallacies: Koralus and Mascarenhas (2014); Mascarenhas (2014)

- Implicit causality: Spenader (2015)

Applications in epistemology.

- The Gettier puzzle for knowledge ascriptions: Uegaki (2012)

- Conversational inquiry: Hamami (2014)

- Belief revision: Ciardelli and Roelofsen (2014)

Connections.

- Dependence logic:
  Väänänen (2007); Yang (2014); Ciardelli (2016)
- Truth-maker semantics:
  Van Fraassen (1969); Fine (2014); Yablo (2014); Ciardelli (2013)
- Possibility semantics for modal logic:
  Humberstone (1981); Holliday (2014, 2015)
- Inferential erotetic logic:
  Wiśniewski and Leszczyńska-Jasion (2015)

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