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Comparatives Revisited: Downward-Entailing Differentials Do Not Threaten Encapsulation Theories

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Than-clause-internal quantifiers do NOT need to take scope over the matrix clause.

The distribution-based analysis (entanglement theories)

$[\![(1)]\!] \Leftrightarrow \forall x[\text{girl}(x) \rightarrow \text{height(John)} > \text{height}(x)]$

Than-clause-internal quantifiers need to take scope over the matrix clause.

A threat for the endpoint-based analysis?

Do both the endpoint-based and distribution-based analyses empirically adequate?

Fleisher (forthcoming): No, because the endpoint-based analysis cannot account for downward-entailing differentials.

The distribution-based analysis

$[\![$than every girl is (tall)$]\!] \approx \forall x[\text{girl}(x) \rightarrow \text{height}(x)...]$

(2)
a. John is taller than every girl is.
b. John is **exactly 4 inches** taller than every girl is
c. John is **less than 4 inches** taller than every girl is.

The endpoint-based analysis

Does ⟦than every girl is (tall)⟧ have a unified meaning?

(3) a. John is taller than every girl is.
MAX reading:
height(John) $>$ height(the tallest girl)

b. John is **exactly 4 inches** taller than every girl is.
MAX=MIN reading:
height(John) $>$ height(the tallest/shortest girl)
$\rightsquigarrow$ Girls are of the same height.

c. John is **less than 4 inches** taller than every girl is.
MAX-&-MIN reading:
height(the shortest girl) $+4'' >$ height(John) $>$ height(the tallest girl)

Overview

We propose a new way to implement the endpoint-based analysis.

- We show that DE or non-monotone differentials do not threaten the endpoint-based analysis.
- Thus *than*-clause-internal quantifiers **do not have to** take scope.

Innovation

Semantics of comparatives

Comparatives are analyzed as the expression about the distance (i.e., result of subtraction) between two intervals.

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Comparatives are analyzed as the expression about the distance (i.e., result of subtraction) between two intervals.

Semantics of comparative morphemes

Comparative morphemes (e.g., *more, -er*) are analyzed as intervals that serve as differentials.

Outline

Observation 1: Interval scales vs. ratio scales

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Interval scales

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They do not necessarily have a meaningful, non-arbitrary and unique zero point.

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Ratio scales

E.g., duration ...

They necessarily have a meaningful, non-arbitrary and unique zero point.

Observation 1: Interval scales vs. ratio scales

(4) We arrived 2 hours earlier than the check-in time.

a. On an **interval scale**:
(i) the time when we arrived; (ii) the check-in time.

b. On a **ratio scale**: the differential *2 hours*.

(5) FSU ranked 3 spots higher than UNC. (from Twitter)

a. On an **interval scale**:
(i) the position of FSU; (ii) the position of UNC.

b. On a **ratio scale**: the differential *3 spots*.

The essential meaning of comparatives

A relation among three things

A relation among **two positions on an interval scale** (i.e., the one representing the comparative standard, e.g., the check-in time, and the one representing the comparative subject, e.g., our arrival time) and **the distance between them**.

Using intervals to express positions

Following Schwarzschild and Wilkinson 2002, we use **intervals** (i.e., convex sets of points), instead of degrees (i.e., points), to represent positions.

Interval

An **interval** represents a value as a range of possibilities (cf. the studies on vagueness or super valuation).

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Notation

An interval $\lambda\delta_d.\{\delta | D_{\text{min}} \leq \delta \leq D_{\text{max}}\}$ can be written as $[D_{\text{min}}, D_{\text{max}}]$.
Type of **degree**: d
Type of **interval**: $\langle dt \rangle$

Obervation 2: some uses of comparative morphemes

(6) He drank till he blacked out. Then he drank (a bit) **more**.

(7) War brings depression; **what's more**, it brings chaos.

The essential meaning of comparative morphemes

Augend + **Addend** = Sum

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Sum - Augend = **Addend/Differential**

The essential meaning of comparative morphemes

More means an addend/differential.

In comparatives, the augend, the addend (or differential) and the sum are all in the same sentence:
(i) the **comparative standard** plays the role of **augend**;
(ii) the **comparative subject** the role of **sum**;
(iii) the **comparative morpheme, i.e., more**, the role of **differential**.

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In comparatives, since the augend and the sum are intervals, here the differential is also an interval.
In other words, we analyze *more* as an interval.

Entities vs. intervals

x_e	$D_{\langle dt \rangle}$
someone	*some*
other	*more*
the other	*the more*
another	*one more*
John	*3 feet*
John, another boy, (will come).	*3 feet more/-er*

Interval operations (Ramon E. Moore. *Methods and Applications of Interval Analysis*. 1979)

$[x_1, x_2]\langle\text{op}\rangle[y_1, y_2]$
$= [\text{MIN}(x_1\langle\text{op}\rangle y_1, x_1\langle\text{op}\rangle y_2, x_2\langle\text{op}\rangle y_1, x_2\langle\text{op}\rangle y_2),$
$\text{MAX}(x_1\langle\text{op}\rangle y_1, x_1\langle\text{op}\rangle y_2, x_2\langle\text{op}\rangle y_1, x_2\langle\text{op}\rangle y_2)]$

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Interval operations

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Examples of interval subtraction

$[5, 8] - [1, 2] = [3, 7]$
$[5, 8] - [3, 7] = [-2, 5]$

Interval subtraction

If $X - [a, b] = [c, d]$, then what is X?

Generally speaking, it is not the case that $X = [a + c, b + d]$.

If $X - [a, b] = [c, d]$,
X is undefined when $b + c > a + d$ (i.e., when the lower bound of X is larger than the upper bound of X);
when defined, $X = [b + c, a + d]$.

Scalar adjectives

$[\![\text{tall}]\!]_{\langle dt,et\rangle} \stackrel{\text{def}}{=} \lambda D_{\langle dt\rangle}.\lambda x_e.[\text{height}_{\langle e,dt\rangle}(x) \subseteq D]$
i.e., the height of the individual x is in the interval D.

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$[\![\text{John is D}_c \text{ tall}]\!] \Leftrightarrow \text{height(John)} \subseteq D_c$
i.e., the height of John is in the contextually salient interval of being tall.

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$[\![\text{John is 6 feet tall}]\!] \Leftrightarrow \text{height(John)} \subseteq [6', 6']$
i.e., the height of John is at the position '6 feet' on the height scale.

Lexical entries

$[\![\text{more/-er}]\!]_{\langle dt \rangle} \stackrel{\text{def}}{=} (0, +\infty)$

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$$[\![\text{than}]\!]_{\langle dt, \langle dt, dt \rangle \rangle} \stackrel{\text{def}}{=} \lambda D_{\text{standard}} . \lambda D_{\text{differential}} . \iota D[D - D_{\text{standard}} = D_{\text{differential}}]$$

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$[\![\text{5 inches ... -er than Mary is}]\!]$
$= [\![\text{than}]\!]([D_{\text{Lower-Mary}}, D_{\text{Upper-Mary}}])([5'', 5''])$
$= \iota D.[D - [D_{\text{Lower-Mary}}, D_{\text{Upper-Mary}}] = [5'', 5'']]$

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After simplification:
$\text{height(John)} \subseteq [D_{\text{Upper-Mary}} + 5'', D_{\text{Lower-Mary}} + 5'']$

Inside of the than-clause

$[\![\text{every girl is } D \text{ (tall)}]\!] \Leftrightarrow \forall x.[\text{girl}(x) \rightarrow \text{height(x)} \subseteq D]$
i.e., for each girl x, x's height is situated in the interval D on the height scale.

Inside of the than-clause

$[\![$every girl is D (tall)$]\!] \Leftrightarrow \forall x.[\text{girl}(x) \rightarrow \text{height(x)} \subseteq D]$
i.e., for each girl x, x's height is situated in the interval D on the height scale.

After a lambda abstraction and the application of a silent $[\![\text{THE}]\!]$, it becomes
$[\![\text{THE}]\!][\lambda D.[\forall x.[\text{girl}(x) \rightarrow \text{height(x)} \subseteq D]]]$
i.e., $[D_{\text{Lower-Girls}}, D_{\text{Upper-Girls}}]$

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[[John is taller than every girl is (tall)]]
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If $X - [a, b] = [c, d]$, when defined, $X = [b + c, a + d]$.

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$\Leftrightarrow$ height(John) $\subseteq \iota D.[D - [D_{\text{Lower-Girls}}, D_{\text{Upper-Girls}}] = (0, +\infty)]$

If $X - [a, b] = [c, d]$, when defined, $X = [b + c, a + d]$.

After simplification:
height(John) $\subseteq (D_{\text{Upper-Girls}}, +\infty)$

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If $X - [a, b] = [c, d]$, when defined, $X = [b + c, a + d]$.

After simplification: $\text{height(John)} \subseteq (D_{\text{Upper-Girls}}, D_{\text{Lower-Girls}} + 4'')$

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If $X - [a, b] = [c, d]$, when defined, $X = [b + c, a + d]$.

After simplification: [height(John) $\subseteq (D_{\text{Upper-Girls}}, D_{\text{Lower-Girls}} + 4'']$

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If $X - [a, b] = [c, d]$, when defined, $X = [b + c, a + d]$.

After simplification:
$\text{height(John)} \subseteq [\text{D}_{\text{Upper-Girls}} + 2'', D_{\text{Lower-Girls}} + 2'']$

John is between 2 and 4 inches taller than every girl is.

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$\text{height(John)} \subseteq [D_{\text{Upper-Girls}} + 2'', D_{\text{Lower-Girls}} + 4'']$

Fewer/less than

$[\![\text{few- than}]\!]_{\langle dt,\langle dt,dt\rangle\rangle}$
$\overset{\text{def}}{=} \lambda D_{\text{standard}}.\lambda D_{\text{differential}}.\iota D[D_{\text{standard}} - D = D_{\text{differential}}]$

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$\stackrel{\text{def}}{=} \lambda D_{\text{standard}}.\lambda D_{\text{differential}}.\iota D[D_{\text{standard}} - D = D_{\text{differential}}]$

If $[a, b] - X = [c, d]$, then X is undefined when $b + c > a + d$; when defined, $X = [b - d, a - c]$

John is more than 4 inches less tall than every girl is.

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$\Leftrightarrow \text{height(John)} \subseteq \iota D[[D_{\text{Lower-Girls}}, D_{\text{Upper-Girls}}] - D = (4'', +\infty)]$

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After simplification: $\text{height(John)} \subseteq (-\infty, D_{\text{Lower-Girls}} - 4'')$

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After simplification: $\text{height(John)} \subseteq [D_{\text{Upper-Girls}} - 4'', D_{\text{Lower-Girls}})$

Summary

We have shown how to compositionally derive the correct truth condition of comparatives containing various kinds of differentials in a precise and very natural way.

- No distributive operation is needed.
- No scope taking out of than-island is envolved.

Thanks! Questions?