

# Modality, Weights, and Inconsistent Premise Sets<sup>\*</sup>

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Penultimate draft. Please consult the official version  
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## Abstract

It is a commonplace that some of our desires are stronger than others; that certain values and norms are more important than others; and that states of affairs can be more likely than others. Some authors have claimed that the classic premise semantics for modals in linguistic semantics fails to capture how the truth-conditions of modal sentences can be sensitive to such differences in strength and priority. I develop an interpretation of the classic premise semantic framework that captures various ways in which weights and priorities can affect the interpretation of modals. Modals, on the standard semantics, receive their interpretation from contextually supplied functions from worlds to premise sets (“conversational backgrounds”). I propose that we understand these functions as encoding the content of an intuitively relevant body of considerations (conditional norms, preferences, expectations, etc.). The resulting world-indexed premise sets that figure in the truth-conditions of modal sentences represent what follows from these considerations given the relevant circumstances in the evaluation world. Facts about weights and priorities are encoded not in the individual premise sets (or in additional operations defined on them), but in how premise sets are assigned across worlds by the given conversational background. Next I offer one way of extending the account to certain comparative modal sentences. The proposed analysis captures various inference patterns involving modal auxiliaries, comparatives, and equatives, improving on certain previous approaches to graded modality within the classic framework. The paper concludes with several theoretical reflections on the relation between the semantics of modals and the logic of weights and priorities. The account developed in this paper locates a crucial role for research on proper reasoning involving weights and priorities without building the findings of this research into the conventional meaning of modal language.

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# 1 Introduction

It is a commonplace that some of our desires are stronger than others; that certain values, norms, and rules are more important than others; and that states of affairs can be more likely or typical than others. These differences in strength and priority can affect which modal claims are true. Suppose you promised Alice that you would meet her for lunch, but you also promised your ailing mother that you would drive her to the hospital for a critical operation. You realize that you can't keep both promises, but it is much more important that you keep your promise to your mother. Suppose also that there are no other normatively relevant factors. Intuitively, (1) is true and (2) is false. (Here and throughout, unless otherwise noted, assume that the modals are given a uniform type of normative reading.)

- (1) You must take your mother to the hospital.
- (2) You must meet Alice for lunch.

Call this case 'WEIGHTED PROMISES'.

Cases such as WEIGHTED PROMISES raise a prima facie challenge for the classic premise semantic framework for modals in linguistic semantics. Modals, in this framework, are treated as receiving their reading or interpretation from a contextually determined set of premises (KRATZER 1977, 1981a, 1991; see also VAN FRAASSEN 1973, LEWIS 1973, VELTMAN 1976). Since modals can themselves occur in intensional contexts, premise sets are indexed to a world of evaluation (written ' $P_w$ '). To a first approximation, given a consistent set of premises  $P_w$ , 'Must  $\phi$ ' says that the prejacent  $\phi$  follows from  $P_w$ . Making room for non-trivial interpretations given inconsistent premise sets, 'Must  $\phi$ ' says that  $\phi$  follows from every maximally consistent subset of  $P_w$ . Slightly less roughly, given a "modal base" premise set  $F_w$  that describes some set of relevant background facts in  $w$ , and an "ordering source" premise set  $G_w$  that represents the content of some ideal (morality, your goals, etc.) in  $w$ , 'Must  $\phi$ ' says that  $\phi$  follows from every maximally consistent subset of  $F_w \cup G_w$  that includes  $F_w$  — or, using the simplifying notation in Definition 1, that  $\phi$  follows from every set in  $\text{MAX}(F_w, G_w)$ .<sup>1</sup>

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<sup>1</sup>KRATZER 1981a, 1991; cf. FRANK 1996, GOBLE 2013: 280–281. This premise semantic implementation is equivalent to the perhaps more familiar implementation in KRATZER 1981a, 1991 which uses the ordering source to induce a preorder on the worlds compatible with the modal base (LEWIS 1981). For simplicity I make the limit assumption (LEWIS 1973: 19–20) and assume that ordering the consistent subsets of  $F_w \cup G_w$  (that include  $F_w$  as a subset) by set inclusion  $\subseteq$  yields a set of subsets that are  $\subseteq$ -maximal. For semantics without the limit assumption, see LEWIS 1973, KRATZER 1981a, 1991, SWANSON 2011. For expository purposes I focus only on strong necessity modals, like 'must' and 'have to'; weak necessity modals, like 'ought' and 'should', raise complications orthogonal to our discussion here (see SILK 2012b for my preferred account). I use unitalicized capital letters or  $w$  for worlds, italicized lowercase (English or Greek) letters for propositions, and italicized capital letters

**Definition 1.**  $\text{MAX}(F_w, G_w) := \{P: \text{cons}(P) \wedge \forall P' [P \subset P' \subseteq F_w \cup G_w \rightarrow \neg \text{cons}(P')]\} \wedge F_w \subseteq P \subseteq F_w \cup G_w\}$ , where, for a set of propositions  $S$ ,  $\text{cons}(S)$  iff  $\bigcap S \neq \emptyset$

**Definition 2.** ‘Must  $\phi$ ’ is true at  $w$  iff  $\forall P \in \text{MAX}(F_w, G_w) : \bigcap P \subseteq \phi$

What premise sets are called for in a case like WEIGHTED PROMISES? As David Lewis counsels us, “We must be selective in the choice of premises... By judicious selection, we can accomplish the same sort of discrimination as would result from unequal treatment of premises” (1981: 220–221; cf. KRATZER 1981b: 210). The question is whether we can do so in a way that captures the full range of data and reflects our intuitive views about the norms relevant in the context.

The ‘must’s in (1)–(2) are interpreted with respect to the relevant norms. The normative force of your promise to Alice should be represented, even if this promise is ultimately outweighed by the promise to your mother. Intuitively, the norms that figure in the interpretation of ‘must’ in (1)–(2) are the same as the norms that figure in the interpretation of ‘have to’ in (3).

(3) If you didn’t help your mother, you would have to meet Alice for lunch.

So, we might treat WEIGHTED PROMISES as calling for premise sets like the ones in (4) that describe the relevant circumstances and the contents of the relevant practical norms, where  $l$  is the proposition that you meet Alice for lunch, and  $h$  is the proposition that you take your mother to the hospital.

$$(4) \quad \begin{aligned} F_w &= \{\neg(h \cap l)\} \\ G_w &= \{h, l\} \\ P &= F_w \cup G_w = \{\neg(h \cap l), h, l\} \end{aligned}$$

But this won’t do.  $P$  is inconsistent. Holding fixed the relevant circumstance that you can’t keep both promises,  $P$  has maximally consistent subsets  $P_1 = \{\neg(h \cap l), h\}$  and  $P_2 = \{\neg(h \cap l), l\}$ . Since neither  $h$  nor  $l$  follows from both  $P_1$  and  $P_2$ , this incorrectly predicts that each of (1) and (2) is false (though it correctly predicts that ‘You must keep your promise to Alice or your mother’ is true). The problem is that there seems to be no room for representing how your promise to your mother is stronger than your promise to Alice, or how the premise  $h$  has priority over the premise  $l$ . There seems to be no mechanism for breaking the tie between the maximally consistent subsets of  $P$ .

It is rare to find explicit articulations of the premises that figure in the interpretations of modals in specific examples. Perhaps for this reason problems concern-

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for sets of propositions. For ease of exposition I often use lowercase Greek letters both as schematic letters to be replaced by declarative sentences as well as for the possible-worlds propositions they express. I assume that the prejacent of modals and the elements of premise sets are sets of possible worlds. I use  $\neg\phi$  as an abbreviation for  $\phi' = W \setminus \phi$ , and  $\phi \supset \psi$  as an abbreviation for  $\phi' \cup \psi = W \setminus \phi \cup \psi$ .

ing weights and priorities among premises have received little attention in formal semantics.<sup>2</sup> Daniel Lassiter (2011) is one of the few to address these issues; his assessment is not optimistic: “The problem is fundamentally that the theory makes no room for one expectation or norms [sic] being stronger than another; instead any conflict of expectation [/norm] leads to incomparability”; “the theory doesn’t leave any room for expectations [/norms] being stronger or weaker than another... expectations [/norms] are all-or-nothing” (2011: 61–62, 63–64; see also 147–149, 179). Likewise here is Stephen Finlay: “the problem is that Kratzer’s semantics... doesn’t provide a way of weighting ends; the ordering source treats every end as on a par” (2014: 155). Considering a semantics equivalent to the one in Definition 2, Lou Goble comes to a similar conclusion: “it fails to take the relative weight or significance of *prima facie* oughts into account” (2013: 287–288).<sup>3</sup>

There are various ways we might respond to this problem posed by WEIGHTED PROMISES. There are rich literatures in deontic logic on capturing priorities among default rules and *prima facie* obligations. One response would be to revise the classic semantics by importing additional apparatus from these theories into our semantics for modals.<sup>4</sup> There may be reasons in the end for doing so. However, I will argue that with a more nuanced characterization of the considerations that figure in the interpretation of modals, we can capture a wide range of linguistic data concerning weights and priorities within the classic premise semantic framework.

The structure of the paper is as follows: First I develop an interpretation of certain formal apparatus used in the classic premise semantic framework. This interpretation provides a systematic way of encoding the intuitive considerations which figure in the interpretation of modals, and it generalizes across flavors of modality

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<sup>2</sup>Though they have received extensive investigation in logic and computer science (see, e.g., MAKINSON 1993, GOBLE 2013 and references therein). I return to this in §5; see also note 4.

<sup>3</sup>In context Lassiter is considering the implications of this alleged failure to represent weights for Kratzer’s account of (epistemic and deontic) comparatives and equatives. Finlay and Goble are focusing on broadly normative readings.

<sup>4</sup>For instance, abstracting away from details of implementation, we might introduce a ranking  $\leq_{G_w}$  on the propositions in a given premise set  $G_w$ . We could then introduce a function  $\mathcal{G}_w$  that takes  $F_w$  and  $\leq_{G_w}$  as argument and returns a preorder on the elements of  $\text{MAX}(F_w, G_w)$ . Making an analogue of the limit assumption, we could then say that ‘Must  $\phi$ ’ is true iff  $\phi$  follows from every set in  $\text{MAX}(F_w, G_w)$  that ranks among the highest in the preorder  $\mathcal{G}_w(F_w, \leq_{G_w})$ . In WEIGHTED PROMISES,  $\mathcal{G}_w$  and  $\leq_{G_w}$  would privilege the maximally consistent subset  $P_1$  over  $P_2$ ; it would only be  $P_1$  from which the modal’s prejacent would be said to follow. This would correctly predict that (1) is true but (2) is false. This is essentially the route taken by Goble (2013: 287–294), following work by HANSEN (2006) and HORTY (2007, 2012); cf. ASHER & BONEVAC 1996, BROWN 1996, BELZER & LOEWER 1997, VAN DER TORRE & TAN 1998, HANSSON 2001. See KATZ ET AL. 2012 for a related account that represents cascades of priorities via an operation of ordered merging of ordering sources that mirrors the lexicographical product for posets. This latter account can be seen as developing Kratzer’s (1981b, 1989, 2002) strategy of “lumping” propositions into additional premises to capture the relative importance of facts in the interpretation of counterfactuals.

(deontic, epistemic, etc.). Understanding the premise semantic framework in the proposed way captures various ways in which weights and priorities can affect the truth values of modal claims (§3). Next I offer one way of extending the account to certain comparative modal constructions (§4). The proposed analysis avoids problems facing certain previous approaches to graded modality within the classic framework, and it provides an improved account of various inference patterns involving modal auxiliaries, comparatives, and equatives. I conclude with several methodological reflections on the relation between the semantics of modals and the logic of weights and priorities (§5). The account developed in this paper locates a crucial role for research on proper reasoning involving weights and priorities without building the findings of this research into the conventional meaning of modal language.

My aim in this paper isn't to argue against alternative frameworks for capturing the linguistic data concerning weights and priorities. Nor is it to provide a complete account of talk about priorities in natural language. The aim is simply to motivate one way of capturing within a standard premise semantic framework how weights and priorities can affect the interpretation of modals. An alternative, less methodologically conservative approach may ultimately prove superior. As we will see, the issues broached in this paper raise more general questions concerning (e.g.) the logic and semantics of priorities, the unity (or lack thereof) in the semantics of different readings of modals, and the proper treatment of gradability across syntactic categories and linguistic constructions. Further empirical and theoretical investigation may call for reconsidering certain foundational assumptions. I hope that the account developed in this paper can help us make progress on these difficult issues and refine our understanding of the space of possible theories. I leave subsequent progressions of the dialectic to future research.

## 2 Weights, priorities, and applicability conditions

Let's start by examining the sorts of considerations that figure in the interpretation of modals. It is a commonplace that norms, values, preferences, expectations, etc. often come with conditions under which they apply. If I want to go for a run, my desire needn't be that I go for a run, come what may. More plausibly it is that I go for a run given that it's sunny, that I'm not injured, that I didn't just eat a burrito, and so on. Our preferences are often conditional, preferences *for* certain circumstances. Similarly with moral norms. Suppose you promised Alice that you would help her move. A norm against breaking your promise might be something to the effect that you help Alice unless you made a conflicting promise to Bert, or keeping your promise would lead to some serious harm, or.... Norms can thus be understood on the model of *conditional* imperatives, imperatives that enjoin an action or state of affairs given that certain circumstances obtain. This captures the intuitive

idea that depending on the circumstances only certain norms apply, or are “in force.” Fixing terminology, I will call the content of a conditional norm, preference, etc. a *consideration*. Given a consideration  $\phi$  if  $C$ ,  $C$  is the consideration’s *applicability condition*, and  $\phi$  is the consideration’s *premise*, or what the consideration *enjoins* given  $C$ . (Categorical considerations can be treated as conditional on the tautology.)

There are a number of ways applicability conditions might be integrated into the semantics. One option, as suggested by the few explicit remarks in the literature on the contents of ordering sources in concrete examples, would be to build them into the premises themselves. The elements of ordering sources would be identified with considerations, construed as material conditionals  $C \supset \phi$ .<sup>5</sup> Simplifying quite a bit, the relevant deontic premise set in WEIGHTED GOALS might be something like in (5), where  $p_a$  is the proposition that you promise to meet Alice for lunch;  $p_m$  is the proposition that you promise to take your mother to the hospital;  $l$  is the proposition that you meet Alice for lunch; and  $h$  is the proposition that you take your mother to the hospital.

$$(5) \quad G_w = \{(p_a \cap \neg p_m) \supset l, p_m \supset h\}$$

One might worry that this approach will require the elements of ordering sources to be extraordinarily complex. In our simplistic example there were only two promises at play. But in more realistic cases there may be many competing norms, and there may be no simple way to describe their interaction and the conditions under which they apply.<sup>6</sup> Even given the sorts of idealizations common in semantics, it is hard to see context as supplying such bodies of premises. A more serious problem with this way of capturing the role of applicability conditions is that it makes incorrect predictions. It treats all normative considerations as categorical commands of conditionals rather than as conditional commands (and all bouletic considerations as preferences that a conditional be satisfied rather than as conditional preferences, and so on). This raises problems familiar from discussions of material conditional analyses of conditionals.

Suppose Little Timmy has some free time after school, followed by a piano recital and a basketball game. Timmy’s homeroom teacher orders him to finish his homework when he gets home from school, before he does anything else. Then in music class Timmy’s teacher tells him to practice piano if he finishes his homework, but in gym class his teacher tells him to practice basketball if he finishes his homework.

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<sup>5</sup>For example: “one should take the propositions that make up the relevant deontic background context  $l(w)$  to consist just of such *conditional* propositions,” like the proposition that “*If someone owns a car and is not handicapped, he must pay taxes*” (FRANK 1996: 54); see also pp. 51–52, 180–181, PORTNER 2009: 83, VON FINTEL 2012: 25–26 (following unpublished notes by Kratzer on information-sensitivity), SILK 2012a: 46–47, 50.

<sup>6</sup>Cf. HART 1961/1994: 125–132; AL-HIBRI 1980: 80; NICKEL 1982: 257; HANSSON 2013: 226–227; also the “qualification problem” in AI (MCCARTHY & HAYES 1969, MCCARTHY 1977).

Given the current way of capturing applicability conditions, this set of commands would seem to call for a deontic premise set like in (6) that describes Timmy’s teachers’ commands, where  $h$  is the proposition that Timmy finishes his homework,  $p$  is the proposition that he practices piano, and  $b$  is the proposition that he practices basketball.

$$(6) \quad G_w = \{h, h \supset b, h \supset p\}$$

Alas, when Timmy gets home from school, it becomes clear that he won’t be able to practice both piano and basketball before needing to leave. This yields the following modal base and elements of  $\text{MAX}(F_w, G_w)$  — the maximally consistent subsets of  $F_w \cup G_w$  that include  $F_w$  (n. 1).

$$(7) \quad F_w = \{\neg(b \cap p)\}$$

$$(8) \quad \begin{aligned} P_1 &= \{\neg(b \cap p), h, h \supset b\} \\ P_2 &= \{\neg(b \cap p), h, h \supset p\} \\ P_3 &= \{\neg(b \cap p), h \supset b, h \supset p\} \end{aligned}$$

Intuitively, in light of his teachers’ orders, Timmy must finish his homework. However, since  $\neg h$  is compatible with  $P_3$ , the deontic premise set in (6) incorrectly predicts that the sentences in (9) are true, assuming that ‘may’ and ‘is permissible’ are duals of ‘must’.

**Definition 3.** ‘May  $\phi$ ’ is true at  $w$  iff  $\exists P \in \text{MAX}(F_w, G_w) : \bigcap (P \cup \{\phi\}) \neq \emptyset$

- (9)    a. It’s permissible for Timmy not to finish his homework.  
       b. Timmy may refrain from finishing his homework.  
       c. Timmy doesn’t have to finish his homework.

If Timmy doesn’t complete his homework when he gets home, he can’t defend himself by saying that he complied with his gym teacher’s and music teacher’s conditional orders by making their antecedents false. Timmy would never be so crafty.

The case of Little Timmy highlights that we must only consider considerations whose applicability conditions are satisfied when evaluating modal claims. I noted above that it is standard in linguistic semantics to index premise sets to a world of evaluation. This feature of the classic semantics makes available an alternative way of distinguishing considerations whose applicability conditions are satisfied.<sup>7</sup>

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<sup>7</sup>Another strategy would be to revise the classic semantic framework by treating the elements of ordering sources not as propositions, but as pairs of applicability conditions (propositions) and premises (propositions). (See the input/output logic of MAKINSON & VAN DER TORRE 2000, 2001 for related representations of conditional norms.) A consideration  $\phi$  if  $C$  would be represented by a premise set that includes the pair  $\langle C, \phi \rangle$ . Only those considerations whose applicability conditions are entailed by the relevant background facts would figure in the interpretation of the modal: ‘Must  $\phi$ ’

Which premise set is relevant for the evaluation of a modal sentence can depend on how things are in the actual world, or on how things could be or could have been. What Little Timmy’s teachers order might change from one world to the next. They could have told Timmy to practice piano first thing after school. This motivates treating the meaning of ‘what Timmy’s parents prescribe’ in (10) as a function that assigns to every possible world the set of propositions describing the house rules in that world.

(10) In light of Timmy’s teachers’ orders, he must finish his homework.

Likewise for the meanings of phrases like ‘the relevant circumstances’, ‘what U.S. law provides’, and so on. It is these functions from worlds to premise sets which context supplies for the interpretation of modals and which determines their intended readings. Following Kratzer, I will call these functions *conversational backgrounds* (written ‘S’). Call the value of a conversational background at a world of evaluation a *premise set* (written ‘S<sub>w</sub>’).

Conversational backgrounds afford a natural way of encoding the contents of bodies of considerations. We can capture the role of applicability conditions in terms of variability in the values of conversational backgrounds at different worlds. Suppose we have a consideration which enjoins  $\phi$  given that conditions  $C$  obtain. We can represent the content of this consideration with a conversational background that assigns to every relevant  $C$ -world a premise set that includes  $\phi$ . For example, we can encode the content of your desire to go for a run, mentioned above, with a (bouletic) conversational background that assigns a premise set that includes the proposition that you go for a run to worlds in which the weather is nice (among other things). Similarly we might represent a norm to donate to charity with a (deontic) conversational background that assigns a premise set that includes the proposition that you donate to charity to worlds in which you have a job, the means of supporting your family, and so on. In this way, the contents of considerations are represented not at the level of individual premises but at the level of the conversational backgrounds supplied by context. An indexed premise set  $S_w$  represents the *conclusions* of the relevant considerations, given the facts in  $w$ . The premises in a premise set reflect what follows from a body of considerations — what is enjoined by a body of conditional norms, what is preferred in light of a body of conditional preferences, what is the case in light of a body of evidential relations, etc. — given the relevant circumstances in the evaluation world.

*Objection:* “The characterization of applicability conditions will be problematically circular in certain cases. For example, the applicability condition for the norm

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could be treated as saying that  $\phi$  follows from every set in  $\text{MAX}(F_w, G'_w)$ , where  $G'_w$  is the set of premises  $\psi$  such that there is a pair  $\langle C, \psi \rangle$  in  $G_w$  such that  $\bigcap F_w \subseteq C$ . Since my aim here is to capture the role of weights and priorities within the classic semantic framework, I won’t pursue this approach.



enjoining you to keep your promise to Alice in WEIGHTED PROMISE won't just be that you haven't promised to take your mother to the hospital. It will presumably be some more general condition, like that you haven't made any more important promises that are incompatible with your keeping your promise to Alice. But this makes reference to weights and the relative importance of various norms, which is precisely what needs to be explained."

*Reply:* We as theorists might use modal notions and talk of weights and priorities in describing the intuitively relevant considerations in a given circumstance, and in explaining what makes it the case about a concrete situation that such-and-such premises are enjoined. But it is the ways things are in the world of evaluation — the features of the world on which the facts about priorities and applicability conditions supervene — that determines what premises are enjoined. Information about applicability conditions, weights, and priorities is encoded in, but not itself recoverable from, a conversational background. It isn't explicitly delineated in the semantics. An intuitive body of considerations determines a conversational background, but not vice versa. (More on this in §5.)

Capturing the role of applicability conditions in this way may also stave off the complexity worry above with treating the elements of premise sets as material conditionals. The complexities in the material conditional antecedents are reflected in the current framework in how distinctions among worlds affect the value of a conversational background at those worlds. Descriptions of the elements of premise sets need only be as complex as the descriptions of what the relevant considerations enjoin.

To recap, in this section I have proposed a systematic way of encoding the intuitive bodies of considerations that figure in the interpretation of modals using the classic premise semantic formal apparatus. Take the deontic case. We *might* have taken the premises in an indexed deontic premise set  $G_w^d$  to represent the prima facie (defeasible) obligations potentially relevant at  $w$ . Or we *might* have taken the premises themselves to describe the precise conditions under which the relevant norms apply in different circumstances. On these lines, worlds  $w, w'$  determining the same general prima facie or conditional norms would be assigned equivalent indexed deontic premise sets  $G_w^d = G_{w'}^d$ . (In the limiting case where the general normative facts are metaphysically necessary (cf., e.g., MOORE 1903, GIBBARD 2003, PARFIT 2011),  $G^d$  would be a constant function.) But these choices of representation aren't forced upon this. Instead I have suggested that we encode the contents of the intuitive norms at play in the contextually supplied *function* from worlds to premise sets, i.e. the deontic conversational background  $G^d$ . How the relevant non-normative circumstances and priority relations affect which imperatives are ultimately in force is reflected in how premise sets are assigned across worlds. The premises in an indexed deontic premise set  $G_w^d$  describe what is ultimately enjoined given the full range of normative and non-normative facts in  $w$ . The indexed deontic premise set represents what is *to be done*. The content of the normative view intuitively relevant in

the context is reflected not in the indexed deontic premise at the evaluation world, but in the *family* of indexed premise sets, i.e. the conversational background  $G^d$  supplied by context. Analogous points hold for epistemic, teleological, and other flavors of modality.

This has all been pretty abstract. So far I have simply proposed a way of interpreting certain formal apparatus used in premise semantics for modals. It will be helpful in clarifying the proposal to see how it applies in specific examples. In the following sections I examine several empirical advantages of the proposed way of understanding the classic premise semantic framework. The interpretation offered in this section helps capture various ways in which weights and priorities can affect the truth values of modal claims.<sup>8</sup>

### 3 Applications

#### 3.1 Little Timmy

Start with the case of Little Timmy. Timmy’s homeroom teacher, recall, orders him to finish his homework first thing after school. His music teacher tells him to practice piano if he finishes his homework, whereas his gym teacher tells him to practice basketball if he finishes his homework. For simplicity, assume that Timmy will in fact either work on his homework, practice basketball, or practice piano. As before, let  $h$  be the proposition that Timmy finishes his homework,  $p$  be the proposition that he practices piano, and  $b$  be the proposition that he practices basketball. Let  $w_C$  be a world in which Timmy completes his homework with time to spare before needing to leave for the game and recital, and let  $w_{\bar{C}}$  be a world in which Timmy doesn’t complete his homework. (These can be treated as representatives of suitable equivalence classes of worlds.) To a first approximation (see below), we can capture the relevant circumstances and Timmy’s teachers’ commands with conversational backgrounds with the following properties:

$$(11) \quad \begin{aligned} F_{w_C} &= F_{w_{\bar{C}}} = \{\neg(b \cap p), h \cup b \cup p\} \\ G_{w_C} &= \{h, b, p\} \\ G_{w_{\bar{C}}} &= \{h, \neg(b \cup p)\} \\ &\vdots \end{aligned}$$

The conditional nature of the gym teacher’s and music teacher’s orders is reflected in how deontic premise sets are assigned across worlds in which their applicability

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<sup>8</sup>See SILK 2012b, 2013a, 2015 for further developments and applications of the proposed interpretation (e.g., concerning weak necessity modals, information-sensitivity, and disjunction in expressivist semantics); cf. SCHROEDER 2013.

condition — that Timmy finishes his homework — is (is not) satisfied. This correctly predicts that (12) is true and (13) is false at both  $w_C$  and  $w_{\bar{C}}$ .

(12) Timmy must finish his homework.

(13) Timmy may refrain from finishing his homework.

Even if it isn't settled whether Timmy will complete his homework before needing to leave — even if  $w_C$  and  $w_{\bar{C}}$  are both live possibilities — it can be known that he must finish his homework.<sup>9</sup>

### 3.2 Weights and contrary-to-duty imperatives

Return to WEIGHTED PROMISES. What we need to capture is that (1) is true and (2) is false; your promise to your mother is more important than your promise to Alice.

(1) You must take your mother to the hospital.

(2) You must meet Alice for lunch.

Consider the following relevant worlds,  $AM$ ,  $\overline{AM}$ ,  $\overline{AM}$ ,  $\overline{AM}$ , characterized with respect to whether or not you promise to meet Alice and whether or not you promise to help your Mother in them. We can capture the normative import of your promises as follows, again where  $h$  is the proposition that you take your mother to the hospital and  $l$  is the proposition that you meet Alice for lunch.<sup>10</sup>

$$(14) \quad \begin{aligned} G_{AM} &= \{h\} \\ G_{\overline{AM}} &= \{l\} \\ G_{\overline{AM}} &= \{h\} \\ G_{\overline{AM}} &= \emptyset \\ &\vdots \end{aligned}$$

The normative import of your promise to Alice is reflected in  $G$ 's assigning a premise set that includes  $l$  to some world in which you make this promise, namely  $\overline{AM}$ . The normative import of your promise to your mother is reflected in  $G$ 's assigning a

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<sup>9</sup>What matters for present purposes is simply that we capture how Timmy must finish his homework before practicing piano or basketball. What one says about Timmy's obligations with respect to practicing piano and practicing basketball if he does complete his homework will depend on one's views about the possibility of irresolvable practical dilemmas. I leave this issue open here, since the aim in this paper is to examine cases where the relevant norms are comparable to one another and hence any apparent dilemmas can be resolved.

<sup>10</sup>For simplicity assume that in  $AM$  you cannot keep both promises; and assume that the worlds are otherwise relevantly normal, e.g. that there is no emergency in  $\overline{AM}$  that would be more pressing than your promise to Alice. As above these may be treated as representatives of suitable equivalence classes of a relevant set of worlds.

premise set that includes  $h$  to some world in which you make this promise, e.g.  $\overline{AM}$  (or  $AM$ ). And the priority of keeping your promise to your mother over your promise to Alice is reflected in  $G$ 's assigning a premise set that includes  $h$  to some world in which you make both promises, namely  $AM$ . (Had your promises been of equal importance, this would be reflected in a premise set that contains the disjunction  $h \cup l$ .) This conversational background  $G$  correctly predicts that (1) is true and (2) is false in the given context, i.e. at  $AM$ :  $h$ , but not  $l$ , follows from every set in  $\text{MAX}(F_{AM}, G_{AM})$ .

$$(15) \quad \text{MAX}(F_{AM}, G_{AM}) = \{\{-(h \cap l), h\}\}, \text{ where}$$

$$F_{AM} = \{-(h \cap l)\}$$

$$G_{AM} = \{h\}$$

Prima facie conflicts among norms needn't lead to incomparabilities.

One might worry that this treatment of WEIGHTED PROMISES makes incorrect predictions concerning contrary-to-duty imperatives. Consider (16)–(17).<sup>11</sup>

- (16) You have to take your mother to the hospital.
- (17) If you don't take your mother to the hospital, you have to meet Alice for lunch.

Assuming a standard Kratzerian (1991) treatment of conditionals (Definition 4), the antecedent of (17) adds the proposition  $\neg h$  to  $F_{AM}$ , and the modal 'have to' is interpreted with respect to this updated modal base  $F_{AM}^+ = F_{AM} \cup \{\neg h\}$ . Given the premise sets for  $F_{AM}$  and  $G_{AM}$  in (15), this seems to incorrectly predict that (17) is false, as reflected in (18)–(19):

**Definition 4.** 'If  $\psi$ ,  $\phi$ ' is true at  $w$  iff  $\forall P \in \text{MAX}(F_w^+, G_w) : \cap P \subseteq \phi$ , where  $F_w^+ = F_w \cup \{\psi\}$

$$(18) \quad F_{AM}^+ = \{-(h \cap l), \neg h\}$$

$$G_{AM} = \{h\}$$

$$\text{MAX}(F_{AM}^+, G_{AM}) = \{F_{AM}^+\}$$

$$(19) \quad (17) \text{ is true at } AM \text{ iff}$$

$$\forall P \in \text{MAX}(F_{AM}^+, G_{AM}) : \cap P \subseteq l \text{ iff}$$

$$\{-(h \cap l), \neg h\} \subseteq l$$

There are subtle issues concerning time, and what information is taken for granted, which may complicate the interpretation of pairs of claims like (16)–(17). Intu-

<sup>11</sup>I use 'have to' instead of 'must' to avoid potential complications from the entailingness of 'must'. (Many speakers find 'Must  $\phi$ , but  $\neg\phi$ ' and 'Must  $\phi$ , but might  $\neg\phi$ ' to be marked, even for deontic readings of 'must'. For such speakers, accepting 'Must  $\phi$ ' has the potential to violate the presupposition of conditionals 'If  $\neg\phi$ ...' that  $\neg\phi$  is a live possibility.)

itively, in interpreting the modal in (16) one assumes that the acts of keeping your promise to your mother and keeping your promise to Alice are both available to you. In interpreting the modal in the consequent of the conditional in (17), by contrast, one assumes that the act of keeping your promise to your mother is no longer an option. The norms encoded in  $G$  must be conditional not only what promises you have made but also on what acts are available to you. Let AMB be a circumstance in which Both the option of keeping your promise to your mother and the option of keeping your promise to Alice are available, and let AMA be a circumstance in which only the option of keeping your promise to Alice is available.<sup>12</sup> A more fine-grained deontic conversational background  $G^*$  can be given as follows:

$$(20) \quad \begin{aligned} G_{AMB}^* &= \{h\} \\ G_{AMA}^* &= \{l\} \\ &\vdots \end{aligned}$$

Assuming that (16) is evaluated at a circumstance like AMB at which keeping your promise to your mother is still available, we continue to predict that (16) is true. Adopting a double modal analysis of deontic conditionals delivers the correct interpretation for (17).<sup>13</sup> On such analyses, the antecedent clause is treated as restricting the modal base not of the overt deontic modal, but of a posited covert necessity modal, in whose nuclear scope the overt deontic modal is embedded.

$$(21) \quad [ \text{NEC if } \psi ] [ \text{must } \phi ]$$

Roughly, in interpreting a deontic conditional one checks whether the modalized consequent clause is verified at all relevant (circumstantially accessible, epistemically accessible, closest) worlds in which the antecedent holds. This delivers the following simplified truth conditions for (17):

$$(22) \quad (17) \text{ is true at } w \text{ iff for all relevant } \neg h\text{-worlds } w': \forall P \in \text{MAX}(F_{w'}, G_{w'}) : \cap P \subseteq l$$

The ordering source for the overt modal is determined in light of the circumstances and norms in the relevant antecedent-worlds, like AMA, yielding  $G^*(AMA) = \{l\}$ . This correctly predicts that (17) is true at AMB, and thus consistent with (16): roughly, (17) is true at AMB iff ‘you have to meet Alice’ is true at AMA iff  $\cap \{ \neg(h \cap l), \neg h, l \} \subseteq l$ .

<sup>12</sup>As before, AMB and AMA can be treated as representatives of suitable equivalence classes of worlds. For simplicity I continue to treat indices of evaluation as worlds. Alternatively, one might capture the sensitivity to time or available acts via independently represented parameters, such as a time parameter, or by treating indices as situations. See, e.g., ARREGUI 2010 for discussion of a situation-based approach to time-sensitive deontic conditionals. For discussion of timeless contrary-to-duty cases, see, e.g., PRAKKEN & SERGOT 1996, 1997.

<sup>13</sup>See MAKINSON 1993, FRANK 1996, VON FINTEL & IATRIDOU 2005, ARREGUI 2010, KAUFMANN & SCHWAGER 2011. I leave open what kind of reading the covert necessity modal is to be given.

### 3.3 Outweighing and undercutting

WEIGHTED PROMISES was a case where one consideration was *outweighed* by another conflicting consideration. Now consider a case where the applicability of one premise *undercuts*, or *excludes*, the applicability of another premise.<sup>14</sup> Suppose Betty is a cadet, and her Captain orders her to clean the barracks. Ordinarily, this would imply that Betty has to clean the barracks. But the Major, who outranks them both, orders Betty to ignore the Captain’s command. Intuitively, (23) is true.

(23) Betty doesn’t have to clean the barracks.

The Major’s command, it is often claimed, isn’t an ordinary, weightier first-order reason; rather, it undercuts the consideration about the Captain’s command from bearing on Betty’s deliberation.

Let  $b$  be the proposition that Betty cleans the barracks,  $c$  be the proposition that the Captain ordered Betty to clean the barracks, and  $m$  be the proposition that the Major ordered Betty to ignore the Captain’s command. Let  $CM$  be the world as it is described by the case, and  $\overline{CM}$  be an otherwise similar world in which the Major doesn’t order Betty to ignore the Captain. We can capture the contents of the relevant norms at play with a conversational background with the following properties:

(24)  $G_{CM} = \emptyset$   
 $G_{\overline{CM}} = \{b\}$   
 $\vdots$

The normative import of the Captain’s command is reflected in  $G$ ’s assigning a premise set that includes  $b$  to  $(c \wedge \neg m)$ -worlds in which the Major doesn’t interfere. The undercutting role of the Major’s command is reflected in  $G$ ’s assigning a premise set that fails to include  $b$  to  $(c \wedge m)$ -worlds. This correctly predicts that (23) is true in the given context (i.e., at  $CM$ ):  $\bigcap\{c, m\} \not\subseteq b$ .

That the Captain’s command is undercut, and not outweighed, is reflected in the fact that the Major needn’t forbid Betty from cleaning the barracks or order her to perform some alternative action. He might just want to undermine the Captain’s authority. The undercutting role of the Major’s command can be further reinforced by considering a minor extension of the case. Adapting an example from HORTY (2012: 131–134), suppose that the situation is as before, but Betty also received an order from her Lieutenant to do drills. Since the Captain outranks the Lieutenant, ordinarily Betty would have to obey the Captain’s command and not do drills (assuming

<sup>14</sup>Compare the distinction between “rebutting” defeat and “undercutting” defeat in the literature on epistemic reasons (POLLOCK 1970, 1995), and the notions of “exclusionary” reasons (RAZ 1975/1990, GANS 1986, PERRY 1989) and “overridden” requirements (CHISHOLM 1964, 1974) in ethics.

she can't both clean the barracks and do drills). But given the Major's command to ignore the Captain, there is now nothing excluding the Lieutenant's command from applying. The contents of the relevant norms can be reflected as follows, where  $d$  is the proposition that Betty does drills,  $l$  is the proposition that the Lieutenant ordered Betty to do drills, and worlds  $LXX$  are  $l$ -worlds.

$$(25) \quad \begin{aligned} G_{LCM} &= \{d\} \\ G_{LC\bar{M}} &= \{b\} \\ G_{\bar{L}CM} &= \emptyset \\ &\vdots \end{aligned}$$

This correctly predicts that (23) and (26) are true in the revised context (i.e., at LCM):  $\{c, m, l, \neg(b \cap d), d\}$  entails  $d$  but doesn't entail  $b$ .

(26) Betty must do drills.

More generally, the contrasting ways in which considerations can be undercut and outweighed is represented in terms of what premise sets are assigned at certain minimally different worlds. Let  $a$  and  $b$  be relevant conditions,  $AB$  be a relevant  $(a \wedge b)$ -world, and  $A\bar{B}$  be a relevant  $(a \wedge \neg b)$ -world. (For simplicity suppose that  $G_{AB}$  and  $G_{A\bar{B}}$  are each consistent, and bracket the role of  $F$ .) Suppose that  $\bigcap G_{A\bar{B}} \subseteq \phi$ , reflecting that given  $a$  and absent some defeating condition  $b$ ,  $\phi$  is necessary. The premise that  $\phi$ , with applicability condition  $a$ , is *outweighed* if there is a premise  $\psi$  with applicability condition  $b$ , where  $\phi \cap \psi = \emptyset$ , such that  $\bigcap G_{AB} \subseteq \psi$  and  $\bigcap G_{AB} \not\subseteq \phi$ . By contrast, the premise that  $\phi$  is *undercut* by a background condition  $b$  simply if  $\bigcap G_{AB} \not\subseteq \phi$ .

### 3.4 Epistemic readings

So far we have been focusing on root modals. The proposals for capturing priorities among premises, and for distinguishing outweighing from undercutting, apply to epistemic readings of modals as well. First, consider a familiar epistemic analog of WEIGHTED PROMISES. Suppose you are looking at a ball. It seems red to you, but a peer tells you that the ball isn't actually red. Your sense perception is general reliable; typically, the fact that an object seems red is good evidence that the object is red. But your peer is eminently trustworthy, and she might have access to information that you don't have access to and that she hasn't made available to you (e.g., that there are cleverly masked unusual lighting conditions). The reliability of your peer, let's suppose, is even greater than that of your sense perception. Intuitively, this outweighs your reason for thinking that the ball is red — indeed, it gives you reason for thinking the ball isn't red. Suppose that there is no other relevant possible evidence that may bear on the color of the ball. (27) seems true in this scenario.

(27) The ball must not be red.

We can capture this as follows. Let  $l$  be the proposition that the ball looks red;  $r$  be the proposition that the ball is red; and  $t$  be the proposition that your peer told you that the ball isn't red. Consider the worlds  $LT$  and  $L\bar{T}$ , characterized in the expected way as above. We can capture the priorities among the relevant evidential norms with a conversational background with the following properties.

$$(28) \quad \begin{aligned} G_{LT} &= \{-r\} \\ G_{L\bar{T}} &= \{r\} \\ &\vdots \end{aligned}$$

The evidential import of the object's looking red is reflected in  $G$ 's assigning a premise set that includes  $r$  to  $\neg t$ -worlds where the ball looks red. The priority of your peer's testimony is reflected in  $G$ 's assigning a premise set that includes  $\neg r$  to  $t$ -worlds. This predicts that (27) is true in the given context (i.e., at  $LT$ ):  $\bigcap\{l, t, \neg r\} \subseteq \neg r$ .

Now consider a case of epistemic undercutting. Suppose that rather than hearing from your peer that the object is not red, you realize that you have taken a drug that makes everything look red. Intuitively, this undercuts your reason for concluding that the ball is red. (29) is false in this scenario.

(29) The ball must be red.

Let  $d$  be the proposition that you took the drug, and consider the worlds  $LD$  and  $L\bar{D}$ , characterized in the expected way. We can capture the probabilistic information encoded in the relevant epistemic norms with a conversational background with the following properties:

$$(30) \quad \begin{aligned} G_{LD} &= \emptyset \\ G_{L\bar{D}} &= \{r\} \\ &\vdots \end{aligned}$$

The evidential import of the object's looking red is reflected in  $G$ 's assigning a premise set that includes  $r$  to  $\neg d$ -worlds in which the ball looks red. The undercutting role of taking the drug is reflected in  $G$ 's assigning the empty set to  $d$ -worlds. This predicts that (29) is false in the given context (i.e., at  $LD$ ):  $\bigcap\{l, d\} \not\subseteq r$ .

## 4 Comparatives

An adequate general treatment of priorities must extend to overt comparative language, such as in (31).



- (31) It is better for me to keep my promise to my mother than for me to keep my promise to Alice.

Indeed, in the passages from LASSITER 2011 cited in §1, Lassiter’s central criticism of Kratzer’s (1981a, 1991, 2012) semantics is that its proliferation of inconsistencies in premise sets leaves it unable to capture the truth of such comparatives. The primary aim of this paper has been to provide a way of capturing facts about weights and priorities in a classic premise semantic framework, and to show how such facts can affect the truth values of sentences with possibility and necessity modals. The proposed strategy is compatible with various views on the semantics of comparatives like (31) and the semantic relation between modals and comparatives. Nevertheless I would like to briefly mention one way of extending the account from §§2–3 to capture certain data involving comparatives. I focus specifically on three problems Lassiter raises for Kratzer’s theory. More thorough investigation must be left for future work.

I suggest that comparatives like (31) have a counterfactual element to their meaning. Intuitively, (31) seems to mean something like “If I had to keep my promise to my mother or to Alice — and conditions were otherwise normal, expected, or as desired — I would have to keep my promise to my mother, not Alice.” I propose that in assessing whether a possibility  $\phi$  is as good as another possibility  $\psi$ , one looks at relevant worlds in which  $\phi \vee \psi$  is necessary (in the relevant sense), and checks whether  $\phi$  is possible (in the relevant sense) in those worlds.<sup>15</sup> Roughly, ‘ $\phi$  is at least as good as  $\psi$ ’ is true at  $w$  iff for all closest (maximally similar) relevant worlds  $w'$  to  $w$  at which ‘Must  $\phi \vee \psi$ ’ is true, ‘May  $\phi$ ’ is true. More formally, where  $s$  is a selection function that selects the set of closest relevant  $\chi$ -worlds to the evaluation world  $w$ , for some proposition  $\chi$ :<sup>16</sup>

**Definition 5.** A proposition  $\phi$  is at least as good a possibility as a proposition  $\psi$  in  $w$  (written:  $\phi \leq_w \psi$ ) iff  $\forall w' \in s(w, \lambda u. \forall P \in \text{MAX}(F_u, G_u) : \cap P \subseteq \phi \cup \psi) : \exists P \in \text{MAX}(F_{w'}, G_{w'}) : \cap(P \cup \{\phi\}) \neq \emptyset$ .

**Definition 6.** A proposition  $\phi$  is a better possibility than a proposition  $\psi$  in  $w$  (written:  $\phi <_w \psi$ ) iff  $\phi \leq_w \psi \wedge \psi \not\leq_w \phi$ .

Following Kratzer (1981a, 1991, 2012; cf. LEWIS 1973), these definitions provide general notions of comparative possibility for propositions, which may then figure in

<sup>15</sup>Cf. LEHMANN & MAGIDOR 1992, VAN DER TORRE 1997.

<sup>16</sup>There may be reasons for strengthening Definition 6 to require not simply that  $\psi \not\leq_w \phi$ , but rather that for *all* closest relevant  $w'$  to  $w$  at which ‘Must  $\phi \vee \psi$ ’ is true, ‘Must  $\neg\psi$ ’ is true. This would follow on the assumption that all the worlds  $w'$  being considered agree in what is possible and necessary (permitted and required) — perhaps understood as a sort of “comparability” assumption between  $\phi$  and  $\psi$ . The two definitions would collapse given conditional excluded middle (LEWIS 1973: 79–81). I assume that the relevant sense of “closeness” is the same (context-dependent) sense used in interpreting subjunctive conditionals, however it is ultimately to be cashed out.

the semantics of particular object language comparative modal constructions. Different flavors of modality (epistemic, deontic, etc.) correspond to different choices of  $F$  and  $G$ , as in the definitions for the grammatical modals (§1). (For convenience I use ‘ $\leq$ ’ and ‘ $<$ ’ both in denoting the relations defined in Definitions 5–6, and in abbreviating object language comparative expressions. World-indexing occurs only in the former use; I include superscript letters for the latter (a superscript ‘ $o$ ’ when neutral on the relevant reading). As we are only considering simple comparative constructions, no harm should come from this. Yet it is important not to conflate comparative modal expressions and the comparative possibility relations which figure in their semantics. My writing ‘ $\phi \leq^o \psi$ ’ to abbreviate object language sentences shouldn’t be taken to prejudge questions about the internal syntax and compositional semantics of the comparative. We will return to these points below.)

The above definitions, interpreted in terms of deontic possibility, correctly predict that (31) is true. Recall the premise sets for WEIGHTED PROMISES at world AM in which you both promise to meet Alice and promise to help your mother:

$$(15) \quad \text{MAX}(F_{AM}, G_{AM}) = \{\{-(h \cap l), h\}\}$$

The set of closest relevant worlds to the evaluation world AM at which ‘Must  $h \vee l$ ’ is true is  $\{AM\}$ , given strong centering for the similarity relation among worlds (i.e., given that if  $u$  is a  $\phi$ -world,  $s(u, \phi) = \{u\}$ ). The truth of the comparative (31) follows straightaway: ‘May  $h$ ’ and ‘Must  $\neg l$ ’ are both true at AM.

Importantly, in evaluating  $\phi \leq^o \psi$ , the selected closest worlds are possibly counterfactual worlds in which  $\phi \vee \psi$  is *necessary* (in the relevant sense). The proposal has no difficulty with examples where  $\phi$  and  $\psi$  are actually prohibited, like (32)–(33), which Lassiter (2011: 147–148) uses in objecting to Kratzer’s theory.

(32) It is better to trespass than it is to murder. (true)

(33) It is better to murder than it is to trespass. (false)

(LASSITER 2011: ex. 5.59)

Plausibly, the closest relevant worlds  $u$  in which ‘One must trespass or murder’ is true are worlds in which murder is still forbidden. A model in which the deontic premise set at  $u$  entails that one doesn’t murder and that one trespasses correctly predicts that (32) is true and (33) is false.

YALCIN (2010) and LASSITER (2010, 2011, 2014) argue that the following problematic inference is validated on Kratzer’s (1981a, 1991, 2012) semantics for comparative modals (see also HOLLIDAY & ICARD 2013 for discussion).

(34) Disjunctive Inference:

- $p$  is as likely as  $q$ .            (written:  $p \leq^e q$ )  
 $p$  is as likely as  $r$ .            (written:  $p \leq^e r$ )  
 $\therefore p$  is as likely as  $q \vee r$ .    (written:  $p \leq^e (q \vee r)$ )

The semantics in Definition 5 avoids validating this inference. This is for the familiar more general reason that weakening of the antecedent is invalidated on the standard semantics for counterfactuals. Here is a model: Suppose four roughly similar players are left in a tournament. We think the referees are dirty and will cheat two of the players in the semifinals, though they will let the finals play out fairly. For all the closest worlds  $w'$  where we know that Chip or Dorothy will win the tournament, we get a tip that Emma and Frank will get screwed by the referees and lose in the semifinals, but our evidence in  $w'$  still leaves open that Chip will beat Dorothy in the finals. Likewise for all the closest worlds  $w''$  where we know that Chip or Emma will win the tournament, we get a tip that Dorothy and Frank will get screwed by the referees and lose in the semifinals, but our evidence in  $w''$  still leaves open that Chip will beat Emma in the finals. But for some of the closest worlds  $w'''$  where we know that Chip or one of the women will win, we get a tip that it is Chip who will get screwed by the referees and lose — just as for some of these worlds we get a tip that Dorothy will get screwed, and for some we get a tip that Emma will get screwed. So, (35a) and (35b) are true, but (35c) is false (letting  $c$  be the proposition that Chip will win the tournament,  $d$  be the proposition that Dorothy will win, etc.).

- (35)    a.  $c \leq^e d$ . (=It is as likely that Chip will win the tournament as it is that Dorothy will win.)  
           b.  $c \leq^e e$ . (=It is as likely that Chip will win the tournament as it is that Emma will win.)  
           c.  $\not\Rightarrow c \leq^e (d \vee e)$ . (=It is as likely that Chip will win the tournament as it is that Dorothy or Emma will win (/that one of the women will win.)

That is, for all worlds  $w' \in s(w, \lambda u. \cap F_u \subseteq c \cup d): \cap (F_{w'} \cup \{c\}) \neq \emptyset$ ; and for all worlds  $w'' \in s(w, \lambda u. \cap F_u \subseteq c \cup e): \cap (F_{w''} \cup \{c\}) \neq \emptyset$ ; but for some worlds  $w''' \in s(w, \lambda u. \cap F_u \subseteq c \cup (d \cup e)): \cap (F_{w'''} \cup \{c\}) = \emptyset$ . (For simplicity I bracket the role of the ordering source  $G$ .)

LASSITER (2011, 2014) also objects to Kratzer's semantics on the grounds that it fails to capture certain apparent entailment relations among modals and comparatives. For instance, Lassiter objects that it fails to validate the inference pattern in (36). (Lassiter focuses specifically on epistemic readings, but the points extend to deontic readings as well.)<sup>17</sup>

<sup>17</sup>Lassiter also considers inferences from 'Must  $\phi$ ' to ' $\phi$  is (much) more likely than  $\neg\phi$ '. Whether the semantics in Definition 6 validates this inference depends on broader issues concerning the logic of counterfactuals. The inference is validated given conditional excluded middle (n. 16) or strong centering for the closeness relation.

- (36) a. Must  $\phi$   
 b.  $\psi \leq^o \phi$   
 c.  $\therefore$  Must  $\psi$

The semantics proposed here also doesn't validate this inference (though slight variants would). However, contrary to Lassiter, I take this to be a feature, not a bug. I am doubtful about whether we should treat the inference in (36) as *semantically* valid. First, Lassiter focuses specifically on epistemic interpretations of 'must' and the comparative in (36). Though (36) may seem compelling with epistemic readings, it is important to observe that the inference is intuitively invalid for deontic readings. It excludes the possibility of supererogatory acts — acts that go “beyond the call of duty,” acts that are permitted but not required and better than what is minimally required — as reflected in (37).

- (37) a. I must referee papers.  
 b. My giving 40% of my income to famine relief is at least as morally good as my refereeing papers.  
 c.  $\therefore$  I must give 40% of my income to famine relief.

Even if there are no supererogatory acts, this should be determined on the basis of substantive normative theory, not logic or semantics. This counts against treating the inference in (36) as valid in any context.

Second, the inference pattern in (36) is arguably invalid even in the case of epistemic readings. Epistemic 'must' sentences make claims about what follows from a relevant body of evidence. Considered in this light, there may seem something puzzling about the constraint on bodies of evidence semantically required by (36): why should we expect the fact that  $\psi$  is at least as likely as  $\phi$  to imply that, necessarily, any body of evidence that entails  $\phi$  must also entail  $\psi$ ? This is abstract; consider a concrete example. Suppose you are inside and see a bunch of people coming in with wet umbrellas. You infer, plausibly enough, that it must be raining:

- (38) It must be raining.

You also accept (39) as true.

- (39) It's at least as likely that it's cloudy (now) as it is that it's raining (now).

After all, it's nearly always cloudy when it rains, and you have no positive evidence that today is unusual in this regard. Nevertheless you are inside with no access to windows. Your evidence doesn't itself say anything about whether it is cloudy. For all your evidence suggests, there is a sun shower. It is hard to see why it would be incoherent, or a reflection of semantic incompetence, for you to fail to accept (40) and think that it might not be cloudy.

(40) It must be cloudy.

We should be cautious about building apparently plausible connections between modal verbs and comparative modal expressions into the semantics itself (cf. KLECHA 2014).

Before moving on I would like to make one further remark about the logic of comparative possibility generated by the proposed analyses. The derived relations  $\leq_w$ ,  $<_w$  over propositions are quite weak; notably, they are not transitive (e.g.,  $p \leq_w r$  cannot be derived from  $p \leq_w q$  and  $q \leq_w r$ ). I regard this as a feature, not a bug. It is contentious whether transitivity should be encoded in the logics of various comparative modal notions. For instance, many preference logics don't assume that the betterness relation is transitive. Indeed, in discussions of ideality (world-)orderings and betterness (proposition-)relations, it is often assumed that the latter is *not* transitive (see HANSSON 1990, VAN DER TORRE 1997).

Consider the following adaptation of a familiar style of example (cf. SEN 1993). Suppose (i) taking no dessert ( $n$ ) is at least as good as (preferable to) taking the last brownie on a dessert tray ( $l$ ), since taking the last brownie would be impolite. And (ii) taking the last brownie on a dessert tray ( $l$ ) is at least as good as (preferable to) taking a dry piece of cake ( $d$ ), since the relative deliciousness of the brownie over the cake surely makes up for any impoliteness. And yet (iii) taking a dry piece of cake ( $d$ ) is better than (preferable to) taking no dessert ( $n$ ); after all, gross cake would never be the last item left, and some dessert is better than none. Hence  $n \leq_w l$ , and  $l \leq_w d$ , but  $n \not\leq_w d$ , indeed  $d <_w n$ .<sup>18</sup> The analyses in Definitions 5–6 nicely capture what is going on in such a case: the relevant closest worlds in which  $n \vee d$  is bouletically necessary (necessary in light of one's desires) aren't among the closest worlds in which  $n \vee l$  is bouletically necessary. Specifically, (i) the closest worlds in which  $n \vee l$  is bouletically necessary are worlds in which there is a single dessert left, the brownie, and taking nothing is compatible with one's desires; (ii) the closest worlds in which  $l \vee d$  is bouletically necessary include worlds in which there is a single dessert left, the brownie, and taking it is compatible with one's desires; but (iii) the closest worlds in which  $n \vee d$  is bouletically necessary are worlds where there are multiple desserts left, and one wants at least something for dessert.

It is important to be clear about the import of such apparent violations of transitivity. First, what is not transitive are the *comparative possibility* relations — the derived relations over propositions — not the ideality orderings over worlds deter-

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<sup>18</sup>Examples with incomparabilities or incomplete preferences would provide another familiar kind of case. For instance, suppose that when comparing apples and bananas, one considers sweetness; and when comparing bananas and oranges, one considers color; but one has no settled basis for comparing apples and oranges (they're just so different!). Then it may be that taking an apple is preferred to taking a banana, and taking a banana is preferred to taking an orange, but it's not the case that taking an apple is preferred to taking an orange.

mined by the contextually supplied premise sets  $F, G$  (see n. 1). That the analyses in Definition 5–6 do not validate transitivity needn't imply (e.g.) that betterness relations over maximally specific possibilities (point propositions) can violate transitivity. Second, it is a substantive question how exactly to interpret the comparative possibility relations in specific contexts. Cases like the one above, characterized in terms of preferences over non-maximal possibilities, may admit of a transitive description, e.g. in terms of a relation on worlds or point propositions.<sup>19</sup> Third, it is worth reiterating that  $\leq_w$  and  $<_w$  constitute *comparative possibility* relations. It is an open question, at least in principle, how exactly these relations figure in the semantics. For all I have said, additional structure could be imposed in the lexical semantics of particular expressions or readings. Indeed, utilizing comparative relations such as those defined in Definitions 5–6 could be regarded as capturing a common semantic core in various comparative modal notions, where these relations could be strengthened or supplemented in particular cases. That the logics for  $\leq_w, <_w$  are weak needn't imply that the logic of any comparative modal expression is weak.

My aim in this section has been modest. I have briefly outlined one strategy for extending the premise semantic treatment of priorities from §§2–3 to provide a semantics for certain comparative modal sentences. The preliminary analyses broached here avoid certain problems facing previous accounts. However, first, like Kratzer, I haven't attempted to show how the proposed truth conditions are to be compositionally derived. Integrating the analyses and derived comparative possibility relations into more general treatments of gradability, scalarity, and modality will be essential for developing a more complete account. Further, the challenges considered in this section certainly aren't the only challenges to be addressed by a classic premise semantic account of graded modality. It is a non-trivial question how the proposed comparative possibility relations (or some alternative/additional relations) might figure in the semantics of the broader range of graded modal expressions. Introducing additional structure into the semantics, as by giving graded epistemic/deontic modal expressions an overtly probabilistic/utility-based semantics, may ultimately be called for. In addition, the arguments in this section notwithstanding, there is reason to be cautious about the project of giving a uniform underlying semantics for graded modal expressions. The logics of likelihood, preference, desirability, etc. are plausibly different. It would be surprising if it was encoded in the very conventional meanings of graded modal expressions that their various readings had the same underlying logic, even at some high level of abstraction. Thorough investigation of (e.g.) inferences involving modals and comparatives, similarities and differences between epistemic readings and deontic readings, and the relation between expressions of comparative possibility and overt probabilistic language is required. Substantial progress has been made on these issues, prompted in particular

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<sup>19</sup>See, e.g., ANAND 1990, 1993, SEN 1993, KIRCHSTEIGER & PUPPE 1996 for relevant discussion.

by Lassiter,<sup>20</sup> but our understanding of the relevant data and the space of possible theories remains incomplete.

## 5 The semantics of modals and logic of weights

Let’s recap. Contrary to certain assumptions in the literature, the classic premise semantic framework for modals *can* capture priorities among intuitive considerations that figure in the interpretation of modals — though not necessarily in the premises themselves. Modals, in this framework, are interpreted with respect to contextually supplied functions from worlds to premise sets. These functions, or “conversational backgrounds,” determine the intended reading of the modal. In this paper I have offered a way of encoding the intuitively relevant bodies of norms, values, preferences, expectations, etc. (“considerations”) in these conversational backgrounds. On this interpretation the value of a conversational background  $P$  at a world of evaluation  $w$  — an indexed premise set  $P_w$  — represents what follows from the body of considerations given the circumstances in  $w$ ; it represents what is ultimately enjoined, given the facts. This way of thinking about premise sets captures various ways in which weights and priorities can affect the truth conditions of modal sentences. The project hasn’t been to argue that no other linguistic theory can get the data right. It has been to motivate one way of doing so that is empirically adequate, methodologically conservative, and theoretically attractive. The hope is that this understanding may be developed in a more complete account of weights and priorities in natural language.

In this spirit I would like to close by briefly returning to an alternative strategy for capturing the role of weights and priorities raised in §1. Consider a prioritized default theory in the style of, say, HORTY (2007, 2012), consisting of a set of background facts  $F$ , a set of default rules representing defeasible norms or generalizations, and an ordering relation  $<$  on the set of default rules reflecting their relative priority. Following Horty’s terminology, say that a default rule  $A \rightarrow B$  whose antecedent is entailed by  $F$  is *triggered*, and that a triggered default is *defeated* if there is some other triggered default that is given higher priority according to  $<$ . To a very rough first approximation, we could then say that ‘Must  $\phi$ ’ is true iff every maximally consistent set of conclusions of non-defeated triggered defaults entails  $\phi$  (cf. nn. 4, 7). For example, in WEIGHTED PROMISES, let  $p_m \rightarrow h$  be the default rule  $d_1$  that you help your mother given that you promised to do so; let  $p_a \rightarrow l$  be the default rule  $d_2$  that you meet Alice for lunch given that you promised to do so; and suppose  $d_1 < d_2$ , reflecting that your promise to your mother has priority over your promise to Alice. Since both defaults are triggered, but  $d_2$  is defeated by  $d_1$ , the set of conclusions of

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<sup>20</sup>See PORTNER 2009, KATZ ET AL. 2012, KLECHA 2014 in addition to the references cited above.

non-defeated triggered defaults is  $\{h\}$ . Thus, ‘You must help your mother’ is true, and ‘You must meet Alice for lunch’ is false.

The premise semantics in §2 simply utilizes a function from worlds to premise sets, where the premises in a premise set represent what is ultimately enjoined given the circumstances that obtain in the evaluation world. Information about priority relations and applicability conditions is encoded in, but not recoverable from, a conversational background. The above default-theoretic analysis, by contrast, explicitly represents a factual condition for each conclusion and a priority relation among defaults. And it makes explicit how the set of conclusions of non-defeated triggered defaults is generated. Given the independent motivation for introducing this extra structure into a logic of practical and theoretical reasoning, why not introduce it into our semantics for modals?

There may ultimately be empirical reasons for incorporating the above sort of apparatus into our semantics for modals. The strategy taken up in §3 may not generalize to more complex cases. But if it does, we should be cautious about building the extra structure into the *conventional* meanings of modals. At least for the examples considered here, we have been able to capture the intuitively correct truth conditions utilizing conversational backgrounds which encode the content of a body of considerations via what premise sets are returned across worlds. We have been able to bracket the substantive normative and logical questions of how to generate conversational backgrounds from intuitive bodies of considerations and priorities, and how to derive premise sets from sets of background conditions and considerations. This isn’t simply a point about theoretical economy. By being neutral on these issues, we can avoid building controversial logical and normative views into the conventional meanings of modals. For instance, unlike the above Horty-style implementation, the present semantics avoids presupposing specific views on the proper logical representations of certain cases, the proper order in which to apply defaults, and consequence (non)monotonicity, and it is compatible with thoroughly “unprincipled” normative frameworks such as particularism (DANCY 2004). Settling on these issues is plausibly not required for semantic competence with modals.<sup>21</sup>

The focus in this paper has been on developing a linguistic semantic theory of how facts about weights and priorities can affect the truth conditions of modal sentences. Nevertheless the theory locates a precise place for theories in logic on proper reasoning involving weights and priorities. Our best logical theories can be seen as providing an explicit account of something that is taken as given by the compositional semantics: how conversational backgrounds are generated from the intuitive

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<sup>21</sup>The strategy begun in KATZ ET AL. 2012, insofar as it is committed to a particular view about how to derive premise sets from ordered bodies of priorities (see n. 4), will inherit this worry when applied to progressively more complicated cases of the sort discussed in the default logic literature. For further discussion of related methodological issues, see FORRESTER 1989: chs. 2, 13, CARR 2012, KRATZER 2012: ch. 2, SILK 2013b, 2016; cf. GIBBARD 1990: ch. 1, 2003: ch. 2.



considerations and priorities relevant in concrete discourse contexts — e.g., in the deontic case, how which imperatives are in force is derived from intuitive bodies of prima facie norms, priority relations among them, and the relevant non-normative facts. This suggests an attractive way of situating the respective work in logic and the semantics of modals in an overall theory of modality and modal language.

To be clear, I am not saying that neutrality on logical issues is in general preferable for a linguistic account. Capturing certain entailment relations is one of the central desiderata for a semantic theory. Nor am I saying that sorting out which entailments are encoded in conventional meaning, or even what the correct entailments are, is an easy matter. Assessing how much logical apparatus to import into the semantics turns not only on difficult empirical questions about truth-value and entailment facts, but also on more general theoretical issues concerning the proper task of a semantic theory and the nature of semantic competence. Explanatoriness and predictive power can sometimes mitigate against a theory's plausibility as a representation of semantic competence. The account developed in this paper strikes the balance between these potentially competing desiderata in one particular way, but this choice is by no means obvious or uncontroversial.

We shouldn't understate the account's potential for using premise sets in an explanatory way, as intended in Kratzer's own ongoing research in premise semantics.<sup>22</sup> The semantics offered in this paper abstract away from precisely how the intuitively relevant considerations and circumstances determine what premises apply in different situations. But premise sets aren't simply "reverse engineered" from our intuitions about the truth values of modal sentences. Context is still treated as supplying conversational backgrounds which determine the readings of modals. What is distinctive about the present approach is its understanding (i) of how the contents of the intuitive considerations (norms, values, etc.) at play are encoded in these conversational backgrounds, and (ii) of what the resulting indexed premise sets represent. As emphasized in §2, it is the conversational background, not its value at a world, that encodes the content of the relevant body of norms, evidence, etc. An indexed deontic premise set  $G_w^d$ , for instance, simply represents which imperatives are ultimately in force; it represents what is ultimately enjoined, or to be done, given the facts. The deontic conversational background  $G^d$  encodes the content of the normative view — what circumstances are normatively relevant, what prima facie norms are weightier than others, etc. Speakers' implicit commitments about weights and priorities thus *are* encoded in the formal semantics, namely via what conversational backgrounds are contextually supplied. We as theorists may not be able to uniquely recover the specific nature of these commitments by examining the formal properties of these supplied conversational backgrounds. But proceeding at this level of abstraction, I have suggested, is sufficient — indeed appropriate — for the purposes

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<sup>22</sup>See, e.g., the preface and introductions to chapters 3, 5, and 6 of her 2012 collection.

of compositional semantics. Conversational backgrounds reflect what is necessary for specifically *semantic* competence with modals: a capacity to deliver truth-value judgments given a full specification of the relevant facts; this includes facts about priorities and other applicability conditions. The task of the metasemanticists, logicians, descriptive linguists, and psychologists is then to detail the implicit structure in conversational backgrounds and how speakers arrive at them, and ought to arrive at them, in concrete contexts. The approach to premise semantics developed here is compatible with Kratzer's project of finding empirical constraints for premise sets, and using premise sets to explain various properties of modals and conditionals in discourse and reasoning.

A primary aim in this paper has been to delineate new avenues for developing the classic (premise) semantic framework for modals, and to begin assessing their prospects. The focus has concerned weights and priorities, but our discussion highlights pressing general questions relevant for constructing an overall theory of the logic and semantics of modals. More general empirical and theoretical investigation, including detailed comparison with alternative theories, is called for. I hope the reflections offered here may enhance our understanding of the space of possible theories and shed light on how the dialectic may proceed.

## References

- AL-HIBRI, AZIZAH. 1980. Conditionality and Ross's deontic distinction. *Southwestern Journal of Philosophy* 11, 79–87.
- ANAND, PAUL. 1990. Interpreting axiomatic (decision) theory. *Annals of Operations Research* 23, 91–101.
- ANAND, PAUL. 1993. *Foundations of rational choice under risk*. Oxford: Clarendon Press.
- ARREGUI, ANA. 2010. Detaching *if*-clauses from *should*. *Natural Language Semantics* 18, 241–293.
- ASHER, NICHOLAS & DANIEL BONEVAC. 1996. Prima facie obligation. *Studia Logica* 57, 19–45.
- BELZER, MARVIN & BARRY LOEWER. 1997. Deontic logics of defeasability. In DONALD NUTE (Ed.), *Defeasible deontic logic*, pp. 45–57. Dordrecht: Kluwer.
- BROWN, MARK A. 1996. A logic of comparative obligation. *Studia Logica* 57, 117–137.

- CARR, JENNIFER. 2012. Deontic modals without decision theory. In EMMANUEL CHEMLA, VINCENT HOMER, & GRÉGOIRE WINTERSTEIN (Eds.), *Proceedings of Sinn und Bedeutung* 17, pp. 167–182. ENS Paris.
- CHISHOLM, RODERICK. 1964. The ethics of requirement. *American Philosophical Quarterly* 1, 147–153.
- CHISHOLM, RODERICK. 1974. Practical reason and the logic of requirement. In STEPHAN KÖRNER (Ed.), *Practical Reason*, pp. 1–17. New Haven: Yale University Press.
- DANCY, JONATHAN. 2004. *Ethics without principles*. Oxford: Clarendon Press.
- FINLAY, STEPHEN. 2014. *Confusion of tongues: A theory of normative language*. New York: Oxford University Press.
- VON FINTEL, KAI. 2012. The best we can (expect to) get? Challenges to the classic semantics for deontic modals. Paper presented at the 2012 Central APA, Symposium on Deontic Modals.
- VON FINTEL, KAI & SABINE IATRIDOU. 2005. What to do if you want to go to Harlem: Anankastic conditionals and related matters. MS, MIT.
- FORRESTER, JAMES WILLIAM. 1989. *Why you should: The pragmatics of deontic speech*. Hanover: Brown University Press.
- VAN FRAASSEN, BAS C. 1973. Values and the heart's command. *The Journal of Philosophy* 70, 5–19.
- FRANK, ANETTE. 1996. *Context dependence in modal constructions*. Ph.D. thesis, University of Stuttgart.
- GABBAY, DOV, JOHN HORTY, XAVIER PARENT, RON VAN DER MAYDEN, & LEENDERT VAN DER TORRE (Eds.). 2013. *Handbook of deontic logic and normative systems*. Milton Keynes: College Publications.
- GANS, CHAIM. 1986. Mandatory rules and exclusionary reasons. *Philosophia* 15, 373–394.
- GIBBARD, ALLAN. 1990. *Wise choices, apt feelings: A theory of normative judgment*. Cambridge: Harvard University Press.
- GIBBARD, ALLAN. 2003. *Thinking how to live*. Cambridge: Harvard University Press.
- GOBLE, LOU. 2013. Prima facie norms, normative conflicts, and dilemmas. In GABBAY ET AL. (2013), pp. 241–352.

- HANSEN, JORG. 2006. Deontic logics for prioritized imperatives. *Artificial Intelligence and the Law* 14, 1–34.
- HANSSON, SVEN OVE. 1990. Defining “good” and “bad” in terms of “better”. *Notre Dame Journal of Formal Logic* 31, 136–149.
- HANSSON, SVEN OVE. 2001. *Structures of values and norms*. Cambridge: Cambridge University Press.
- HANSSON, SVEN OVE. 2013. The varieties of permission. In GABBAY ET AL. (2013), pp. 195–240.
- HART, H.L.A. 1961/1994. *The concept of law*. Oxford: Oxford University Press, 2nd edn.
- HOLLIDAY, WESLEY H. & THOMAS F. ICARD, III. 2013. Measure semantics and qualitative semantics for epistemic modals. In TODD SNIDER (Ed.), *Semantics and Linguistic Theory (SALT)* 23, pp. 514–534. Ithaca: CLC Publications.
- HORTY, JOHN F. 2007. Defaults with priorities. *Journal of Philosophical Logic* 36, 367–413.
- HORTY, JOHN F. 2012. *Reasons as defaults*. New York: Oxford University Press.
- KATZ, GRAHAM, PAUL PORTNER, & AYNAT RUBINSTEIN. 2012. Ordering combination for modal comparison. In ANCA CHERECHES (Ed.), *Proceedings of SALT* 22, pp. 488–507. Ithaca: CLC Publications.
- KAUFMANN, STEFAN & MAGDALENA SCHWAGER. 2011. A uniform analysis of conditional imperatives. In ED CORMANY & SATOSHI ITO (Eds.), *Proceedings of Semantics and Linguistic Theory (SALT)* 19, pp. 223–238. Ithaca: CLC Publications.
- KIRCHSTEIGER, GEORG & CLEMENS PUPPE. 1996. Intransitive choices based on transitive preferences: The case of menu-dependent information. *Theory and Decision* 41, 37–58.
- KLECHA, PETER. 2014. *Bridging the divide: Scalarity and modality*. Ph.D. thesis, University of Chicago.
- KRATZER, ANGELIKA. 1977. What ‘must’ and ‘can’ must and can mean. *Linguistics and Philosophy* 1, 337–355.
- KRATZER, ANGELIKA. 1981a. The notional category of modality. In HANS-JÜRGEN EIKMEYER & HANNES RIESER (Eds.), *Words, worlds, and contexts: New approaches in word semantics*, pp. 38–74. Berlin: de Gruyter.

- KRATZER, ANGELIKA. 1981b. Partition and revision: The semantics of counterfactuals. *Journal of Philosophical Logic* 10, 201–216.
- KRATZER, ANGELIKA. 1989. An investigation of the lumps of thought. *Linguistics and Philosophy* 12, 607–653.
- KRATZER, ANGELIKA. 1991. Modality/Conditionals. In ARNIM VON STECHOW & DIETER WUNDERLICH (Eds.), *Semantics: An international handbook of contemporary research*, pp. 639–656. New York: de Gruyter.
- KRATZER, ANGELIKA. 2002. Facts: Particulars or information units? *Linguistics and Philosophy* 25, 655–670.
- KRATZER, ANGELIKA. 2012. *Modals and conditionals: New and revised perspectives*. New York: Oxford University Press.
- LASSITER, DANIEL. 2010. Gradable epistemic modals, probability, and scale structure. In NAN LI & DAVID LUTZ (Eds.), *Proceedings of Semantics and Linguistic Theory (SALT) 20*, pp. 197–215. Ithaca: CLC Publications.
- LASSITER, DANIEL. 2011. *Measurement and modality: The scalar basis of modal semantics*. Ph.D. thesis, New York University.
- LASSITER, DANIEL. 2014. Epistemic comparison, models of uncertainty, and the disjunction puzzle. *Journal of Semantics* pp. 1–36.
- LEHMANN, DANIEL & MENACHEM MAGIDOR. 1992. What does a conditional knowledge base entail? *Artificial Intelligence* 55, 1–60.
- LEWIS, DAVID. 1973. *Counterfactuals*. Cambridge: Harvard University Press.
- LEWIS, DAVID. 1981. Ordering semantics and premise semantics for counterfactuals. *Journal of Philosophical Logic* 10, 217–234.
- MAKINSON, DAVID. 1993. Five faces of minimality. *Studia Logica* 52, 339–379.
- MAKINSON, DAVID & LEENDERT VAN DER TORRE. 2000. Input/output logics. *Journal of Philosophical Logic* 29, 383–408.
- MAKINSON, DAVID & LEENDERT VAN DER TORRE. 2001. Constraints for input/output logics. *Journal of Philosophical Logic* 30, 155–185.
- MCCARTHY, JOHN. 1977. Epistemological problems of artificial intelligence. In *Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI)*, pp. 1038–1044. Cambridge: MIT Press.

- MCCARTHY, JOHN & PATRICK J. HAYES. 1969. Some philosophical problems from the standpoint of artificial intelligence. *Machine Intelligence* 4, 463–502.
- MOORE, G.E. 1903. *Principia ethica*. New York: Cambridge University Press.
- NICKEL, JAMES W. 1982. Are human rights utopian? *Philosophy and Public Affairs* 11, 246–264.
- PARFIT, DEREK. 2011. *On what matters*, vol. 1-2. Oxford: Oxford University Press.
- PERRY, STEPHEN. 1989. Second-order reasons, uncertainty, and legal theory. *Southern California Law Review* 62, 913–994.
- POLLOCK, JOHN. 1970. The structure of epistemic justification. *American Philosophical Quarterly* 4, 62–78.
- POLLOCK, JOHN. 1995. *Cognitive carpentry: A blueprint for how to build a person*. Cambridge: MIT Press.
- PORTNER, PAUL. 2009. *Modality*. Oxford: Oxford University Press.
- PRAKKEN, HENRY & MAREK SERGOT. 1996. Contrary-to-duty obligations. *Studia Logica* 57, 91–115.
- PRAKKEN, HENRY & MAREK SERGOT. 1997. Dyadic deontic logic and contrary-to-duty obligations. In DONALD NUTE (Ed.), *Defeasible deontic logic*, pp. 223–262. Dordrecht: Kluwer.
- RAZ, JOSEPH. 1975/1990. *Practical reason and norms*. Oxford: Oxford University Press, 2nd edn.
- SCHROEDER, MARK. 2013. Tempered expressivism. In RUSS SHAFER-LANDAU (Ed.), *Oxford studies in metaethics*, vol. 8, pp. 283–314. Oxford: Oxford University Press.
- SEN, AMARTYA. 1993. Internal consistency of choice. *Econometrica* 61, 495–521.
- SILK, ALEX. 2012a. Modality, weights, and inconsistent premise sets. In ANCA CHERECHES (Ed.), *Proceedings of SALT 22*, pp. 43–64. Ithaca: CLC Publications.
- SILK, ALEX. 2012b. Weak and strong necessity. URL <http://goo.gl/xY9roN>. MS, University of Michigan.
- SILK, ALEX. 2013a. Deontic conditionals: Weak and strong. In *Proceedings of the 19th International Congress of Linguists*. Département de Linguistique de l'Université de Genève. URL <http://goo.gl/xSGbcJ>.

- SILK, ALEX. 2013b. Truth-conditions and the meanings of ethical terms. In RUSS SHAFER-LANDAU (Ed.), *Oxford studies in metaethics*, vol. 8, pp. 195–222. New York: Oxford University Press.
- SILK, ALEX. 2015. How to be an ethical expressivist. *Philosophy and Phenomenological Research* 91, 47–81.
- SILK, ALEX. 2016. *Discourse Contextualism: A framework for contextualist semantics and pragmatics*. Oxford: Oxford University Press.
- SWANSON, ERIC. 2011. On the treatment of incomparability in ordering semantics and premise semantics. *Journal of Philosophical Logic* 40, 693–713.
- VAN DER TORRE, LEENDERT. 1997. *Reasoning about obligations: Defeasibility in preference-based deontic logic*. Ph.D. thesis, Erasmus University Rotterdam.
- VAN DER TORRE, LEENDERT & YAO-HUA TAN. 1998. An update semantics for prima facie obligations. In *Proceedings of the 13th European conference on artificial intelligence (ECAI '98)*, pp. 38–42. New York: Wiley-Blackwell.
- VELTMAN, FRANK. 1976. Prejudices, presuppositions, and the theory of conditionals. In JEROEN GROENENDIJK & MARTIN STOKHOF (Eds.), *Amsterdam papers in formal grammar*, vol. 1, pp. 248–281. Central Interfaculteit, University of Amsterdam.
- YALCIN, SETH. 2010. Probability operators. *Philosophy Compass* 5, 916–937.